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De-risking of Green Investments through a Green Bond Market – Empirics and a Dynamic Model

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Abstract

A substantial increase of green investments is still required to reach the Paris Agreement's emission targets. Yet, capital markets to expedite green investments are generically constrained. Literature has shown that governments could de-risk such investments. Empirical beta pricing and yield estimates reveal some public involvement in the green bonds market, especially for long maturity bonds. We provide empirical evidence that Governments and Multilateral organizations can de-risk green investments by supporting the issuance of green bonds in contrast to private green bonds - that show higher yields, volatility and beta prices - and conventional energy bonds, that are more volatile due to oil price variations. Since lower betas also mean lower capital costs, we use those empirical results and run a dynamic model with two types of firms, modeling the economic behavior of innovators (renewable energy firms) and incumbents (fossil fuel firms). The simulations of our model show that de-risked interest rates help to phase in renewable energy firms in the market and avoid a sharp debt increase. However, when the new entrants carry negative pay-offs for a longer time, it might not be sufficient to keep the debt low and to avoid a shake-out in the market. Subsidies and carbon taxation can complement the role of the de-risked interest rates and expedite the energy transition. Beside deterministic model variants, we also explore a stochastic version of the model.

JEL classification: C610, G120, 0380, Q580

Keywords: green bonds, innovation, climate finance, de-risking

1 Introduction

Since 2009, when the 15th Conference of the Parties (COP 15) to the United Nations Framework Convention on Climate Change (UNFCCC) took place in Copenhagen, climate finance has come to the forefront. This movement was followed by international climate agreements[1] that fostered the public and private sector mobilization of financial resources and the development of new financial tools.

Governments have a role in providing funding and in risk-bearing investments that exhibit higher externalities and uncertainties by reducing risk premia for such projects (Arrow & Lind, 1970; Stiglitz, 1993). There is often uncertainty on environmental costs of projects which provides incentives for reducing the risk premia through public investment (Arrow & Fisher, 1974). This is also true for the implementation of renewable energies, especially in developing countries: the higher fixed and upfront costs vis-a-vis fossil fuel projects demand a de-risking effort for green investments (Ondraczek et al., 2015; Sweerts et al., 2019; Waissbein et al., 2013).

Green bonds can play a relevant role for this purpose (Flaherty et al., 2017; Orlov et al., 2018). It provides an instrument to implement Sachs' (2014) idea of "intertemporal burden sharing". The cost of climate policies can be shared by current and future generations through debt finance. Governments and Multilateral organizations are key agents with respect to phasing in green bonds into the asset markets. Moreover, asset holders need to be induced to hold green bonds into their portfolio which in turn depends on the performance of green bonds in the financial market. This in turn is likely to reduce the capital cost of green investments and aid to transform the energy system.

The issuance of green bonds has risen as an innovative instrument to finance sustainable projects. Since 2007, more than 3,000 bonds were issued by Governments, Private and Multilateral organizations mobilizing more than US$ 414 billion. The green bonds are fixed-income securities, usually certified by a third-party, to leverage resources in the capital market. The external certification guarantees that the proceeds are used for sustainable projects only, such as renewable energy, green buildings and clean transport. The green bonds

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[1]In 2009, through the Copenhagen Accord, the international community agreed on financing US$ 100 billion per year for sustainable projects in developing countries. In 2010, the Cancun Agreement mobilized Governments to keep global temperatures well-bellow 2⁰C above the pre-industrial level. In 2015, the Paris Agreement stressed this temperature goal, keeping the target but encouraging a further effort to reach 1.5⁰C.

decrease portfolio risks and solve investors' information constraints which can attract resources owned by private institutional investors[2], especially those with better ESG practices. One has also become aware of the financial instability risks of holding carbon-intensive assets (Carney, 2015). While climate change increases financial risks and investment needs, the macro environment and quantitative easing policy (QE) induce lower asset returns (Morana & Sbrana, 2019). Nevertheless, institutional investors can be crowded-in if public agents use its de-risk potential (IFA WG, 2017).

This paper discusses the role of green bonds and verifies whether or not the Governments and Multilateral organizations can de-risk these bonds as a strategy to increase green investments. We calculate bond yields and beta prices and find that Governments and Multilateral organizations can de-risk green projects by acting as an issuer of green bonds or by initiating policies supporting green bonds.

The paper also studies the impact of this strategy on the implementation path of renewable energy in the context of a dynamic model. Our model is influenced by the evolutionary approach in economics (Arthur, 1989). It is also related to the work of dynamic limit pricing - as in Judd & Petersen (1985), Gaskins (1971) and Kato & Semmler (2011) where, however, the incumbent is dominantly pursuing an intertemporal strategy of profit maximizing. We run a small-scale model of two types of firms studying the performance of the innovators (renewable firms) and the incumbents (fossil fuel firms). We assume that the market entrants (innovators), pursuing the supply of renewable energy, exhibit an intertemporal pay-off function. We also introduce a debt dynamics for the innovators and explore analytically the debt sustainability. The model is designed to explore the market impact of the de-risking strategy on the implementation of renewable technologies. We also evaluate a stochastic version of the model and a model variant with taxes on the carbon sector and subsidies for green activities[3].

The paper is organized as follows. In section 2, we present the theoretical background that justifies the role of the public sector in capital markets and in environmental projects. Section 3 gives a brief overview of the Bloomberg database of corporate green bonds and studies the beta prices and returns for

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[2]Institutional investors hold around US$ 120 trillion in assets (Bielenberg et al., 2016) while only 1.5% of climate finance is provided by this type of agent (CPI, 2019).

[3]A discussion about the interaction of carbon taxation and green bonds is also set by Heine et al. (2019) and Steckel & Jakob (2018).

de-risked bonds. Based on those results, section 4 introduces the dynamic model of the two types of firms and proves under what conditions debt sustainability can be achieved. Section 5 presents the results from the numerical simulations and introduces a stochastic version of the model. Section 6 concludes the paper. The appendix presents the solution procedure of the model, the data background and an evaluation of the volatility of the returns of green and fossil fuel bonds.

2   The role of the public sector in climate finance

Although the role of the public sector in climate finance has increased, the great needs for climate finance demands complementary credit sources. Credit dynamics is key for understanding investment and growth (Faulwasser et al., 2018; Gertler & Bernanke, 1989). However, asymmetric information, moral hazard and adverse selection can explain credit costs and credit rationing given the relationship between borrowers and lenders and the existence of information constraints (Akerlof, 1970; Stiglitz & Weiss, 1981). Due to market imperfections, the Government may intervene in the credit market to reduce credit constraints and foster investment for certain types of projects.

Indeed, Governments are able to provide funding and fix market failures associated with costly information in credit markets. When markets are missing and incomplete, the Government can also act as a risk-bearing agent (Stiglitz, 1993). According to the Arrow-Lind Theorem, under uncertainty, projects with social benefits and with publicly born risks can have lower cost of risk-bearing as the State can distribute it across taxpayers (Arrow & Lind, 1970). Furthermore, public sector's equity and bond issuing can reduce the risk premia and generate a liquidity premium in contrast to private agents (Grant & Quiggin, 2003; Holmström & Tirole, 1998).

However, the role of the public sector and its capacity to buffer risk-bearing projects is unequal between countries. Capital market imperfections and distinct sovereign risk perceptions impact the weighted average cost of capital (WACC)[4]. It also limits the public capacity to de-risk activities with high

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[4]The WACC for renewable energy projects in Africa varies from 8% to 32% in a sample of 46 countries (Sweerts et al., 2019). For better rated European countries (Figure B.1, Appendix B), the capital cost in 2017 varies from 1.43% (France) to 4.53% (Greece). For non-European countries (Figure B.2, Appendix B), 25% of the countries have credit costs above 14% while only 9% of them have a credit cost below 4%.

externalities. Nevertheless, the public sector can help to direct financial market resources toward the implementation of green technologies.

First, the cost of capital depends also on firms' asset prices and on the industry life cycle. Small, medium and start-up firms in innovative industries are frequently financially constrained and face a higher cost of capital (Hall & Lerner, 2010). Innovative small firms follow a financial growth cycle in which financial needs change as the business grows or the investment needs increase[5]. Moreover, there is evidence that the bond market, instead of the equity market, explains better the investment behavior of firms (Philippon, 2009; Semmler & Mateane, 2012).

Second, as to environmental investments and credit markets, Fisher (1973) reviews the Arrow-Lind Theorem and finds that there is an uncertain cost of such projects that may affect the performance of investment. This uncertainty entails an adjustment of an investment's expected benefits and, as these costs are hard to measure and to identify, public policy should attempt to internalize them (Arrow & Fisher, 1974).

Third, initially, the monetary cost of green investments can be high which reinforces the need of public policy in de-risking those investments. The implementation of renewable technologies faces higher fixed upfront costs in comparison with fossil fuel investments, especially in developing countries (Ondraczek et al., 2015; Sweerts et al., 2019; Waissbein et al., 2013). These new technologies are operated at a lower scale of production and are usually expensive in terms of set-up costs. Yet, as to recent trends of green technology, the global costs for renewable energy have decreased and tend to be cheaper than the fossil fuel cost of production (Figure 1)[6]. This price decrease is due to economies of scale and to the infinite supply of renewable energy but also due to public policy support aimed at reducing credit risk and guaranteeing the implementation of new technologies with high externalities and significant uncertainties[7].

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[5]From seed capital and venture capital to debt and equity (Berger & Udell, 1998) or from internal to external finance, using first debt and then equity (Semmler, 2011).

[6]Gimon & O'Boyle (2019) find that, for the US in 2018, 74% of the national coal supply is at risk.

[7]For environmental projects, Governments and international institutions often pursue loan guarantees, new regulatory frameworks, risk insurance, investment in portfolios with higher risk technologies and the issuance and purchase of green bonds (Steckel & Jakob, 2018, Mazzucato & Semieniuk, 2018). In 2017, Governments expenditures to implement renewable-based electricity were around US$ 143 billion, which represented 19% of the total investment employed in the electricity sector (IEA, 2018). Most public support was for solar and wind energy (80%). China, Germany, United States, Japan and Italy employed 2/3 of the total support.
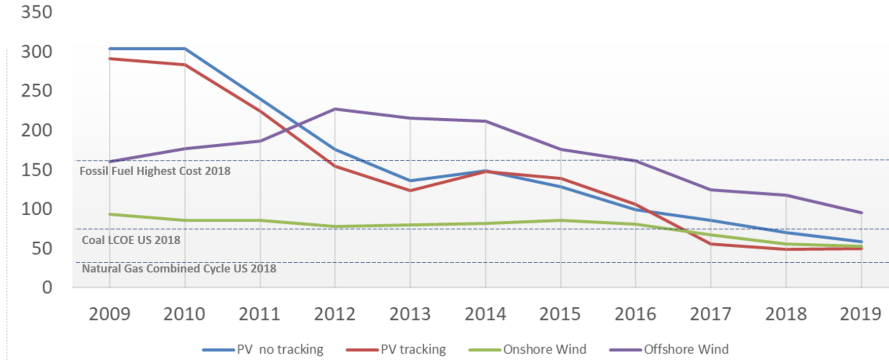
Figure 1: Levelized cost of electricity (LCOE) for renewable energy sources versus fossil fuels (USD per MwH - 2009/2019), Note: The LCOE was obtained through Bloomberg. The references for coal and natural gas are for the US only while the others are global assessments. The "Fossil Fuel Highest Cost" for 2018 was estimated by IRENA (2019)

However, we should note that public and private actors interact in the financial markets. Some authors argue that investors pay the same price for green and conventional bonds, i.e. there is a zero "green premium" (Larcker & Watts, 2019; Hyun et al., 2019). We discuss, in the next section, that a green bond yield analysis should take into account the different issuers profiles. This debate sheds light on the yield sensitivity of investors for green bonds and how its related with the bond profile.

Several factors - such as maturity, bond rating, countries or issuers debt, market conditions and liquidity - determine the bond yields. Investment grade green bonds perform differently than other green bonds (Kuhn et al., 2018; Hachenberg & Schiereck, 2018). Green bonds can be more liquid than conventional bonds, depending on the bonds profile (Bachelet et al., 2019; Febi et al., 2018). The nature of the issuer (if it's public or not) and whether the green bond is certified by a third-part or not also matter for the liquidity and yields, i.e. the green reputation of the bond allows lower yields (Bachelet et al., 2019; Fatica et al., 2019; Kapraun & Scheins, 2019). Furthermore, green bond issuing attracts long-term investors who value environmental gains which impacts liquidity, demand and lower yields (Flammer, 2018; Baker et al., 2018;

In the United States, explicit federal subsidies to renewable sources were US$ 15 billion in 2013 and US$ 6.7 billion in 2016, representing 46% of the total subsidies for the energy sector (EIA, 2018). Due to this effort, the levelized cost of electricity (LCOE) for renewable energy is from 2% to 9% lower than a similar non-subsidized investment in the country (Lazard, 2018).

Karpf & Mandel, 2018; Partridge & Medda, 2018; Nanayakkara & Colombage, 2018; Zerbib, 2019; Hachenberg & Schiereck, 2018). Therefore, the green bond's characteristics and the nature of the issuer matters for green bonds analysis. For that reason, our empirics and modeling analysis in the following section consider the different yields for Private, Public and Multilateral issuers.

As information constraints are relaxed for green bonds, new institutional investors concerned with ESG practices and aware of climate transition risks can be attracted[8]. Market agents report that green bonds attract new investors (Climate Bonds Initiative, 2018) and allow known institutional investors to gain exposure to climate-friendly assets (Venugopal, 2015). The use of de-risking tools by public agents - with higher rating and credibility - can turn green bonds to be even more attractive to institutional investors (IFA WG, 2017).

Investors' pro-environmental preferences add up to the hedging role of green securities as an incentive to attract institutional investors. The literature shows that green bonds protect investors from the volatility associated with energy and commodities fluctuations, which reduces portfolio risks (Horsch & Richter, 2017, Reboredo, 2018). The purchase of green bonds by private investors can reduce their exposure to riskier carbon-intensive bonds as the volatility of green bond returns is disconnected from fluctuations driven by oil prices, as we empirically demonstrate in Appendix C.

The incentives for institutional investors help to solve saving-investment imbalances, as investors hold assets on portfolios with lower return-risk impacted by QE - as Morana & Sbrana (2019) report for catastrophe bonds. Indeed, QE has been widely implemented in advanced countries after global financial crisis but should also be analyzed in the context of endogenous and exogenous risks for financial stability. In order to address those risks, a green QE (with green bonds purchase, eg.) can accelerate the transition (De Grauwe, 2019; Matikainen et al., 2017).

Overall, recent literature seems to support the view of Arrow and his co-authors who have argued from early on that, given the yield sensitivity of environmental projects, public organizations have a role in supporting such projects which, otherwise, would not be implemented by private firms' bond issuance

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[8]The CPI (2019) shows that, for climate finance, private investors accounts for 56% of total investment but only 1.5% is financed by institutional investors. Bielenberg et al. (2016) suggests that increasing institutional investors role is key for financing the sustainable infrastructure gap: they estimate a potential increase of US$ 1 trillion to US$ 1.5 trillion a year (these investors currently hold around US$ 120 trillion in assets).

only. Furthermore, green bonds have recently emerged as a relevant instrument for public issuers, private investors and portfolio holdings: green bonds can decrease portfolio risks, in particular in the light of oil price driven volatility of other assets returns (see Appendix C).

3 Governments and multilateral organizations in the green bond market

Though Governments and Multilateral organizations are relevant drivers of the green bond market, there are also significant private corporate green bonds issued and traded. Beside ownership, one has also to take account of bonds maturity, ratings and countries risk (and income groups).

3.1 The green bond market – An overview of the Bloomberg database

The Bloomberg terminal provides a special label identifying the bonds issued as "green bonds"[9]. From 2010 to 2018, 1,452 green bonds were issued, with an average maturity of 7.78 years[10]. Table 1 shows how these bonds are distributed by capital ownership[11], rating, maturity[12], countries income group[13] and country of risk[14].

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[9]This label is based on the issuer self-declaration while other sources, such as the Climate Bonds Initiative, publish only certified bonds.

[10]We are not taking account US municipalities bonds, given its specificity and the fact that we were not able to get monthly yields to this type of bonds (in order to calculate the beta price). During this period, around 4,000 municipalities bonds were issued in the US.

[11]The Government bonds consider bonds issued by Governments or state-owned firms and banks. The Multilateral bonds are issued by international financial institutions such as multilateral and regional development banks (listed Table B.2).

[12]Although the sample maturity mean is 7.78 years and its median is 5 years, capital market agents define that long-term bonds have more than 10 years while short-term has less than 5 years and intermediate between 5 and 10 (Kenny, 2019).

[13]Following the World Bank classification available at: https://datahelpdesk.worldbank.org/knowledgebase/articles/906519-world-bank-country-and-lending-groups .

[14]Bonds face different risk premia depending of their country of risk. The yields data shown in section 3.2 are likely to reflect the interest rates and country's risk premia.

Capital ownership		Maturity		Rating S&P/Moody`s		Countries' income group		Country of risk	
Private	53.3%	Short	75.0%	Investment grade	50.0%	High income	59.9%	China	14.9%
Governments	33.0%	Long	25.0%	Other / NA	50.0%	Upper-middle income	24.0%	Multilateral	13.7%
Multilateral	13.7%					Lower-middle income	2.4%	United States	12.0%
						Multilateral	13.7%	Sweden	11.1%
								France	9.6%
								Malaysia	6.6%
								Germany	5.6%
								India	1.9%
								Others	24.6%

Table 1: Green bonds database Bloomberg (2010-2018), Note: The Bloomberg sample contains 1,452 bonds, except US municipalities bonds.

Table 1 shows that the sample has a larger share of private issuers, short-term bonds and a well-balanced risk profile. Also, the bonds are mostly issued in high-income countries (United States and European Countries) although China is also relevant[15]. We also classified each bond by the "use of proceeds" and found that mitigation projects can be financed by 98.12% of the bonds (75% are allowed to invest in renewable energy, 40% in low-carbon transport and 35% in green building or water management) while only 26% of the bonds can be used for adaptation projects[16].

Since 2010, green bonds leveraged US$ 442 billion in the market (US$ 316 million in 2010 and US$ 143 billion in 2018). The Figure 2 shows the evolution along time of the green bonds issuance and how this amount is distributed across distinct issuers. We observe that the growing path of the green bonds' volume is driven by the increasing role of the private and public agents. In the next section, we analyze in detail the yield and beta prices of these bonds.

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[15]Banga (2019) lists market barriers that prevent developing countries from entering the green bonds market such as: the lack of knowledge, inappropriate institutional arrangements, minimum size requirements, the currency of issuance and high transaction costs.

[16]It adds up to more than 100% as the data is based on the issuer self-declaration of potential investments at the time of the issuance. The definitive allocation of resources is defined ex-post.
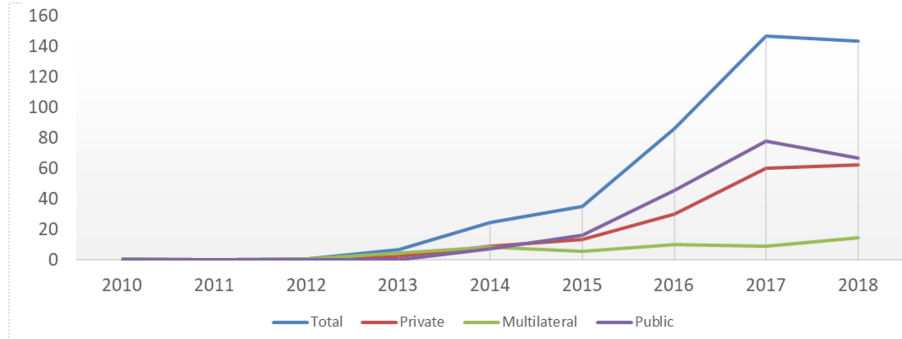
Figure 2: Green bonds: volume issued by capital ownership in billions USD (2010- 2018), Note: The green bonds data were obtained through Bloomberg.

3.2 Green bond yields and capital costs

Capital Markets are characterized by credit constraints and there are uncertainties and costs associated with environmental projects and with the implementation of new green technologies. Given those features, we should verify whether Governments and Multilateral organizations can help to re-price risk and de-risk financial assets that are used for green investments. For this purpose, we study the yields and the beta price for green bonds.

The yield is the return an investor gets on a bond. Usually, investors accept taking more risk when bonds exhibit higher yields. The Bloomberg database provides the current yield for each bond. The beta price is a measure of relative risk of an asset in relation to the overall market. The higher the risk, the higher the beta (roughly, a beta greater than 1 indicates that the bond is more volatile, and thus more risky, than the market). The beta price for the green bonds is calculated based on the monthly yields for each bond (using the last 12 months observations) and on a stock markets index (S&P 500). It's measured through the ratio of the covariance between the green bond and market returns and the variance (risk) of the bonds monthly yields. The average beta price is weighted by value. We also control its risk and yield by maturity, grouping the bonds by short and long maturity.

The current yield distribution, obtained through a Kernel density[17], is

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[17]For Kernel density methods in R, see http://lmdvr.r-forge.r-project.org/figures/figures.html

shown in Figure 3 by issuers' capital ownership (Private, Governments or Multilateral) and maturity (short or long). Based on this density, we evaluate the bonds returns (current yield) and its volatility (measured by the distribution of standard deviations, a proxy for risk). A first check gives us the following results: Private bonds have, on average, higher return-risk ratio in comparison with Governments and Multilateral bonds[18]: for long maturities, this ratio is 1.96 for Private, 1.27 for Governments and 1.47 for Multilateral; for short maturities, this ratio is 1.34 for Private, 1.26 for Governments and 1.18 for Multilateral. Thus, this difference is greater for long maturity bonds[19]. We should also observe in Figure 3 that, for Private bonds, the yield increases with the maturity which is not observed for Governments and Multilateral bonds. For these two types of bonds, we observe lower yields and less volatile for long-term bonds[20].
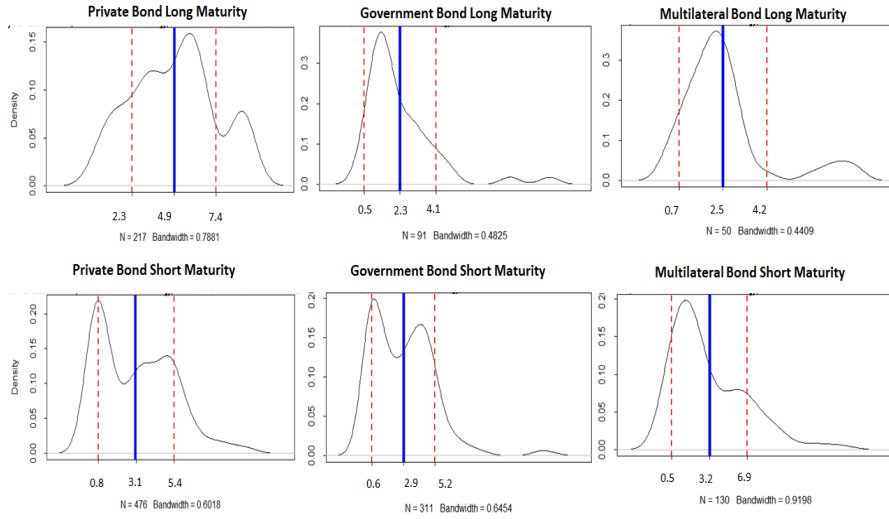
Figure 3: Green bonds current yield (%): density estimation by issuers' capital ownership (mean and standard deviation per maturity – short and long)

Sustainable infrastructure projects are known for being long-term projects

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[18]We measure this ratio evaluating the returns over the volatility (Current Yield/Standard Deviation), a proxy for the sharpe ratio.

[19]For long maturities, the sharpe ratios are driven by the lower returns and lower volatility of public bonds. For short maturities, it's driven by the lower returns for Government bonds and by the higher volatility of Multilateral bonds.

[20]This reversal of the yields of long and short-term Government bonds presumably is arising also from the fact that there is a reversal of the term structure of sovereign bonds in many countries in recent times.

that demand more stable and long-term finance sources. Also, as seen in section 2 (and in Figures B.1 and B.2, Appendix B), borrowers pay significantly higher interest rates in developing countries. For the green bonds database, we find the same conclusion: upper middle-income countries bonds exhibit an average yield of 7.93 while high-income countries have a yield of 1.51 (Table B.1, in Appendix B). Indeed, the distribution for Private and Government bonds have double-peaks also due to the distinct risk profile of issuers or countries in the sample as there is a group of non-investment grade bonds (and bonds issued in middle-income countries) that are paying higher yields.

Overall, Governments and Multilateral agents seem to act de-risking investment projects by issuing green bonds – resulting in lower return-risk ratio - to support projects that otherwise would not be undertaken or would pay higher risk premia. Note that these are still general results whereby we do not compare conventional and renewable energy bonds. This issue is studied in the end of this section and in Appendix C.

The beta price is another relevant measurement to assess bond risk. This is particular important for green investments' capital cost. The average beta price, weighted by volume, is shown in Table 2 grouped by the issuers' capital ownership and by maturity. Although the whole sample consists of 1,452 bonds, we have 690 bonds with available monthly yields for 12 months. The sample shows that the average beta price is low (0.17) and get lower for short-term bonds, especially for those issued by Governments and Multilateral organizations. However, for longer maturities, the Governments bonds are riskier than Private and Multilateral bonds. This presumably comes from the fact that there are countries with high sovereign risk ratings which spillover to green bonds risk. We apply a detailed analysis of the beta distribution in the observations for each type of bond, considering the effect of a bond duration.

Maturity	Multilateral (n=119)	Government (n=163)	Private (n=408)	Total (n=690)
Short	0.13	0.09	0.16	0.12
Long	0.15	0.30	0.25	0.26
Total	0.16	0.15	0.19	0.17

Table 2: Green bonds: beta prices by issuers' capital ownership and maturity (weighted mean for the 12 months beta price), Note: The beta prices are based on data obtained through Bloomberg and Standard & Poors.

The distributions for the beta prices, obtained via a density estimation based on the non-parametric kernel smoothing method proposed by Racine (2008), are shown in the Figure 4 by capital ownership and maturity[21]. We observe that the distribution of Governments and Multilateral bonds differs from the whole sample. Multilateral bonds have a beta price mean lower than the Private bonds (0.09 versus 0.14) and a slightly lower standard deviation (0.29 versus 0.43). Government bonds have a beta mean slightly lower than Private bonds (0.12 versus 0.14) and a much lower standard deviation (0.19 versus 0.43). Nevertheless, we observe a heterogeneity between countries which means that the sovereign risk of countries also matters and impacts a bonds beta price and thereafter the capital costs. Table B.1 (Appendix B) shows that the WACC, the beta prices and the yields are usually lower for high-income countries and higher for middle and low-income countries. For Government bonds, the sovereign rating matters and should be taken into account as countries with better financial market access and better rating have a greater capacity to de-risk green investment.

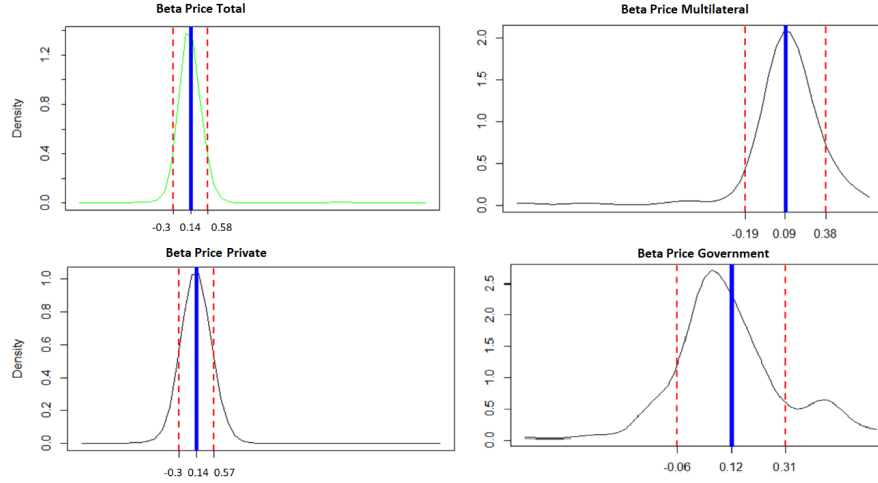Figure 4: Green bonds: beta price density estimation per issuers' capital ownership (mean and standard deviation, 12 monthly yields)

However, the greatest beta price differences are observed in the Multilateral bonds distribution. It seems that, although the public sector may act

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[21]We should observe that the beta for each group is calculated by the weighted mean while the distributions in Figure 4 report the simple mean.

de-risking green investments, a greater de-risking effort has been undertaken by Multilateral organizations if we look only at the beta prices. Although the density estimation shows a lower risk for Multilateral bonds, we should also verify if the term-profile of the bonds is impacting the bonds' beta. For this purpose, we generate the distributions for the Government and Multilateral bonds' beta with long and short maturities (Figure 5). We observe that Government bonds with longer maturities are riskier than those with short maturities (have higher betas). However, we do not find the same pattern for Multilateral bonds. The beta distribution remains very similar for both maturities. It reinforces the role of Multilateral organizations in fostering green investment also in middle and low-income countries. Banga (2019) recommends the use of development banks as intermediary institutions for green bond management to solve existent constraints for developing countries. Indeed, countries with lower capacity to de-risk bonds, due to their poor financial situation, may access loans and grants supplied by these institutions.
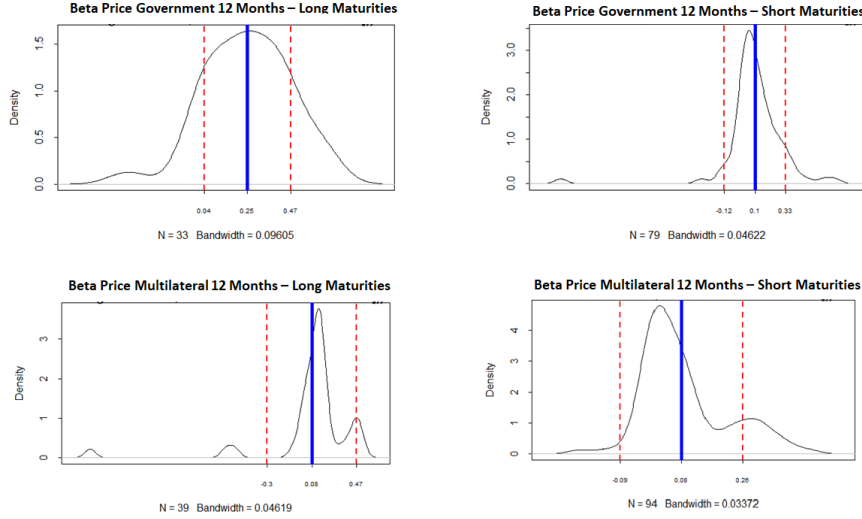
Figure 5: Green bonds density estimation by maturity: Multilateral and Government (mean and standard deviation, 12 monthly yields)

The analysis of yields and beta prices by capital ownership and maturity provides evidence that Multilateral organizations and Governments can play a role in de-risking green investments through green bonds issuance. In addition, green bonds are also less impacted by oil price fluctuation which decreases these

bonds' volatility due to economic cycles. Reboredo (2018) shows that the green bond market only weakly co-moves with the fossil fuels markets which brings hedge and diversification opportunities to investors.

In Appendix C, we apply harmonic estimations for the oil price changes, for the returns of the S&P Green Bonds Index and for the returns of the S&P 500 Energy Corporate Bond Index (a more comprehensive index that also includes carbon intensive energy assets). It shows that the swings in volatility of the oil prices mainly spillover to fossil fuel based bonds. We also run a linear regression model using these estimations and find that the oil price variations have a greater impact on the energy corporate bond returns than on green bond returns. Thus, green bonds are good instruments for risk hedging against certain market fluctuations and for de-risking of investments.

We add to this fact the empirical evidence that Government and Multilateral bonds show lower yields and also lower volatility for long-term bonds in comparison to Private bonds. These yields are rather low if we compare them with the credit cost in many developing countries. Furthermore, the beta analysis shows that Multilateral bonds exhibit lower beta prices and that maturity does not increase their risk profile. Based on this analysis, we run a model to simulate the market impact of de-risking bonds for renewable energy firms who are entering the energy market in which the incumbent firms are still using fossil fuel technologies.

4 A Dynamic Model

Next, we introduce a dynamic evolutionary model of technical change and firm competition. As mentioned, our model is particular influenced by the evolutionary approach in economics (Arthur, 1989), following a Schumpeterian view of innovation dynamics. It also incorporates features of the work of Gaskins (1971), Judd & Petersen (1985) and Kato & Semmler (2011).

Our work is distinct in several aspects from traditional studies that use a static theory of the firm[22]. First, renewable energy firms (innovators)

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[22]In recent modeling efforts of modeling the energy sector, a static profit-maximizing theory of firm competition is a widely used method. Kotlikoff et al. (2019), for instance, present an energy sector, represented by firms extracting non-renewable resources and firms producing clean energy through a production function using capital, labor and land. From the static maximization function, they derive the profit maximization conditions for both types of firms.

enter the market and compete with existing energy firms (incumbents) for energy production. Their success depends on the initial conditions, interaction effects with the incumbents, financing constraints and debt level. Second, the innovating firms pursue an intertemporal pay-off function. Their optimization problem is not based on a static production function. Third, our model allows us to detail the innovating firms' operational and financial costs together with their debt management while they expand in the market. Though the innovating firms can temporarily have negative cash flows, we give a proof under what conditions the sustainability of debt dynamics is provided.

Some of these distinctions can also be found in energy firms modeling in the climate-change literature (Kotlikoff et al., 2019; Acemoglu et al., 2012). On the third difference, we should note that we introduce finance as an instrument of public de-risking effort. We don't observe finance in other climate models. Yet, our model shares a common theoretical background with others models in climate economics - see Acemoglu et al. (2012). Similarly to our approach, those models are adapted for the case of two sectors (green and brown energy). Our model is inspired by models in which both energy sources are substitutes and returns to scale of the new technology matters. Note that climate models allow the existence of negative externalities from carbon-intensive energy use. This raises the issue of how fiscal policy should counteract the negative externalities. Given these effects, we explore the role of the public sector in solving this market failure by the implementation of green energy, based on carbon taxation and subsidies (as in Acemoglu et al., 2012). Furthermore, we consider only set up costs in the renewable energy firm pay-off function as its main input (wind and sun light, eg.) is free while fossil fuel firms face environmental and input costs (oil and coal prices).

4.1 Model Specification

We present a small-scale model of two types of firms modeling the behavior of the innovators (renewable energy firms) and the incumbents (fossil fuel firms). We thus assume that there are heterogeneous firms in the energy sector. One type of firms are the incumbents: the large scale fossil fuel energy firms that behave passively. Another group of firms enters the energy market implementing

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While Kotlikoff et al. (2019) have a finite decision horizon for the households' optimization horizon, we presume this for the firms' optimization problem.

low-carbon technologies, possibly leading a less carbon intensive energy sector. We assume that the market entrants (innovators), pursuing the supply of renewable energy, exhibit an intertemporal pay-off function. This approach is related to some models of dynamic limit pricing[23], although it is distinct due to the fact that the incumbent is not dominantly pursuing an intertemporal strategy of profit maximization. We thus presume that the established incumbents are passively reacting to the new innovations in energy supply. However, we propose that they can learn and adopt partially the new technology for low-carbon energy supply.

While established incumbents are passively reacting to the new innovations in energy supply, we assume that the entrants (the low-carbon energy firms) undertake innovations to increase their market share by expanding the number of firms. They may follow a joint pay-off maximizing strategy, $g(x_2, x_3, u)$, whereby x_2 is the number of innovating firms, x_3 is the external debt and u is their effort toward green innovations, with $u \in \Omega_+$. Note that we could make the proposition that both types of firms have an intertemporal pay-off function but this would lead us to a complicated differential game set up.

As mentioned, our model of such heterogeneous firms in the energy sector, and their quite complex interactions, is inspired by the evolutionary approach in economics, developed by Brian Arthur (Arthur, 1989). This is frequently called the Schumpeterian view of innovation dynamics. Since much modern theory of this direction relies on the replicator dynamics, we will stylize the interaction of our heterogeneous firms in such a way. We thus may assume different types of interaction effects between the firms: a *predator-prey* relation between the innovators and incumbents, a *cooperative effect*; and a *competition (or crowding) effect*[24].

The multi-period pay-off function of the innovators, subject to con-

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[23]See Judd & Petersen (1985) and Gaskins (1971), for models in which the dominant firms determine prices through entry preventing price setting. See also Kato & Semmler (2011) for a model in which dominant firms combat new entrants by building up entry-preventing capital.

[24]The predator-prey relation occurs when innovators grow at the expense of the incumbents. The competition effect results when the new technology becomes known by others and quickly diffuses. The excess profit, for example, falls because of reduced prices and compressed markups. We use an inverse demand function to specify this effect. The two groups of firms also gain from each others' success. Finally, the cooperative effect relies on spillovers or learning effects that bounds the number of incumbents away from zero, so that, although firms exit, complete extinction of incumbents does not occur.

straints, looks like the following[25]:

$$\max_u V = \int_0^T e^{-\gamma t} g(x_2, x_3, u) dt$$

s.t.

$$\dot{x}_1 = k - ax_1 x_2^2 + bx_2 - x_1 e/\mu \tag{1}$$

$$\dot{x}_2 = x_2(ax_1 x_2 + vg(x_2, x_3, u) - \beta) \tag{2}$$

$$\dot{x}_3 = rx_3 - g(x_2, x_3, u) - \tau x_3^2 \tag{3}$$

The three types of interaction effects among the two types of firms are incorporated in the state equs. (1) - (3). The pay-off function of the innovating firms also plays a role in the state equs. (2) and (3) and is given by: $g(x_2, x_3, u) = \mu(x_2, u)x_2 u - cu - c_0 x_2 - rx_3$, where $\mu(x_2, u)x_2 u$ is the net revenue ($\mu(x_2, u)$, being the (net) price, or markup) and the remaining terms are the costs. The cost cu is independent of the number of firms and cox_2 is a cost depending on the number of firms: $cu + c_0 x_2$ is the total amount of resources spent to innovate and rx_3 is the interest on the external debt x_3.

The equs. (1) - (2) depend on the mark-up $\mu = \alpha/(\Phi + x_2 u)$ which represents the effect the entrants have on the incumbents in (1) and also on the innovators in (2). The terms $k, \alpha, \beta, c, r, \Phi, \gamma$ and v are constants and positive. Further information on parameters is summarized in Table 3. Furthermore, x_1 represents the number of incumbents, x_2 the number of innovators, x_3 the external debt and u is the effort to create new technologies (e.g., hiring engineers, buying patents, running research labs), a decision variable related to the introduction of renewable energy. This investment is usually risky since there are uncertainties and the technological and market risks involved over time.

We first limit our model to a deterministic version: if the pay-off increases, x_2 rises proportionally to the pay-off (excess profit attracts entry); if the x_2 increases, it impacts negatively the pay-off (and reduce the excess profits). In the equ. (2) the term $vg(\cdot)$, in which v is a constant, means that there is an increase in the number of innovators proportional to their excess profit. This is a quite conventional determination of the entry dynamics, whereby excess profits attracts entry, and the excess profits erode if the number of those firms

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[25]For the detailed numerical procedure to solve our model variants see Appendix A.

rises.

Parameter	Definition	Value
r	Interest rate	0.02 (de-risked) or 0.2 (not de-risked)
γ	Discount rate	0.03
k	Constant parameter for x_1 dynamics	0.1
a	Technology diffusion parameter	0.163
b	Cooperative effect parameter	0.02
v	Entrant respond to profit	1
ß	Constant parameter to x_2 dynamics	0.02
c	Cost per unit of effort	0.1
c_0	Cost per unit of firm	0.04
α	Fixed parameter in Mark-up	1
T	The Bohn Term	0.2
φ	Fixed parameter in mark-up	3.5 (high mark-up) or 5 (low mark-up)

Table 3: Simulation parameters

We should have a further look at the equs. (1) - (3). The term ax_1x_2 means that when the number of firms applying the new technology grows, the accessibility of the incumbents to that technology also grows. Therefore, the rate of decrease of the incumbents in (1) may increase innovators in (2). The term bx_2 in (1) reflects the cooperative effect of x_2 on x_1. This represents the learning gains of the incumbents when they improve their performance as the information about the new technology spreads and the competitive pressure on the incumbents increases due to the new technology. The term $ax_1x_2^2$ represents the predator-prey interaction where the adoption of the new technology is supposed to take place proportionally to the product of x_1 and x_2^2. The last term x_1e/μ is the crowding effect for x_1: when x_2 increases this term increases and x_1 decreases.

The state equ. (3) represents the evolution of the external finance of renewable energy firms through loans from banks or bonds issuing (r is a fixed return on debt given by x_3). If $g(x_2, x_3, u)$ is positive, there is a repayment of liabilities; if it is negative, there is an increase of liabilities of the innovating firms. The latter can generate perils of debt non-sustainability. In order to avoid this, we employ a type of Bohn term (Bohn, 1998) that prevents debt instability and generates debt sustainability.

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4.2 Derivation of debt sustainability

In the basic model above we have added to the equ. (3) the term $-\tau x_3^2$ that represents the firm's behavior when it is threatened by debt non-sustainability. This is a type of Bohn term [26] and generates a mean-reversion of the debt. For private firms with a chosen investment plan, our model defines that the debt increase changes the firm financial strategy towards a debt control strategy at "refinancing points" (Strebulaev, 2007). Also, as advocated by the financial hierarchy theory, financial needs change as the business grows: firms switch to more costly sources, from internal to external finance, using first debt and then equity (Semmler, 2011). On the other side, this strategy may lead firms to turn fixed assets into liquidity, repay part of the debt and issue equity instead, which decreases firms' leverage. In the following, we derive that the debt term $-\tau x_3^2$ matters for our debt dynamics in the sense that it stabilizes the evolution of the debt.

For the mathematical proof that the term $-\tau x_3^2$ is relevant for the debt stabilization we rewrite our dynamic system (1)-(3) in a compact form. We define

$$V(x_o, u(.)| := \int_0^\infty e^{-rt} g(x_2(t), x_3(t), u(t)) dt \tag{4}$$

and

$$V^*(x_o) := \max_{u(.)} V(x_o, u(.)) \tag{5}$$

s.t.

$$\dot{x_1}(t) = k - a x_1(t) x_2(t)^2 + b x_2(t) - e x_1(t)/\mu(x_2(t), x_3(t), u(t)) - \beta \tag{6}$$

$$\dot{x_2}(t) = x_2(t)(a x_1(t) x_2(t)^2 + v g(x_2(t), x_3(t), u(t)) - \beta) \tag{7}$$

$$\dot{x_3}(t) = r x_3(t) - g(x_2(t), x_3(t), u(t)) - \tau x_3^2 \tag{8}$$

with $u(t) \geq 0$, and

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[26] Bohn (1998) adds this type of term to include the effect of a change of government behavior due to the debt increase. There is a positive response of the primary surplus to changes in debt level.

$$x_3(t) \geq 0, t \geq 0 \tag{9}$$

and $x(0) = x_0$.

Furthermore, $g(x_2, x_3, u) = \mu(x_2, u)x_2 u - cu - c_0 x_2 - rx_3$; $\mu(x_2, u) = \alpha/(\phi + x_2 u)$, and $x := (x_1, x_2, x_3)'$.

It is possible that the state constraints can become active in our model. Such an issue is discussed as a continuation of solutions where the equ. (9) is active, see Bonnans & Hermant (2008), Bonnans & Hermant (2009) and Bonnans & Shapiro (2000). Considering the specification of the model with $\tau = 0$ and without the pure state constraint, equ. (9), the objective value either becomes $-\infty$ or $+\infty$, as is shown in the following. Yet, note that a positive τ (a Bohn term) stabilizes the evolution of the debt. The objective function (4) can be rewritten as follows, using equ. (8):

$$V(x_o, u(.))| := \int_0^\infty e^{-rt} g(x_2(t), x_3(t), u(t)) dt \tag{10}$$

$$= \int_0^\infty e^{-rt} (rx_3(t) - \dot{x_3}(t)) dt \tag{11}$$

$$= e^{-rt} x_3(t)|_0^\infty + \int_0^\infty e^{-rt} \dot{x_3}(t) dt - \int_0^\infty e^{-rt} \dot{x_3}(t) dt \tag{12}$$

$$= x_3(0) - \lim_{t \to \infty} e^{-rt} x_3(t) \tag{13}$$

Next, we show that $x_2(.)$ and $u(.)$ remain bounded. Therefore we note that

$$\lim_{u, x_2 \to \infty} (x_2 u \alpha)/(\phi + x_2 u) = \alpha \tag{14}$$

yielding for $x_3 \in R$. Note that we have:

$$\lim_{u, x_2 \to \infty} g(x_2, x_3, u) = \lim_{u, x_2 \to \infty} (\alpha - c_u - c_0 x_2 - rx_3) = -\infty. \tag{15}$$

Moreover, for x_2 or u bounded the expression $\mu(x_2, u)$ results in:

$$\lim_{t \to \infty} (x_2 u \alpha)/(\phi + x_2 u) = \alpha \tag{16}$$

Hence equ. (8) yields

$$\lim_{t \to \infty} \dot{x}_3(t) \geq 2r x_3(t) \tag{17}$$

and therefore

$$\lim_{t \to \infty} x_3(t) \geq \epsilon * \lim_{t \to \infty} e^{2rt} \tag{18}$$

Plugging this into equ. (13) we find

$$x_3(0) - \lim_{t \to \infty} e^{-rt} x_3(t) \leq x_3(0) - \epsilon * \lim_{t \to \infty} e^{rt} \tag{19}$$

Depending on the sign of ϵ we either find for equ. (5) that $V^*(x_o) = -\infty$ for $\epsilon > 0$ and $V^*(x_o) = \infty$ for $\epsilon < 0$. Thus if we add the term $\tau > 0$ this would generate mean reversion and the system stabilizes.

5 Economic effects of de-risking green investments – Numerical results

Next, we undertake numerical explorations of dynamic variants where we assume the term $\tau > 0$ and thus presume the system stabilizes. Yet, before we get to the numerical results let us discuss what interest rates we will use. As demonstrated, interest rates can be de-risked. We thus will explore the effects of low (de-risked) and high (not de-risked) interest rates. We also will allow for endogenously generated interest rates depending on the level of debt. We thus will discuss two versions of the model: one in where the variable r is fixed and an alternative version in where $r = f(x_3)$ and is variable, depending on the level of x_3. In the latter case, we represent r as a logistic function of x_3 :

$$r = 0.04 + (0.3 - 0.04)/(1 + e^{-10(x_3 - 0.3)}) \tag{20}$$

In equ. (20), $r = 0.04$ is the lower bound, $r = 0.3$ is the upper bound and $x_3 = 0.3$ is the debt turning point in which r increases faster. This logistic function is shown in the Figure 6.
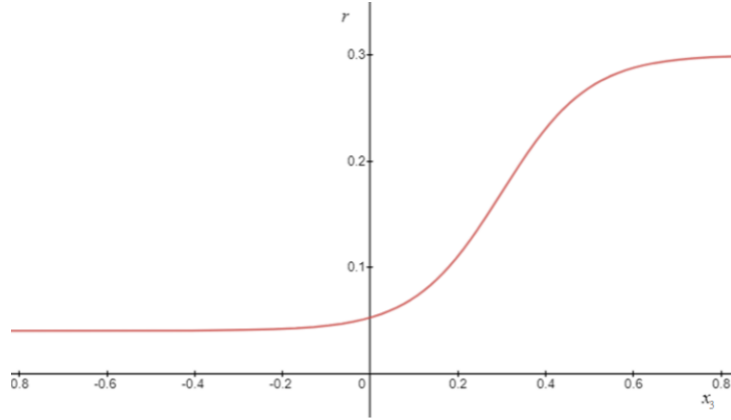
Figure 6: Variable r from a logistic function

Considering two types of behavior of the interest rate, we verify the impact of a de-risking strategy based on the cases of fixed and varying interest rates when firms have low or high mark-ups.

5.1 Case 1: Fixed low interest rate versus fixed high interest rate

We solve the maximization problem using NMPC for a deterministic case (see Appendix A). For one case, we have a low fixed de-risked interest rate and, for the other case, the investors face a very high interest rate. For the former, we follow the Government and Multilateral long-term green bonds average current yield ($r = 0.02$).[27] For the latter, we follow the lending interest rate data available for non-European countries (Figure B.2, Appendix B) in which we find that in 25% of these countries the borrowing cost is between 0.14 and 0.6, being several of them concentrated around 0.2. Therefore, we use $r = 0.2$. We run the model for an initial number of incumbent firms ($x_1(0)$) equal to 5 [28]and

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[27]Regarding the inflation rate: we neglect the inflation rate as a driver for the real interest rate, since most countries are in a low inflationary environment. For the use of a real interest rate to drive the real debt dynamics, see Ernst et al.(2017).

[28]For a reference to set the number of incumbent firms, we consider, in the European OECD countries, the share of renewable energies in the total capacity of electricity generation (excluding hydro-energy). For 2017, it was around 16% of the total capacity. We also assume that the incumbents have a multiple of the productive capacity of the entrants. See: https://data.oecd.org/energy/renewable-energy.html .

an initial number of innovator firms ($x_2(0)$) equal to 1, for distinct levels of initial debt (High or Low) and for different mark-up levels (High or Low).

For the case of a high mark-up (Figure 7), the model shows that de-risking the interest rate stabilizes the debt level at lower levels (closer to zero) while it keeps the number of innovator firms high. In a non de-risked scenario with a high initial required debt, the debt sharply increases and stabilizes at 2 while the number of entrant firms (x_2) increases at the initial periods but shrinks right after. –This movement is accompanied by a sharp decrease in the number of incumbent firms (x_1). In a de-risked scenario, it is always the case that x_2 increases and x_1 decreases while the system stabilizes at a lower debt level (close to zero). The lower interest rate allows the system to stabilize with a lower debt level at the same time it keeps the number of innovator firms high.
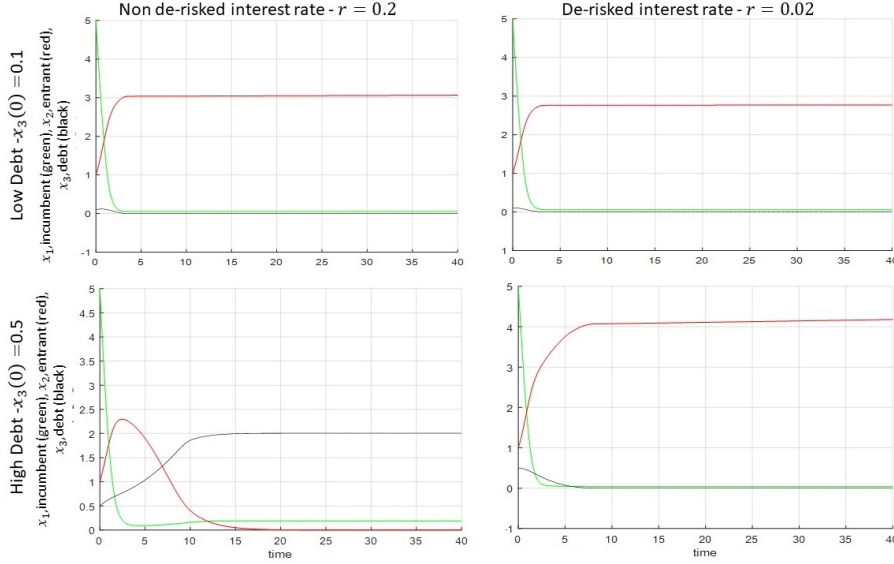

Figure 7: Non-derisked versus de-risked interest rate with a high mark-up in a high and low debt scenario ($x_1(0) = 5$, $x_2(0) = 1$)

When firms operate with a low mark-up, the role of a de-risked interest rate is also relevant (Figure 8). However, the low interest rate as such may not be enough to keep the number of innovating firms (entrants) high as they may carry negative pay-offs for a longer time: in all the simulations, x_1 decreases rapidly and x_2 increases initially but sharply decreases later. This movement is faster if the amount of the debt is higher at the initial period or if the interest

rate is not de-risked. However, the low credit cost avoids the debt increase that is observed in the non de-risked scenario.

If the interest rate is high, x_3 accelerates and reaches a value equal to 2. This movement is also faster when the debt is high at the initial stage. If the interest rate is de-risked, x_3 increases to a level lower than 1 and greater than 0.5 but decreases when x_2 reaches levels lower than 1. Therefore, in case of a long period of negative pay-offs, de-risking of the interest rates decreases the likelihood of debt explosion but is not sufficient to keep the number of innovator firms high. There is a shake-out of the number of renewable energy firms.
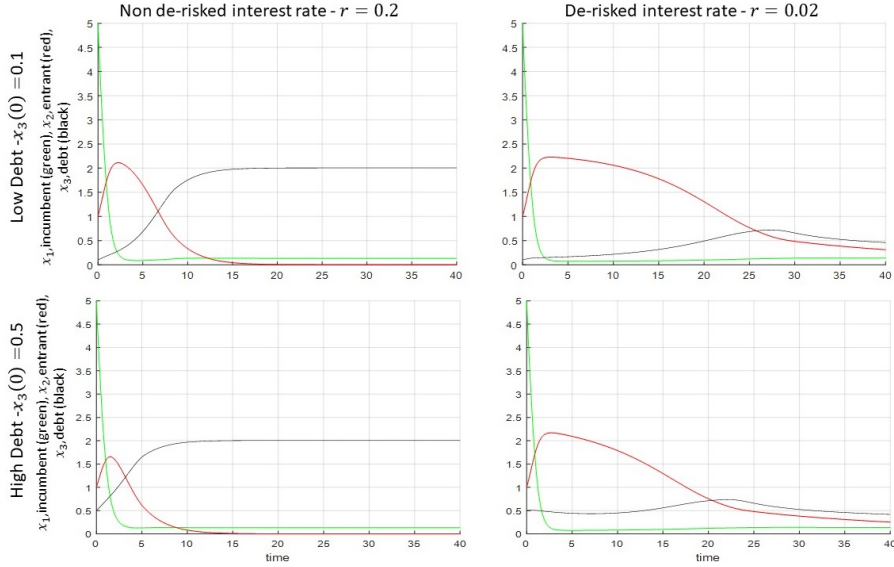
Figure 8: Non-derisked versus de-risked interest rate with a low mark-up in a high and low debt scenario ($x_1(0) = 5$, $x_2(0) = 1$)

5.2   Case 2: Fixed interest rate versus variable interest rate

Next, in Case 2, we test the effect of de-risking investment through a fixed interest rate versus a variable interest rate, given by a logistic function such as depicted in Figure 6. We can compare the outcome of this new simulation with those relying on a fixed interest rate (Case 1, Figure 7 and 8). We simulate the

model for the case in which $x_1(0) = 5$, $x_2(0) = 1$, the debt is high or low and the mark-up is high or low. The results are shown in Figure 9. Analyzing the impact of the new interest rate on the debt, we observe that the debt increases and reaches $x_3 = 3$ in almost all the simulations - except when we have a high mark-up and a low debt. The debt is more likely to increase and the system reaches higher debt levels when the interest rate is given by a logistic function. In Case 1, the debt does not sharply increase when the interest rate is de-risked. When it is not de-risked, the debt increases to $x_3 = 2$ in a low mark-up scenario and to $x_3 = 3$ in a high mark-up scenario with high debt. This different result has to do with the fact that the new interest rate moves together with the debt level, as the risk-premium increases.

Furthermore, the number of innovators (x_2) and of incumbents (x_1) decrease in almost all the simulations shown in Figure 9. In Case 1, it happens only when the mark-up is low. The interest rate movement also impacts negatively the pay-off function via the financial cost increase. At a certain moment of time, when x_2 increases, the debt growth damages the innovators' profits and the new firms leave the market. The outcomes obtained in the Figure 9 show that a fixed interest rate (in contrast to a variable interest rate) can guarantee the existence of innovator firms in the market and avoid a sharp debt increase. On the other hand, the variable interest rate movements increase strongly the debt which impacts the pay-off function of the renewable energy firms and may induce them to leave the market.
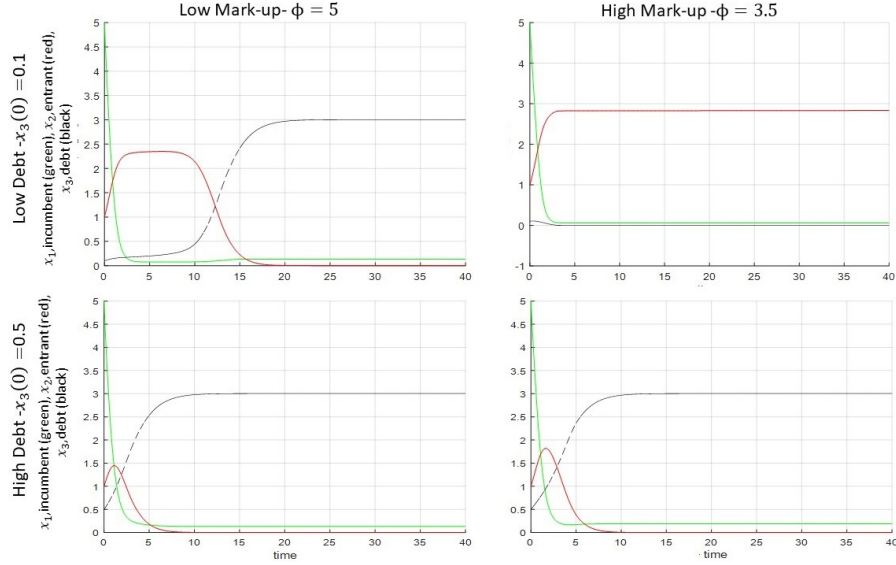
Figure 9: Variable interest rate with a high and low mark-up in a high and low debt scenario $(x_1(0) = 5, x_2(0) = 1)$

5.3 Case 3: De-risked bonds and green fiscal reform

Several countries have been using fiscal incentives in order to disincentivize carbon intensive activities (through carbon pricing) and incentivize green energy (subsidies for investments or current expenses), as discussed in Section 2. New green investments can be fostered by decreasing the interest rate paid on the debt (e.g., de-risked green bonds) but also by reducing the future operating cost (e.g., subsidies to decrease operational cost). Although carbon pricing can induce low-carbon transition, high capital and upfront costs demand the combination of green bonds and carbon taxation as de-risking instruments (Steckel & Jakob, 2018; Heine et al., 2019), since an increasing scale is likely to lead to decreasing cost, see Figure 1. We adapt the model to verify the effect of de-risking bonds (or not) in an economy in which the Government taxes the carbon industry and provides subsidies for renewable energy activities.[29]

In order to do this, we change the equ. (1) and the pay-off function of

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[29]For a similar approach see Acemoglu et al. (2012), where fossil fuel firms that are generating negative externalities are taxed and non-polluting firms are subsidized.

the model, decreasing the mark-up for the incumbent and increasing the pay-off for the entrants, with the new parameter ρ, which is the carbon taxation, equal to the subsidies for green investments. The new equations are bellow:

$$g(x_2, x_3, u) = \mu(x_2, u)x_2 u - (1 - \rho)(cu + c_0 x_2) - rx_3 \qquad (21)$$

$$\dot{x}_1 = k - ax_1 x_2^2 + bx_2 - x_1 e/(1 - \rho)\mu \qquad (22)$$

For the adapted model, we set $\rho = 0.09$. This level is based on the US current subsidies to renewable energy activities, published by Lazard (2018)[30]. Also, we simulate two scenarios, one in which we have a low fixed de-risked interest rate and the other in which the innovators face a very high fixed interest rate. We run the model for a low mark-up when, at the initial stage, debt is high or low and $x_1(0) = 5$ and $x_2(0) = 1$. We find a different outcome from the last section for the case in which the interest rate is de-risked. In this case, the subsidies enhance the outcomes obtained by a de-risked interest rate, avoiding a decrease in the number of innovators and keeping the debt level around zero.

The Figure 10 shows the model simulations under a low mark-up scenario. When the interest rate is not de-risked, we obtain outcomes similar with the results shown in the Figure 8. When the interest rate is de-risked, the debt remains around zero and the number of entrant firms remains greater than 2 when the debt is low or high. Therefore, the number of renewable energy firms does not decrease at a certain point in time and the debt remains on a stable path, at a very low level. Therefore, as the pay-offs do not remain negative for a long time, the x_2 path changes and the subsidies avoid a shake-out in the market.

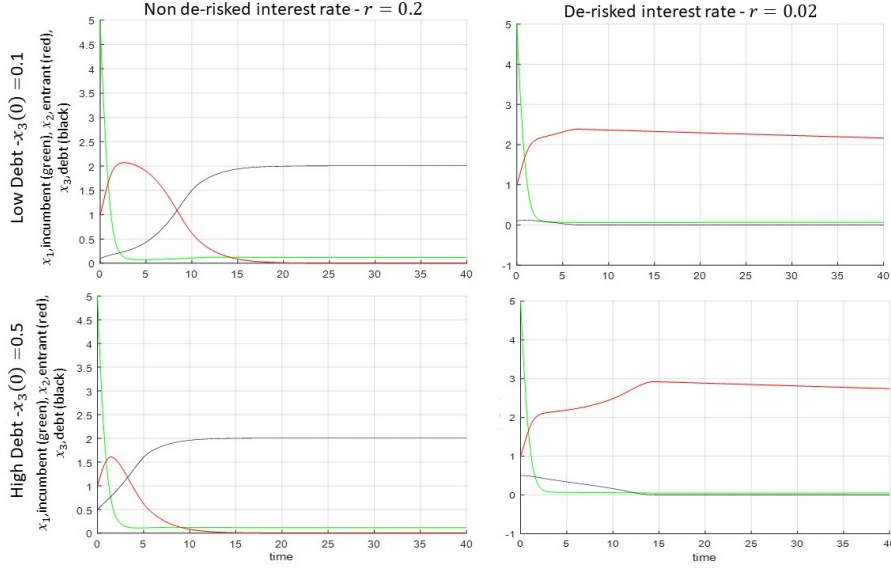[30]For solar energy, the subsidies are up to 9% of the operational cost.

Figure 10: Non-derisked versus de-risked interest rate with a low mark-up in a high and low debt scenario with subsidies ($x_1(0) = 5$, $x_2(0) = 1$)

Overall, as all of our cases show, the debt dynamics is always stabilized and, in that sense, is sustainable, due to the term $-\tau x_3^2$. Yet, in some cases the stabilized path of debt may exceed the acceptable level of debt for the creditors and may trigger unpleasant responses from the creditors. At what threshold this will occur is more of an empirical and institutional issue not treated in this paper.[31]

5.4 Outlook for a stochastic version

In the previous simulations, we solve the maximization problem using NMPC for a deterministic case only. Nevertheless, the literature shows that market risks - the risk of losing market share - also matters for the success of new technologies. To address those risks, we introduce a stochastic version of the model using a NMPC algorithm for a stochastic case (Appendix A). This is done by a simplified version of the model, without debt dynamics but including a new state equation that generates shocks that allow us to simulate the market

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[31]For an extensive discussion on this issue, see Semmler (2011), chapter 20.

success or failure of the innovator firms. We get, however, similar results as for the deterministic case, in the previous sections.

The multi-period pay-off function of the innovators in discrete time form, subject to constraints, looks like the following[32]:

$$E(\max_{u\in} \sum_{t=0}^{N-1} \delta^t g(x_2(t), u(t))) \tag{23}$$

s.t.

$$x_1(t+1) = x_1(t) + 0.01(-ax_1(t)x_2^2(t) + bx_2(t) - x_1(t)e/\mu) \tag{24}$$

$$x_2(t+1) = x_2(t) + 0.01x_2(t)(ax_1(t)x_2(t) + v(g(x_2(t), u(t)) - \beta)) \tag{25}$$
$$+ \psi + \delta log(x_3(t))x_2(t)$$

$$x_3(t+1) = e^{\bar{\rho}log(x_3(t))+\sigma z} \tag{26}$$

We include a new objective function of the innovating firms with the discount factor δ and with a pay-off function without the debt equation: $g(x_2(t), u(t)) = \mu(x_2(t), u(t))x_2(t)u(t) - cu(t) - c_0x_2(t)$. Therefore, we have a new variable x_3, now representing the exogenous shocks given by z (an i.i.d. random variable), amplified by σ (the standard deviation) and depending on $\bar{\rho}$ (the persistent parameter for shocks)[33] . Those shocks impact the performance of the innovator firm, thus impacting the dynamics of $x_2(t+1)$ through the term $\psi + \delta log(x_3(t))x_2(t)$.

We solve this new model for the variant case with a low mark-up (Figure 11). We observe that the model behavior is similar to the case of the deterministic model for a lower mark-up and a de-risked interest rate (Figure 8), in which the number of fossil fuel firms (x_1) decreases rapidly and the number of renewable energy firms (x_2) increases initially but decreases later. However, we should note that the stochastic model is based on a discrete-time system and is

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[32]For the detailed numerical procedure to solve our model variants see Appendix A.

[33]For this new version, improvements also in $x_1(t+1)$ and $x_2(t+1)$ were implemented to guarantee the model stability. Due to these improvements, we have the following new parameters: $\delta = 0.95$, $\bar{\rho} = 0.9$, $\sigma = 0.5$, $\psi = 0.05$, and $\delta = 0.05$. Furthermore, z is an i.i.d. random variable.

solved with small steps which demands more iterations to reach a similar out-
come. Nevertheless, the use of the deterministic case, as shown in Figure 8, is
a good proxy for the market dynamics of the stochastic case. In the stochastic
case, there are additive market shocks which can generate multiple paths for the
evolution of the innovator firms. Yet the overall direction of outcomes shows
close similarity to the deterministic case. We can compare Figure 8 and Figure
11 and see how similar the stochastic case with no debt is to the deterministic
case with a de-risked interested rate, in which r is closer to zero.
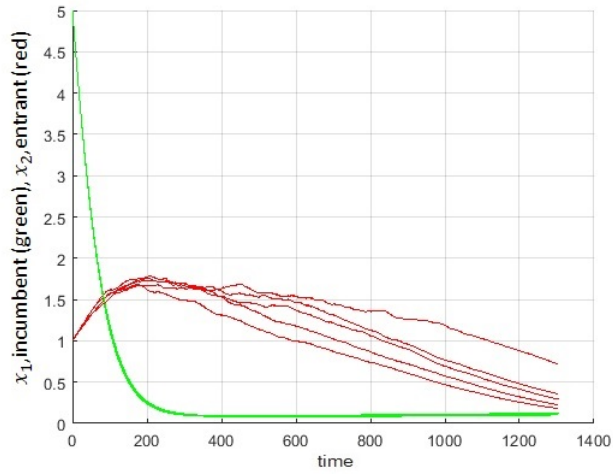

Figure 11: Stochastic version of the model with a low mark-up and $\sigma = 0.5$
$(x_1(0) = 5, x_2(0) = 1)$

6 Conclusions

The dynamics of the credit market is key for the investment behavior, but agents
risk evaluation can increase costs and limit borrowing and investment. These
constraints tend to be higher in developing countries, smaller firms and for
projects with higher uncertainty, such as environmental projects. In the latter
case, the role of Governments in the financial market is relevant in risk-bearing
investments and attracting institutional investors to climate resilient securities.
We verify that Governments and Multilateral organizations can act de-risking
green bonds and support the transition to a low carbon economy.

We find that Governments and Multilateral bonds have lower volatility
and higher yields, especially for long maturities, which keeps the return-risk

ratio higher for Private bonds. In particular, long-term Private bonds exhibit higher yields than long-term Government and Multilateral bonds. Sustainable infrastructure projects are known for being long-term. We later add to these findings empirical evidence that green bonds are a good hedging instrument against assets exposed to oil price fluctuations - as conventional energy bonds are, see Appendix C. It reinforces the green bond's role as a de-risking tool.

We also find that borrowers pay significantly higher interest rates in developing countries, depending on the country risk classification. Therefore, our results add to the evidence that Governments and Multilateral agents can act de-risking projects by issuing green bonds to support projects that otherwise would be left aside. On the investors' side, it brings with it a lower return-risk investment opportunity and reduces the climate and financial instability risks due to climate change. It opens some space for a green QE that can de-risk assets exposed to climate risks. Yet, we should note that whereas Multilateral agents do not seem to depend so much on a country's risk classification, particular countries do.

As concerning capital costs for investments, we find that the mean for the beta price is higher for Private bonds and lower for Multilateral and Governments bonds. However, analyzing the beta distribution, we observe that, in some countries, long-term Government bonds can be riskier than Private bonds. Indeed, we find that the beta price, the current yields and the WACC vary between countries. Countries with lower sovereign risk have lower capacity of de-risking green investments. On the other hand, the maturity does not influence the beta price for Multilateral bonds. Thus, these organizations can de-risk green investments for middle and low-income countries that have capital markets and sovereign debt constraints.

Based on the above considerations, we run a dynamic model with two types of firms - innovating firms (renewable energy firms) and incumbents firms (fossil fuel firms) - and verify the impact of a de-risking strategy with a low fixed de-risked interest rate versus a variable or higher fixed interest rate. For fixed interest rates, the de-risking strategy stabilizes the debt level and keeps renewable energy firms in the market. However, when the innovating firms exhibit negative pay-offs for a longer time, de-risking the interest rate may not be sufficient to keep the number of innovators high and avoid a shake-out in the market. For a variable interest rate, the dynamics is more likely to reach high debt levels due to the increase in the interest rate with the debt evolution. The debt increase also impacts the pay-off function and the number of innovating

firms sharply decreases when the debt reaches a certain level. Therefore, a de-risked and fixed interest rate increases the likelihood of reaching a sustainable debt scenario (with debt control and positive number of new innovating firms) in case of negative pay-offs. Though, in all cases the debt level is stabilized at some level, yet in some cases the level might not be sufficient for creditors.

Furthermore, given that several countries have provided fiscal incentives in order to foster the transformation of the energy sector, we adapt the model to verify the effect of de-risking bonds in an economy in which the Government taxes the carbon industry and provides subsidies for renewable energy activities. We find that subsidies combined with a de-risked interest rate can avoid a shakeout of innovating firms and keep the debt at lower levels when firms have low mark-ups. An active fiscal policy complement the benefits of a de-risked interest rate as the pay-offs do not remain negative for a long time.

Finally, we should note that the deterministic case does not take account of market risks. We run a stochastic version, in section 5.4, to address this issue and find a result similar to the deterministic case with a very low interest rate. Furthermore, we have treated the incumbents as passively behaving firms. We have left out their pay-offs and debt dynamics since this would have made our model overly complex. It can be considered in a future extension of the paper.

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Appendix A: Deterministic and stochastic numerics

Deterministic case

For the numerical solution of the deterministic model presented in section 4 and the numerical results in section 5 we do not apply here the dynamic programming (DP) approach as presented in Grüne & Semmler (2004). DP faces the problem of the curse of dimension. We here use a procedure that is easier to implement. We are using what is called nonlinear model predictive control (NMPC) as proposed in Grüne & Pannek (2012) and Grüne et al. (2015). NMPC only computes single (approximate) optimal trajectories at a time. To describe the NMPC procedure for the deterministic case we can write the optimal decision problem as:

$$\text{maximize} \quad \int_0^\infty e^{-\rho t} \ell(x(t), u(t)) dt, \tag{A.1}$$

where $x(t)$ satisfies

$$\dot{x}(t) = g(x(t), u(t)), \quad x(0) = x_0 \tag{A.2}$$

By discretizing this problem in time, we obtain an approximate discrete time problem of the form

$$\text{maximize} \quad \sum_{i=0}^\infty \delta^i \ell(x_i, u_i), \tag{A.3}$$

where the maximization is now performed over a sequence u_i of control values and the sequence x_i that satisfies $x_{i+1} = \Phi(h, x_i, u_i)$. Hereby $h > 0$ is the discretization time step.

The procedure of NMPC consists in replacing the maximization of the infinite horizon functional (A.3) by the iterative maximization of finite horizon functionals

$$\text{maximize} \quad \sum_{k=0}^N \delta^i \ell(x_{k,i}, u_{k,i}), \tag{A.4}$$

for a truncated finite horizon $N \in N$ with $x_{k+1,i} = \Phi(h, x_{k,i}, u_{k,i})$. Hereby the index i indicates the number of iterations. Note that neither δ^i nor ℓ nor Φ

changes when passing from (A.3) to (A.4). The procedure works by moving ahead with a receding horizon.

The decision problem (A.4) is solved numerically by converting it into a static nonlinear program and solving it by efficient NLP solvers, see Grüne & Pannek (2012). In our simulations, we have used a modification of NMPC, as developed by Grüne & Pannek (2012), in their routine `nmpc.m,` available from `www.nmpc-book.com`, which uses MATLAB's `fmincon` NLP solver in order to solve the static optimization problem. Our modification employs a discounted variant of the NMPC MATLAB version, see Grüne et al. (2015).

Given an initial value x_0, an approximate solution of the system (A.1)-(A.2) can be obtained by iteratively solving (A.4) such that for i=1,2,3, that solves for the initial value $x_{0,i} := x_i$ the resulting optimal control sequence by $u_{k,i}^*$, but uses only the first control $u_i := u_{0,i}^*$ and iterates forward the dynamics $x_{i+1} := \Phi(h, x_i, u_i)$ by employing only the first control. Thus, the algorithm yields a trajectory x_i, $i = 1, 2, 3, \ldots$ whose control sequence u_i consists of all the first elements $u_{0,i}^*$ of the optimal control sequences of the finite horizon problem (A.4). Under appropriate assumptions on the problem, it can be shown that the solution (x_i, u_i),which depends on the choice of N in (A.4), converges to the optimal solution of (A.3) as $N \to \infty$, see Grüne et al. (2015). A demonstration that this "turnpike property" holds is shown in Grüne et al (2015). It operates as a receding horizon problem, such that for example in step $i = 1$ with the decision horizon $N = 4$, it is iterated forward 6 times, thus we have $i = 1....6$. The solution is then the outer envelop of the piecewise solutions using the horizon $N = 4$ multiple times, in our case 6 times.

The main requirement in these assumptions is the existence of an optimal equilibrium for the infinite horizon problem (A.1)-(A.2). If this equilibrium is known, it can be used as an additional constraint in (A.4), in order to improve the convergence properties. In our solution of the model in section 5, we did not use the terminal condition to solve the model but moved forward with a receding horizon to find the (approximate optimal) trajectories. Thus, without a priori knowledge of this equilibrium this convergence can also be ensured.

Stochastic case

The stochastic case is illustrated here by a simple problem. We show here how this works for a discrete time pay-off function and discrete time one state

variable dynamics, for details see Grüne et al. (2015). The NMPC is applied to a stochastic problem using the certainty equivalence principle. Since there is not much work of a NMPC type algorithm for this problem we only sketch here a currently available NMPC algorithm. Due to the fact that the control generated by the NMPC algorithm is in feedback form, the basic concept can be extended to a stochastic problem as follows

$$V_\infty(x_0) = E\left(\max_{u\in} \sum_{k=0}^{\infty} \delta^k g(x(k), u(k))\right) \tag{A.5}$$

with the discrete time stochastic dynamics

$$x(k+1) = \varphi(x(k), u(k), z_k), \quad x(0) = x_0, \tag{A.6}$$

where the z_k is an i.i.d. random variable. This problem could a priori be given in discrete time or it could be given as the time discretization of a continuous time stochastic optimal control problem as in the above deterministic case of section 4.

From a computational point of view, the main difficulty in stochastic NMPC is the efficient solution of the corresponding finite horizon problem (A.4) which now becomes a stochastic optimal control problem whose solution is computationally considerably more expensive than in the deterministic case. While some NMPC approaches in the literature indeed solve stochastic optimal control problems, here we follow the simpler certainty equivalence approach, see Grüne et al. (2015) which does in general not compute the true stochastic optimum but in the case of stochastic perturbations with low intensities may still yield approximately optimal results. In this procedure then, we replace the stochastic dynamics by its expected counterpart

$$x^e(k+1) = E\Big(\varphi(x^e(k), u(k), z_k)\Big), \quad x^e(0) = x_0 \tag{A.7}$$

and in each iteration we solve

$$\max_{u\in} \sum_{k=0}^{N-1} \delta^k g(x^e(k), u(k)). \tag{A.8}$$

Note that we only use (A.7) in order to solve (A.8) in step (1) of the NMPC algorithm. In step (2) we simulate the closed loop solution using the original

stochastic dynamics (A.7) with z_k realized by appropriate random numbers.

We illustrate the performance of this approach by a two dimensional stochastic version of the stochastic growth model. We extend a discrete time one dimensional dynamics using a second variable modeling a stochastic shock. The model is given by two discrete time equations

$$x_1(k+1) = x_2(k)Ax_1(k)^\alpha - u(k) \tag{A.9}$$
$$x_2(k+1) = \exp(\bar\rho \ln x_2(k) + z_k) \tag{A.10}$$

where A, α and $\bar\rho$ are real constants and the z_k is an i.i.d. random variable with zero mean. The pay-off function in this simple case of a stochastic growth model can be $g(x,u) = \ln u$.

In a numerical computation in Grüne et al (2015) the following parameters were used: $A = 5$, $\alpha = 0.34$, $\bar\rho = 0.9$ and $\delta = 0.95$ and z_k is an i.i.d. Gaussian random variables with zero mean and variance $\sigma^2 = 0.008^2$. Using that $E(\exp(a + z_k)) = \exp(a + \sigma^2/2)$, the model used for the open loop optimization is then given by

$$x_1^e(k+1) = E(x_2^e(k)Ax_1^e(k)^\alpha - u(k)) = x_2^e(k)Ax_1^e(k)^\alpha - u(k) \tag{A.11}$$
$$x_2^e(k+1) = E(\exp(\rho \ln x_2^e(k) + z_k)) = \exp(\rho \ln x_2^e(k) + \sigma^2/2). \tag{A.12}$$

The optimally controlled dynamics is given by $x_1(k+1) = \alpha\beta Ax_2(k)x_1(k)^\alpha$.

The above setup is then extended to solve our stochastic case of section 5.4, however leaving aside the debt dynamics. We include a pay-off function that is more complex then in the above stochastic growth model, and two state variables, the dynamics of the innovative and incumbent firms. We also add a third state variable, the stochastic process, as (A.10) in the optimal control problem of the stochastic growth model, and (A.11) and (A.12) in the simulation of it using a random number generator.
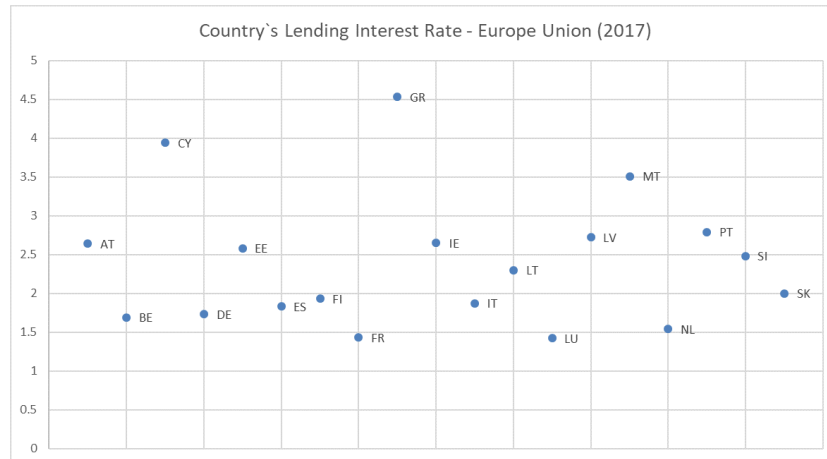
Appendix B: Data Presentation

Figure B.1: European countries' lending interest rates, Source: Eurosystem - European Central Bank. Available at https://sdw.ecb.europa.eu/home.do
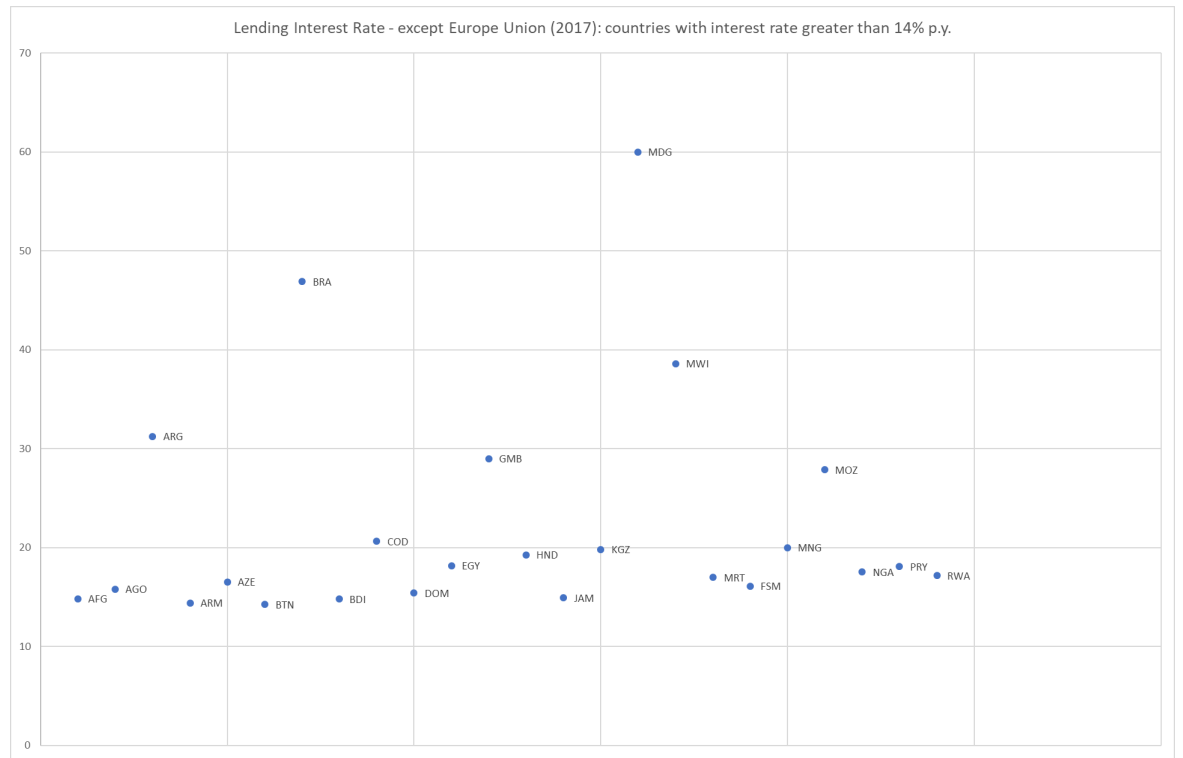
Figure B.2: Non-European countries' lending interest rates - greater than 14%,
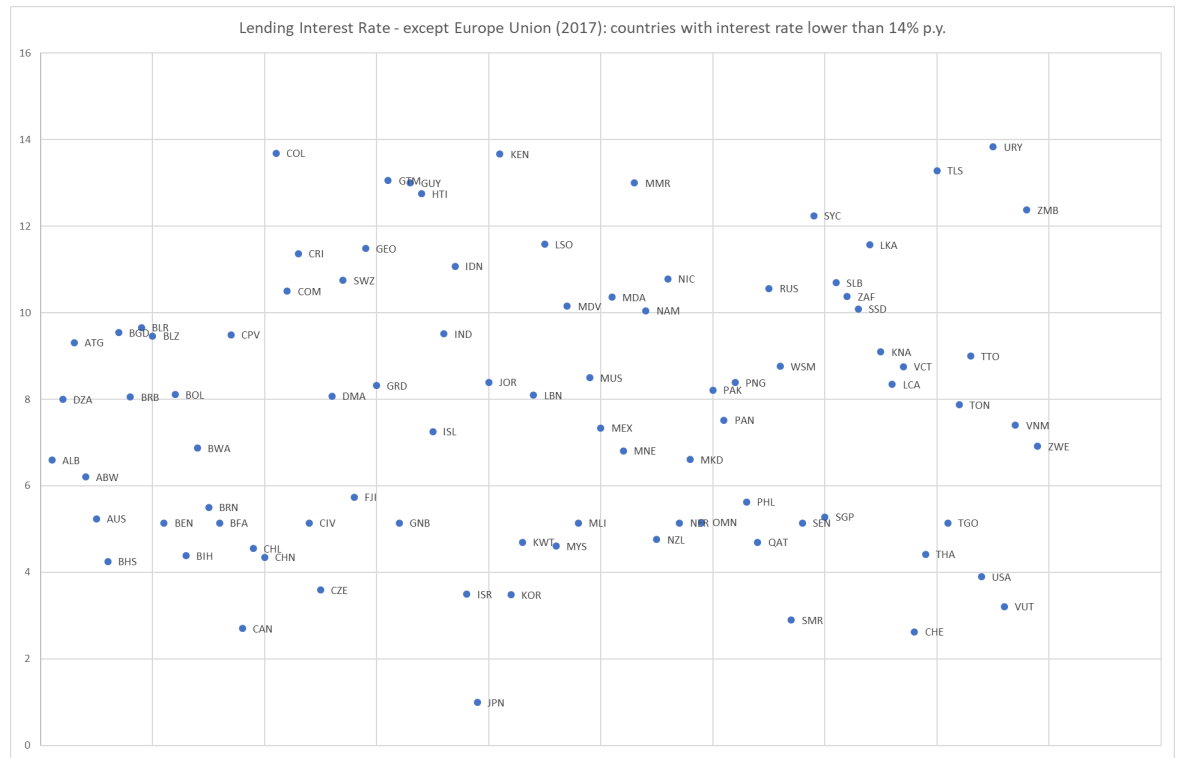Source: World Bank. Available at https://data.worldbank.org/

Figure B.3: Non-European countries' lending interest rates - lower than 14%,
Source: World Bank. Available at https://data.worldbank.org/

Country	Green bonds		WACC	
	Average beta	Current yield	Total	Sample
High Income	0.21	1.51	7.31	6.49
Sweden	0.01	0.97		4.90
Germany	0.11	1.34		9.20
France	0.33	1.17		6.30
Upper Middle-Income (without China)	0.23	7.93	13.70	14.25
China	0.04	4.03		8.30
Costa Rica	0.09	5.83		14.60
Argentina	0.52	12.83		13.90
Lower Middle-Income	0.30	5.83	14.17	13.23
India	0.32	5.25		11.60
Nigeria	-0.10	13.80		13.80
Low Income	N.A.	N.A	16.57	NA
Ethiopia	N.A.	N.A.		11.00
Madagascar	N.A.	N.A.		32.00

Table B.1: Green bonds: average beta, average current yield and WACC per country

Notes: (1) The highlights were chosen based on the lower and highest value; (2) WACC estimated by Ondraczek et al. (2015). Total is based on the complete list of 143 countries, eliminating four who are not classified by the World Bank by income. Sample is the average WACC for the 22 countries, in which the WACC and beta were calculated; (3) In the lower middle-income group, the beta was obtained only for three countries: India, Nigeria and Indonesia. In the upper middle-income, for 5 countries: Argentina, Brazil, Costa Rica, Mexico, South Africa. In the high-income group, for 17 countries.

African Development Bank
Asian Development Bank
Central American Bank for Economic Integration
Corporacion Andina de Fomento
Eurofima
European Bank for Reconstruction & Development
European Investment Bank
International Bank for Reconstruction & Development
International Finance Corp
New Development Bank BRICS
The Nordic Investment Bank
North American Development Bank

Table B.2: List of Multilateral organizations issuing green bonds (2010-2018)

Appendix C: Harmonic estimations of bond returns and oil price changes

In order to assess the volatility of green bonds and other types of bonds due to energy price fluctuations, we compare market indices for different types of assets with oil price variations. The oil price changes are given by the European Brent oil spot price annual variation for every month from February/2011 to September/2019. For asset returns, we use, for the same period, the annual monthly total returns of the S&P Green Bonds Index (which contains renewable energy and others green assets) and the S&P 500 Energy Corporate Bond Index (a more comprehensive index that also includes carbon intensive energy assets).

Given that asset allocation decisions are based on low frequency movements in asset returns, we use the securities data to estimate low frequency movements in asset returns by using harmonic estimations (see Chiarella et al., 2016). We apply the Fast Fourier Transformation (FFT) method on the detrended real bond returns and oil price variation. We get empirical estimations based on linear regressions constructed with trigonometric functions: we fit each time series using a linear combination of sine and cosine functions, as shown in the equation C.1.

$$y(t) = \sum_{i=1}^{k} (a_i sin(\frac{2\pi}{\tau_i}(t - t_o)) + b_i cos(\frac{2\pi}{\tau_i}(t - t_o))) \tag{C.1}$$

We estimate the harmonic regression model for different values of k, from 1 to 6, which represents different frequencies - from low frequencies to high frequency data. For our analysis, we selected the estimation with the lower squared error term (shown in Figures C.1, C.2 and C.3). Although we observe similarities between the features of the three cyclical movements, yet the downturns and upturns of the S&P Green Bond Index returns are clearly less associated with oil price changes over time. Thus, green bonds appear to be better hedging instruments than oil price driven assets.
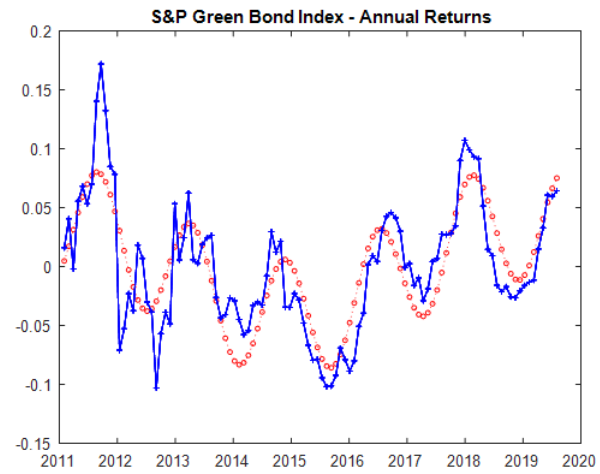
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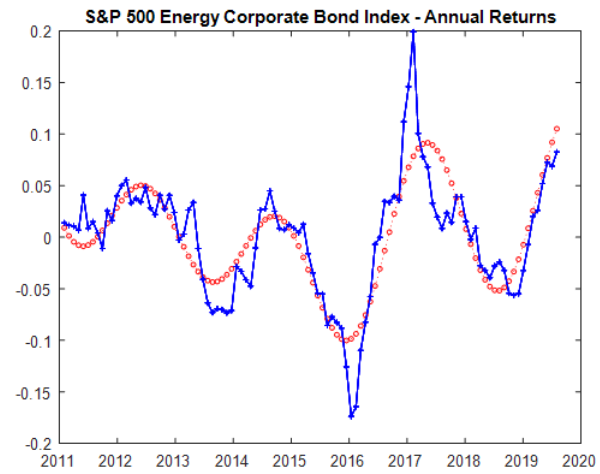Figure C.1: S&P Green Bond Index Annual Total Returns and Harmonic Estimation

Figure C.2: S&P 500 Energy Corporate Bond Index Total Returns and Harmonic Estimation
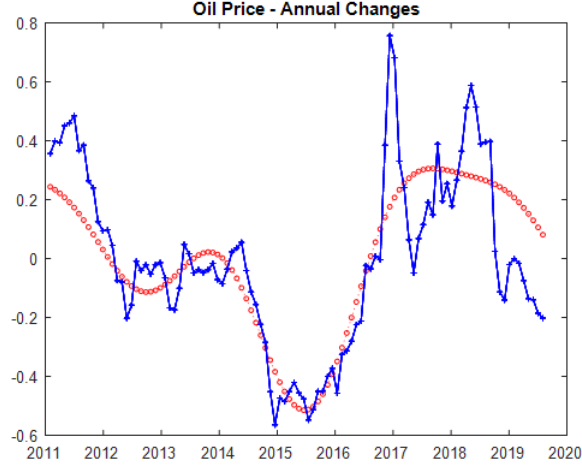
Figure C.3: Oil price Annual Changes and Harmonic Estimation

In order to verify this relationship, we run the following linear regressions using the harmonic estimation values. These regressions evaluate the influence of oil prices cycles in the returns to investors of green bonds and energy corporate bonds[34]:

$$GreenBonds = \alpha_0 + \alpha_1 \triangle OilPrices + Trend \qquad (C.2)$$

$$EnergyBonds = \beta_0 + \beta_1 \triangle OilPrices + Trend \qquad (C.3)$$

For the period from January/2012 to September/2019, the estimated coefficients for the equations C.2 and C.3 show that the fluctuations of oil prices have a greater impact in the energy bond returns as shown in Table C.1. The estimated coefficient for green bonds is $\alpha_1 = 0.06$ while the one for energy corporate bonds is $\beta_1 = 0.12$. Although the green bonds have been implemented in 2010, we start the regressions in 2012 as it took a while for market agents to adjust the issue of a new product, as shown in Figure 2 (Section 3.1).

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[34]We also add a simple trend model to the regression, having a time index t as a dependent variable, affected also by a random noise. It generates a predictor for the dependent variable for the next periods if a clear trend is observed.

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Dependent Variable	Coefficients	Estimate (Standard Error)
Green Bonds (C.2)	α_0	-0.03** (0.01)
	$\alpha 1$	0.06** (0.02)
	Trend	0.00* (0.00)
Energy Bonds (C.3)	β_0	0.02* (0.01)
	β_1	0.12*** (0.02)
	Trend	0.00* (0.00)

* p<0.05, ** p<0.01, *** p<0.001

Table C.1: Green bonds and Energy Bonds - estimated coecients for the regressions C.2 and C.3