A method of successive approximations for constructing guiding program package in the problem of guaranteed closed-loop guidance

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To the memory of our beloved Mentor Arkady Kryazhimskiy

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Arkady's work on control problems with incomplete information

In myriads of Arkady's scientific interests control problems with incomplete information were prominent throughout his career.

> «*The problem of constructing optimal closed-loop control strategies under uncertainty is one of the key problems of the mathematical control theory. Its solution would give a new impetus to the theory's development and create the foundation for its new applications.*» Arkady Kryazhimskiy (2013)

- A. V. Kryazhimskiy. *A differential approach game under conditions of incomplete information about the system.* Ukrain. Mat. Zh., 27:4 (1975), 521–526.
- A. V. Kryazhimskiy, S. D. Filippov. *On a game problem on the convergence of two points on a plane under incomplete information.* Control Problems with Incomplete Information. Trudy IMM Ural. Nauchn. Centr Akad. Nauk SSSR, 19 (1976), 62–77.
- A. V. Kryazhimskiy. *An alternative in a linear approach-deviation game with incomplete information.* Dokl. Akad. Nauk SSSR, 230:4 (1976), 773–776.
- A. Kryazhimskiy, V. Maksimov. *On exact stabilization of an uncertain dynamical system.* J. Inverse Ill-Posed Probl., 12:2 (2004), 145–182

Arkady's work on control problems with incomplete information

Program packages method

An innovative approach for solving control problems with incomplete information about states of the dynamic system developed by Arkady Kryazhimskiy and Yurii Osipov

- Yu. S. Osipov. *Control Packages: An Approach to Solution of Positional Control Problems with Incomplete Information.* Usp. Mat. Nauk 61:4 (2006), 25–76.
- A. V. Kryazhimskiy, Yu. S. Osipov. *Idealized Program Packages and Problems of Positional Control with Incomplete Information*. Trudy IMM UrO RAN 15:3 (2009), 139–157.
- A. V. Kryazhimskiy, Yu. S. Osipov. *On the solvability of problems of guaranteeing control for partially observable linear dynamical systems*. Proc. Steklov Inst. Math., 277 (2012), 144–159
- A. V. Kryazhimskiy, N. V. Strelkovskii. *An open-loop criterion for the solvability of a closed-loop guidance problem with incomplete information. Linear control systems.* Trudy IMM UrO RAN, 20:3 (2014), 132–147.
- A. V. Kryazhimskii, N. V. Strelkovskii. *A problem of guaranteed closed-loop guidance by a fixed time for a linear control system with incomplete information. Program solvability criterion.* Trudy IMM UrO RAN, 20:4 (2014), 168–177

Guaranteed positional guidance problem at pre-defined time

The case for linear systems and finite initial states set was studied by Arkady in 2012-2014.

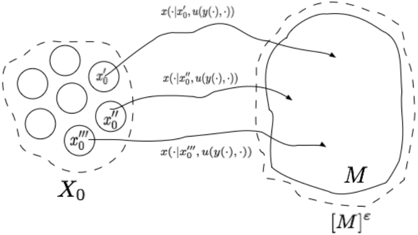
$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + c(t), t_0 \leq t \leq \vartheta \tag{1}$$

Open-loop control (program) $u(\cdot)$ is measurable.

$u(t) \in P \subset \mathbb{R}^r$, P is a convex compact set

$x(t_0) = x_0 \in X_0 \subset \mathbb{R}^n$, X_0 is a **finite** set

$x(\vartheta) \in M \subset \mathbb{R}^n$, M is a **closed and convex** set



Observed signal $y(t) = Q(t)x(t)$, $Q(\cdot) \in \mathbb{R}^{q \times n}$ is left piecewise continuous

Problem statement

Based on the given arbitrary $\varepsilon > 0$ choose a closed-loop control strategy with memory, **whatever the system's initial state x_0 from the set X_0**, *the system's motion $x(\cdot)$ corresponding to the chosen closed-loop strategy and starting at the time t_0 from the state x_0 reaches the state $x(\vartheta)$ belonging to the ε-neighbourhood of the target set M at the time ϑ.*

Homogeneous signals

Homogeneous system, corresponding to (1)

$$\dot{x}(t) = A(t)x(t)$$

For each $x_0 \in X_0$ its solution is given by the Cauchy formula:

$$x(t) = F(t, t_0)x_0; \; F(t, s) \; (t, s \in [t_0, \vartheta]) \text{ is the fundamental matrix.}$$

Homogeneous signal, corresponding to an admissible initial state $x_0 \in X_0$:

$$g_{x_0}(t) = Q(t)F(t, t_0)x_0 \; (t \in [t_0, \vartheta], \; x_0 \in X_0).$$

Let $G = \{g_{x_0}(\cdot) | x_0 \in X_0\}$ be the set of all homogeneous signals and let $X_0(\tau | g(\cdot))$ be the set of all admissible initial states $x_0 \in X_0$, corresponding to the homogeneous signal $g(\cdot) \in G$ *till time point* $\tau \in [t_0, \vartheta]$:

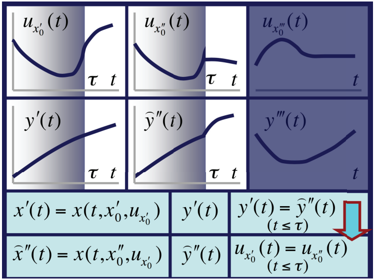
$$X_0(\tau | g(\cdot)) = \{x_0 \in X_0 : g(\cdot)|_{[t_0, \tau]} = g_{x_0}(\cdot)|_{[t_0, \tau]}\}.$$

Method milestone

These terms were introduced in [Kryazhimskiy, Osipov (2012)].

Package guidance problem

Program package is an open-loop controls family $(u_{x_0}(\cdot))_{x_0 \in X_0}$, satisfying **non-anticipatory condition**: for any homogeneous signal $g(\cdot)$, any time $\tau \in (t_0, \vartheta]$ and any admissible initial states $x_0', x_0'' \in X_0(\tau | g(\cdot))$ the equality $u_{x_0'}(t) = u_{x_0''}(t)$ holds for almost all $t \in [t_0, \tau]$.



Program package $(u_{x_0}(\cdot))_{x_0 \in X_0}$ is **guiding**, if for all $x_0 \in X_0$ holds $x(\vartheta | x_0, u_{x_0}(\cdot)) \in M$.
Package guidance problem is solvable, if a guiding program package exists.

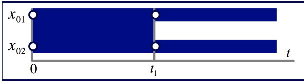
Theorem 1 (Osipov, Kryazhimskiy, 2006)

The problem of positional guidance is solvable if and only if the problem of package guidance is solvable.

Homogeneous signals splitting

For an arbitrary homogeneous signal $g(\cdot)$ let

$$G_0(g(\cdot)) = \left\{ \tilde{g}(\cdot) \in G : \lim_{\zeta \to +0} (\tilde{g}(t_0 + \zeta) - g(t_0 + \zeta)) = 0 \right\}$$



be the set of **initially compatible** homogeneous signals and let

$$\tau_1(g(\cdot)) = \max \left\{ \tau \in [t_0, \vartheta] : \max_{\tilde{g}(\cdot) \in G_0(g(\cdot))} \max_{t \in [t_0, \tau]} |\tilde{g}(t) - g(t)| = 0 \right\}$$

be its **first splitting moment**.

For each $i = 1, 2, \ldots$ let

$$G_i(g(\cdot)) = \left\{ \tilde{g}(\cdot) \in G_{i-1}(g(\cdot)) : \lim_{\zeta \to +0} (\tilde{g}(\tau_i(g(\cdot)) + \zeta) - g(\tau_i(g(\cdot)) + \zeta)) = 0 \right\}$$

be the set of all homogeneous signals from $G_{i-1}(g(\cdot))$ equal to $g(\cdot)$ in the right-sided neighbourhood of the time-point $\tau_i(g(\cdot))$ and let

$$\tau_{i+1}(g(\cdot)) = \max \left\{ \tau \in (\tau_i(g(\cdot)), \vartheta] : \max_{\tilde{g}(\cdot) \in G_i(g(\cdot))} \max_{t \in [\tau_i(g(\cdot)), \tau]} |\tilde{g}(t) - g(t)| = 0 \right\}$$

be the $(i+1)$**-th splitting moment** of the homogeneous signal $g(\cdot)$.

Initial states set clustering

Let

$$T(g(\cdot)) = \{\tau_j(g(\cdot)) : j = 1, \ldots, k_{g(\cdot)}\}$$

be the set of all splitting moments of the homogeneous signal $g(\cdot)$ and let

$$T = \bigcup_{g(\cdot)\in G} T(g(\cdot))$$

be the set of all splitting moments of all homogeneous signals. T is finite and $|T| \leq |X_0|$. Let us represent this set as $T = \{\tau_1, \ldots, \tau_K\}$, where $t_0 < \tau_1 < \ldots < \tau_K = \vartheta$.

Lemma 2 (Kryazhimskiy (2013))

Programs family $(u_{x_0}(\cdot))_{x_0\in X_0}$ is a program package if and only if for any homogeneous signal $g(\cdot)$, any time $\tau \in T(g(\cdot))$ and any initial states $x_0', x_0'' \in X_0(\tau|g(\cdot))$ equality $u_{x_0'}(t) = u_{x_0''}(t)$ holds for almost all $t \in [t_0, \tau]$.

Initial states set clustering

For every $k = 1, \dots, K$ let the set

$$\mathcal{X}_0(\tau_k) = \{X_0(\tau_k | g(\cdot)) : g(\cdot) \in G\}$$

be the **cluster position** at the time-point τ_k, and let each its element $X_{0j}(\tau_k)$, $j = 1, \dots, J(\tau_k)$ be a **cluster of initial states** at this time-point; $J(\tau_k)$ is the number of clusters in the cluster position $\mathcal{X}_0(\tau_k)$, $k = 1, \dots, K$.

Lemma 3 (Kryazhimskiy (2013))

Open-loop control family $(u_{x_0}(\cdot))_{x_0 \in X_0}$ is a program package if and only if for any $k = 1, \dots, K$, any $X_{0j}(\tau_k) \in \mathcal{X}_0(\tau_k)$, $j = 1, \dots, J(\tau_k)$ and arbitrary initial states $x_0', x_0'' \in X_{0j}(\tau_k)$ the equality $u_{x_0'}(t) = u_{x_0''}(t)$ holds for almost all $t \in (\tau_{k-1}, \tau_k]$ in case $k > 1$ and for almost all $t \in [t_0, \tau_1]$ in case $k = 1$.

Extended space

Arkady proposed to use a special Euclidean space. Let $\mathcal{R}^h$ $(h = 1, 2, \ldots)$ be a

finite-dimensional Euclidean space of all families $(r_{x_0})_{x_0 \in X_0}$ from $\mathbb{R}^h$ with a scalar product $\langle \cdot, \cdot \rangle_{\mathcal{R}^h}$ defined as

$$\langle r', r'' \rangle_{\mathcal{R}^h} = \langle (r'_{x_0})_{x_0 \in X_0}, (r''_{x_0})_{x_0 \in X_0} \rangle_{\mathcal{R}^h} = \sum_{x_0 \in X_0} \langle r'_{x_0}, r''_{x_0} \rangle_{\mathbb{R}^h} \quad ((r'_{x_0})_{x_0 \in X_0}, (r''_{x_0})_{x_0 \in X_0} \in \mathcal{R}^h).$$

For each non-empty set $\mathcal{E} \subset \mathcal{R}^h$ $(h = 1, 2, \ldots)$ let us define its *lower* $\rho^-(\cdot|\mathcal{E}) : \mathcal{R}^h \mapsto \mathbb{R}$ and *upper* support functions $\rho^+(\cdot|\mathcal{E}) : \mathcal{R}^h \mapsto \mathbb{R}$:

$$\rho^-((l_{x_0})_{x_0 \in X_0} | \mathcal{E}) = \inf_{(e_{x_0})_{x_0 \in X_0} \in \mathcal{E}} \langle (l_{x_0})_{x_0 \in X_0}, (e_{x_0})_{x_0 \in X_0} \rangle_{\mathcal{R}^h} \quad ((l_{x_0})_{x_0 \in X_0} \in \mathcal{R}^h),$$

$$\rho^+((l_{x_0})_{x_0 \in X_0} | \mathcal{E}) = \sup_{(e_{x_0})_{x_0 \in X_0} \in \mathcal{E}} \langle (l_{x_0})_{x_0 \in X_0}, (e_{x_0})_{x_0 \in X_0} \rangle_{\mathcal{R}^h} \quad ((l_{x_0})_{x_0 \in X_0} \in \mathcal{R}^h)$$

Extended open-loop control control

Let $\mathcal{P} \subset \mathcal{R}^m$ be the set of all families $(u_{x_0})_{x_0 \in X_0}$ of vectors from P.
Extended open-loop control control is a measurable function $t \mapsto (u_{x_0}(t))_{x_0 \in X_0} : [t_0, \vartheta] \mapsto \mathcal{P}$.
Let us identify arbitrary programs family $(u_{x_0}(\cdot))_{x_0 \in X_0}$ and an extended open-loop control $t \mapsto (u_{x_0}(t))_{x_0 \in X_0}$.

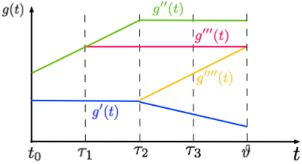
For each $k = 1, \ldots, K$ let $\mathcal{P}_k$ be an **extended admissible control set** on $(\tau_{k-1}, \tau_k]$ in case $k > 1$ and on $[t_0, \tau_1]$ in case $k = 1$ as a set of all vector families $(u_{x_0})_{x_0 \in X_0} \in \mathcal{P}$ such that, for each cluster $X_{0j}(\tau_k) \in \mathcal{X}_0(\tau_k), j = 1, \ldots, J(\tau_k)$ and any $x_0', x_0'' \in X_{0j}(\tau_k)$ holds $u_{x_0'} = u_{x_0''}$.

Extended open-loop control control $(u_{x_0}(\cdot))_{x_0 \in X_0}$ is **admissible**, if for each $k = 1, \ldots, K$ holds $(u_{x_0}(t))_{x_0 \in X_0} \in \mathcal{P}_k$ for almost all $t \in (\tau_{k-1}, \tau_k]$ in case $k > 1$ and for almost all $t \in [t_0, \tau_1]$ in case $k = 1$;

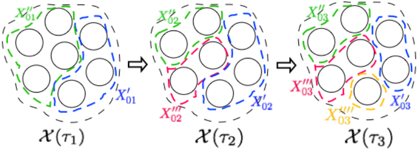
Lemma 4 (Kryazhimskiy (2013))

Extended open-loop control control $(u_{x_0}(\cdot))_{x_0 \in X_0}$ is a control package if and only if it is admissible.

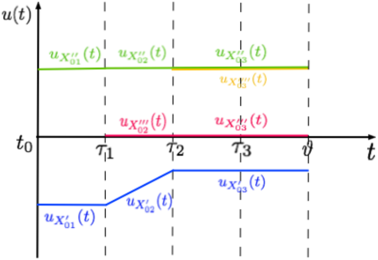
Homogeneous signals, cluster positions and extended open-loop control controls



Homogeneous signals splitting



Initial states set clustering



Extended open-loop control control

Extended problem of program guidance

Extended system (in the space $\mathcal{R}^n$):

$$\begin{cases} \dot{x}_{x_0}(t) = A(t)x_{x_0}(t) + B(t)u_{x_0}(t) + c(t) \\ x_{x_0}(t_0) = x_0 \end{cases}$$

$$(x_0 \in X_0)$$

Extended target set $\mathcal{M}$ is the set of all families $(x_{x_0})_{x_0 \in X_0} \in \mathcal{R}^n$ such, that $x_{x_0} \in M$ for all $x_0 \in X_0$.

An admissible extended open-loop control $(u_{x_0}(\cdot))_{x_0 \in X_0}$ is **guiding the extended system**, if $(x(\vartheta | x_0, u_{x_0}(\cdot)))_{x_0 \in X_0} \in \mathcal{M}$.

The **extended problem of open-loop guidance** is solvable, if there exists an admissible extended open-loop control which is guiding the extended system.

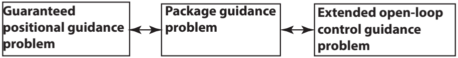
Attainability set of the extended system at the time ϑ:
$\mathcal{A} = \{(x(\vartheta | x_0, u_{x_0}(\cdot)))_{x_0 \in X_0} : (u_{x_0}(\cdot))_{x_0 \in X_0} \in \mathcal{U}_{ext}\}$, where $\mathcal{U}_{ext}$ is the set of all admissible extended open-loop control controls.

Solvability criterion

Theorem 5 (Kryazhimskiy, Strelkovskii (2014))

1) The package guidance problem is solvable if and only if the extended problem of open-loop guidance is solvable. 2) An admissible extended open-loop control is a guiding program package if and only if it is guiding extended system.

Arkady's original solution scheme:



Let us denote $D(t) = B^{\mathrm{T}}(t)F^{\mathrm{T}}(\vartheta, t) \quad (t \in [t_0, \vartheta])$ and set the function $p(\cdot,\cdot) : \mathbb{R}^n \times X_0 \mapsto \mathbb{R}$:

$$p(l, x_0) = \langle l, F(\vartheta, t_0)x_0\rangle_{\mathbb{R}^n} + \left\langle l, \int\limits_{t_0}^{\vartheta} F(\vartheta, t)c(t)dt \right\rangle_{\mathbb{R}^n} \qquad (l \in \mathbb{R}^n,\ x_0 \in X_0).$$

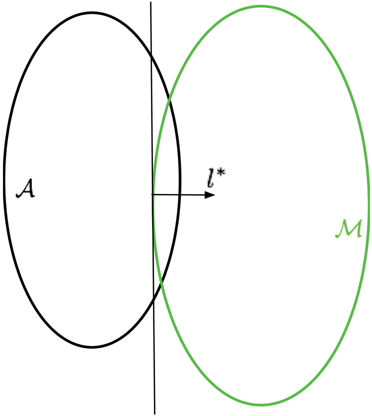
Let us set

$$\gamma((l_{x_0})_{x_0\in X_0}) = \rho^-\left((l_{x_0})_{x_0\in X_0}|\mathcal{A}\right) - \rho^+\left((l_{x_0})_{x_0\in X_0}|\mathcal{M}\right) =$$

$$= \sum_{x_0\in X_0} p(l_{x_0}, x_0) - \sum_{x_0\in X_0} \rho^+(l_{x_0}|M) + \sum_{k=1}^{K} \int\limits_{\tau_{k-1}}^{\tau_k} \sum_{X_{0j}(\tau_k)\in \mathcal{X}_0(\tau_k)} \rho^-\left(\sum_{x_0\in X_{0j}(\tau_k)} D(t)l_{x_0} \Big| P\right) dt.$$

Solvability criterion

Let $\mathcal{L}$ be a compact set in $\mathcal{R}^n$, containing an image of the unit sphere $\mathcal{S}^n$ — for some positive r_1 and $r_2 \geq r_1$ for each $\ell \in \mathcal{S}^n$ there is $r \in [r_1, r_2]$, for which $r\ell \in \mathcal{L}$.



Theorem 6 (Kryazhimskiy, Strelkovskii (2014))

Each of the three problems – (i) the extended open-loop control guidance problem, (ii) the package guidance problem and (iii) the guaranteed positional guidance problem – is solvable if and only if

$$\max_{(l_{x_0})_{x_0 \in X_0} \in \mathcal{L}} \gamma((l_{x_0})_{x_0 \in X_0}) \leq 0. \qquad (2)$$

Construction of the guiding program package

Assuming that the solvability criterion (2) is satisfied, let us introduce the function $\hat{\gamma}(\cdot,\cdot) : \mathcal{R}^n \times [0,1] \mapsto \mathbb{R}$:

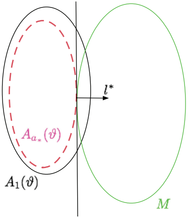
$$\hat{\gamma}((l_{x_0})_{x_0 \in X_0}, a) = \sum_{x_0 \in X_0} \langle l_{x_0}, F(\vartheta, t_0) x_0 \rangle_{\mathbb{R}^n} + \left\langle l_{x_0}, \int_{t_0}^{\vartheta} F(\vartheta, t) c(t) dt \right\rangle_{\mathbb{R}^n} - \sum_{x_0 \in X_0} \rho^+(l_{x_0} | M) -$$
$$- \sum_{k=1}^{K} \int_{\tau_{k-1}}^{\tau_k} \sum_{X_{0j}(\tau_k) \in \mathcal{X}_0(\tau_k)} \rho^- \left(\sum_{x_0 \in X_{0j}(\tau_k)} D(t) l_{x_0} \middle| aP \right) dt. \quad (3)$$

Program package $(u^0_{x_0}(\cdot))_{x_0 \in X_0}$ is **zero-valued**, if $u^0_{x_0}(t) = 0$ for almost all $t \in [t_0, \vartheta], x_0 \in X_0$.

Lemma 7 (Kryazhimskiy (2014))

If the solvability criterion (2) *holds and zero-valued program package is not guiding the extended system, then exists* $\mathbf{a}_* \in (0,1]$ *such, that*

$$\max_{(l_{x_0})_{x_0 \in X_0} \in \mathcal{L}} \hat{\gamma}((l_{x_0})_{x_0 \in X_0}, \mathbf{a}_*) = 0. \quad (4)$$



Construction of the guiding program package

For each program package $(u_{x_0}(\cdot))_{x_0 \in X_0}$, arbitrary cluster $X_{0j}(\tau_k) \in \mathcal{X}(\tau_k)$, $j = 1, \dots, J(\tau_k)$, $k = 1, \dots, K$ and arbitrary $t \in [\tau_{k-1}, \tau_k)$ let us denote $u_{X_{0j}(\tau_k)}(t)$ program values $u_{x_0}(t)$, which are equal for all $x_0 \in X_{0j}(\tau_k)$.
Let $(\mathbf{l}^*_{\mathbf{x_0}})_{\mathbf{x_0} \in \mathbf{X_0}}$ be the maximizer of the left handside of (4). Cluster $X_{0j}(\tau_k)$ is **regular**, if

$$\sum_{x_0 \in X_{0j}(\tau_k)} D(t)\mathbf{l}^*_{\mathbf{x_0}} \neq 0, \; t \in [\tau_{k-1}, \tau_k).$$

Otherwise the cluster is **singular**.

Theorem 8 (Kryazhimskiy (2014))

Let P be a strcitly convex compact set, containing the zero vector; condition (4) holds and the program package $(u^*_{x_0}(\cdot))_{x_0 \in X_0}$ satisfies the condition $u^*_{x_0}(t) \in \mathbf{a}_* P \quad (x_0 \in X_0, \; t \in [t_0, \vartheta])$. Let the clusters $X_{0j}(\tau_k) \in \mathcal{X}_0(\tau_k)$, $k = 1, \dots, K$, $j = 1, \dots, J(\tau_k)$ be regular, and for each of them the following equality holds

$$\left\langle D(t) \sum_{x_0 \in X_{0j}(\tau_k)} \mathbf{l}^*_{\mathbf{x_0}}, u^*_{X_{0j}(\tau_k)}(t) \right\rangle_{\mathbb{R}^m} = \rho^- \left(D(t) \sum_{x_0 \in X_{0j}(\tau_k)} \mathbf{l}^*_{\mathbf{x_0}} \,\middle|\, \mathbf{a}_* P \right) \; (t \in [\tau_{k-1}, \tau_k)).$$

Then the program package $(u^*_{x_0}(\cdot))_{x_0 \in X_0}$ is guiding.

Method of successive approximations. Stage 0

Arkady proposed to use this well-known method for numerical solution of the extended open-loop control guidance problem.

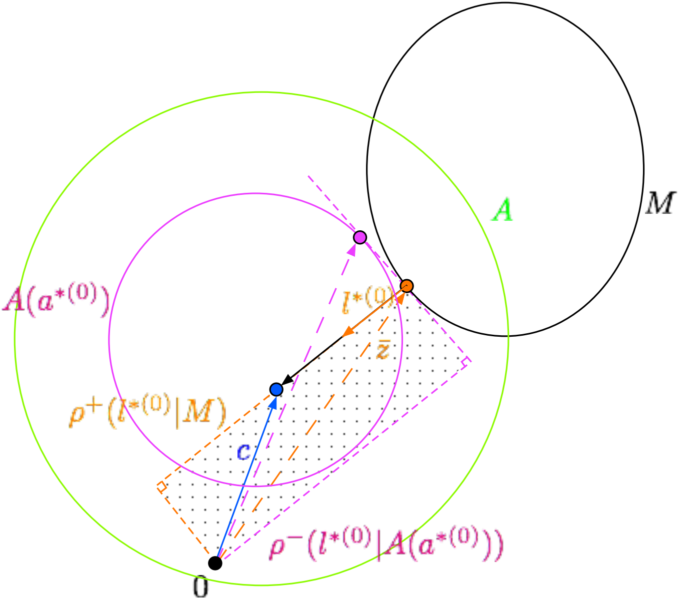
- Let $c = F(\vartheta, t_0)x_0 + \int\limits_{t_0}^{\vartheta} F(\vartheta, t)c(t)dt \quad (c \in \mathbb{R}^n)$ be the terminal state of the system's motion under zero-valued control. Obviously $c \in A$, but $c \notin M$.
- Let us find the point
$$\bar{z} = \arg\min_{z \in M} \|c - z\|_{\mathbb{R}^n}.$$
- Let us create the zero approximation of the support vector $l^{*(0)} = \frac{c-\bar{z}}{\|c-\bar{z}\|_{\mathbb{R}^n}}$.
- It is clear that $\hat{\gamma}(l^{*(0)}, 0) > 0$.
- From the solvability criterion it follows that $\hat{\gamma}(l^{*(0)}, 1) \leq 0$. Since $\hat{\gamma}(l^{*(0)}, 0) > 0$ and the function $\hat{\gamma}(\cdot, \cdot)$ is continuous, such $a^{*(0)} \in (0, 1]$ exists that $\hat{\gamma}(l^{*(0)}, a^{*(0)}) = 0$. Let us find it:

$$a^{*(0)} = \frac{\|c - \bar{z}\|_{\mathbb{R}^n}}{\int\limits_{t_0}^{\vartheta} \rho^{-}\left(D(t)l^{*(0)} \middle| P\right) dt}.$$

Method of successive approximations. Stage 0

- Using the minimum condition let us derive the zero approximation of the guiding control
$$u^{*(0)} \in a^{*(0)} \operatorname{Arg} \min_{u \in P} \left\langle D(t) l^{*(0)}, u \right\rangle_{\mathbb{R}^m} \quad (t \in [t_0, \vartheta)). \tag{5}$$
assuming $D(t) l^{*(0)} \neq 0, t \in [t_0, \vartheta)$.
- Let us derive the zero approximation of the system's motion value at the moment ϑ:
$$x^{(0)} = x(\vartheta | x_0, u^{*(0)}(\cdot)) = c + \int_{t_0}^{\vartheta} F(\vartheta, t) B(t) u^{*(0)}(t) dt$$
- If $x^{(0)} \in M$ (or $d(x^{(0)}, M) \leq \varepsilon$) then the algorithm ends with the output (5). Otherwise assuming that $\bar{z}^{(0)}$ is the upper support vector of M for vector $l^{*(0)}$, namely,
$$\bar{z}^{(0)} \in \operatorname{Arg} \max_{z \in M} \langle l^{*(0)}, z \rangle_{\mathbb{R}^n}$$
the algorithm procceds to the Stage 1.

Method of successive approximations. Stage 0



Method of successive approximations. Stage i $(i = 1, 2, \ldots)$

- Let us find the vector $l^{*(i)}$ such, that $\hat{\gamma}(l^{*(i)}, a^{*(i-1)}) > 0$.
- From the solvability criterion it follows, that $\hat{\gamma}(l^{*(i)}, 1) \leq 0$. Since $\hat{\gamma}(l^{*(i)}, a^{*(i-1)}) > 0$ and the function $\hat{\gamma}(\cdot, \cdot)$ is continuous, such $a^{*(i)} \in (a^{*(i-1)}, 1]$ exists that $\hat{\gamma}(l^{*(i)}, a^{*(i)}) = 0$. Let us find it:

$$a^{*(i)} = \frac{\rho^{+}(l^{*(i)}|M) - \langle c, l^{*(i)} \rangle_{\mathbb{R}^n}}{\int\limits_{t_0}^{\vartheta} \rho^{-}\left(D(t)l^{*(i)} \middle| P\right) dt}.$$

- Using the minimum condition let us derive the i-th approximation of the guiding control

$$u^{*(i)} \in a^{*(i)} \operatorname{Arg} \min_{u \in P} \left\langle D(t)l^{*(i)}, u \right\rangle_{\mathbb{R}^m} \quad (t \in [t_0, \vartheta)). \tag{6}$$

  assuming $D(t)l^{*(i)} \neq 0, t \in [t_0, \vartheta)$.
- Let us derive the i-th approximation of the system's motion value at the moment ϑ:

$$x^{(i)} = x(\vartheta | x_0, u^{*(i)}(\cdot)) = c + \int\limits_{t_0}^{\vartheta} F(\vartheta, t)B(t)u^{*(i)}(t)dt$$

- If $x^{(i)} \in M$ (or $d(x^{(1)}, M) \leq \varepsilon$) then the algorithm ends with the output (6). Otherwise assuming that $\bar{z}^{(i)}$ is the upper support vector of M for vector $l^{*(i)}$, namely,

$$\bar{z}^{(i)} \in \operatorname{Arg} \max_{z \in M} \langle l^{*(i)}, z \rangle_{\mathbb{R}^n}$$

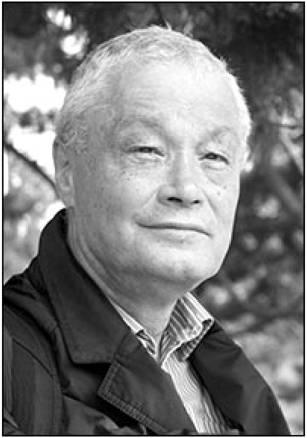
  the algorithm procceds to the Stage $(i + 1)$.

Afterword

Dozens of great Arkady's ideas which he had shared are waiting for us to be implement...

«Ideas never die»

Wilhelm von Humboldt



Arkady Kryazhimskiy (1949 – 2014)