MODELS OF NATIONAL SETTLEMENT SYSTEMS: 2. HOW CAN ONE APPROACH POLICY ORIENTED MODELLING OF POORLY UNDERSTOOD SYSTEMS?

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July 1974 WP-74-28

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The aim of this note is to highlight some of the difficulties in the dynamical modelling of poorly understood systems, and to call for a strategy that integrates fundamental research on the structure and workings of the urban system with the policy need for methodologies that analyse through time the effects and repercussions of alternative policies. In urban and regional analysis it is of little value to develop simulation or optimization approaches as if dealing with well structured engineering systems. Large scale optimization and simulation methods have been largely ineffective over two decades of development in urban planning. For poorly understood systems, it is necessary that analytical modelling be integrated into a structured learning process. Research into planning methodology requires a dialogue between hypothesis and data, and between theory and practice that can only be achieved in an iterative ongoing process. This cyclical learning process has been well documented in the development and application of simple comparative-static land use models in the U.K. (Massey and Cordey-Hayes, 1971; Barras et al., Broadbent 1973). Such a view has also been put forward by Boyce (1971), and indirectly and in much more polemical form by Lee (1973). In the current vogue for dynamical modelling (both simulation and optimization) much of this earlier experience is being ignored. But this note will go no further into this debate, here the aim is simply to emphasize how important it is that dynamic modelling be integrated with a serious programme of experimental research devoted to the formal understanding of urban processes. A framework which is useful in this respect is outlined.

The approach is derived from dynamical systems theory, which itself is historically a direct outgrowth of the Lagranian viewpoint of classical mechanics (see for example, Rosen 1970). It can be used as a dynamic optimizing approach, or adapted as a heuristic or simulation method. However, in urban systems it is probably more useful simply as a general structuring framework for the experimental analysis of growth and change. Two research projects that are based upon the approach are outlined --one uses the framework to provide an analytical strategy for a dynamical study of inter-urban migration (Cordey-Hayes and Gleave, 1973), and the second focuses more on policy and the optimization of inter-regional economic growth (Paelinck, 197[illegible]).

In concluding this introduction it is perhaps worth noting that his approach is derived from a state space approach to dynamical systems theory in preference to the classical transform function approach. The latter is based on a Laplace transform of a linear input-output differential equation and this gives a high level of abstraction that is suitable for classical control problems in engineering but which is rather opaque to the mechanisms and behavioural changes that occur within the system. In urban and regional planning the explicit manner in which the system changes and the intermediate states through which it passes are of vital importance. An approach which is based on states variables and direct rates of change is préferred because it has greater transparency to the processes of change, and this makes it a much more useful conceptual framework for policy oriented dynamic analyses of urban systems.

2. The Analysis of Urban Growth and Change

The dynamical study of any system has two basic aspects: first, it must be decided what constitutes an instantaneous description of the system of interest; and secondly, the mechanisms that translate this information from one point in time to another must be understood and expressed in formal terms. An ordered n-tuple of variables $(x_1, x_2, \ldots, x_n)$ arising from a finite set of measurements represents a possible instantaneous state of the system of interest and this notationally expresses the first step in the dynamic description. But now the manner in which this system changes over time must be specified, this is much more difficult. Sometimes it is possible to give conditions that help in the specification of the functional dependencies that express the rates of change. For example, the rate at which a particular state variable $x_i(t)$ is changing at time t may depend only on the existing state

$(\chi_1(t), \chi_2(t) \ldots\ldots \chi_n(t))$

ie. $$\frac{d\chi_i}{dt} = f_i(\chi_1 \ldots\ldots \chi_n) \qquad i = (1 \ldots\ldots n) \qquad (1)$$

Thus in this case the dynamics of the system are determined by specifying the instantaneous description $(\chi_1 \ldots\ldots \chi_n)$ and the functions $f_1 \ldots\ldots f_n$. It is very rare that we are able to specify these functional dependencies adequately for urban systems, and it is considered here that the experimental deduction of these functions is the fundamental long term problem in the analysis of growth and change. The functions are essentially an expression of the exogenous 'forces' that are acting upon the system and which are responsible for its dynamical behaviour. The problem of interest is how to structure experimental analysis in order to deduce these functional dependencies.

If the functions f_i were known then, in principle, it would be possible to consider how the inputs to the system could be chosen such that the trajectory to some pre-assigned state is made in a 'best possible' way. This leads to variational principles and 'optimal controls' as outlined in Section 4. But we restate that for urban systems these functional dependencies are mostly unknown, and therefore an important problem is how to structure analyses in order to deduce these functions whilst concurrently addressing policy questions.† The next two sections outline attempts to do this.

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†For further discussion of this point in relation to a critical appraisal of the dynamic simulation methods of Forrester see Cordey-Hayes, 1972.

3. A Framework for the Analysis of Migration

The system considered here for migration analysis is conceptually a relatively simple one. We are interested in the rates of change of population of a set of city regions in terms of the probability of transitions (migration) between the city regions. Thus let χ_i denote the population of city region i and $\frac{d\chi_i}{dt}$ its rate of change over time due to migration. Denote the probability per unit time of a transition from category i to j as a_{ij}. Similarly, χ_j is the occupation number of j and a_{ji} is the transition coefficient which represents the probability of a j to i transition in unit time. The rate of change of the occupation number of category i is thus simply related to the difference between the inward and outward flows for that category.

$$\frac{d\chi_i}{dt} = \sum_j (a_{ji}\chi_j - a_{ij}\chi_i) \qquad i = (1 \ldots n) \qquad (2)$$

Various constraints will normally restrict the solutions to this set of equations, but Rosen (1970) describes how the solutions have the general form

$$e^{u_i t} \, e^{\sqrt{-1}\, v_i t}$$

where the second exponential represents an undamped oscillatory function (u_i and v_i are related to the transition parameters a_{ij}; they are the eigenvalues of the a_{ij} matrix).

The time variation of the population of a city region (χ_i) can thus exhibit a variety of behaviours depending on the sign of

u_i and its magnitude relative to v_i. If u_i is positive and much larger than v_i, then χ_i increases almost exponentially with time; when u_i is negative and larger than v_i the χ_i decrease exponentially, and when u_i is small but v_i large then χ_i exhibits damped oscillations as shown in Figure 1.

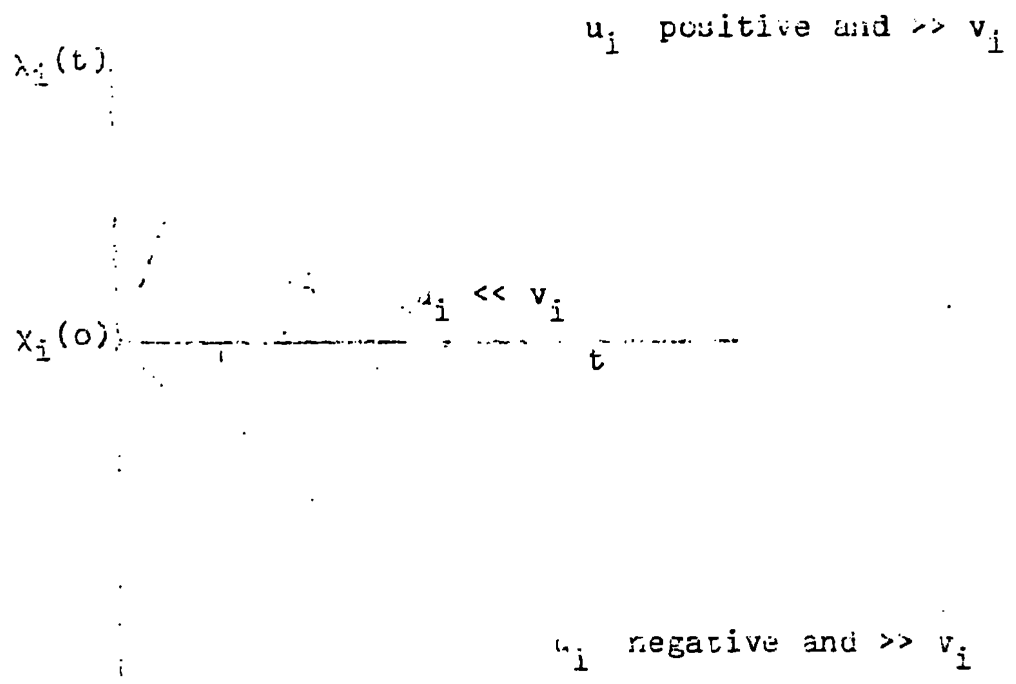

Figure 1. Some possible variation of χ_i through time.

Thus a variety of trajectories are possible and the explicit properties of the dynamical system are determined by the matrix (a_{ij}) which represents the external forces acting upon the system. Given a specific functional form for (a_{ij}), then it is possible to obtain a particular solution of the basic equation 2 above, which would then, for example, analytically describe the population changes due to migration within a system of interacting city regions.

This analytical approach to urban growth and change processes suggests that a useful strategy for structuring <u>experimental</u> work (in this case for migration analysis) is:[†]

i) To construct an accounting matrix comprising transition probabilities from observations of past behaviour. This provides a <u>description</u> of the migration process through time in a useful summary form, but this must be followed by:

ii) An interpretation of these parameters in terms of hypothesized causal relationships which represent the external forces.

iii) These tested hypotheses for the particular form of a_{ij} could then give specific solutions to equation 2 which would then, in principle, give the future distributions of population over time, or at least the 'behaviour modes' (approximate time path) of that system of city regions. It should also begin to provide the understanding necessary for the implementation of policies aimed at steering the system of city regions to some planned national settlement pattern.

Thus initially our aim is to interpret the a_{ij} for city regions in terms of the characteristics of i and j. Considerable simplification of the solution procedure of equation (2) is possible if the a_{ij} are decomposed into two components, which have been called in earlier papers an 'escape probability per unit time' (ε_i), equivalently an escape frequency, and a 'capture cross-section' (μ_j). This simplification separates

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[†] The first two steps are similar to the strategy described by Ginsberg (1972) in his work on a Semi-Markov approach to migration analysis.

the migration interaction into dynamic 'mover pool' and 'differential attraction' components. For example, from conventional spatial interaction theory it is hypothesized that the 'differential attraction' depends not only upon the intrinsic attributes of city region j as perceived from i, but also upon the competing attractions from all other possible destinations j. That is, the probability of a migrant from i selecting a destination j from a competing set of city regions is

$$\mu_{ij} = \frac{q_j P_j f(c_{ij})}{\sum_k q_k P_k f(c_{ik})} \tag{3}$$

where q_j represents some 'intrinsic attractiveness' of city region j for potential migrants, the population (P_j) and the function $f(c_{ij})$ weight this intrinsic attraction in relation to its size and distance from the origin region i. Hence, the probability of an individual in city region i migrating to city region j in unit time becomes

$$a_{ij} = \varepsilon_i \frac{q_j P f(c_{ij})}{\sum_k q_k P_k f(c_{ik})} \tag{4}$$

Cordey-Hayes and Gleave have described progress to date on the interpretation of ε_i and q_j in terms of the changing intrinsic characteristics of i and j for 20 city regions in the U.K. These experimental results do suggest that there exists a concept of intrinsic attractiveness that strongly influences the differential allocation of migrants, but the results also indicate that there exists a strong feedback relationship between q_j and ε_i that dominates the properties of the 'mover pool' and it is this that may have caused difficulties and 'anomalous' results in many migration analyses. These results have been outlined in Section 5 of the note 'Models of National Settlement Systems: 1. A preliminary perspective', and there it is discussed how the results disagree with the traditional migration equations that have been used in recent dynamic simulation models. This is important because not only is migration the most volatile component of population change, but also in most models provides the lynch-pin interaction with the employment growth sector. To some extent this demonstrates the need for the iterative structured approach to model building of poorly understood systems.

The strong feedback mechanism between ε_i and μ_j noted above makes the approach analytically intractable and a simulation model based upon these experimental analyses has been developed. In due course this model will be used to explore policy questions relating to regional imbalances and national settlement patterns, and this will then begin to integrate structured analytical research on the workings of the system with policy analysis. But on the whole the above research has been concerned with the analysis of the dynamics of the interdependent interactions between the demographic and employment sectors and has had relatively little to say on policy. The next section considers a research project that has a similar analytical framework but which is much more directly orientated towards policy analysis.

4. A Model for Multiregional Economic Policy[†]

Paelinck in a series of papers (1973A, 1973B, 1974) has used what is essentially a dynamical systems theory approach to construct a model for multiregional economic policy. It is largely structured on concepts of 'locational profiles' (Guigou, 1971) and 'attraction analysis' (Klaassen, 1969). Essentially locational profiles analysis consists of comparing the profiles offered (supplied) by a region to the optimally desired (demanded) profiles of a firm. It considers that both the growth in output and the probability of a new firm locating in a particular region is proportional to some matching of these regional and sectoral profiles.

Adapting Paelinck's notation into differential equation form, we have

$$\frac{dy_{ij}}{dt} = f_i(s_i, r_j, x_{ij}) \qquad (5)$$

where $\frac{dy_i}{dt}$ is the rate of growth of production in sector i in region j; s_i and r_j represent the sectoral and regional profiles respectively, and x_{ij} are a set of policy variables that operate on the profiles. Paelinck mentions that the functional dependencies f_i should be related to a "framework of consistent theory ... and ... not a simple (spurious) correlation exercise". However, he does not consider the problems which are associated with this due to the primitive state of urban and regional theory.

[†]This is a very free adaptation of Paelinck's work to suit the aims of this note. Readers interested in these multi-regional models should, of course, refer to the cited papers.

Instead he goes on to consider the problem of how the inputs to the system (particularly the policy variables) can be chosen such that the transit to a pre-assigned state is made in some optimum way. (This essentially involves the use of the calculus of variations to minimize an objective function over a variety of possible trajectories through time).

Paelinck defines the above functional dependencies through a divergence index (d_{ij}) which is a weighted comparison of the match between sectoral and regional profiles, and then hypotheses that:

$$\frac{dx_{ij}}{dt} = \rho_i - \alpha_i d_{ij} \qquad (6)$$

where ρ_i is some potential rate of growth of sector i which is related, for example, to the national growth rate in that sector. Thus, the rate of growth of sector i in region j is assumed proportional to this sectoral potential for growth moderated by the regional-sectoral divergence index d_{ij}. Other policy variables, and stochastic elements can be included in the equation. Equations of this type are then written to include firstly a set of sectoral policies and then a set of regional policies, and in each case an expression for the "acceleration" of production in sector i in region j is calculated. Acceleration is a second differential which is used by Paelinck to mean the sensitivity of the rate of growth of production to changes in the policy variables ("acceleration" being the differential of a rate). On the basis of this set of equations, a mathematical programming format is written down to calculate an optimal combination of regional and sectoral policies to maximize the change of growth rate subject to various constraints. It is suggested that the constraints could be expressed in terms of a multi-regional, multi-sectoral accounting model, and Paelinck argues that this optimising approach then contains "economic behavioural relations as well as policy constraints".

This approach is described and solutions obtained for a hypothetical two region example, but even in this case it was necessary, because of analytical complexity to modify the optimising approach into a simulation methodology. The framework has also been used to guide the empirical studies of the long term growth behaviour of the regions of the EEC, and a Ruhr study which aimed at selecting industrial activities which are "chain efficient" in the sense that they promote a clustering of other activities. From these hypothetical tests and empirical studies Paelinck concludes that the marginal effects of either regional or sectoral policy alone are small, but that the combined effect of complimentary sectoral and regional policies is significant. This certainly begins to illustrate the potentiality of this method for policy analysis, but it must be emphasized again that the properties of the components out of which the system is constructed are not at all well understood. Almost in passing, Paelinck observes that "the functional forms under which these measures have to be integrated in the model appear to be important." It was the thesis of the last section on migration that these functional dependencies are of very fundamental importance, and at present dynamical simulation and optimization methods consider only very simple (mostly linear) dependencies and that these are based on untested hypothesis. Plausible relationships of the type given above in equation 6 may contain a fifty percent error (and possibly much more) and as a result of compounding these errors the simulations are of limited use. This is why this approach must be integrated with a careful systematic, experimental analysis of the transition matrices (or dynamic multipliers) that represent the dynamic properties of the system. If this is done then the approach allows an explicit exploration of how urban and regional systems work and how they may react to the implementation of various planning policies, and this begins to provide the structured framework for the researcher and policy analyst alike.

Perhaps the strategy that is called for in our work on national settlement policy is one which is more policy orientated than the migration analysis outlined in Section 3, but which is also more sensitive and cautious to the problems of writing the dynamical equations for poorly understood systems than is the inter-regional economic growth study described above. The approach should be one of structured learning as well as identifying and testing policy options.†

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†Within this iterative process, there are of course many specific problems that may be amenable to move direct programming methods and some detailed consideration should be given to these. For example, Alonso (1973) has suggested that the consideration of the best investment strategy for national settlement policy is such a problem--should there be a simultaneous growth of several small centres or a big push to one?

Our next task will be to describe how to implement this strategy--this will be the topic of another note.

5. The Approach Simplified to Give Comparative-Static Models

In the absence of a formal theoretical understanding of how urban aréas evolve and change through time it has been usual to develop comparative-static models. Such models are useful in the sense that they give the long term implications of changes made to input variables; but they are atemporal devices in the sense that they do not consider the process of moving from one situation to another (Cordey-Hayes, 1972). This section illustrates how the dynamical systems approach outlined in Section 3 can if required be adapted to give simple comparative static models. The system considered is again a set of city regions between which there are continuous inter-changes of population, and we are interested in the question: What would be the implications for national settlement patterns if the current migration movement were continued over long term?

The rate of change of population (n_i) of city region i due to migration is the difference between the inward and outward rates of flow of population.

$$\frac{dn_i}{dt} = \sum_j (a_{ji} n_j - a_{ij} n_i)$$

At equilibrium n_i remains unchanged

ie. $\sum_j (a_{ji} n_j - a_{ij} n_i) = 0$

As in Section 3, it is useful to make the simplification that migration is a two stage process--an individual takes a decision to move and then selects a destination. This allows us to decompose the transition parameters a_{ij} into two components which may be called an 'escape' or departure frequency (ε_i) and a 'capture' or destination cross-section (μ_j) i.e. write

$$a_{ij} = \varepsilon_i \mu_j$$

$$\therefore \sum_j (\varepsilon_j \mu_i n_j - \varepsilon_i \mu_j n_i) = 0$$

We consider a closed system in which an individual who leaves i must enter one of the regions j.

$$\text{i.e.} \quad \sum_j \mu_j = 1$$

$$\therefore \mu_i \sum_j \varepsilon_j n_j - \varepsilon_i n_i = 0$$

$$\therefore \frac{\varepsilon_i}{\mu_i} n_i = \sum_j \varepsilon_j n_j = \text{constant}$$

$$\therefore n_i = k \, {}^{\mu_i}/_{\varepsilon_i}$$

Since $\sum_i n_i = n$

we have

$$n_i = n \frac{{}^{\mu_i}/_{\varepsilon_i}}{\sum_i {}^{\mu_i}/_{\varepsilon_i}}$$

This gives that at equilibrium the number of individuals in city region i depends on the total population within the whole system (n), the ratio of the capture cross section and escape frequency, and inversely on the sum of these ratios for all other city regions. This last component ($\Sigma_i \mu_i / \varepsilon_i$) represents the 'competition' amongst destinations. The above equation is similar in format to comparative static spatial interaction equations used in transportation analysis and retailing etc, but this equation has now been derived from a dynamic framework and has time implicit in the concepts of escape frequency (the mover pool). The equation can be used to assess some 'end state' to which an existing unstable distribution is moving, the disturbance being caused, for example, by some planned change that influences the ε_i or μ_j.

Here we take a somewhat simpler approach and assess the long term equilibrium distribution implied by current migration patterns. This involves calculating ε_i and μ_j from a recent Census and entering these into the above equation. The results for the U.K. based upon the 1961 Census are given in Table 1 . The figures for the equilibrium population were calculated for a system-wide population total which is the same as at present, this enables the current and equilibrium totals to be more easily compared--a modification to allow for natural increase in population is straight forward. The twenty city regions given in the Table are functional labour market areas, and these are considerably larger than the administrative regions.

Table 1 indicates that if current migration patterns continued then Newcastle would decline by approximately 30%

before an equilibrium situation is reached, and that Southhampton would increase in population by a similar percentage; all other city regions would grow or decline by an intermediate amount. Such changes would have important consequences for the individual cities but do not indicate a fundamental re-structuring of the U.K. settlement pattern, and when one allows for natural increase in population it appears unlikely that any city region will actually decline in total population. However, there may be large scale re-structuring within these city regions. For example, Lever (1973) has studied the equilibrium population of 1,000 towns in the U.K. using a Markov analysis approach. His results imply a strong clustering of city sizes around populations of about 150,000. This result can be misleading if taken too literally in a normative sense. Lever is probably observing decentralization and suburbanization effects at his scale of resolution and these sub-urban towns have very strong functional ties to each other and to the nearby metropolitan centre. They would not evolve with such frequency if isolated.

These results will not be discussed in anymore detail here, because they are extremely tentative but also they are part of a note on analytical style and research strategy. They are given here mainly to indicate the versatility of the dynamical systems framework and to illustrate that analytical research and policy issues need not be widely disparate.†

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† A working Paper that is presently being prepared develops a kinematic equation which interpolates between the current and equilibrium population totals given in Table 1, and then compares these with equivalent values obtained from a Markov approach.

TABLE 1

City Region	Current Population	Equilibrium Population
	(in thousands)	
1. Growth regions		
Southampton	1,563	2,059
Plymouth	1,145	1,379
Bristol	1,888	2,349
Coventry	902	1,178
Norwich	1,015	1,226
Leicester	849	926
Nottingham	1,494	1,583
Derby	531	578
Hull	927	962
London	12,579	12,915
2. Declining regions		
Newcastle	2,139	1,348
Sheffield	1,540	1,153
Manchester	3,540	3,074
Leeds	2,508	2,286
B'ham	3,616	3,605
Liverpool	3,473	3,392
Cardiff	1,299	1,118
Swansea	744	629
Middlesborough	925	889
Stoke	765	730

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