HYDROGEN MARKET PENETRATION: FURTHER REFINEMENTS ON THE STRATEGY FOR RESEARCH

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FURTHER REFINEMENTS ON THE STRATEGY FOR RESEARCH.

by

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1. Introduction

At its early stage a research and development program is a risky venture. Numerous alternative approaches have to be tested in order to determine a successful one if any. Clearly enthusiasm and even stubbornness will play a signifant role, but economic considerations may also help to efficiently allocate the effort and in particular to specify a somewhat "reasonable" time-cost trade-off for the completion of the project. "Reasonable" can only be properly defined once the main features of the situation have been quantified and related to each other within a model. Then logical analysis of the model may be used to provide guidelines for action.

The objective of this paper is to briefly review the analysis of a sampling process which appears to be used as a model in the research and development literature [see Manne-Marchetti 1974 but also Scherer 1966].

This sampling process may be simply described by a set of five assumptions:

(i) each approach will either result into a failure, with subjective probability p ($o < p < 1$), or a success, with subjective probability 1-p.

(ii) all approaches are stochastically independent,

(iii) one or more successful approaches yield a global benefit b (taken as unity),

(iv) all approaches have the same cost c (expressed in percentage of the benefit),

(v) all approaches require the same amount of time (taken as unity) to yield any result.

The review of this model will be made along two lines of inquiry; first the choice of the decision criterium and in particular the signifiance of risk aversion; second the role of discounting in sequential sampling. The result of the analysis will show that the optimal sample size may vary widely if the parameters of the problem happen to be in a certain range. This will call for a very careful model specification whenever it is suspected that such values are relevant.

Before turning to the analysis let us define some notations:

x = number of parallel approaches,

i = discount rate between two successive periods,

β = discount factor between two successive periods.

= $1/(1+i)$

p = probability of failure of any approach,

q = probability of success of any approach,

= $1-p$

c = cost of any approach expressed in percentage of the global benefit associated with one or more successful approaches

p^x = overall probability of failure in one time period.

$1-p^x$ = probability that at least one approach is a success in one time period,

$f(x)$ = expected benefit in one time period,

= $1-p^x-cx$

$g(x)$ = discounted expected benefit with an infinite horizon.

= $f(x)/(1-\beta p^x)$.

2. The Choice of the Decision Criterium

In the last ten years, decision under uncertainty has been the object of a considerable amount of theoretical and empirical research [Raiffa 1968, Edwards 1964]. Whereas simple criteria such as maximization of expected benefit have been under critical scrutiny, behavioral considerations such as "aversion towards risk" have led to the more general utility maximization theory.

In this section we wish to investigate the implications of explicitly introducing risk considerations into the model. To somewhat enhance the results and simplify the analysis, we shall restrict our attention to the one time period decision problem. Now, is this decision problem a risky venture at all? Let us pour out some numbers. The cost of one approach c may be assumed small relative to the benefit, say c=.001. Under any criteria one should not start more than 1000 approaches and by starting 100 one has used only 10% of the benefit associated with success. Now if the probability of success of any approach p is larger than .1, by starting 100 approaches the overall probability of success will be more than $1 - (.9)^{100} \simeq .99997$! This is not what we would call a risky venture. On the other hand if q is of the same order of magnitude as c, say 5c, this number would only be $1 - (.995)^{100} \simeq .4$. The prospect seems much dimmer and attitude towards risk becomes crucial. Should one use up 90% of the potential benefit to obtain what is left of it (a mere

10%, but this might still be a large sum of money) with a reasonable probability of success (now $1 - (.995)^{900} \simeq .989$) or just forget about the whole matter? This is the question we wish to answer from a theoretical point of view. As a utility function for the benefit w expressed in money terms, we shall take

$$u(w) = (1 - e^{-\rho w})/\rho$$

in which ρ is a parameter related to the decision maker's risk aversion. Note that for $\rho=0$, $u(w)=w$. It will be convenient to use as a reference point the certainty equivalent r of the lottery (0 with probability 1/2 and 1 with probability 1/2). Then ρ and r are related by the following table:

r	.5	.4	.3	.2	.1	.0
ρ	0	.82	1.8	3.3	7	∞

As an illustration, if $\rho=1.8$, the decision maker would be indifferent between receiving

(i) an amount r=.3 with probability 1,

(ii) an amount 0 with probability 1/2 or 1 with probability 1/2. Hence, the smaller r (or ρ) the more risk averse the decision maker. This class of utility functions is widely used in decision analysis. (This key underlying assumption is the following. Suppose that your present wealth is W. You are offered a risky venture that you are prepared to accept. Now if your present wealth were modified by a positive or negative amount ΔW, would you still be prepared to accept the venture? If the answer is yes whatever the value of ΔW then it may be shown that the utility function belongs to the class described above).

Under the utility maximization assumption the decision problem becomes

$$\max_{x \text{ integer}} \left[u(f(x))\right]$$

$$u[f(x)] = p^x \left(\frac{1 - e^{\rho c x}}{\rho}\right) + (1 - p^x)\left(\frac{1 - e^{\rho c x - \rho}}{\rho}\right)$$

$$= \begin{bmatrix}\text{probability}\\ \text{of}\\ \text{failure}\end{bmatrix} \cdot \begin{bmatrix}\text{utility}\\ \text{of}\\ -cx\end{bmatrix} + \begin{bmatrix}\text{probability}\\ \text{of}\\ \text{success}\end{bmatrix} \cdot \begin{bmatrix}\text{utility}\\ \text{of}\\ 1 - cx\end{bmatrix}$$

After some manipulations this problem may be equivalently written as

$$\max_{x \text{ integer}} \left\{- cx - \frac{1}{\rho} \operatorname{Log} \left[p^x + e^{-\rho}(1 - p^x)\right]\right\}$$

The results are summarized in Table 1.

c = cost of 1 approach q = success probability of 1 approach	r = .5		r = .4		r = .3		r = .2		r = .1		r = .0	
	$\hat{x}$ (optimal sample)	$1-p^{\hat{x}}$ (success probability)	$\hat{x}$	$1-p^{\hat{x}}$	$\hat{x}$	$1-p^{\hat{x}}$	$\hat{x}$	$1-p^{\hat{x}}$	$\hat{x}$	$1-p^{\hat{x}}$	$\hat{x}$	$1-p^{\hat{x}}$
c = .001, q = c	1	.001	0	0	0	0	0	0	0	0	0	0
q = 2c	345	.501	300	.454	0	0	0	0	0	0	0	0
q = 3c	365	.667	405	.704	405	.704	0	0	0	0	0	0
q = 4c	345	.751	400	.797	455	.838	430	.821	0	0	0	0
q = 5c	320	.801	370	.846	438	.889	520	.926	0	0	0	0
q = 6c	300	.834	345	.875	410	.915	510	.953	0	0	0	0
q = 7c	280	.858	320	.895	380	.931	480	.966	190	.740	0	0
q = 8c	260	.876	300	.910	355	.942	450	.973	630	.993	0	0
q = 9c	240	.890	280	.921	335	.950	420	.978	640	.997	0	0
q = 10c	230	.901	265	.930	310	.956	395	.981	610	.998	0	0

TABLE 1

Risk Aversion, Optimal Sample Size and Overall Probability of Success

3. The Role of Discounting in Sequential Sampling

If sequential sampling is allowed, that is, waiting one time period to see the results of the approaches before undertaking any new ones, then there is a basic trade off between the arrival date of the first success and the amount of R & D expenditures spent in parallel approaches. More precisely, since more than one success is redundant, engaging into parallel approaches might lead to spending money unnecessarily and not engaging into parallel approaches might lead to a waste of time before obtaining the first success. This trade-off is theoretically resolved by comparing future streams of money in terms of their discounted present values [Koopmans 1960]. A constant discount rate is somehow equivalent to an impatient behavior which does not depend on the current wealth of the decision maker. The more impatient the larger the discount rate (the smaller the discount factor).

In this section we want to study numerically the relationship between discounting and expected arrival date of the first success within the sampling model described in the introduction.

This problem may be formulated as follows:

$$\max_{x \text{ integer}} \left[g(x)\right]$$

$$g(x) = (1 - p^x - cx)/(1 - \beta p^x).$$

Let x^* be the optimal size then the expected arrival date of the first success T^* is such that

$$T^* = 1(1 - p^{x^*}) + 2p^{x^*}(1 - p^{x^*}) +, \ldots, + n\, p^{nx^*}(1 - p^{x^*}) +, \ldots$$

$$= 1/(1 - p^{x^*}).$$

The numerical results are summarized in Table 2, assuming $c = .001$ and $p = .99$.

c = .001 p = .99			
Discount Rate i	Optimal Sample Size x^*	Success Probability in 1st Period $1-p^{x^*}$	Expected Arrival Date of Success $1/(1-p^{x^*})$
0	1	.01	100
1%	40	.33	3.02
2%	53	.41	2.42
3%	62	.46	2.15
4%	71	.51	1.96
5%	78	.54	1.84
6%	83	.56	1.77
7%	88	.59	1.70
8%	92	.61	1.65
9%	96	.62	1.61
10%	100	.63	1.57

TABLE 2

Discounting in Sequential Sampling and
Expected Arrival Date of Success

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