



## Erratum to: *Daphnia* revisited: local stability and bifurcation theory for physiologically structured population models explained by way of an example

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In the original publication, the addresses of the authors Dr. J.A.J. Metz and Dr. S. Nakaoka were incorrectly published. The correct address list for the authors are:

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While working on their paper (Breda et al. 2015), Philipp Getto and Julia Sánchez Sanz found that formula (4.22) is incomplete (indeed, the solution operator of the homogeneous version of the linearized equation (4.3) should act on the second term at the right hand side of (4.22)). As a consequence (4.23), (4.24), (4.25), (4.31) and (4.32) are incorrect as well. Actually, as it turned out, there were additional inaccuracies. The correct formulas are

$$D_3\xi(\tau; a, \bar{S})\psi = \eta(\tau) + e^{\int_{\bar{\tau}}^{\tau} \frac{\partial g}{\partial \xi}(\bar{\xi}(\sigma), \bar{S})d\sigma} \left( \frac{g(\xi_{A+}, \bar{S})}{g(\xi_{A-}, \bar{S})} - 1 \right) \eta(\bar{\tau}) \tag{4.22}$$

$$D_3\xi(\tau; a, \bar{S})\psi = \eta(\tau) + e^{\int_{\bar{\tau}}^{\tau} \frac{\partial g}{\partial \xi}(\bar{\xi}(\sigma), \bar{S})d\sigma} \left( \frac{g(\xi_{A+}, \bar{S})}{g(\xi_{A-}, \bar{S})} - 1 \right) \eta(\bar{\tau})H(\tau - \bar{\tau}) \tag{4.23}$$

$$D_2\Xi(a; \bar{S})\psi = \int_0^a K(a, \alpha)\psi(-a + \alpha)d\alpha + \left( \frac{g(\xi_{A+}, \bar{S})}{g(\xi_{A-}, \bar{S})} - 1 \right) \int_0^{\bar{\tau}} K(a, \alpha)\psi(-a + \alpha)d\alpha \cdot H(a - \bar{\tau}). \tag{4.24}$$

$$D_2\mathcal{F}(a, \bar{S})\psi = \int_0^a L(a, \theta)\psi(-a + \theta)d\theta - \frac{\mu(\xi_{A+}, \bar{S}) - \mu(\xi_{A-}, \bar{S})}{g(\xi_{A-}, \bar{S})} \bar{\mathcal{F}}(a) \times \int_0^{\bar{\tau}} K(\bar{\tau}, \alpha)\psi(-a + \alpha)d\alpha \cdot H(a - \bar{\tau}) - \left( \frac{g(\xi_{A+}, \bar{S})}{g(\xi_{A-}, \bar{S})} - 1 \right) \bar{\mathcal{F}}(a) \times \int_0^{\bar{\tau}} \int_{\bar{\tau}}^a \frac{\partial \mu}{\partial \xi}(\bar{\xi}(\sigma), \bar{S}) K(\sigma, \alpha)d\sigma\psi(-a + \alpha)d\alpha H(a - \bar{\tau}). \tag{4.25}$$

$$k_{12}(a) = \bar{b} \int_{\max\{0, \bar{\tau}-a\}}^{\infty} \left\{ \beta(\bar{\xi}(a + \theta), \bar{S})L(a + \theta, \theta) + \frac{\partial \beta}{\partial \xi}(\bar{\xi}(a + \theta), \bar{S})K(a + \theta, \theta)\bar{\mathcal{F}}(a + \theta) \right\} d\theta + \bar{b} \left( \frac{g(\xi_{A+}, \bar{S})}{g(\xi_{A-}, \bar{S})} - 1 \right) \int_{\max\{0, \bar{\tau}-a\}}^{\bar{\tau}} K(a + \theta, \theta) \times \frac{\partial \beta}{\partial \xi}(\bar{\xi}(a + \theta), \bar{S})\bar{\mathcal{F}}(a + \theta)d\theta$$

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$$\begin{aligned}
 & + \bar{b} \frac{\mu(\xi_{A-}, \bar{S}) - \mu(\xi_{A+}, \bar{S})}{g(\xi_{A-}, \bar{S})} \int_{\max\{0, \bar{\tau}-a\}}^{\bar{\tau}} \beta(\bar{\xi}(a + \theta), \bar{S}) \\
 & \times \bar{\mathcal{F}}(a + \theta) K(\bar{\tau}, \theta) d\theta - \bar{b} \left( \frac{g(\xi_{A+}, \bar{S})}{g(\xi_{A-}, \bar{S})} - 1 \right) \\
 & \times \int_{\max\{0, \bar{\tau}-a\}}^{\bar{\tau}} \beta(\bar{\xi}(a + \theta), \bar{S}) \bar{\mathcal{F}}(a + \theta) \int_{\bar{\tau}}^{a+\theta} \frac{\partial \mu}{\partial \xi}(\bar{\xi}(\sigma), \bar{S}) K(\sigma, \theta) d\sigma d\theta \\
 & + \chi_{[0, \bar{\tau}]}(a) \bar{b} \frac{\beta(\xi_{A+}, \bar{S}) \bar{\mathcal{F}}(\bar{\tau})}{g(\xi_{A-}, \bar{S})} K(\bar{\tau}, \bar{\tau} - a). \tag{4.31}
 \end{aligned}$$

$$\begin{aligned}
 k_{22}(a) = & -\bar{b} \int_0^\infty \left\{ \gamma(\bar{\xi}(a + \theta), \bar{S}) L(a + \theta, \theta) \right. \\
 & \left. + \frac{\partial \gamma}{\partial \xi}(\bar{\xi}(a + \theta), \bar{S}) K(a + \theta, \theta) \bar{\mathcal{F}}(a + \theta) \right\} d\theta \\
 & - \bar{b} \left( \frac{g(\xi_{A+}, \bar{S})}{g(\xi_{A-}, \bar{S})} - 1 \right) \int_{\max\{0, \bar{\tau}-a\}}^{\bar{\tau}} K(a + \theta, \theta) \frac{\partial \gamma}{\partial \xi}(\bar{\xi}(a + \theta), \bar{S}) \bar{\mathcal{F}}(a + \theta) d\theta \\
 & - \bar{b} \frac{\mu(\xi_{A-}, \bar{S}) - \mu(\xi_{A+}, \bar{S})}{g(\xi_{A-}, \bar{S})} \int_{\max\{0, \bar{\tau}-a\}}^{\bar{\tau}} \gamma(\bar{\xi}(a + \theta), \bar{S}) \bar{\mathcal{F}}(a + \theta) K(\bar{\tau}, \theta) d\theta \\
 & + \bar{b} \left( \frac{g(\xi_{A+}, \bar{S})}{g(\xi_{A-}, \bar{S})} - 1 \right) \int_{\max\{0, \bar{\tau}-a\}}^{\bar{\tau}} \gamma(\bar{\xi}(a + \theta), \bar{S}) \bar{\mathcal{F}}(a + \theta) \\
 & \times \int_{\bar{\tau}}^{a+\theta} \frac{\partial \mu}{\partial \xi}(\bar{\xi}(\sigma), \bar{S}) K(\sigma, \theta) d\sigma d\theta \\
 & - \chi_{[0, \bar{\tau}]}(a) \bar{b} \frac{(\gamma(\xi_{A+}, \bar{S}) - \gamma(\xi_{A-}, \bar{S})) \bar{\mathcal{F}}(\bar{\tau})}{g(\xi_{A-}, \bar{S})} K(\bar{\tau}, \bar{\tau} - a). \tag{4.32}
 \end{aligned}$$

We thank Philipp Getto and Julia Sánchez Sanz for discovering the mistake and pointing it out to us. We also thank Francesca Scarabel for checking the correctness of the formulas above. Moreover, we are very happy with the continuation of our work in [Breda et al. \(2015\)](#).

**References**

Breda D, Getto P, Sánchez Sanz J, Vermiglio R (2015) Computing the eigenvalues of realistic Daphnia models by pseudospectral methods. *SIAM J Sci Comput* 37:A2607–A2629