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EFFICIENT USE OF PRICES AND QUANTITY CONSTRAINTS FOR CONTROL AND COORDINATION OF LINEAR SECTORAL PRODUCTION MODELS

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PREFACE

A number of complex national agricultural models with linear programming description of the sectors of farms have been developed. Intended as tools for policy analysis, these models do not clearly endognize the government's behaviour. However, it is interesting to explore the consequences and possibilities of an efficient utilization of the sectoral programming models for a fully conscious governmental decision making. The topic is especially important for those centrally planned economies where the failure of the administrative (command) methods of management of agriculture has been unequivocally acknowledged and yet no consistent alternative mechanism has so far emerged.

The paper presents a scheme for the control and coordination of production plans generated by the linear aggregative (sectoral) programs. Thereby prices and, if need be, quotas and constraints on input availabilities may be set in such a way as to guarantee the satisfaction of the governmental goals while fully respecting sovereignity of the profit-motivated producers.

It has been demonstrated that prices alone, even if fully controlled by the government, need not lead to the satisfaction of the achievable government's goals. It is only with the introduction of quotas on outputs and limits on available inputs that the possibility of a reconciliation of the government's goals with the behaviour of profit-motivated sectors is recovered. However, the introduction of quantity restrictions imposes additional conditions that cannot be violated for fear that this would unleash uncontrollable speculation with respect to licences for production and inputs. The analytical framework for the consistent simultaneous determination of prices and quantity restrictions has been given. This implies solving linear programming models with some nonconcave quadratic constraints.

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Introduction

The paper presents a scheme for the control and coordination of production plans generated by the linear aggregative (sectoral) programs. Thereby prices and, if need be, quotas and constraints on input availabilities may be set in such a way as to guarantee the satisfaction of the governmental goals while fully respecting sovereignity of the profit-motivated producers.

Key words: sectoral aggregative models, two-level planning, production control through prices and quantity constraints.

A number of complex national agricultural models with linear programming description of the sectors of farms have been developed (for a list of references see McCarl and Spreen, 1980). Intended as tools for policy analysis, these models do not clearly endognize the government's behaviour. However, it is interesting to explore the consequences and possibilities of an efficient utilization of the sectoral programming models for a fully conscious governmental decision making. The topic is especially important for those centrally planned economies where the failure of the administrative (command) methods of management of agriculture has been unequivocally acknowledged and yet no consistent alternative mechanism has so far emerged.

As long as the objectives that can be ascribed to the governments of the centrally planned economies (and the instruments available to them) differ from those typical in the framework of a market economy, the analytical framework to be of some appeal cannot be easily borrowed from the existing studies. Some of these studies (e.g. Meister, Chen and Heady, 1978) do not envisage any place for a governmental intervention in agriculture and address themselves to the context where any governmental intervention is in fact prohibited. Others (e.g. Candler, Fortuny-Amat and McCarl) consider the problem as typical in the concept of coordination in hierarchical systems. By concentrating on interaction between the separate decision makers at various levels of hierarchical ladder they do not pay sufficient attention to the possibilities of cooperation (exchange) among formally unconnected (equally ranked) elements of the hierarchy.

All in all, the subject of a coordinated use of both price and quota instruments in agricultural production policies in the linear programming context has not - to the author's knowledge - been analytically tackled so far. (Of course there has been a lot of constructive work on the subject of the use of these instruments in control of domestic consumption and international trade (see Keyzer, 1980).

The question whether one specific set of governmental objectives and instruments make more sense than the other will certainly not be resolved in this paper. However, some conclusions concerning the limitations on both price and price-and-quota policies will be reached.

1. Direct vs Indirect Control of Economic Activities

One of the most important issues related to the operation of a centrally planned economy has been the design of workable mechanisms for allocation of resources and production targets among production units. At early stages of the economic development the allocation is decided administatively by government agencies (ministries) supervising the performance of production units. Growing sophistication of production processes (proliferation of products, inputs, technologies; deepening of specialization and cooperation among production units) makes the task of ministries increasingly difficult. Not infrequently the decisions appear incorrect, leading to wastage of resources in production units and therefore reducing the overall well-being of the society.

The decline in the efficiency of the administative allocation system is accompanied by growing difficulties in enacting the decisions. Under the system, the production units get rewarded for subordination, i.e., for fulfilment of production targets while keeping the use of inputs within pre-determined confines. Accordingly, the failure to meet production-input targets is penalized.

As the allocation decisions become less and less accurate, the corresponding reward-penalty system becomes less and less fair. (In particular the production targets for some units may not be possible to fulfil at all at given availability of inputs.) The "unfairness" of the reward system unleashes, in turn, destructive invention on the part of the production units which specialize in playing down their production possibilities. (By this they try to secure lowest possible production targets and highest possible input endowments so as to guarantee high reward for "subordination" without much real effort.) Also all sorts of tricks are played to rationalize failures in meeting the production targets (blaming weather, cooperation, incorrect ministry's decisions). Needless to say this way of controlling production causes tremendous losses at the level of particular production units--there is no real coherent motive for economic, let alone, positively innovative behaviour on the part of enterprises. Typically the enterprises purposefully do not develop let alone utilize their potential for fear that this would cause an increase in expectations concerning their future activities.

There are several theoretical proposals concerning the issue of improving the efficiency of the administrative system of production control. Most of them provide for the creation of a "centrally operated" optimization model which would generate the allocation decisions free of mistakes and inefficiences typical of the traditional bureaucratic routines (Lange, 1967)[1]

While agreeing that the introduction of computers and mathematical modelling might significantly enhance the efficiency of the administrative allocation decisions, one must not overlook the fact that this will leave some basic disadvantages of the original approach pretty much unchanged.[2] There is every reason to believe that even under computerized and optimization-oriented administrative allocation procedure, the reward system favoring subordination (and therefore punishing insubordination) will push enterprises' creativity in the wrong direction.

The real theoretical alternative to the administrative allocation procedure would require the introduction of the profit or a profit-like motive as the sole basis of the actual reward system for separate, otherwise independent enterprises.

However, the profit motive has always operated within the framework of a market economy, where prices emerge out of rather uncontrollable processes with many factors (speculative, monopolistic) contributing to their imperfection and leading to some economically and socially undesirable phenomena.

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[1] A very large number of studies have been concerned with algorithms for solving the "master" optimization model in an iterative way that may be interpreted as an interactive planning requiring exchange of information between the governmental agency and the subordinate production units. Usually, the topic is covered by the keyword "decomposition of linear programs" (see Dantzig and Wolfe, 1960; Kornai and Liptak, 1965; Johansen, 1978). It should be observed that the "interactive" interpretation does not allow for purposeful misinformation. (This has been rightly pointed out by Malinvaud, 1972.)

[2] Since mathematical models are hypotheses, their optimum solutions may not work in practice just the same way as the simple administrative decisions have not.

Since this way of generating prices seems incompatible with the basic ideals of a socialist economy, the question arises of how to set the prices so as to make the profit motive work, yet without really losing control over basic economic and social events.

Theoretically, this question was "constructively" answered in an early work by Lange (1936). The prescription required construction of a mathematical model of the market, with separate groups of producers and consumers being described by separate supply-demand models derived from the assumption of economic behavior (i.e. maximization of profit or utility).

Then, the computation of the economic equilibria, executed in a computer, would provide the government with correct equilibrium prices to be announced to the populations of producers and consumers.

In this way, the discrepancy between "marketplace" and "central planning" would be eliminated. All the participants of the processes of the production and exchange would retain decision sovereignty and yet the overall result of their actions would be fully consistent with the social preferences as reflected in central planner's objectives.

Prof. Lange's prescription implicitly accepted neoclassical assumptions with respect to both producers and consumers (smooth convex production and utility functions). Only under these assumptions the computation algorithm considered (tâtonnement method) may (for some special cases) be shown to be convergent to a (possibly) unique equilibrium price.

However, because in many non-neoclassical situations the equilibrium prices may not exist at all, or be non-unique, this prescription must not be taken literally. In the following considerations the general idea of Prof. Lange is followed while, at the same time, some non-neoclassical assumptions that can be attributed to the agricultural practice are fully respected.

2. Institutional and Technical Context

Let us consider a situation where a very large number of separate production units may produce the same goods while using the same inputs and applying virtually similar technologies. This is a typically agricultural context, wherein a very large number of farms may produce the same products (bundles of crop and animal produce) while using the same inputs (fertilizers, feedstuffs, machinery, etc.). Because of the sheer number of separate farms, the assumption ruling out any monopolization may usually be easily accepted.

To operationalize the considerations it is necessary to have all the farms grouped into possibly small sets of "sectors" characterized by similar technological and behavioral characteristics (constraints) and similar resources of land, labor and capital goods.[3] As a result of the specification of homogenous sectors of farms it is possible to set the analysis in manageable dimensions without giving up the assumption concerning the "atomistic" character of the whole agriculture.

Next, it is assumed that the Government Agency supervising agricultural production is in fact a monopsonistic buyer of agricultural produce and a monopolistic supplier of intermediate inputs for agricultural production. More precisely, the farms ("sectors") are free to exchange both products and inputs at free market prices. Yet, it is assumed that the Agency is the ultimate buyer of the agricultural produce and the primary seller of the non-agricultural and some agricultural (notably imported feedstuffs) inputs for agriculture. Thus, the sectors are insulated from either real consumers of the agricultural produce and real producers of the inputs for agriculture. It is worth noting that some of the products (e.g. feedstuffs such as hay or grass, manure) may be treated as non-marketable intermediate inputs to be used by the producer. The Agency's and free-market demand for these products is equal to zero and so are their prices.

In the following considerations the problems concerned with only short-run (yearly) situations are studied. Thus, only plans on production and use of intermediate inputs are assumed to be flexible. Questions of allocation of capital goods, reallocation of land, stocks of animals and labor force are therefore not included. However, the control mechanism for the long-run decisions does not, mathematically, look very different. From the economic point of view it is nonetheless desirable to consider it in some separation from the currently discussed problem of the short-term control of agriculture. Although the separation is rather partial (for the information on the profitability of various capital inputs must be taken into account while determining the demand for various capital goods) the Agency's postulates concerning goals of control will clearly be different in the long-run situations. (These goals may relate more to the desired transformation of rural structure than is the case in the short-run situation, where they correspond rather to current size and structure of outputs and inputs attainable at given capital, land and labor availabilities.) Accordingly, the reallocation, investment and change-of-job decisions by the farmers (migration) cannot be assumed to be governed by the same motives as the ones that are presumably predominant in the short-run contexts.

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[3] The problems of construction, identification and applications of "aggregative" agricultural programming models are extensively discussed in McCarl, Spreen (1980).

3. Production Potential of Sectors and Whole Agriculture

Let us suppose that the set of feasible plans on production and purchase of intermediate inputs is--for any sector--determined by a system of linear inequalities. Generally speaking, i-th sector's plans have to satisfy the following constraints:

$$A_i x_i + B_i s_i \leqslant b_i \tag{1}$$

$$-C_i x_i + I s_i \leqslant 0 \tag{2}$$

$$x_i ; s_i \geqslant 0 \tag{3}$$

where x_i is a vector of production activities; s_i is a vector of purchase of inputs;

A_i, B_i are matrices of fixed coefficients for technical constraints (1) representing utilization of fixed assets (land, buildings, labor, etc.); b_i is a vector of available fixed assets. Some constraints in (1) may represent behavioral restraints which rule out plans that the farmers consider either too risky or unacceptable in view of the longer-term perspective (e.g. regard for crop-rotation); C_i is a matrix of fixed coefficients for technical constraints (2) representing use of intermediate inputs. It is assumed that the sectors do not have significant reserves of intermediate inputs--thus the right-hand side of (2) is equal to zero. (In practice the sectors may have some reserves (especially of feedstuffs) that are kept for weather emergencies and are not taken into account while planning the utilization of production potential under "normal" conditions.) On the basis of the descriptions of each sector's production potentials it is possible to describe the potential of the whole agriculture. This is given by the system of inequalities (1) ÷ (3) for all sectors:

$$\left.\begin{array}{r} A_i x_i + B_i s_i \leqslant b_i \\ -C_i x_i + I s_i \leqslant 0 \\ x_i , s_i \geqslant 0 \end{array}\right\} \quad i = 1,2,\cdots,m \tag{4}$$

with additional constraints on the total use of intermediate inputs:

$$\sum_{i=1}^{m} s_i - gI\sum_{i=1}^{m} x_i \leqslant s \qquad (5)$$

where s is a vector of total availabilities of intermediate inputs at the Agency's disposal, g is a vector of fixed coefficients. (These coefficients are equal one for the products that can be used as marketable intermediate inputs--i.e. as feedstuffs such as wheat, and zero for all other products.) Thus, the inequalities (5) represent the balances of intermediate inputs--both those which are assumed to be given irrespectively of the agricultural production and those whose appearance depends on the course of the processes of agricultural production.

4. Socially Optimal Utilization of the Production Potential of Agriculture

It is assumed that the Agency has clear-cut objectives with respect to total purchases of agricultural products and total sales of intermediate inputs to agriculture.

It may be supposed that the primary objective of the Agency may be stated in the form of a criterion function:

$$\text{Maximize } d \sum_{i=1}^{m} x_i - h \sum_{i=1}^{m} s_i \qquad (6)$$

where d is vector of parameters valuing particular products;
h is vector of parameters valuing particular inputs.
The paramaters of vectors d, h may be identified with the world market prices of products and inputs. (The activities representing production of intermediate non-marketable inputs (hay, manure) are not included in (6), i.e. the corresponding parameters are equal zero.

Alternatively, the primary goal of the Agency may be stated in a multi-objective fashion, as the postulate of achieving total production Σx_i approximately some vector of "ideal" total production $\hat{x}$ while keeping total use of inputs (Σs_i) "as small as possible". In this case it is also possible to formulate a linear criterion function reflecting the preferences expressed in the statement of the "ideal" production and use of inputs.[4]

[4] The simplest formula allowing the transformation of the multi-objective statement of the goal into a single-objective one is provided by the concept of goal programming (Kornbluth, 1973). A more flexible approach has been developed by Wierzbicki (1981).

Independently of the main objective, as reflected in the linear criterion function (6), or any other relevant criterion function (whether linear or not) the Agency may require some minimum and maximum levels for production:

$$x_{min} \leqslant \sum_{i=1}^{m} x_i \leqslant x_{max} \qquad (7)$$

where x_{min}, x_{max} are vectors of fixed minima and maxima levels of production of particular commodities.

By solving the optimization ("master") model with the criterion function (6) and the constraints (4), (5), (7) the Agency determines plans for production and use of inputs in all sectors. These plans, denoted as (x_i^0, s_i^0), if carried through by the sectors would guarantee a best (from Agency's point of view) utilization of the production potential of agriculture. Now, the problem emerges of how to make the sectors behave in such a way that their performance will eventually lead to (6) equal f^0 while respecting both (5) and (7). Since we are not interested in the sending of direct orders to the sectors, it is now necessary to define a universal (applicable to all sectors) price system under which sectors' own optima would be also optimal for the Agency.

It is interesting to note that if producers were offered world market prices (d, h) and (7) could be, if necessary, balanced through foreign trade, then maximization of (6) subject to (4) and (5) would yield the same solution as would be attained through perfect competition. However, in the presence of restrictions on international trade (whether implied by other countries import quota or the Agency's desire to preserve a certain level of self-sufficiency) the adoptation of world market prices may not, in general, be accepted even if the objective function (6) is specified precisely with the world market prices. (Later, see (16), additional requirements will be specified which once again may imply the need for introduction of domestic prices differing from the world market ones - even in the absence of restriction on the world market prices).

5. Analytical Conditions for Feasible Prices

The i-th sector's optimum plan on production and use of inputs may be stated in the following way:

$$\text{maximize } px_i - \pi s_i \tag{8}$$

s.t.

$$A_i x_i + B_i s_i \leqslant b_i \tag{9}$$

$$- C_i x_i + I s_i \leqslant o \tag{10}$$

$$x_i , s_i \geqslant o \tag{11}$$

where p , π are vectors of prices for products and inputs respectively.

The objective function (8) is in fact net revenue: value of production minus value of intermediate inputs. (Because capital endowments, land and employment are given (and are not supposed to be subject to any change within the production period) the maximization of (8) is equivalent to maximization of both profit and income. Therefore, although the long-term objective functions for traditional agriculture may be assumed to be income rather than profit, in the short-term context considered, the acceptance of (8) for any set of agricultural sectors seems justified.

Let $p^0 > 0$, $\pi^0 > 0$ be such that the following conditions are satisfied for some non-negative values for vectors $\hat{x}_i, \hat{s}_i, \hat{y}_i, \hat{z}_i$:

$$\begin{aligned} A_i x_i + B_i s_i &\leqslant b_i \\ - C_i x_i + I s_i &\leqslant 0 \\ y_i A_i - z_i C_i &\geqslant p^0 \\ y_i B_i + z_i I &\geqslant -\pi^0 \\ px_i - \pi s_i - y_i b_i &= 0 \end{aligned} \tag{12}$$

Then, by virtue of the Fundamental Theorem of the duality theory for linear programs, $(\hat{x}_i, \hat{s}_i)$ is an optimum solution to the problem (8)-(11) specified with prices (p^0, π^0). (It is worth noting that if (p^0, π^0) satisfies (12), (13), then any price vector $(\alpha p^0, \alpha\pi^0)$ with $\alpha > o$ would also result in optimality of $(\hat{x}_i, \hat{s}_i)$. Thus, the most that can be expected from the analysis is (at least for the time being) the structure of prices, and not their absolute levels.) It follows from (12) that in order to find prices (p^0, π^0) at which the rational behavior of the sectors may be consistent [5] with the Agency's goals one has to find a solution to the system of conditions:

[5] At prices (p^0, π^0) satisfying (13)-(15) some sectoral optimization models (8)-(11) may have multiple optima. According to (13)-(15), at least one of these optima is also good from the Agency's point of view.

$$
\begin{aligned}
& A_i x_i + B_i s_i \leqslant b_i \\
& -C_i x_i + I\, s_i \leqslant 0 \\
& y_i A_i - z_i C_i \geqslant p \qquad\qquad i=1,2,\cdots,m \qquad (13) \\
& y_i B_i + z_i I \geqslant -\pi
\end{aligned}
$$

$$
\begin{aligned}
& p x_i - s_i - y_i b_i = 0 \\
& h \sum_{i=1}^{m} x_i - d \sum_{i=1}^{m} s_i = f^0 \\
& \sum_{i=1}^{m} s_i - g\, I \sum_{i=1}^{m} x_i \leqslant s \qquad (14) \\
& x_{min} \leqslant \sum_{i=1}^{m} x_i \leqslant x_{max}
\end{aligned}
$$

$$p, \pi > o \ , \ x_i, s_i, y_i, z_i \geqslant 0, \ i=1,2,\cdots,m \qquad (15)$$

It is interesting to note that similar conditions developed by Fortuny-Amat and McCarl, 1981, do not stipulate the attainment of the Agency's feasible optimum f^0. Instead of the constraint $h \sum_{i=1}^{m} x_i - d \sum_{i=1}^{m} s_i = f^0$, the authors introduce the objective function (6).

The conditions (13)-(15) may be additionally accompanied by some other constraints expressing auxiliary (and yet important in practice) goals related to price stability and profitabilities of the sectors. These constraints may be stated in the form of inequalities:

$$p_{min} \leqslant p \leqslant p_{max} \ , \ \pi_{min} \leqslant \pi \leqslant \pi_{max}$$

and $\qquad (16)$

$$p x_i - \pi s_i = R_i \quad i=1,2,\cdots,m$$

where p_{min}, p_{max}, π_{min}, π_{max} denote limits for the acceptable movements in the prices of particular products and inputs; R_i is postulated level of revenue earned by i-th sector[6]. (The preservation of stability in prices may be important even in the short-term context assumed in the analysis. Namely, it is implied by (8)÷(11), and then carried over through the following analysis, that the sectors do not "store" production or inputs in expectation of a rise in prices. This, however, may not be correct should the price changes determined by the Agency exhibit too much of a variability over time.)

[6] With the introduction of (16), the price solutions to (13)-(15) cease to be homogenous.

6. The Existence of Feasible Prices

The system of inequalities (13)-(15), and therefore (13)-(16) may not have any solution. What happens in this case is that for one (or more) sectors any solution (x_i, s_i) satisfying (14) is placed in the interior of the set of optimum solutions defined by (9)-(11). Other "unpleasant" situations occur when the solution (x_i, s_i) satisfying (14) is placed on the boundary of the set of sector's feasible solutions (9)-(11), yet not at its vertex. (This is the case of multiplicity of optimum solutions mentioned already.)

Figures 1 and 2 illustrate possible cases for a pair of two-good examples. For the sake of simplicity it is assumed that for some sector i there is only one $(x^0, s^0) = (x_i^0, s_i^0)$ satisfying (14). (This is then the only i-th sector's plan acceptable to the Agency.)

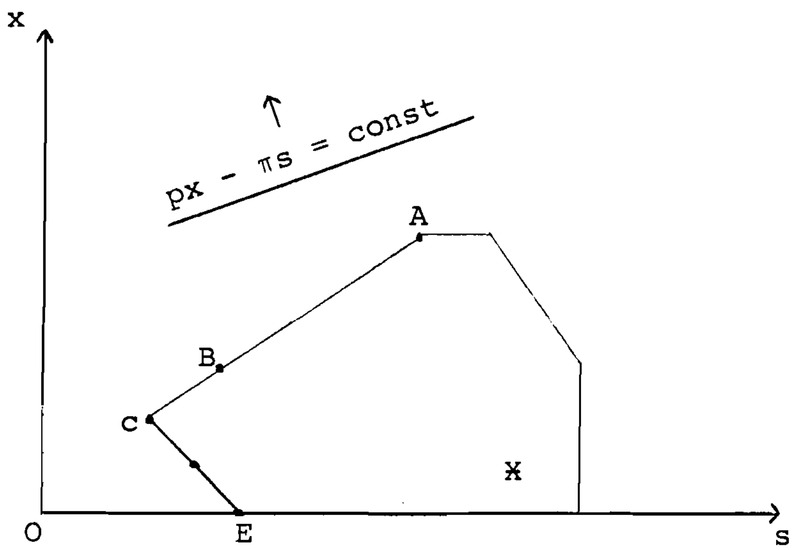

Figure 1. Possible locations of optimum (x^0, s^0) in a sector's set of feasible solutions $\mathfrak{X}$.

Optima (x^0, s^0) may only be - in the case considered - located on lines EC and CA. (All other positions would imply non-optimality of (x^0, s^0) as a part of the solution to the master problem.) If optimum is located at A, then any relative price p/π ranging from 1 to infinity is satisfactory. At C any relative price p/π ranging from zero to one is satisfactory. At B there is exactly one satisfactory price p/π, equal to one. However, at $p/\pi = 1$ any solution placed on CA is also the sector's optimum. At D there is no satisfactory price structure. (Formally, one would have to accept $p/\pi = -1$.)

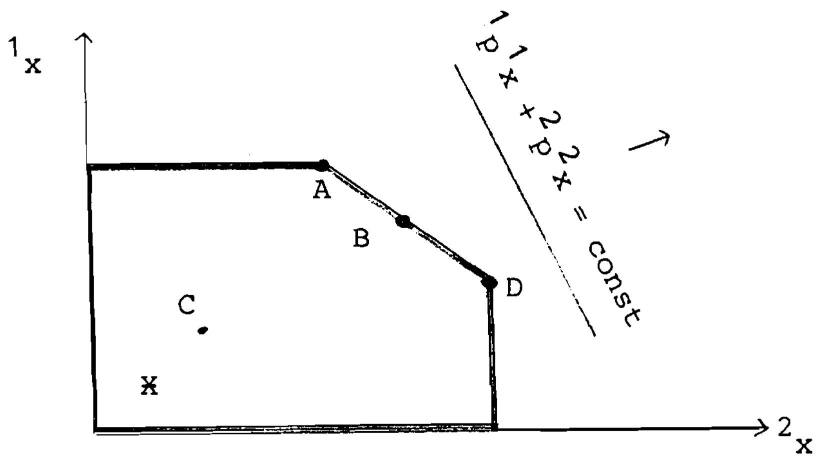

Figure 2. Possible locations of optima for products $({}^{1}x^{0}, {}^{2}x^{0})$ in sector's set of feasible solutions $\mathbf{X}$ in a two-dimensional case.

If optimum is located at A, then any relative price p^{1}/p^{2} ranging from infinity to one is satisfactory. At B there is exactly one satisfactory relative price $(p^{1}/p^{2} = 1)$. However, if $p^{1}/p^{2} = 1$, then any solution ranging from A to D is also the sector's own optimum. At C there is no satisfactory price structure at all.

7. Introduction of Production Quotas and Availability Limits

Since we have shown that prices alone may not, in general, be expected to guarantee the identity of sector's own optima with the optima for sectors[7], there is a need for the application of some additional measures. There is one set of measures which are both very simple in nature and yet not contradictory to the internal sovereignty of the sectors and the preservation of the profit motive. This provides for the possibility of constraining the access to some inputs and for contractual arrangements with respect to some agricultural products. More precisely, the Agency may resort to the rationing with respect to inputs and to introduction of quota with respect to products to be bought from the sectors.

While the meaning of the rationing is rather obvious, the contracting may require some more interpretation. Namely, by setting a contract $\overset{*}{x}_{ij}$ on production of good j of sector i, the Agency obliges itself to buy from sector i any amount of good j not exceeding $\overset{*}{x}_{ij}$ at given price ${}^{j}p$ of that good. The surplus of production does not have any guarantee of being bought by the Agency at any price. At the same time, the production being less

[7] The Dantzig-Wolfe (1960) decomposition algorithm and many more that have followed, are sometimes interpreted in terms of a two-level control procedure for "allocation through prices". In view of the presented considerations, this interpretation is misleading.

than $\bar{x}_i$ does not imply any penalty to be applied to the sector. Now, i-th sector's optimum planning on production and use of inputs may be stated in the following way:

$$\text{maximize } px_i - \pi s_i$$

s.t.

$$\begin{aligned} A_i x_i + B_i s_i &\leqslant b_i \\ -C_i x_i + I\, s_i &\leqslant \overset{*}{s}_i \\ x_i\ ,\ s_i &\geqslant 0 \\ x_i &\leqslant \overset{*}{x}_i \end{aligned}$$

where $\overset{*}{x}_i, \overset{*}{s}_i$ are quotas on purchases of production and limits on the sales of inputs respectively.

Having the possibility of setting both prices (p^0, π^0) and constraints $(\overset{*}{x}_i, \overset{*}{s}_i)$ for all goods and for all sectors, the Agency would have no problems in an efficient control of any sector (see Figures 3 and 4). More precisely, the constraints of the Agency's master program (14)-(16) would still have to be satisfied, with the following modification of (13):

$$\begin{aligned} A_i x_i + B_i s_i &\leqslant b_i \\ -C_i x_i + I\, s_i &\leqslant \overset{*}{s}_i \\ x_i &\leqslant \overset{*}{x}_i \\ y_i A_i - z_i C_i + w_i I &\geqslant p \qquad\qquad i=1\div m \qquad\qquad (17) \\ y_i B_i + z_i I &\geqslant -\pi \\ px_i - \pi s_i &= y_i b_i + z_i \overset{*}{s}_i + w_i \overset{*}{x}_i \\ x_i \geqslant 0,\ \overset{*}{x}_i \geqslant 0,\ s_i \geqslant 0,\ \overset{*}{s}_i \geqslant 0,\ y_i &\geqslant 0\ ,\ z_i \geqslant 0,\ w_i \geqslant 0 \end{aligned}$$

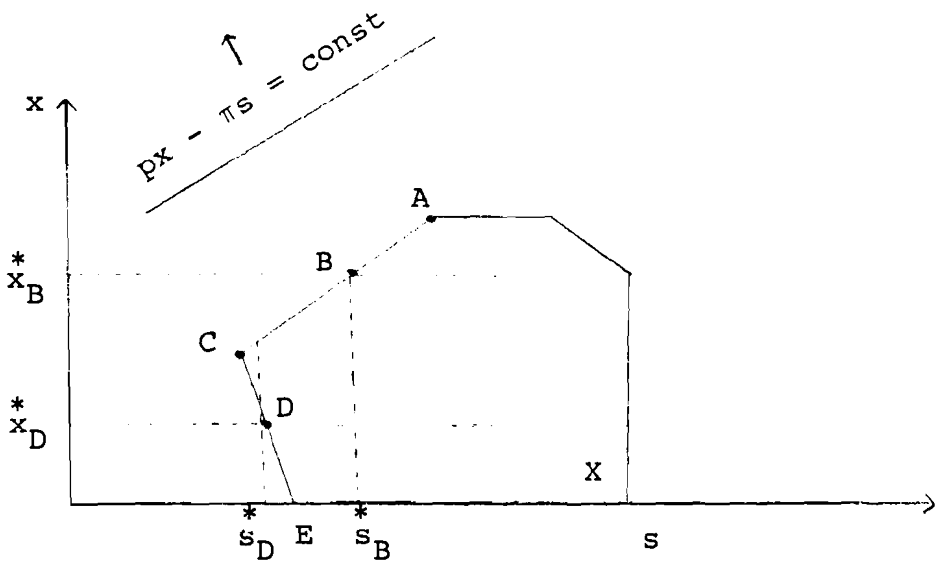

Figure 3. The application of a quota and (or) a limit in the two-dimensional case.

By introducing quota $\overset{*}{x}_D$ and limit $\overset{*}{s}_D$ one transforms D into a (single) vertex of the set of feasible solutions. Any relative price structure p/π ranging from zero to infinity results now in D's unique optimality. With respect to B one achieves the same by introducing quota $\overset{*}{x}_B$ and limit $\overset{*}{s}_B$. (In the case considered the application of either quota $\overset{*}{x}_B$ or the limit $\overset{*}{s}_B$ is sufficient to transform B into a vertex of the set of feasible solutions.)

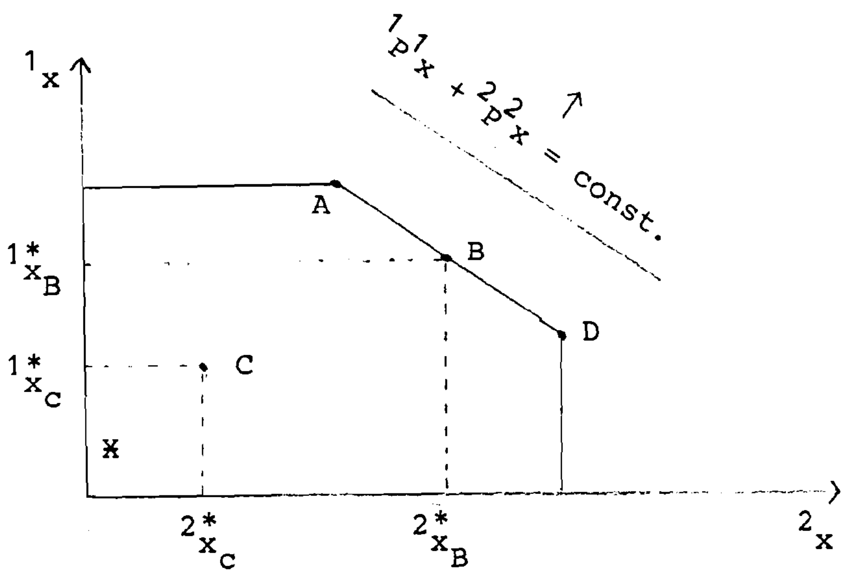

Figure 4. The application of quotas in the two-dimensional case.

By introducing quotas ${}^{1}\overset{*}{x}_C$ and ${}^{2}\overset{*}{x}_C$ one transforms C into a vertex of the set of feasible solutions. With respect to B the same is achieved through introduction of quota ${}^{2}\overset{*}{x}_B$ or (and) ${}^{1}\overset{*}{x}_B$.

It is quite obvious that the Agency introducing quota and limits for all goods and for all sectors is fully controlling whole agriculture. Yet, this may be both costly in terms of organizational effort and, in fact, unnecessary. In most cases one can expect only some fraction of sectoral optima to be displaced from the vertices of the respective sets of feasible solutions. Moreover, in the cases of optima located on hyperplanes it is enough to introduce constraints on some goods only (see both cases B, Figures 3 and 4). Therefore, the question emerges of how to control the sectors with possibly small numbers of quantity restrictions.

8. Preventing Undesirable Cooperation among the Sectors

Under the complete system of contracts and rationing, whereby all sectors face definite quota on all products and definite limits for all inputs, the sectors do not have any reason to enter any exchange trade in inputs or products. (Of course, there may be a definite reason for the exchange if the prices are not the same for all sectors. The same may happen if the quotas and contracts for some sector exceed their own production possibilities or input needs while actively constraining some other sectors' activities.)

Under a partial system of contracts and limits, whereby the sectors would not face quota on all products and limits on all inputs, one could expect the development of an exchange market for products and inputs. Should such a market develop, the Agency would lose effective control over the sectors. The prices (p^0, π^0) announced and the quotas and limits $\overset{*}{x}_i, \overset{*}{s}_i$ imposed would not be taken at face value by the sectors. First, along with (p^0, π^0) there would exist intersectoral prices $(\hat{p}, \hat{\pi})$ affecting sector's plans. Second, the quotas and limits would be violated by the exchange trade (which would have to be labelled as speculation with respect to licences for production and use of inputs. As a result, the overall production actually realized, together with actually used up inputs may have nothing to do with any optimum solution to the master-program. Moreover, the constraints (5) and (or) (7) may be violated.

Prevention of inter-sectoral speculation through administrative measures seems plainly impossible--and even not advisable on purely economic grounds. It would require the creation of a huge army of "inspectors" supervising the transportation of products and inputs and seeing that no consignment missed its "legitimate" destination. (A crude alternative with respect to crop production would provide for contractual arrangements with respect to the acreages. In this case, there is a need for inspectors supervising land usage.) In actual fact, the introduction of administrative measures to enact the Agency's decisions with respect to a partial system of quotas and limits could easily lead to a restoration of a penalty-reward system favoring subordination and not economic behavior.

Besides administrative, there are also economic means by which one can prevent undesirable "cooperation" of the sectors. First, it should be observed that the motive for the "fraudulent" transactions does not appear when the increase in total gain of the whole agriculture coming from the violation of the sectoral quantity restrictions is equal to zero.

The maximum total gain of the whole agriculture with the sectors forming a coalition acting "against" the Agency is determined by the following optimization model:

$$\text{maximize } p \sum_{i=1}^{m} x_i - \pi \sum_{i=1}^{m} s_i \tag{18}$$

s.t.

$$\left.\begin{array}{l} A_i x_i + B_i s_i \leqslant b_i \quad (19) \\ -C_i x_i + I s_i \leqslant 0 \quad (20) \end{array}\right\} \quad i = 1 \div m$$

$$-gI \sum_{i=1}^{m} x_i + \sum_{i=1}^{m} s_i \leqslant \sum_{i=1}^{m*} s_i \tag{21}$$

$$\sum_{i=1}^{m} x_i \leqslant \sum_{i=1}^{m} {}^{*} x_i \tag{22}$$

$$x_i \,,\, s_i > 0 \qquad i = 1 \div m \tag{23}$$

The maximum total gain of the whole agriculture with the sectors following the quantity constraints imposed by the Agency is equal (see 16)

$$R^0 = \sum_{i=1}^{m} R_i$$

Now, it is possible to form additional conditions on prices and quantity restrictions which will prevent the emergence of the motive for the fraudulent transactions.

Namely, if $p, \pi \sum_{i=1}^{m} {}^{*} s_i \sum_{i=1}^{m} {}^{*} x_i$ satisfy the following inequalities:

$$\left.\begin{array}{l} \rho_i A_i + \mu_i C_i - \delta I g - \sigma I \geqslant p \\ \rho_i B_i - \mu_i I + \delta I \geqslant -\pi \end{array}\right\} \quad i=1 \div m \tag{24}$$

$$\sum_{i=1}^{m} \rho_i b_i + \delta \sum_{i=1}^{m} {}^{*} s_i + \sigma \sum_{i=1}^{m} {}^{*} x_i = R^0 \tag{25}$$

for some non-negative vectors ρ_i, μ_i, $(i=1\div m)$, δ, σ - these are dual variables for problem (18)-(23)-then, by virtue of the Fundamental Theorem of the duality theory for linear programs, any optimum solution to (18)-(23) does not provide the coalition of sectors with any additional gain due to any exchange of licences for production and inputs.

9. A Procedure for Determination of Prices and Quantity Constraints: A Restatement

The analytical considerations of various aspects of price/quantity constraints setting presented so far may--after proper interpretation--serve as a basis for alternative computational procedures. A general procedure which seems to follow the considerations in the most natural way would require the formation and numerical analysis of three master programs.

Master Program I

This is given by (4)÷(7),(16). Solving it one obtains f^0. If $(x_i^0, x_2^0, \cdots, x_m^0, s_i^0, s_2^0, \cdots, s_m^0)$ is unique then the formation and analysis of the remaining Master Programs may be greatly simplified.) (If Master Program I has no feasible solution then some of the original Agency's goals cannot be satisfied. Some changes in the parameters representing targets must follow.)

Master Program II

This is given by (13)-(16) and some (quite arbitrary) objective functions such as

$$\text{minimize} \sum_{i=1}^{m} \sum_{j=1}^{n} z^{j,i}$$

where $(z^{1,i}, z^{2,i}, \ldots, z^{n,i}) = z_i$

($z^{j,i}$ is dual price of the limit for i-th input in the i-th sector). If there is a feasible solution to Master Program II, then pricing alone virtually[8] leads to the consistency between the Agency's goals and the profit-maximizing behavior of the sectors.

Master Program III (Formed when Master Program II does not have any feasible solution.)

This is given by (14)÷(17), (24), (25). Any feasible solution to Master Program III defines prices and quantity constraints which virtually[8] lead to the consistency between the Agency's goals and the profit-maximizing behavior of the sectors. The final selection of the feasible prices and quantity constraints

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[8] Although it may be necessary to ask the sectors to restrict the choice to be finally made from the sets of sectors' own optima.

satisfying the conditions of Master Program III may be facilitated by the choice of a suitable objective function. Let us notice that since we are interested in reducing the amount of quantity restrictions actively constraining the sectors' behavior, we can require that as many dual prices for these constraints as possible be equal to zero. This requirement is reflected by the objective function

$$\text{minimize} \sum_{i=1}^{m} \left(\sum_{j=1} z^{j,i} + \sum_{j=1}^{k} w^{j,i} \right)$$

where $(w^{1,i}, w^{2,i}, \ldots, w^{k,i}) = w_i$. ($w^{j,i}$ is dual price of the quota on j-th product for sector i).

CONCLUSION

The paper has presented a scheme for control and coordination of production plans generated by the linear aggregative (sectoral) programs. It has been demonstrated that prices alone, even if fully controlled by the government, need not lead to the satisfaction of the achievable government's goals. It is only with the introduction of quotas on outputs and limits on available inputs that the possibility of a reconciliation of the government's goals with the behavior of profit-motivated sectors is recovered. However, the introduction of quantity restrictions imposes additional conditions that cannot be violated for fear that this would unleash uncontrollable speculation with respect to licences for production and inputs. The analytical framework for the consistent simultaneous determination of prices and quantity restrictions has been given. This implies solving linear programming models with some nonconcave quadratic constraints. (See Hansen, Manne, 1978.)

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