BASIC PROBLEMS OF LONG-TERM INFERENCE INTO THE FUTURE

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PREFACE

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis, including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modelling techniques, to generate inputs for planning, design, and operational decisions.

During the year of 1978 it was decided that parallel to the continuation of demand studies, an attempt would be made to integrate the results of our studies on water demands with water supply considerations. This new task was named "Regional Water Management" (Task 1, Resources and Environment Area).

Although this paper does not refer explicitly to water resources, it is concerned with the problems of primary importance to water resources planning. In several countries very significant capital investments are being made and contemplated for the future for water supply projects of increasing size. The purpose of these projects is to satisfy future water demands which often are estimated (predicted) on the basis of statistically derived demand relationships.

The paper examine some of the major problems and difficulties involved in the use of statistically derived relationships for long-term inference into the future.

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1. THE DEFINITION OF LONG-TERM INFERENCE INTO THE FUTURE

We shall consider the problem of long-term inference into the future. Let

$$Y = f(X_1, X_2, \ldots, X_k, \xi) \tag{1}$$

be the model which is to be used for this purpose. In (1) Y denotes the endogenous variable whose value at the future time T is to be foreseen while the X_i's are the explanatory variables and ξ is the random component of the model. Let t_1 denote the present time period and let T_o be the time interval from which statistical data were used for the estimation of model (1).

It must be observed that the problem of long-term inference into the future occurs not only when the difference $T - t_1$ is large but such may be also the situation when $T - t_1$ is relatively small, provided T is very distant from T_o. In the latter case we are confronted by a special case of long-term inference into the future due to the fact that the model to be used is outdated.

Long-term inference into the future creates special problems because very often one can doubt if in fact it will fulfill the basic preconditions for such inference. For the sake of clarity of exposition let us remind the reader these five preconditions (see Pawlowski (1973a)).

1) If prediction or forecast[1] for Y is sought then a model must be had, such that Y plays the role of the endogenous variable of that model.
2) The model must be valid not only for T_o but for the whole time interval from the beginning of T_o up to the end of T.
3) The probability distribution of ξ must be the same in T_o and in time T.
4) The values of the explanatory variables $X_1, X_2, \ldots, X_k$ at time T must be known at time t_1.
5) The model used for inference into the future can be extrapolated outside the sample-observed range of variation of its explanatory variables.

Obviously, when the distance between T and T_o is large, it is quite likely that a number of these preconditions[2] will not be met, and this is especially true of preconditions (2), (3) and (4). As it will be shown in the subsequent sections of this paper, long-term inference into the future is still

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[1] The distinction between a prediction and a forecast is connected with the type of model used for inference into the future and will be explained in section 3.

[2] Let us note that these preconditions can be looked upon as the necessary conditions for making any inference into the future. While they do not guarantee a good result, one can see that the inference will be void of logical and probabilistic grounds if at least one of these preconditions is not fulfilled.

possible if the model gets obsolete in a slow way, i.e. when the changes which occur in the true relation between Y and $X_1, X_2, \ldots, X_k$ and in the distribution of ξ are rather regular and smooth. Also the problem of precondition (4) requiring an exact knowledge of the values of explanatory variables at time T can be circumvented. For this reason it is sometimes said that the long-term inference into the future from a model can be made under relaxed basic preconditions.

2. SOME MAJOR PROBLEMS AND DIFFICULTIES

There is a number of pitfalls a statistician is confronted with when using a model of type (1) for long-term inference into the future. Restricting our attention to the most important ones, one must take into account the following possibilities:

a) The functional form of the model, i.e. the type of function f, may change in time.

b) While the type of function represented by f remains the same from T_o up to time T, there are changes in the values of the parameters which enter function f.

c) The values of the explanatory variables $X_1, X_2, \ldots, X_k$ (or of some of them) are not known at time t_1 when inference is made.[3)]

d) The probability distribution of the random component ξ of the model may change in time, changing thus the degree of accuracy with which the model describes the behavior of Y.

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3) Let us observe that this precondition is automatically satisfied when (1) represents a trend model since the only explanatory variable is then time variable whose value at time T is obviously known.

e) New factors may arise and influence Y while some of the explanatory variables appearing in the model may cease to be relevant.

In any practical circumstances the statistician may be confronted with just one of the dangers elicited here or he may be faced with a combination of a number of them. Unfortunately, one cannot tell in advance which of the situations is most likely to occur nor it would be correct to claim that there is any general relation between the size of probability of such a danger and the distance between T_o and T. There are variables whose behavior in time is very regular and there are variables so erratic that it is most difficult to foresee their values even in the case of short-run inference.

3. TERMINOLOGY

Before we start to discuss the five cases listed in the previous section, we shall introduce a number of terminological definitions which will facilitate our exposition.

By prediction or forecast will be denoted henceforth the numerical result of inference into the future, the term "prediction" referring to the case when the model (1) is a causal one and the term "forecast" referring to all other types of models.[4] Accordingly, the variable to which refers the process of inference into the future will be called the predicted (or the forecast) variable.

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[4] These may be, for instance, trend- and periodic-movement models, stochastic process models, adaptive models, etc.

Future time period T for which prediction or forecast is sought will be referred to as the predicted time period.

The distance between the predicted time period and the present one, i.e. the difference $T - t_1$, is known as the prediction (or forecast) lead while the distance between T and T_o will be termed the predictive delay of the model.[5)]

4. VARIABILITY OF STRUCTURAL PARAMETERS

First, we shall discuss the problem of variability of structural parameters of the model. Three typical situations must be considered, namely: a) structural parameters exhibit continuous and rather regular shifts in time, b) time-changes of parameters are so erratic that they can be considered random, c) structural parameters change in time, in relation to changes of a third (observable) variable.

4.1. Regular Shifts of Parameters in Time

For the sake of easier argument let us assume model (1) to be a linear one, so that there is

$$Y = \sum_{i=1}^{k} \beta_i X_i + \xi \quad . \tag{2}$$

The simplest way to cope with this type of parameter shifts is to assume β_i's to be some explicit function of time. Most often linear changes are assumed

$$\beta_i = \beta_{io} + \beta_{i1} t \quad , \tag{3}$$

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5) To shorten the argument we shall use henceforth only the term prediction unless the model explicitly has the form which permits using it for forecasting only.

where t denotes time variable. Substituting (3) into (2) leads to a new model

$$Y = \sum_{i=1}^{k} (\beta_{io} + \beta_{i1}\, t) \cdot X_i + \xi \quad , \tag{4}$$

or

$$Y = \sum_{i=1}^{k} \beta_{io} X_i + \sum_{i=1}^{k} \beta_{i1} t X_i + \xi \quad . \tag{5}$$

As is easily seen, this is again a linear model in which, besides the original explanatory variables X_i's, appear new ones, namely tX_i's which can be interpreted as interaction of time and X_i. Once the model (5) is estimated, the estimates b_{i1}'s of β_{i1}'s provide information about the direction and magnitude of structural parameter changes.

For prediction purpose the structural parameters must be adjusted for time T and hence, prediction value is computed from the model

$$Y = \sum_{i=1}^{k} \beta_{io} X_{iT} + \sum_{i=1}^{k} \beta_{i1} T X_{iT} + \xi \quad , \tag{6}$$

where X_{iT} denotes the value of X_i assumed for period T. If the principle of unbiased prediction is used[6)] then prediction is equal to the right-hand side of (6) in which ξ has been put equal to zero.

Sometimes one does not have enough a priori grounds to assume linear (or any other specific) time variation of parameters. If the length of time series used for estimation of

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[6)] The principle of unbiased prediction consists in setting prediction equal to the expected value of the predicted variable in time T.

the model is large enough one can infer about the character of such changes by analyzing sample data.

Let n be the number of sample observations and let m be a positive integer much smaller than n. The existing time series data are then used to generate n-m+1 subsamples. The first subsample includes the data referring to m periods of time - from the first to the m-th sample period, the second subsample includes the data from the second till the (m+1)th sample period, etc., and the last subsample is based on the data from the (n-m+1)th period up to the last sample period.

Each subsample is used to estimate the model (2). Thus, for every parameter β_i there are n-m+1 estimates ordered in time. Plotting these estimates against time can usually give an idea as to how the various β_i's change in time - if they change at all.[7)]

4.2. Random Variation of Structural Parameters

Such variation can best be detected by using the approach of consecutive subsamples, described above. If the sequence of parameter estimates is erratic and the variance of observed estimates is high[8)] then one can conclude that the corresponding structural parameter changes its value from period to period in a random way.

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7) It is quite conceivable that while some structural parameters vary in time other remain constant.

8) The assumption of high variance is essential. In the case of small differences of consecutive estimates one should rather conclude that these estimates vary in a random way while the true parameter value remains constant in time.

Prediction can then be made using one of the following two approaches: 1) averaging prediction with respect to random variation of parameters, 2) using the optimistic and pessimistic predictions. The two approaches will be briefly outlined in section 6 of this paper.[9)]

4.3. Variation Induced by a Third Variable

There are cases when a structural parameter is a function (deterministic or stochastic) of a third variable. In practice, such variable can be usually identified by theoretical argument using available subject-matter knowledge or empirical evidence referring to the area to which belongs the predicted variable. Once such third variable has been identified - let us denote it by Z - the problem reduces to a simple one.

One may either use the approach similar to that described in (a) using, instead of equation (3) a similar model, namely

$$\beta_i = \beta_{io} + \beta_{i1} Z \quad , \tag{7}$$

which eventually leads to the model with interactions of X_i's with Z

$$Y = \sum_{i=1}^{k} \beta_{io} X_i + \sum_{i=1}^{k} \beta_{i1} X_i Z + \xi \quad . \tag{8}$$

Alternatively - and this is especially useful in non-linear models - one can assume a structural parameter to be proportional to variable Z. As an example, we present a simple

9) The method of building optimistic and pessimistic predictions has been described in a detailed way in Pawlowski (1978a).

modification of Cobb-Douglas production function (see Pawlowski, 1970) where there are two auxiliary variables Z_1 and Z_2:

$$Q = \beta_1 L^{\beta_2 Z_1} \cdot K^{\beta_3 Z_2} , \tag{9}$$

where Q denotes output, L is labor input, K stands for capital, Z_1 for technical equipment of labor and Z_2 is a variable measuring the level of managerial ability. As is seen, contrary to the classical Cobb-Douglas production function, the exponents of L and K are not constant but vary according to changes of Z_1 and Z_2.

As in the case of time-variation of parameters, when making prediction, one must set the values of third-variable-dependent parameters at the level corresponding to the value assumed for Z variable in time T.

5. UNCERTAINTY ABOUT THE FUNCTIONAL FORM OF THE MODEL

While there are usually few reasons why the functional form of the model (i.e. the class of f function) should change in time, one is very often confronted with the serious problem of uncertainty if the estimated functional form is really the correct one. The problem is ever present in any econometric analysis but it is especially important when a long-term inference into the future is sought. With large prediction (or forecasting) lead even small estimation errors of function f within the observed sample variation of explanatory variables may become very substantial when using the model well out of the observed region of variation of explanatory variables.[10)]

[10)] And this precisely happens quite often when long-term inference is needed. As most of the explanatory variables exhibit monotonic trends, their values corresponding to periods in a distant future will obviously coincide with values observed in the sample.

Since the situation is especially typical when a trend model is used the following arguments will assume the model to have the form

$$Y = f(t,\xi) \quad . \tag{10}$$

Generalization of the results to other classes of models is straightforward.

Two approaches at least can be recommended to cope with the problem. The first of them consists in constructing the region of functional uncertainty of inference. Let us suppose that using sample data it is possible to find a number of trend functions which fit these data with roughly the same degree of accuracy. Let these trend functions be $f_1(t), f_2(t), \ldots, f_s(t)$.

The next step is to seek among these trend functions such a one which gives the highest forecast of Y at time T and another trend function which, for the same predicted time period, gives the lowest forecast. Since the way the $f_i(t)$ functions are allotted their numbers is arbitrary, we can assume the function giving the highest forecast to be $f_1(t)$ and that giving the lowest one to be $f_s(t)$. Let us now put $f_1(T) = y_T^{**}$ and $f_s(T) = y_T^{*}$. The interval $[y_T^{*}, y_T^{**}]$ is the region of functional uncertainty of inference [11] and provides the range of values of the predicted variable which one must take into account because of doubts as to the right form of the trend function. Obviously, the situation is the better as the interval is narrower.

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[11] The term "region of uncertainty" is due to the fact that, generally speaking, one may consider a number of predicted time periods $T_1, T_2, \ldots, T_s$ and one is lead to consider then a region comprised between the graphs of functions giving for $T_1, T_2, \ldots, T_p$ the highest and the lowest values of forecasts.

Let $\bar{y}_T$ denote the average forecast[12)] computed by using the functions $f_i(t)$. The ratio

$$\lambda = \frac{\bar{y}_T}{y_T^{**} - y_T^{*}} , \tag{11}$$

can be regarded as a measure of accuracy of information.

The second approach applies again to trend functions $f_1(t), f_2(t), \ldots, f_s(t)$. For each one of them, the corresponding difference or differential equation is obtained. Once this is done, such equation is subject to theoretical and empirical analysis. The purpose of the first one is to find if the equation is consistent with the existing theoretical knowledge about the predicted variable. This theoretical analysis is supplemented by an empirical one, the aim of which is to provide information if the empirical data are consistent with the dynamic characteristics resulting from the corresponding difference (or differential) equation.

Two short examples will supplement the theoretical argument. First, let us suppose the trend function to be exponential. A well-known property of exponential function is that for Δt = constant, the corresponding relative changes of $f(t)$ are also constant. Hence, the empirical analysis should consist in the application of an appropriate statistical test for the hypothesis that the observed relative changes of the endogenous variable of the model differ among each other only in a random way.

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12) On the other hand $\bar{Y}_T$ can be thought of as synthetic forecast based on the various considered trend functions $f_i(t)$.

The second example refers to power function $Y = \alpha t^{\beta}$. As can easily be verified for this class of function, there is

$$\frac{dY}{Y} = \beta \frac{dt}{t} \quad . \tag{12}$$

Hence, to check if the trend is represented by this type of function one may test if the relative changes of Y are inversely proportional to the value of time variable.

The approach consisting in the analysis of dynamic properties of various functions, derived from their difference or differential equation, results in an a priori elimination of a number of $f_i(t)$ functions. This usually leads to narrower region of functional uncertainty of inference.

6. UNCERTAINTY ABOUT THE VALUES OF EXPLANATORY VARIABLES OF THE MODEL

Here, a number of different approaches are possible, their character varying according to the amount of information available about the predicted time period and to the desired level of sophistication of the analysis.

The simplest procedure which can be used when the exact values of the explanatory variables are not known consists in computing a set of predictions, each of them corresponding to a different assumption about X_{iT}'s. Although very simple, this procedure is not to be highly recommended since, in fact, it does not provide a straightforward answer to the question: what will be the value of the predicted variable at time T? Instead, this approach provides a number of answers leaving open the question of the determining conditions for Y_T.

Another simple approach, used often in econometrics, is to rely on the observed trends of explanatory variables.[13] Let $g_1(t), g_2(t), \ldots, g_k(t)$ be the observed trends of the variables X_i entering model (1). By extrapolating these trends for $t = T$, one gets the approximate values of X_{iT}'s. Obviously, the efficiency of this method depends on the fit of trend functions $g_i(t)$ with real data and on the validity of such trends also for periods posterior to sample interval T_o, and this in turn depends on the predictive delay of the model.

In countries with planned economies one uses sometimes plan targets as X_{iT} values. A better approximation, however, is provided by products $c_i X_{iT}^{(p\ell)}$, where $X_{iT}^{(p\ell)}$ denotes plan target for period T and c_i is a positive coefficient expressing the expected degree of fulfillment of such target.

If the explanatory variables (or at least some of them) can be considered as random[14] and their probability distributions in time T is known, it is possible to build the so-called predictions averaged with respect to the distribution of explanatory variables. For the sake of simplicity, let us assume the model to be linear (and hence of form (2)) and let $G_T(x)$ be the probability distribution function of the explanatory

[13] The relatively common use of this approach in econometrics stems from the fact that economic variables very often exhibit well pronounced trends.

[14] In particular, only some of the explanatory variables may be random. Let us note also that, as a rule, a lagged endogenous variable appearing as explanatory must be treated as a random one since in time t-L it is influenced by ξ_{t-L} - by definition a random variable.

variables in time T. Prediction averaged with respect to this distribution is defined as (see Pawlowski (1968))

$$\bar{y}_{Tp,x} = \int_{\Delta_x}\!\!\cdots\!\int \Big(\sum_{i=1}^{k} \beta_i X_i\Big) \cdot dG_T(x) \quad , \tag{13}$$

where Δ_x denotes the region of possible variation of the explanatory variables of the model. After a number of easy transformations (13) can be expressed in the following simple form

$$\bar{y}_{Tp,x} = \sum_{i=1}^{k} \beta_i E(X_{iT}) \quad . \tag{14}$$

This means that getting an unbiased prediction in the presence of random explanatory variables consists in substituting for these variables their expected values at time T and equating the random component to zero. It can be shown also (see Pawlowski (1973b)) that the variance of prediction (14) is equal to

$$D^2(\bar{y}_{Tp,x}) = \sum_{i=1}^{k} \beta_i^2 D^2(X_{iT}) + 2\sum_{i=1}^{k} \sum_{j>i} \beta_i \beta_j \mathrm{Cov}(X_{iT}, X_{jT}) + \sum_{i=1}^{k} D^2(b_i)\,[E(X_{iT})]^2 + 2\sum_{i=1}^{k} \sum_{j>i} \mathrm{Cov}(b_i b_j)\,E(X_{iT}) \cdot E(X_{jT}) + \sigma_T^2 \quad . \tag{15}$$

In this formula $D^2(X_{iT})$ stands for the variance of X_{iT}, $D^2(b_i)$ is the variance of the estimate of β_i while the symbol $\mathrm{Cov}(\cdot)$ denotes covariance. Finally σ_T^2 represents the variance of the random component ξ in time T.

By similar argument as that leading to prediction (13) one can also build predictions in the case when X_{iT}'s are known while the structural parameters are random.[15] If $H(\beta)$ is the probability distribution function of structural parameters then prediction averaged with respect to variation of parameters is defined as

$$\bar{y}_{Tp,\beta} = \int_{\Delta\beta} \cdots \int \left(\sum_{i=1}^{k} \beta_i X_{iT} \right) \cdot dH(\beta) \quad , \tag{16}$$

which finally leads to the formula

$$\bar{y}_{Tp,\beta} = \sum_{i=1}^{k} X_{iT} \cdot E(\beta_i) \quad . \tag{17}$$

The variance of prediction in this case is provided by the obvious modification of (15). One might consider also predictions averaged both with respect to random variation of structural parameters and of explanatory variables

$$\bar{y}_{Tp} = \int_{\Delta_x \times \Delta_\beta} \cdots \int \left(\sum_{i=1}^{k} \beta_i X_i \right) dG_T(x) \cdot dH(\beta) \quad . \tag{18}$$

The search for the variance of prediction leads now, however, to a very complicated formula.

Finally, two other approaches can be mentioned. If the values of the explanatory variables in time T are unknown when prediction is made, one can also make use of the concept of optimistic and pessimistic prediction or that of alternative

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[15] The situation of random variation of parameters has already been introduced in subsection 4b of this paper.

predictions. These two methods have been described in detail by Pawlowski (1978b) so there seems no point to repeat the respective algorithms in full details.

It must be pointed, however, that the rationale of optimistic and pessimistic predictions can also be used in the case of random structural parameters. To this generalization, we shall devote now some space.

Since the explanatory variables assume positive values there is no need - as in the classical case of pessimistic and optimistic predictions - to subdivide the set of explanatory variables into two subsets. In order to build an optimistic prediction such values are substituted for structural parameters which are favorable from the viewpoint of formation of the predicted variable, the probability of getting still "better" values for each of these parameters being equal to a predetermined number ε. Similarly, for building a pessimistic prediction such values are substituted for structural parameters that the probability of getting still worse value for each parameter is equal to ε.

For the sake of example let us assume the model

$$\underline{Y} = \beta_1 X + \beta_2 + \xi \quad , \qquad (19)$$

in which β_1 and β_2 are assumed to be random variables with rectangular probability density functions over the intervals (o.4, o.8) and (1.0, 2.0), respectively. Let us assume further that $\varepsilon = 0.1$ and that the value of X for time T is assumed to be equal to 5.0 and that utility is an increasing function of y.

Since under the assumed distributions there is

$$P\{\beta_1 \geqslant 0.76\} = 0.1 \text{ and } P\{\beta_2 \geqslant 1.9\} = 0.1 \ ,$$

hence, the optimistic prediction is

$$y_{opt} = 0.76 \times 5 + 1.9 = 5.7 \quad .$$

On the other hand, as one finds that

$$P\{\beta_1 \leqslant 0.44\} = 0.1 \text{ and } P\{\beta_2 \leqslant 1.1\} = 0.1 \quad ,$$

so the pessimistic prediction is

$$y_{pess} = 0.44 \times 5 + 1.1 = 3.3 \quad .$$

Thus, the interval of uncertainty of prediction is (3.3, 5.7).

7. THE EFFECT OF NON-STATIONARITY OF THE PROBABILITY DISTRIBUTION OF ξ_t

Another difficulty one is likely to come across when making a long-term inference into the future is the risk of coming across the non-stationarity of distribution of the random component of the model. This non-stationarity does not preclude computing a prediction but interferes with getting a correct information about the level of its accuracy. Especially serious is the situation when the variance of ξ_t increases in time since this means that the accuracy of prediction will decrease steadily as the predictive delay of the model becomes greater.

If, for some reason, the model cannot be changed so as to achieve the stationarity of distribution of ξ_t, one should at least attempt to estimate how the main distribution characteristics depend on time. Since among these characteristics the most important one is the variance of ξ_t we shall concentrate on various methods of analysis of time-dependence of $D^2(\xi_t)$.

The first method consists in observing the residuals $u_t = y_t - \hat{y}_t$, where y_t denotes observed value of Y variable in time $t \varepsilon T_o$ and $\hat{y}_t$ is the corresponding theoretical value computed from the model. Once the residuals are computed and ordered according to their sequence in time, their absolute values are then considered

$$|u_1|,\ |u_2|,\ \ldots,\ |u_n| \quad . \tag{20}$$

If the sequence (20) can be accepted as a random one[16] there is no ground to reject the hypothesis that $D^2(\xi_t)$ is constant. If on the contrary, this sequence shows an increasing trend one must conclude that the variance increases in time. Time trend fitted to the elements of (20) provides then information about the relation of standard deviation of ξ_t with respect to time variable. Extrapolation of this relation provides an estimate of $D(\xi_t)$ for the predicted time period.

The second possible approach to the analysis of time behavior of the variance $D^2(\xi_t)$ consists in using consecutive subsamples, as it was explained in subsection 4a. Since the model is estimated for each subsample, an estimate of $D^2(\xi_t)$ is also available and the statistician gets a sequence $s_1^2,\ s_2^2,\ \ldots,\ s_{n-m+1}^2$ of such estimates. An appropriate analysis of this sequence permits to infer if the sequence can be regarded as a random one or if it exhibits a time-dependence of subsample variances.[17] Discovery of such dependence allows for extrapolation of the variance for time period T.

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16) There are many tests for testing the hypothesis of randomness of a sequence of observations, such as, for instance, the various run tests (see A.M. Mood [1940] or any major text-book of mathematical statistics).

17) When looking for a trend in the $s_1^2,\ s_2^2,\ \ldots,\ s_{n-m+1}^2$ sequence it is sometimes more advisable to consider the sequence of standard deviations $s_1, s_2, \ldots, s_{n-m+1}$ since the latter is less subject to random fluctuations.

If $D^2(\xi_t)$ does increase in time it may sometimes happen that $D^2(\xi_t)$ will prove to be so large that the prediction is virtually useless. This calls for a change of the model and for substitution of the former one by another with smaller random variance.

8. EMERGENCE OF NEW FACTORS

This is certainly the most difficult problem since it cannot be dealt with by statistical methods. The best method known so far to cope with it is to use experts' judgements and to introduce accordingly an appropriate correction to the prediction obtained from the formal model.

The weak point of this method consists in the impossibility to measure a priori the degree of accuracy of such corrected prediction which may prove later - when the period T occurs - to be very poor. If a practical action is based on prediction the danger of such situation is obvious.

From the formal point of view the corrected prediction is defined as

$$y_{Tp}^{(corr)} = y_{Tp} + e_{corr} \quad , \tag{21}$$

where y_{Tp}^{corr} denotes the corrected prediction, y_{Tp} is the prediction computed by using the model and e_{corr} stands for experts' judgements about the impact of the new factor which is to influence the behavior of the predicted variable. Alternatively, if experts formulate their opinion with reference to relative changes, the corrected prediction is defined as

$$y_{Tp}^{corr} = y_{Tp}(1 + e_{corr}) \quad . \tag{22}$$

9. HORIZON OF PREDICTION

To conclude the argument about long-term inference into the future, one should emphasize the fact that such inference cannot go indefinitely far into the future. To have any practical meaning predictions must have an adequate level of accuracy.[18)] Although there are many ways to measure this accuracy, the most commonly used method of measurement in the case of unbiased prediction consists in using the variance of prediction defined as

$$D^2(y_{Tp}) = E(Y_T - y_{Tp})^2 \quad . \tag{23}$$

In most practical cases this variance increases with prediction lead and for most types of commonly used models it is found that $D^2(y_{Tp})$ increases as far as the square of prediction lead.[19)] In consequence , beginning with a future time period t_2, the variance is larger than a predetermined number d corresponding to the limiting admissible order of prediction error. Besides, for very distant future time periods also the basic preconditions of prediction, even in their relaxed form, are not met, precluding thus the possibility of rational inference into the future. These two limitations call, therefore, for a careful choice of the model to be used for prediction. Such model must not only include all the relevant (or decision) factors but must be time-robust in the sense of ensuring the possibility of prediction even for distant time periods.

[18)] This level is obviously determined a priori by the user of prediction.

[19)] This is, for instance, the case of polynomial trend models.

The set of all such future time periods for which simultaneously two conditions are fulfilled, namely: a) the basic preconditions of prediction are true, b) the degree of accuracy of prediction is admissible, determines the so-called horizon of prediction (see Pawlowski, 1978b).

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