# THE IMPLEMENTATION OF THE MULTICRITERIA REFERENCE POINT OPTIMIZATION APPROACH TO THE HUNGARIAN REGIONAL INVESTMENT ALLOCATION MODEL

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## PREFACE

This paper reports the results of the implementation of the multicriteria optimization approach to the Hungarian Regional Investment Allocation Model, a component of IIASA's Food and Agriculture Program's (FAP) Hungarian Task 2 Case Study of the "Analysis of the Impacts of Technological Development on Production and the Environment".

The reference point approach of Wierzbicki (1979) has been used. Several types of objective functions to be optimized have been considered. Primary numerical results are presented. Description of the implemented packages and instructions are given. Suggestions for further research directions are stated.

# THE IMPLEMENTATION OF THE MULTICRITERIA REFERENCE POINT OPTIMIZATION APPROACH TO THE HUNGARIAN REGIONAL INVESTMENT ALLOCATION MODEL

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## INTRODUCTION

This paper is to report the contribution in implementing the IIASA/FAP Task 2 Case Study Hungarian Agriculture Investment Policy Model (Harnos 1981). It was prepared during the author's stay at IIASA with SDS in 1981.

The task included creation of the program package for multicriteria optimization approach for the given version of the model, the model's numerical properties investigation and evaluation of the reference point approach (Wierzbicki 1979), as well as its implementation (Lewandowski 1981) validity in such problems.

Since the computational investigation of the model was in the early stage of research and further work in this direction will be continued, the following requirements to be created for the package arose:

- -- package should provide the possibility of fairly easy model developing and changing,
- -- model expansion by increasing the number of regions and time periods under consideration should be guaranteed in a straightforward way,

-- package should be as easy as possible to work with both in manual sense as well as by supporting the decision-maker with some solution analysis features.

Furthermore, it appeared to be necessary to support the research with some suggestions how to handle the package and the model itself and how to utilize the advantages of the reference point (curve) approach in order to meet the model creator's expectations.

In Chapter 1 a brief introduction to the model is given. Details can be found in (Harnos 1981). Due to the different goal functions under consideration our two main approaches will be described in Chapter 2. Chapter 3 contains results of some selected runs which illustrate both the properties of our approaches and suggestions on exploring the reference point approach properties. In Chapter 4 a short description of the package is given. This chapter can serve as a brief user's manual for the actually existing package and is to be helpful for its further development due to the development of the model. Chapter 5 contains some concluding remarks.

The author hopes that his work will be a step forward in further developing the Hungarian Agriculture Investment Policy Model and will help to introduce the powerful and useful tools of multicriteria optimization to a broad field of their possible applications in systems analysis.

The author wishes to thank Istvan Valyi for his contribution during model generator implementing and Andrzej Lewandowski for the useful introduction to his program package for interactive linear multicriteria optimization.

## 1. THE MODEL

In the sequel the model version under consideration will be briefly presented. For the simplicity of the presentation all quantities and equations used will first be listed and then those used in any particular given case will be stated. Also note that the enumeration of the constraints equations given below is consequently used in the package.

*Indices:*

| | | |
|---|---|---|
| $k = 1,\dots,km$ | -- | index of the regions |
| $a = 1,\dots,am$ | -- | investments type index with $am = 3$ and<br>$a = 1$ for chemicals (fertilizers)<br>$a = 2$ for melioration<br>$a = 3$ for irrigation |
| $s = 1,\dots,sm$ | -- | land classes index with $sm = 3$ and<br>$s = 1$ for present state<br>$s = 2$ for the state after melioration<br>$s = 3$ for the state after melioration and reclamation |
| $n = 1,\dots,nm$ | -- | crops index with $nm = 6$ |
| $j = 1,\dots,jm$ | -- | groups of crops index with $jm = 2$ and<br>$I_1 = \{1,2,3\}$, $I_2 = \{4,5,6\}$ |
| $t = 1,\dots,tm$ | -- | time periods index |

*Variables:*

| | | |
|---|---|---|
| $u_{k,a}(t)$ | -- | magnitude of type a investments in region k in time period t, |
| $x_{k,s}(t)$ | -- | amount of class s land in region k in time period t, |
| $z_{k,s,n}(t)$ | -- | amount of class s land used in region k for production of crop n in time period t, |
| $y$ | -- | additional artificial variable (used only in the single-criterion approach, see §2). |

It is necessary to underline that for each k and s and for $t = 1$ $x_{k,s}(1)$ is a constant, *not* a variable as one may deduce from the above notation which was introduced for simplicity of presentation.

*Parameters:*

| | | |
|---|---|---|
| $b(t)$ | -- | total amount of investments available in time period t, |
| $\beta_a$ | -- | coefficient for defining the upper limit of type a investments in time period t as a percentage of the total amount of available investments b(t), |

| | | |
|---|---|---|
| $\alpha_a$ | -- | coefficient for defining the lower limit of type a investments in time period t as a part of the total amount of available investments b(t), |
| $k_j(t)$ | -- | lower limit of j-th group of crops production in time period t, |
| $h_{k,s,n}(t)$ | -- | expected yields of crop n on class s land in region k in time period t, |
| $p_{k,s,n}(t)$ | -- | cost of fertilizers (per unit of area) for production of crop n on the class s land in region k in time period t, |
| $c_k$ | -- | maximal area of arable land in region k, |
| $d_k$ | -- | maximal area of land suitable for irrigation in region k, |
| $1/J_{k,s}(t)$ | -- | reclamation cost per unit area of land of class s-1 (to class s) in region k during time period t, |
| $R_{k,s,n}(t)$ | -- | deterioration coefficient of class s land in region k due to the production of crop n during time period t, |
| e | -- | artificial coefficient (used only in the single-criterion approach as a lower limit of land used for the group of crops production), |
| $Y_j^0(t)$ | -- | reference j-th crops group yields trajectory. |

*Constraints:*

(1) $$\sum_{k=1}^{km} \sum_{a=1}^{am} u_{k,a}(t) \leq b(t) \quad ; \quad t = 1,\ldots,tm$$

(2) $$\sum_{k=1}^{km} u_{k,a}(t) \leq \beta_a b(t) \quad ; \quad t = 1,\ldots,tm \ , \quad a = 1,\ldots,am$$

(3) $$-\sum_{k=1}^{km} u_{k,a}(t) \leq -\alpha_a b(t) \quad ; \quad t = 1,\ldots,tm \;, \; a = 1,\ldots,am$$

(4) $$-\sum_{k=1}^{km} \sum_{s=1}^{sm} \sum_{n \in I_j} h_{k,s,n}(t) \cdot z_{k,s,n}(t) \leq -k_j(t) \quad ; \quad t = 1,\ldots,tm \;, \; j = 1,\ldots,jm$$

(5) $$\sum_{s=1}^{sm} \sum_{n=1}^{nm} p_{k,s,n}(t) \cdot z_{k,s,n}(t) \leq u_{k,3}(t) \quad ; \quad t = 1,\ldots,tm \;, \; k = 1,\ldots,km$$

(6) $$\sum_{s=1}^{3} x_{k,s}(t) \leq c_k \quad ; \quad t = 2,\ldots,tm \;, \; k = 1,\ldots,km$$

(7) $$x_{k,3}(t) \leq d_k \quad ; \quad t = 2,\ldots,tm \;, \; k = 1,\ldots,km$$

(8a) $$-\sum_{k=1}^{km} \sum_{s=1}^{sm} \sum_{n \in I_2} z_{k,s,n}(t) + e \cdot y \leq 0 \quad ; \quad t = 1,\ldots,tm$$

(8b) $$-\sum_{k=1}^{km} \sum_{s=1}^{sm} \sum_{n \in I_1} z_{k,s,n}(t) + e \cdot y \leq 0 \quad ; \quad t = 1,\ldots,tm$$

(9) $$\sum_{n=1}^{nm} z_{k,s,n}(t) - x_{k,s}(t) = 0 \quad ; \quad t = 1,\ldots,tm \;, \; s = 1,\ldots,sm \;, \; k = 1,\ldots,km$$

(10) $$x_{k,1}(t+1) - x_{k,1}(t) + \sum_{n=1}^{nm} R_{k,1,n}(t) \cdot z_{k,1,n}(t) - \sum_{n=1}^{nm} R_{k,2,n}(t) \cdot z_{k,2,n}(t) + J_{k,2}(t) \cdot u_{k,1}(t) = 0 \quad ; \quad t = 1,\ldots,tm-1 \;, \; k = 1,\ldots,km$$

(11)
$$x_{k,2}(t+1) - x_{k,2}(t) + \sum_{n=1}^{nm} R_{k,2,n}(t) \cdot z_{k,1,n}(t) - \sum_{n=1}^{nm} R_{k,3,n}(t) \cdot z_{k,3,n}(t) + J_{k,3}(t) \cdot u_{k,2}(t) - J_{k,2}(t) \cdot u_{k,1}(t) = 0 \quad ; \quad t = 1,\ldots,tm-1 \; , \; k = 1,\ldots,km$$

(12)
$$x_{k,3}(t+1) - x_{k,3}(t) + \sum_{n=1}^{nm} R_{k,3,n}(t) \cdot z_{k,3,n}(t) - J_{k,3}(t) \cdot u_{k,z}(t) = 0 \quad ; \quad t = 1,\ldots,tm-1 \; , \; k = 1,\ldots,km$$

(13a)
$$\sum_{k=1}^{km} \sum_{s=1}^{sm} \sum_{n \in I_1} z_{k,s,n}(t) + y = 1 \quad ; \quad t = 1,\ldots,tm$$

(13b)
$$\sum_{k=1}^{km} \sum_{s=1}^{sm} \sum_{n \in I_2} z_{k,s,n}(t) + y = 1 \quad ; \quad t = 1,\ldots,tm$$

(14)
$$x_{k,s}(t) \geq 0 \quad ; \quad t = 1,\ldots,tm \; , \; s = 1,\ldots,sm \; , \; k = 1,\ldots,km$$

$$u_{k,a}(t) \geq 0 \quad ; \quad t = 1,\ldots,tm \; , \; a = 1,\ldots,am \; , \; k = 1,\ldots,km$$

$$z_{k,s,n}(t) \geq 0 \quad ; \quad t = 1,\ldots,tm \; , \; n = 1,\ldots,nm \; , \; s = 1,\ldots,sm \; , \; k = 1,\ldots,km$$

$$y \geq 0$$

Inequalities (1), (2) and (3) express the global upper bound for investments and upper and lower bounds for different types of investments in each time period, respectively. Inequality (4) states the lower limit for each group of crops production while (5) points out that the total amount of fertilizers (chemicals) to be used in a region cannot exceed its upper bound due to the type 3 investments allocation. Inequalities (6) and (7) define the amount of arable land and land suitable for irrigation in each region, respectively. Equalities (8a), (8b), (13a) and (13b) are additional constraints used in one of the approaches (see §2). Equalities (10), (11) and (12) describe the dynamic changes of the quality of land due to the simultaneous process of land reclamation and deterioration. In variable (14) nonnegativity requirements are stated.

*Goals:*

(15) $$\Psi_j(z) = \frac{\sum_{k=1}^{km} \sum_{s=1}^{sm} \sum_{n \in I_j} \sum_{t=1}^{tm} h_{k,s,n}(t) \cdot z_{k,s,n}(t)}{\sum_{k=1}^{km} \sum_{s=1}^{sm} \sum_{n \in I_j} \sum_{t=1}^{tm} z_{k,s,n}(t)} \; ;$$

$$j = 1,\ldots,jm$$

(16) $$\Phi_{j,t}(z) = \frac{\sum_{k=1}^{km} \sum_{s=1}^{sm} \sum_{n \in I_j} h_{k,s,n}(t) \cdot z_{k,s,n}(t)}{\sum_{k=1}^{km} \sum_{s=1}^{sm} \sum_{n \in I_j} z_{k,s,n}(t)} \; ;$$

$$t = 1,\ldots,tm \quad , \quad j = 1,\ldots,jm$$

(17) $$F_j(z) = \sum_{k=1}^{km} \sum_{s=1}^{sm} \sum_{n \in I_j} \sum_{t=1}^{tm} h_{k,s,n}(t) \cdot z_{k,s,n}(t) \; ;$$

$$j = 1,\ldots,jm$$

$$f_{j,t}(z) = \sum_{k=1}^{km} \sum_{s=1}^{sm} \sum_{n \in I_j} h_{k,s,n}(t) \cdot z_{k,s,n}(t) \quad ; \tag{18}$$

$$t = 1,\ldots,tm \quad , \quad j = 1,\ldots,jm$$

Each of the goals $\Psi_j(z)$, $j = 1,\ldots,jm$ in (15) express the average j-th group of crops yield while $\Phi_{j,t}(z)$, $t = 1,\ldots,tm$, $j = 1,\ldots,jm$ in (16) stands for the actual j-th group of crops yield in a given time period t. Goal functions given in (17) and (18) represent the average production and the j-th group of crops production in a given time period t, respectively. Note that (15) and (16) produce nonlinear (hyperbolic/fractional) while (17) and (18) linear functions to be optimized. All goals defined in (15)-(18) are to be maximized. It was assumed that the decision-maker will express his preferences by setting for each set of objectives the reference points (curves, trajectories) which will describe his aspiration levels.

## 2. APPROACHES

While considering the possible computational approaches, some of the available software limitations had to be taken into account. Choosing an approach in the case of linear goal functions was straightforward since an excellent package for interactive linear multicriteria reference point optimization (Lewandowski 1981) is available at IIASA. Unfortunately, it is not the case for nonlinear problems. The choice was restricted to the following alternatives only:

1. create a package for interactive nonlinear multicriteria optimization similar to this for the linear case,
2. provide nonlinear (noninteractive) multicriteria optimization using MINOS/Augmented directly,
3. solve a sequence of linear multicriteria problems due to a sequence of nonlinear goal functions linearizations,
4. consider one of the nonlinear goals only, transform the fractional programming problem which arises into an equivalent linear programming problem (see Appendix A) setting some additional linear constraints (suggested by the model's creators) to control remaining nonlinear goals.

Unfortunately, realization of the first alternative, which is the best solution since it provides an interactive mode of work, would exceed the time limit of the author's stay with IIASA. Nevertheless, further attempts should be made in this direction. The second alternative leads to a nonlinear programming problem solving which would require a significant computational effort increase (with comparison to the linear programming case) since about 75% of variables would have to be declared as the nonlinear ones in MINOS. The third alternative seemed to be a good problem solution since it preserves the interactive mode of work property and linearity of the problems to solve, but the simple version of this approach implementation (with linearization of the objectives set up in a large loop: file *"sert"* ÷ file *"mpsxfil"*, (see Chapter 4 and Figure 8 for details) happened to be inefficient while the more sophisticated version (one with linearization in a small loop: "fil-6" ÷ "fil-9", see Chapter 4) would have met the same time limitations as the first alternative. For reasons mentioned above the following approaches were finally implemented.

Approach IA:

Goal function $\Psi_1(z)$ defined in (15) with j = 1 is to be maximized subject to constraints (1)-(7), (9)-(12), (14). This fractional programming problem is transformed to a linear programming problem (see Appendix A). This transformation adds constraints (13a). To prevent ignoring the importance of goal function $\Psi_2(z)$ constraint (7a) is added.

Approach IB:

Goal function $\Psi_1(z)$ defined in (15) with j = 2 is to be maximized subject to constraints (1)-(7), (9)-(12), (14). Transformation adds (13b) while (7b) is due to taking goal $\Psi_1(z)$ into consideration. This approach was suggested by Harnos (1981) and both IA and IB are implemented in package P1 (see Chapter 4).

The following approach is based on the reference point approach of Wierzbicki (1979) and its implementation for the linear multicriteria case due to Lewandowski (1981). See Appendix B for brief description, and Lewandowski (1981) for details.

Approach IIA:

Goal functions $F_j(z)$, $j = 1,\ldots,jm$ defined in (17) with constraints (1)-(7), (9)-(12), (14) are under consideration. The linear penalty scalarizing function is optimized, the reference point is being changed in an interactive mode.

Approach IIB:

Objectives $f_{j,t}(z)$, $t = 1,\ldots,tm$, $j = 1,\ldots,jm$ defined in (18) are used together with constraint (1)-(7), (9)-(12), (14) again.

Both IIA and IIB are implemented in package P2 (see Chapter 4).

## 3. PRELIMINARY NUMERICAL RESULTS

In this chapter the results of some selected runs based on the presently available data will be presented. Since it was not the aim of our computational experiments to answer the question whether or not and to what extent the actual version of the model reflects the reality of Hungarian agriculture we will confine ourselves to a brief discussion of the properties of our approaches and numerical features of the model itself. Also, we will make an attempt to illustrate a simple example how the features of the reference point approach can be utilized in order to bring the solution of the problem close to the aspiration level of the decision-maker.

Figure 1 displays the computational results of Approach IA and Approach IB. Observe that if the yield of the first crop group is maximized then, despite taking the second objective into account by setting constraint (8a)--land use limit for production of the second crop group--the second crop group yield is relatively low. The same phenomenon can be observed in the case of Approach IB when the yield of the second crop group is maximized. Moreover, the situation in crop groups production is unbalanced too. The production level of the crop group whose yield is actually directly maximized, is low in both cases. So for each group of crops we get either high yields but low production level, or, low yields with high level of production.

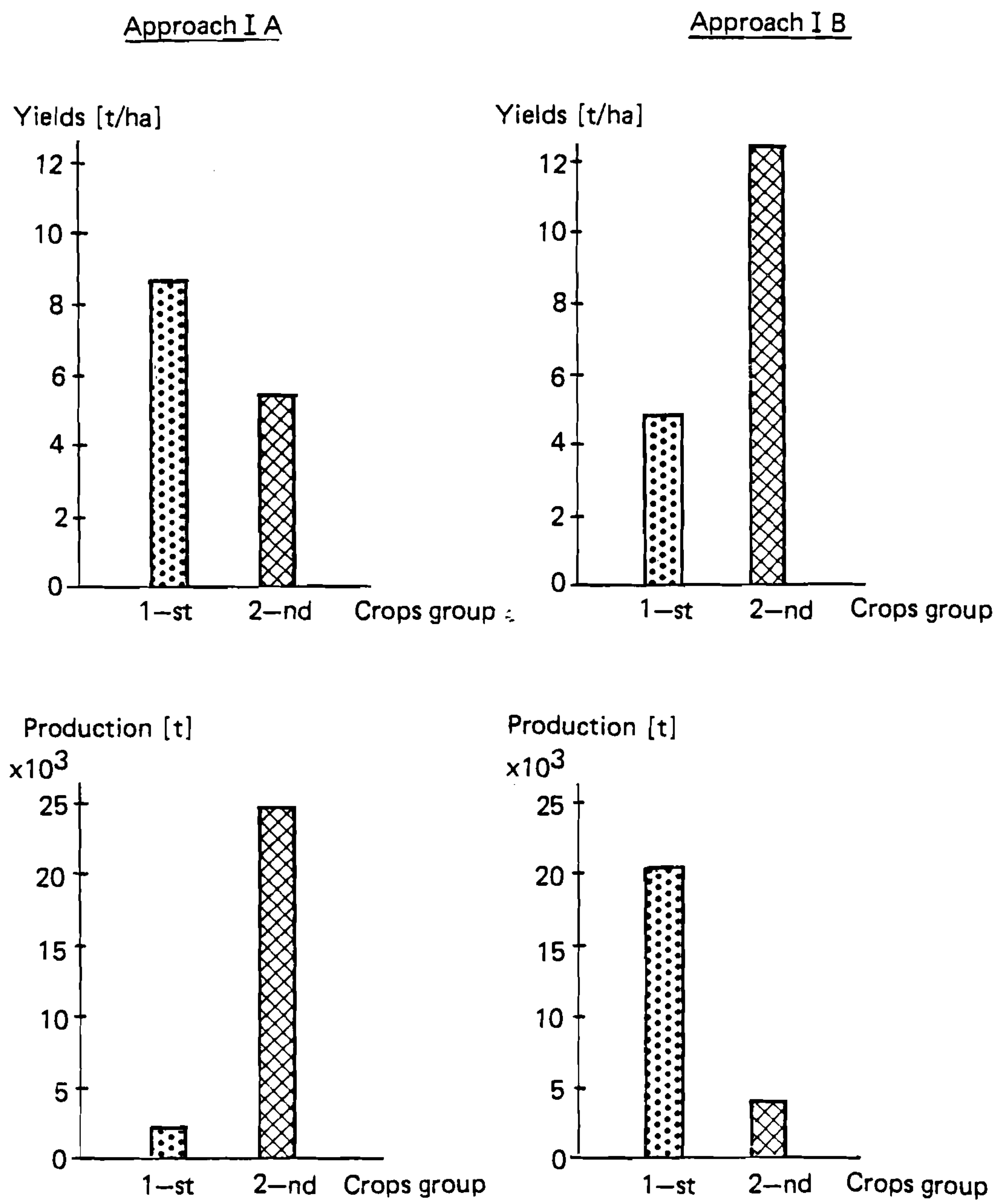
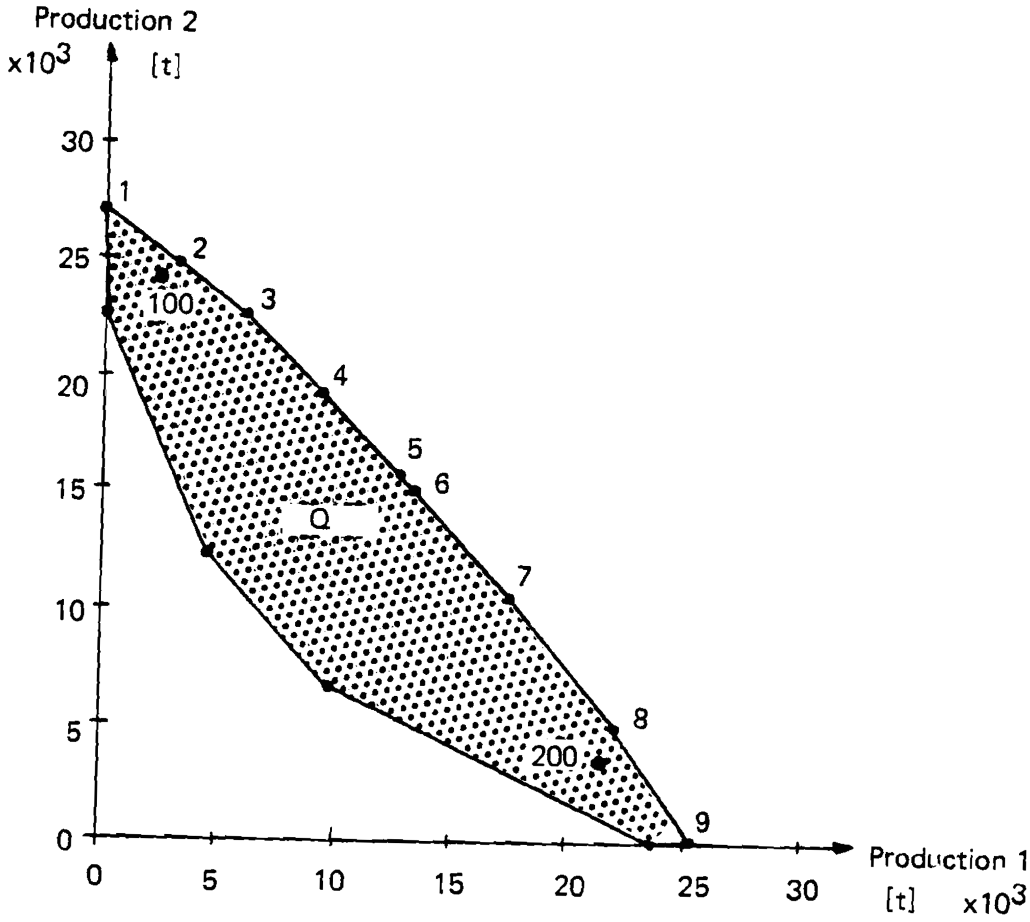

Approach I A
Approach I B
Yields [t/ha]
12
10
8
6
4
2
0
1–st
2–nd
Crops group
Yields [t/ha]
12
10
8
6
4
2
0
1–st
2–nd
Crops group
Production [t]
x10^3
25
20
15
10
5
0
1–st
2–nd
Crops group
Production [t]
x10^3
25
20
15
10
5
0
1–st
2–nd
Crops group

Figure 1.

This is due to the fact that for the production of the crops group whose yield is actually directly maximized the model assigns pieces of highest quality land and restricts the production of this crop group to the best land only in order to achieve high yields. Of course, the more balanced situation can be obtained by setting some additional constraints for each of the crop group production levels and/or the land use structure. But since the second approach does not cause such troubles, the feature of Approach I described above must be its disadvantage.

Now let us discuss the results obtained by Approach IIA. Note that objectives defined by (17) with jm = 2 are under consideration. They are presented in Figures 2, 3 and 4. Figure 2 gives an impression of the shape of the set Q of the attainable solutions (see Appendix B for definition) in the space of objectives (production levels of the crop groups). Observe how easily the whole set of Pareto solutions can be scanned by using the reference points. This would not be the case with some other methods of multicriteria optimization, like the method of weighing coefficients, for example. Depending on the requirements, scanning may be uniform or some more attention may be given to certain subsets of the Pareto set. Figure 2, as well as Figures 3 and 4, contains the results of Approach IIA with different reference points numbered from 1 to 9 and also the results of Approach IA and B numbered by 100 and 200, respectively. Figure 2 indicates that the results of Approach IA and B, being Pareto-optimal in the objective space of yields, are not Pareto-optimal in the objective space of production. It shows that Approaches I and II are not equivalent in this sense and points out that for this reason a careful goal selection must be made in the pre-optimization stage of research. Figures 3 and 4 present production levels and yields of two crop groups obtained by Approach IA as functions of time, respectively. Note that the uniform distribution in the space of objectives--crop groups production summed up overtime--do not correspond to a uniform distribution of the production trajectories (Figure 3). Optimization of aggregated functions simply cannot extort a desirable behaviour of the disaggregated quantities. Therefore if one is



Production 2
x10^3
[t]
30
25
20
15
10
5
0
1
2
3
4
5
6
7
8
9
100
Q
200
0
5
10
15
20
25
30
Production 1
[t]
x10^3


Figure 2.

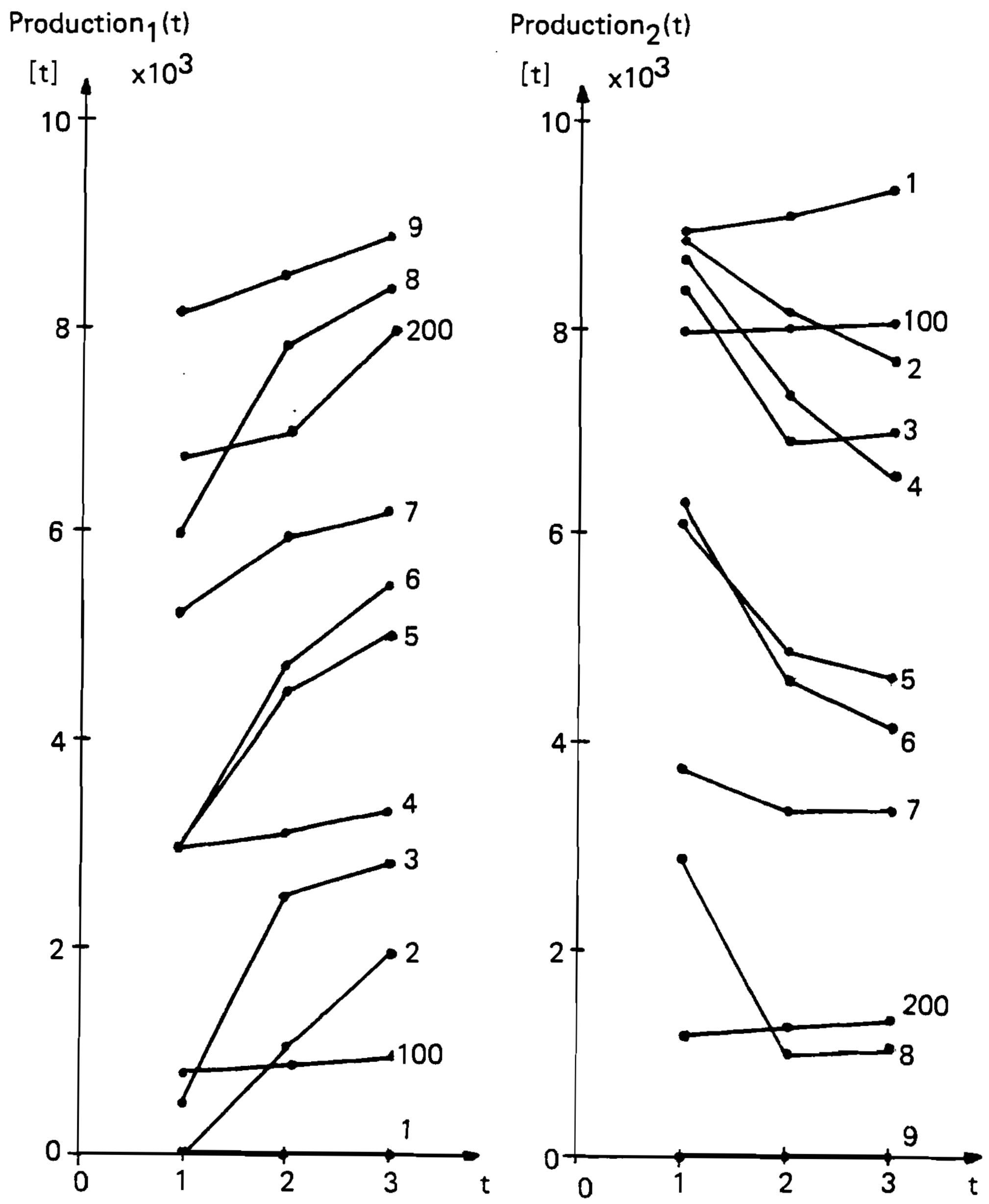
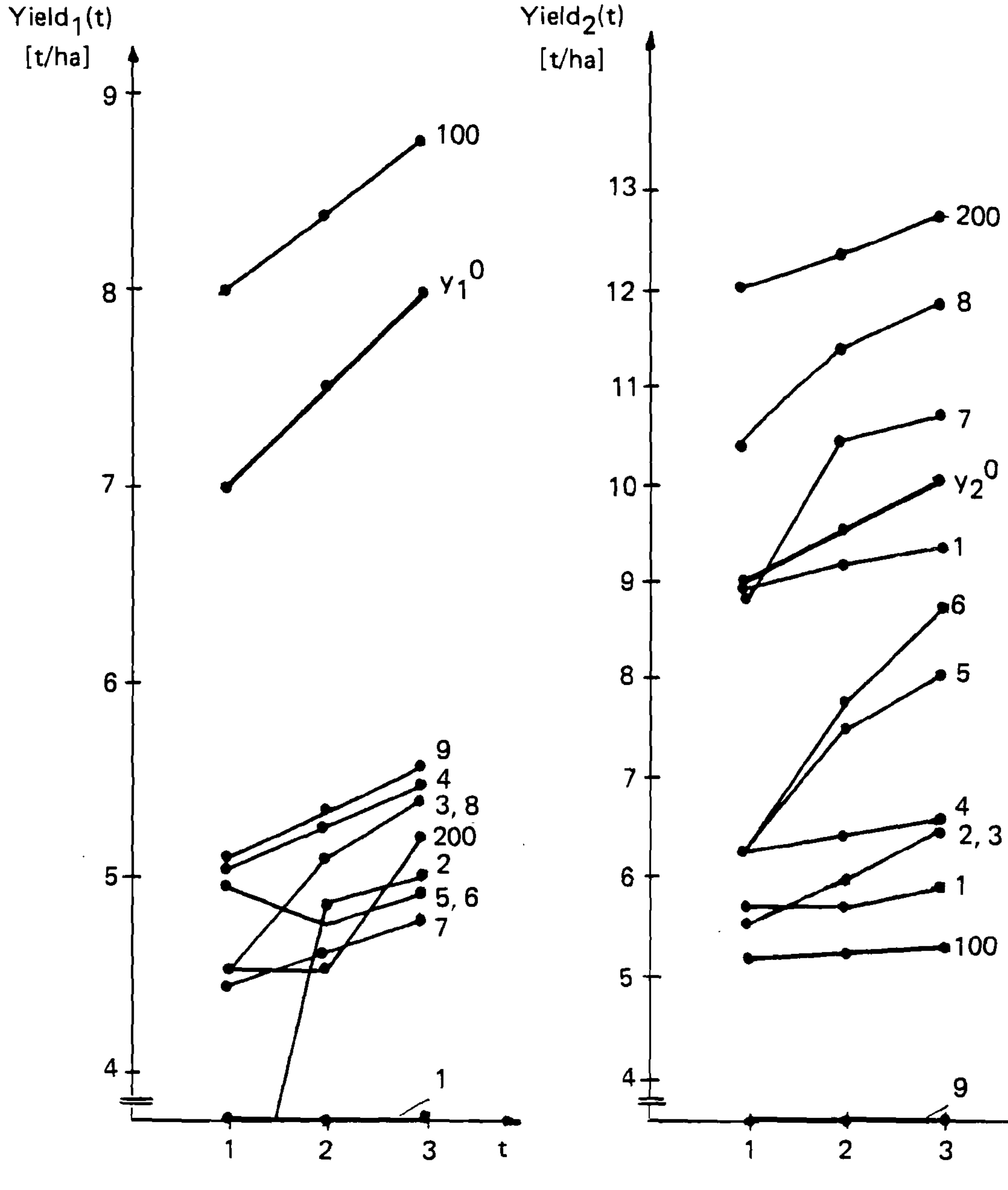

Production1(t)
[t] x10^3
10
8
6
4
2
0
0 1 2 3 t
9
8
200
7
6
5
4
3
2
100
1
Production2(t)
[t] x10^3
10
8
6
4
2
0
0 1 2 3 t
1
100
2
3
4
5
6
7
200
8
9

Figure 3.

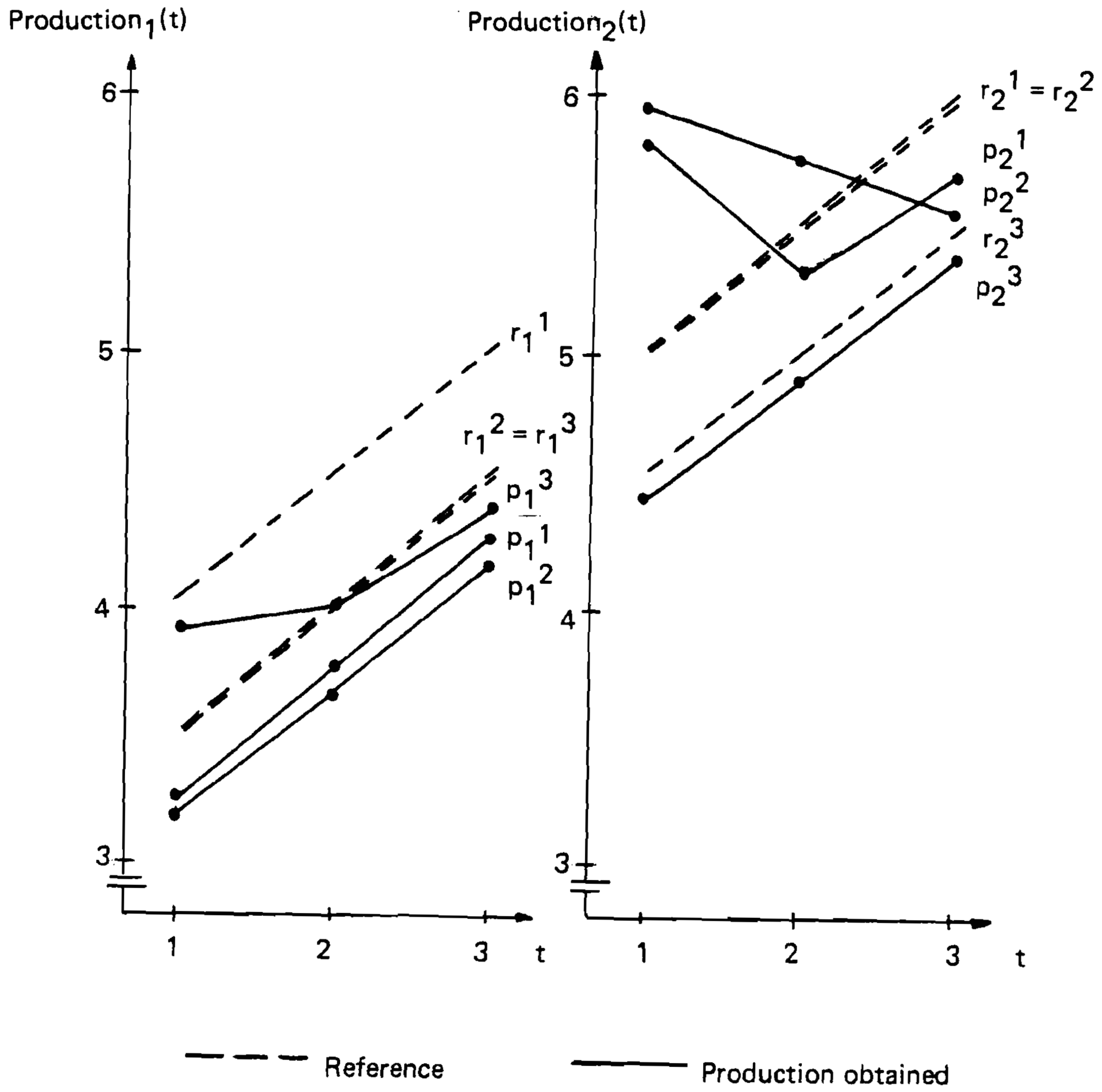


Yield$_1$(t)
[t/ha]
9
8
7
6
5
4
100
$y_1^0$
9
4
3, 8
200
2
5, 6
7
1
1
2
3
t
Yield$_2$(t)
[t/ha]
13
12
11
10
9
8
7
6
5
4
200
8
7
$y_2^0$
1
6
5
4
2, 3
1
100
9
1
2
3
t


Figure 4.

not only interested in the global effects but also in providing some demanded trends of the system, then Approach IIA is not the right way to proceed. Moreover, this approach does not enable to control the yields while the productions are being optimized. It is seen if one compares the distributions of productions trajectories in Figure 3 and yields trajectories in Figure 4. (Figure 4 contains also the reference yields $Y^0_{100}$ and $Y^0_{200}$ and obtained yields trajectories denoted by 100 and 200 from Approach IA and B, respectively, for comparison.

Let us discuss the results of Approach IIB. An attempt to show how, by changing the reference point (trajectory), the decision-maker's aspiration levels for the objective function can be reached as closely as possible. Suppose that the decision-maker expresses his aspiration levels by setting the reference trajectories for production of each crop groups. They are represented in Figure 5 by broken lines denoted by $r^1_j$, $j = 1,2$. As a result of multicriteria reference point optimization production trajectories $p^1_j$ are obtained. Suppose that the decision-maker is not satisfied with this solution because of the monotonic production $p^1_2$. Now, still having the reference trajectories $r^1_j$, $j = 1,2$ in mind as his aspiration level he can change the current reference trajectories in order to obtain more satisfactory production trajectories. It is a rule that to extort the desired shape of the solution trajectory one should bring the reference point closer to the boundary of the feasible set. It means that the reference point should be realistic. Let us see what happens if the decision-maker lowers the reference trajectory for the first crop group production only, i.e., if he sets $r^2_j$, $j = 1,2$ with $r^2_2 = r^2_1$. As one can see in Figure 5, the production of the first group of crops $p^2_1$ lowered is still preserving the demanded shape of the aspiration level $r^1_1$ while the second crop group production $p^2_2$ is now decreasing in time. Now on the one hand, the decision-maker wants to bring the first crop group production trajectory to a higher level, since it is far below his aspiration level, and on the other hand, he wants the second crop group production trajectory to increase monotonically what is possible only on its lower level. So the current reference



Production1(t)
6
5
4
3
1
2
3
t
r11
r12 = r13
p13
p11
p12
Production2(t)
6
5
4
3
1
2
3
t
r21 = r22
p21
p22
r23
p23
Reference
Production obtained


Figure 5.

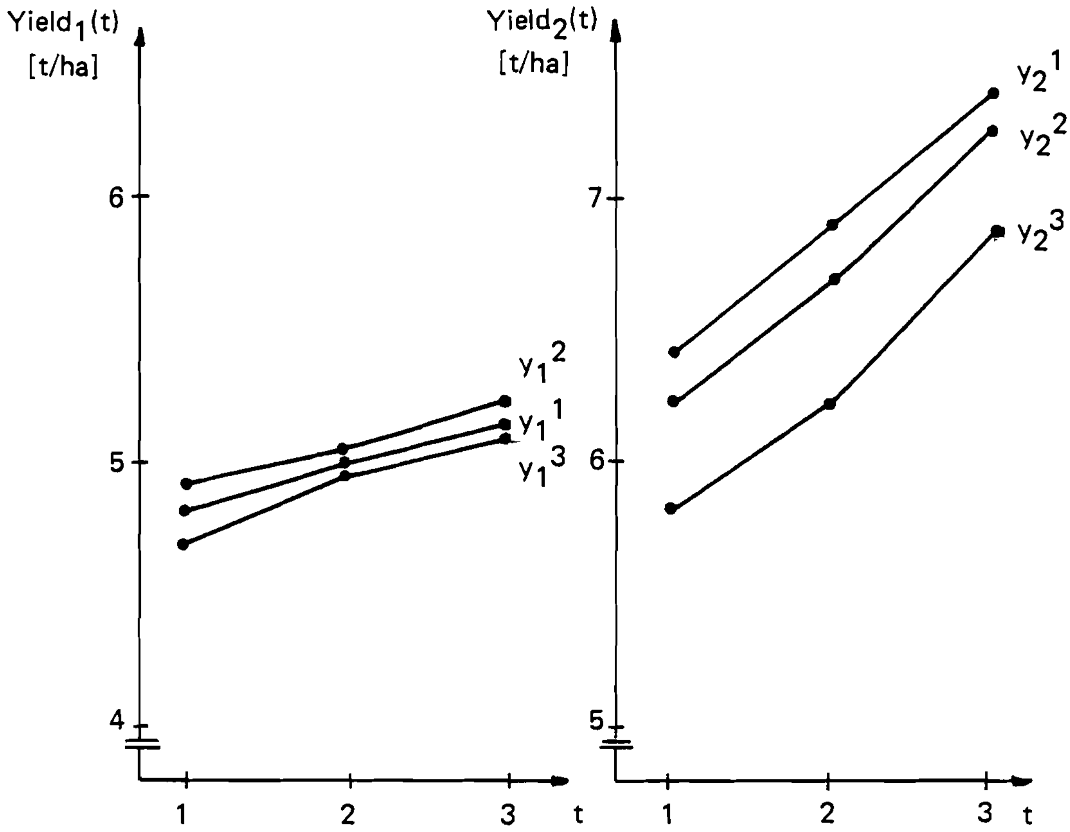
trajectories $r_j^3$, j = 1,2 are set and new solutions $p_j^3$, j = 1,2 are obtained. This process of changing the reference curves is to be continued until the decision-maker is satisfied. It is worth mentioning that the shape of the reference curves must not be necessarily that of the aspiration level trajectories. Any modification of the references can be made if only the solution obtained satisfies the decision-maker. However, it is the author's advice to start with the references simply shifted up or down the aspiration level trajectories as in the example described above. If improvement cannot be achieved in this way anymore, then one can start with more sophisticated changing of references.

Another useful feature of Approach IIB (to this very model at least) can be observed in Figure 6 which presents the yields trajectories $Y_j^i$, j = 1,2, i = 1,2,3 corresponding with the production trajectories $p_j^i$ and references $r_j^i$, j = 1,2, i = 1,2,3 from Figure 5. Note that the levels of yields follow the levels of productions, specially when references are close to the set of Pareto solutions, that means, that by changing the production reference trajectories and deserving the resulting production and yield curves one is able to choose not only the satisfactory production trajectories but also, to some extent, the yields trajectories. This of course is an indirect yield control. The direct one, however, as it was mentioned in Chapter 2 in the discussion of the possible approaches for nonlinear goals, requires the creation of a new package similar to that of Lewandowski (1981), but for the nonlinear case. Therefore, it seems to be a useful feature of Approach IIB that using existing software dealing with the linear production goals one has an inconvenient but still a possibility to control the nonlinear yield trajectories to a certain extent.

## 4. DESCRIPTION OF THE PACKAGES

This chapter contains a brief description of implemented packages due to the model's expansion and changing.

Package 1 covers Approach IA and B while Package 2 is the Approach IIA and B implementation. Flow-diagrams of the packages



Yield₁(t)
[t/ha]
6
5
4
1
2
3
t
$y_1^2$
$y_1^1$
$y_1^3$
Yield₂(t)
[t/ha]
7
6
5
1
2
3
t
$y_2^1$
$y_2^2$
$y_2^3$


Figure 6.

are presented in Figures 7 and 8. Details of the commands are exhibited in Appendix C. Appendix D presents the primary input data used as well as the rules of the whole input data set generation.

Let us briefly discuss Package 1 which is an implementation of Approach IA and B. File *mpsx1.f* is the most important part of the package. By typing on the terminal *dom1* command which is as all commands in both packages, the name of an executable shell-file (see Appendix C) the primary input data contained in *dat1* file are read by a Fortran-program in mpsx1.f file. First, the whole set of the model's input data is generated (see Appendix D) and next it is written into a temporary data file *tdata* and into a file *spraw* together with some other calculated model parameters like number of rows, columns, etc. File *spraw* is to enable checking the model's input data before the optimization run. Later on the standard MINOS input file *fil-9* is generated. This part of the model's generator contains comments which indicates the number of the constraint (see Chapter 1) actually under consideration. So if any changes of one of the constraints are to be made it requires a change in only a few, strictly specified by the constraint number in comments lines of program. This feature seems to be important since further development of the packages as well as the model itself will be continued only with limited contribution by the author. At the end, the program contained in *mpsx1.f* automatically generates an appropriate MINOS specification file *specs*. When the generation of the model is completed the command *aminos* provides an optimization run, results of which can be found in the MINOS standard output file *fil-6*. After the solution is obtained, commands sequence *dor;dop1* has to be executed. The command dor causes the run of the program contained in file *zas.f*. This program simply looks for the first line of solution printout in file *fil-6*. Then the program contained in *zaw1.f* reads the solution from *fil-6* values of all model's coefficients from the temporary data file *tdata* and produces file *sert* in which final results of the run are printed in an easy-to-read-and-analyze form. In particular the left-hand constraints and objectives are recalculated. Comparing these results with

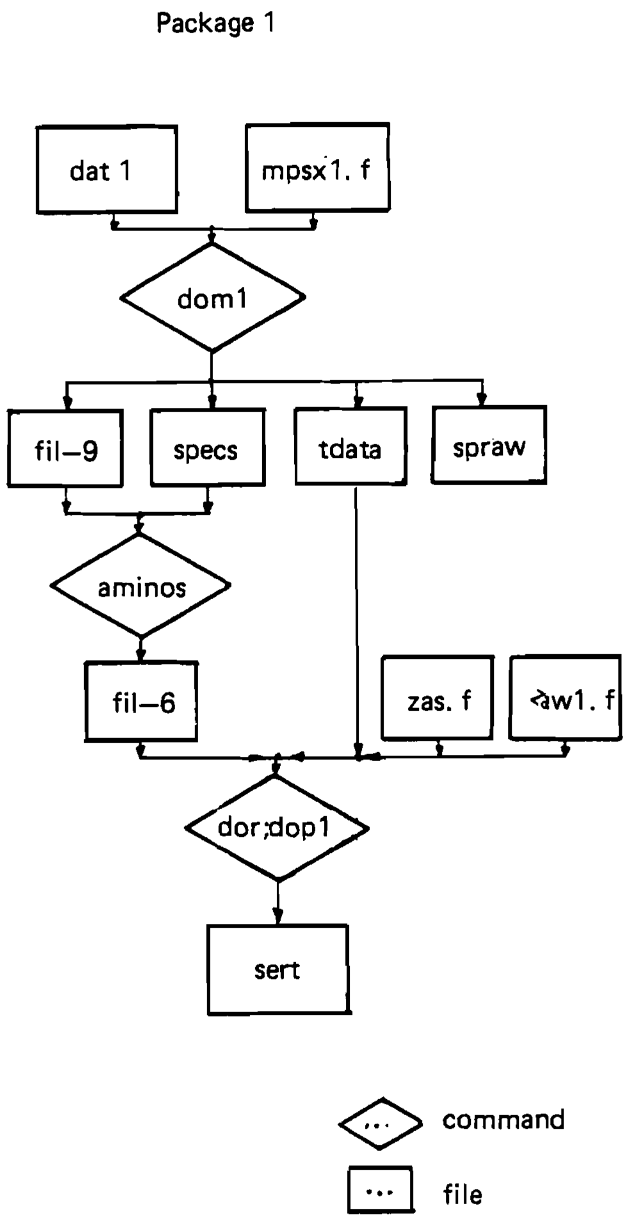

Package 1
dat 1
mpsx1. f
dom1
fil—9
specs
tdata
spraw
aminos
fil—6
zas. f
aw1. f
dor;dop1
sert
command
file

Figure 7.

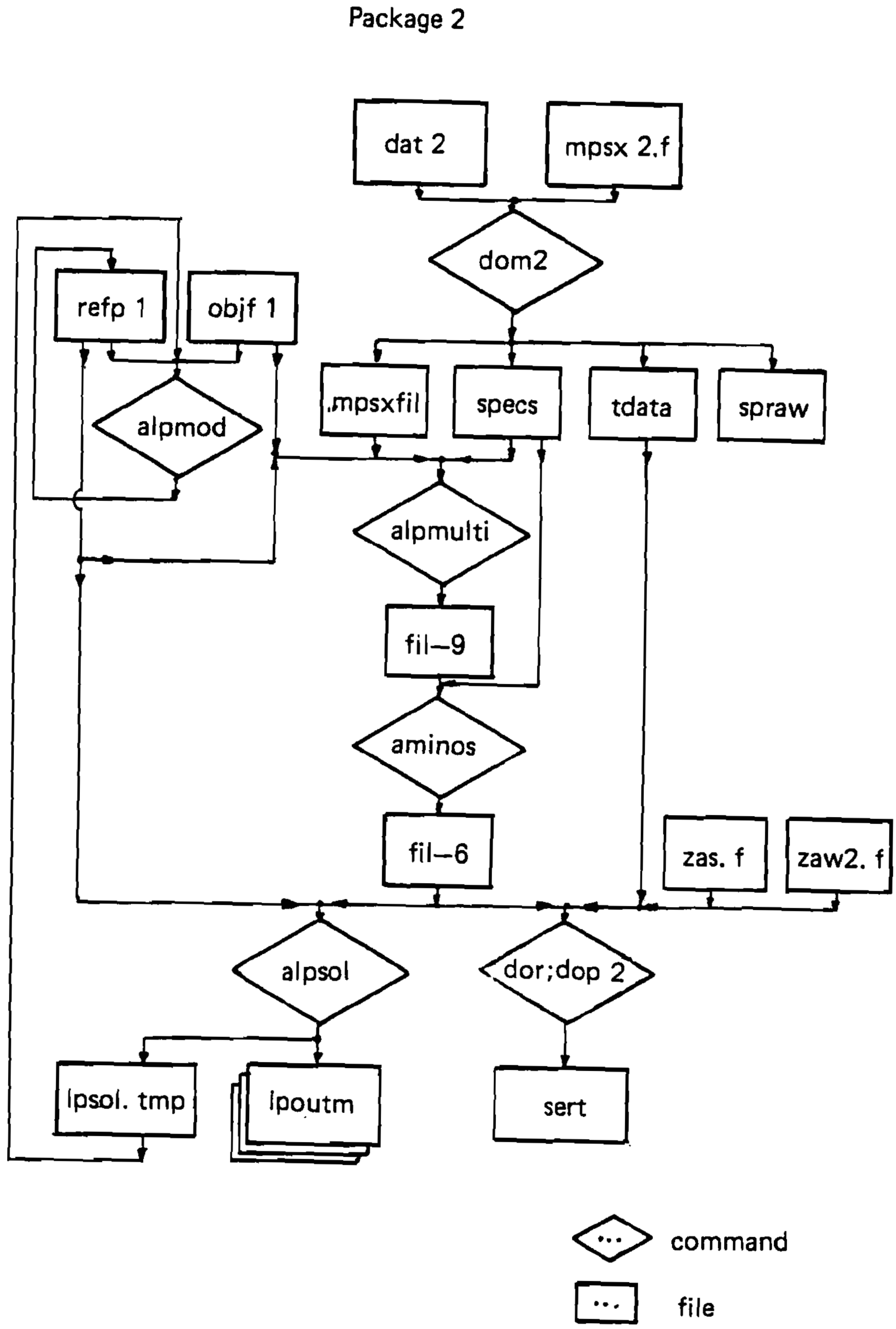

Package 2
dat 2
mpsx 2.f
dom2
refp 1
objf 1
alpmod
.mpsxfil
specs
tdata
spraw
alpmulti
fil–9
aminos
fil–6
zas. f
zaw2. f
alpsol
dor;dop 2
lpsol. tmp
lpoutm
sert
command
file

Figure 8.

the values in *fil-6* one easily check if the generation of the model was correct. Generation of file *sert* completes the run. For the following run one usually has to change the data and/or the model. This is an advantage of the packages that any changes are fairly easy to make. Instructions for data changing are in Appendix D. Right-hand changes can be made by using usual MINOS facilities. Major model improvements require some changes in files *mpsx1.f* and *zaw1.f*. See Appendix E for a short guide. To switch from Approach IA to B or vice versa one needs to change the value of one parameter in file dat1--see Appendix D.

Now let us briefly discuss the structure of Package 2. Its flow-diagram is presented in Figure 8. The first step of the run, i.e., the execution of the *dom2* command is just the same as in Package 1 with *dom1* command. The only difference is, that, instead of the standard MINOS input file *fil-9* the input file for the package for linear multicriteria reference point optimization by Lewandowski (1981), *mpsxfil*, is produced in this step. As a next step, the command *alpmod* may be executed if one is willing to change the reference point. While executing the commands alpmod and alpmulti some extra typing will be required by the user but we will not go into details here since it is enough to follow the instructions that will appear on the screen or see Lewandowski (1981) for details. The alpmod command deals with the *refp1* and *objfil* files which contain the current reference point and the names of objective functions, respectively. These two files have to be produced by the user. Again, see Lewandowski (1981) for instructions. Additional requirement for the *objfil*, arising from the present version of the model generator in *mpsx2.f* file, is that the names of objectives must consist of the word objec followed by a three-digit number (*objec001*, *objec012*, *objec123*, for example). So the number of objectives is limited to one thousand. In the next step, the *alpmulti* command has to be executed. Its input files are *refp1*, *objf1*, *mpsxfil* and *specs* and it produces the MINOS standard input file *fil-9*. Execution of *aminos* and *dor;dop2* is the same as in Package 1. Additionally, before or after *dor;dop2* the command *alpsol* can be executed. It produces the file *lpsol.tmp* in which the values of the objectives in the solution, the current reference point and other information are saved. The contents of this file also appears on the screen. File *lpoutm*, also produced by *alpsol* command, contains the history of the runs, i.e., all *lpsol.tmp* files produced up to the current moment. See Lewandowski (1981) for more details about his package and Appendix D for instructions how to switch from Approach IIA to Approach IIB.

## CONCLUSIONS

The results of the implementation of the multicriteria optimization to the Hungarian Regional Investment Allocation Model have been reported briefly. One type of nonlinear and two types of linear objective functions have been finally taken into consideration. Two program packages which cover four approaches due to these different types of objectives to be optimized were created and tested. The features, advantages and disadvantages of the approaches have been discussed. As a most convenient presently available method to treat the model, Approach IIB is suggested, in which the production of each crop group in each period of time are the objective functions to be simultaneously maximized with respect to the production reference trajectories.

The multicriteria reference point optimization approach (Wierzbicki 1979) has proved again to be a useful and powerful tool for dealing with problems in which many objectives have to be taken into consideration. Some suggestions as how to utilize its features have been given for the users. The package for linear multicriteria reference point optimization (Lewandowski 1981) has been found to be very convenient for the user. The necessity to create a package for the nonlinear multicriteria optimization, having features similar to those of the linear case, has been indicated. This work could not actually be done by the author because of his short stay at IIASA.

It is the author's hope that his contribution will help develop the final model of investment allocation in Hungarian agriculture and introduce the useful methods for multicriteria optimization into the new fields of their applications.

APPENDIX A:

Problem I:

$$\frac{c^T x + \alpha}{d^T x + \beta} \rightarrow \max$$

s.t.

$$Ax \leq b$$

$$x \geq 0$$

Problem II:

$$c^T y + \alpha \cdot t \rightarrow \max$$

s.t.

$$Ay - bt \leq 0$$

$$d^T y + \beta t = 1$$

$$y \geq 0$$

$$t \geq 0$$

If $\hat{x}$ solves Problem I, then $(\hat{t}\hat{x},\hat{t})$ solves Problem II, where $\hat{t} = (d^T\hat{x} + \beta)^{-1}$.

If $(\bar{t},\bar{y})$ solves Problem II, then $x = \frac{1}{\bar{t}} \cdot \bar{y}$ solves Problem I.

APPENDIX B:

[Lewandowski 1981]

Let A be in $R^{m\times n}$, C in $R^{p\times n}$ and b in $R^m$ and consider the following linear multicriteria optimization problem (LMOP)

$$\begin{aligned} & Cx = q \rightarrow \max \\ \text{s.t.}\quad & Ax = b \\ & x \geq 0 \end{aligned}$$

Def. 1 An objective vector value $q = \bar{q}$ is attainable if there exists a feasible x for which $Cx = \bar{q}$.

Def. 2 An attainable point $\bar{q}$ is Pareto-optimal if there is no attainable point q such that $q \geq \bar{q}$ with a strict inequality for at least one component.

Denote $w = q - \tilde{q}$ where $\tilde{q} = [\tilde{q}_1, \ldots, \tilde{q}_p]$ is a reference point (attainable or not) and introduce one of the forms of the penalty scalarizing function which results in a linear programming formulation

$$s(w) = -\min\left\{\rho \cdot \min_{i=1,p} w_i \, , \, \sum_{i=1}^{p} w_i\right\} - \varepsilon \cdot w$$

Further denote by $W \equiv \{w : -w + Cx = q,\ Ax = b,\ x \geq 0\}$ the feasible set of variables vector w and let

$$z = \max_{i=1,p} (-\rho w_i - \varepsilon w) \quad , \quad z \in R \quad , \quad y = z + \varepsilon w \quad , \quad y \in R$$

then

$$s(w,y) = \min_{\substack{w \in W \\ y \in R}} \left\{ y - \varepsilon w : -y - \rho w_i \leq 0 \text{ for all } i,\ -y - \sum_{i=1}^{p} w_i \leq 0 \right\} .$$

If $(\hat{y}, \hat{w}, \hat{x})$ is a minimizer of the penalty scalarizing function then $\tilde{q} = C\hat{x}$ is a Pareto-optimal solution of LMOP. (See Lewandowski (1981), Kallio et al. (1980) and Wierzbicki (1979) for details.) The following figure is an illustration of the approach.

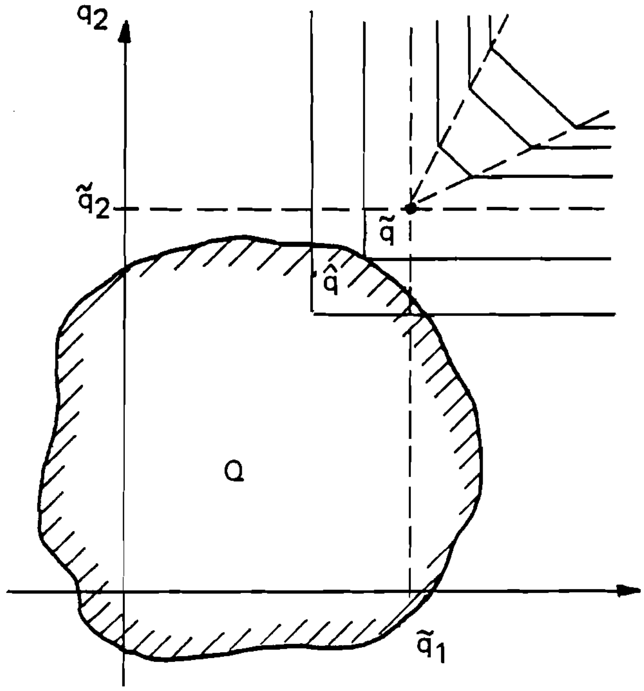



Figure 8a

APPENDIX C:

Commands

```
dom1:      f77  mpsx1.f >& errs1  -0  mpsx1.out
           mpsx1.out  6=+fil-9  4=+spraw  5=+dat1  8=tdata
           7=+specs

dom1-a:    mpsx1.out  6=+fil-9  4=+spraw  5=+dat1  8=+tdata
           7=+specs

dom2:      f77  mpsx2.f >& errs2  -0  mpsx2.out
           mpsx2.out  6=+mpsxfil  4=+spraw  5=+dat2  8=tdata
           7=+specs

dom2-a:    mpsx2.out  6=+mpsxfil  4=+spraw  5=+dat2  8=tdata
           7=+specs

alpmod:    lpmod  objf1  refp1

alpmulti:  lpmulti  mpsxfil  objf1  refp1

aminos:    minos  5=specs

alpsol:    lpsol  objf1  refp1

dor:       f77  zas.f >& errzas  -0  zas.out
           zas.out  4=+rnu111  7=fil-6
```

```
dor-a:     zas.out  4=+rnu111  7=fil-6

dop1:      f77  zaw1.f >& errzaw  -0 zaw1.out
           zaw1.out  4=+sert  7=fil-6  9=rnu111  8=tdata

dop1-a:    zaw1.out  4=+sert  7=fil-6  9=rnu111  8=tdata

dop2:      f77  zaw2.f >& errzaw  -0  zaw2.out
           zaw2.out  4=+sert  7=fil-6  9=rnu111  8=tdata

dop2-a     zaw2.out  4=+sert  7=fil-6  9=rnu111  8=tdata
```

APPENDIX D:

The data file *dat1* is presented in Figure 9. It seems to be easily readable because of the comments lines. All names of parameters are either identical or similar to these from model description in Chapter 1. They are identical with those used inside the whole package. To avoid any mistakes let us just point out that all parameters with indices exist as matrices inside the package, for example:

| in model description (Chapter 1) | inside the package |
| --- | --- |
| $Y_j^0(t)$ | yy(ijm,itm) |
| $J_{k,s}(t)$ | ajot(ikm,ism,itm) |
| $K_j(t)$ | ak(ijm,itm) |
| $\beta_a$ | beta(iam) |

Maximal values of indices have a prefix i inside the package. Now let us discuss how the complete model's input data set is generated from the primary data presented in Figure 9. Coefficients h and r values are given for t = 1. Denote

```
   3 , 3 , 3 , 2 , 6 , 2 , 2  , 101 ,
c itm iam ism ijm inm ikm nobj lprob
  3.e6 ,0.15e6,
c b(1)  delb
  0.5 ,0.33333,0.66666,
c beta(1), (2) ,  (3)
  0.1, 0.1, 0.4,
c alpha(1),(2),(3)
  0.7e1, 0.5e0,
c yy(1,1) delyy
  0.9e1, 0.5e0
c yy(2,1) delyy
  800., 1., 0.8,
c e    ro(1) ro(2)
  20000., 1.05,
c ajot(1,2,1) prajot
  40000., 1.05,
c ajot(1,3,1) prajot
  15000., 1.05,
c ajot(2,2,1) prajot
  50000., 1.05,
c ajot(2,3,1) prajot
  1000., 600.,
c c(1)   c(2)
  250., 200.,
c d(1)  d(2)
  2000., 1.05,
c ak(1,1) prak
  2500., 1.05,
c ak(2,1) prak
  700., 200., 100., 400., 100., 100.,
c x(ik,is,1),is=1,ism),ik=1,ikm)
  4. , 5. ,6. ,5. ,6.5 , 8. ,
  5.,6.,7.,6.,8.,10.,
  6.,7.,8.,9.,11.,12.,
  4.5,5.5,6.5,4.5,6.,7.,
  5.5,6.5,7.5,5.5,7.,8.,
  6.,7.,8.,6.5,8.,9.,
c   h(ik,is,in,1),in=1,inm),is=1,inm),ik=1,ikm)
  1000.,1500.,2000.,1200.,1800.,2400.,
c   p(1,1,in,1),in=1,inm)
 0., 0.1, 0.15, 0., 015. 0.2,
 0.2, 0.2, 0.3, 0.1, 0.15, 0.2,
 0.2, 0.25, 0.3, 0.15, 0.2, 0.25,
 0., 0.1, 0.1, 0., 0.2, 0.2,
 0.1, 0.15, 0.2, 0.15, 0.25, 0.3,
 0.15, 0.2, 0.25, 0.2, 0.3, 0.35,
c r(ik,is,in,1),in=1,inm),is=1,ism),ik=1,ikm)
```

Figure 9. File dat1

$$H_1(t) = h_{k,s,n}(t) \text{ for } n \in I_1$$

$$H_2(t) = h_{k,s,n}(t) \text{ for } n \in I_2$$

$$R(t) = r_{k,s,n}(t) \text{ for all } n$$

then

$$H_1(t) = 1.05 \cdot H_1(t-1) \quad , \quad H_2(t) = 1.03 \cdot H_2(t-1) \quad , \qquad t = 2,\dots,tm$$

$$R(t) = R(t-1) \qquad t = 2,\dots,tm$$

Values of coefficient p are given for $k = s = t = 1$

$$p_{k,s,n}(1) = p_{1,1,n}(1)$$

$$p_{k,s,n}(t) = 1.05 \cdot p_{k,s,n}(t-1) \quad , \quad t = 2,\dots,tm$$

Furthermore

$$Y_j(t) = Y_j(t-1) + \Delta_Y^j \quad , \qquad t = 2,\dots,tm$$

$$J_{k,s}(t) = \delta_{k,s} \cdot J_{k,s}(t-1) \quad , \quad t = 2,\dots,tm$$

$$K_j(t) = \gamma_j \cdot K_j(t-1) \quad , \qquad t = 2,\dots,tm$$

In the dat1 file the values of $\Delta$, $\delta$, $\gamma$ are set in the same lines with the initial values of Y, J, K.
Parameter nobj (first line of dat1) switches the model generator:

nobj = 1 - Approach IA
nobj = 2 - Approach IIB .

The next parameter lprob stands simply for the number of run.
Its value will be printed at the beginning of the files *spraw* and *sert* to help with the registration of the results.

In the case of file *dat2* in Package 2 only two parts are different: values of reference yields $Y_j(t)$ are not used and if

nobj = 2 - Approach IIA (with 2 objectives),
nobj = a > 2 - Approach IIB (with tm $\cdot$ a objectives).

APPENDIX E:

Here, we will briefly show how the model is generated, i.e., how the input file for linear optimization (in standard MPSX form) is created. Let us take constraint (6) into consideration, for example. Figure 10 presents parts of the program contained in *mpsx2.f* file which deal with this constraint. In the columns section the name of the row in which a given variable occurs and the value of its coefficient must be stated. The names of all rows precede a four-digit number *(row␣␣␣1, row␣␣12, row␣123, row1234,* for example). The number of rows each constraint produces are calculated and set in matrix k(•):

$$k(1) = tm \quad , \quad k(2) = tm * am \quad , \quad k(6) = (tm-1) \cdot km$$

for example. In matrix l(i) the number of the last row that i-th constraint produce. Therefore

$$\begin{aligned} l(1) &= k(1) \quad , \\ l(2) &= l(1) + k(2) \quad , \\ &\vdots \\ l(i) &= l(i-1) + k(i) \quad . \end{aligned}$$

Note that indices vary like in implicit Fortran DO-loops for constraint (6), for example, first the value of t increases from 2 until tm with k=1 then k=2 and t grows again from 2 to tm. So, the number of row in which variable $x_{k,s}(t)$ in constraint (6) will occur is:

$$ir = l(5) + (k-1) \cdot (tm-1) + it-1 \quad .$$

In a similar manner, the right-hand section and the rows-section are generators. Therefore any model change can be done quite easily since it requires only some changes in the specified parts of the program.

```
      ⋮
c          columns section / variables x
       do 40 ik=1,ikm
       do 40 is=1,ism
       do 40 it=1,itm
c                  in (6)
       if(it.eq.1)goto 31
       ir=l(5)+(ik-1)*(itm-1)+it-1
       aa=1.
       write(6,300)'x',ik,is,it,'row',ir,aa
   300 format(1x,4x,a,3i1,6x,a,i3,4x,f12.6)
    31 continue
      ⋮
c          rhs section
      ⋮
c                in (6)
       do 45 ik=1,ikm
       do 45 it=2,itm
       ir=l(5)+(ik-1)*(itm-1)+it-1
       aa=-c(ik)
       write(6,1007)'rhs',1,'row',ir,-aa
    45 continue
      ⋮
```

Figure 10. In *mpsx2.f* file