Working Paper

APPLYING THE BALATON SECTOR MODEL
FOR ANALYSIS OF PHOSPHORUS DYNAMICS
IN LAKE BALATON, 1976-1978

A.V. Leonov

September 1981
WP-81-118

International Institute for Applied Systems Analysis
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PREFACE

One of the principal themes of the Task on Environmental Quality Control and Management in IIASA's Resources and Environment Area is a case study of eutrophication management for Lake Balaton, Hungary. The case study is a collaborative project involving a number of scientists from several Hungarian institutions and IIASA (for details, see WP-80-187 and WP-81-108).

This paper, originally prepared for the Third Task Force Meeting on Lake Balaton Modeling (Veszprem, Hungary, August 1981), is a further contribution to the Lake Balaton case study. The report describes a mathematical model BALSECT (Balaton Sector Model) of the phosphorus transformations in the lake. The model is one of three models that have been developed for the analysis of data characterizing recent variations of water quality within the Lake. The report gives further details on the simulation of the phosphorus transformation processes and phytoplankton growth in Lake Balaton (see also WP-80-88 and WP-80-149). The results reported make possible a comparison of the performance of the model with the observations recorded for 1976-1978.

ACKNOWLEDGMENTS

I gratefully acknowledge the support and the encouragement, the assistance and sound advice given by my colleagues, members of IIASA's Resources and Environment Area (REN), Drs. M.B. Beck and L. Somlyody, as well as Dr. G. van Straten, who worked in REN two years ago. I also wish to thank Drs. R. Anderson and A. Smyshlyaev (both from IIASA) for their useful advice on the application of the statistical methods employed. The technical assistance offered by Serge Medow in programming much of this work is gratefully acknowledged. Finally, I express my gratitude to A. John for suggesting many corrections to my English.

ABSTRACT

The Balaton Sector Model was developed at IIASA. It includes the interaction between five phosphorus fractions (dissolved organic P, dissolved inorganic P, nonliving particulate P, phytoplankton-P, and bacterial-P) and takes into account the wind- and temperature-regulated phosphorus exchange between sediment and water as well as the horizontal transport of phosphorus fractions from basin to basin by wind induced and advective water flow. This model was applied to a real set of field observations on the state of the environment, such as temperature, radiation, wind, water balance, and phosphorus loading, in order to examine the feasibility of the model to represent the phosphorus dynamics in different parts of Lake Balaton for the environmental conditions from 1976-1978. The model adequacy in describing phosphorus measurements is analyzed by statistical methods which show that the simulated phosphorus dynamics agree sufficiently with the available phosphorus measurements for Lake Balaton. The results of sensitivity analysis to determine the relative importance of measurements (temperature, radiation, and phosphorus loading) or the quality of input data determining the conditions of simulated phosphorus transformation are discussed in terms of changes in phosphorus concentrations, averaged on a monthly and annual basis. Some preliminary information on the phosphorus exchange in the sediment-water layer, extracted from the simulation results, is presented for discussion in this report. Furthermore, the analysis of phosphorus fluxes, external as well as internal, and the conditions of phosphorus cycling in 1976-1978 were conducted in order to clarify the specificity of phosphorus transformation within the Lake Balaton ecosystem.

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INTRODUCTION

The study of eutrophication in any water body by modeling techniques presupposes the understanding of the overall trends in nutrient cycling and of the transformation of the major system compounds within the water body, organic as well as inorganic. This is considered to be a necessary, quantitative base in assessing the current status of water quality and in identifying the possible direction of the trophic changes in the given water body, at different nutrient loads. It is possible to formulate some recommendations for a general solution to the eutrophication problem which would help to prevent the development of eutrophication (brought about by excessive nutrient loading) or to retain the current water quality.

In preliminary testing it became apparent that the model of phosphorus transformation, BALSECT (Balaton Sector Model), which was developed for studying the phosphorus transformation and eutrophication in Lake Balaton, had to be complemented by the sediment-water phosphorus interactions and wind-induced interbasin phosphorus transfer (Leonov 1980). These processes, in combination with biochemical ones, are of major interest in the analysis of the present status of water quality and for the prediction of future eutrophication trends in Lake Balaton. The basic ideas and the theoretical background in modeling the sediment-water phosphorus exchange as well as the phosphorus interbasin transfer by wind-induced water flow were recently formulated by van Straten and Somlyody (1980).

This report deals with an improved version of BALSECT and describes the results of simulation of the phosphorus dynamics for the environmental conditions of 1976-1978. By using the modeling results in combination with the data analysis, further steps in the ecological modeling were examined: (i) the assessment of model adequacy through statistics; (ii) sensitivity analysis; (iii) the role of sediment in the phosphorus dynamics and (iv) phosphorus cycling within the Lake Balaton ecosystem.

THE MODEL

The model equations are constructed on the basis of mass conservation principles for phosphorus compartments--nonliving particulate organic phosphorus (P_D), dissolved organic phosphorus (DOP), bacterial phosphorus (B), dissolved inorganic phosphorus (DIP) and phytoplankton phosphorus (F)--and it is given by a set of coupled ordinary differential equations. The general form of the model equations in the improved version of BALSECT is:

$$\frac{dC_{ij}}{dt} = R_{i,j} + CZ_{i,j} + \frac{(Q_{in_j} + Q^a_{win_j})}{V_j} \cdot C_{i,j-1} - \frac{(Q_{out_j} + Q^b_{wout_j})}{V_j} \cdot C_{i,j}$$

$$- \frac{(Q^a_{wout_j})}{V_j} \cdot C_{i,j} - \frac{(Q^b_{win_j})}{V_j} \cdot C_{i,j+1} + \frac{(Q_{pr_j})}{V_j} \cdot C_{r_i} + S_{i,j} \qquad (1)$$

where i is equal to 1, 2, 3, 4 and 5 for P_D, DOP, B, DIP and F respectively;

j is the number of basins considered and the hypothesis on four-basin segmentations used in the simulation study;

$C_{i,j}$, $C_{i,j-1}$ and $C_{i,j+1}$ are concentrations of particular phosphorus compounds in the basins under examination (in mgP/ℓ);

Q_{in_j}, Q_{out_j} and Q_{pr_j} are input, output flow rates and precipitation rate respectively (all m^3/day);

$Q^a_{win_j}$ and $Q^b_{win_j}$ are input rates of wind-induced flows through left and right interbasin cross section areas respectively (both m^3/day);

$Q^a_{wout_j}$ and $Q^b_{wout_j}$ are output rates of wind-induced flows through left and right interbasin cross section areas respectively (both m^3/day);

C_{r_i} is phosphorus concentrations in rain water and it is taken into account for DOP(i=2) and DIP(i=4) (mgP/ℓ);

$R_{i,j}$ is the sum of reaction rates of biochemical processes taken into account in the model (mgP/ℓ-day);

$CZ_{i,j}$ is the direct phosphorus loading rates from the watershed (mgP/ℓ-day) and it is taken into acount for P_D(i=1) and for DIP(i=4);

$S_{i,j}$ is the direct phosphorus loading rates due to sediment-water interactions (mgP/ℓ-day) and it is taken into account for P_D(i=1) and DIP(i=4);

V_j is volume (m^3) of the basin considered.

Thus in the spatial mass transport modeled, two basic mechanisms are taken into consideration. These are: the net hydrological transport based on the water balance data (weekly data for the River Zala and monthly data for interbasin exchange) and the wind-induced exchange flows through interbasin cross-sections. The net hydrological transport was already modeled previously (Leonov 1980) while the wind-induced exchange between basins has now been included in the improved version of BALSECT. The latter should simulate the longitudinal transport of phosphorus as a consequence of wind action, important for the regulation of phosphorus levels in different parts of Lake Balaton. Eight measurements for wind regime per day, which include wind speed and wind direction, were used for the calculation of rates of wind-induced flows through interbasin cross sections as shown below:

$$Q_w = \text{abs}|k \cdot W \cdot A_j \cdot \cos(\alpha - 30)| \tag{2}$$

where Q_w is the rate of wind-induced flow (m^3/day);
k is the proportionality coefficient, equal to 0.0018 (unitless);
W is wind speed (m/sec);
A_j is the interbasin areas (m^2) equal to 8125 m^2, 12500 m^2 and 7500 m^2 for I-II, II-III and III-IV respectively;
α is wind direction;
30 is the angle of deviation of Lake Balaton's longitudinal axis from the space coordinate axis.

Another improvement was made to the model so it could take into account the sediment-water interactions. The sediments in water bodies act as a potential nutrient source and the rate of nutrient exchange through the sediment-water interface is regulated by environmental factors. Among different mechanisms of phosphorus exchange reactions in the sediment-water interface, the most interesting (from the point of view of importance) are the sedimentation and resuspension of particulate phosphorus and the release of mineral phosphorus. They are modeled on the basis of the approaches suggested by Somlyody (1980) who studied the influence of wind action on the exchange processes in the sediment-water interface in the central part of Lake Balaton, the Szemes Basin, with a mean depth of about 4.3 m. Thus in this study, it has been assumed that additional quantities of phosphorus increase the levels of nonliving particulate phosphorus and dissolved mineral phosphorus, as a result of the sediment-water interactions. The rate of the nonliving particulate phosphorus load, S_{PD_j} in mgP/ℓ-day, due to the combined effect of resuspension and sedimentation, is given by

$$S_{PD_j} = P_{Dres} \cdot (4.3/d_j)^2 \cdot W^U - K_{sed} \cdot (4.3/d_j) \cdot P_{D_j} \qquad (3)$$

where K_{sed} is the rate constant of P_D sedimentation, which is assumed to be equal to 0.25 day^{-1};
d_j is the depth of the basins: d_I=2.28 m; d_{II}=2.87 m; d_{III}=3.22 m; d_{IV}=3.68 m;
W is wind speed in m/sec;
U is the empirical coefficient which is assumed to be equal to 1;

P_{Dres} is time-averaged flux of particulate-P from the sediment which is assumed to be similar for all the basins (mgP/ℓ-day);
4.3 is the depth of the Szemes Basin (m).

The flux of mineral phosphorus from the sediment is given as

$$S_{DIP_j} = DIP_{r_j} \cdot \exp(K_{tr} \cdot T) \cdot W \qquad (4)$$

where K_{tr} is the rate constant of phosphorus transformation in sediment which is assumed to be equal to 0.125 day^{-1};
T is water temperature in ^oC;
DIP_{r_j} is the time-average flux of DIP from the sediment (mgP/ℓ-day).

Among the biochemical processes which are important in the phosphorus transformations, this model takes into account:

(i) phytoplankton production and nutrient uptake which are characterized by a function of temperature, light and DIP content;

(ii) bacterial production which is temperature-dependent and is an important step of DOP transformation and DIP regeneration;

(iii) metabolic excretion of DOP and DIP by phytoplankton and bacteria respectively;

(iv) nonpredatorial mortality of bacteria and phytoplankton which are essential factors in phosphorus cycling;

(v) decomposition of nonliving particulate phosphorus, because this is an important stage in phosphorus transformation in the release of chemical energy stored in detritus.

Mathematical equations describing these biochemical phosphorus transformations are given in Appendix A (Leonov 1980). Together with equations (1)-(4) they give a complete set of equations for the slightly modified version of BALSECT.

DATA BASE

This report gives only a brief description of the data used for the simulation of the phosphorus dynamics in Lake Balaton, for the period of 1976-1978. All existing data on the lake at IIASA used in the simulation runs may be subdivided into three groups:

(i) physical, meteorological and hydrological data;

(ii) tributary stream influence and watershed nutrient loading data;

(iii) phosphorus, nitrogen and phytoplankton data in open water.

The first group of data contains the measurements of water temperature, solar radiation, wind and water balance characteristics. The dynamics of daily mean values of water temperature and solar radiation for 1976-1978 are presented in Figures 1 and 2 respectively. The fluctuations in wind speeds measured every three hours during 1976-1978 is shown in Figure 3. The water balance data includes the weekly measurements of the River Zala discharge flow rates and monthly average input-output rate and precipitation rates for all the basins. Figures 4 and 5 show the fluctuations of input and output flow rates respectively. Monthly mean precipitation rates are presented for 1976-1978 in Table 1. All the data from the first group is used in the simulation of the phosphorus dynamics as environmental factors regulating the rates of phosphorus transformations.

The second group of data contains information on the phosphorus load. Sources of phosphorus load are River Zala discharge water, watershed runoff, rainfall, sewage and sediments. The fluctuations in the concentrations of nonliving particulate phosphorus are obtained by the difference between weekly measurements of total phosphorus fractions and dissolved inorganic phosphorus and phytoplankton phosphorus in the River Zala discharge water for 1976-1978, and all of these are presented in Figure 6. The concentrations of the bacterial-P in the River Zala discharge water are assumed to be constant and equal to $4 \cdot 10^{-4}$ mgP/ℓ while the DOP concentrations, because the information is absent, are assumed to be negligibly low (Leonov 1980). The DIP and DOP contents in the rainfall were assumed to be constant for the different years and equal to 0.1 and 0.06 mgP/ℓ respectively. Together with the water balance data presented above, these phosphorus loads allowed inclusion of the direct influence of the

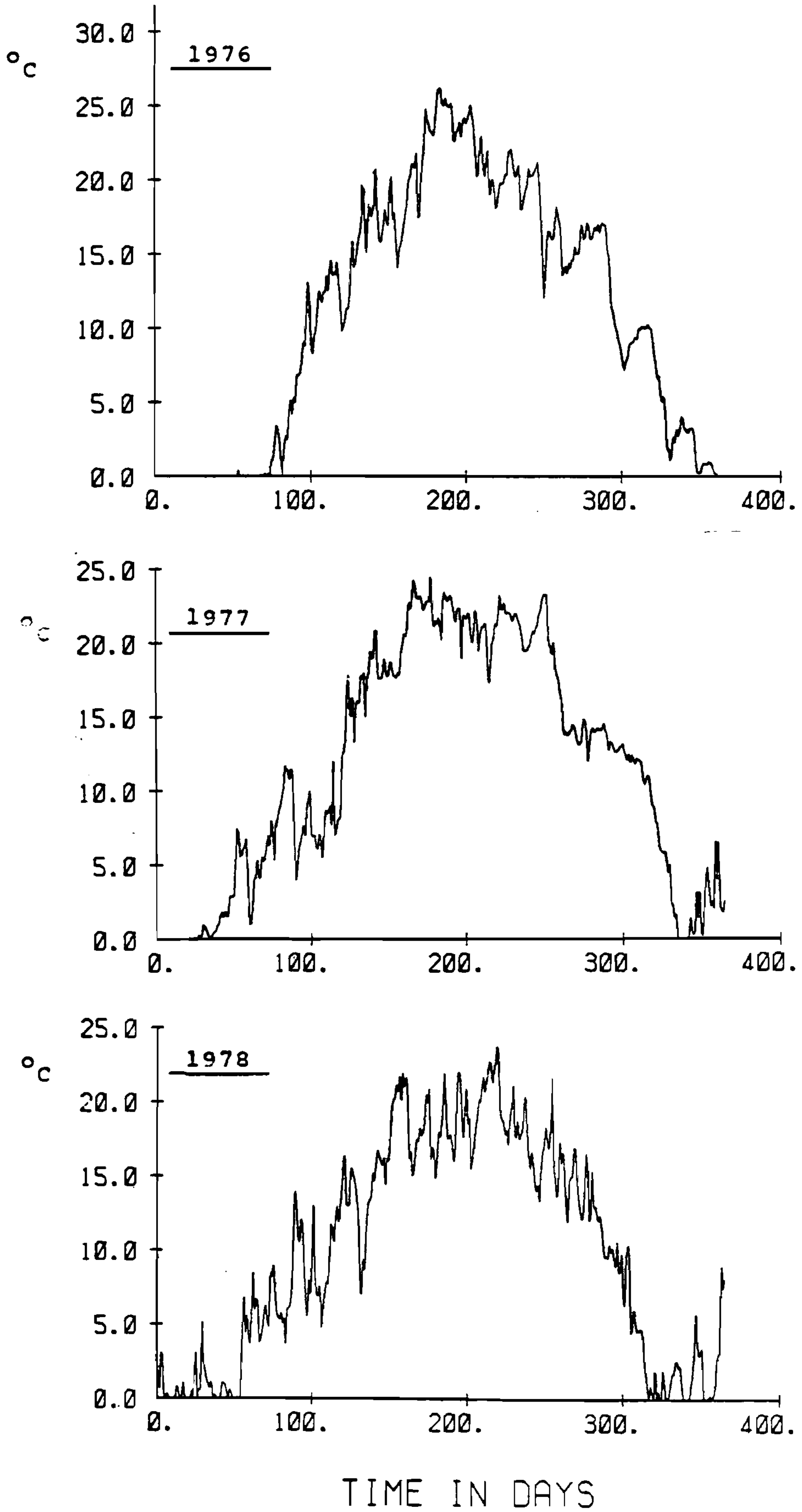

Figure 1. Dynamics of daily average water temperature in Lake Balaton for 1976-1978.

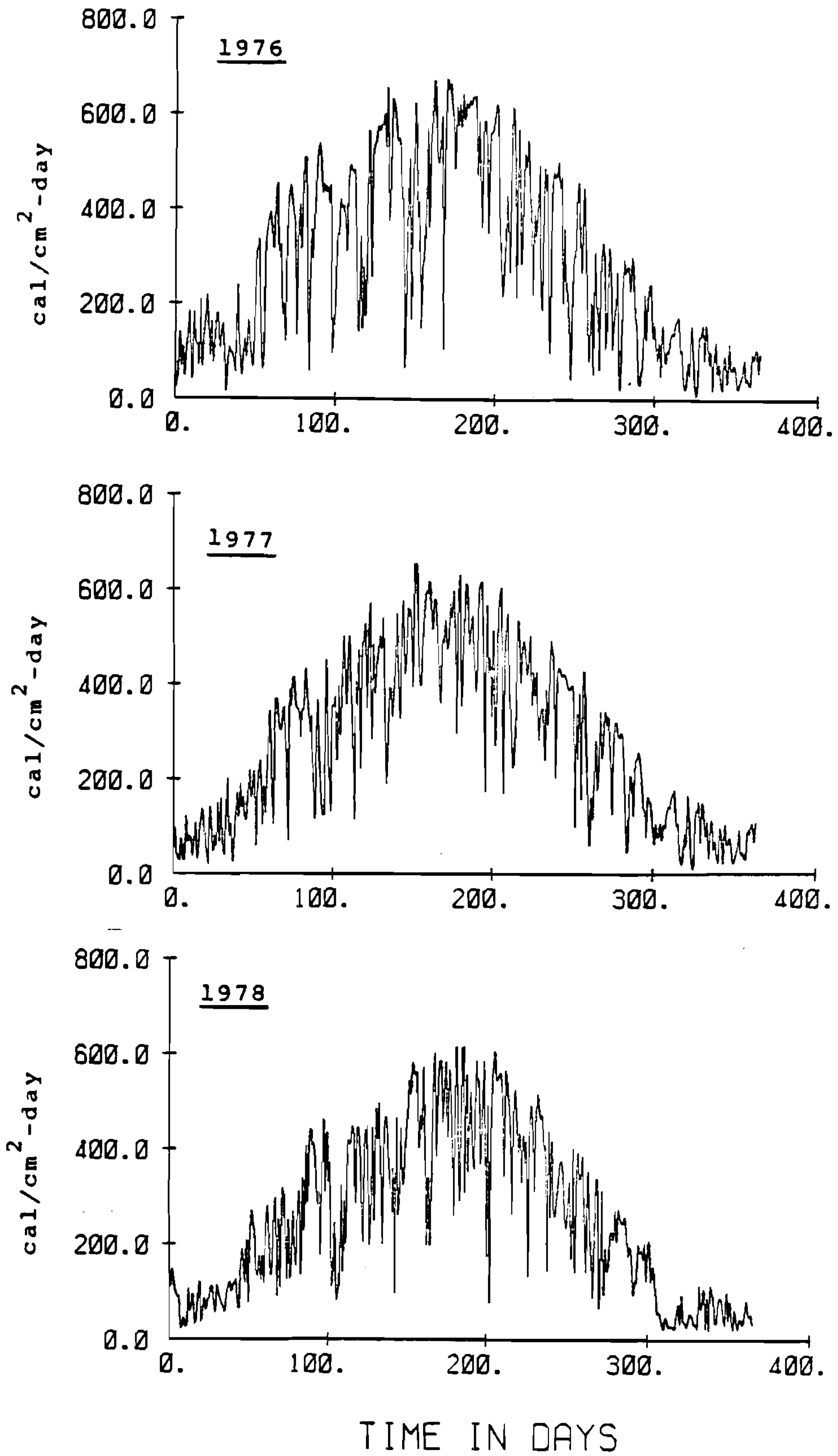

Figure 2. Dynamics of daily average values of solar radiation for 1976-1978.

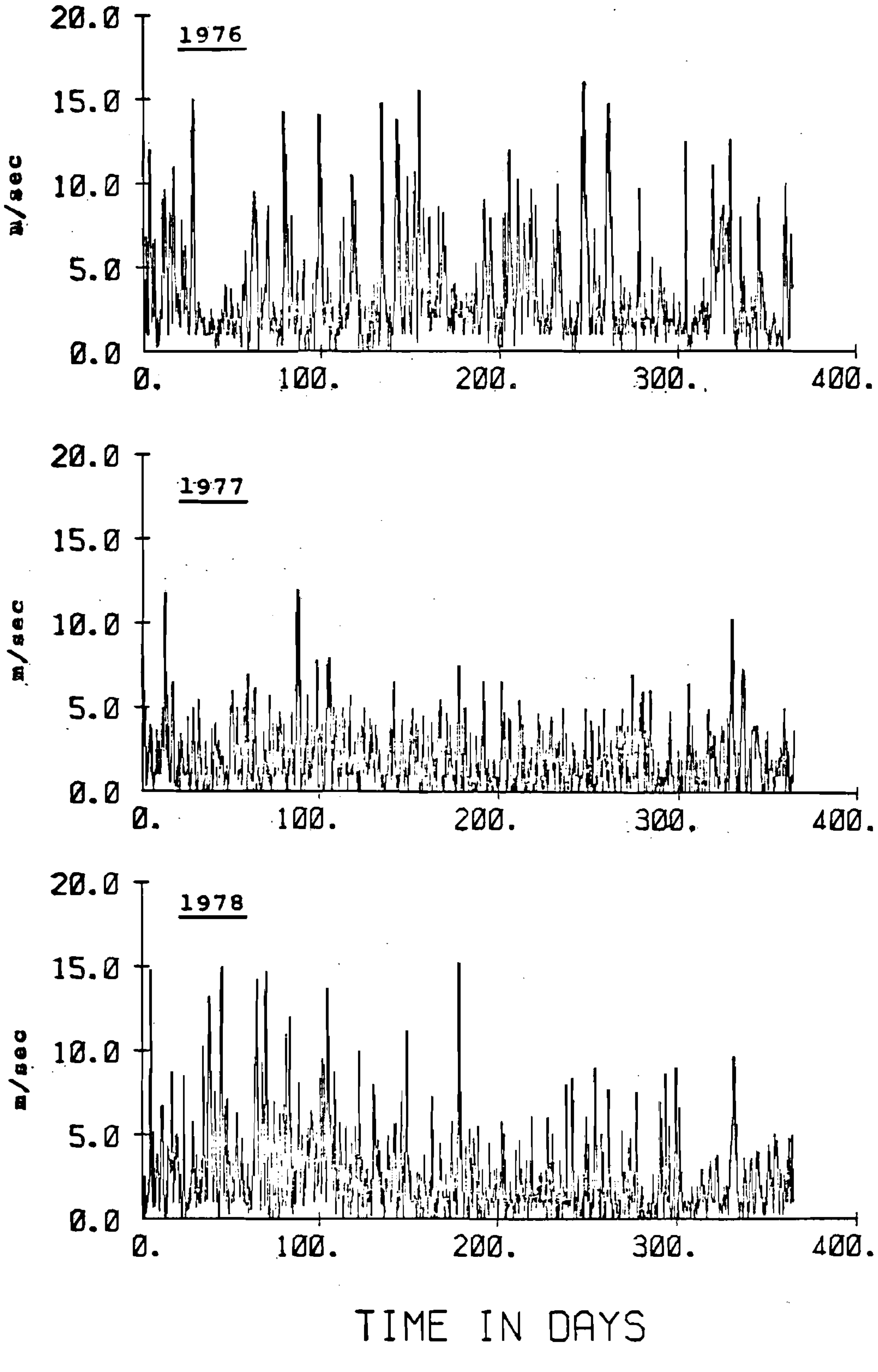

Figure 3. Directly measured wind speeds for Keszthely Bay (1976-1978).

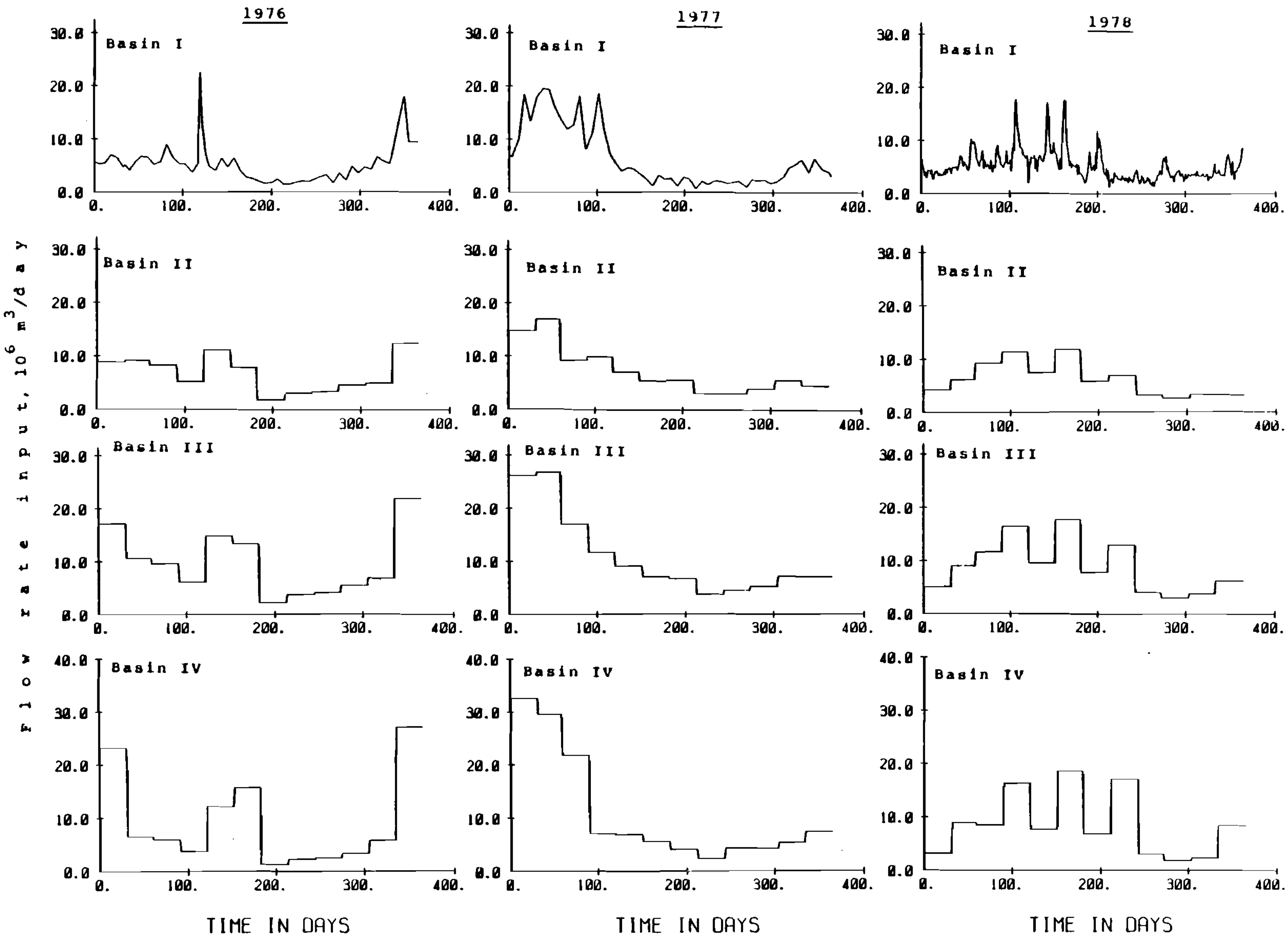

Figure 4. Water balance data: flow rate input for Lake Balaton Basins (1976-1978).

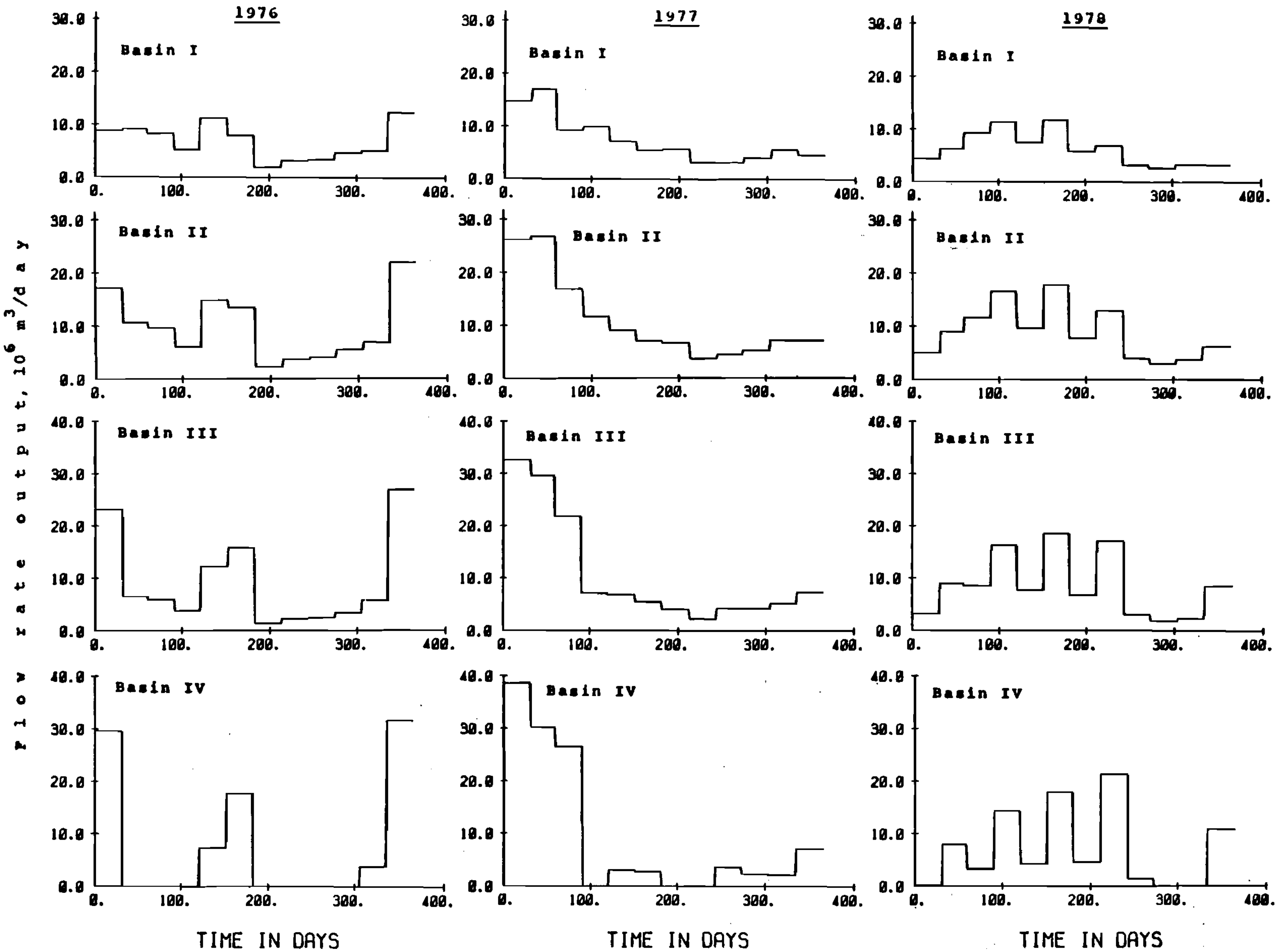

Figure 5. Water balance data: flow rate output for Lake Balaton Basins (1976-1978).

Table 1. Water balance data: monthly average precipitation rates (10^6 m^3/day) for 1976-1978.

Months	Basins											
	I			II			III			IV		
	1976	1977	1978	1976	1977	1978	1976	1977	1978	1976	1977	1978
Jan	0.576	0.650	0.135	2.183	2.462	0.511	2.820	3.180	0.660	3.457	3.898	0.809
Feb	0.131	0.760	0.421	0.496	2.880	1.594	0.641	3.720	2.059	0.786	4.560	2.524
Mar	0.429	0.601	0.319	1.626	2.276	1.208	2.100	2.940	1.560	2.574	3.604	1.912
Apr	0.697	0.583	0.608	2.640	2.208	2.304	3.410	2.852	2.976	4.180	3.496	3.648
May	0.417	0.319	0.993	1.579	1.208	3.763	2.040	1.560	4.860	2.501	1.912	5.957
June	0.545	0.975	1.165	2.064	3.696	4.416	2.667	4.774	5.704	3.268	5.852	6.992
July	0.723	0.613	1.226	2.741	2.323	4.645	3.540	3.000	6.000	4.339	3.677	7.355
Aug	0.613	0.797	0.355	2.323	3.019	1.347	3.000	3.900	1.740	3.677	4.781	2.133
Sept	0.912	0.557	0.304	3.456	2.112	1.152	4.464	2.728	1.488	5.472	3.344	1.824
Oct	0.563	0.233	0.404	2.137	0.883	1.532	2.760	1.140	1.980	3.380	1.397	2.427
Nov	0.659	0.899	0.165	2.496	3.408	0.624	3.224	4.402	0.806	3.952	5.396	0.988
Dec	1.213	0.429	0.404	4.599	1.626	1.532	5.940	2.100	1.980	7.281	2.574	2.427

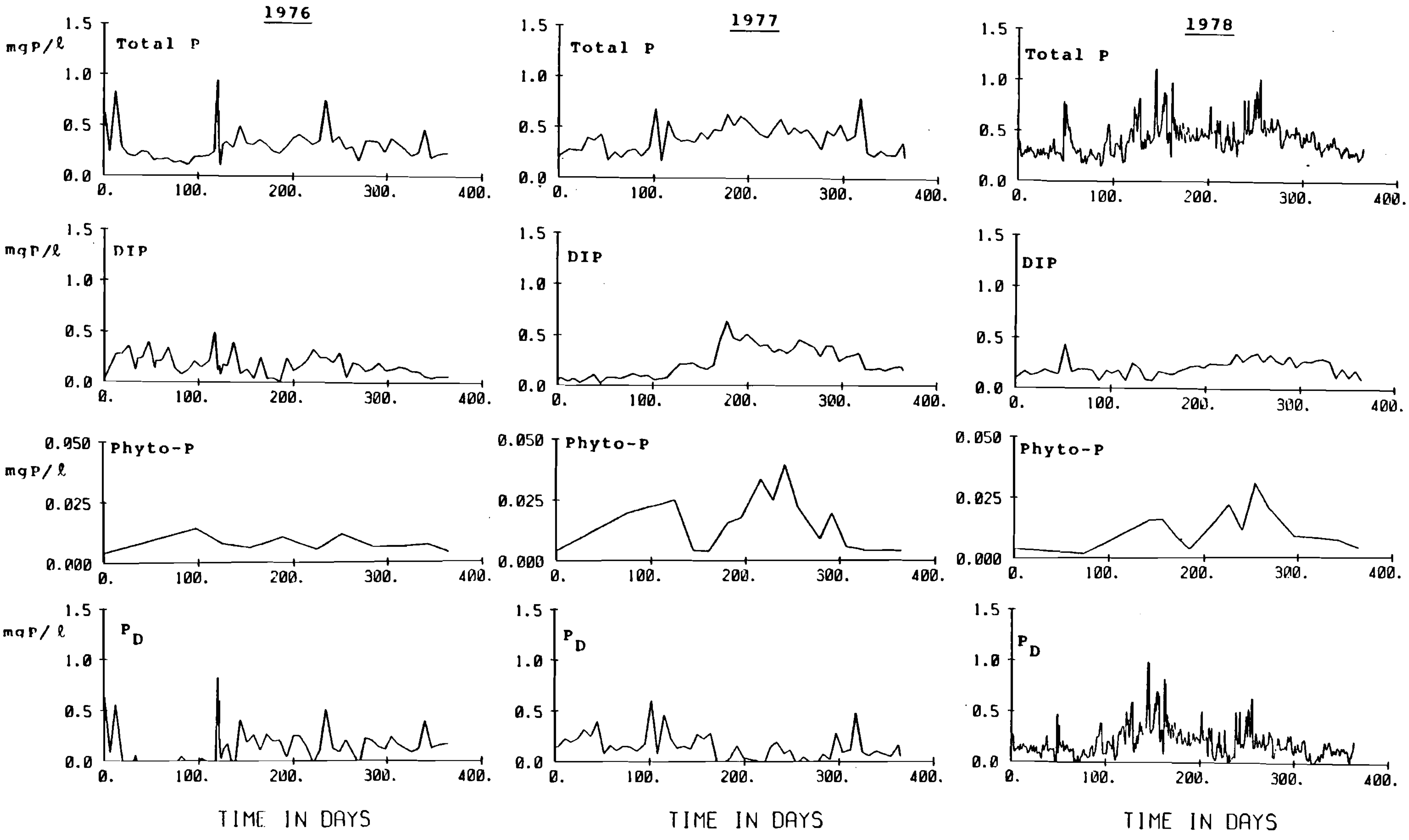

Figure 6. Dynamics of phosphorus concentrations in River Zala discharge in 1976-1978.

River Zala discharge and precipitation on the phosphorus dynamics in Lake Balaton for 1976-1978.

The influence of sediment as the DIP source is taken into account as being equal to $1.45 \cdot 10^{-5}$, $5.2 \cdot 10^{-6}$, $4.2 \cdot 10^{-6}$ and $3.3 \cdot 10^{-6}$ mgP/ℓ-day for Basins I through IV respectively. Time-averaged flux of particulate-P from the sediment to water was assumed to be equal to $7 \cdot 10^{-4}$ mgP/ℓ-day for all the basins during 1976-1978.

Table 2 gives the rates of DIP load used in the simulation runs from other nutrient sources, primarily from sewage. These rates were evaluated on the basis of the hypotheses of the four-basin extrapolation from the River Zala DIP load distribution (van Straten and Somlyody 1980).

Additional entrance of nonliving particulate-P from the watershed was also taken into account in the simulation runs, using the hypothesis on the longitudinal distribution of non-point sources over

Table 2. Time distribution of sewage DIP load rates (in mgP/ℓ-day) used in simulation runs.

Months	Basins			
	I	II	III	IV
Jan	.00021	.00010	.00007	.00009
Feb	.00021	.00010	.00007	.00009
Mar	.00021	.00010	.00007	.00009
Apr	.00021	.00010	.00007	.00009
May	.00021	.00010	.00007	.00009
June	.00021	.00010	.00007	.00009
July	.00042	.00020	.00014	.00018
Aug	.00042	.00020	.00014	.00018
Sept	.00021	.00010	.00007	.00009
Oct	.00021	.00010	.00007	.00009
Nov	.00021	.00010	.00007	.00009
Dec	.00021	.00010	.00007	.00009

the four basins (from Keszthely Bay to Siofók) discussed by van Straten and Somlyody (1980). The basis of this approach is the River Zala load for nonliving particulate-P (or River Zala runoff) evaluated on the data of the River Zala P_D concentration fluctuations and River Zala discharge flow rates. Figure 7 presents the time series of the River Zala runoff according to data used. Particulate phosphorus load for Basins II-IV is calculated using the formula:

$$\text{Runoff}_{(II-IV)} = \varepsilon \cdot \text{Runoff}_{(I)} \cdot V_{(I)}/V_{(II-IV)} \qquad (5)$$

where ε is the proportionality coefficient equal to 1, 0.45 and 0.3 for Basins II-IV respectively;

$V_{(I-IV)}$ are the volumes of the basin considered which are $82 \cdot 10^6\ m^3$, $413 \cdot 10^6\ m^3$, $600 \cdot 10^6\ m^3$ and $802 \cdot 10^6\ m^3$ for basins I-IV respectively;

Runoff_I is the River Zala runoff, mgP/ℓ-day.

The third group of data contains the phosphorus concentrations in different parts of the lake. Directly measured phosphorus compounds are dissolved inorganic phosphorus or orthophosphate phosphorus (PO_4), total dissolved phosphorus (TDP), particulate inorganic phosphorus (PIP)* and total phosphorus (TP). The concentrations of other phosphorus fractions were calculated from those directly measured:

(i) dissolved organic phosphorus, DOP = TDP - PO_4

(ii) particulate phosphorus, PP = TP - TDP

(iii) particulate organic phosphorus, POP = PP - PIP.

On account of varying the number of sampling stations, the average concentrations of phosphorus fractions and chlorophyll "a" were calculated for each basin considered (van Straten et al. 1979). All the data from the third group was used for a comparison with the modeling results for different basins in 1976-1978.

*This phosphorus fraction is not taken into account in the given model.

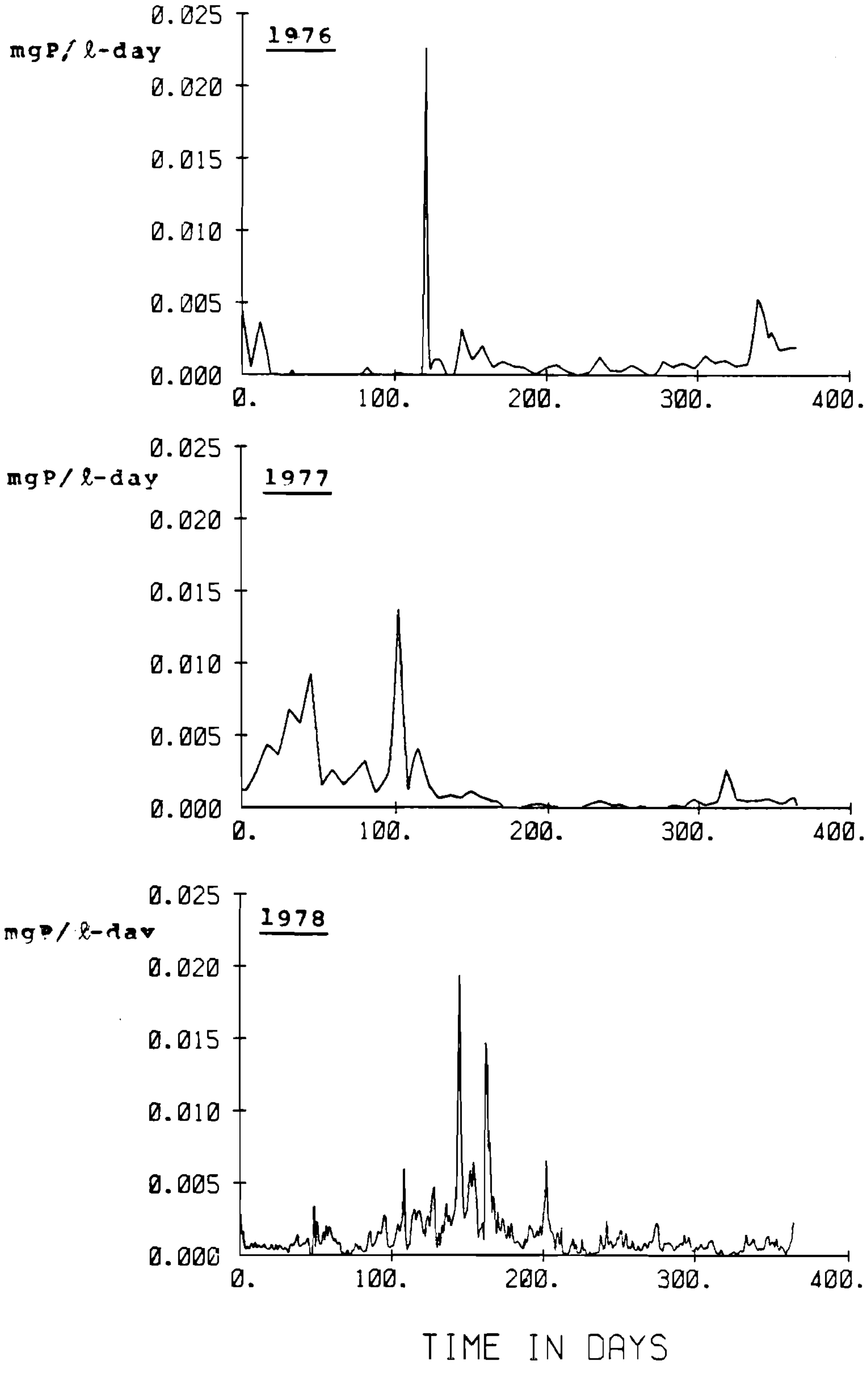

Figure 7. Time series for River Zala load for nonliving particulate phosphorus in 1976-1978.

SIMULATION

The ordinary nonlinear differential equations of this model were coded in FORTRAN and run on IIASA's computer. The equations were solved numerically, using the Runge-Kutta-4 algorithm. The time step used was 0.1 day for all the differential equations.

The initial concentrations of phosphorus fractions selected, which correspond to the environmental conditions of January 1, 1976, are given in Appendix B. All the model coefficients used were determined earlier during model application for the simulation of the phosphorus dynamics in Lake Balaton, for the environmental conditions of 1977 (Leonov 1980). In this study, the same model coefficients were used for the simulation of the phosphorus transformations for the three year period 1976-1978. All model coefficients used are given in Appendix B.

The modeling results were compared to the phosphorus concentrations after the averages were obtained for the direct measurements taken in the four basins. A comparison of the model calculations and 1976-1978 data are presented in Figures 8-12 (for particulate organic phosphorus, DIP, DOP, total soluble phosphorus and total phosphorus respectively).

All the phosphorus observation data are plotted in Figures 8-12 as points, each point being an arithmetic mean of the range of minimum and maximum observations.* All the curves in the figures are the result of model calculations and they show the phosphorus dynamics in the different basins of the lake for the three year period, 1976-1978, starting from January 1, 1976.

A preliminary analysis of Figures 8-12 allows one to conclude that the model quantitatively describes the major tendencies in the phosphorus concentration changes in the various basins during 1976-1978 and the modeled phosphorus concentrations are

*A possible error in phosphorus measurements may be expected in the range of ±10%. Because in the Basin I there was only one sampling station, this range is indicated in Figures 8-12 for phosphorus concentrations in Basin I.

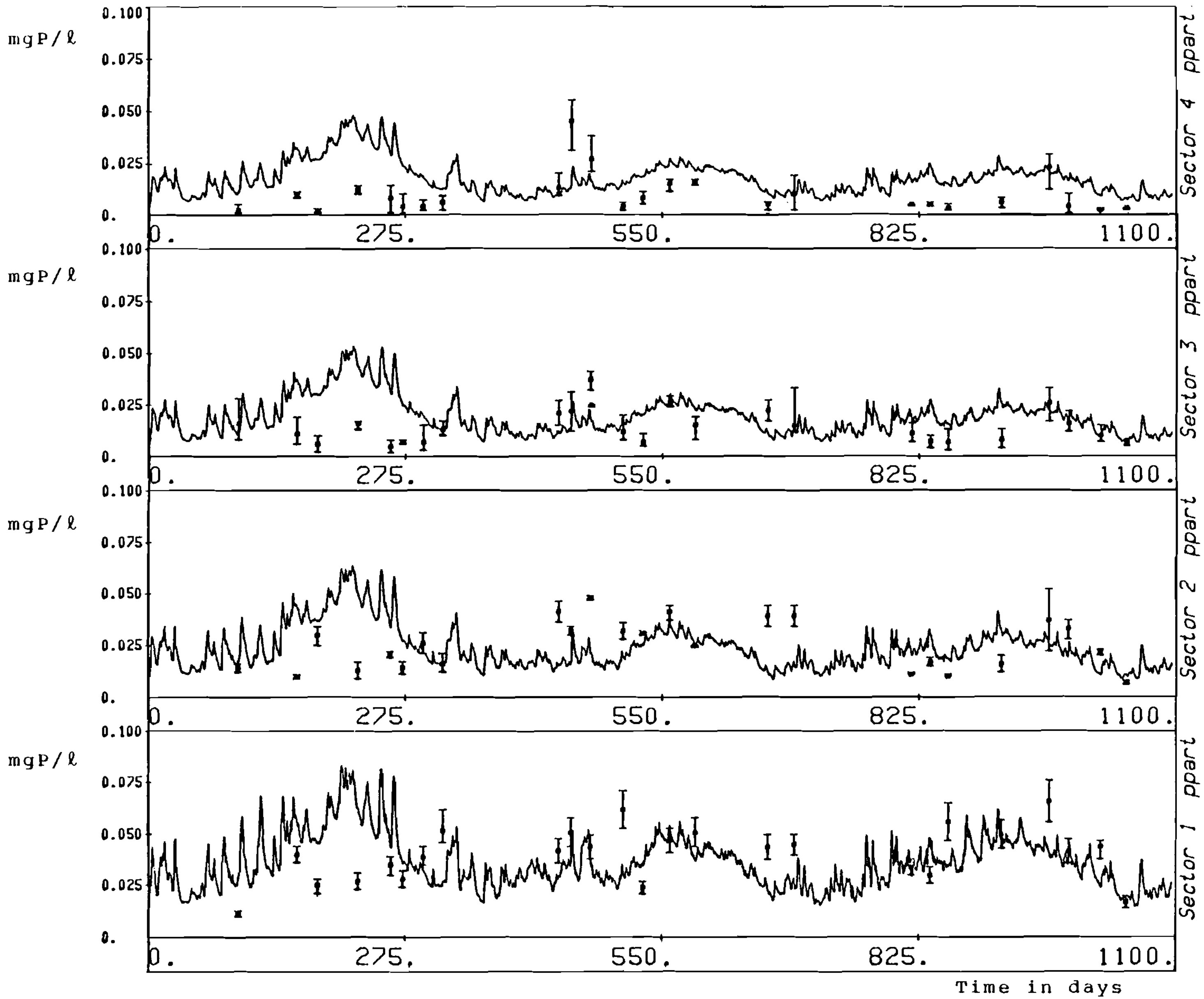

Figure 8. Comparison of model calculations (curves) and observed data for particulate organic phosphorus. Lake Balaton Basins I-IV, 1976-1978.

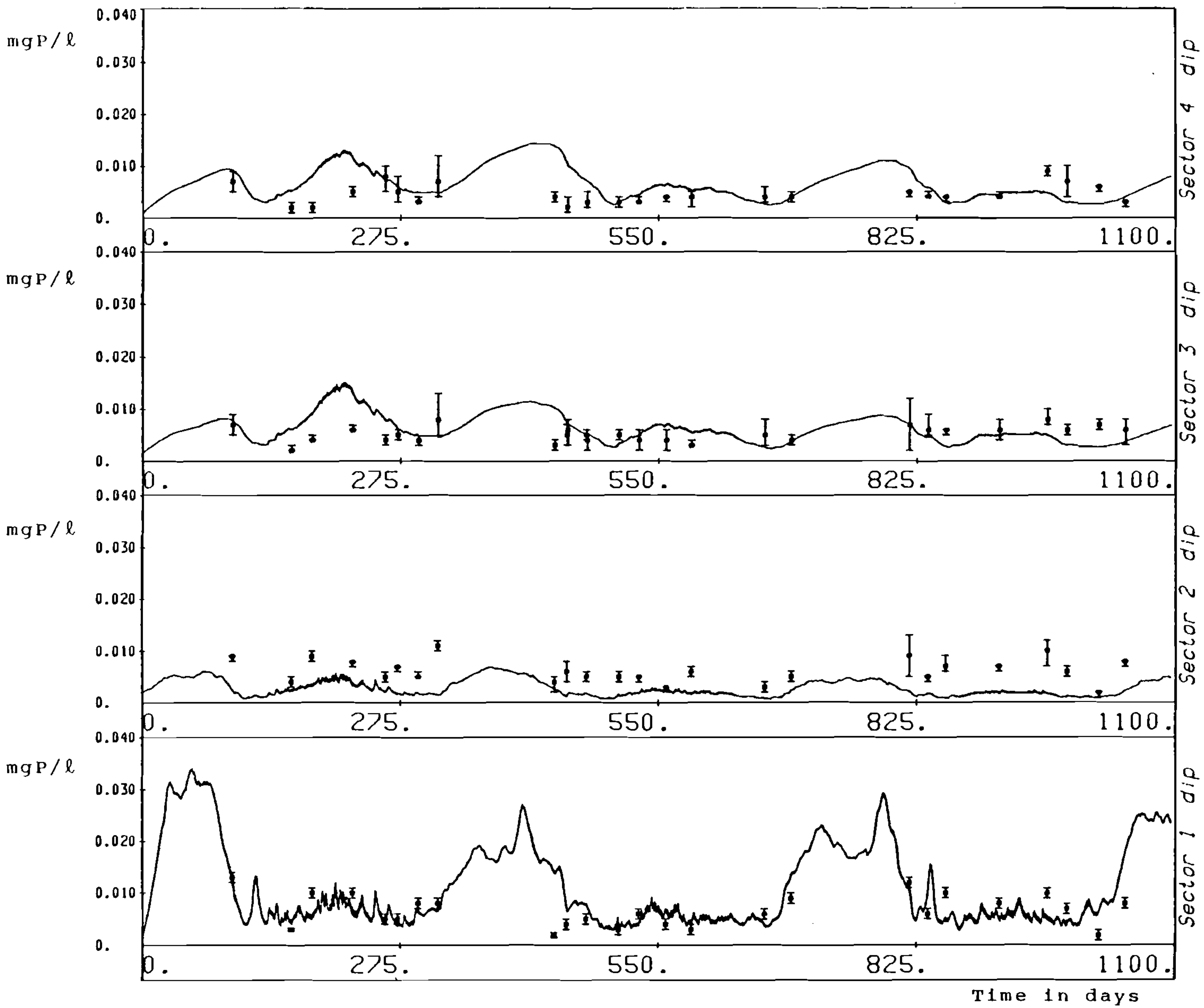

Figure 9. Comparison of model calculations (curves) and observed data for dissolved inorganic phosphorus. Lake Balaton Basins I-IV, 1976-1978.

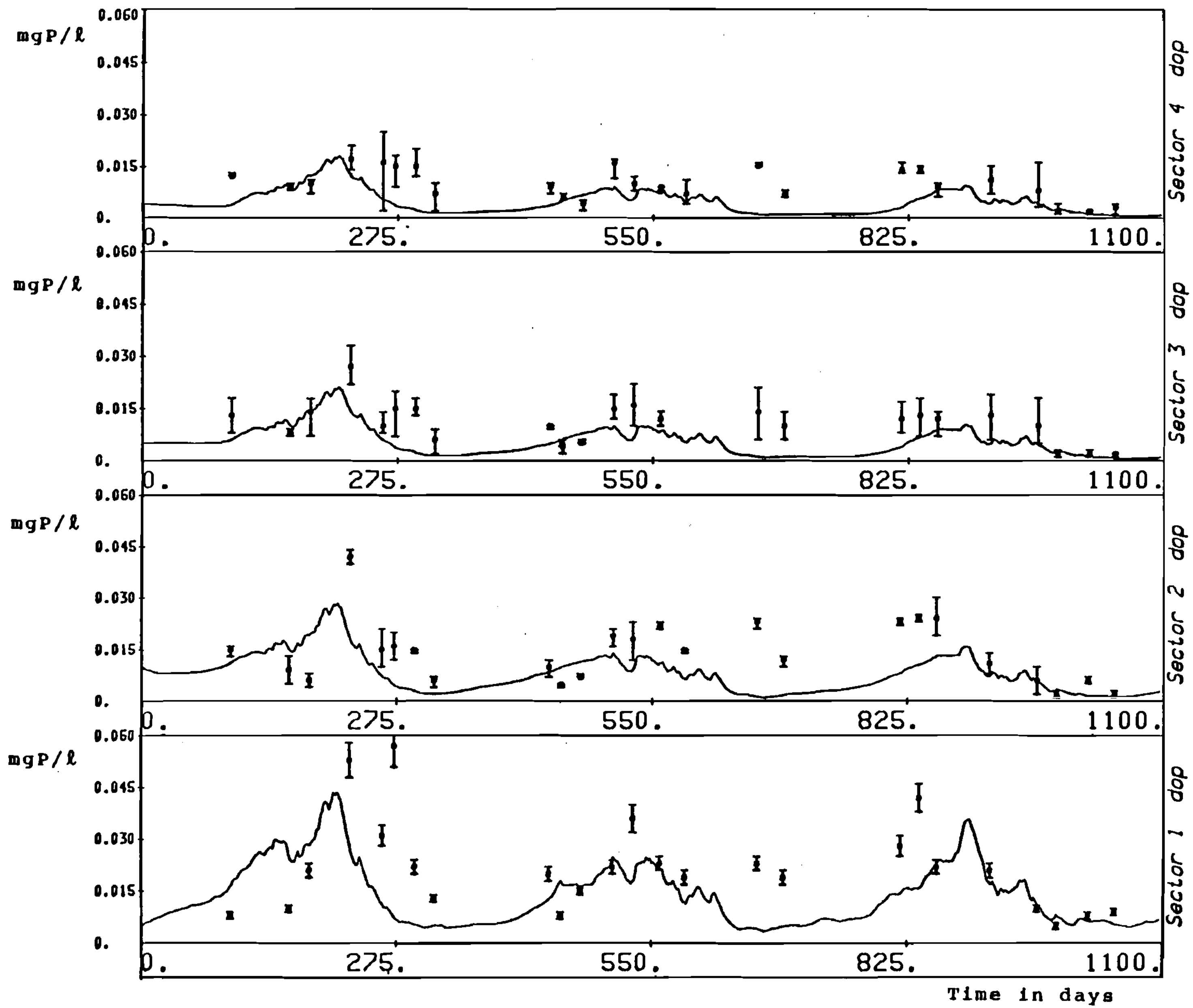

Figure 10. Comparison of model calculations (curves) and observed data for dissolved organic phosphorus. Lake Balaton Basins I-IV, 1976-1978.

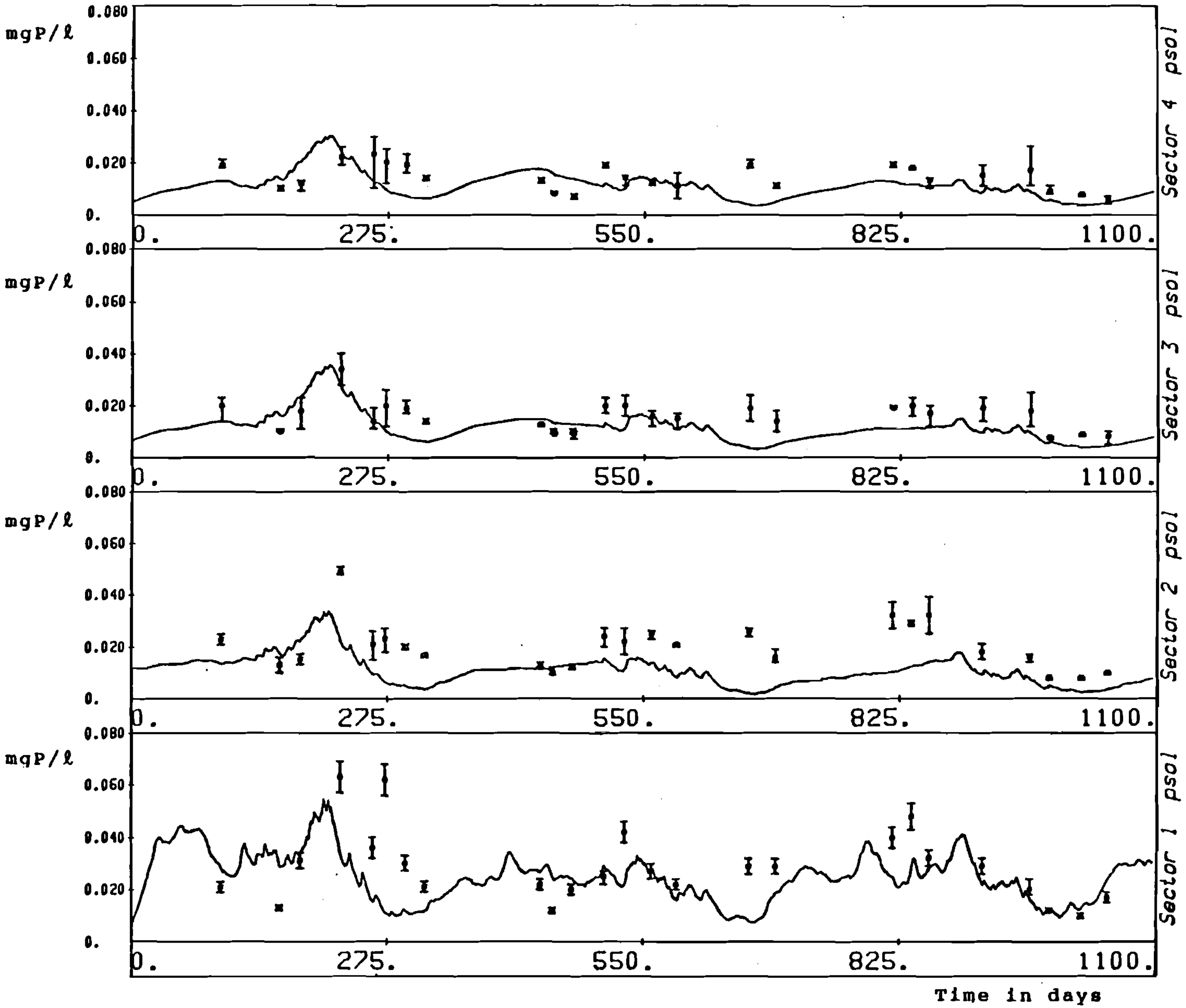

Figure 11. Comparison of model calculations (curves) and observed data for total soluble phosphorus. Lake Balaton Basins I-IV, 1976-1978.

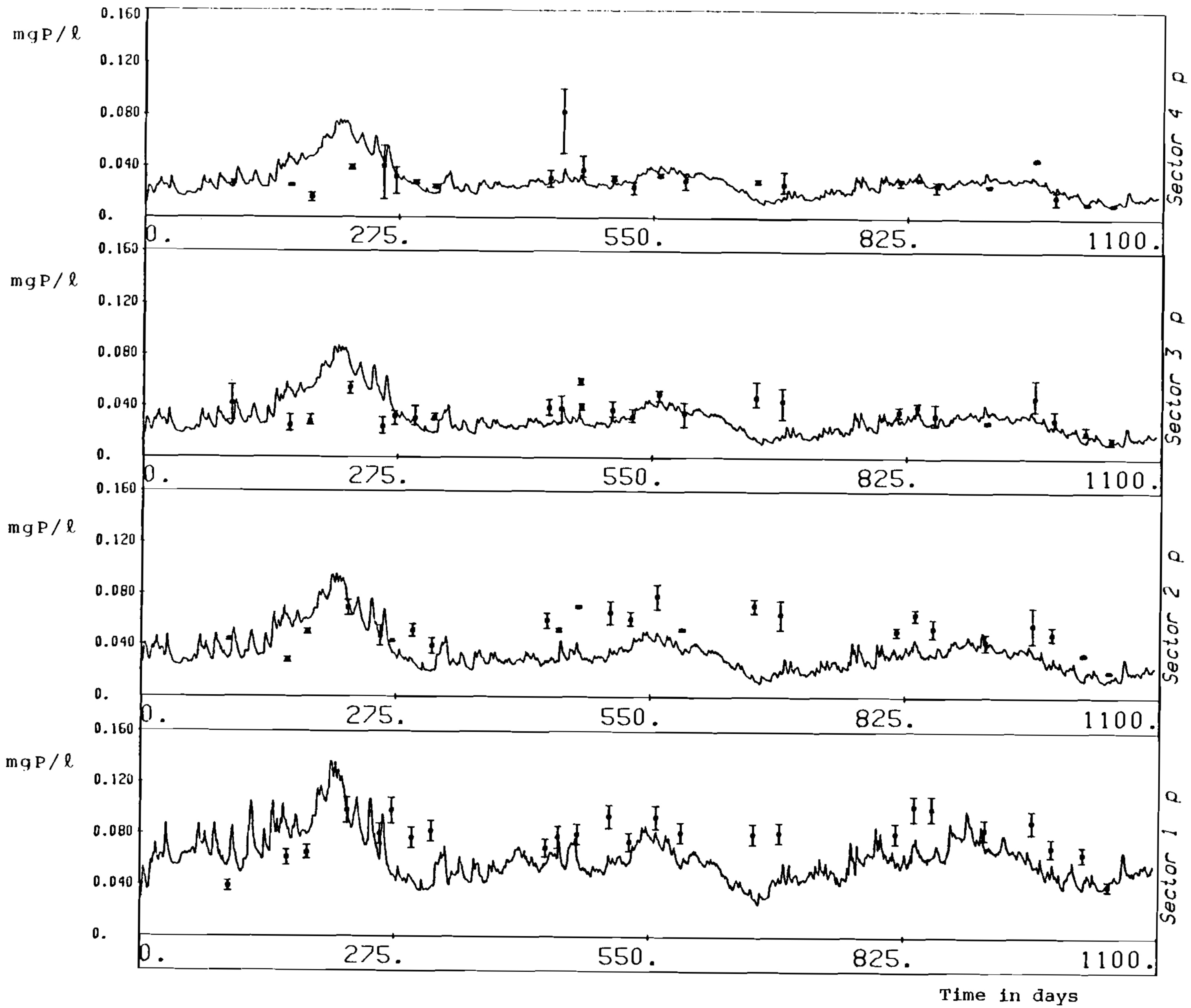

Figure 12. Comparison of model calculations (curves) and observed data for total phosphorus. Lake Balaton Basins I-IV, 1976-1978.

close to those in the observations. To obtain the criteria for showing how the simulation results correspond to the observations available and are used for comparison, statistical methods should be applied. This is discussed in the following section of this paper.

MODEL ADEQUACY

It is clear that the method for the quantitative assessment of model adequacy should be very flexible, and it must take into account, to a certain extent, the uncertainties which exist in the initial set of data (in this case in observations) used for the comparison with the modeling results.

The preliminary analysis of raw measurements generalized by van Straten et al. (1979) shows that phosphorus observations in Lake Balaton may be characterized by the following features:

(i) the observations do not provide a similar degree of information for each year studied because:
  (a) they were performed erratically and obviously in accordance with weather conditions, so that some important extremes in the phosphorus compound concentrations may have been omitted and
  (b) the date on which the first observations were made each year as well as the interim period between observations differ somewhat, therefore it is reasonable to assume that the time interval (or time step) between observations is inaccurately related with the course of the phenomena (the phosphorus transformation) under examination;

(ii) the observations are not similarly informative for the individual basins (i.e., in a space scale) because the number of the sampling stations per unit area is varied for each basin, so that "the density coefficient of observation" is 1.9, 1.6 and 2 times lower for Basins II-IV respectively than for Basin I;

(iii) when analyzing raw measurements for 1976-1978, there are significant fluctuations in the values of relative deviations of the individual phosphorus observations from their mean, in the different basins (Table 3).

Taking into account the features of the original data, we have to ensure the appropriateness of applying some of the statistical criteria to the observations. They acquire a specific meaning when the process under study is described by some of the statistical rules.

The set of observations used for the comparison with modeling results includes the averages from the measured concentrations of the phosphorus fractions in different parts of the lake selected at random from the spring-autumn period in 1976-1978. Thus we have a random sample of time-variable measurements from the general population of points that illustrate the properties of individual phosphorus fractions and the phosphorus system as a whole. The next sample of data is the results of modeling which includes the phosphorus compound concentrations from the other general population of points, described by the continuous curves and showing the temporary changes in the concentrations of all phosphorus fractions. However for the comparison with observations, only a limited number of phosphorus concentrations from modeling results, with correspondence in the time of measurement, is used. Therefore, we should examine how the data on two samples, observations and modeling results, correspond with each other.

First of all, we must estimate the quality of data in observations* to know how the sample of individual observations is representative of the general population, as it may be characterized by extensive files of data; however only a small sample of measurements are used which are representative of a certain moment of time. One should ensure that decreasing the observations series or the removal of a few samples will have little effect on the distribution of the remaining group of points (observations).

For the given random sample of observations, we can calculate

*Average values from measured phosphorus concentrations at a certain time for each basin.

Table 3. Possible ranges of relative deviations in individual phosphorus measurements from mean values (in %%)* for Basins II-IV**.

Phosphorus fraction	Basin	Relative deviations (%%)		
		minimum	average	maximum
Total P	II	2.3	18.0	50.0
	III	7.1	35.3	64.3
	IV	2.2	29.4	105.0
Total dissolved P	II	4.8	22.8	52.4
	III	5.3	35.8	72.2
	IV	5.6	33.0	90.9
Particulate organic P	II	4.0	27.2	81.1
	III	4.0	80.3	171.4
	IV	18.7	93.1	237.5
Dissolved organic P	II	6.7	43.0	133.3
	III	10.0	67.2	130.0
	IV	6.7	50.0	162.5
Dissolved inorganic P	II	11.1	36.6	88.9
	III	16.7	60.5	142.9
	IV	16.7	59.4	150.0

Note : *) Calculations were made by the formula:

$$\text{Rel.dev.} = \frac{P^t_{max} - P^t_{min}}{P^t_m} \cdot 100\%$$

where P^t_{max}, P^t_{min} and P^t_m are maximum, minimum and mean concentrations in time t in raw set of measurements;

**) In Basin I there was only one measurement station.

the sample mean value and its variance (or standard deviation). The result of these calculations are given in Table 4. Analysis of Table 4 shows that:

(i) mean values of the phosphorus fractions in an observation series in the different basins of the lake may change from 2 to 5 times during the three-year period of study (1976-1978);

(ii) within basins, the mean values of phosphorus fractions are changed in a relatively narrow range for the individual year considered;

(iii) the values of standard deviations may differ from 1.5 to 2.5 times in a comparison of their values for the individual basins and years of study; however, they are slightly lower than the mean value;

(iv) values of mean and standard deviations computed for the whole lake are close to those computed for each year.

Thus the general conclusion is that the observations for the individual basins may be generalized in one set of data for the entire period of study (1976-1978) and the statistical characteristics of this generalized observation series may be considered as representative for the process of phosphorus transformation that is under study.

The statistics for the time-based observation series that characterize the behavior of individual phosphorus fractions in each of the lake's basins are given in Table 5. Using the data from Table 5, we can arrive at an understanding of the gradient of changes in the concentrations of all phosphorus fractions and the possible order of their fluctuations according to the observations available.

Now it is possible to estimate how the mean sample for the individual phosphorus fractions in observations may correspond to the unknown mean for the general population. If the observations in the general population and in a random sample have a normal distribution, i.e., concentrations within both series have a particular kind of standard mathematical shape, then using

Table 4. Summary statistics of phosphorus concentrations observed in Lake Balaton Basins (1976-1978).

Phosphorus fractions	Years	Basins 1		Basins 2		Basins 3		Basins 4		Whole lake	
		mean	std. dev.	mean	std. dev.	mean	std. dev.	mean	std. dev.	mean	std. dev.
Total phosphorus	1976	0.0750	0.0197	0.0464	0.0117	0.0333	0.0100	0.0288	0.0080	0.0458	0.0222
	1977	0.0811	0.0080	0.0634	0.0087	0.0421	0.0082	0.0357	0.0178	0.0552	0.0211
	1978	0.0790	0.0204	0.0461	0.0139	0.0311	0.0104	0.0245	0.0114	0.0452	0.0255
Particulate organic phosphorus	1976	0.0323	0.0120	0.0180	0.0070	0.0101	0.0045	0.0060	0.0037	0.0166	0.0124
	1977	0.0456	0.0101	0.0364	0.0070	0.0202	0.0084	0.0159	0.0129	0.0293	0.0153
	1978	0.0424	0.0155	0.0191	0.0109	0.0114	0.0067	0.0064	0.0068	0.0198	0.0173
Total dissolved phosphorus	1976	0.0346	0.0186	0.0226	0.0113	0.0186	0.0072	0.0174	0.0048	0.0263	0.0157
	1977	0.0253	0.0082	0.0188	0.0061	0.0141	0.0043	0.0127	0.0042	0.0175	0.0075
	1978	0.0260	0.0136	0.0191	0.0105	0.0148	0.0054	0.0131	0.0049	0.0182	0.0102
Dissolved organic phosphorus	1976	0.0269	0.0189	0.0155	0.0115	0.0135	0.0064	0.0125	0.0038	0.0171	0.0125
	1977	0.0206	0.0075	0.0146	0.0065	0.0096	0.0046	0.0092	0.0040	0.0134	0.0072
	1978	0.0181	0.0126	0.0122	0.0099	0.0082	0.0053	0.0079	0.0051	0.0116	0.0094
Dissolved inorganic phosphorus	1976	0.0078	0.0033	0.0072	0.0024	0.0050	0.0019	0.0049	0.0024	0.0062	0.0028
	1977	0.0047	0.0021	0.0047	0.0011	0.0044	0.0009	0.0034	0.0007	0.0043	0.0014
	1978	0.0079	0.0030	0.0068	0.0025	0.0065	0.0008	0.0052	0.0020	0.0066	0.0023

Table 5. Summary statistics for the time series of phosphorus observations generalized for three years, 1976-1978.

phosphorus fraction	basin	mean	standard deviation	minimum	maximum
Total phosphorus	I	0.0785	0.0162	0.039	0.102
	II	0.0524	0.0139	0.020	0.078
	III	0.0360	0.0104	0.015	0.060
	IV	0.0299	0.0136	0.011	0.082
	whole lake	0.0492	0.0232	0.011	0.102
Particulate organic P	I	0.0403	0.0134	0.012	0.066
	II	0.0250	0.0119	0.007	0.048
	III	0.0144	0.0081	0.005	0.037
	IV	0.0097	0.0098	0.002	0.045
	whole lake	0.0223	0.0160	0.002	0.066
Total dissolved P	I	0.0285	0.0140	0.010	0.063
	II	0.0201	0.0092	0.008	0.049
	III	0.0156	0.0057	0.008	0.034
	IV	0.0143	0.0049	0.006	0.023
	whole lake	0.0196	0.0106	0.006	0.063
Dissolved organic P	I	0.0218	0.0135	0.005	0.057
	II	0.0141	0.0091	0.002	0.042
	III	0.0104	0.0056	0.002	0.027
	IV	0.0098	0.0046	0.002	0.017
	whole lake	0.0140	0.0100	0.002	0.057
Dissolved inorganic P	I	0.0067	0.0031	0.002	0.012
	II	0.0062	0.0023	0.002	0.011
	III	0.0052	0.0015	0.002	0.008
	IV	0.0045	0.0019	0.002	0.009
	whole lake	0.0056	0.0024	0.002	0.012

the formal statistical method we can find the confidence interval for unknown expectations of the general population on the known mean in a sample considered (Cowden 1957). Thus we must now estimate how the observations are distributed in samples characterizing the individual phosphorus fractions.

In order to do this, we will operate by values of the mean and the standard deviation, for each of the phosphorus fractions that were calculated for the individual basins. These characteristics are called the parameters of the distributions (Allard 1977). The results of these estimates are presented in Table 6. It shows that the concentrations of all phosphorus fractions in observations lie in the range ±3 σ. Thus on the basis of formal statistics we have received evidence that the mean values of all phosphorus fractions from a random sample of observations are representative and characterize the process of phosphorus transformation in Lake Balaton's ecosystem.

A similar test for modeling results also presented in Table 6 shows that the distribution of the phosphorus concentrations, calculated by the model, is very close to a normal distribution. Thus we can say with certainty that the given model describes the process of phosphorus transformation in accordance with available observations. Therefore, this test illustrates that two independent sets of data, drawn at random from the observations and modeling results, have a similar (or close to similar) distribution, which is normal. Each phosphorus fraction in these samples of data have a specific value of mean, μ, and a variance σ^2 and also a standard error of mean $\bar{\sigma}$. These statistical criteria are calculated using the formulae:

$$\mu = \sum_{i=1}^{n} P_i / n \qquad (6)$$

$$\sigma^2 = \sum_{i=1}^{n} (P_i - \mu)^2/(n-1) \qquad (7)$$

$$\bar{\sigma} = \sigma/\sqrt{n} \qquad (8)$$

Table 6. Test on normality of distribution of phosphorus concentrations in observations and model output data generalized for 1976-1978.

Phosphorus fraction	Basin	Observations: Number of points(%) in range			Model output data: Number of points(%) in range		
		$\pm 1\ \sigma$	$\pm 2\ \sigma$	$\pm 3\ \sigma$	$\pm 1\ \sigma$	$\pm 2\ \sigma$	$\pm 3\ \sigma$
Total phosphorus	I	72.0	92.0	100.0	20.0	80.0	100.0
	II	72.0	96.0	100.0	24.0	80.0	100.0
	III	65.4	92.3	100.0	65.4	84.6	96.2
	IV	80.0	96.0	96.0	76.0	96.0	100.0
	whole lake	72.3	94.1	99.0	46.5	85.1	99.1
Particulate organic phosphorus	I	72.0	96.0	100.0	80.0	96.0	100.0
	II	56.0	100.0	-	80.0	100.0	-
	III	73.1	96.1	100.0	69.2	88.5	96.2
	IV	88.0	96.0	96.0	60.0	92.0	96.0
	whole lake	72.3	97.0	99.0	72.3	94.0	98.0
Total dissolved phosphorus	I	72.0	92.0	100.0	76.0	100.0	-
	II	72.0	96.0	100.0	44.0	100.0	-
	III	77.8	96.3	100.0	63.0	92.6	100.0
	IV	68.0	100.0	-	52.0	92.0	100.0
	whole lake	72.5	96.0	100.0	58.8	96.0	100.0
Dissolved organic phosphorus	I	68.0	92.0	100.0	68.0	100.0	-
	II	76.0	96.0	100.0	64.0	100.0	-
	III	80.0	96.1	100.0	61.5	100.0	-
	IV	60.0	100.0	-	36.0	100.0	-
	whole lake	71.3	96.0	100.0	57.4	100.0	-
Dissolved inorganic phosphorus	I	56.0	96.0	100.0	76.0	96.0	100.0
	II	68.0	96.0	100.0	20.0	100.0	-
	III	70.4	96.3	100.0	81.5	96.3	96.3
	IV	68.0	96.0	100.0	76.0	92.0	96.0
	whole lake	65.7	96.1	100.0	65.7	95.1	98.1

where P_i is the concentrations of phosphorus compound in observations or modeling results and n is the total number of components in the series considered.

With these statistics, it is possible to calculate the variance of mean, μ, for observations and modeling results with a 95% confidence interval using a simple statistical expression $\mu \pm 1.96\ \bar{\sigma}$ (Allard 1977). The results of these calculations, given in Table 7, show that 95% confidence intervals of mean values for all phosphorus fractions are in reasonable agreement for two samples, excluding the data for Basin II, where phosphorous loading seems to be lower than expected, according to observed phosphorus levels. Thus it allows one to conclude that mean values for phosphorus fractions estimated on the basis of modeling results in the same degree as observations correspond to their general population and therefore the process of phosphorus transformation is similarly explained by two independent sets of data--observed and simulated phosphorus concentrations.

As the model gives detailed information on the continuous temporary changes of phosphorus concentrations (for the three-year period, in the different basins) it is possible to define more exactly the mean values of the phosphorus fractions and their variations for an individual year and for various basins within the lake. In order to be able to compare these new and more precisely evaluated statistical values with those obtained on the limited observations available, only the phosphorus concentrations modeled, which cover the spring-autumn period, i.e., when the observations were actually made, will be analyzed.

The results of calculations of mean and variances from the modeling results on the basis of data for each five-day period within the spring-autumn months or between day 90 and 320, are presented in Table 8. These values differ slightly from those estimated previously (Table 7). The analysis of the data in Table 8 shows that

(i) for the environmental conditions of 1976, the mean values of phosphorus fractions as well as their standard deviations are higher by about 1.5-2 times than those for 1977 and 1978;

Table 7. Summary statistics of phosphorus data.

Year	Basin	Phosphorus fraction	Data set	μ	σ	$\bar{\sigma}$	95% Confidence Interval on Mean	
							Lower limit	Upper limit
1976	whole lake	Total phosphorus	Observed	0.0458	0.0222	0.0039	0.0381	0.0535
			Simulated	0.0466	0.0205	0.0036	0.0395	0.0537
		Particulate organic phosphorus	Observed	0.0166	0.0124	0.0022	0.0123	0.0209
			Simulated	0.0302	0.0131	0.0023	0.0257	0.0347
		Total dissolved phosphorus	Observed	0.0263	0.0157	0.0028	0.0209	0.0317
			Simulated	0.0181	0.0095	0.0017	0.0148	0.0214
		Dissolved organic phosphorus	Observed	0.0171	0.0125	0.0022	0.0128	0.0214
			Simulated	0.0101	0.0079	0.0014	0.0074	0.0128
		Dissolved inorganic phosphorus	Observed	0.0062	0.0028	0.0005	0.0052	0.0072
			Simulated	0.0059	0.0021	0.0004	0.0052	0.0066
1977	whole lake	Total phosphorus	Observed	0.0552	0.0211	0.0035	0.0484	0.0620
			Simulated	0.0356	0.0157	0.0026	0.0305	0.0407
		Particulate organic phosphorus	Observed	0.0293	0.0153	0.0025	0.0244	0.0342
			Simulated	0.0221	0.0098	0.0016	0.0189	0.0253
		Total dissolved phosphorus	Observed	0.0175	0.0075	0.0012	0.0151	0.0199
			Simulated	0.0136	0.0067	0.0011	0.0115	0.0157
		Dissolved organic phosphorus	Observed	0.0134	0.0072	0.0012	0.0111	0.0157
			Simulated	0.0085	0.0057	0.0009	0.0067	0.0103
		Dissolved inorganic phosphorus	Observed	0.0043	0.0014	0.0002	0.0039	0.0047
			Simulated	0.0050	0.0026	0.0004	0.0042	0.0058
1978	whole lake	Total phosphorus	Observed	0.0452	0.0255	0.0045	0.0364	0.0540
			Simulated	0.0331	0.0164	0.0029	0.0274	0.0388
		Particulate organic phosphorus	Observed	0.0198	0.0173	0.0031	0.0136	0.0258
			Simulated	0.0218	0.0109	0.0019	0.0180	0.0258
		Total dissolved phosphorus	Observed	0.0182	0.0102	0.0018	0.0150	0.0214
			Simulated	0.0113	0.0068	0.0012	0.0089	0.0137
		Dissolved organic phosphorus	Observed	0.0116	0.0094	0.0017	0.0083	0.0149
			Simulated	0.0067	0.0053	0.0009	0.0049	0.0085
		Dissolved inorganic phosphorus	Observed	0.0066	0.0023	0.0004	0.0058	0.0074
			Simulated	0.0051	0.0029	0.0005	0.0041	0.0061
1976-8	I	Total phosphorus	Observed	0.0785	0.0162	0.0032	0.0722	0.0908
			Simulated	0.0582	0.0168	0.0034	0.0515	0.0649
		Particulate organic phosphorus	Observed	0.0403	0.0134	0.0027	0.0351	0.0455
			Simulated	0.0364	0.0116	0.0023	0.0319	0.0409
		Total dissolved phosphorus	Observed	0.0285	0.0140	0.0028	0.0230	0.0340
			Simulated	0.0216	0.0078	0.0016	0.0185	0.0247
		Dissolved organic phosphorus	Observed	0.0218	0.0135	0.0027	0.0165	0.0271
			Simulated	0.0144	0.0076	0.0015	0.0114	0.0174
		Dissolved inorganic phosphorus	Observed	0.0067	0.0031	0.0006	0.0055	0.0079
			Simulated	0.0071	0.0033	0.0007	0.0058	0.0084
	II	Total phosphorus	Observed	0.0524	0.0139	0.0028	0.0469	0.0579
			Simulated	0.0344	0.0154	0.0031	0.0283	0.0405
		Particulated organic phosphorus	Observed	0.0250	0.0119	0.0024	0.0203	0.0297
			Simulated	0.0241	0.0099	0.0020	0.0202	0.0280
		Total dissolved phosphorus	Observed	0.0201	0.0092	0.0018	0.0165	0.0237
			Simulated	0.0106	0.0067	0.0013	0.0080	0.0132
		Dissolved organic phosphorus	Observed	0.0141	0.0091	0.0018	0.0105	0.0177
			Simulated	0.0085	0.0059	0.0012	0.0062	0.0108
		Dissolved inorganic phosphorus	Observed	0.0062	0.0023	0.0005	0.0053	0.0071
			Simulated	0.0032	0.0009	0.0002	0.0028	0.0036
	III	Total phosphorus	Observed	0.0360	0.0104	0.0020	0.0321	0.0399
			Simulated	0.0315	0.0132	0.0026	0.0264	0.0366
		Particulate organic phosphorus	Observed	0.0144	0.0081	0.0008	0.0128	0.0160
			Simulated	0.0197	0.0085	0.0008	0.0181	0.0213
		Total dissolved phosphorus	Observed	0.0156	0.0057	0.0011	0.0134	0.0178
			Simulated	0.0115	0.0053	0.0010	0.0095	0.0135
		Dissolved organic phosphorus	Observed	0.0104	0.0056	0.0011	0.0082	0.0126
			Simulated	0.0060	0.0036	0.0007	0.0046	0.0074
		Dissolved inorganic phosphorus	Observed	0.0052	0.0015	0.0003	0.0046	0.0058
			Simulated	0.0056	0.0017	0.0003	0.0050	0.0062
	IV	Total phosphorus	Observed	0.0299	0.0136	0.0027	0.0246	0.0352
			Simulated	0.0294	0.0117	0.0023	0.0249	0.0339
		Particulate organic phosphorus	Observed	0.0097	0.0098	0.0020	0.0058	0.0136
			Simulated	0.0182	0.0079	0.0016	0.0150	0.0214
		Total dissolved phosphorus	Observed	0.0143	0.0049	0.0010	0.0124	0.0162
			Simulated	0.0109	0.0049	0.0010	0.0090	0.0128
		Dissolved organic phosphorus	Observed	0.0098	0.0046	0.0009	0.0080	0.0116
			Simulated	0.0046	0.0032	0.0006	0.0033	0.0059
		Dissolved inorganic phosphorus	Observed	0.0045	0.0019	0.0004	0.0038	0.0052
			Simulated	0.0055	0.0022	0.0004	0.0046	0.0064
	whole lake	Total phosphorus	Observed	0.0491	0.0232	0.0023	0.0446	0.0536
			Simulated	0.0383	0.0183	0.0018	0.0347	0.0419
		Particulate organic phosphorus	Observed	0.0223	0.0160	0.0016	0.0192	0.0254
			Simulated	0.0246	0.0118	0.0012	0.0222	0.0270
		Total dissolved phosphorus	Observed	0.0196	0.0106	0.0011	0.0175	0.0217
			Simulated	0.0136	0.0077	0.0008	0.0121	0.0151
		Dissolved organic phosphorus	Observed	0.0140	0.0100	0.0010	0.0120	0.0160
			Simulated	0.0084	0.0065	0.0006	0.0071	0.0097
		Dissolved inorganic phosphorus	Observed	0.0056	0.0024	0.0002	0.0051	0.0061
			Simulated	0.0053	0.0026	0.0003	0.0048	0.0058

Table 8. Summary statistics for data on concentrations of phosphorus fractions computed by model (n = 46).

Phosphorus fractions	Years	Basins								Whole lake	
		1		2		3		4			
		mean	std. dev.	mean	std. dev.	mean	std. dev.	mean	std. dev.	mean	std. dev.
Total phosphorus	1976	0.0759	0.0254	0.0510	0.0211	0.0466	0.0186	0.0415	0.0162	0.0537	0.0244
	1977	0.0562	0.0136	0.0324	0.0092	0.0299	0.0084	0.0280	0.0075	0.0366	0.0151
	1978	0.0608	0.0140	0.0314	0.0095	0.0277	0.0077	0.0255	0.0066	0.0364	0.0174
	1976-8	0.0643	0.0202	0.0383	0.0169	0.0347	0.0151	0.0317	0.0130	0.0422	0.0210
Particulate organic phosphorus	1976	0.0482	0.0160	0.0349	0.0136	0.0294	0.0115	0.0266	0.0102	0.0348	0.0154
	1977	0.0364	0.0084	0.0229	0.0064	0.0193	0.0053	0.0180	0.0048	0.0241	0.0097
	1978	0.0388	0.0091	0.0228	0.0064	0.0186	0.0051	0.0171	0.0045	0.0243	0.0108
	1976-8	0.0412	0.0127	0.0268	0.0110	0.0224	0.0092	0.0205	0.0082	0.0277	0.0132
Total dissolved phosphorus	1976	0.0277	0.0119	0.0157	0.0083	0.0170	0.0083	0.0150	0.0070	0.0188	0.0104
	1977	0.0198	0.0070	0.0096	0.0045	0.0106	0.0039	0.0100	0.0037	0.0125	0.0065
	1978	0.0220	0.0079	0.0087	0.0046	0.0090	0.0034	0.0084	0.0031	0.0120	0.0077
	1976-8	0.0232	0.0097	0.0113	0.0068	0.0122	0.0066	0.0111	0.0056	0.0145	0.0089
Dissolved organic phosphorus	1976	0.0208	0.0112	0.0131	0.0075	0.0093	0.0055	0.0077	0.0047	0.0127	0.0091
	1977	0.0144	0.0067	0.0080	0.0042	0.0059	0.0030	0.0052	0.0026	0.0084	0.0057
	1978	0.0152	0.0083	0.0072	0.0045	0.0048	0.0029	0.0043	0.0026	0.0079	0.0067
	1976-8	0.0168	0.0093	0.0094	0.0061	0.0067	0.0044	0.0057	0.0037	0.0097	0.0076
Dissolved inorganic phosphorus	1976	0.0069	0.0025	0.0026	0.0012	0.0076	0.0033	0.0072	0.0029	0.0061	0.0033
	1977	0.0055	0.0016	0.0016	0.0005	0.0048	0.0015	0.0049	0.0017	0.0042	0.0021
	1978	0.0068	0.0032	0.0015	0.0004	0.0041	0.0011	0.0041	0.0013	0.0041	0.0026
	1976-8	0.0064	0.0026	0.0019	0.0009	0.0055	0.0027	0.0054	0.0025	0.0048	0.0028

(ii) for 1977 and 1978 the mean values of phosphorus fractions as well as the variances are close to each other;

(iii) gradients of phosphorus concentrations changes similar to those in observations for 1976-1978, have been obtained by simulation.

A comparison of mean values and variances for observations (Tables 4-5) and modeling results (Table 8) shows that there is some difference in these characteristics. Since we received these data independently of each other, it is interesting to check how statistically significant the differences obtained by the so-called variance ratio of F-test are. It is defined as the ratio of larger to smaller, of the two variance estimates for two small data sets, σ_1^2 and σ_2^2:

$$F = \sigma_1^2/\sigma_1^2 \qquad (9)$$

Calculated values of the F-ratio presented in Table 9 should be compared with the statistical variance ratio taken from tables of the F-distribution (Bailey 1959). These values for the 5% level of significance and known degrees of freedom are also shown in Table 9. The comparison of F values shows that computed F-ratios are, as a rule, smaller than statistical F-distribution. Therefore, the general conclusion of the analysis is that the variances of means in two group of data, observations and modeling results, are homogenous so far as we can tell by comparing data sets with a different number of components. Further:

(i) differences obtained for DIP and DOP variances in observations and simulation results for Basin II are statistically significant while for other basins it may be considered quite reasonable; DIP dynamics are simulated better for 1976-1977 than for 1978, while DOP dynamics are better simulated for 1976 and 1978 than for 1977; the mean values of DIP and DOP as shown by the modeling results are smaller than in observations;

(ii) differences obtained for total dissolved phosphorus in all cases are in acceptable agreement with the statistical point of view as a whole, excluding the results for Basin

Table 9. Comparison of variances in the phosphorus concentrations.

Phosphorus fraction	Year	Data set	Basins I σ	I F-ratio	I F-distr.	II σ	II F-ratio	II F-distr.	III σ	III F-ratio	III F-distr.	IV σ	IV F-ratio	IV F-distr.
Total P	1976	Observed Simulated	0.0197 0.0254	1.66	3.33	0.0117 0.0211	3.25	3.33	0.0100 0.0186	3.45	3.33	0.0080 0.0162	4.10	3.33
	1977	Observed Simulated	0.0080 0.0136	2.89	3.03	0.0087 0.0092	1.12	3.03	0.0082 0.0084	1.05	2.82	0.0178 0.0075	5.63	2.16
	1978	Observed Simulated	0.0204 0.0140	2.12	2.23	0.0139 0.0095	2.14	2.23	0.0104 0.0077	1.82	2.23	0.0114 0.0066	2.98	2.23
	1976-1978	Observed Simulated	0.0162 0.0202	1.55	1.78	0.0139 0.0169	1.48	1.78	0.0104 0.0151	2.11	1.75	0.0136 0.0130	1.09	1.60
Particulate organic P	1976	Observed Simulated	0.0120 0.0160	1.78	3.33	0.0070 0.0136	3.77	3.33	0.0045 0.0115	6.53	3.33	0.0037 0.0102	7.59	3.33
	1977	Observed Simulated	0.0101 0.0084	1.45	2.16	0.0070 0.0064	1.20	2.16	0.0084 0.0053	2.51	2.10	0.0129 0.0048	7.22	2.16
	1978	Observed Simulated	0.0155 0.0091	2.90	2.23	0.0109 0.0064	2.90	2.23	0.0067 0.0051	1.73	2.23	0.0068 0.0045	2.28	2.23
	1976-1978	Observed Simulated	0.0134 0.0127	1.11	1.60	0.0119 0.0110	1.17	1.60	0.0081 0.0092	1.29	1.75	0.0098 0.0082	1.43	1.60
Total dissolved P	1976	Observed Simulated	0.0186 0.0119	2.44	2.23	0.0113 0.0083	1.85	2.23	0.0072 0.0083	1.33	3.33	0.0048 0.0070	2.12	3.33
	1977	Observed Simulated	0.0082 0.0070	1.37	2.16	0.0061 0.0045	1.84	2.16	0.0043 0.0039	1.22	2.10	0.0042 0.0037	1.29	2.16
	1978	Observed Simulated	0.0136 0.0079	2.96	2.23	0.0105 0.0046	5.21	2.23	0.0054 0.0034	2.52	2.23	0.0049 0.0031	2.49	2.23
	1976-1978	Observed Simulated	0.0140 0.0097	2.08	1.60	0.0092 0.0068	1.83	1.60	0.0057 0.0066	1.34	1.75	0.0049 0.0056	1.31	1.77
Dissolved organic P	1976	Observed Simulated	0.0189 0.0112	2.85	2.23	0.0115 0.0075	2.35	2.23	0.0064 0.0055	1.35	3.33	0.0038 0.0047	1.53	3.33
	1977	Observed Simulated	0.0075 0.0067	1.25	2.16	0.0065 0.0042	2.39	2.16	0.0046 0.0030	2.35	2.10	0.0040 0.0026	2.37	2.16
	1978	Observed Simulated	0.0126 0.0083	2.30	2.23	0.0099 0.0045	4.84	2.23	0.0053 0.0029	3.34	2.23	0.0051 0.0026	3.85	2.23
	1976-1978	Observed Simulated	0.0135 0.0093	2.11	1.60	0.0091 0.0061	2.23	1.60	0.0056 0.0044	1.62	1.59	0.0046 0.0037	1.54	1.60
Dissolved inorganic P	1976	Observed Simulated	0.0033 0.0025	1.74	2.23	0.0024 0.0012	4.00	2.23	0.0019 0.0033	3.02	3.33	0.0024 0.0029	1.46	3.33
	1977	Observed Simulated	0.0021 0.0016	1.72	2.16	0.0011 0.0005	4.84	2.16	0.0009 0.0015	2.77	2.82	0.0007 0.0017	5.90	3.03
	1978	Observed Simulated	0.0030 0.0032	1.14	3.33	0.0025 0.0004	39.1	2.23	0.0008 0.0011	1.89	3.33	0.0020 0.0013	2.37	2.23
	1976-1978	Observed Simulated	0.0031 0.0026	1.42	1.60	0.0023 0.0009	6.53	1.60	0.0015 0.0027	3.24	1.75	0.0019 0.0025	1.73	1.77

II, 1978; the dynamics of this phosphorus fraction are described with approximately similar accuracy for all years studied;

(iii) differences obtained for particulate organic phosphorus in two samples are statistically significant for Basin II, 1976 and Basin III-IV, 1976-1977, and as a whole the dynamics of this phosphorus fraction are described better for 1977-1978 than for 1976;

(iv) differences obtained for total phosphorus variances are statistically significant for Basin IV; the dynamics of total phosphorus are better described for 1976 and 1978 than for 1977.

In the next statistical test, all the phosphorus data available for individual phosphorus fractions were combined and the variances calculated for both samples, observations and modeling results, were compared. As a result of this test, a model error (β) is calculated by

$$\beta = (\sigma_e^2/\sigma_d^2) \cdot 100\% \tag{10}$$

where σ_e and σ_d are standard deviations for modelling results and observations, respectively and these are

$$\sigma_e^2 = \sum_{i=1}^{n} (\Delta P - \mu_e)^2/(n-1) \tag{11}$$

$$\sigma_d^2 = \sum_{i=1}^{n} (P_{obs} - P_{obs}^m)^2/(n-1) \quad ; \tag{12}$$

P_{obs} and P_{sim} are the observed and simulated concentrations of phosphorus fractions;

ΔP is the difference between observed and simulated values of phosphorus fractions;

P_{obs}^m is the mean phosphorus concentration in observation;

μ_e is the mean difference in phosphorus concentrations in the observed and simulated time series and it is equal to

$$\mu_e = \sum_{i=1}^{n} (P_{obs} - P_{sim})/n \tag{13}$$

Criteria β allows one to estimate how the model describes the dynamic changes of phosphorus concentrations as a whole in both samples and to determine how fluctuations in phosphorus fractions in both samples correspond with each other. A reasonable agreement of modeling results to observations may be assumed for the cases when the model errors calculated by formulas (10)-(13) are between 25-75% (Beck 1978). Results of calculations of this model error presented in Table 10 show that evaluated errors lie in the range 20.4-73.2% excluding two cases with 115.5% and 139.2% for Basins III-IV, 1976. Table 10 also shows that a better model description of observed phosphorus concentrations is obtained for 1978 than for 1976-1977 and for the entire period covering 1976-1978, the phosphorus dynamics for Basins I-IV is simulated with an accuracy evaluated to be equal to 34.7-77.9% (with the mean error for the whole lake being 41.6%).

In order to seek a quantitative relationship between phosphorus concentrations in the observed and simulated time series, the method of regression analysis was also used. The simplest form of the relationship is presented by the simple regression equation

$$P_{obs} = a + b \cdot P_{sim} \qquad (14)$$

where a and b are regression coefficients, intercept and slope respectively.

Taking into account the features of the original phosphorus data in observations mentioned above, the following assumptions were formulated as important for the regression analysis:

(i) each phosphorus observation should have an individual weight in keeping with the peculiarities in raw measurements;

(ii) the weight of an observation with a large variance should be lower than for one with a small variance;

(iii) the weight of an observation in the basin where there are a larger number of sampling stations per unit area (i.e., when a value of "density coefficient of observation" is higher) should be higher than for one in a basin with a lower density of observations;

(iv) the weight of an observation with a high mean value should be higher than for one with a low mean value.

Table 10. Review of model errors calculated on equations (10-13) for samples with all the phosphorus data.

Year	Basin	Sample	μ mgP/ℓ	σ mgP/ℓ	σ_e^2/σ_d^2 %%
1976	1	Observed Simulated	0.0353 0.0300	0.0268 0.0211	61.9
	2	Observed Simulated	0.0219 0.0207	0.0162 0.0138	73.2
	3	Observed Simulated	0.0161 0.0191	0.0116 0.0125	115.5
	4	Observed Simulated	0.0139 0.0171	0.0100 0.0118	139.2
	whole lake	Observed Simulated	0.0218 0.0218	0.0192 0.0155	65.4
1977	1	Observed Simulated	0.0354 0.0270	0.0276 0.0136	24.3
	2	Observed Simulated	0.0276 0.0146	0.0218 0.0136	38.8
	3	Observed Simulated	0.0177 0.0135	0.0143 0.0088	38.3
	4	Observed Simulated	0.0154 0.0128	0.0147 0.0111	57.1
	whole lake	Observed Simulated	0.0238 0.0169	0.0215 0.0124	33.3
1978	1	Observed Simulated	0.0347 0.0255	0.0285 0.0129	20.4
	2	Observed Simulated	0.0207 0.0133	0.0168 0.0099	34.4
	3	Observed Simulated	0.0144 0.0122	0.0108 0.0065	36.0
	4	Observed Simulated	0.0114 0.0115	0.0096 0.0071	54.4
	whole lake	Observed Simulated	0.0203 0.0156	0.0200 0.0101	25.2
1976-8	1	Observed Simulated	0.0352 0.0276	0.0274 0.0162	34.7
	2	Observed Simulated	0.0235 0.0162	0.0187 0.0134	51.5
	3	Observed Simulated	0.0162 0.0148	0.0125 0.0099	63.1
	4	Observed Simulated	0.0136 0.0138	0.0118 0.0104	77.9
	whole lake	Observed Simulated	0.0221 0.0180	0.0203 0.0131	41.6

The application of weight is generally accepted in the regression analysis when the observations include some measurement errors or when state variables do not quite correspond to the one specified in the model (Allard 1977). According to the assumptions formulated above, the equation for the computation of the weight, WG, of individual phosphorus observations may be written as

$$WG = (N/S) \cdot \frac{P_m^t}{P_{max}^t - P_{min}^t} \qquad (15)$$

where N is the number of sampling stations in the basin considered;
S is a square of the basin considered;
P_{max}^t , P_{min}^t and P_m^t are maximum, minimum and mean phosphorus concentrations at time t in raw sets of measurements.

The uncertainties in the observations (or the measurement errors in the original data) will increase the dispersion of the P_{obs} values around their expected value at each value of P_{sim} in the regression relationship. The standard linear regression statistics correlated for the weight of the individual observations were computed with equation (15). The adequacy of the model as a whole may be evaluated on the statistical values of mean, standard deviation, minimal and maximal concentrations of phosphorus fractions in two sets of independent data, as well as on the values of r-squared, b and standard error of estimate for the regression relationship between both data series.

The results of this analysis for the time series joined for all phosphorus compounds and for the whole lake in 1976-1978 are summarized in Tables 11 and 12. The following conclusions may be extracted from an analysis of the statistics in Tables 11 and 12:

(i) the given model reasonably describes the range of the fluctuations of all phosphorus fractions observed in the measurements;

(ii) the weighting increases the level of mean values of all phosphorus fractions and slightly changes the values of standard deviations in the time series of phosphorus observations;

Table 11. Statistics from regression analysis (calculated from regression of "Observation" on "Simulation") of entire phosphorus data.

Year	Basin	Amount of rows in samples	Data	Mean	Std. deviation	Regression coefficients: in simple regression a	in simple regression b	in simple regression r^2	in weighted regression a	in weighted regression b	in weighted regression r^2
1976	1	40	Observed	0.0359	0.0269	0.0307	0.589	0.328	0.0145	0.727	0.433
			Simulated	0.0305	0.0242		(4.2)			(5.2)	
	2	40	Observed	0.0222	0.0162	0.0146	0.633	0.552	0.0132	0.674	0.544
			Simulated	0.0212	0.0187		(6.7)			(6.6)	
	3	40	Observed	0.0163	0.0116	0.0115	0.488	0.499	0.0066	0.545	0.560
			Simulated	0.0194	0.0162		(6.0)			(6.8)	
	4	40	Observed	0.0141	0.0100	0.0111	0.425	0.499	0.0094	0.413	0.554
			Simulated	0.0173	0.0150		(6.0)			(6.7)	
	whole lake	160	Observed	0.0225	0.0197	0.0203	0.667	0.378	0.0114	0.690	0.428
			Simulated	0.0222	0.0191		(9.7)			(10.8)	
1977	1	45	Observed	0.0362	0.0274	0.0112	1.124	0.732	0.0028	1.230	0.771
			Simulated	0.0274	0.0196		(10.6)			(11.8)	
	2	45	Observed	0.0281	0.0217	0.0156	1.296	0.574	0.0065	1.491	0.680
			Simulated	0.0149	0.0117		(7.4)			(9.3)	
	3	52	Observed	0.0180	0.0143	0.0089	1.039	0.538	0.0004	1.328	0.702
			Simulated	0.0136	0.0098		(7.5)			(10.6)	
	4	45	Observed	0.0156	0.0148	0.0064	1.225	0.332	0.0076	0.704	0.415
			Simulated	0.0129	0.0096		(4.5)			(5.4)	
	whole lake	187	Observed	0.0239	0.0215	0.0116	1.133	0.692	0.0047	1.242	0.705
			Simulated	0.0169	0.0143		(20.3)			(20.9)	
1978	1	40	Observed	0.0353	0.0287	0.0047	1.327	0.838	0.0002	1.376	0.861
			Simulated	0.0259	0.0193		(13.6)			(14.9)	
	2	40	Observed	0.0210	0.0170	0.0093	1.181	0.700	0.0075	1.142	0.578
			Simulated	0.0136	0.0120		(9.2)			(7.0)	
	3	40	Observed	0.0146	0.0109	0.0046	0.994	0.743	0.0055	0.819	0.754
			Simulated	0.0123	0.0099		(10.2)			(10.5)	
	4	40	Observed	0.0195	0.0122	0.0044	0.884	0.626	0.0033	0.974	0.640
			Simulated	0.0170	0.0109		(7.8)			(8.0)	
	whole lake	160	Observed	0.0203	0.0201	0.0029	1.341	0.855	0.0025	1.235	0.780
			Simulated	0.0156	0.0143		(30.4)			(23.5)	
1976-8	1	125	Observed	0.0353	0.0275	0.0183	0.939	0.576	0.0071	1.046	0.644
			Simulated	0.0276	0.0209		(12.8)			(14.8)	
	2	125	Observed	0.0237	0.0186	0.0184	0.823	0.432	0.0117	0.953	0.526
			Simulated	0.0163	0.0146		(9.6)			(11.6)	
	3	132	Observed	0.0163	0.0125	0.0117	0.672	0.446	0.0062	0.795	0.548
			Simulated	0.0148	0.0123		(10.1)			(12.5)	
	4	125	Observed	0.0137	0.0119	0.0106	0.676	0.277	0.0076	0.657	0.484
			Simulated	0.0138	0.0115		(6.8)			(10.7)	
	whole lake	507	Observed	0.0221	0.0203	0.0047	0.964	0.585	0.0072	0.988	0.608
			Simulated	0.0180	0.0161		(26.6)			(28.0)	

Table 12. Weighted regression analysis for phosphorus concentrations in observed and simulated phosphorus concentrations combined for the three-year period (1976-1978) and for the entire lake.

Phosphorus fraction	Weighted regression equation	r^2	Standard error of regression
Total phosphorus	$P_{obs} = 0.0192 + 0.827 \cdot P_{sim}$ (8.7)	0.441	0.0174
Particulate organic phosphorus	$P_{obs} = 0.0172 + 0.464 \cdot P_{sim}$ (3.9)	0.135	0.0142
Total dissolved phosphorus	$P_{obs} = 0.0103 + 0.747 \cdot P_{sim}$ (6.0)	0.269	0.0105
Dissolved organic phosphorus	$P_{obs} = 0.0105 + 0.722 \cdot P_{sim}$ (5.3)	0.224	0.0101
Dissolved inorganic phosphorua	$P_{obs} = 0.0049 + 0.261 \cdot P_{sim}$ (3.1)	0.100	0.0024

Note: T-statistics is shown in brackets

(iii) the satisfactory correlation between phosphorus concentrations in observed and simulated time series were evaluated with a tendency for regression coefficient a to be slightly larger than zero* and the relationship between phosphorus fractions in both series may be considered as distinctly positive, with b changing from 0.261 (for DIP) to 0.827 (for total phosphorus) and significant T-statistics (3.1-8.7);

(iv) the values of r-squared, show that the given model also acceptably described the trend in the temporary changes in concentrations of the individual phosphorus fractions, such as total phosphorus, dissolved phosphorus and dissolved organic phosphorus; this trend is described by the model with less accuracy for particulate organic phosphorus and dissolved inorganic phosphorus.

*Order of a is comparable with a standard error of regression.

Finally, in an examination of model adequacy, Theil's inequality coefficient (Theil 1971) was calculated by

$$\rho = \frac{\sqrt{I/n \sum_{i=1}^{n} (P_{obs} - P_{sim})^2}}{\sqrt{I/n \sum_{i=1}^{n} (P_{obs})^2} + \sqrt{I/n \sum_{i=1}^{n} (P_{sim})^2}} \qquad (16)$$

This coefficient is the index which measures the degree to which a simulation model describes the observation. This index varies between 0 and 1 and if $\rho = 0$, the model description of the observations is perfect. The values of this coefficient, ρ, computed for individual phosphorus fractions, individual basins and for each year studied, as well as for combined phosphorus data and the total period of study covering 1976-1978, are presented in Table 13. Summarizing the results in Table 13, it is possible to conclude that:

(i) the range of errors in the simulation of the dynamics of individual phosphorus fractions are 0.154-0.221 (mean 0.2) for total P, 0.214-0.291 (mean 0.251) for DIP; 0.243-0.353 (mean 0.283) for particulate organic P; 0.269-0.325 (mean 0.296) for total dissolved P and 0.303-0.405 (mean 0.369) for DOP;

(ii) the range of errors in the simulation of phosphorus dynamics in Basin I is 0.203-0.261 (mean 0.225) while for Basins II-IV it is equal to 0.250-0.347 (0.295), 0.204-0.284 (0.252) and 0.237-0.307 (0.290) respectively;

(iii) the error in the simulation of phosphorus dynamics is estimated to be equal to 0.267 for 1976, 0.262 for 1977, 0.223 for 1978 and 0.253 for the three-year period of study, 1976-1978.

Thus the various statistical methods applied in this study allow one to conclude that as a whole, the simulation results may be considered as representing the phenomena of phosphorus transformation, so far as we can tell from relatively sparse phosphorus measurements available for each of the years in the period 1976-1978.

Table 13. Model assessment by Theil's inequality coefficient.

Year	Basin	TP	PPART	PSOL	DIP	DOP	All P-fractions
1976	1	0.197	0.249	0.377	0.135	0.449	0.261
	2	0.200	0.360	0.302	0.319	0.309	0.250
	3	0.267	0.496	0.209	0.222	0.345	0.284
	4	0.292	0.581	0.255	0.197	0.407	0.307
	whole lake	0.221	0.353	0.325	0.214	0.405	0.267
1977	1	0.157	0.198	0.233	0.322	0.281	0.203
	2	0.302	0.306	0.373	0.281	0.400	0.347
	3	0.187	0.248	0.249	0.181	0.360	0.248
	4	0.181	0.359	0.273	0.361	0.395	0.304
	whole lake	0.206	0.255	0.272	0.291	0.333	0.262
1978	1	0.142	0.200	0.194	0.233	0.303	0.205
	2	0.176	0.250	0.428	0.456	0.374	0.276
	3	0.135	0.304	0.277	0.149	0.353	0.204
	4	0.185	0.443	0.226	0.194	0.418	0.237
	whole lake	0.154	0.243	0.269	0.258	0.336	0.223
1976-8	1	0.168	0.214	0.295	0.233	0.373	0.225
	2	0.239	0.305	0.356	0.362	0.353	0.295
	3	0.229	0.352	0.238	0.187	0.351	0.252
	4	0.231	0.438	0.254	0.253	0.406	0.290
	whole lake	0.200	0.283	0.296	0.251	0.369	0.253

SENSITIVITY ANALYSIS

In providing additional insight on the model's behavior, 20 sensitivity analysis model runs were conducted, using all the available data for 1977. The primary purpose of these runs is to understand how the quality of input data used in the model for calculating the rates of phosphorus transformation may change the model output. Among all the data used, most attention in these runs was given to considering the role of factors defining the state of the environment. They may be subdivided into non-controllable factors such as temperature and radiation, as well as controllable ones such as nutrient loading. In sensitivity analysis runs, different degrees of time averaging were done for the raw measurements of temperature, radiation and phosphorus loading, which were used as input data. The observations available included the daily average measurements of temperature and radiation and weekly measurements of phosphorus loading from the River Zala for 1977. The dynamics of these characteristics for this year are shown in Figures 1, 2 and 3. During simulation of the annual phosphorus dynamics given above, the interpolated values of phosphorus concentrations in River Zala discharge water for each day is used so that the time scale of averaging all variables defining the state of the environment is a day.

In sensitivity analysis runs a different time scale of averaging was used, corresponding to day, week, month and season, of the values of the characteristics mentioned. Appendixes C, D and E show the values of temperature, radiation and phosphorus concentrations in River Zala discharge water, averaging for week, month and season, starting from January 1, 1977. These values were used in sensitivity model runs.

Tables 14-15 present some of the results for Basin I, obtained in various sensitivity runs where different combinations of averaged characteristics were used. Under the column "Averaging data" in Tables 14-15 capital letters T, R, and L refer to the temperature, radiation and loading respectively and the indexes d, w, m, and s mean the time scale of averaging, that is day, month or season, respectively.

Table 14. Monthly and annual mean values of phosphorus concentrations evaluated at the examination of model sensitivy to changes of input data on temperature, radiation and River Zala phosphorus load (Basin I, 1977).

Averaging data	Phosphorus fractions	Months: Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Annual mean
$T_dR_dL_d$	DIP	0.0189	0.0219	0.0137	0.0071	0.0036	0.0053	0.0063	0.0049	0.0048	0.0046	0.0091	0.0197	0.0100
	DOP	0.0057	0.0080	0.0136	0.0168	0.0212	0.0210	0.0199	0.0139	0.0102	0.0043	0.0045	0.0052	0.0121
	PPART	0.0271	0.0323	0.0326	0.0361	0.0289	0.0405	0.0487	0.0423	0.0387	0.0273	0.0257	0.0203	0.0334
	PSOL	0.0246	0.0300	0.0273	0.0239	0.0249	0.0263	0.0262	0.0188	0.0088	0.0150	0.0136	0.0249	0.0221
	Total P	0.0517	0.0623	0.0599	0.0600	0.0538	0.0668	0.0749	0.0611	0.0538	0.0362	0.0394	0.0452	0.0555
	Chl."a"	13.8	20.2	27.7	23.2	27.4	37.7	45.0	37.6	33.5	24.0	20.0	14.5	27.1
$T_wR_wL_w$	DIP	0.0186	0.0214	0.0138	0.0069	0.0036	0.0052	0.0062	0.0048	0.0047	0.0044	0.0089	0.0190	0.0098
	DOP	0.0058	0.0081	0.0135	0.0168	0.0207	0.0205	0.0190	0.0135	0.0099	0.0042	0.0045	0.0054	0.0119
	PPART	0.0272	0.0322	0.0324	0.0361	0.0287	0.0401	0.0486	0.0418	0.0385	0.0272	0.0257	0.0203	0.0333
	PSOL	0.0244	0.0295	0.0272	0.0237	0.0242	0.0257	0.0261	0.0183	0.0146	0.0086	0.0134	0.0244	0.0217
	Total P	0.0517	0.0618	0.0597	0.0599	0.0529	0.0658	0.0747	0.0601	0.0531	0.0358	0.0392	0.0448	0.0550
	Chl."a"	14.0	20.1	27.5	23.6	27.0	37.4	45.0	37.2	33.2	23.8	20.0	14.6	27.0
$T_mR_mL_m$	DIP	0.0182	0.0208	0.0140	0.0075	0.0035	0.0053	0.0059	0.0048	0.0041	0.0043	0.0092	0.0187	0.0097
	DOP	0.0059	0.0083	0.0137	0.0164	0.0221	0.0218	0.0175	0.0137	0.0076	0.0045	0.0046	0.0065	0.0119
	PPART	0.0271	0.0316	0.0324	0.0361	0.0288	0.0413	0.0461	0.0422	0.0363	0.0276	0.0251	0.0214	0.0330
	PSOL	0.0240	0.0291	0.0277	0.0239	0.0256	0.0271	0.0234	0.0185	0.0118	0.0088	0.0138	0.0252	0.0216
	Total P	0.0511	0.0607	0.0601	0.0600	0.0544	0.0684	0.0695	0.0607	0.0481	0.0364	0.0389	0.0466	0.0546
	Chl."a"	14.0	20.1	27.2	24.0	27.2	38.5	42.8	37.4	31.6	23.8	19.7	15.8	26.9
$T_dR_dL_s$	DIP	0.0207	0.0238	0.0127	0.0158	0.0042	0.0049	0.0058	0.0050	0.0049	0.0039	0.0092	0.0270	0.0115
	DOP	0.0059	0.0085	0.0137	0.0193	0.0275	0.0226	0.0184	0.0139	0.0104	0.0039	0.0040	0.0055	0.0128
	PPART	0.0273	0.0302	0.0332	0.0379	0.0313	0.0409	0.0468	0.0421	0.0391	0.0262	0.0244	0.0216	0.0335
	PSOL	0.0266	0.0323	0.0264	0.0351	0.0317	0.0274	0.0242	0.0189	0.0153	0.0078	0.0133	0.0325	0.0243
	Total P	0.0539	0.0625	0.0597	0.0730	0.0630	0.0683	0.0710	0.0610	0.0544	0.0340	0.0376	0.0542	0.0578
	Chl."a"	15.1	21.3	25.8	31.8	30.4	36.7	42.5	38.0	33.9	22.0	19.2	16.4	27.8
$T_dR_sL_d$	DIP	0.0170	0.0199	0.0129	0.0073	0.0036	0.0050	0.0059	0.0048	0.0049	0.0046	0.0085	0.0165	0.0092
	DOP	0.0062	0.0086	0.0138	0.0168	0.0212	0.0210	0.0200	0.0139	0.0102	0.0043	0.0046	0.0060	0.0122
	PPART	0.0277	0.0323	0.0323	0.0360	0.0289	0.0401	0.0482	0.0422	0.0388	0.0273	0.0258	0.0212	0.0334
	PSOL	0.0232	0.0284	0.0267	0.0240	0.0249	0.0260	0.0259	0.0187	0.0151	0.0089	0.0131	0.0225	0.0215
	Total P	0.0509	0.0607	0.0590	0.0601	0.0538	0.0662	0.0741	0.0609	0.0538	0.0362	0.0390	0.0437	0.0549
	Chl."a"	14.8	20.1	27.0	23.1	27.5	36.8	43.9	37.4	33.7	23.8	20.0	15.9	27.1
$T_sR_dL_d$	DIP	0.0175	0.0199	0.0185	0.0049	0.0041	0.0040	0.0049	0.0044	0.0047	0.0079	0.0110	0.0127	0.0095
	DOP	0.0062	0.0087	0.0120	0.0223	0.0166	0.0062	0.0121	0.0118	0.0103	0.0070	0.0071	0.0089	0.0108
	PPART	0.0280	0.0325	0.0299	0.0399	0.0337	0.0333	0.0367	0.0384	0.0375	0.0259	0.0254	0.0241	0.0321
	PSOL	0.0238	0.0286	0.0305	0.0272	0.0207	0.0103	0.0170	0.0162	0.0149	0.0150	0.0181	0.0216	0.0203
	Total P	0.0518	0.0610	0.0603	0.0670	0.0545	0.0436	0.0537	0.0546	0.0525	0.0408	0.0435	0.0457	0.0525
	Chl."a"	15.5	20.2	23.1	30.1	28.8	28.2	37.2	35.0	34.0	21.0	20.6	20.8	26.3
$T_sR_sL_s$	DIP	0.0171	0.0193	0.0159	0.0083	0.0043	0.0031	0.0042	0.0045	0.0048	0.0067	0.0100	0.0132	0.0093
	DOP	0.0070	0.0098	0.0125	0.0255	0.0209	0.0064	0.0106	0.0117	0.0104	0.0066	0.0065	0.0108	0.0116
	PPART	0.0289	0.0302	0.0307	0.0428	0.0359	0.0331	0.0337	0.0381	0.0380	0.0249	0.0242	0.0270	0.0324
	PSOL	0.0240	0.0291	0.0284	0.0338	0.0253	0.0095	0.0148	0.0162	0.0152	0.0133	0.0165	0.0240	0.0209
	Total P	0.0529	0.0593	0.0592	0.0766	0.0612	0.0426	0.0485	0.0543	0.0532	0.0382	0.0407	0.0510	0.0532
	Chl."a"	17.8	21.0	21.4	40.2	31.0	26.0	33.1	35.2	34.7	19.0	19.9	24.7	27.1
$T_wR_wL_s$	DIP	0.0205	0.0233	0.0128	0.0154	0.0040	0.0048	0.0057	0.0049	0.0047	0.0037	0.0090	0.0260	0.0112
	DOP	0.0059	0.0086	0.0136	0.0194	0.0268	0.0220	0.0185	0.0135	0.0101	0.0038	0.0041	0.0057	0.0127
	PPART	0.0274	0.0302	0.0331	0.0380	0.0311	0.0405	0.0468	0.0417	0.0389	0.0261	0.0244	0.0218	0.0334
	PSOL	0.0264	0.0319	0.0264	0.0348	0.0308	0.0268	0.0242	0.0184	0.0148	0.0076	0.0131	0.0318	0.0239
	Total P	0.0538	0.0620	0.0596	0.0728	0.0618	0.0673	0.0709	0.0601	0.0537	0.0336	0.0375	0.0535	0.0573
	Chl."a"	15.3	21.2	25.7	32.3	30.0	36.3	42.5	37.6	33.6	21.7	19.2	16.6	27.7
$T_wR_sL_w$	DIP	0.0170	0.0199	0.0133	0.0073	0.0035	0.0049	0.0059	0.0047	0.0048	0.0046	0.0085	0.0167	0.0092
	DOP	0.0062	0.0085	0.0136	0.0168	0.0207	0.0206	0.0200	0.0135	0.0099	0.0042	0.0046	0.0060	0.0121
	PPART	0.0277	0.0322	0.0321	0.0361	0.0288	0.0397	0.0481	0.0417	0.0386	0.0272	0.0258	0.0211	0.0333
	PSOL	0.0232	0.0284	0.0269	0.0240	0.0242	0.0255	0.0259	0.0182	0.0147	0.0088	0.0131	0.0226	0.0213
	Total P	0.0509	0.0606	0.0590	0.0601	0.0530	0.0652	0.0740	0.0600	0.0533	0.0361	0.0389	0.0438	0.0546
	Chl."a"	14.8	19.9	26.8	23.7	27.1	36.4	43.8	37.1	33.5	23.9	19.9	15.8	27.0
$T_sR_wL_w$	DIP	0.0173	0.0192	0.0180	0.0047	0.0041	0.0040	0.0049	0.0044	0.0046	0.0077	0.0107	0.0120	0.0093
	DOP	0.0063	0.0088	0.0121	0.0223	0.0167	0.0063	0.0121	0.0117	0.0103	0.0070	0.0071	0.0091	0.0108
	PPART	0.0281	0.0325	0.0299	0.0397	0.0337	0.0333	0.0366	0.0384	0.0375	0.0258	0.0254	0.0241	0.0321
	PSOL	0.0236	0.0281	0.0301	0.0270	0.0207	0.0102	0.0169	0.0161	0.0149	0.0147	0.0178	0.0211	0.0201
	Total P	0.0516	0.0605	0.0600	0.0667	0.0544	0.0436	0.0536	0.0545	0.0523	0.0405	0.0432	0.0452	0.0522
	Chl."a"	15.7	20.3	23.2	30.0	28.8	28.2	37.2	34.9	34.0	20.9	20.6	20.8	26.3
$T_mR_mL_s$	DIP	0.0202	0.0224	0.0126	0.0158	0.0039	0.0048	0.0055	0.0049	0.0042	0.0037	0.0091	0.0235	0.0109
	DOP	0.0060	0.0088	0.0137	0.0190	0.0282	0.0230	0.0167	0.0136	0.0078	0.0042	0.0043	0.0068	0.0127
	PPART	0.0274	0.0302	0.0329	0.0379	0.0312	0.0411	0.0449	0.0420	0.0367	0.0267	0.0242	0.0223	0.0332
	PSOL	0.0262	0.0313	0.0262	0.0348	0.0321	0.0278	0.0222	0.0185	0.0120	0.0079	0.0135	0.0302	0.0236
	Total P	0.0536	0.0615	0.0591	0.0727	0.0633	0.0689	0.0671	0.0606	0.0487	0.0346	0.0376	0.0526	0.0568
	Chl."a"	15.4	21.1	20.6	32.2	30.2	36.7	41.0	37.7	32.0	22.1	19.3	17.1	27.6
$T_mR_sL_m$	DIP	0.0169	0.0196	0.0134	0.0079	0.0035	0.0050	0.0056	0.0048	0.0043	0.0046	0.0089	0.0162	0.0092
	DOP	0.0062	0.0086	0.0139	0.0164	0.0221	0.0220	0.0176	0.0137	0.0076	0.0045	0.0047	0.0071	0.0121
	PPART	0.0275	0.0315	0.0321	0.0360	0.0289	0.0409	0.0458	0.0420	0.0364	0.0277	0.0253	0.0220	0.0330
	PSOL	0.0231	0.0282	0.0273	0.0243	0.0256	0.0269	0.0232	0.0184	0.0119	0.0091	0.0136	0.0234	0.0213
	Total P	0.0506	0.0597	0.0594	0.0603	0.0545	0.0679	0.0689	0.0605	0.0483	0.0367	0.0389	0.0453	0.0543
	Chl."a"	14.7	19.9	26.5	24.1	27.3	37.5	41.9	37.3	31.9	24.0	19.8	16.7	26.9
$T_sR_mL_m$	DIP	0.0168	0.0191	0.0185	0.0047	0.0041	0.0040	0.0048	0.0044	0.0045	0.0074	0.0105	0.0130	0.0093
	DOP	0.0064	0.0089	0.0123	0.0223	0.0167	0.0065	0.0114	0.0118	0.0103	0.0070	0.0070	0.0095	0.0108
	PPART	0.0280	0.0318	0.0303	0.0395	0.0336	0.0340	0.0356	0.0383	0.0374	0.0256	0.0250	0.0251	0.0321
	PSOL	0.0232	0.0280	0.0308	0.0269	0.0207	0.0105	0.0162	0.0162	0.0148	0.0143	0.0175	0.0224	0.0202
	Total P	0.0511	0.0598	0.0610	0.0665	0.0544	0.0445	0.0518	0.0545	0.0522	0.0400	0.0425	0.0475	0.0522
	Chl."a"	15.7	20.3	23.6	30.1	28.7	29.1	35.8	35.0	33.9	20.5	20.3	22.2	26.3

Table 15. Seasonal values of phosphorus concentrations evaluated at the examination of model sensitivity to changes of input data on temperature, radiation and River Zala phosphorus load (Basin I, 1977).

Averaging data	P fraction	Winter (Jan-Mar)					Spring (Apr-June)					Summer (July-Sept)					Autumn (Oct-Dec)				
		Minimum		Mean	Maximum		Minimum		Mean	Maximum		Minimum		Mean	Maximum		Minimum		Mean	Maximum	
		date	value		date	value	date	value		date	value	date	value		date	value	date	value		date	value
$T_dR_dL_d$	DIP	29 M	.0063	.0181	10 F	.0276	17 M	.0027	.0054	28 J	.0092	4 A	.0035	.0053	1 J	.0085	27 O	.0032	.0114	25 D	.0241
	DOP	8 J	.0054	.0092	28 M	.0181	19 A	.0165	.0197	28 J	.0248	30 S	.0051	.0147	2 J	.0244	2 N	.0033	.0121	31 D	.0062
	PPART	8 J	.0190	.0306	30 M	.0516	10 J	.0230	.0351	27 J	.0540	30 S	.0313	.0433	23 J	.0565	21 D	.0146	.0247	27 N	.0387
	PSOL	12 J	.0230	.0272	10 F	.0348	11 J	.0209	.0250	28 J	.0332	30 S	.0089	.0201	2 J	.0322	31 O	.0072	.0161	31 D	.0297
	Total P	8 J	.0432	.0578	30 M	.0754	4 M	.0461	.0601	28 J	.0862	30 S	.0403	.0634	1 J	.0868	31 O	.0250	.0407	27 N	.0571
	Chl."a"	8 J	12.1	20.6	25 M	40.1	29 A	19.9	29.4	30 J	52.8	30 S	25.0	38.8	2 J	52.6	12 D	12.5	19.6	16 O	27.7
$T_wR_wL_w$	DIP	31 M	.0065	.0178	11 F	.0264	17 M	.0026	.0052	28 J	.0087	4 A	.0034	.0052	2 J	.0085	27 O	.0033	.0110	27 D	.0232
	DOP	6 J	.0054	.0092	28 M	.0176	10 A	.0165	.0193	30 J	.0241	30 S	.0050	.0145	2 J	.0247	2 N	.0033	.0048	31 D	.0064
	PPART	8 J	.0193	.0306	30 M	.0505	18 M	.0228	.0349	27 J	.0535	30 S	.0312	.0430	23 J	.0562	21 D	.0149	.0246	27 N	.0389
	PSOL	12 J	.0228	.0270	11 F	.0338	10 J	.0206	.0245	30 J	.0316	30 S	.0086	.0197	2 J	.0326	1 N	.0071	.0158	31 D	.0293
	Total P	8 J	.0431	.0576	30 M	.0746	4 M	.0467	.0595	28 J	.0837	30 S	.0398	.0628	1 J	.0857	31 O	.0248	.0404	27 N	.0570
	Chl."a"	6 J	12.3	20.5	25 M	37.1	29 A	20.6	29.3	30 J	51.6	30 S	24.9	38.5	2 J	52.6	12 D	13.5	19.6	14 O	26.5
$T_mR_mL_m$	DIP	31 M	.0087	.0175	18 F	.0225	17 M	.0027	.0054	15 A	.0094	30 S	.0034	.0049	23 J	.0081	30 O	.0033	.0110	30 D	.0224
	DOP	3 J	.0055	.0093	28 M	.0165	1 A	.0156	.0201	16 M	.0240	24 S	.0062	.0130	2 J	.0204	5 N	.0033	.0053	31 D	.0072
	PPART	8 J	.0202	.0303	30 M	.0481	18 M	.0232	.0353	27 J	.0524	23 S	.0319	.0416	23 J	.0536	31 D	.0158	.0249	27 N	.0430
	PSOL	12 J	.0221	.0269	18 F	.0314	30 A	.0216	.0255	5 J	.0291	30 S	.0099	.0179	2 J	.0273	30 O	.0071	.0162	31 D	.0295
	Total P	8 J	.0431	.0572	30 M	.0732	30 A	.0471	.0609	30 J	.0781	23 S	.0425	.0595	1 J	.0777	31 O	.0256	.0412	27 N	.0621
	Chl."a"	1 J	12.6	20.5	25 M	30.5	30 A	19.9	29.9	30 J	44.9	30 S	27.9	37.3	5 J	47.4	12 D	14.5	19.9	1 O	28.1
$T_dR_dL_s$	DIP	28 M	.0051	.0189	10 F	.0267	17 M	.0027	.0082	15 A	.0232	4 A	.0035	.0052	26 J	.0077	27 O	.0030	.0137	25 D	.0335
	DOP	6 J	.0054	.0094	28 M	.0172	2 A	.0159	.0232	23 M	.0301	30 S	.0051	.0143	2 J	.0228	2 N	.0029	.0045	31 D	.0071
	PPART	8 J	.0198	.0302	30 M	.0504	18 M	.0240	.0366	16 A	.0518	30 S	.0317	.0427	23 J	.0546	21 D	.0165	.0243	27 N	.0392
	PSOL	31 M	.0217	.0283	18 F	.0349	1 A	.0219	.0314	19 A	.0426	30 S	.0090	.0195	2 J	.0293	31 O	.0064	.0182	25 D	.0394
	Total P	8 J	.0442	.0585	30 M	.0723	18 M	.0552	.0680	16 A	.0927	30 S	.0408	.0622	1 J	.0805	31 O	.0229	.0426	26 D	.0639
	Chl."a"	1 J	12.6	20.7	25 M	34.1	2 A	19.3	33.0	28 J	44.5	30 S	25.3	38.2	6 J	47.4	12 D	14.0	19.4	1 O	26.0
$T_dR_sL_d$	DIP	30 M	.0061	.0165	10 F	.0248	11 J	.0028	.0033	28 J	.0089	4 A	.0035	.0052	2 J	.0081	31 O	.0033	.0101	21 D	.0189
	DOP	2 J	.0055	.0096	28 M	.0183	17 A	.0164	.0197	28 J	.0248	30 S	.0050	.0147	12 J	.0245	2 N	.0034	.0050	31 D	.0070
	PPART	8 J	.0200	.0307	30 M	.0510	18 M	.0230	.0350	27 J	.0534	30 S	.0316	.0431	23 J	.0562	21 D	.0162	.0250	27 N	.0389
	PSOL	13 J	.0216	.0261	10 F	.0327	11 J	.0205	.0250	28 J	.0330	30 S	.0089	.0199	2 J	.0321	1 N	.0071	.0151	25 D	.0255
	Total P	8 J	.0430	.0567	30 M	.0747	4 M	.0463	.0599	28 J	.0854	30 S	.0407	.0631	1 J	.0861	31 O	.0251	.0401	27 N	.0565
	Chl."a"	1 J	12.5	20.6	25 M	39.1	29 A	20.0	29.1	29 J	51.6	30 S	25.6	38.4	2 J	51.8	12 D	14.0	20.1	17 O	27.3
$T_sR_dL_d$	DIP	31 M	.0154	.0186	10 F	.0246	13 J	.0025	.0043	1 A	.0158	4 A	.0032	.0047	26 J	.0070	1 O	.0046	.0107	22 D	.0147
	DOP	1 J	.0055	.0090	28 M	.0142	16 J	.0050	.0151	21 A	.0257	1 J	.0070	.0114	27 J	.0148	2 N	.0062	.0078	1 O	.0103
	PPART	8 J	.0198	.0300	30 M	.0470	4 M	.0284	.0356	16 A	.0558	10 J	.0319	.0375	23 J	.0465	31 O	.0164	.0254	27 N	.0409
	PSOL	12 J	.0223	.0276	25 M	.0356	15 J	.0078	.0194	19 A	.0295	1 J	.0126	.0161	26 J	.0208	2 N	.0134	.0185	25 D	.0241
	Total P	8 J	.0435	.0576	30 M	.0770	14 J	.0372	.0550	16 A	.0843	10 J	.0473	.0536	23 J	.0653	15 D	.0299	.0438	27 N	.0612
	Chl."a"	1 J	12.8	19.6	25 M	27.5	1 A	22.4	29.0	4 A	38.6	30 S	31.1	35.4	27 J	42.9	7 N	17.6	21.0	1 O	29.9
$T_sR_sL_s$	DIP	31 M	.0117	.0174	1 J	.0213	13 J	.0023	.0052	1 A	.0121	9 J	.0030	.0045	23 J	.0063	1 O	.0047	.0101	3 D	.0152
	DOP	1 J	.0055	.0098	28 M	.0141	28 J	.0051	.0176	23 A	.0319	1 J	.0052	.0109	27 J	.0140	2 N	.0054	.0081	30 D	.0134
	PPART	8 J	.0217	.0299	30 M	.0465	25 J	.0279	.0372	16 A	.0594	4 J	.0274	.0366	23 J	.0442	31 O	.0151	.0257	27 N	.0421
	PSOL	12 J	.0216	.0271	18 F	.0313	26 J	.0078	.0229	19 A	.0391	1 J	.0085	.0154	27 J	.0191	1 N	.0116	.0182	21 D	.0264
	Total P	8 J	.0443	.0571	30 M	.0724	25 J	.0362	.0602	17 A	.0970	4 J	.0391	.0520	26 J	.0617	31 O	.0269	.0439	4 D	.0648
	Chl."a"	1 J	12.9	20.0	25 M	23.8	1 A	19.2	32.4	14 A	51.4	1 J	24.2	34.3	27 J	40.0	5 N	16.6	21.5	1 O	30.8
$T_wR_wL_s$	DIP	31 M	.0054	.0187	10 F	.0258	17 M	.0026	.0080	15 A	.0226	4 A	.0033	.0051	23 J	.0075	27 O	.0030	.0132	27 D	.0321
	DOP	6 J	.0054	.0094	28 M	.0168	1 A	.0160	.0228	24 M	.0282	30 S	.0050	.0140	2 J	.0231	2 N	.0029	.0046	31 D	.0073
	PPART	8 J	.0200	.0302	30 M	.0495	18 M	.0238	.0365	16 A	.0525	30 S	.0316	.0425	23 J	.0545	31 O	.0164	.0243	27 N	.0394
	PSOL	31 M	.0215	.0281	18 F	.0345	1 A	.0217	.0308	15 A	.0416	30 S	.0087	.0192	2 J	.0298	31 O	.0063	.0179	30 D	.0388
	Total P	8 J	.0442	.0583	30 M	.0715	18 M	.0546	.0673	16 A	.0927	30 S	.0403	.0617	1 J	.0796	31 O	.0227	.0422	26 D	.0629
	Chl."a"	1 J	12.6	20.7	24 M	31.9	13 M	21.7	32.8	30 J	43.4	30 S	25.2	37.9	6 J	47.4	12 D	15.2	19.3	1 O	26.0
$T_wR_sL_w$	DIP	31 M	.0066	.0166	11 F	.0244	17 M	.0027	.0052	14 A	.0090	4 A	.0034	.0052	1 J	.0082	31 O	.0034	.0101	21 D	.0190
	DOP	2 J	.0055	.0095	28 M	.0177	17 A	.0164	.0193	30 J	.0242	30 S	.0049	.0145	2 J	.0249	2 N	.0034	.0050	31 D	.0076
	PPART	8 J	.0201	.0306	30 M	.0501	18 M	.0227	.0348	27 J	.0531	30 S	.0315	.0428	23 J	.0562	21 D	.0162	.0249	27 N	.0389
	PSOL	12 J	.0215	.0261	11 F	.0323	10 J	.0204	.0246	30 J	.0314	30 S	.0087	.0197	2 J	.0323	31 O	.0071	.0151	27 D	.0257
	Total P	8 J	.0429	.0567	30 M	.0742	4 M	.0470	.0594	28 J	.0831	30 S	.0404	.0625	1 J	.0851	31 O	.0251	.0401	27 N	.0567
	Chl."a"	1 J	12.6	20.5	25 M	36.0	29 A	20.8	29.1	30 J	50.6	30 S	25.6	38.2	2 J	51.6	11 D	14.2	20.1	15 O	26.8
$T_sR_wL_w$	DIP	31 M	.0149	.0181	11 F	.0234	11 J	.0025	.0042	1 A	.0152	4 A	.0031	.0046	23 J	.0069	1 O	.0045	.0102	24 D	.0139
	DOP	2 J	.0055	.0091	28 M	.0142	17 J	.0050	.0151	21 A	.0256	1 J	.0069	.0114	27 J	.0148	5 N	.0062	.0079	31 D	.0102
	PPART	8 J	.0202	.0301	30 M	.0471	4 M	.0285	.0355	16 A	.0537	10 J	.0319	.0375	23 J	.0462	31 O	.0163	.0254	27 N	.0415
	PSOL	12 J	.0221	.0272	25 M	.0352	15 J	.0078	.0193	18 A	.0294	1 J	.0127	.0160	27 J	.0205	1 N	.0132	.0181	25 D	.0234
	Total P	8 J	.0435	.0573	30 M	.0767	14 J	.0372	.0549	16 A	.0822	10 J	.0473	.0535	23 J	.0650	31 O	.0297	.0435	27 N	.0613
	Chl."a"	1 J	12.8	19.7	27 M	27.6	1 A	22.5	29.0	4 A	39.0	30 S	30.9	35.3	27 J	42.1	5 N	17.4	21.0	1 O	29.7
$T_mR_mL_s$	DIP	31 M	.0075	.0183	7 F	.0241	17 M	.0028	.0081	15 A	.0228	30 S	.0034	.0049	23 J	.0077	30 O	.0028	.0124	31 D	.0301
	DOP	3 J	.0055	.0095	28 M	.0159	1 A	.0150	.0234	16 M	.0302	24 S	.0063	.0127	2 J	.0195	2 N	.0029	.0052	31 D	.0078
	PPART	8 J	.0203	.0302	30 M	.0478	18 M	.0241	.0367	16 A	.0528	23 S	.0323	.0413	23 J	.0523	12 N	.0167	.0246	27 N	.0422
	PSOL	31 M	.0225	.0278	18 F	.0331	1 A	.0227	.0316	15 A	.0415	30 S	.0100	.0176	2 J	.0256	30 O	.0060	.0176	31 D	.0379
	Total P	8 J	.0442	.0580	30 M	.0709	3 A	.0557	.0683	16 A	.0927	30 S	.0430	.0589	1 J	.0748	31 O	.0230	.0422	4 D	.0621
	Chl."a"	1 J	12.6	20.6	10 M	28.0	1 A	21.0	33.0	30 J	41.9	30 S	28.2	37.0	27 J	45.5	5 N	14.8	19.7	1 O	28.2
$T_mR_sL_m$	DIP	31 M	.0087	.0165	18 F	.0215	17 M	.0027	.0054	15 A	.0099	4 A	.0035	.0049	23 J	.0078	30 O	.0035	.0101	21 D	.0187
	DOP	2 J	.0055	.0096	28 M	.0166	1 A	.0156	.0202	16 M	.0240	24 S	.0062	.0130	2 J	.0205	3 N	.0034	.0055	31 D	.0082
	PPART	8 J	.0206	.0303	30 M	.0478	18 M	.0231	.0352	27 J	.0519	23 S	.0321	.0414	23 J	.0534	31 D	.0165	.0252	27 N	.0431
	PSOL	12 J	.0212	.0261	18 F	.0307	30 A	.0218	.0256	5 J	.0289	30 S	.0101	.0179	2 J	.0271	31 O	.0073	.0156	27 D	.0265
	Total P	8 J	.0429	.0565	30 M	.0729	30 A	.0476	.0608	27 J	.0774	23 S	.0428	.0593	1 J	.0770	31 O	.0259	.0408	27 D	.0618
	Chl."a"	1 J	12.6	20.4	25 M	29.6	30 A	20.3	29.6	24 J	43.7	30 S	28.6	37.1	27 J	46.7	12 D	15.2	20.4	1 O	28.5
$T_sR_mL_m$	DIP	31 M	.0139	.0181	1 J	.0213	29 A	.0024	.0042	1 A	.0142	4 A	.0031	.0045	23 J	.0070	1 O	.0045	.0104	30 N	.0156
	DOP	1 J	.0055	.0092	28 M	.0144	17 J	.0056	.0152	23 A	.0258	1 J	.0061	.0112	27 J	.0145	5 N	.0061	.0079	30 D	.0109
	PPART	8 J	.0210	.0300	30 M	.0469	4 M	.0275	.0357	16 A	.0535	4 J	.0302	.0371	23 J	.0457	31 O	.0162	.0255	27 N	.0428
	PSOL	12 J	.0213	.0273	25 M	.0337	15 J	.0089	.0194	19 A	.0296	1 J	.0105	.0157	27 J	.0204	2 N	.0131	.0183	3 D	.0241
	Total P	8 J	.0434	.0572	30 M	.0756	25 J	.0402	.0552	17 A	.0822	4 J	.0436	.0528	26 J	.0645	31 O	.0297	.0439	27 N	.0645
	Chl."a"	1 J	12.8	19.8	25 M	27.3	1 A	21.8	29.3	4 A	37.1	1 J	28.6	34.9	27 J	42.2	5 N	17.5	21.2	1 O	29.9

The results in Tables 14-15 of the time scale averaging of all data corresponding to one day, i.e. $T_d R_d L_d$, may be considered a control run because exactly the same time scale was used for the averaging in the simulation of phosphorus dynamics for the three-year period, 1976-1978, discussed above. The analysis of data in Tables 14-15 shows that:

(i) weekly averaging of the characteristics mentioned above in the input model data hardly change the model output according to monthly, seasonal and annual mean concentrations of phosphorus fractions; this procedure may just slightly shift the dates of extreme concentrations with ± 2 days in the winter and spring months;

(ii) monthly averaging of the same input data gives slightly lower values for monthly and seasonal mean concentrations of phosphorus fractions in the winter and summer months and slightly higher levels of these mean values for the spring and autumn months; however, the mean annual concentrations of phosphorus fractions are practically the same as in the control run;

(iii) the procedure of averaging the same input data for a season have a significant effect on the model output for the summer months (especially for July and August) so that monthly mean concentrations of DIP, DOP, particulate organic-P, total soluble P and total P are smaller by 31.3-45%, 46.7-69%, 18.3-31%, 44-64% and 35-36% respectively than in the control run; the essential change of model output may also be observed by the date of minimum and maximum phosphorus concentrations during the spring and summer months, as well as in mean phosphorus concentrations for these seasons;

(iv) from three input data characteristics studied, i.e., temperature, radiation and phosphorus loading, the temperature and phosphorus loading have an express influence on the model output, unlike radiation; however, the influence of averaging both these input data (temperature and phosphorus loading) affect the model output to different degrees; for example, in comparison with the control run, the seasonal average of temperature may change mean concentrations of total phosphorus by 0.5%, 8.5%, 15.4% and 7.6% for winter, spring, summer and autumn respectively, while the seasonal average of phosphorus loading data changes the mean total phosphorus by 1.2%, 13.1%, 1.9% and 4.7% respectively, for the same seasons; the influence of averaging the input radiation data is that it decreases the mean total phosphorus concentration for winter by 1.9%, for the spring-summer period by 0.4-0.5% and for autumn by 1.5%;

(v) interesting results are also obtained by the comparison of the dates on which minimum and maximum phosphorus concentrations occur during the different seasons, as a result of averaging the input data discussed; for example, seasonal averaging of phosphorus loading data has a significant influence on the change of these dates for the winter-spring months, while the same time scale of averaging the temperature data influences the dates of extreme phosphorus concentrations during the spring, summer and autumn months; the seasonal averaging of data on the radiation counted only for the dates of minimum phosphorus concentrations during the winter and autumn months.

It is apparent that the results of sensitivity analysis runs presented here have a rather preliminary character. For a complete picture of the model behavior, the effect of the same time scale averaging of other input data, such as wind and water balance data, should also be evaluated. This is considered to be important for applying the model in the management framework, as well as for the solution of the problem of how the model and input data used could be simplified, but without any significant disturbance in the accuracy of the model output.

PHOSPHORUS EXCHANGE PROCESSES IN THE SEDIMENT-WATER LAYER

Among the phosphorus transformation processes developed in the water bodies, the phosphorus interactions between sediments and water, as well as their quantitative relationships have not yet been studied sufficiently well. It is known that the sediments play a significant role in the nutrient cycling and their importance as a potential nutrient source in the development of water body eutrophication has been generally recognized. However, the quantitative measurements of the sediment effect on the nutrient balance within the water body is a difficult task because of the complexity of the analytical techniques used. Nevertheless, an attempt has been made to simulate the contributions of phosphorus from the Lake Balaton sediments to the water's biochemical and dynamic cycle as a variable factor defined by physical and chemical environmental characteristics. It follows that the model gives one the possibility of quantitatively estimating the influence of sediment on the phosphorus dynamics in Lake Balaton. Furthermore, it is interesting to note how the simulation results obtained for 1976-1978 may explain the influence of the environment on the phosphorus exchange through the sediment-water interface, in accordance with the model hypothesis and the measurements used.

An analysis of the simulation results obtained may be made to elucidate the dynamics of the phosphorus exchange in the sediment-water layer, as well as to clarify the role of the sediment-water phosphorus fluxes in a total phosphorus balance within the given lake. The first question which arose in the analysis of the simulation results concerning the sediment-water phosphorus exchange was on the interaction between the processes regulating the phosphorus exchange in the sediment-water interface. In accordance with the hypothesis used during model construction, these processes are the resuspension and sedimentation of particulate phosphorus, as well as the sediment release of the dissolved inorganic phosphorus. The development of these processes depends on the wind and temperature, as well as on the physical-chemical-biological reactions within the sediment-water layer.

According to the data used and the assumptions made, the simulated time patterns of the nonliving particulate phosphorus transport through sediment-water interface for the environmental conditions of 1976-1978 are shown in Figures 13-16, for Basins I-IV respectively. These pictures explain quantitatively, how much phosphorus in the particulate form may enter from the

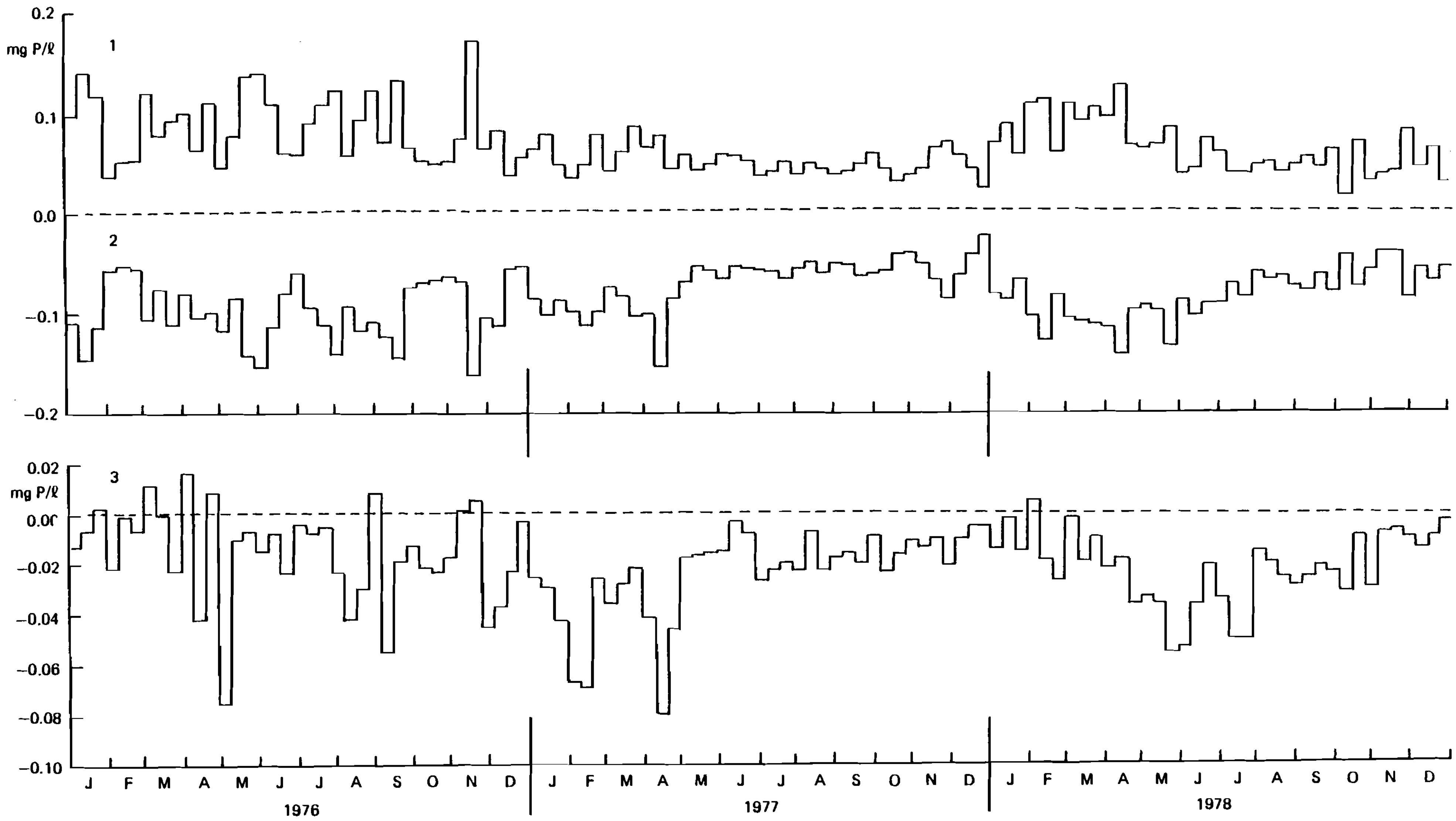

Figure 13. Exchange of nonliving particulate-P in sediment-water interface in Basin I as calculated by model:
1. nonliving particulate-P resuspension;
2. nonliving particulate-P sedimentation;
3. net nonliving particulate-P losses to sediment.

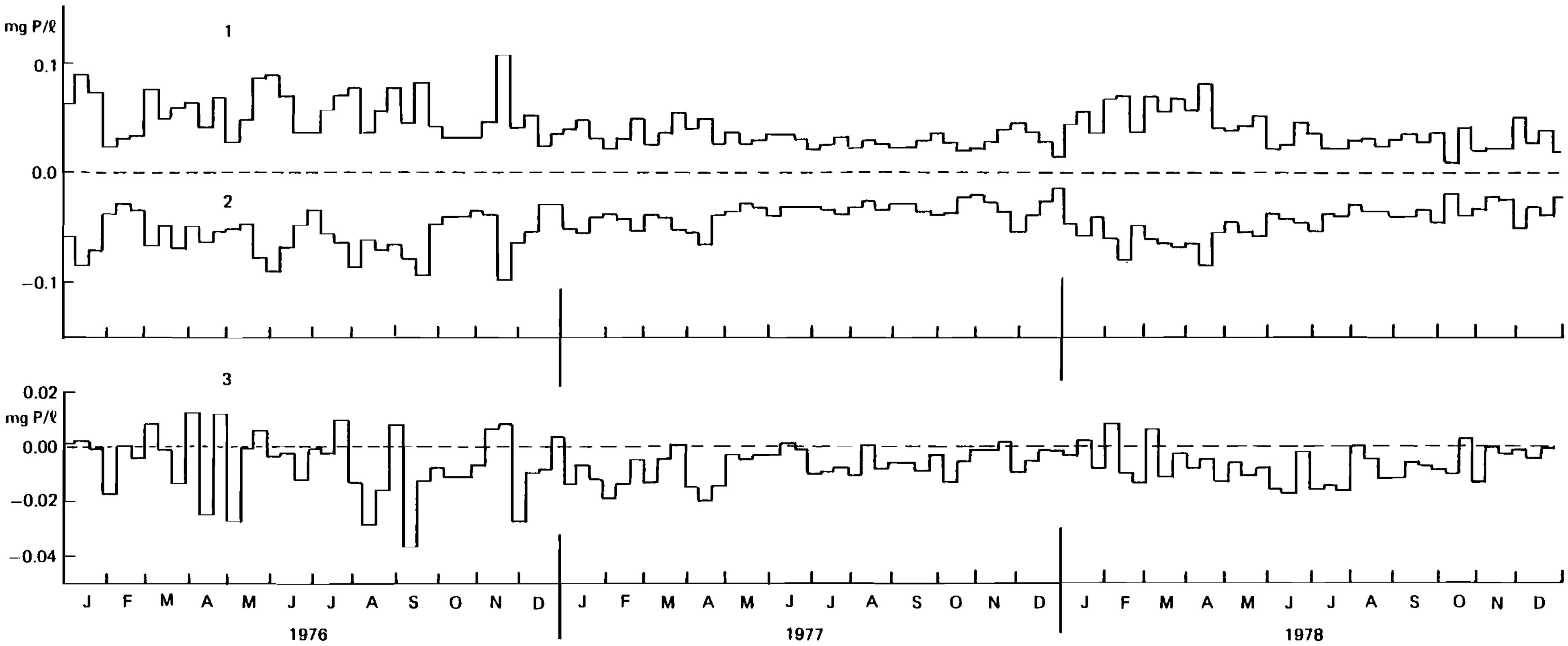

Figure 14. Exchange of nonliving particulate-P in sediment-water interface in Basin II as calculated by model:
1. nonliving particulate-P resuspension;
2. nonliving particulate-P sedimentation;
3. net nonliving particulate-P losses to sediment.

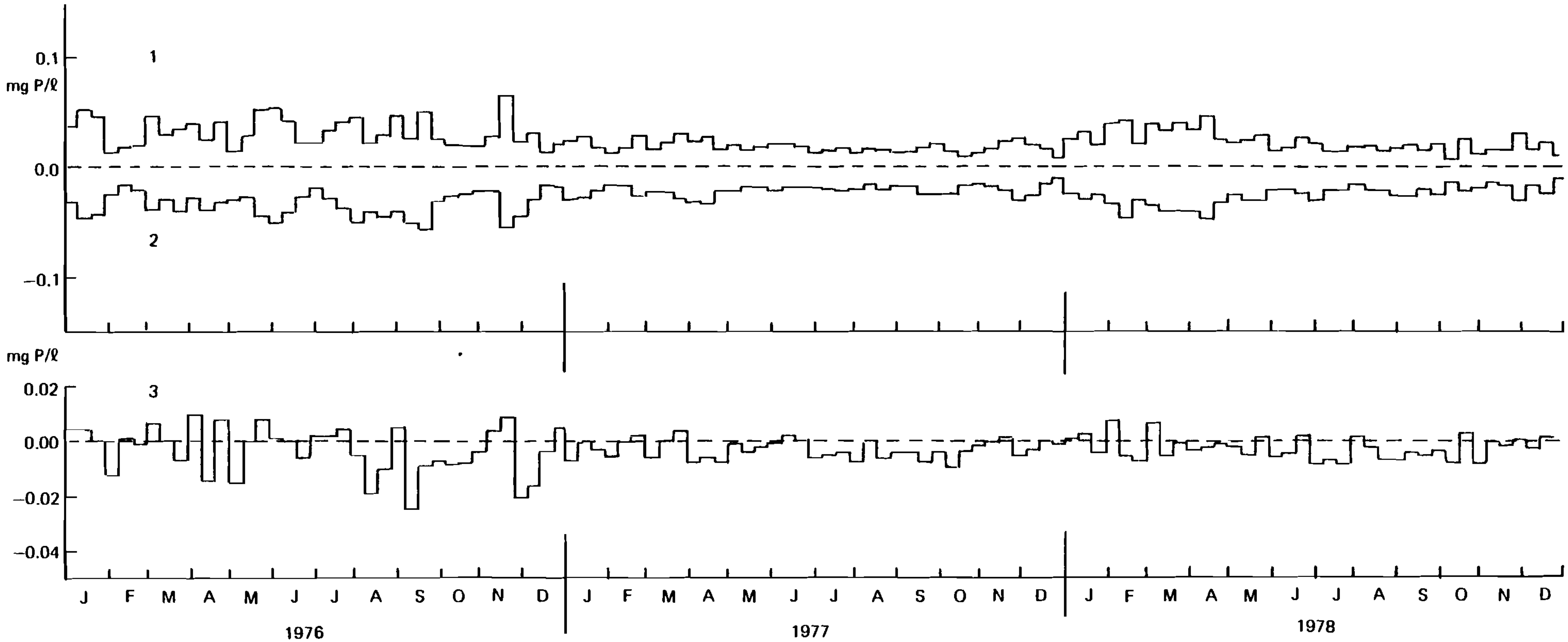

Figure 15. Exchange of nonliving particulate-P in sediment-water interface in Basin III as calculated by model:
1. nonliving particulate-P resuspension;
2. nonliving particulate-P sedimentation;
3. net nonliving particulate-P losses to sediment.

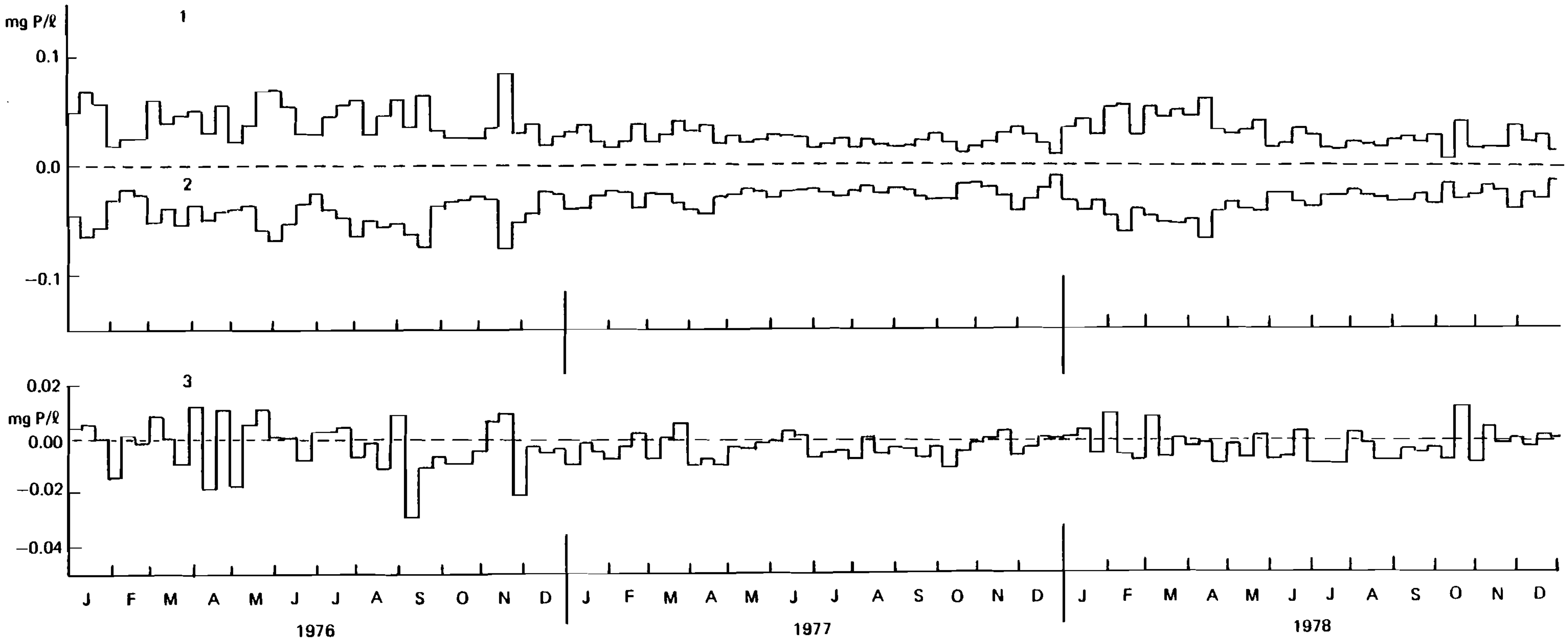

Figure 16. Exchange of nonliving particulate-P in sediment-water interface in Basin IV as calculated by model;
1. nonliving particulate-P resuspension;
2. nonliving particulate-P sedimentation;
3. net nonliving particulate-P losses to sediment.

sediments and settle into the sediment and what the net phosphorus losses to sediment are for every 10 day interval during 1976-1978. They also allow one to estimate the order of the rates in the phosphorus exchange processes in the sediment-water layer for the period of time mentioned.

Further conclusions may be obtained on the basis of the analysis of simulation results representing the exchange of particulate phosphorus through the sediment-water interface:

(i) the total amount of phosphorus which settled into the sediments was larger than that resuspended in 1976-1978. The Figures 13-16 show that only during relatively small time intervals, when wind speed was higher than 12-15 m/sec, phosphorus entered water by resuspension and in these cases, this phosphorus flux could have been higher than phosphorus losses determined by sedimentation. These cases were observed during the spring and autumn of 1976 as well as during the winter of 1978;

(ii) for the environmental conditions of 1976, the exchange of particulate phosphorus through the sediment-water interface was more active than in 1977-1978 as a result of more frequent winds, with speeds higher than 10-12 m/sec. Therefore, the annual particulate phosphorus fluxes to and out of the sediment were higher for 1976 by 1.5 and 1.3 times for sedimentation than for 1977 and 1978 respectively and they were also higher by 1.7 and 1.4 times for resuspension in 1976 in comparison with 1977 and 1978 respectively. Evaluated values of annual mean phosphorus sedimentation rates for Basin I were $10.1 \cdot 10^{-3}$, $7.4 \cdot 10^{-3}$, and $8.6 \cdot 10^{-3}$ mgP/ℓ-day for 1976-1978 respectively. The corresponding values of rates for the same years were $6.0 \cdot 10^{-3}$, $3.9 \cdot 10^{-3}$ and $4.6 \cdot 10^{-3}$ mgP/ℓ-day for Basin II, $4.6 \cdot 10^{-3}$, $2.8 \cdot 10^{-3}$ and $3.4 \cdot 10^{-3}$ mgP/ℓ-day for Basin III and $3.5 \cdot 10^{-3}$, $2.2 \cdot 10^{-3}$ and $2.7 \cdot 10^{-3}$ mgP/ℓ-day for Basin IV. The annual mean rates of particulate phosphorus resuspension calculated on the basis of simulation results were equal to $8.5 \cdot 10^{-3}$, $4.9 \cdot 10^{-3}$ and $6.3 \cdot 10^{-3}$ mgP/ℓ-day for Basin I in 1976-1978 respectively. For the other basins and the same years, these rates were equal to $5.4 \cdot 10^{-3}$, $3.1 \cdot 10^{-3}$ and $3.0 \cdot 10^{-3}$ mgP/ℓ-day (Basin II); $4.3 \cdot 10^{-3}$, $2.5 \cdot 10^{-3}$ and $3.1 \cdot 10^{-3}$ mgP/ℓ-day (Basin III); and $3.3 \cdot 10^{-3}$, $1.9 \cdot 10^{-3}$ and $2.4 \cdot 10^{-3}$ mgP/ℓ-day (Basin IV). The comparison of net losses of particulate phosphorus for the three year period, 1976-1978, shows that it was lowest in 1976 as a result of active resuspension of particulate phosphorus brought about by unusually strong winds.

(iii) evaluating the seasonal rates of the particulate phosphorus exchange shows that in 1976, phosphorus losses by sedimentation were highest in the spring-summer months. Calculated rates of phosphorus sedimentation for these seasons in 1976 were equal to $(10.9\text{-}11.7) \cdot 10^{-3}$ mgP/ℓ-day for Basin I, $(6.3\text{-}7.3) \cdot 10^{-3}$ mgP/ℓ-day for Basin II, $(4.7\text{-}5.6) \cdot 10^{-3}$ mgP/ℓ-day for Basin III and $(3.6\text{-}4.4) \cdot 10^{-3}$ mgP/ℓ-day for Basin IV. The order of rates of phosphorus sedimentation for the winter and autumn 1976 were $(9.1\text{-}9.2) \cdot 10^{-3}$ mgP/ℓ-day (Basin I), $(5.3\text{-}5.7) \cdot 10^{-3}$ mgP/ℓ-day (Basin II), $(4\text{-}4.3) \cdot 10^{-3}$ mgP/ℓ-day (Basin III) and $(3.1\text{-}3.3) \cdot 10^{-3}$ mgP/ℓ-day (Basin IV). In 1977 and 1978, the seasonal phosphorus losses by sedimentation were estimated to be highest in the winter-spring months. In 1977, for these months, the rates of phosphorus sedimentation evaluated were equal to $(8\text{-}9.6) \cdot 10^{-3}$ (Basin I), $(4.1\text{-}4.6) \cdot 10^{-3}$ (Basin II), $(3\text{-}3.2) \cdot 10^{-3}$ (Basin III) and $(2.3\text{-}2.4) \cdot 10^{-3}$ (Basin IV (all mgP/ℓ-day), while for 1978, for the same periods, they were $(9.8\text{-}10.9) \cdot 10^{-3}$ (Basin I), $(5.4\text{-}5.9) \cdot 10^{-3}$ (Basin II), $(3.9\text{-}4.4) \cdot 10^{-3}$ (Basin III) and $(3\text{-}3.4) \cdot 10^{-3}$ (Basin IV). The intensity of the phosphorus sedimentation in the summer and autumn months were estimated to be similar in 1977-1978 with the following rates $(6.2\text{-}7.8) \cdot 10^{-3}$, $(3.4\text{-}4.1) \cdot 10^{-3}$, $(2.5\text{-}3) \cdot 10^{-3}$ and $(2\text{-}2.4) \cdot 10^{-3}$ mgP/ℓ-day for Basins I-IV respectively.

(iv) the seasonal changes of phosphorus resuspension rates were similar to changes of sedimentation rates for the three year period of study. For 1976, resuspension rates were highest in the spring-summer months and they were estimated to be equal to $(9.2\text{-}9.6) \cdot 10^{-3}$ (Basin I), $(5.8\text{-}6.1) \cdot 10^{-3}$ (Basin II), $(4.6\text{-}4.8) \cdot 10^{-3}$ (Basin III) and $(3.5\text{-}3.7) \cdot 10^{-3}$ (Basin IV), while in the winter-autumn months the range of resuspension rates were evaluated to be equal to $(7.2\text{-}8.6)\cdot 10^{-3}$, $(4.6\text{-}5.4)\cdot 10^{-3}$, $(3.6\text{-}4.3)\cdot 10^{-3}$ and $(2.8\text{-}3.3)\cdot 10^{-3}$ mgP/ℓ-day for Basins I-IV respectively. The intensity of resuspension in 1977-1978 was estimated to be higher in the winter-spring months than in the summer-autumn months. For 1977 it was lower than in 1978, so that for the winter-spring months, the resuspension rates calculated were equal to $(5.3\text{-}5.7)\cdot 10^{-3}$, $(3.4\text{-}3.6)\cdot 10^{-3}$, $(2.6\text{-}2.9)\cdot 10^{-3}$ and $(2.1\text{-}2.2)\cdot 10^{-3}$ mgP/ℓ-day for Basins I-IV respectively. In 1978, for the winter-spring months, the resuspension rates constituted $(7.1\text{-}8.7)\cdot 10^{-3}$, $(4.5\text{-}5.5)\cdot 10^{-3}$, $(3.5\text{-}4.3)\cdot 10^{-3}$ and $(2.7\text{-}3.4)\cdot 10^{-3}$ mgP/ℓ-day for Basins I-IV respectively. For the summer and autumn months in 1977-1978, the order of resuspension rates evaluated were similar, so that they were $(4.2\text{-}4.9)\cdot 10^{-3}$, $(2.6\text{-}3.1)\cdot 10^{-3}$, $(2.1\text{-}2.4)\cdot 10^{-3}$ and $(1.6\text{-}1.9)\cdot 10^{-3}$ mgP/ℓ-day for Basins I-IV respectively.

It is also possible to evaluate the intensity of DIP releases by sediment. Figure 17 shows the dynamics of sediment DIP release rate for Basin I in 1976-1978. According to modeling results, the annual mean values of sediment DIP release in this basin were estimated to be equal to $6.24\cdot 10^{-4}$, $3.38\cdot 10^{-4}$ and $3.05\cdot 10^{-4}$ mgP/ℓ-day for 1976-1978 respectively. For Basins II-IV these rates were 3.5, 4.9 and 7.2 times smaller than in Basin I.

Seasonal changes of sediment DIP release rates for the three year period show that the range of these rates in the winter months for Basin I was $(1.01\text{-}1.54)\cdot 10^{-3}$ mgP/ℓ-day as evaluated for Basin I. During the spring and summer months the rates of DIP release by sediment could significantly differ from year to year as a result of the prevailing environmental conditions, primarily wind and temperature. The simulation results show that the rate of DIP release from the sediment in Basin I during the spring was equal to $8.37\cdot 10^{-4}$, $4.99\cdot 10^{-4}$ and $4.44\cdot 10^{-4}$ mgP/ℓ-day for 1976-1978 respectively. In the summer months these rates were $13.56\cdot 10^{-4}$, $5.99\cdot 10^{-4}$ and $5.28\cdot 10^{-4}$ mgP/ℓ-day for the same years. During the autumn months the rate of sediment DIP release was smaller than in the summer months and it was estimated to be equal to $2.36\cdot 10^{-4}$,

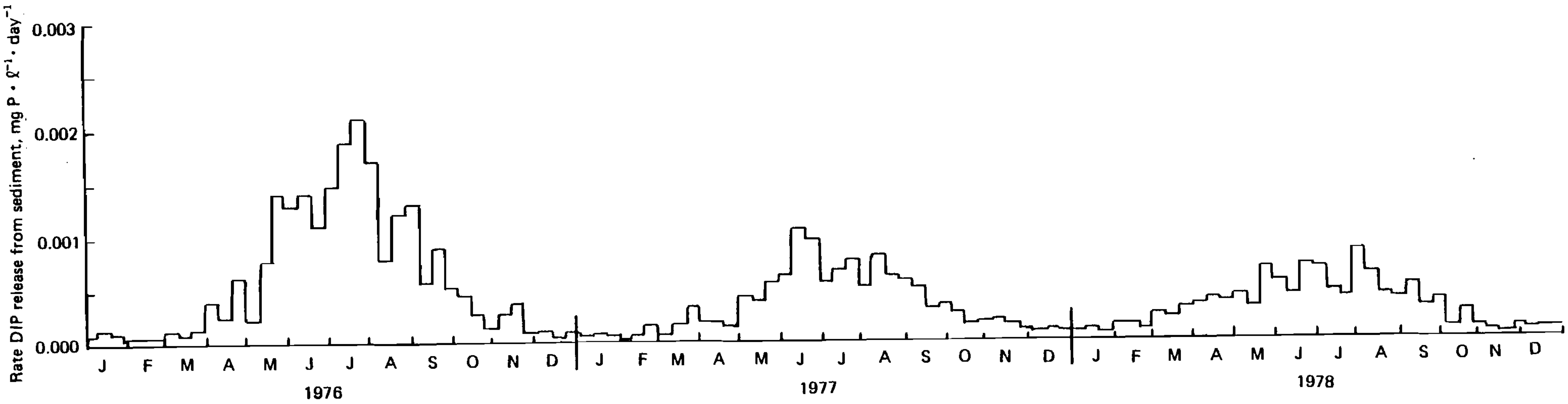

Figure 17. The dynamics of sediment DIP release rate in Basin I as calculated by model.

$1.49 \cdot 10^{-4}$, $1.10 \cdot 10^{-4}$ mgP/ℓ-day in 1976-1978. The total amount of DIP released by sediment per year, according to simulation, was estimated to be equal to 0.2278, 0.1227 and 0.1112 mgP/ℓ for 1976-1978 respectively in Basin I, 0.0643, 0.0349 and 0.0317 mgP/ℓ for Basin II, 0.0467, 0.0252 and 0.0228 mgP/ℓ for Basin III and 0.0321, 0.0173 and 0.0157 mgP/ℓ for Basin IV.

Taking into account the simulation results presented in Figures 13-17, it is possible to give a total assessment of sediment influence on the phosphorus cycling in Lake Balaton. Table 16 summarizes by season for 1976-1978 the data obtained by the model on the phosphorus exchange in the sediment-water layer for each basin studied. The comments below follow from the analysis of the data in Table 16:

(i) in 1976, the amount of phosphorus resuspended was higher than in 1977-1978; this may be explained by the more significant influence of wind action as the wind speeds were much stronger in 1976 than in 1977-1978;

(ii) particulate phosphorus resuspension was higher in the winter-spring months than in the summer-autumn months;

(iii) phosphorus sedimentation was higher in the spring-summer months than in the winter and autumn months;

(iv) net particulate phosphorus losses to sediment due to the interaction between resuspension and sedimentation were higher in the spring-summer months (as with sedimentation) when the rate of ecological phosphorus transformation was rapid and the amount of particulate phosphorus formed by biochemical processes was significant; the strong winds such as occurred in 1976, could decrease the general particulate phosphorus losses by increasing the intensity of its resuspension;

(v) the amount of DIP released by sediment was highest during the spring-summer months, as a result of favorable conditions in phosphorus transformation in the sediment, primarily defined by temperature; during the winter-autumn months the amount of the sediment DIP release was smaller by 5-10 times than in the spring-summer months;

(vi) the amount of DIP released by sediment was not sufficient

Table 16. Phosphorus exchange flows through sediment-water interface as shown by model calculation.

Basin	Season	Year	P_D resuspension, mgP/ℓ-3 months	P_D sedimentation, mgP/ℓ-3 months	Net P_D losses to sediment, mgP/ℓ-3 months	DIP release from sediment, mgP/ℓ-3 months	Net phosphorus losses to sediment, mgP/ℓ-3 months	Net phosphorus losses to sediment, kgP/day
I	Winter (Jan-Mar)	1976	.77189	.83215	.06026	.00913	.05113	46.6
		1977	.51694	.86532	.34838	.01033	.33805	308.0
		1978	.78269	.88066	.09797	.01383	.08414	76.7
	Spring (Apr-June)	1976	.82540	.98435	.15895	.07536	.08359	76.2
		1977	.47708	.72328	.24620	.04492	.20128	183.4
		1978	.63778	.97834	.34056	.03998	.30058	273.9
	Summer (July-Sept)	1976	.86137	1.05344	.19107	.12207	.06900	62.9
		1977	.37952	.56854	.18902	.05395	.13507	123.1
		1978	.43936	.70623	.26687	.04752	.21935	199.8
	Autumn (Oct-Dec)	1976	.65226	.82179	.16953	.02127	.14826	135.1
		1977	.42066	.53807	.11741	.01346	.10395	94.7
		1978	.42280	.56043	.13763	.00991	.12772	116.4
II	Winter (Jan-Mar)	1976	.48949	.51410	.02461	.00260	.02201	101.0
		1977	.32782	.41807	.09025	.00294	.08731	400.7
		1978	.49634	.52826	.03192	.00394	.02798	128.4
	Spring (Apr-June)	1976	.52343	.56706	.04363	.02147	.02216	101.7
		1977	.30254	.36922	.06668	.01280	.05388	247.2
		1978	.40445	.48965	.08520	.01139	.07381	338.7
	Summer (July-Sept)	1976	.54623	.65493	.10870	.03478	.07392	339.2
		1977	.24066	.31808	.07742	.01537	.06205	284.7
		1978	.27862	.36855	.08993	.01354	.07639	350.5
	Autumn (Oct-Dec)	1976	.41363	.47417	.06054	.00606	.05448	250.0
		1977	.26676	.30674	.03998	.00383	.03615	165.9
		1978	.26811	.30626	.03815	.00282	.03533	162.1
III	Winter (Jan-Mar)	1976	.38594	.39202	.00608	.00187	.00421	28.1
		1977	.25847	.28785	.02938	.00212	.02726	181.7
		1978	.39135	.39874	.00739	.00284	.00455	30.3
	Spring (Apr-June)	1976	.41270	.42551	.01281	.01546	-.00265	-17.7
		1977	.23854	.26895	.03041	.00921	.02120	141.3
		1978	.31888	.35436	.03548	.00820	.02728	181.9
	Summer (July-Sept)	1976	.43069	.50079	.07010	.02504	.04506	300.4
		1977	.18976	.24258	.05282	.01107	.04275	278.3
		1978	.21968	.27423	.05455	.00974	.04481	298.7
	Autumn (Oct-Dec)	1976	.32613	.35968	.03355	.00436	.02919	194.6
		1977	.21033	.23514	.02481	.00276	.02205	147.0
		1978	.21141	.23149	.02008	.00203	.01805	120.3
IV	Winter (Jan-Mar)	1976	.29732	.29888	.00156	.00129	.00027	2.4
		1977	.19912	.21477	.01565	.00146	.01419	126.4
		1978	.30148	.30526	.00378	.00195	.00183	16.3
	Spring (Apr-June)	1976	.31793	.32664	.00871	.01062	-.00191	-17.0
		1977	.19096	.20956	.01860	.00633	.01227	109.3
		1978	.24566	.27213	.02652	.00564	.02088	186.1
	Summer (July-Sept)	1976	.33179	.39232	.06053	.01722	.04331	385.9
		1977	.14619	.19415	.04796	.00761	.04035	359.6
		1978	.16923	.21660	.04737	.00670	.04067	362.4
	Autumn (Oct-Dec)	1976	.25123	.27933	.02810	.00299	.02511	223.8
		1977	.16202	.18449	.02247	.00190	.02057	183.3
		1978	.16285	.17977	.01692	.00139	.01553	138.4

to compensate for the phosphorus losses due to sedimentation of particulate phosphorus and only during the spring-summer months was the contribution of sediment DIP release to the total phosphorus balance considered important from the quantitative point of view;

(vii) there is a general tendency toward phosphorus accumulation in the sediment throughout all the seasons and it may be disturbed in periods with strong winds which occurred, for example, in the winter-spring months of 1976 when the fluxes of particulate phosphorus by resuspension and sedimentation were almost in balance.

Table 16 includes the rates of net phosphorus losses to sediment in kgP/day, which allow one to compare the general phosphorus removals from water to sediments in the different parts of the lake during the various seasons within the individual years studied. Thus the simulation results make it possible to receive a quantitative explanation of the phosphorus exchange processes in the sediment-water interface on the basis of assumptions used and data available for Lake Balaton.

PHOSPHORUS CYCLING WITHIN LAKE BALATON

In this study phosphorus transformation was considered from the point of view of the functioning of the phosphorus system in Lake Balaton. The principal goal of the study was to understand how the conversion of phosphorus forms developed in this water body, in order to obtain the quantitative information of the phosphorus dynamics and phosphorus cycling as a whole. These aspects of the study are important for an understanding of the relative importance of the natural phosphorus cycle in aquatic ecosystem behavior and in water body eutrophication in particular, defined by surplus phosphorus entering the water body from the watershed area.

The mathematical model of phosphorus transformation used in this study and composed from phosphorus compartments such as DIP, DOP, nonliving particulate-P, phyto- and bacterial-P, allow one to evaluate the quantities of phosphorus in the individual phosphorus compartments (or phosphorus pools) and the phosphorus recycling from one phosphorus form to another. Furthermore, on the basis of the data used, the model also estimates the various fluxes of phosphorus during each year studied. Considering the following phosphorus fluxes is interesting for the analysis of the phosphorus system:

(i) external input-output phosphorus fluxes (EF_i-OF_i) which take into account the total transport of phosphorus through the Lake Balaton basins;

(ii) system phosphorus flux (SPF) which takes into account the external phosphorus input and total amount of phosphorus transported by advective and wind-induced water flows;

(iii) compartment fluxes of phosphorus (CP_i) which take into account the amount of phosphorus transferred through the given compartment by internal transformation mechanisms and input from all external sources;

(iv) total phosphorus flux ($CF_{tot.P}$) which is the sum of flows for all phosphorus compartments.

Information on the external input-output phosphorus fluxes based on the tentative estimates of present phosphorus loading of Lake Balaton as well as on the direct phosphorus measurements in the River Zala in 1976-1978 is summarized in Table 17. According to the data used, the external input fluxes, considered in Table 17, include:

(i) DIP with River Zala discharge water, watershed runoff (sewage load), rainfall load as well as sediment load;

(ii) DOP with rainfall;

(iii) P_D with River Zala discharge water, watershed runoff and from sediment;

(iv) phyto- and bacterial-P with River Zala discharge water.

It should be noted that the role of the external sources mentioned in the phosphorus loading is quite different. For example, it is possible to evaluate the role of sediment as a potential phosphorus source. In 1976, the sediment in Basins I-IV contributed 31.9%, 50.5%, 49.1% and 36.7% respectively from the overall DIP load; for 1977 these estimates for the same basins were 19.4%, 35.5%, 34.3% and 23.9% and for 1978 they were equal to 16.5%, 34.3%, 33.2% and 22.9% for Basins I-IV respectively. In the total balance of nonliving particulate-P in 1976 the sediments provided 90.6%, 96.8%, 98.7% and 99.3% of particulate-P for Basins I-IV respectively; in 1977 these number were 76.1%, 91.1%, 96.2% and 97.8% for the same Basins and in 1978 they were estimated to be equal to 81.6%, 93.4%, 97.3% and 98.4% for Basins I-IV respectively.

In a comparison of the external input-output fluxes of phosphorus, presented in Table 17, we can see that phosphorus is removed from the water in Basins I-IV by sedimentation and in Basin IV also by advective water flow. Thus the estimated values of total balance in the external input-output fluxes of phosphorus show that it is stably positive for Basin I; for Basins II-IV it may be positive as well as negative, in accordance with the data and hypothesis used for the phosphorus load.

It is interesting to note that in Basin I the total external input of phosphorus is larger by 1.12-1.13 times than the output for 1976-1978. Estimated values of the total phosphorus balance for the whole lake show that in 1976 the external input flux was higher than the output flux and this difference is estimated to be equal to 74.7 kgP/d; in 1977 the output flux was higher by 65.9 kgP/d than the input phosphorus flux and in 1978, both fluxes were almost in balance and the difference is only 0.6 kgP/d. The data in Table 17 allows one to assume that the external input of phosphorus should be revised, especially for Basin II. This is considered to be important for obtaining a correct picture of the phosphorus interactions in the lake and using the model for the prediction of the future phosphorus levels (as a result of the changeable phosphorus loads).

On the basis of the simulation results obtained, the conditions of the phosphorus cycling may be estimated for 1976-1978 as a result of all the interactions considered in the model. The phosphorus flux through the system depends on the total flux

Table 17. Comparison of the input-output phosphorus fluxes for Lake Balaton Basins in 1976-1978 as calculated by the model.

Year	1976					1977					1978				
Dimension	mg P/ℓ-year				kg P/d	mg P/ℓ-year				kg P/d	mg P/ℓ-year				kg P/d
Basins	I	II	III	IV	whole lake	I	II	III	IV	whole lake	I	II	III	IV	whole lake
Input fluxes (EF_i):															
DIP	.7134	.1283	.0950	.0874	652.9	.6311	.0980	.0733	.0724	532.0	.6753	.0923	.0687	.0687	519.7
DOP	.0166	.0125	.0111	.0102	58.4	.0164	.0124	.0110	.0101	58.0	.0144	.0109	.0097	.0088	50.7
P_D	3.4341	2.0368	1.5755	1.2070	8312.6	2.3569	1.2492	.9320	.7063	5023.5	2.7941	1.5488	1.1730	.8930	6266.3
F	.0184	-	-	-	4.1	.0413	-	-	-	9.3	.0217	-	-	-	4.9
B	.0009	-	-	-	0.2	.0012	-	-	-	0.3	.0009	-	-	-	0.2
Total P	4.1834	2.1776	1.6815	1.3046	9028.2	3.0469	1.3596	1.0163	.7888	5623.1	3.5064	1.6520	1.2514	.9705	6841.8
Output fluxes (OF_i):															
DIP	-	-	-	.0020	4.4	-	-	-	.0051	11.2	-	-	-	.0018	3.9
DOP	-	-	-	.0015	3.3	-	-	-	.0012	2.6	-	-	-	.0014	3.1
P_D	3.6907	2.2103	1.6780	1.3014	8941.9	2.6952	1.4120	1.0345	.8065	5671.9	3.1257	1.6927	1.2588	.9768	6828.5
F	-	-	-	.0011	2.4	-	-	-	.0011	2.4	-	-	-	.0016	3.5
B	-	-	-	.0007	1.5	-	-	-	.0004	0.9	-	-	-	.0010	2.2
Total P	3.6907	2.2103	1.6780	1.3067	8953.5	2.6952	1.4120	1.0345	.8143	5689.0	3.1257	1.6927	1.2588	.9826	6841.2
Total P balance	.4927	-.0327	.0035	-.0020	74.7	.3517	-.0524	-.0182	-.0255	-65.9	.3807	-.0407	-.0074	-.0121	0.6

through all the phosphorus compartments and coupling between the compartments. Table 18 shows the results of the comparison of the phosphorus fluxes taken into account in the model. Ratios between compartment fluxes to external fluxes presented in Table 18, demonstrate the role of the internal transformation processes in the phosphorus cycling. This ratio for DIP, CF_{DIP}/EF_{DIP} was rather constant for 1976-1978 and equal to 1.3-1.4 in Basin I where the role of external sources in the DIP load was considered to be much more important than in the other Basins. For the Basins II-IV the ratio, CF_{DIP}/EF_{DIP} was changed from 2.4 to 3 for 1976-1977 and from 2 to 2.4 for 1978. For DOP, the role of internal cycling was assumed to be quite significant in comparison to the external load because only a single external source, precipitation, was used and the corresponding range of the ratio CF_{DOP}/EF_{DOP} was 14-42 for Basins I-IV during 1976-1978. The ratio CF_{P_D}/EF_{P_D} was quite stable for all the basins considered in 1976-1978 and it was equal to 1.2-1.3. For total phosphorus, this ratio was more or less constant in 1976-1977 and equal to 1.8-2.0 while in 1978 it decreased to 1.6.

The ratio $CF_{Total\ P}/SPF$ shows the role of phosphorus cycling by chemical-biological transformation in contrast to phosphorus loading from external sources and as a result of hydrodynamical transport. This ratio was changed within the range 1.6-1.9, 1.7-2.0 and 1.5-1.6 for 1976-1978 respectively.

Finally the ratio $SPF/EF_{Total\ P}$ shows the role of hydrodynamical transport of phosphorus fractions in the phosphorus load in the individual basins for 1976-1978. This ratio, being changeable in a small range, was taken as 1.06-1.14 for Basins I-III and 1.03-1.04 for Basin IV during the three years studied.

A better understanding of phosphorus recycling in the system may be obtained through an analysis of turnover time of total phosphorus as well as its individual fractions (Pomeroy 1970). As Watson and Loucks (1979) indicated, the turnover time of the system may be considered as a function of the turnover time of its parts and the nature of the coupling between compartments. It is defined as the time required for the amount of the substance

Table 18. Comparison of the phosphorus fluxes determined by internal phosphorus cycling within Lake Balaton in 1976-1978.

Y e a r	I976					I977					I978				
Dimension	mg P/ℓ-year				kg P/d	mg P/ℓ-year				kg P/d	mg P/ℓ-year				kg P/d
B a s i n s	I	II	III	IV	whole lake	I	II	III	IV	whole lake	I	II	III	IV	whole lake
I.Compartment fluxes (CF_i):															
DIP	I.0I28	.3897	.2803	.2426	I66I.2	.8436	.2607	.I890	.I735	II75.7	.8483	.22I4	.I506	.I377	990.7
DOP	.69I2	.4338	.3309	.27I5	I785.4	.4765	.264I	.2022	.I738	III9.4	.4I07	.I99I	.I4I8	.II97	28I3.I
P_D	4.357I	2.59I2	I.97I5	I.5I77	I0479.3	3.0787	I.6063	I.I832	.92II	6473.6	3.479I	I.8534	I.3745	I.0574	7456.6
F	.9446	.422I	.275I	.234I	I655.2	.8265	.2880	.I899	.I73I	I203.3	.7985	.2435	.I505	.I370	I002.6
B	.53I0	.4028	.3226	.27I2	I700.2	.3752	.2474	.I979	.I73I	I069.2	.2977	.I826	.I38I	.II89	76I.2
Total P flux	7.5367	4.2396	3.I804	2.537I	I728I.3	5.6005	2.6665	I.9622	I.6I46	II04I.2	5.8343	2.7000	I.9555	I.5707	I3024.2
II.System P flux(SPF)	4.6576	2.4928	I.85I4	I.3579	9887.3	3.26I0	I.5338	I.I048	.8202	6082.2	3.7I59	I.8257	I.3355	.999I	7286.2
III.Ratios :															
a. CF_i/EF_i															
DIP	I.4	3.0	2.9	2.8	2.5	I.3	2.7	2.6	2.4	2.2	I.3	2.4	2.2	2.0	I.9
DOP	4I.6	34.7	29.8	26.6	30.6	29.0	2I.3	I8.4	I7.2	I9.3	28.5	I8.3	I4.6	I3.6	50.5
P_D	I.3	I.3	I.3	I.3	I.3	I.3	I.3	I.3	I.3	I.3	I.2	I.2	I.2	I.2	I.2
Total P	I.80	I.95	I.89	I.94	I.9I	I.84	I.96	I.93	2.05	I.96	I.66	I.63	I.56	I.62	I.90
b. $CF_{Total\ P}/SPF$	I.62	I.70	I.72	I.87	I.75	I.72	I.74	I.78	I.97	I.82	I.57	I.48	I.46	I.57	I.79
c. $SPF/EF_{Total\ P}$	I.II	I.I4	I.I0	I.04	I.09	I.07	I.I3	I.09	I.04	I.08	I.06	I.I0	I.07	I.03	I.06

transferred into or out of a compartment to be equal to the amount present in the compartment (Robertson 1957).

The mean annual turnover time of all phosphorus compartments were calculated for each basin and for individual years studied by dividing the phosphorus pool sizes averaged for the year considered, by the rate of phosphorus fluxes through the pools, for the same period of time. Flux rate in this case was defined as the average of the input and output rates of the pools. The results of calculating the turnover time of the phosphorus compartments and total phosphorus obtained with the aid of the model are summarized in Table 19.

The analysis of the turnover time of the individual phosphorus compartments may help to explain the role of these compartments in phosphorus cycling. It is recognized that compartments may act as a holder or a mover of nutrients inside a system (Pomeroy 1974). The analysis of the data in Table 19 shows that the fastest turnover within the phosphorus compartments in the lake as estimated by by the model, concern only the bacterial-P and nonliving particulate-P (or detritus) formed as a result of the action of microorganisms. The turnover time of bacterial-P is estimated to be changeable in a relatively small range, 3.2-6.7 days (mean 5.5 day for the whole lake during 1976-1978). The turnover time of nonliving particulate-P is also estimated as being a characteristic with a small variance for the different basins, so that the possible range of the turnover time is 2.1-3.4 days (mean 2.7 days for the whole lake during 1976-1978). The turnover time for dissolved inorganic phosphorus and phytoplankton-P are estimated to be similar, so that the ranges in estimates of the turnover time are 3.7-13.1 days (mean 8.0 days) for DIP and 5.1-9.7 days (mean 8.0 days) for phytoplankton-P. The assessment of the turnover time for DIP obtained for the Lake Balaton ecosystem by the given model is in accordance with the measured turnover time of 5-10 days for the phosphates in other lakes (Golterman 1975). The range of the turnover time for DOP is 8.1-12.1 days (mean 9.8 days) and as follows from the analysis, this phosphorus compartment may be considered the one with the slowest turnover among all the compounds taken into account in the model. The turnover

Table 19. Annual turnover times (in days) for phosphorus fractions in Basins I-IV as evaluated by the model.

Phosphorus fractions	I			II			III			IV			Range	Mean
	1976	1977	1978	1976	1977	1978	1976	1977	1978	1976	1977	1978		
DIP	4.3	3.9	4.3	3.7	4.8	5.0	9.2	11.4	12.1	11.0	12.8	13.1	3.7-13.1	8.0
DOP	8.6	9.2	11.6	9.2	9.7	11.0	8.8	9.0	11.6	8.1	8.8	12.1	8.1-12.1	9.8
P_D	2.1	2.2	2.1	2.5	3.2	3.3	2.2	3.1	3.4	2.4	3.2	3.2	2.1- 3.4	2.7
F	5.1	5.4	5.8	7.2	8.8	9.7	8.0	9.6	9.7	8.6	8.4	9.4	5.1- 9.7	8.0
B	3.2	4.9	5.0	5.0	5.7	6.0	5.4	6.1	6.4	5.8	6.4	6.7	3.2- 6.7	5.5
Total P	5.4	6.0	5.6	6.7	7.6	6.4	7.8	9.3	7.2	9.6	11.9	9.1	5.4-11.9	7.7

time of the total phosphorus is comparable with the turnover time for DIP and its range equal to 5.4-11.9 days (mean 7.7 days for the whole lake during 1976-1978).

Thus the estimated values of turnover time for the individual phosphorus compartments show that the phosphorus cycling is developed slightly faster in Basins I-II than in Basins III-IV. For the environmental conditions of 1976-1978 the phosphorus system shows similar properties in annual phosphorus cycling which indicates the stability of the way in which Lake Balaton's ecosystem functions. Undoubtedly, some differences in the phosphorus cycling may be expected from year to year for the individual seasons within the year, as a result of the combined influence of the weather conditions and the nutrient loads. Therefore further insight into the functioning of Lake Balaton's ecosystem can be gained by considering the seasonal turnover time of the phosphorus compartments, as well as by analyzing the behavior of the individual phosphorus compartments in the phosphorus transformation.

CONCLUSIONS

1. The emphasis in the given report is on the examination of the improved version of the Balaton Sector Model (BALSECT), which includes the five phosphorus compartments (dissolved inorganic-P, dissolved organic-P, nonliving particulate organic-P, phytoplankton-P and bacterial-P) and takes into account the biochemical interactions between these phosphorus compartments as well as phosphorus exchange through the sediment-water interface and the horizontal interbasin transport of phosphorus by advective and wind-induced water flow.

2. The given model was applied to a real set of original field observations for temperature, wind, radiation, water balance and phosphorus loading to examine the model's ability to describe the phosphorus dynamics in the different parts of Lake Balaton in 1976-1978.

3. The model adequacy in representing the phosphorus dynamics observed in Lake Balaton basins in 1976-1978 was evaluated by

applying statistical methods. The adequacy of the simulation results to the observations was shown by the comparison of mean values of phosphorus compartments as well as their variances. It was found that phosphorus concentrations, observed and simulated, have a distribution close to normal and the fluctuations of both sets of data lie in the range $\pm 3\sigma$ for all phosphorus compartments studied. Estimated values of 95% confidence intervals of mean phosphorus concentrations in observed and simulated phosphorus data are in reasonable agreement, excluding data for Basin II, where phosphorus loading seems to be lower than may be expected by observed phosphorus levels. The comparison of mean phosphorus concentrations evaluated on the different amount of data in samples, observations and simulation results, by the F-test shows that the variances of means in two group of data may be considered as a homogenous whole. According to data on the F-test, the DIP dynamics were simulated better in 1976-1977 than in 1978, while the DOP dynamics were better modeled for 1976 and 1978 than for 1977; however, the mean values of DIP and DOP evaluated on the simulated results, are lower than those estimated on scarce observations covering the spring-autumn periods within 1976-1978. The dynamics of nonliving particulate organic-P is better simulated for 1977-1978 than for 1976 and the total P is modeled better for 1976 and 1978 than for 1977. When a comparison was made between modeling results and observations for combined sets of data on all phosphorus fractions there was reasonable agreement. The ratio of variances in simulated phosphorus data to that observed is estimated to be equal and changeable in the range 34.7-77.9% (mean 41.9%) in Basins I through IV for 1976-1978. The quantitative relationship between phosphorus concentrations in the observed and simulated time series of phosphorus concentrations was also analyzed by regression analysis, simple as well as weighted. Finally, Theil's inequality coefficients were calculated for each phosphorus compartment on the basis of observations available, and simulation results were obtained. This coefficient illustrates that the range of errors in the simulation of the dynamics of individual phosphorus fractions is 0.154-0.221 (mean 0.2) for total P, 0.214-0.291 (mean 0.251) for DIP, 0.243-0.353

(mean 0.283) for nonliving particulate organic-P, 0.269-0.325 (mean 0.296) for total dissolved P and 0.303-0.405 (mean 0.369) for DOP. The range of errors in the simulation of the phosphorus dynamics is estimated to be 0.203-0.261 (mean 0.225) for Basin I, 0.250-0.347 (mean 0.295) for Basin II, 0.204-0.284 (mean 0.252) for Basin III and 0.237-0.307 (mean 0.290) for Basin IV. The errors in the simulation of the phosphorus dynamics are estimated as 0.267, 0.262 and 0.223 for 1976-1978 respectively and 0.253 for the total three year period, 1976-1978.

4. A series of sensitivity runs on the basis of data available for 1977 were conducted to understand how the quality of input data on the temperature, radiation and phosphorus loading may change a model output for Basin I. In these runs for the values of input data, a different degree of time scale averaging, corresponding to day, week, month and season was used. The results of the sensitivity analysis compared with a control run where the original set of observations was used, show that: (a) weekly averaging of the mentioned input data do not practically change the model output; (b) monthly averaging of the same input data gives slightly lower values of mean monthly and seasonal mean phosphorus concentrations for the winter and summer periods and slightly higher ones for the spring and autumn periods; (c) seasonal averaging significantly changes the model output for the summer months, so that mean monthly concentrations of DIP, DOP, particulate organic-P, total soluble P and total P are lower by 31.3-45%, 46.1-62%, 18.3-31%, 44-64% and 35-36% respectively than in a control run; this procedure also essentially alters the dates of extreme phosphorus concentrations for the spring and summer months; (d) the averaging of temperature and phosphorus loading data have an express influence on the model output in contrast to averaging of the radiation data; for example, the seasonal averaging of the temperature data may change the mean seasonal concentrations of total P to 0.5%, 8.5%, 15.4% and 7.6% for the winter, spring, summer and autumn respectively; the same averaging of the phosphorus loading data gives the change of the mean total P concentrations on 1.2%, 13.1%, 1.9% and 4.7% for similar seasons, while the averaging of the radiation data change the model output only

by 1.9% for winter, by 0.4-0.5% for the spring-summer months and by 1.5% for the autumn months; (e) seasonal averaging of the phosphorus loading data has a significant influence on the dates of the minimum and maximum phosphorus concentrations during the winter-spring months while the same time scale of averaging of the temperature data has an influence on the dates of extreme phosphorus concentrations during the spring, summer and autumn months; the seasonal averaging of radiation data show the influence only on the dates of the minimum phosphorus concentrations during the winter and autumn months.

5. The role of the sediments as a potential phosphorus source was evaluated in the given report. In 1976, the sediments provided 31.9%, 50.5%, 49.1% and 36.7% of DIP from all DIP sources in Basin I through IV respectively; for 1977 the same estimates were 19.4%, 35.5%, 34.3% and 23.9% and for 1978 they were 16.5%, 34.3%, 33.2% and 22.9% for Basins I-IV respectively. As for nonliving particulate-P, the model estimates the sediment contribution in 1976 as 90.6%, 96.8%, 98.7% and 99.3% for Basins I-IV respectively. For 1977, these estimates were 76.1%, 91.1%, 96.2% and 97.8% and for 1978 81.6%, 93.4%, 97.3% and 98.4% for Basins I-IV respectively. According to the simulation results, the general tendency of the phosphorus to accumulate in the sediment was expressed for all the seasons within 1976-1978 and it was disturbed just in the periods with strong winds, such as occurred in the winter-spring months of 1976. The net particulate-P losses to sediment due to the interaction between resuspension and sedimentation were estimated to be higher in the spring-summer months when the rate of the ecological phosphorus transformation is highest and the total amount of particulate-P biochemically formed is significant. The amount of DIP released by the sediment is considered as having an influence only during the spring-summer months, but this process may only slightly compensate the total phosphorus losses due to sedimentation, even during that period.

6. The conditions of phosphorus cycling in Lake Balaton were estimated by considering phosphorus fluxes such as external input-output (EF_i-OF_i) phosphorus fluxes, system phosphorus flux (SPF),

compartment phosphorus fluxes (CF_i) and total phosphorus flux ($CF_{tot.P}$). In the given analysis, the ratio CF_i/EF_i demonstrates the role of the internal phosphorus transformation in providing the nutrients as opposed to that of the external loading. This ratio for DIP looks quite stable (1.3-1.4) for 1976-1978 in Basin I, where the role of external loading is significantly higher than in Basins II-IV. For the latter basins the ratio CF_{DIP}/EF_{DIP} is 2.4-3 for 1976-1977 and 2-2.4 for 1978. For DOP the role of the internal transformation is considered much more important than external loading and the ratio CF_{DOP}/EF_{DOP} is estimated to be equal to 14-42 for Basins I-IV during 1976-1978. For nonliving particulate-P this ratio appears to be constant and equal to 1.2-1.3 for Basins I-IV for 1976-1978, which testifies to the definite balance between all the processes providing external and internal input of particulate-P. The ratio $CF_{tot.P}/SPF$ showing the role of phosphorus biochemical cycling in contrast to the loading by external input and hydrodynamical transport, is estimated to be in the range 1.6-1.9, 1.7-2 and 1.5-1.6 for 1976-1978 respectively. The role of hydrodynamical transport in phosphorus loading may be evaluated by the ratio $SPF/EF_{tot.P}$ and the range of this ratio is relatively small, 1.06-1.14 for Basins I-III and 1.03-1.04 for Basin IV during 1976-1978.

7. The properties of the lake's ecosystem as well as the role of the phosphorus compartments in the phosphorus cycling were investigated through the analysis of the simulated phosphorus dynamics and phosphorus fluxes. Flux rates, pool sizes and turnover times were computed for the individual phosphorus fractions. The analysis of turnover time indicates that the bacterial-P and nonliving particulate organic-P are the fastest phosphorus compartments within Lake Balaton's ecosystem and their annual mean turnover time was estimated to be equal to 5.5 day and 2.7 days respectively. The turnover time of DIP is comparable with those for the phytoplankton-P and it is equal, for both of these fractions, to 8.0 days. The turnover time of DOP is estimated to be slightly higher than for DIP and is equal to 9.8 days. The turnover time of the total P is close to DIP and phytoplankton-P turnover time and it is estimated to be 7.7 days. The turnover

of total P occurs in 5.4-7.6 days within Basins I-II and in 7.2-11.9 days within Basins III-IV. On an annual basis the values of turnover time of the individual phosphorus compartments indicate stability and also show that similar conditions existed in the phosphorus cycling in the period 1976-1978.

Model equations of biochemical phosphorus transformation.

$$R_{DIP_j} = L_{B_j} \cdot B_j - UP_{F_j} \cdot F_j$$

$$R_{DOP_j} = K_3 \cdot P_{D_j} + L_{F_j} \cdot F_j - UP_{B_j} \cdot B_j$$

$$R_{PD_j} = M_{F_j} \cdot F_j + M_{B_j} \cdot B_j - K_3 \cdot P_{D_j}$$

$$R_{B_j} = (UP_{B_j} - L_{B_j} - M_{B_j}) \cdot B_j$$

$$R_{F_j} = (UP_{F_j} - L_{F_j} - M_{F_j}) \cdot F_j$$

1.Microorganism growth (or substance uptake):

- for phytoplankton $$UP_{F_j} = \frac{K_1 \cdot R_{TF} \cdot R_{IF}}{1 + F_j/(\gamma \cdot DIP_j)}$$

K_1 is maximum uptake rate (day^{-1});

γ is coefficient of substrate conversion per unit biomass (unitless);

R_{IF} is light reduction factor (unitless) :

$$R_{IF} = (e/K_e \cdot h)\left[\exp(-r_x) - \exp(-r_1)\right]$$

$h = 0.5$ m;

$r_1 = I/I_{opt}$; $I_{opt} = 350$ cal/cm^2-day

$$r_x = r_1 \cdot \left[\exp(-K_e \cdot h)\right]$$

K_e is light extinction coefficient (m^{-1}):

$$K_e = K_a + K_b \cdot (\mu gChl/\ell)$$

I is daily course of light intensity (cal/cm^2-day):

$$I = I_{max} \cdot h \cdot \left[1 + \cos \frac{2\pi \cdot (t_{now} - t_{peak})}{f}\right]$$

t_{now} is current time of day in hours;

t_{peak} is I2 o'clock when light intensity is maximum;

f is photoperiod in hours;

I_{max} is maximum light intensity

(cal/cm^2-day):

$$I_{max} = 2 \cdot I_{av}/f$$

I_{av} is mean daily light intensity (cal/cm^2-day).

R_{TF} is temperature reduction factor(unitless)

$$R_{TF} = 0.2 + \frac{0.022(e^{0.21 \cdot T} - 1)}{1 + 0.028e^{0.21 \cdot T}}$$

T is water temperature in ^oC.

- for bacteria

$$UP_{B_j} = \frac{K_2 \cdot R_{TB}}{1 + B_j/DOP_j}$$

K_2 is maximum uptake rate (day^{-1});

R_{TB} is temperature reduction factor (unitless):

$$R_{TB} = 0.3 + \frac{3.68 \cdot 10^{-3}(e^{0.403 \cdot T} - 1)}{1 + 5.25 \cdot 10^{-3}e^{0.403 \cdot T}}$$

2.Microorganism's metabolical excretion:

- for phytoplankton

$$L_{F_j} = r_{F_j} \cdot UP_{F_j}$$

r_{F_j} is the coefficient representing the fraction of excretion over uptake:

$$r_{F_j} = \frac{(a_1/a_2) \cdot UP_{F_j}}{(1/a_2) + UP_{F_j}} + (1 - a_1/a_2)$$

- for bacteria

$$L_{B_j} = r_{B_j} \cdot UP_{B_j}$$

r_{B_j} is the coefficient representing the fraction of excretion over uptake:

$$r_{B_j} = \frac{(a_3/a_4) \cdot UP_{B_j}}{(1/a_4) + UP_{B_j}} + (1 - a_3/a_4)$$

a_i are coefficients with dimensions(day).

3.Microorganism mortality :

- for phytoplankton $M_{F_j} = (v_1 \cdot F_j)/UP_{F_j}$

v_1 is constant $(mg\ P/\ell)^{-1} \cdot (day)^{-2}$

- for bacteria $M_{B_j} = v_2 + (v_3 \cdot B_j)/UP_{B_j}$

v_2 and v_3 are constants with dimensions day^{-1} and $(mg\ P/\ell)^{-1} \cdot (day)^{-2}$ respectively.

4.Temperature-dependent rate of detritus decomposition:

$$K_3 = \frac{1.2 \cdot 10^{-4}(e^{0.351 \cdot T} - 1)}{1 + 3.0 \cdot 10^{-4} \cdot e^{0.351 \cdot T}}$$

T is water temperature in $^{\circ}C$.

Appendix B. Values of initial phosphorus concentrations and rate coefficients.

Parameters	Units	Symbols	Basins			
			I	II	III	IV
<u>State variables(Jan I 1976)</u>:						
Dissolved inorganic phosphorus	mg P/ℓ	DIP	.002	.002	.0015	.001
Dissolved organic phosphorus	- " -	DOP	.005	.010	.005	.004
Phytoplankton phosphorus	- " -	F	.005	.003	.0025	.002
Bacterial phosphorus	- " -	B	.001	.0008	.0007	.0006
Non-living particulate organic phosphorus	- " -	P_D	.010	.004	.003	.002
Chlorophyll "a"	μg/ℓ	Chl	10.6	6.4	5.3	4.3
<u>Rate constants and other parameters</u>						
Maximum uptake rate for phytoplankton	day^{-1} at 20^oC	K_1	2.8	2.8	0.9	0.9
Excretion efficiency of phytoplankton	day	a_1	.057	.057	.057	.057
- " - " - " - " - " - " - " - " - "	- " -	a_2	.075	.075	.075	.075
Phytoplankton mortality as function of biomass and nutrient levels	$(mgP/\ell)^{-1}day^{-2}$	v_1	.2	.2	.2	.2
Coefficient of substrate conversion by phytoplankton	unitless	γ	.6	.6	.6	.6
Maximum uptake rate for bacteria	day^{-1} at 20^oC	K_2	.3	.3	.3	.3
Excretion efficiency of bacteria	day	a_3	.3	.3	.3	.3
- " - " - " - " - " - " - " -	- " -	a_4	.45	.45	.45	.45
Natural mortality of bacteria	day^{-1}	v_2	.053	.053	.053	.053
Bacterial mortality as a function of biomass and nutrient levels	$(mgP/\ell)^{-1}day^{-2}$	v_3	1.0	1.0	1.0	1.0
Particulate Phosphorus decomposition rate	day^{-1} at 20^oC	K_3	.1	.1	.1	.1
Extinction coefficient	m^{-1}	K_a	1.8	1.8	1.5	1.5
- " - " - " - " - " -	-	K_b	.0088	.0088	.0088	.0088

Appendix C. Weekly average temperature,radiation and phosphorus concentrations in River Zala discharge water(data for I977

Week starting from 1st of January	Temperature °C	Radiation cal/cm^2-day	Orthophosphate-P mgP/ℓ	Total P mgP/ℓ
1	0.01	58.43	0,0675	0.2261
2	0.01	68.57	0,0523	0.2658
3	0.01	91.86	0.0614	0.2694
4	0.05	70.43	0.0398	0.2806
5	0.47	110.43	0.0700	0.3749
6	0.70	94.0	0.0943	0.3790
7	2.01	157.86	0.0399	0.3926
8	5.34	182.29	0.0729	0.2154
9	3.87	210.29	0.0788	0.2373
10	4.71	301.86	0.0737	0.2129
11	6.51	315.14	0.0909	0.2584
12	9.30	353.86	0.1104	0.2732
13	9.49	272.43	0.0944	0.2327
14	6.97	256.57	0.0956	0.3037
15	7.54	285.29	0.0663	0.5649
16	7.46	414.71	0.0653	0.2928
17	8.37	324.00	0.0845	0.4979
18	13.77	449.14	0.1484	0.4189
19	15.77	465.29	0.2026	0.3701
20	17.64	328.29	0.2112	0.3701
21	18.76	465.57	0.2139	0.3672
22	18.27	538.14	0.1824	0.4334
23	18.69	515.43	0.1673	0.4076
24	22.51	534.86	0.2279	0.4688
25	22.76	488.00	0.4553	0.4898
26	22.33	499.29	0.5921	0.5980
27	22.21	533.29	0.4862	0.5440
28	22.37	501.14	0.4522	0.5984
29	21.37	403.43	0.4941	0.5668
30	20.70	451.14	0.4500	0.4990
31	19.74	367.00	0.3991	0.4382
32	21.44	459.57	0.3932	0.4194
33	22.03	378.43	0.3427	0.4985
34	21.13	353.00	0.3520	0.5579
35	20.01	401.00	0.3300	0.4674
36	22.26	401.43	0.3658	0.4896
37	19.70	304.86	0.4383	0.4642
38	15.13	173.86	0.4202	0.4710
39	13.89	274.57	0.3859	0.4014
40	13.66	268.71	0.3181	0.3242
41	14.09	187.86	0.3757	0.4431
42	13.67	215.29	0.3716	0.4484
43	12.86	134.00	0.2633	0.5032
44	12.26	93.00	0.2772	0.4038
45	11.60	76.86	0.2979	0.4437
46	9.74	98.43	0.2975	0.6803
47	6.13	111.29	0.1796	0.3218
49	0.01	71.86	0.1668	0.2670
50	1.66	62.43	0.1537	0.2423
51	2.49	36.14	0.1730	0.2382
52	3.61	58.63	0.1836	0.3093

Appendix D. Monthly average temperature, radiation and phosphorus concentrations in River Zala discharge water(data for I977)

Month	Temperature °C	Radiation cal/cm^2-day	Orthophosphate-P mgP/ℓ	Total P mgP/ℓ
Jan	0.06	76.87	0.0556	0.2698
Feb	2.77	138.79	0.0716	0.3270
Mar	6.91	304.90	0.0904	0.2440
Apr	7.49	324.70	0.0798	0.4111
May	16.97	438.06	0.1958	0.3851
June	21.26	519.70	0.3353	0.4824
July	21.61	466.61	0.4698	0.5469
Aug	20.87	385.74	0.3625	0.4790
Sept	17.94	297.57	0.3977	0.4563
Oct	13.45	189.68	0.3260	0.4292
Nov	8.55	95.77	0.2454	0.4279
Dec	1.92	58.74	0.1698	0.2642

Appendix E. Seasonally average temperature, radiation and phosphorus concentrations in River Zala discharge water(data for I977)

Season	Temperature °C	Radiation cal/cm^2-day	Orthophosphate-P mgP/ℓ	Total P mgP/ℓ
Winter (Jan - Mar)	3.27	175.96	0.0728	0.2783
Spring (Apr - June)	15.45	428.04	0.2087	0.4295
Summer (July - Sept)	20.15	384.82	0.4084	0.4935
Autumn (Oct - Dec)	7.97	114.93	0.2471	0.3732

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