AUTOMATED ISOCHRONES AND THE LOCATION OF EMERGENCY MEDICAL SERVICES IN CITIES: A NOTE

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FOREWORD

The principal aim of health care research at IIASA has been to develop a family of submodels of national health care systems for use by health service planners. The modeling work is proceeding along the lines proposed in the Institute's current Research Plan. It involves the construction of linked submodels dealing with population, disease prevalence, resource need, resource allocation, and resource supply.

The work introduced here is an approach that is designed to aid the location and scheduling of emergency medical facilities in large cities. Although travel time to a medical center is not critically important for routine medical services, it is in the case of emergencies. Thus health authorities must pay close attention to the spacing of facilities. Unfortunately, because of congestion effects, seasonal factors, staff shortages, and other reasons, it is difficult to supply these services efficiently and cheaply. The idea developed in this paper is based on *isochrones*--the locus of points around a center that can be reached in a given time. Isochrones delimit the emergency response areas under varying traffic conditions for different sets of centers, so that geographical gaps in provision may be determined and, if necessary, remedied.

Related publications in the Health Care Systems Task are listed at the end of this paper.

Andrei Rogers
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Human Settlements
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ABSTRACT

This paper describes a method for the automatic calculation and graphical reproduction of isochrones that are set for different time standards and for varying traffic conditions around emergency medical centers in large cities. The technique is based on the concept of a velocity field. It permits a rapid evaluation of coverage standards under different operating conditions.

CONTENTS

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INTRODUCTION

In the planning and operation of accident and emergency medical centers, the distance of travel between the site of an accident and the nearest treatment center is recognized as being important. The types of coverage problems that this travel constraint generates have been the subject of several studies (for example, Church and Revelle 1974; Mayhew 1979; Toregas et al. 1971; and Toregas and Revelle 1973). In urban areas the time of travel is certainly more important than distance because of the variation in congestion levels on different roads in the transport network. Also, a study of a city's emergency medical centers often indicates a dimension in their behavior that perhaps is not reflected in the location models developed to date. Very important, for instance, are the opening hours, which are different (and sometimes unscheduled) among the centers during the day and night, thus allowing, in a complicated way, the substitution of one center for another. The object of this note is to suggest how the nature of travel in a city and the operating behavior of the medical centers interact and how they may be monitored. The results given are preliminary and are intended as an introduction to a more general approach that will be developed in subsequent publications.

The idea developed here is based on the notion of automated isochrones to be drawn on a visual display unit for any time standard and set of locations. Isochrones are the locus of points about an emergency center that can be reached in a given time. The areas the isochrones encompass are called response areas, because they can be reached by emergency vehicles in less than the given time. When knowledge of these areas is centrally located in a command and control center, it becomes possible to obtain a prompt indication of which parts of the city are adequately covered by accident and emergency services and which are not.

The time standard defined by the isochrone is simply the maximum desirable travel time taken to reach the site of an emergency. Typically, different operating authorities will have different views on what constitutes an adequate time standard. Once selected, however, the standard strongly influences the character of the system, particularly the scheduling of services and the number of centers that are open at any time. Nevertheless, with the response areas easily displayed on a screen, any significant change that would alter the accident and emergency coverage would then be quickly identified and an alternative plan could be devised using this method.

The premise for mapping the isochrones in the way to be discussed is that published data on journey times are often unreliable and do not reflect the daily variations in traffic conditions, which in most cities are an important factor influencing journey time. The results shown from this study are exploratory, but they are encouraging and capable of further development. The first part of this note gives an outline of the methods; the second part details the mathematics for a particular example.

1. METHOD

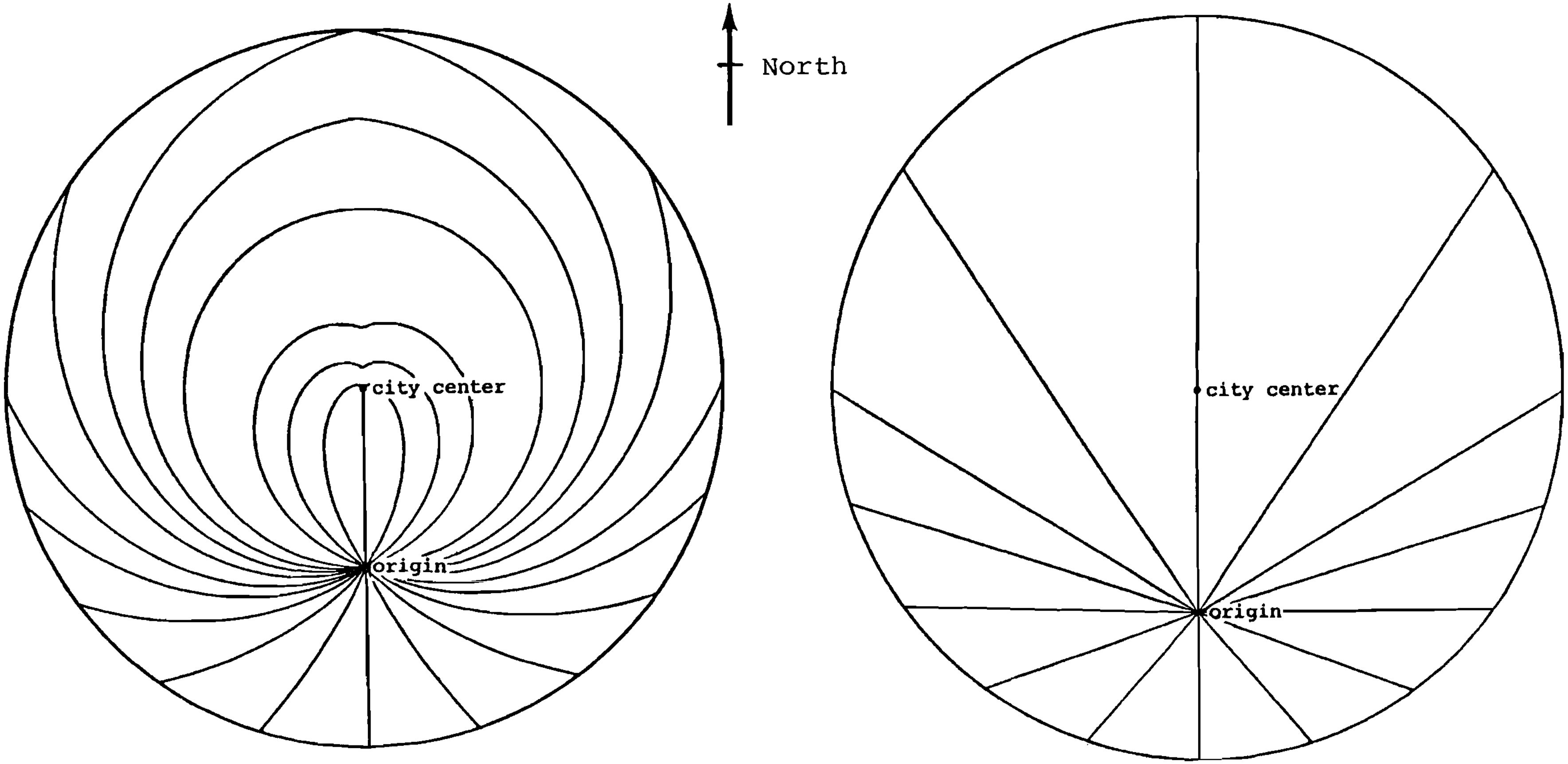
The basis for this approach to determining response areas is the concept of a velocity field introduced by Angel and Hyman (1970, 1971, 1977) for obtaining measures of journey times between pairs of points in an urban area. In their work, it is observed that, in cities, the average speed of travel by road shows an increase from the center to the periphery, and it is with reference to this property that the authors solve for minimum journey times. At this level of abstraction the twists and turns of the actual network are ignored, which seems reasonable providing the road network in the city is sufficiently dense. An analysis shows that, if certain conditions are met, the minimum time paths between two non-centrally located points will reproduce on a diagram as smooth curves that spiral around the city center, trading off the increased travel distance against the delays caused by traffic congestion. Figure 1 compares this effect for one city against another where the speed is considered to be uniform for a journey origin that is south of the city center. In the second case the shortest paths are mapped as straight lines.

In general a minimum time path between A and B can be evaluated by finding the smallest value of the integral,

$$\int_A^B \frac{1}{V(r)}\, dr \qquad (1)$$

where V(r) is the average speed of travel expressed as a function of r the distance from the city center. If the parameters of V(r) are z and p, then the time t can be written in functional form as

$$t = t(r_1\theta_1,\ r_2\theta_2,\ z,\ p) \qquad (2)$$



(a) non-uniform

(b) uniform

Figure 1. Minimum time journey paths in a non-uniform and uniform velocity field.

where $r_1\theta_1$ and $r_2\theta_2$ are the points A and B expressed in polar coordinates. If we re-express time as a function of r_2 instead, then in principle we can obtain,

$$r_2 = r_2(r_1\theta_1, \theta_2, t, z, p) \tag{3}$$

the equation of the isochrone. By fixing $r_1\theta_1$, z and p and then allowing θ_2 to range through 2Π radians, the desired isochrones about a facility located at $r_1\theta_1$ can thus be mapped for any desired value of t.

It remains to parameterize the function V(r) by empirically determining values for p and z under varying road and weather conditions, and then solving for r_2 in the above equations. The first part of this would be achieved using special surveys to determine average speeds under different traffic conditions; the theoretical part is considered for one class of function in part two of this note. The accuracy obtained by our method depends on our ability to characterize journey speeds with the function we choose for V(r). If a city cannot be characterized this way at all (that is if speed is clearly not a function of r) then the method is, of course, invalid. In any event the results can never be perfect as there is always a stochastic component in the time attached to any journey. This is true regardless of how the measurement is made. There may also be barriers to travel such as rivers for which corrections will be necessary. Nevertheless, with the method's development, the measure of time necessary for the journey can be brought to a level of accuracy that is sufficient for the application intended and certainly superior to the alternative measure of distance. The following example will show how the method works and what possibilities there are for extensions.

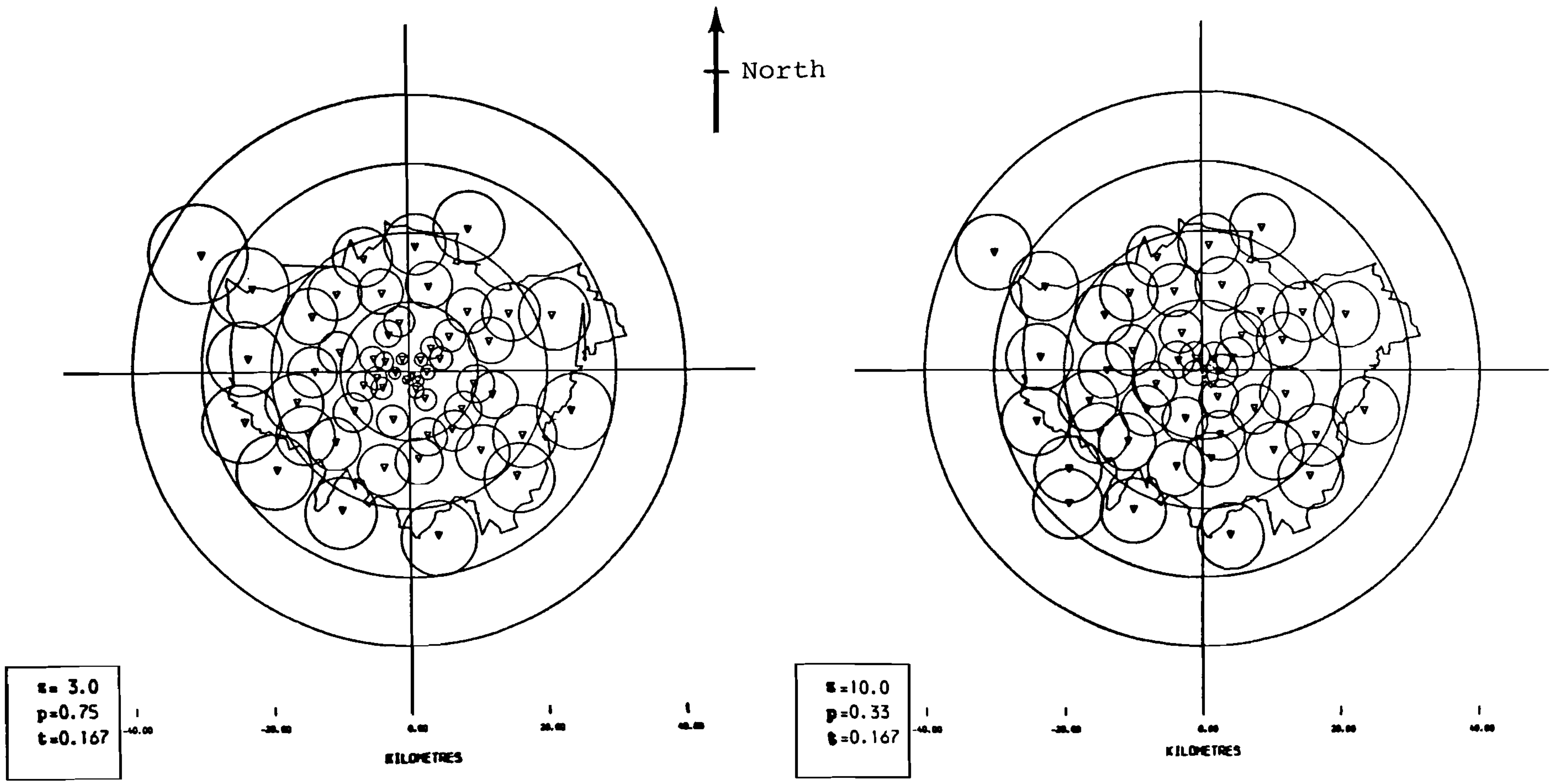
Assume that the relationship between distance and the average speed of travel is given by

$$V(r) = z\ r^p \qquad 0 < p < 1 \qquad (4)$$

This function is flexible, analytically convenient (see part two), and easily fitted to empirical data. There are two drawbacks in the performance of this application: firstly, the journey speed at the city center is defined as zero, which is arguably impossible even in the most congested of cities; secondly, speeds can increase indefinitely with no limit for a large r. Often ambulances are not bound by local speed limits but only by their physical capabilities, so the second criticism may not apply over the range of r that we would want to consider. Nevertheless, this specification of V(r) is not exhaustive, and it is given here simply as an example.

Figure 2 shows how the results will be produced. The area covered by the map is the administrative area of the Greater London Council. The locations are drawn from an existing subset of hospitals currently providing emergency treatment facilities. The entire image is reproduced from a microfilm linked to a graphics package. The values of z and p are, respectively, 3.0 and 0.75 in example (a) and 10.0 and 0.33 in example (b), whereas the value of the plotted isochrone t has been set to ten minutes in both cases.

As is seen the response area served by each center increases with distance from the city center. Although each area appears to be circular, a closer inspection shows that the side nearest the city center is, in fact, slightly compressed. For certain values of p, z, r_1, and t those areas closest to the city center will distort to cardiods, the cusp lying astride a radial through $r_1\theta_1$. This would simply be a reflection of the higher congestion effects near the center where velocities approach zero. As will be shown, those facilities at the city center itself possess a circular response area; this is the only point in the city where this can occur.



(a) (b)

Figure 2. Ten-minute (0.167 hours) isochrones around selected emergency medical centers in London for two different parameter sets.

The question arises in practice: which is the optimum covering set for our parameter values and time standard? To answer this the approach must be extended and then linked to the existing methods of solution while retaining the same graphical output. In the "set coverage problem" (Toregas et al. 1971), for example, the objective in our formulation would be to identify from the feasible set the minimum number of centers necessary to cover demand points within a given time. (Demand points within a city can be represented on a grid.) For very strict time standards, this may be impossible. Alternatively, we might choose to solve the "maximum covering problem" (Church and ReVelle 1974), which has the slightly less noble goal of maximizing the total population covered in a time t from a fixed number of centers. This would be valuable for nighttime coverage when staff shortages or budgetary considerations greatly constrain the feasible set.

2. ANALYSIS

In this final section we derive the equations for the isochrones associated with the velocity field represented in equation (4). The technique for determining shortest journey times, and hence the required isochrones, is based on the concept of a two-dimensional time surface, set in three dimensions, on which the distance between two points A' and B' is equal to the journey time between the image points, A and B, in the urban plane. This distance is a geodesic (shortest path) with respect to the surface on which it lies. Thus its reflection in the urban plane--a smooth curve connecting A and B--is also the required shortest path between these two points. It is shown by Angel and Hyman (1977) that for the velocity relationship described by $V(r) = z\,r^{p}$ the appropriate time surface is a cone whose apex is the image point of the city center. Using the transformation given by them, it can be shown that the journey time between two points in a city characterized by this field is given by

$$t(r_1, r_2, \theta_{12}) = \frac{1}{z(1-p)} \left\{ r_1^{\,2-2p} + r_2^{\,2-2p} - 2r_1^{1-p}\, r_2^{\,1-p} \cos\left[(1-p)\ \theta_{12}\right] \right\}^{\frac{1}{2}} \qquad (5)$$

where r_1 and r_2 are the respective distances from the city center and θ_{12} is their angle of separation $(0 \leq \theta_{12} \leq \Pi)$. Rearranging (5) and making r_2 the subject, we obtain

$$r_2 = \left\{ m \cos\left[(1-p)\ \theta_{12}\right] \pm \sqrt{z^2\ t^2 (1-p) - m^2 \sin^2\left[(1-p)\ \theta_{12}\right]} \right\}^{\frac{1}{1-p}} \qquad (6)$$

where $m = r_1^{\,(1-p)}$. Equation (6) is thus of the form specified in equation (3). We now consider three special cases of it.

Case A

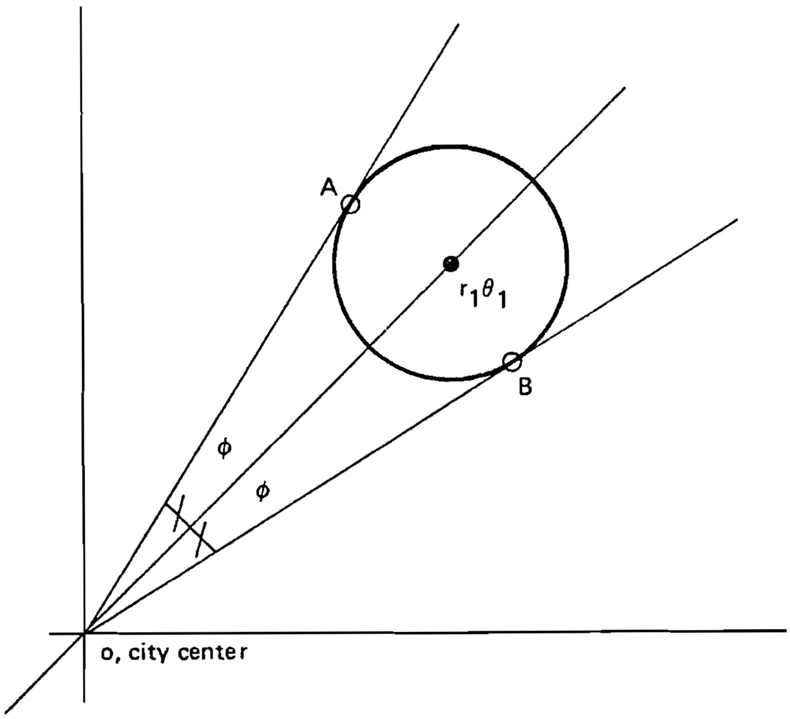
From (6) for a real solution,

$$z^2\ t^2 (1-p)^2 \geq m^2 \sin^2\left[(1-p)\ \theta_{12}\right] \qquad (7)$$

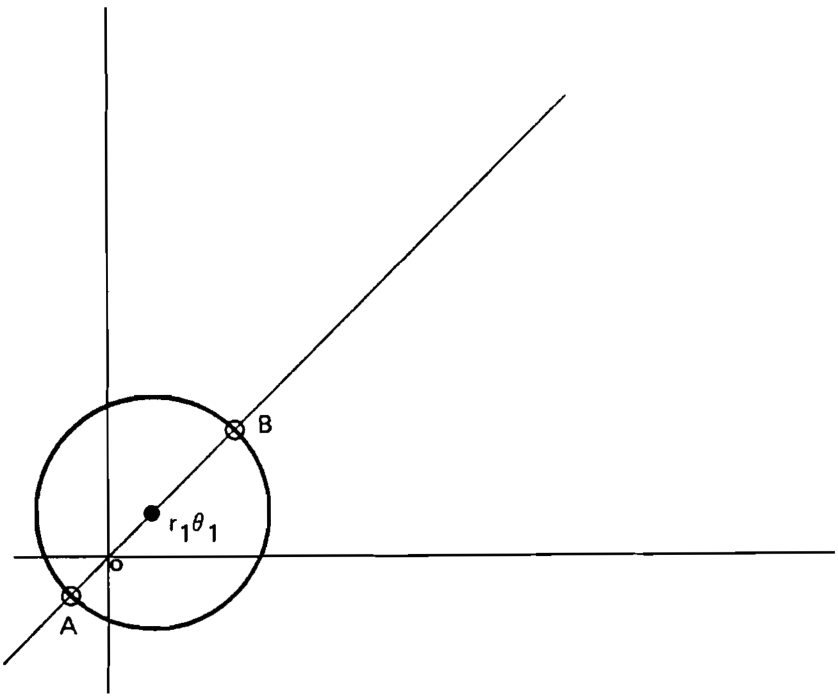
Thus, at the limits A and B shown in Figure 3a

$$_{12} = \frac{1}{(1-p)} \sin^{-1}\left[\frac{zt(1-p)}{m}\right] = \phi \qquad (8)$$

On the side furthest from the city center (6) is taken in the positive, θ_{12} ranging from $(\theta_1 - \phi)$ to $(\theta_1 + \phi)$. On the opposite side (6) is taken in the negative over the same range for θ_{12}.



(a) a non-central facility



(b) a central facility

Figure 3. Case (a) and (b).

Case B

Also for a real solution, if the negative of (6) is taken,

$$m \cos \left[(1-p)\ \theta_{12}\right] \geq \sqrt{z^2\ t^2 (1-p)^2 - m^2 \sin^2 \left[(1-p)\ \theta_{12}\right]} \qquad (9)$$

implying that

$$r_1 \geq \left[zt(1-p)\right]^{\frac{1}{(1-p)}} \qquad (10)$$

This means that the radial from an emergency center to the city center must cut the isochrone en route. If not, the isochrone cuts all four quadrants as shown in Figure 3b. In this instance, θ_{12} in equation (6) ranges over the interval $0 \leq \theta_{12} \leq \Pi$.

Case C

Suppose $r_1 = 0$. Then the emergency center is located at the city center. Equation (6) then simplifies to

$$r_2 = \left[zt(1-p)\right]^{\frac{1}{(1-p)}} \qquad (11)$$

which is a constant. Thus the associated response area for any value of t is a circle.

CONCLUSION

A method has been shown that allows the automatic delimitation of response areas in cities around emergency medical centers for any time standards and different traffic conditions. In principle, it can be extended to schedule the opening times of different centers and to develop "optimum" configurations for various time standards at different times of the day or year.

REFERENCES

Angel, S., and G. Hyman (1970) Urban Velocity Fields. *Environment and Planning* 2:221-224.

Angel, S., and G. Hyman (1971) Urban Travel Time. *Papers of the Regional Science Association* 26:85-99.

Angel, S., and G. Hyman (1977) *Urban Fields.* London: Pion.

Church, R., and C. Revelle (1974) The Maximal Covering Problem. *Papers of the Regional Science Assocation* 32:101-118.

Mayhew, L.D. (1979) *The Theory and Practice of Urban Hospital Location.* Unpublished Ph.D. Thesis. London: Department of Geography, Berkbeck College, University of London.

Toregas, C., R. Swain, C. Revelle, and L. Bergman, (1971) The Location of Emergency Service Facilities. *Operations Research* 19:1363-1373.

Toregas, C., and C. Revelle (1973) Binary Logic Solutions to a Class of Location Problems. *Geographical Analysis* 5:145-155.

RECENT PUBLICATIONS IN THE HEALTH CARE SYSTEMS TASK

Hughes, D.J. (1979) *A Model of the Equilibrium between Different Levels of Treatment in the Health Care Systems: Pilot Version* (WP-79-15).

Fleissner, P. (1979) *Chronic Illnesses and Socio-Economic Conditions: The Finland Case 1964 and 1968* (WP-79-29).

Shigan, E.N., D.J. Hughes, and P. Kitsul (1979) *Health Care Systems Modeling at IIASA: A Status Report* (SR-79-4).

Rutten, F.F.H. (1979) *Physician Behaviour: The Key to Modeling Health Care Systems for Government Planning* (WP-79-60).

A Committee Report (1979) to IIASA by the participants in an Informal Meeting on *Health Delivery Systems in Developing Countries* (CP-79-10).

Shigan, E.N., P. Aspden, and P. Kitsul (1979) *Modeling Health Care Systems: June 1979 Workshop Proceedings* (CP-79-15).

Hughes, D.J., E. Nurminski, and G. Royston (1979) *Nondifferentiable Optimization Promotes Health Care* (WP-79-90).

Rousseau, J.M., R.J. Gibbs (1980) *A Model to Assist Planning the Provision of Hospital Services* (CP-80-3).

Fleissner, P., K. Fuchs-Kittowski, and D.J. Hughes (1980) *A Simple Sick-leave Model used for International Comparison* (WP-80-42).

Aspden, P., R. Gibbs, and T. Bowen (1980) *DRAM Balances Care* (WP-80-43).

Aspden, P., and M. Rusnak (1980)*The IIASA Health Care Resource Allocation Submodel: Model Calibration for Data from Czechoslovakia* (WP-80-53).

Kitsul, P. (1980) *A Dynamic Approach to the Estimation of Morbidity* (WP-80-71).

Shigan, E.N., and P. Kitsul (1980) *Alternative Approaches to Modeling Health Care Demand and Supply* (WP-80-80).

Hughes, D.J., and A. Wierzbicki (1980) *DRAM: A Model of Health Care Resource Allocation* (RR-80-115).

Aspden, P. (1980) *The IIASA Health Care Resources Allocation Submodel: DRAM Calibration for Data from the South West Health Region, UK* (WP-80-115).

Mayhew, L., and A. Taket (1980) *RAMOS: A Model of Health Care Resource Allocation in Space* (WP-80-125).

Mayhew, L.D. (1980) *The Regional Planning of Health Care Services: RAMOS and* $RAMOS^{-1}$ (WP-80-166).

Pauly, M.V. (1981) *Adding Demand, Incentives, Disequilibrium, and Disaggregation to Health Care Models* (WP-81-4).

Mayhew, L.D. (1981) *DRAMOS: A Multi-category Spatial Resource Allocation Model for Health Service Management and Planning.* (WP-81-39).