IMMIGRATION AND THE STABLE POPULATION MODEL

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ABSTRACT

This paper reports on work aimed at extending stable population theory to include immigration. Its central finding is that, as long as fertility is below replacement, a constant number and age distribution of immigrants (with fixed fertility and mortality schedules) lead to a stationary population. Neither the level of the net reproduction rate nor the size of the annual immigration affects this conclusion; a stationary population eventually emerges. How this stationary population is created is studied, as is the generational distribution of the constant annual stream of births and of the total population.

It is also shown that immigrants and their early descendants may have fertility well above replacement (as long as later generations adopt and maintain fertility below replacement), and the outcome will still be a long-run stationary population.

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Since the beginning of the 20th century, the population of the United States has roughly tripled -- from approximately 75 million in 1900 to about 225 million in 1980. Both natural increase (births minus deaths) and net immigration (immigrants minus emigrants) have contributed to this growth. During the decade 1901-1910 the average annual number of immigrants to the U.S. was nearly 880,000, and net immigration accounted for 40 percent of intercensal population growth.[1] But following 1910 the importance of net immigration relative to natural increase declined, reaching a minimum during the Depression decade, 1931-1940, when emigrants outnumbered immigrants. The 1965 amendments to the 1952 Immigration and Naturalization Law replaced the previous annual ceiling of 154,000 immigrants with a preference system permitting 290,000 immigrants plus about 100,000 relatives of citizens to enter the country each year. The effect of these regulations was to increase substantially the volume of immigration, and for the next decade the annual number of legal immigrants was close to 400,000. Recent statistics indicate a further increase to perhaps 600,000 per year, including refugees. With this growth in numbers, the relative contribution of net immigration to overall U.S. population growth has once again risen; for the period 1971-1978, it was estimated at 22 percent.

Falling birth rates have accentuated the rising comparative importance of net immigration. The U.S. total fertility rate crossed below the replacement level in 1972, for the first time since the Depression, and it has fluctuated around 1.8 or 1.9 ever since. Annual births still exceed annual deaths, but that is due to a temporary phenomenon of large proportions of females in the childbearing ages.

We may ask what the U.S. population would look like if current conditions were to persist into the indefinite future. Specifically, suppose fertility and mortality schedules were held constant so that fertility was permanently below replacement, and suppose that a constant number of persons (with a fixed age distribution) migrate to the United States each year. Would the population continue to grow because of the influx of migrants and the children they would bear? Would the population eventually level off and then experience a long-term decline owing to sub-replacement fertility? Or, would net immigration counterbalance the low fertility rates, causing a stationary population to evolve? This problem takes on added significance since immigration has been and is likely to continue to be an important source of U.S. population growth, and because immigration will be a major policy consideration throughout the 1980s. Moreover, the circumstance of below-replacement fertility plus net immigration is one shared by numerous other industrial nations.

There are two ways to answer the question. One is with a straightforward projection of the U.S. population. To illustrate this approach, we use the estimated U.S. population on July 1, 1977 and project it forward on the assumption that 1977 age-specific fertility and mortality rates remain constant and that net immigration totals 400,000 each year. Given these postulates we arrive ultimately at a stationary population. As seen in Table 1, the eventual stationary population contains 107,903,100 persons, with 1,209,800 annual births and 1,609,800 annual deaths to offset the 400,000 immigrants.

A second approach is to analyze the problem in terms of stable population theory. Typically, by assuming a female population

TABLE 1. U.S. Population, July 1, 1977, and Eventual Stationary Population Achieved with Constant 1977 Fertility and Mortality and with 400,000 Annual Immigrants (all numbers in thousands)

Age	U.S. Population, July 1, 1977		Immigration Assumptions		Eventual Stationary Population	
	Females	Males	Females	Males	Females	Males
0-4	7414.0	7760.0	16.4	17.6	2952.2	3089.8
5-9	8400.3	8759.0	16.4	17.6	3027.3	3168.0
10-14	9413.0	9791.0	10.4	11.2	3090.6	3234.1
15-19	10428.0	10753.0	6.4	5.6	3126.3	3261.1
20-24	9978.0	10111.0	20.4	8.4	3183.5	3266.6
25-29	8909.0	8837.0	48.4	47.2	3344.4	3372.3
30-34	7776.0	7640.0	34.4	44.0	3537.7	3567.5
35-39	6309.0	6030.0	19.2	26.4	3651.9	3703.2
40-44	5735.0	5465.0	10.0	14.0	3692.8	3744.6
45-49	5898.0	5613.0	5.6	8.0	3680.2	3706.2
50-54	6167.0	5714.0	2.8	4.0	3622.3	3590.6
55-59	5766.0	5277.0	1.6	2.0	3517.1	3387.7
60-64	4983.0	4380.0	0.4	0.8	3352.6	3078.6
65-69	4708.0	3732.0	0.2	0.3	3121.4	2660.9
70-74	3543.0	2594.0	0.1	0.1	2798.5	2142.2
75+	5634.0	3219.0	0.1	0.1	5994.5	3236.4
Total	111,061.2	105,675.0	192.8	207.2	55,693.2	52,209.9
Both sexes	216,736.2		400.0		107,903.1	

Summary Demographic Measures

	1977-1982	Stationary Population
Total fertility rate	1.826	1.826
Gross reproduction rate	0.891	0.891
Net reproduction rate (NRR)	0.869	0.869
Male births per 100 female births	105.0	105.0
Female life expectancy at birth (in years)	77.09	77.09
Male life expectancy at birth (in years)	69.32	69.32
Population size	221,241.8	107,903.1
Yearly births	3,449.6	1,209.8
Yearly deaths	2,028.7	1,609.8
Yearly net immigrants	400.0	400.0
Annual rates per 1,000 population		
Birth rate	15.6	11.2
Death rate	9.2	14.9
Natural increase	6.4	-3.7
Net migration	1.8	3.7
Population increase	8.2	0.0

closed to the influence of migration, the stable model has investigated the shape of the long-run age distribution and eventual levels for rates of birth, death, and natural increase when underlying age-specific fertility and mortality schedules are fixed. Here we add the assumption of a fixed annual number and age composition of immigrants.[2] Focusing on females, we may extend the theory to include immigration in the following way.

STABLE THEORY WITH BELOW-REPLACEMENT FERTILITY AND CONSTANT IMMIGRATION[3]

A. *Annual Births*

If we represent the annual number of females immigrating at age a by I(a), the annual rate of bearing daughters for women at age a by m(a), and the probability of surviving from birth to exact age a in the female life table by p(a), then the annual number of births at time t, B(t), can be expressed as the sum across all ages of childbearing of the number of women at age a at times t multiplied by the annual rate of childbearing at age a, or as

$$B(t) = \int_{\alpha}^{\beta} N(a,t) \cdot m(a)da \quad , \qquad (1)$$

where α and β denote the lower and upper limits of the childbearing ages, respectively. Since we are interested in the long-run character of the population, we will restrict our attention to values of $t > \beta$, where $t = 0$ represents the time after which I(a), m(a), and p(a) are held constant. For $t > \beta$, women in the population at time $t = 0$ are no longer bearing children, and the youngest females in the first wave of immigrants after $t = 0$ have reached the end of their childbearing years.

The number of women at age a at time t depends first on the number of women who were born in the population a years earlier and have survived to age a, and second on the number of women who immigrated at all ages less than a and are now age a. The first component can be written as $B(t-a) \cdot p(a)$. To understand the second component, consider a particular age, say age 23.

Then the number of foreign-born women who are now age 23 equals the number of females who migrated at age 0 times the probability of surviving from age 0 to age 23, plus the number of females who migrated at age 1 times the probability of surviving from age 1 to age 23, and so on. Expressing this algebraically, the number of foreign-born women who have attained age a at time t equals $I(0) \cdot \frac{p(a)}{p(0)} + I(1) \cdot \frac{p(a)}{p(1)} + \ldots + I(a-1) \cdot \frac{p(a)}{p(a-1)} + I(a)$. The continuous-form analog of this number is

$$\int_0^a I(x) \, \frac{p(a)}{p(x)} \, dx \quad .$$

Therefore,

$$N(a,t) = B(t-a) \cdot p(a) + \int_0^a I(x) \, \frac{p(a)}{p(x)} \, dx \quad . \tag{2}$$

In words, equation (2) says that the number of women in the population who are age a at time t is the number of native-born women who have attained age a plus the number of foreign-born women who have attained age a.

Since the second term on the right-hand side of (2) depends only on a and not on t, it is simpler to write it as $H_I(a)$. Now we can substitute for $N(a,t)$ in (1) to obtain

$$B(t) = \int_\alpha^\beta B(t-a)p(a)m(a)da + \int_\alpha^\beta H_I(a)m(a)da \quad . \tag{3}$$

This equation tells us that the total number of births at time t is the sum of births to native-born women and births to foreign-born women. Since the second term on the right-hand side of equation (3) does not involve the variable time t, the number of births to foreign-born women is some constant value that is repeated year after year. We can represent it by B_1 so that

$$B(t) = \int_\alpha^\beta B(t-a)p(a)m(a)da + B_1 \quad . \tag{4}$$

We may now ask what the long-run behavior of $B(t)$ will be.

Taking Laplace transforms across (4) in the usual way we have

$$\tilde{B}(s) = \tilde{B}(s) \cdot \tilde{F}(s) + \frac{B_1}{s} \tag{4a}$$

where $\tilde{F}(s)$ is given by

$$\tilde{F}(s) = \int_0^\infty e^{-sa} p(a)m(a)da \quad . \tag{4b}$$

From (4a) we obtain

$$\tilde{B}(s) = \frac{B_1}{s(1-\tilde{F}(s))} \quad . \tag{5}$$

We now invoke the tauberian theorem that, providing $s\tilde{B}(s)$ has no singular points for $s > 0$, then $\lim_{t\to\infty} B(t) = \lim_{s\to 0} s\tilde{B}(s)$. This means in our case that as long as $1 - \tilde{F}(s)$ does not equal zero for any positive s, which from (4b) is guaranteed only if $\int_0^\infty p(a)m(a)da$ is less than 1, then the birth trajectory must reach an asymptotic limit given by

$$\lim_{t\to\infty} B(t) = \lim_{s\to 0} \frac{B_1}{(1-\tilde{F}(s))} = \frac{B_1}{1 - \int_0^\infty p(a)m(a)da} \quad . \tag{6a}$$

We recognize $\int_0^\infty p(a)m(a)da$ as the net rate of reproduction NRR. The theorem thus tells us that providing the NRR is less than 1, births must ultimately level off to a constant B given by

$$B = \frac{B_1}{1 - \text{NRR}} \quad . \tag{6b}$$

The reader may check that a stationary level B does indeed satisfy (4) if

$$B = \int_0^\infty Bp(a)m(a)da + B_1$$

that is, if

$$B = B_1/(1 - NRR)$$

as in (6b).

To summarize, we have shown that the annual number of births eventually becomes stationary, at a level equal to the annual number of births to immigrant women divided by 1-NRR.

B. *Total Population*

To calculate total population size we return to equation (2) and recognize that the total number of females is obtained by adding up the number at each age, or that

$$N(t) = \int_0^\omega N(a,t)da \quad , \tag{7}$$

where N(t) is the total number of females at time *t*, and ω is the oldest age attained by anyone in the population. Substituting from (2) into (7) we have

$$N(t) = \int_0^\omega \{B \cdot p(a) + H_I(a)\}da \quad . \tag{8}$$

Since the right-hand side of equation (8) does not involve the variable *t*, total population size does not change with time. We can therefore drop *t* from the left-hand side, knowing that we have a formula for the size of the eventual stationary population (N).

It is possible to write equation (8) more simply by realizing that $\int_0^\omega p(a)da$ is another way of expressing life expectancy at birth ($\mathring{e}_0$) and by letting H_I represent the total size of the foreign-born population, $\int_0^\omega H_I(a)da$. Thus,

$$N = B\mathring{e}_0 + H_I \tag{9}$$

or

$$N = B_1 \left(\frac{\mathring{e}_0}{1 - NRR}\right) + H_I \quad . \tag{10}$$

Equation (9) shows that the total eventual stationary population is actually composed of two smaller stationary populations. One of these arises from a constant annual number of births and has an exact parallel in the ordinary life table stationary population. There, the crude birth rate (ℓ_0/T_0) equals the reciprocal of life expectancy at birth (T_0/ℓ_0), so that the total population that would ultimately be generated by a constant yearly number of births (B) is $B \cdot \overset{o}{e}_0$.

The second stationary population contains H_I, the stock of foreign-born women. We can compute H_I simply, by summing $H_I(a)$ -- the number of immigrants in the population who are age a -- across all ages. This yields:

$$H_I = \int_0^{\omega} H_I(a)\,da = \int_0^{\omega}\int_0^{a} I(x)\,\frac{p(a)}{p(x)}\,dx\,da \quad . \qquad (11)$$

Substituting for B_1 and H_I in (10) we may write the total population size, in full, as

$$N = \left(\frac{e_0}{1-NRR}\right)\int_{\alpha}^{\beta}\int_0^{a} I(x)\,\frac{p(a)}{p(x)}\,m(a)\,dx\,da + \int_0^{\omega}\int_0^{a} I(x)\,\frac{p(a)}{p(x)}\,dx\,da \quad . \qquad (12)$$

NUMERICAL RESULTS

To see how well our analytic expressions predict the size and other characteristics of the long-run stationary population, we have applied them to U.S. fertility and mortality schedules for 1977 and to the data in Table 1 on immigrants.

The annual number of female births (B) in the stationary population is given by equation (6b) as

$$B = \frac{B_1}{1 - NRR} \quad ,$$

where B_1, the annual female births to immigrants, can be evaluated using the second term on the right-hand side of equation (3). Doing so yields B_1 = 77.29 thousand, and combining this with NRR = 0.869 we have B = 77.29 ÷ .131 = 590 thousand. In Table 1 annual male and female births combined total 1209.8 thousand, but since these projections assume a sex ratio at birth equal to 105 males per 100 females, approximately 0.4878 of all births are female. Therefore, the computer-based projections imply that B = 1209.8 × .4878 or 590.1 thousand.

Total female population size (N) is computed from equation (9) as $N = B\overset{\circ}{e}_0 + H_I$, where H_I, the size of the foreign-born female population, is equal to $\int_0^{\omega} H_I(a)da$. Setting B = 590.1, $\overset{\circ}{e}_0 = 77.09$, and $H_I = 10{,}201.25$, we have N = 55,692.1 thousand. This, except for rounding, is the same as the number in Table 1. For the female population the crude birth rate is 10.60, the crude death rate is 14.06, the immigration rate is 3.46, and the rate of natural increase equals -3.46.

DISCUSSION AND FURTHER RESULTS

If stable theory is expanded to include immigration, we have shown that as long as fertility is below replacement, a stationary population results by combining fixed fertility and mortality schedules with a constant number and age distribution for immigrants. Neither the level of the net reproduction rate nor the size of the annual immigration qualitatively affects this conclusion; a stationary population eventually emerges.

We can both generalize the above result and see how this stationary population is constructed, using a simple heuristic argument. Imagine a country divided into halves in such a way that the population alive at time t = 0 and any of its descendants reside in the western portion, and immigrant arrivals after t = 0 together with their descendants reside in the eastern portion. Concentrating first on the population in the west, we can see that this population eventually dies away. Even though it may continue growing for a while after t = 0 due to the momentum that a youthful age composition imparts to population growth, its below-replacement fertility is sufficient to guarantee a negative stable growth rate and, therefore, long-run extinction.

The eastern portion of the country develops demographically in a more complex way. Any population that exists there must either be direct immigrants or the descendants of immigrants. Hence this population (that is, the female part of it) will consist at any time of surviving immigrant women, native-born women whose mothers were immigrants , native-born women whose grandmothers were immigrants, and so on. It will be useful to call women whose mother immigrated "first generation", whose grandmother immigrated "second generation", whose great-grandmother immigrated "third generation", and so on, tagging each woman in the population by her immigration ancestry.[4] We can assume, in general, that fertility behavior differs for women of different immigration "generations", so that women of "generation" i have fertility schedule $m_i(a)$, with associated net reproduction rate NRR_i.

The eastern population then builds up as follows. In a relatively short time after time zero, two or three generations say, the stock of surviving direct immigrants becomes stationary and stays stationary, building up in exactly the same way as a standard life-table population, except that in this case people can enter the population at all ages. In time then there is a constant number of surviving immigrant women $H_I(a)$ at age a, in any year. In turn, each year thereafter B_1 children are born whose mothers are immigrants, where

$$B_1 = \int_{\alpha}^{\beta} H_I(a)m_0(a)da \quad , \tag{13}$$

and where $m_0(a)$ is the fertility schedule of immigrant women. Since immigrants are constant in number at any age, these annual "first generation" births are constant too. A generation or so after the appearance of "first-generation" births, "second-generation" births B_2 start to appear. Since these are born to the constant flow of "first generation" births, they number

$$B_2 = \int_{\alpha}^{\beta} B_1 p(a)m_1(a)da = NRR_1 B_1 \quad , \tag{14}$$

and each year, they too are born in constant numbers.

Given sufficient time, children of all "generations" up to "generation" N say, are born each year, and generalizing (14), we can show that each year produces a constant flow B_i of "generation i" births, where

$$B_i = NRR_{i-1}B_{i-1} \quad ; \quad i < N \quad . \tag{15}$$

As we move indefinitely into the future, all "generations" are represented in the eastern population, and the annual birth flow can be written as the infinite sum of "generational" births

$$B = B_1 + B_2 + B_3 + \ldots \tag{16}$$

or, substituting from (15)

$$B = B_1(1 + NRR_1 + NRR_1 \cdot NRR_2 + NRR_1 \cdot NRR_2 \cdot NRR_3 + \ldots) \tag{17}$$

This series will converge providing that NRR_i is less than one for all "generations", after some finite number n. In other words, the birth flow in the eastern population eventually becomes stationary, providing only that immigrant-descended women adopt below-replacement fertility a finite number of generations after "arrival".

Now each of these births, whatever its "generational" status, faces the same survival schedule, and so each birth flow B_i generates its own stationary population $B_i\overset{o}{e}_0$. Counting the annual stock of surviving immigrants, H_I, in with the "generational" population stocks, the eastern-half population levels off at the value

$$N = e_0B_1(1 + NRR_1 + NRR_1 \cdot NRR_2 + NRR_1 \cdot NRR_2 \cdot NRR_3 + \ldots) + H_I \quad . \tag{18}$$

We can conclude from this argument that stationarity can still come about even when immigrants and their close descendants have above-replacement fertility. All we require is that from some "generation" on immigrant descendants adopt, like the native population, below-replacement fertility. If so, stationarity is guaranteed.

Returning to the special case of the previous sections, where all net reproduction rates are equal and below one, we see that (17) becomes

$$B = B_1(1 + NRR + NRR^2 + NRR^3 + \ldots) \tag{19}$$

or

$$B = B_1 \cdot \frac{1}{1 - NRR} \quad , \tag{20}$$

which is the same as (6b), so that (18) is a generalization of our previous result, (10).[5]

Equations (16) - (20) provide a basis for determining the "generational" distribution of total births and of total population. In the example in Table 1 there are 590.1 thousand female births each year in the stationary population. Since NRR = 0.869, the fraction 1 - NRR or 13.1 percent are "first-generation" births; 11.4 percent (= 13.1 × .869) will be "second-generation" births, and so on. The total stationary population includes 55,693.2 thousand females, of which 10,201.3 thousand, or 18.3 percent, are immigrants. Since we have assumed that all females are subject to the same age-specific death rates, the size of the native-born population, $B\overset{o}{e}_0 = (B_1 + B_2 + \ldots + B_i + \ldots)\overset{o}{e}_0$, is distributed by generation in the same proportions as total births. Thus, 10.7 percent of all females are "first-generation", 9.3 percent are "second generation", and so forth. The distribution of total population by "generational" status is important because the preservation of native language, tradition, and culture is likely to be influenced by whether one is an immigrant, the child of an immigrant, or the grandchild. Cultural heterogeneity will be more pronounced the lower is the value of NRR.

This kind of analysis can also be of practical significance in helping to formulate immigration policy. The projection in Table 1 shows that 400,000 annual net immigrants lead eventually to a total population of 107.9 million, or 269.76 persons in the stationary population for every annual immigrant. Suppose the United States wanted to arrive at a stationary population as large as the 1980 census of approximately 226 million. Then, assuming 1977 fertility and mortality conditions and the age-sex composition of immigrants in Table 1, almost 840,000 annual immigrants would be needed -- a number that may not be far from the 1980 figure. (Of course, the population would increase to almost 300 million before falling to 226 million).

CONCLUSION

In this paper we have shown that any fixed fertility and mortality schedules with an NRR below one, in combination with any constant annual number and age distribution of immigrants, will lead in the long run to a stationary population. The size and other characteristics of this eventual stationary population depend only upon our assumptions regarding fertility, mortality, and the age-sex composition of immigrants, and are not influenced in any way by the population we begin with.

Moreover, we have shown that this long-run stationary population is actually composed of many smaller stationary populations -- one of immigrants themselves, one of "first-generation" descendants, and so on. The composition of the total stationary population by its so-called "generational status" can be computed from a knowledge of the specific fertility, mortality, and immigration assumptions.

These results, we have shown, can be obtained even when some "generations" have above-replacement fertility. All that is required to establish a stationary population in the long run is that, at some point in the generational chain of immigrant descendants, one generation and all those that succeed it adopt fertility below replacement.

NOTES

1. These and subsequent statistics on the part played by immigration in U.S. population growth are contained in Leon F. Bouvier, "The Impact of Immigration on the Size of the U.S. Population," Washington, D.C.: Population Reference Bureau, Inc.

2. Since immigration is controlled in most countries, assuming that the number of immigrants is constant is preferable to assuming constant rates of immigration.

3. This development parallels earlier work by Ansley J. Coale (1972). Coale approached the problem by starting with a stationary population closed to migration and then inquired how much of a reduction in fertility would be required when immigration is added to maintain a stationary population with the same number of births. We begin at the other end, by assuming below-replacement fertility and show that, with immigration constant both in volume and in age composition, a stationary population evolves. Moreover, any below-replacement fertility schedule, if held constant, leads to a stationary population when constant immigration is included.

4. Where quotation marks are used, "generation" signifies a label on each woman marking her immigration ancestry. Without inverted commas, generation signifies as usual either time elapsed or a particular population as measured reproductively from some initial event or population.

5. Equation (19) gives us an important clue as to how fast convergence to stationarity takes place. If NRR = .75 for example, then the first eight terms of the infinite series (19) account for 84% of the series total. In other words, most births in the population would be attributable to immigrants and the first six or seven "generations" of descendants of immigrants. In turn, this implies that the birth flow, in this case, would settle down six or seven generations after the stock of immigrants stabilizes: in total, about eight to ten generations after time zero.

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