MODELING REGIONAL WATER
SUPPLY: SILISTRA CASE STUDY

V. Chernyatin

July 1981
WP-81-93

PREFACE

Water resource systems have been an important part of re-
sources and environment related research at IIASA since its in-
ception. As demands for water increase relative to supply, the
intensity and efficiency of water resources management must be
developed further. This in turn requires an increase in the
degree of detail and sophistication of the analysis, including
economic, social and environmental evaluation of water resources
development alternatives aided by application of mathematical
modeling techniques, to generate inputs for planning, design,
and operational decisions.

In 1978 it was decided that parallel to the continuation
of demand studies, an attempt would be made to integrate the
results of our studies on water demands with water supply con-
siderations. This new task was named "Regional Water Management"
(Task 1, Resources and Environment Area).

One of the case studies in this Task, carried out in col-
laboration with several Bulgarian institutions and the Regional
Development Task of IIASA, is concerned with water resources
management in the Silistra region of Bulgaria. This paper on
modeling the water supply system in the Silistra region accom-
panies the earlier study on the water demands of agriculture in
the same region.

Murat Albegov   Janusz Kindler
Leader   Chairman
Regional Development Task   Resources & Environment Area

CONTENTS

MODELING REGIONAL WATER
SUPPLY: SILISTRA CASE STUDY

V. Chernyatin

1. INTRODUCTION

The IIASA's water resources research related to the Silistra
Case Study started in 1977 with the modeling of agricultural
water demands (Gouevsky and Maidment, 1977). In many respects,
the water demand model had the character of a general agricul-
tural model for the Silistra region. Later on, the model was
extended to take into account the subdivision of the region into
a number of districts with the various conditions of soil, crop
structure, water supply, etc. (Gouevsky, Maidment, and Sikorski,
RR-80-38, 1980). The crucial point at this stage of the study
was knowing how much water supply costs in total, and what the
shadow prices of water for the various districts would be. Un-
fortunately, it is impossible to answer these questions, even
roughly, without analysis of a regional water supply system.

The second stage of the Silistra water resources related
study, being the main subject of this paper, is a water supply
model. The major problem to be solved here is to determine the
least-cost variant of the water supply system and the shadow
prices of water distributed geographically. The latter very much
influences the intraregional structure and intensity of production.
In this respect the water supply model reported here can be con-
sidered as an essential part of the system of regional models

(Albegov and Chernyatin, 1978). What is especially important is that the water supply model should be interrelated with other regional models. In the Silistra Case, at the end of 1979, such a coordination was done for the agricultural water demand and supply models (Chernyatin and Gouevsky, forthcoming).

This paper sums up IIASA's work on modeling of the Silistra water supply system. The water supply model presented here was developed in close cooperation with the Sofia Institute for Water Projects which is responsible for designing water resources systems in Bulgaria. It must be stressed that the Silistra region is characterized by fairly simple hydrological conditions in the region. Namely, abundance of water in the Danube river-- the only source of water--allows one correctly to confine oneself to within-year regulation of water resources. This property essentially simplifies analysis of a water supply system. The developed optimization model determines basic parameters of the Silistra water supply system--capacities of reservoirs and pumping stations, and discharge capacities of canals.

Although intended primarily for the Silistra water supply system, the model actually had many properties of a general water supply model under conditions of within-year regulation of water resources. Afterwards the model was generalized to cover the whole set of irrigation systems of "Silistra type". In this general form, it is expected to be applied for the planning of many irrigation systems in the Danube lowland in Bulgaria. The first experiment in this field was crowned with success. Namely, the practical application of the modeling results lead to a considerable budget saving for the Silistra irrigation system.

Below, the purposes of modeling, the mathematical model of a regional water supply, and the results of its application for the Silistra region are described in detail.

2. SILISTRA WATER SUPPLY PROBLEMS AND PURPOSES OF MATHEMATICAL MODELING

Silistra is a region covering a 2700 km^2 area, with a population of 200,000 located in the North-Eastern part of Bulgaria. The soil quality and the number of days of sun per

year make this region favorable for intensive agricultural development under irrigation. Unfortunately, it has a pronounced shortage of its internal water resources. Since no other rivers exist in the region, the bordering Danube river is the only source of water for agricultural, domestic and industrial consumption. Groundwater is available, only in small quantities, at a depth exceeding 400 metres which makes it unprofitable for production use. Furthermore, the annual rainfall is rather moderate--500 mm in average--and distributed (somewhat unfavorably) within the year with respect to the growing season.

According to the long-term hydrological forecasts, there will be no deficit of water in the Danube river at least until the year of 2000. Because of the abundance of water in the Danube river, the question of how much water to withdraw for agricultural and industrial production and for municipal use is decided solely by the economics of water use. Suffice it to say, for example, that all the irrigated areas are located at a level varying from 100 m to more than 200 m higher than the Danube river level. This means that the conveyance of irrigation water is rather expensive.

The water supply system for the Silistra region is divided into two separate sub-systems--irrigation water supply and water supply for household and industrial consumption. The reason for making such a division is the essential difference in the level of water quality demanded by different types of water users. With regard to industry these are mainly food enterprises except for some other industrial activities in the city of Silistra. However, being situated along the Danube river they have their own water intakes which are small in comparison to the total regional water requirements. As it is known, the food industry requires that the water quality be of drinking-water standards, which, of course, are higher than that required for irrigation water.

In quantitative respect, about 10 to 15% of the total regional water requirements fall to the share of household and industrial uses. The only source of water for these uses is the Danube terrace water, which is limited in quantity. On the other hand,

irrigation water is withdrawn from the Danube river whose water
quality is much worse than that of terrace water, but it is
admissible for irrigation. The irrigation water supply system,
in turn, represents a system of interconnected reservoirs, canals,
pumping station, culverts, syphons, etc. All the above leads to
the conclusion that the Silistra irrigation system is a separate
and most important part of the regional water supply system.
That is why this study is concerned with the irrigation water
supply system only.

From the geographical point of view, the irrigation system
for the Silistra region is divided into three hydraulically dis-
connected water supply systems for the Tutrakan, Malak Preslavets
and Silistra districts, respectively (see Figure 1). The M.
Preslavets irrigation system is the most representative one with
respect to both the irrigation area (more than 60%) and the
number of typical system elements such as reservoirs, pumping
stations, canals, etc. Regarding the Tutrakan irrigation system,
this project is already underway and half-built. Therefore, the
M. Preslavets irrigation systems have been chosen as a pilot
water project in the Silistra case. The water supply model
developed for the M. Preslavets district is expected to be
transferred afterwards to other irrigation systems in the Silistra
region.

The work on mathematical modeling of water supply systems
has been done by IIASA, in close cooperation with the Sofia
Institute for Water Projects. Of course, the mathematical
modeling by no means replaces the whole work of designing a
water resources system. The best way to understand the purposes
of modeling or, similarly, what IIASA's role was in such a col-
laboration, is to briefly reproduce the sequence of designing
stages in the development of a water supply system as they were
made by the Institute for Water Projects.

As seen from Figure 2, the designing stages for an irriga-
tion system range from preliminary investigations to design work.
We will briefly comment on them. The _first_ _stage_ includes
preliminary geological, topographical, and design investigations,
with the object of roughly outlining what type of water supply

Figure 1. Silistra region.

-6-

Task for Irrigation

```
                         │
                         ▼
    ┌─────────────────────────────────────┐
    │  Preliminary Investigations of a     │
    │  Water Supply System (WSS)           │
    └─────────────────────────────────────┘
                         │
                         ▼
    ┌─────────────────────────────────────┐
    │  Definition of Irrigated Areas and   │
    │  Water Requirements                  │
    └─────────────────────────────────────┘
                         │
                         ▼
    ┌─────────────────────────────────────┐
    │  Definition of Possible              │
    │  Variants for WSS                    │
    └─────────────────────────────────────┘
```

Choice of a Few Variants for WSS

Simulation of Infinite Number of Variants

Determination of "Best" Variant for WSS

Search of Optimal Variant

Mathematical Modelling

Hydraulic and Economic Calculations of Optimal WSS

Final Designing

Constructing Works
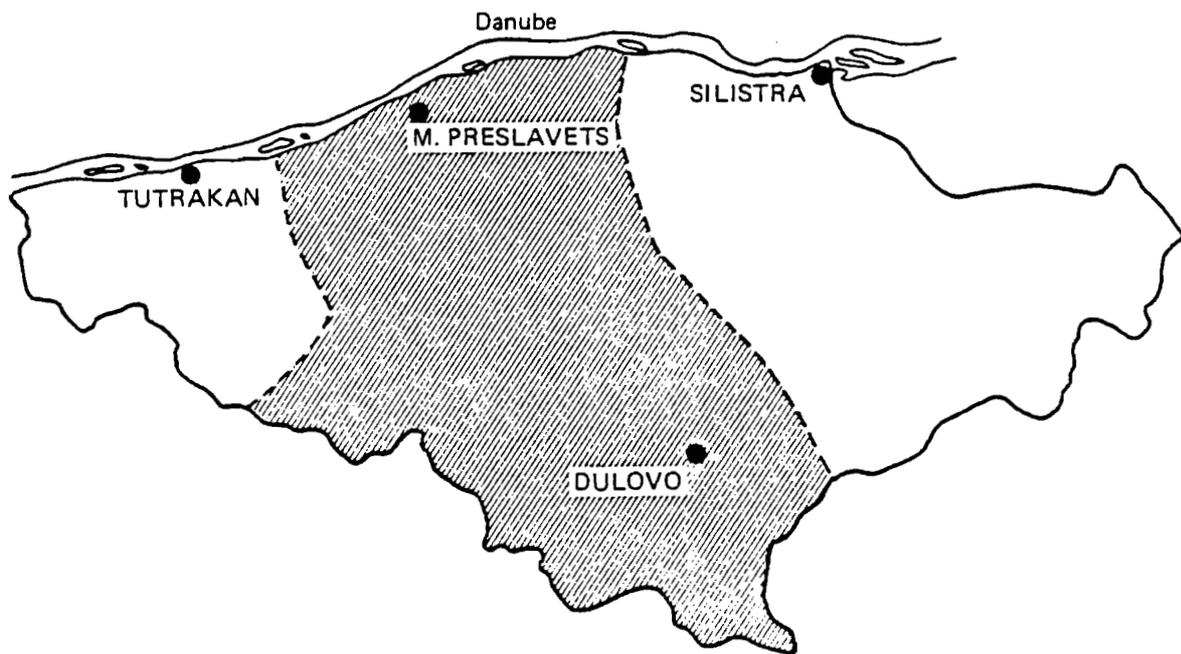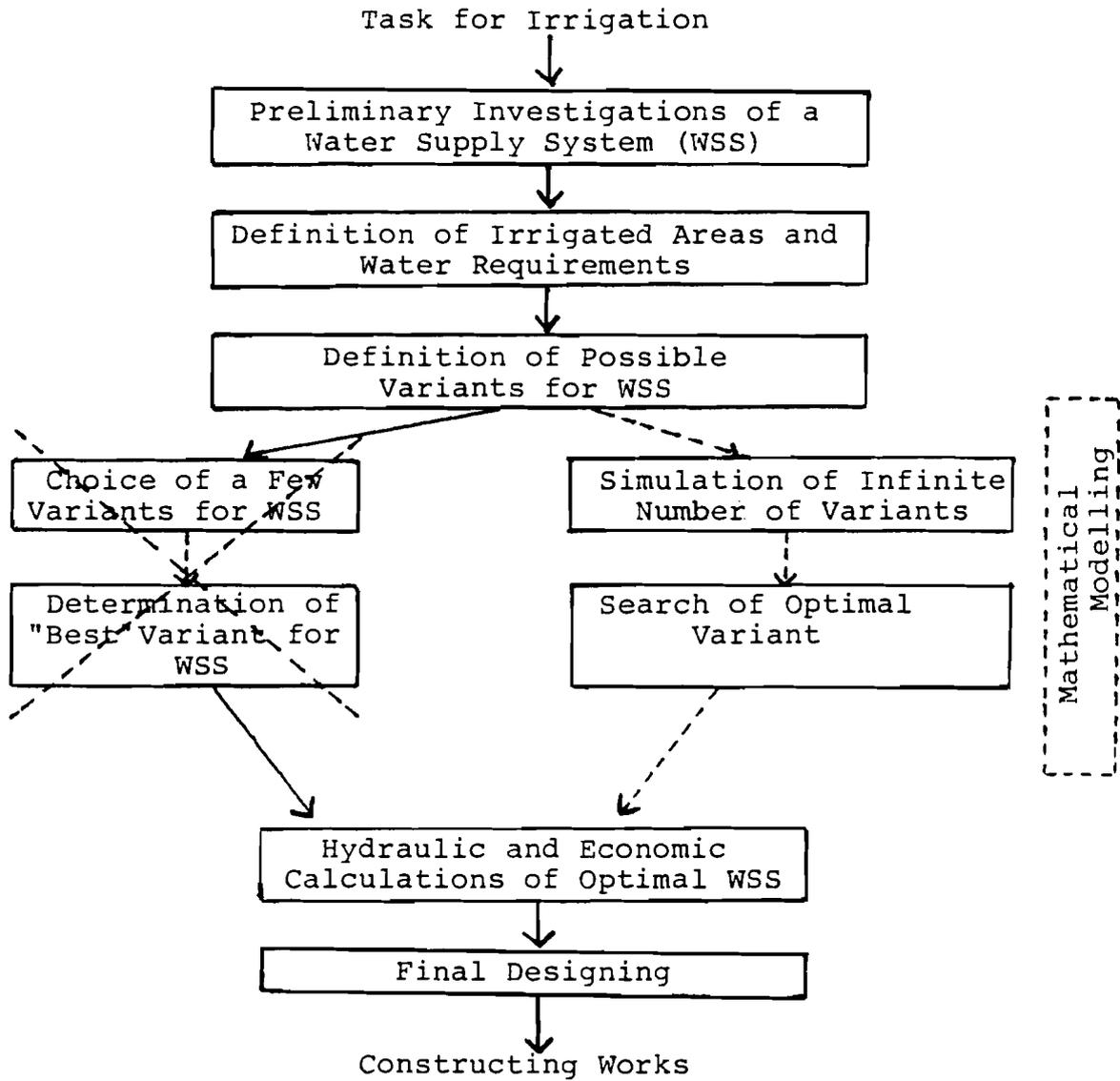
Figure 2. Sequential stages in designing of water supply system.

system will be created--with reservoir or not, with open canals
or pipelines, with water conveyance by gravity or by pumping,
etc. At the second stage, the land suitable for irrigation is
defined and, by doing that, the water requirements are determined.

The third stage consists in the definition of a set of pos-
sible variants for a water supply system. In fact, this alter-
native set is infinite. Nevertheless, in view of pure practical
difficulties--complicated water-balance, engineering, and economic
calculations of the whole irrigation system--in the next stage
the designer has to confine himself to a set of a few variants,
usually no more than 4 or 5. The fifth design stage results in
determination of the "best" variant for an irrigation system.
The "best" here, means the optimal variant in the narrow sense of
the word, as we are dealing with the optimal variant chosen from
among a very limited set of possible variants of water supply
systems. The measure with which to compare the different variants
amongst each other is the total annual cost of a water supply
system.

Looking at the design scheme presented in Figure 2, it is
evident that stages 4 and 5 are rather labor-consuming and, at the
same time, easily formalizable. We can give the mathematical
modeling complete control over these two stages. The essential
advantage of the mathematical-modeling approach here is that it
allows to analyze the infinite number of variants for a water
supply system. In the mathematical model, the simulation of the
infinite number of variants is realized in a fairly simple
manner--by the flow and mass balance constraints in all the
nodes of a water network. Another stage--search for the optimal
variant--is realized by the optimization procedure, which deter-
mines the least-cost variant of water supply system.

Thus, through substitution of the two conventional design
stages (see Figure 2--choice of a few variants and determination
of the "best" variant of a water supply system) for the two
modeling stages--simulation of all potential variants and search
of the optimal variant produces results which:
    o   saves the designer from the multiple, labor-consuming
        calculations of a water supply system;
    o   guarantee the selected variant to be really optimal.

The mathematical modeling effort has two additional objectives which are very important for practical application. Namely, the mathematical model should be:

(a) operational for a wide range of initial data,

(b) suitable for the rather arbitrary configurations of water supply system.

Of course, application of a mathematical model requires that the analyst has access to the computing facilities equipped with a necessary software.

3. GENERALIZED WATER SUPPLY MODEL

3.1 Basic Assumptions

Before describing the mathematical model, it is necessary to outline the range of its applicability. The best way of doing that is to present the main assumptions of the model:

1. The main goal of the water supply system under analysis is to meet water requirements, prespecified both in space and time.

2. The water supply system is determined as it is by the end of the planning period.

3. The available water resources are unlimited and can meet all water requirements.

4. Proceeding from the analysis of topographical and geological conditions, the basic scheme of the water supply system is fixed.

5. The optimal water supply system is considered to be that one which is the least-costly.

6. All water-users consume water resource irreversibly.

7. Only within-year regulation of water resources is considered.

8. The transit time delays for canals are not taken into account.

Actually these assumptions indicate the type of a water supply system which can be analyzed by the model presented below. The first step of the model building process is to construct flow network representation of the irrigation system.

3.2 Flow Network Representation of the System

The flow network consists of the following standard elements:
1. nodes,
2. arcs,
3. inputs (inflows),
4. outputs (outflows).

All of them should be interconnected in a certain sequence as it is in the real irrigation system. Though mapping a real system into a uniform network is not a matter of difficulty, nevertheless, this procedure cannot be entirely formalized. For example, in doing so, sometimes we have to introduce a number of fictitious nodes and arcs, combine a few standard elements into one unit, etc.

Next, this spatial representation has to be expanded to take into account the multi-period operation of the water supply system. This means that the flow network should have two dimensions--space and time. The time representation of the system can be realized as a layered network, where each layer corresponds to a single time period and is connected with the subsequent ones by storage arcs leaving all reservoir nodes. Since the links between the time layers are easy to be accounted for, we can confine ourselves to a detailed consideration of only one time layer of flow network, taking into account the storage arcs entering and leaving the reservoir nodes. Figure 3 shows all standard elements of a flow network and their interpretation in the terms of real elements of an irrigation system.

Any irrigation system we are dealing with in this paper is assumed to be represented by a flow network consisting only of the standard elements presented in Figure 3. By definition, any input can only be at a pumping node, which is called an input pumping node. On the other hand, any internal pumping or distributing node is assumed to have an output. The actual absence of water withdrawal in some internal pumping or distributing nodes is simulated by the output of zero capacity. For explanatory purposes, a simple example of a flow network for a single time period is shown in Figure 4.

	Standard Element of the Model	Corresponding Element of the Real Water Supply System	Legend
1	Input	Water source or water inflow from other system	
2	Reservoir Node	Storage reservoir	
3	Pumping Node	Pumping station	
4	Distributing Node	Junction of two or more water flows	
5	Arc	Open canal, pipeline, culvert, syphon or any combination of those	
6	Storage Arc	Fictitious link for taking into account the transfer of water from one time period to another	
7	Output	Water withdrawal for irrigation or water outflow to other system	
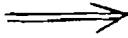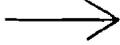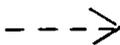
Figure 3. Types and Definitions for Standard Elements of the Flow Network.

Now we should introduce the numbering system for all elements of the flow network and for all time periods. The complete numbering system is shown in Table 1 (the elements of the flow network presented in Figure 4 are numbered following these rules).

For the sake of generality, it is easy to present the uniform network in an analytical form. For these purposes, it is necessary to introduce the following notions describing the links between all nodes.
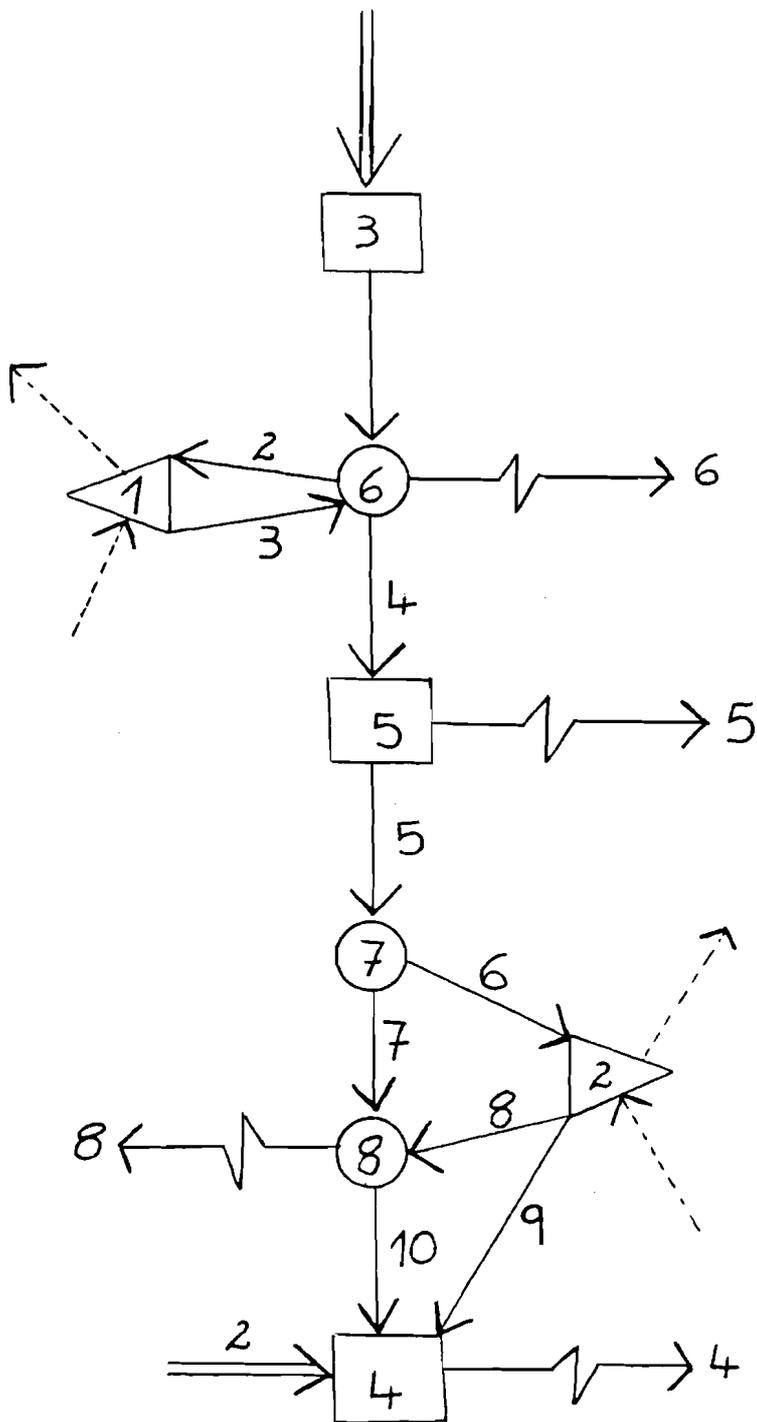
Figure 4. One-Layer Spatial Uniform Network.

Table 1. Numbering System for Standard Elements of Flow Network and for the Time Periods.

	Type of Model Elements		Number of Elements	Numbering Index	Set of Elements of a given type
1.	Inputs		s	α	$1,\ldots,s$
2.	Reservoir Nodes		r	j	$1,\ldots,r$
3.	Pumping Nodes	input	s	j	$r+1,\ldots,r+s$
		internal	m	j	$r+s+1,\ldots,r+s$
4.	Distributing Nodes		l	j	$r+s+m+1,\ldots,r+s+m+l$
5.	Arcs		n	i	$1,\ldots,n$
6.	Outputs *		$m+l$	j	$s+r+1,\ldots,r+s+m+l$
7.	Time Periods		N	k	$1,\ldots,N$

* Distributing or internal pumping nodes and outputs leaving them have the same sequential number.

$I \quad = \{1,\ldots,n\} =$ set of all arc numbers,

$I_j^+ \epsilon I =$ subset of the entering-arc numbers for node j,

$I_j^- \epsilon I =$ subset of the leaving-arc numbers for node j.

Let us assume a real irrigation system which is mapped into a uniform flow network consist of the standard elements given in Figure 3. Then the uniform network is said to be presented analytically, if the following data are specified:
1. numbers s, r, m, l, n,
2. subsets of arc numbers I_j^+ and I_j^- for all $j = 1,\ldots,$ r+s+m+l.

For example, the analytical representation of the flow (Table 2) network shown in Figure 4 is:

$s = 2;\ r = 2;\ m = 1;\ l = 3;\ n = 11.$

Table 2. Subsets I_j^+ and I_j^- in the pattern network

Node Number = j	I_j^+	I_j^-
1	2	3
2	7	8, 11
3	$\emptyset$*	1
4	$\emptyset$	6
5	4	5
6	8, 9	10
7	1, 3	2, 4
8	5, 6	7, 9
9	10, 11	$\emptyset$

* $\emptyset$ is empty set

3.3 Mathematical Description of
     the Generalized Model

   To describe formally the water supply model, it is neces-
sary to define the model variables; for the sake of brevity
they are presented in the following table:

Table 3. Variables in the Model

Definition	Designation	Units	Type of Variable
Input flow α in period k	q_α^k	m^3/s	decision (or initial data)
Flow in arc i in period k	y_i^k	m^3/s	decision
Output flow j in period k	w_j^k	m^3/s	initial data
Capacity of reservoir j	V_j	m^3	decision
Discharge capacity of canal/ arc i	z_i	m^3/s	decision
Capacity of pumping station/ node j	X_j	m^3/s	decision
Duration of time period k	t_k	sec	initial data
Active water storage in reservoir j at the beginning of period k	s_j^k	m^3	decision

   As seen from the above table, input flow q_α^k (k=1,...,N)
from a water source α can be a decision variable or initial data.
The first situation corresponds to the case where input water
flow results from the solution of some optimization problem.
The water sources here can be streamflow, lake or groundwater.
In the second case, the input water flow is prespecified. For
example, it can be an input from another irrigation system
already built. On the other hand, output water flow w_j^k can be
a water requirement for an irrigated area or a fixed input into
another irrigation system. In both cases, this flow should be
prespecified.

3.3.1 Constraints

Now we have at our disposal everything that is required to describe the model mathematically. Let us start with constraints on the decision variables. All constraints of the model are physical ones, and can be divided into the following four groups:

(1) <u>Non-negativity conditions for decision variables</u>

Being inherent in most mathematical programming problems, these constraints require that all decision variables be non-negative.

(2) <u>Flow balances at pumping and distributing nodes</u>

This set of constraints requires that flow continuity be satisfied at the network nodes. For input pumping nodes those are:

$$q_\alpha^k + \sum_{i \in I_j^+} y_i^k - \sum_{i \in I_j^-} y_i^k = 0,$$

$$\alpha = 1, \ldots, s$$

$$j = r + \alpha$$

$$k = 1, \ldots, N \ .$$

Analogously, for the distributing and internal pumping nodes we have:

$$\sum_{i \in I_j^+} y_i^k - \sum_{i \in I_j^-} y_i^k - w_j^k = 0 \ ,$$

$$j = r+s+1, \ldots, r+s+m+\ell$$

$$k = 1, \ldots, N.$$

(3) <u>Mass balances for each reservoir</u>

These constraints describe the release and storage regimes for all reservoirs:

$$s_j^1 = t_N \left(\sum_{i \in I_j^+} y_i^N - \sum_{i \in I_j^-} y_i^N \right) + s_j^N \ ,$$

$$s_j^{k+1} = \left(\sum_{i \in I_j^+} y_i^k - \sum_{i \in I_j^-} y_i^k \right) + s_j^k \quad ,$$

$$j = 1,\ldots,r$$

$$k = 1,\ldots,N-1$$

The first set of these constraints is annual cycle condition for reservoirs. In other words, it reflects the fact that we consider only within-year regulation of water.

(4) Upper bounds

This set of physical constraints requires that canal and pumping station flows and active reservoir storages should not exceed their capacities. Those are:

$$t_k \sum_{i \in I_j^-} y_i^k - s_j^k \leq 0$$

$$s_j^k - V_j \leq 0 \quad \text{(for reservoir nodes)}$$

$$j = 1,\ldots,r; \quad k = 1,\ldots,N$$

$$\sum_{i \in I_j^-} y_i^k - X_j \leq 0 \quad \text{(for pumping nodes)}$$

$$j = r+1,\ldots,r+s+m; \quad k = 1,\ldots,N \quad .$$

$$y_i^k - z_i \leq 0 \quad \text{(for canals/arcs)}$$

$$i = 1,\ldots,n; \quad k = 1,\ldots,N \quad .$$

Furthermore, a number of upper bounds constraining capacities of water supply facilities should be added.

3.3.2 Objective Function

As stated above, the objective of our modeling is to find
a least-cost water supply system. When the hydraulic scheme of
the system is fixed, we should determine capacities of reser-
voirs, pumping stations, canals, and within-year regimes of their
operation. In the model under analysis, the measure for the
total costs associated with the establishment and operation of
the water system is the generalized annual cost caused by:

- construction of reservoirs, pumping stations and canals,
- loss of the submerged arable lands,
- operation of reservoirs and canals,
- maintenance of pumping stations,
- consumption of electric energy for pumping water.

As stated above, we keep the assumption that objective
function is linear with respect to capacities of reservoirs
and pumping stations, and discharge capacities of canals. To
express it formally, the following notions should be introduced:

a_j = increment of annual cost associated with the construc-
tion and operation of reservoir j, due to the unit
increment of its capacity, lv/m^3;[*]

b_j = increment of annual cost associated with the construc-
tion and maintenance of pumping station j, due to the
unit increment of its capacity, $lv/m^3/s$;

γ_i = increment of annual cost associated with the construc-
tion and operation of canal i, due to the unit incre-
ment of its discharge capacity, $lv/m^3/s$;

e_j = unit cost associated with electricity consumption for
water pumping at node j.

---

[*] lv is an abbreviation for leva - Bulgarian monetary unit.

In these terms, the objective function can be written as follows:

$$E = \sum_{j=1}^{r} a_j V_j + \sum_{j=r+1}^{r+s+m} b_j X_j +$$

cost of reservoirs   cost of pumping stations

$$+ \sum_{i=1}^{n} \gamma_i Z_i + \sum_{j=r+\sigma+1}^{r+s+m} e_j \sum_{i \in I_j^+} \sum_{k=1}^{N} t_k y_i^k + \sum_{\alpha=1}^{\sigma} e_{r+d} \sum_{k=1}^{N} t_k q_\alpha^k \cdot$$

cost of canals   cost of electric energy

It must be said that objective function E describes the real annual cost of the whole water supply system with the precision of constant additives. Because constant additives do not influence the solution of an optimization problem, they are omitted.

Thus, the generalized mathematical model of water supply system under analysis is the set of constraints (1)-(4) and objective function (5) to be minimized over all decision variables. It is enough for the user of this model to know the numbers r, s, m, 1, n, N, the sets of arc numbers I_j^+ and I_j^-, and the initial data w_j^k, t_k, a_j, b_j, γ_j, e_j, for all j and k.

4. SILISTRA WATER SUPPLY MODEL*

The mathematical model of the Silistra water supply system will be derived from the general model as a special case. The detailed scheme of the (modeled) water supply system is shown in Figure 5. It consists of the following standard elements: three reservoirs, six pumping stations, twenty canals, and nine distributary nodes. In addition, the irrigation system's only water input comes from the Danube river and the twelve water outputs intended for irrigated areas.

---

* By the Silistra water supply system we mean the M. Preslavets one.
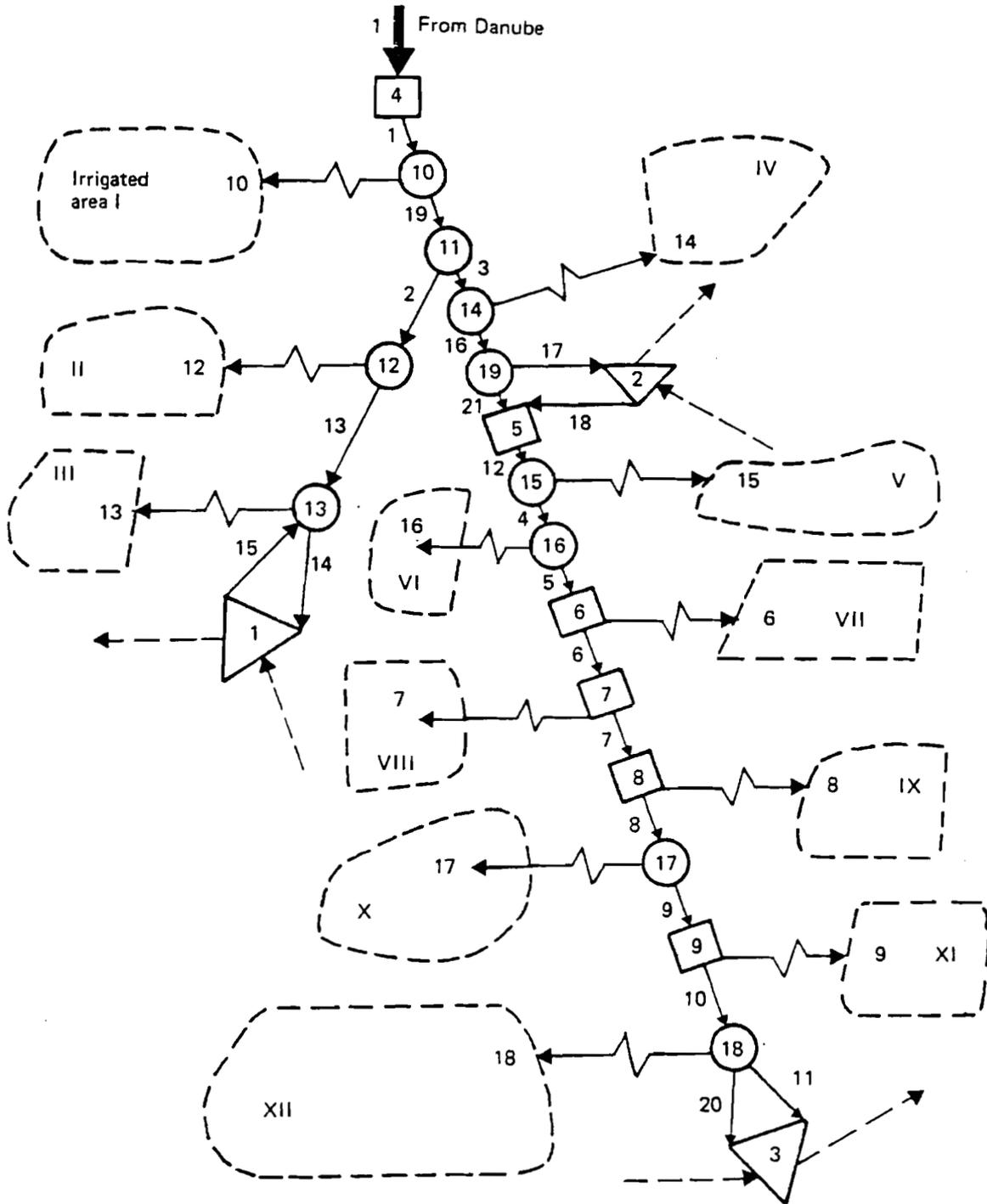
Figure 5. Scheme of the Silistra water supply system.

The Silistra water supply model is constructed under the
same assumptions which we previously stated. We will briefly
comment on some of them in connection with the Silistra Case.
For each irrigated area, the water requirement and the general-
ized irrigation time-table are specified. Although the Silistra
water supply system is expected to be put into operation only
step-wise, the decision is made solely with respect to the
fully completed system which corresponds to the end of the
planning period, the year of 1990. In the Silistra site of the
Danube river, the total water withdrawal does not exceed 5% of
the streamflow, even in the peak period of a dry year. This
allows one to regard the (available) water resources as unlimited.
The irreversible use of water follows from the fact that the
only user of water is irrigation. Finally, because of the small
size of the Silistra region, the transit time delays are not
taken into account.

As stated previously, in order to describe mathematically
the model we should:

(1) specify the numbers s, r, m, l, n, N;

(2) enumerate all the standard elements into a flow network
according to the rules of Figure 5;

(3) define the subsets I_j^+ and I_j^- for all nodes j=1, ...,
r+s+m+l. After doing this (see Figure 1), we obtain:

s = 1 = number of water inputs or input pumping nodes,

r = 3 = number of reservoir nodes,

m = 5 = number of internal pumping nodes,

l = 10 = number of distributing nodes,

n = 21 = number of arcs.

The subsets of the entering (I_j^+) and leaving (J_j^-) arcs
are in Table 4.

It is necessary to stress that in the Silistra Case the
water input q_1^k is a decision variable, and the twelve water
outputs w_j^k (j = 6, ..., 10, 12, ..., 18) are prespecified
irrigation water requirements.

Table 4. Subsets I_j^+ and I_j^- in the Silistra Water Supply System

Node Number j	I_j^+	I_j^-
1	14	15
2	17	18
3	20	11
4	ϕ	1
5	18, 21	19
6	5	6
7	6	7
8	7	8
9	9	10
10	1	12
11	12	2, 3
12	2	13
13	13, 15	14
14	3	16
15	19	4
16	4	5
17	8	9
18	10, 11	20
19	16	17, 21

Thus, the mathematical model of the Silistra water supply system can be presented in the form of constraints (1), ..., (4) and the objective function (5). However, taking into account the peculiarities of the Silistra irrigation scheme, the model can be written much more simply. First, as seen from Figure 5, capacities of all pumping stations are equal to the discharge capacities of the respective canals. This means that some decision variables are unnecessary and can be omitted from the general model. Second, some canals are (intentionally) considered to be of zero cost. This is done for three reasons. The first is that the sizes (costs) of some canals (e.g. 13)

are fully defined by those of the adjacent up-stream canals (respectively 2). The second reason is that some of the canals are artificially introduced into the scheme (e.g. 21) to present it as a uniform network. Finally, the costs of some canals (e.g. 17) are included in the costs of other facilities (reservoir 2). Everything mentioned above means that the expression (5) for objective function in the Silistra water supply model can be essentially simplified in comparison with the general case.

The mathematical model for the Silistra irrigation system is presented below in the reduced form, which corresponds to the general model (1) - (5) with the above mentioned simplifications introduced. The transformations of the general model constraints and the objective function include:

(1) elimination of the decision variables:

$$q_1{}^k, \ y_{21}{}^k, \ y_{19}{}^k, \ x_4, \ \ldots, \ x_9 \ ,$$

by the relations :

$$q_1{}^k = y_1{}^k$$
$$y_{19}{}^k = y_2{}^k + y_3{}^k$$
$$y_{21}{}^k = y_{16}{}^k - y_{17}{}^k$$
$$x_4 = z_1 \tag{1}$$
$$x_5 = z_{12}$$
$$x_6 = z_5$$
$$x_7 = z_6$$
$$x_8 = z_7$$
$$x_9 = z_9$$

(2) omission of the decision variables $z_{12}, \ \ldots, \ z_{21}$ because they correspond to the canals of zero cost as explained above.

With these modifications, the model under analysis is written as follows (constraint sets and the objective function are numbered in accordance with the numbers adopted for the general model presented in section 3 of this paper):

Flow balances at pumping and distributing nodes

$$y_1^k - y_2^k - y_3^k = w_{10}^k$$

$$y_2^k - y_{13}^k = w_{12}^k$$

$$y_{13}^k - y_{14}^k + y_{15}^k = w_{13}^k$$

$$y_3^k - y_{16}^k = w_{14}^k$$

$$y_{12}^k + y_{17}^k - y_{18}^k - y_{16}^k = 0$$

$$y_{12}^k - y_4^k = w_{15}^k \qquad\qquad (2)$$

$$y_4^k - y_5^k = w_{16}^k \qquad\qquad k = 1, \ldots, N$$

$$y_5^k - y_6^k = w_6^k$$

$$y_6^k - y_7^k = w_7^k$$

$$y_7^k - y_8^k = w_8^k$$

$$y_8^k - y_9^k = w_{17}^k$$

$$y_9^k - y_{10}^k = w_9^k$$

$$y_{10}^k + y_{11}^k - y_{20}^k = w_{18}^k \quad .$$

Mass balances for reservoirs

$$s_1^{k+1} - s_1^k + t_k(y_{15}^k - y_{14}^k) = 0 \qquad\qquad k = 1, \ldots, N$$

$$s_1^{n+1} = s_1^1$$

$$s_2^{k+1} - s_2^k + t_k(y_{18}^k - y_{17}^k) = 0 \qquad\qquad (3)$$

$$s_2^{n+1} = s_2^1$$

$$s_3^{k+1} - s_3^k + t_k(y_{11}^k - y_{20}^k) = 0$$

$$s_3^{n+1} = s_3^1 \quad .$$

Upper bounds

$$t_k y_{15}^k - s_1^k \leq 0$$

$$t_k y_{18}^k - s_2^k \leq 0 \qquad\qquad (4)$$

$$t_k y_{11}^k - s_3^k \leq 0 \qquad\qquad k = 1, \ldots, N$$

$$y_{16}^k - y_{17}^k \leq 0$$

$$s_j^k - v_j \leq 0 \qquad\qquad j = 1, 2, 3$$

$$y_p^k - z_p \leq 0 \qquad\qquad p = 1, \ldots, 11, 12$$

The reduced form for the objective function is:

$$E = \sum_{j=1}^{3} a_j v_j \qquad + \qquad \sum_{p=1}^{12} \gamma_p z_p \qquad + \qquad (5)$$

cost of reservoirs   cost of canals and pumping
stations

$$+ \sum_{k=1}^{n} + t_k (e_4 y_1^k + e_5 y_{12}^k + e_6 y_5^k + e_7 y_6^k + e_8 y_7^k + e_9 y_9^k)$$

cost of energy for pumping water

The model consisting of relations (2)-(4) and the objective function (5) was implemented on a computer.

5. RESULTS OF MODELING

The mathematical model presented in Section 4 was run on the IBM 370/165 in Pisa. Before showing the results of modeling, it is necessary to present the full set of model coefficients—time periods t_k, water requirements w_j^i, and the cost coefficients a_j, e_α, γ_p.

Actually, a year was divided into the nine time periods, as shown in Table 5. While modeling, the three-month time period, December, January, and February was omitted, because during these winter months water supply system does not operate. This interruption is caused both by possible freezing of water in canals or reservoirs and by the necessity of carrying out some work on maintenance of the irrigation system.

The prespecified water requirements for all irrigated areas are shown in Table 6. There is no irrigation in the first period of four months, with interruption for the winter season, and it can only be used for storing water in reservoirs, if any. The sixth period—the first ten days of August—is a period of the most intensive irrigation for all areas.

Table 5. Division of a Year into Time-Periods.

Period Number	Months	Duration [month]	Comment
-	December January February	3	out of work
1	October November March April	4	no irrigation
2	May	1	
3	First 20 days of June	2/3	
4	Last 10 days of June	1/3	
5	July	1	
6	First 10 days of August	1/3	the most intensive irrigation
7	Last 20 days of August	2/3	
8	September	1	

Table 6. Water Requirements for All Irrigation Areas [m^3/s].

Period number i Water requirements	1	2	3	4	5	6	7	8
w_1^i	0	0.248	0.317	1.191	0.897	1.294	0.582	0.242
w_2^i	0	0.179	0.228	0.859	0.61	0.961	0.42	0.174
w_3^i	0	4.015	5.127	19.248	13.674	20.927	9.419	3.914
w_4^i	0	0.383	0.488	1.833	1.302	1.993	0.898	0.373
w_5^i	0	0.626	0.8	3.003	2.133	3.264	1.469	0.61
w_6^i	0	0.413	0.528	1.983	1.408	2.155	0.97	0.403
w_7^i	0	0.257	0.328	1.229	0.873	1.337	0.602	0.25
w_8^i	0	0.887	1.135	4.263	3.027	4.634	2.086	0.867
w_9^i	0	1.057	1.35	5.069	3.6	5.51	2.48	1.03
w_{10}^i	0	0.525	0.671	2.522	1.791	2.741	1.234	0.512
w_{11}^i	0	0.414	0.53	1.987	1.411	2.16	0.972	0.404
w_{12}^i	0	7.413	9.467	35.551	25.248	38.64	17.39	6.569

While running the model, the following cost coefficients were used:

$a_1 = 0.0122$ lv/m^3 $\gamma_1 = 118060$ $lv/m^3/sec.$

$a_2 = 0.101$ " $\gamma_2 = 66896$ "

$a_3 = 0.0368$ " $\gamma_3 = 79130$ "

$e_4 = 0.00593$ " $\gamma_4 = 68900$ "

$e_5 = 0.000359$ " $\gamma_5 = 128088$ "

$e_6 = 0.000282$ " $\gamma_6 = 149418$ "

$e_7 = 0.00094$ " $\gamma_7 = 150641$ "

$e_8 = 0.000525$ " $\gamma_8 = 8653$ "

$e_9 = 0.00213$ " $\gamma_9 = 105155$ "

                                   $\gamma_{10} = 17115$ "

                                     $\gamma_{11} = 67754$ "

                                     $\gamma_{12} = 44960$ "

All of the model coefficients presented in this section are calculated on the basis of initial data submitted by the Sofia Institute for Water Projects.

5.1 Basic Characteristics of Water Supply System

One of the main goals of modeling is to determine basic characteristics of the Silistra water supply system--capacities of reservoirs and pumping stations and discharge capacities of canals. Some of these capacities computed under the above coefficients are shown in the following table (Table 7).

As can be seen from Table 6, the time-tables of irrigation for all areas are rather irregular. Analysis of water requirements shows that ratio $w_j^i/max \; w_j^i$ is constant for all irrigated areas (within 2-3%). In other words, all areas have the same irrigation time-table (see Figure 6).

If the water supply system had contained reservoirs, then all canals and pumping stations would have had the same within-year operation regimes as the time-table of irrigation. From this point of view, reservoirs are intended for equalization of the within-year operation regime for the irrigation system.
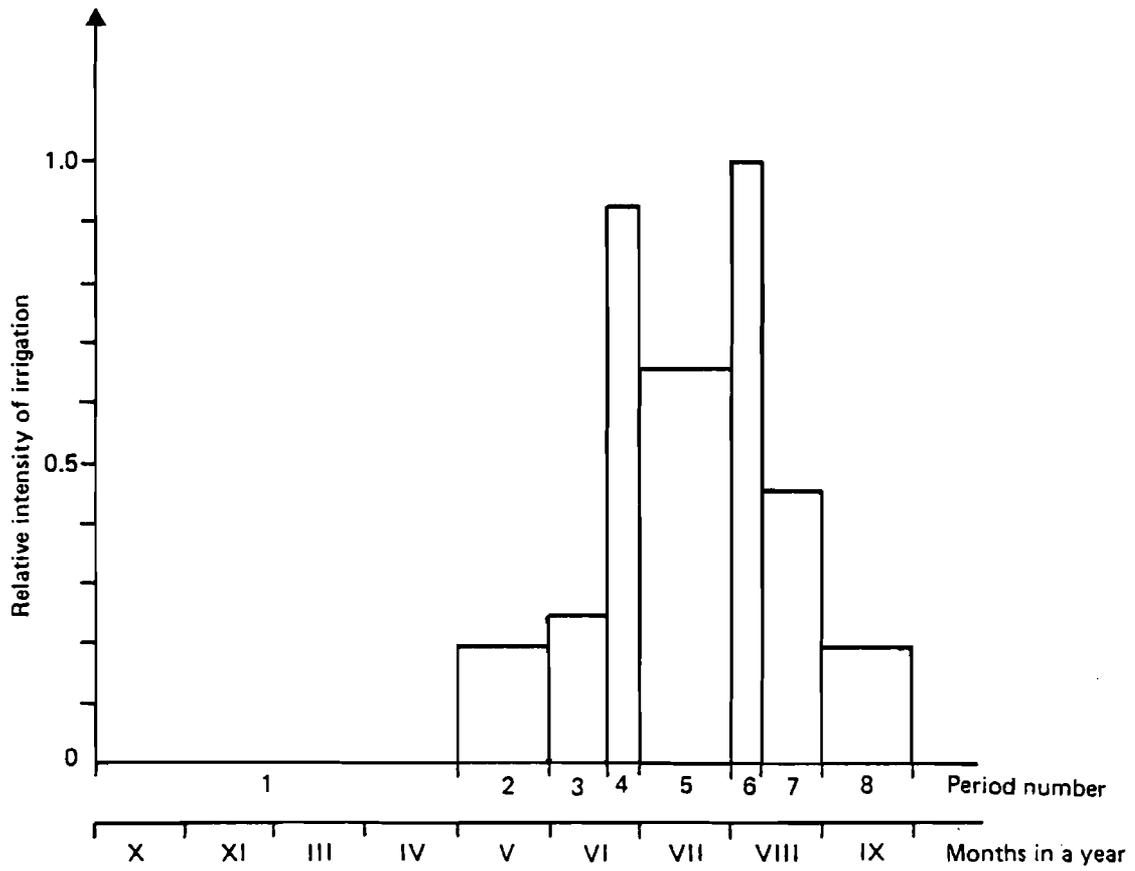
Figure 6. Time-table of irrigation.

Table 7.　Basic Characteristics of the Silistra
　　　　　　Irrigation System

Facilities	Capacities		Units
	Model Notation	Value	
Reservoir 1	V_1	5.248	$10^6 m^3$
Reservoir 2	V_2	2.459	"
Reservoir 3	V_3	131.645	"
Pumping Station 1	Z_1	38.054	m^3/s
Pumping Station 2	X	21.801	"
Pumping Station 3	Z_5	16.382	"
Pumping Station 4	Z_6	15.045	"
Pumping Station 5	Z_7	12.433	"
Pumping Station 6	Z_9	10.891	"
Canal 2	Z_2	15.813	"
Canal 3	Z_3	21.344	"
Canal 11	Z_{11}	38.64	"

Figures 7 and 8 show the operation regimes for some facilities.
Comparison of Figures 6 and 8 lead to the conclusion that the
reservoirs result in:

　　　(1)　equalizing the within-year operation regimes, and

　　　(2)　decreasing the maximum transient water flows,
for pumping stations and canals.　The former occurs during the
four-month non-irrigation period of storing water in reservoirs.
For example, the operation time-table of pumping station 4 is
very close to constant during all the nine working months (see
Figure 8).

　　　The decrease of the maximum transient water flows becomes
possible, because in the peak irrigation periods--last 10 days
of June and first 10 days of August--the water requirements are
met by reservoirs as much as possible.　Quantitative illustration
of this point is made easy with the help of the example of the
basic pumping station 1 situated on the Danube streamflow.　As
seen from Table 6, the total maximum water requirement
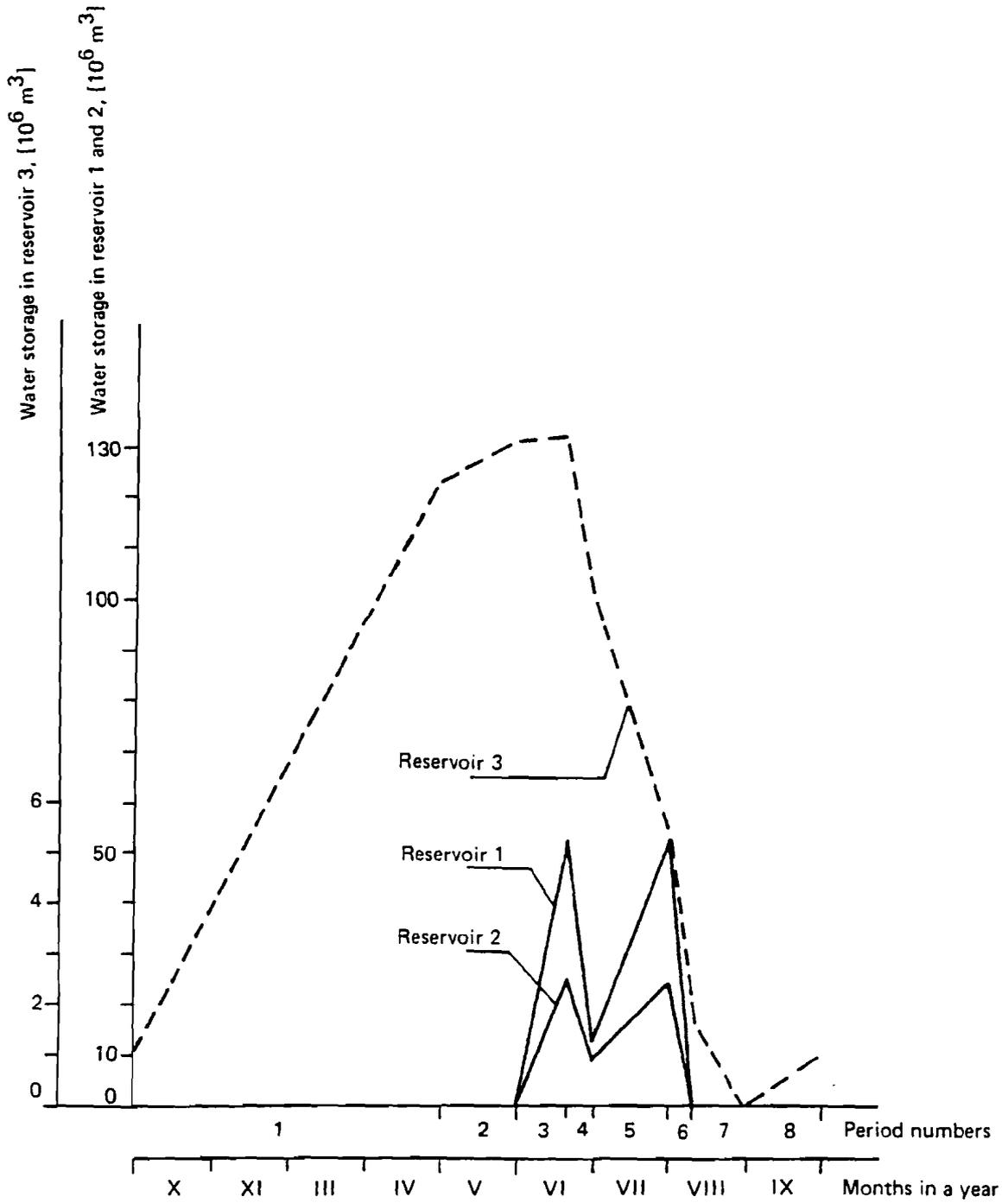
Figure 7. Within-year storing of water in reservoirs.
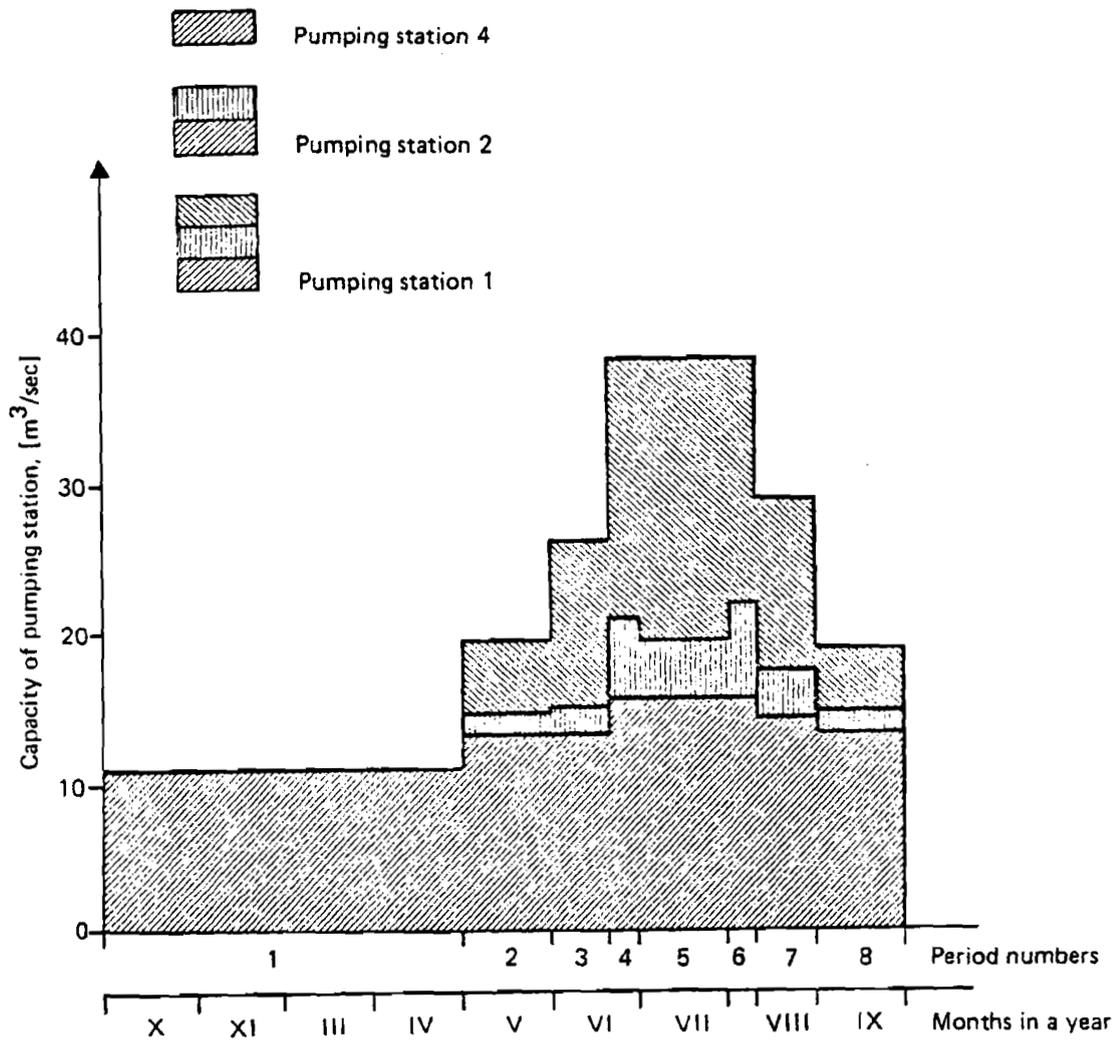
Figure 8. Within-year operation regimes for pumping stations.

corresponding to the peak irrigation period 6 is:

$$w_{max} = \sum_{j=1}^{12} w_j{}^6 = 85.616 \ m^3/s \ .$$

This means if reservoirs had not existed, the capacity of the pumping station would have been equal to 85.626 m^3/s. But, as can be seen from the last table (Table 7), the optimal value for this capacity is only 38.054 m^3/s. In other words, the presence of three reservoirs decreases the capacity of the Danube pumping station by 2.25 times.

5.2 Marginal Costs of Water

Another set of modeling results is concerned with marginal cost of water. The two groups of costs are presented here-- seasonal and mean annual unit costs of water. Both are obtained as the solution of the dual problem with respect to basic one.

By definition, the seasonal marginal cost (c_j^k) of water in the j-th irrigated area is the increment of the optimal value of objective function E caused by the unit increment of water consumption in this area at time period k. In principle, it is an additional cost associated with supplying the j-th area with an additional unit of water at time period k. The seasonal marginal costs of water are shown in the following table.

As can be seen from Table 8, seasonal marginal costs of water depend essentially on the geographical location of an irrigated area and the season of water consumption. For example, at the seventh irrigation period, the seasonal costs vary from 0.0059 to 0.0584 lv/m^3, or about ten times, depending on the location of the irrigated area. Analogously, in the eighth irrigated area, these costs differ about sixtysix-fold--from 0.0075 to 0.0495 lv/m^3--depending on the season.

Three tendencies are clearly observed when analyzing the seasonal marginal costs of water. Firstly, costs depend on the intensity of irrigation during a given period, rather than on

Table 8. Seasonal Marginal Costs of Water

Period Numbers / Area Numbers	1	2	3	4	5	6	7	8	Units
1	0.59	0.59	0.59	2.89	1.01	10.7	0.59	0.59	$10^{-2} lv/m^3$
2	0.59	0.59	0.59	2.89	2.89	12.8	0.59	0.59	"
3	0.59	0.59	0.59	2.89	2.89	12.8	0.59	0.59	"
4	0.59	0.59	0.59	3.77	3.77	10.7	0.59	0.59	"
5	0.63	0.63	0.63	3.81	3.81	15.9	0.63	0.63	"
6	0.63	0.63	0.63	3.81	3.81	23.9	0.63	0.63	"
7	0.66	0.66	0.66	3.84	3.84	38.8	0.66	0.66	"
8	0.75	0.75	0.75	5.58	5.58	49.5	0.75	0.75	"
9	0.8	1.82	1.94	5.63	5.63	49.6	5.63	1.62	"
10	0.8	1.82	1.95	5.63	5.63	49.6	5.63	1.95	"
11	2.0	2.16	2.16	5.84	5.84	49.8	5.84	2.16	"
12	2.16	2.16	2.16	5.84	5.84	14.7	5.84	2.16	"

the period itself. In particular, at the period of the most intensive irrigation--the first 10 days of August ($i=6$)--the unit water costs for all irrigated areas are much higher than all other periods. Secondly, the seasonal costs of water increase, as a rule, when the distance of an irrigated area from the Danube river increases too. Thirdly, it is important that the possibility of withdrawing water for some irrigated areas directly from a reservoir influences the seasonal cost of water very much. For example, for area 12, distant from the Danube river, the seasonal cost (0.0147) is much less than in the adjacent area 11 (0.498 lv/m^3). This happens because irrigated area 12 can use water from reservoir 3, which is impossible for area 11.

For the purposes of economic analysis, the mean annual marginal costs of water are more suitable than the seasonal ones. It is natural enough to define the mean annual cost c_j, for some

irrigated area j, as the weighted-mean sum of seasonal costs over all the time periods; that is:

$$c_j = \sum_{i=1}^{8} \delta_j^k \, c_j^k \quad , \tag{6}$$

where $\delta_j^i > 0$ and $\sum \delta_j^i = 1$. By definition, the weight coefficients δ_j^i are directly proportional to the amount of water taken by irrigated area j in the respective time periods. Namely,

$$\delta_j^k = \frac{t_k w_j^k}{\sum_i t_k w_j^k} \quad . \tag{7}$$

As seen from the above (see Table 6), the weight coefficients for the different irrigated areas are with sufficient accuracy equal, i.e. $\delta_j^k = \delta^k$ for all j=1, ..., 12. This is a trivial consequence of that fact that all irrigated areas have the same time-table of irrigation (see Figure 6).

The weight coefficients calculated by the formula (7) are given below:

Table 9. Weight Coefficients for Marginal Costs of Water

Period Number i	1	2	3	4	5	6	7	8
Weight Coefficient k	0	0.09	0.076	0.144	0.306	0.156	0.141	0.088

Using relation (6), the mean annual unit costs of water
are determined as follows:

Table 10. Mean Annual Costs of Water

Area Numbers	1	2	3	4	5	6	7	8	9	10	11	12
Mean Annual Costs, 10^{-2}lv/m^3	2.67	3.57	3.53	3.6	4.54	5.69	8.03	10.5	11.5	11.7	11.8	6.16

As can be seen from above, the mean annual costs of water
also depends on geographic location of an irrigated area. The
costs vary about 4.5 times--from 0.0267 lv/m^3 in area 1 to
0.118 lv/m^3 in area 11. As stated before, reservoirs decrease
the mean annual costs of water for irrigated areas using water
directly from the reservoir. For example, in area 12, the mean
annual cost equals 0.626 lv/m^3 versus 0.118 lv/m^3 in the eleventh
area, which is closer to the Danube than the previous one. The
same can be said about the third and second irrigated areas--
0.353 lv/m^3 versus 0.0357 lv/m^3, respectively.

All the above-mentioned facts mean that the use of the
average unit cost of water is not correct in economic analysis.

5.3 Sensitivity Analysis

When modeling, the response of optimal solutions to the
variations in some initial data was analyzed. From one point
of view, water requirements, price of land, and price of electric
energy are most uncertain in the Silistra Case. We will try
briefly to explain this point. As a rule, water requirements
are determined proceeding from *a priori* defined unit costs of
water. Since it is impossible to correctly prespecify these
costs, we should make provision for the possibility of varying
water requirements in a wide range. Next, the land price is a

rather subjective value and therefore uncertain. It is suffi-
cient to say that the price of land in Bulgaria is defined as
a net return from the hundred-years crop-yield on this land.
At the same time, in the Silistra region, one has to distinguish
four categories of land depending on soil quality and topograph-
ical conditions. Lastly, the problem of today--energy--is the
reason for widely varying prices of electricity. That is why
we have centered on the sensitivity of the model with respect
to the above-mentioned initial data.

Since meeting water requirements is the main purpose of
water supply system, it should be very sensitive to those. The
two types of variations in water requirements were considered--
coordinated and partial. By coordinated variation in water
requirements, we mean the case where all of the fractional
changes $\dfrac{\Delta w_j^i}{w_j^i}$ in water requirements are equal for all i and j.
The partial variation corresponds to an opposite case.

The response of the modeled water supply system to the
coordinated variations in water requirements is shown in Figure
9. As noted from the figure, the changes in basic parameters
and generalized annual cost of the irrigation system are a
linear function of variation in water requirements. For example,
the coordinated change in water requirements of 20% is identified
with the changes in:

o   capacity of pumping station 1 of 7.6 m^3/s,
o   capacity of reservoir 1 of about 1.05 million m^3,
o   capacity of reservoir 1 of about 26.3 million m^3,
o   generalized annual system cost of 5.88 million lv/year.

Practically speaking, the marginal water costs--seasonal
and mean annual--are insensitive to the coordinated variations
in water requirements.

In a partial manner, water requirements were varied only
for the third and twelfth irrigated areas; separately for each
one. Such a choice of irrigated areas is stipulated by the
fact that, among others, areas 3 and 12 have the largest water
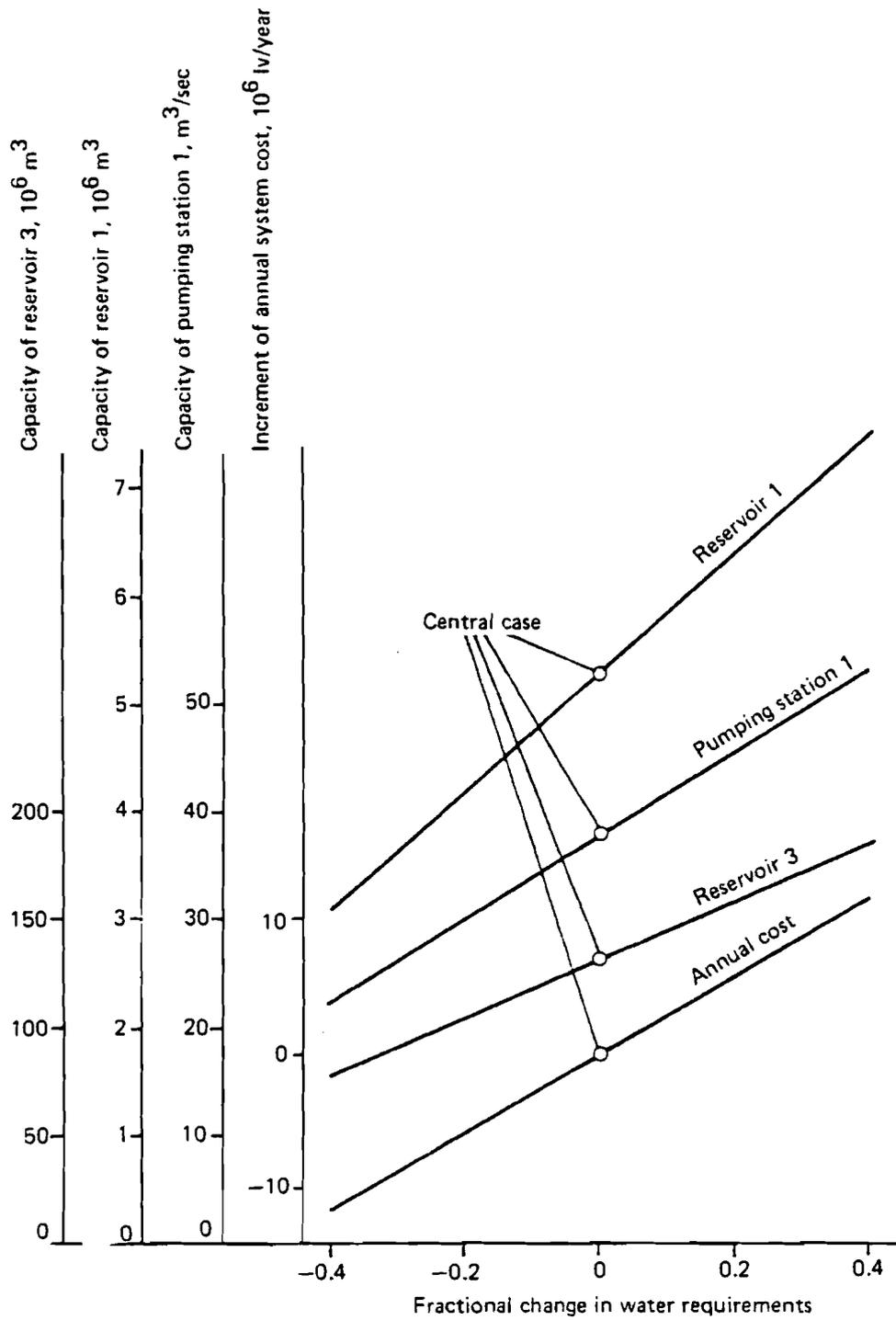
Figure 9.   Sensitivity of the system to coordinated variations
in water requirements.

requirements. Within a year, water requirements for an area were changed proportionally over all time-periods so that the

fractional variations $\dfrac{\Delta w^i_j}{w^i_j}$ (j = 3 or 12) were equal for all i.

Figure 10 illustrates the results of sensitivity of some system parameters to the variations in water requirements for irrigated area 12. As can be seen from this figure, the responses of capacities of pumping station 1 and reservoir 2 to the variations in water requirements w^i_{12} are non-linear functions of those. In particular, a 20% increase of water requirement in area 12 does not influence the capacities of pumping station 1 and reservoir 2. In contrast with this, a 20% decrease in water requirement is identified with changes in capacities of pumping station 1 of 1.254 m^3/s and reservoir 2 of 1.082 million m^3.

As expected, the annual cost of the system and capacity of reservoir 3 are rather sensitive to the variations in the water requirements for area 12. Practically speaking, both are linear functions of those variations with the state coefficients of 0.1355 million lv/year/percent variation and 1.55 million m^3/percent variation, respectively. It must be said that the capacity of the reservoir that is not shown in Figure 10, does not depend on the water requirement variations ranging from -80% to 20%.

The responses of some system parameters to the variations in water requirement for irrigated area 3 are shown in Figure 11. Three curves there corresponding to the generalized annual cost of the system, and capacities of pumping station 1 and reservoir 1, are very close straight lines. Hence, a 20% increase in water requirements increases annual cost by 0.818 million lv/year, capacity of reservoir 1 by 1.012 million m^3 and in capacity of pumping station 1 by 3.013 m^3/s. Notably, capacities of all the water supply facilities situated on the branches of the water network to the right side of node 11 (see Figure 5) do not depend on the variations in water requirements for area 3.
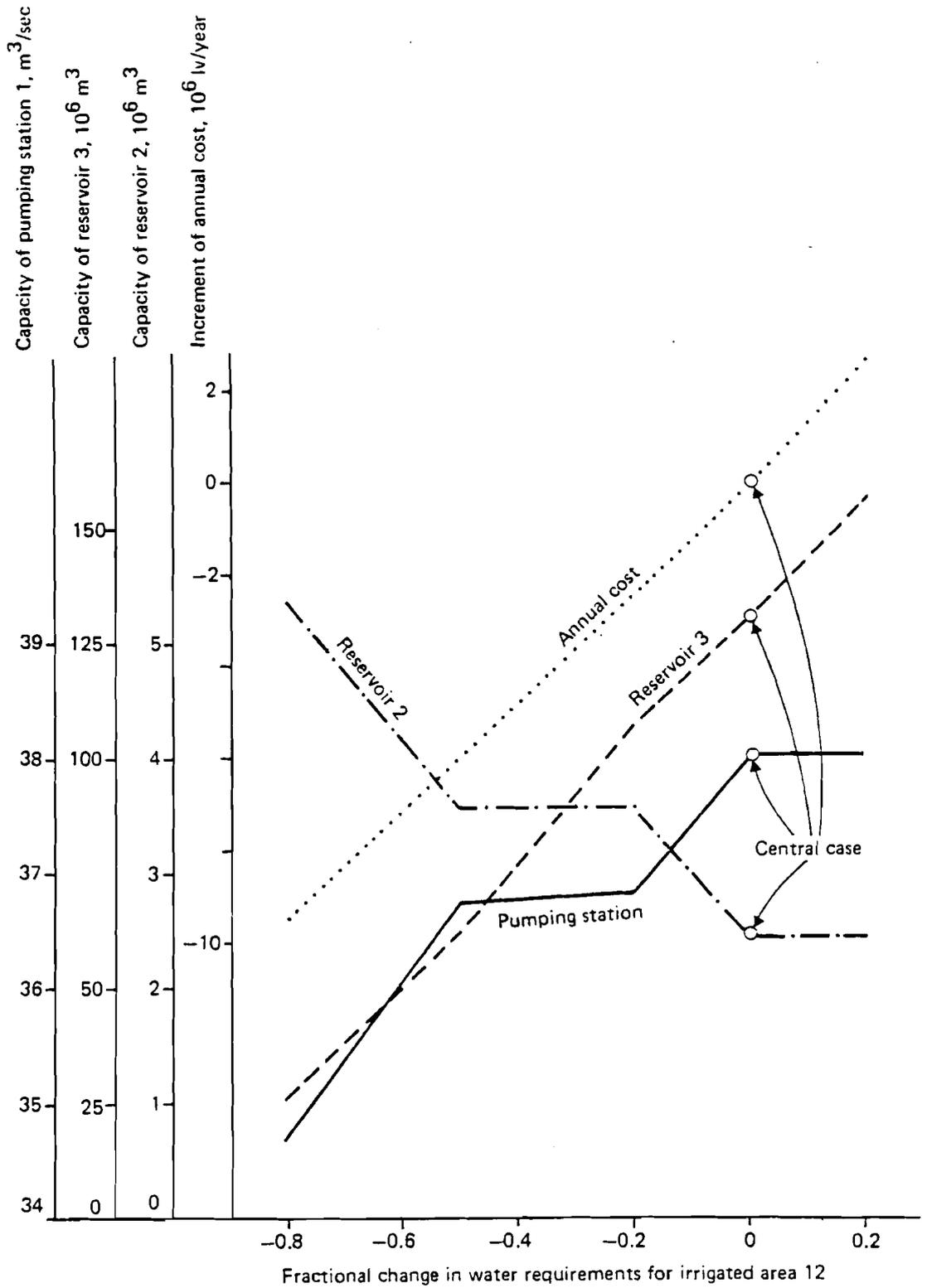
Figure 10. Sensitivity of system to partial variations in water requirements.

-40-
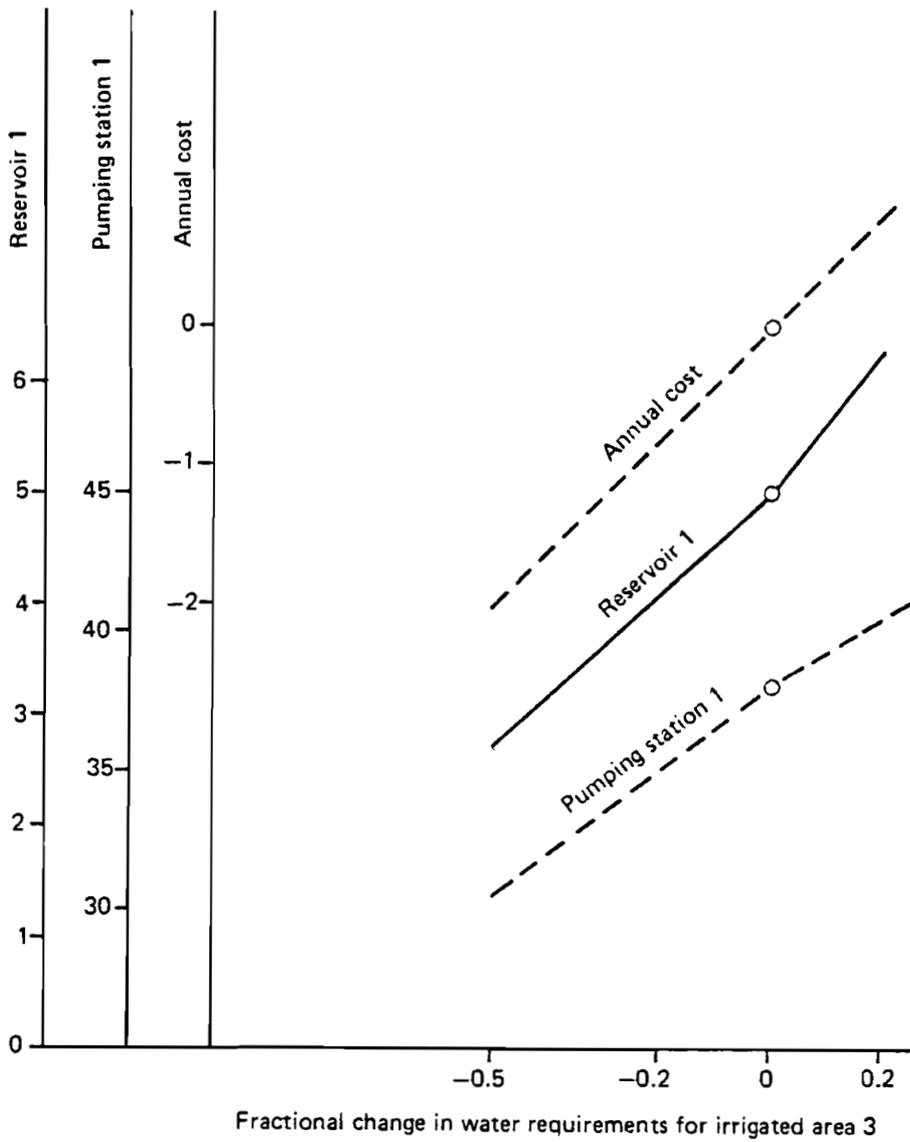
Figure 11.   Sensitivity of system parameters to variations in
water requirements.

We go on with sensitivity analysis of the system under uncertainty in land price. As stated above, it is a subjective value. At the same time, establishment of the water supply system entails the inevitable losses of land needed for the construction of irrigation facilities. According to the data presented by the Sofia Institute for Water Projects, up to 70% of their generalized annual costs are due to the land losses. When modeling, the following two types of lost land are taken into account:

(1) submerged by reservoirs,

(2) needed for construction of canals.

It must be said that, as distinct from reservoirs, the area of land required for construction of canals depends on canal capacities.

Figure 12 illustrates the sensitivity of water supply system with respect to the variations in land price. As can be seen from it, capacity of the Danube pumping station is actually insensitive to land price in its whole range. The capacities of reservoirs 1 and 3 are constant in the range of price ratio from 0.5 to 2.0, but then the decrease in the price ratio from 0.5 to 0.1 is identified with the changes in the capacities of 6.95% and 7.68% for reservoirs 1 and 3, respectively. The capacity of reservoir 2 is most sensitive to land price. For example, a decrease of the price ratio from 0.5 to 0.1 causes the 86.3% increase in the capacity of reservoir 2. Observing the curves in Figure 12, we can state that basic parameters of water supply system are fairly insensitive to land price in the range of price ratio from 0.5 to 1.5.

The capacities of all water supply facilities are quite insensitive to the variations in energy price. This conclusion follows from the structure of objective function (5). As can be seen from it, the total energy cost associated with pumping water is determined only by the following values:

(1) price of electric energy,

(2) amounts of water pumped by each of the six pumping stations within a year.
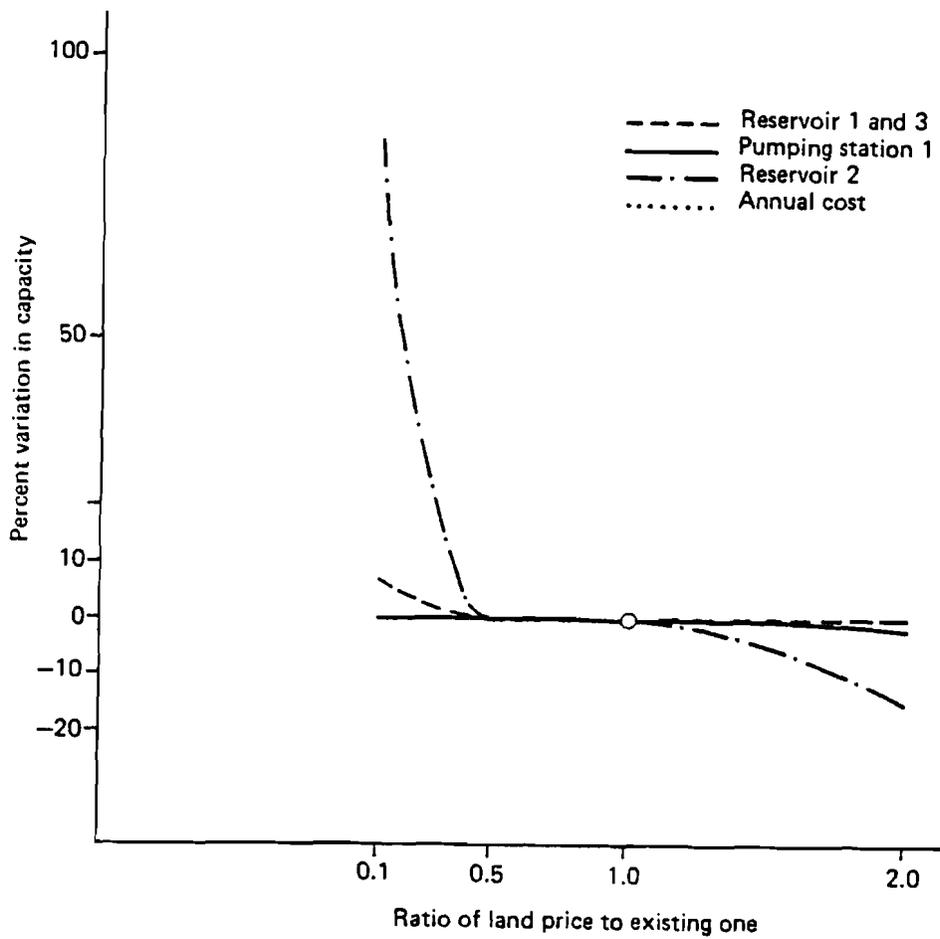
Figure 12. Sensitivity of system to variations in price of land.

Since the scheme of water supply system is fixed and water requirements are prespecified, the latter means that the energy cost does not depend on the variant of the system. In other words, the search of optimal solutions does not depend on energy price. Thus, this results in the insensitivity of the basic system parameters to every price. Of course, the generalized annual cost of the water supply system is influenced by energy price. Specifically, the annual cost of the system is a linear function of energy price, so that a 1% increase in price is identified with an increase in annual cost of 0.039 million lv/year.

Summarizing some results of sensitivity analysis, we can state that the water supply system is rather sensitive to both the coordinated and partial variations in water requirements. As to prices of land and energy, the system is fairly insensitive to them--at least in the long range.

5.4 Practical Application of the Model

The present version of the Silistra Water Supply Model was first implemented on the Pisa IBM 370/165 computer and afterwards transferred to the Sofia ICL 1904 computer. Running the model on the latter allowed the Bulgarian designers to correct basic parameters of the irrigation system due to the variations in some initial data over time.

In 1979, IIASA recommendations on the basic parameters of the Silistra water supply system were given to the Sofia Institute for Water Projects which is in charge of designing water resource systems in Bulgaria. As a result of the above, the main criteria for the practical estimation of the modeling results is the generalized annual cost of the water supply system, in spite of the fact that this annual cost of the system envisaged by the project was known--45.516 million lv/year. The point is that the generalized annual cost under modeling is not a real one due to the following. Firstly, the linear objective function describes only in approximate terms, the changes in real costs depending on the changes in decision variables. Secondly, it takes into account no additive components. As a modeling results, the optimal value of generalized annual cost of the system is equal to 29.4 million lv/year.

That is why the following way was chosen to correctly estimate the optimal solution obtained by the model run. As stated above, the model run results in capacities and operation regimes for all the water supply facilities. On the basis of this data, once more hydraulic and economic calculation of the whole irrigation system was made by the Sofia Institute for Water Projects. The generalized annual cost--39.212 million lv/year--obtained by this calculation, should be compared with that envisaged by the project. Comparison of two annual costs answers the question, which of the two variants of water supply system is better--that envisaged by the project or that determined by the optimization model. The following table illustrates some results of comparing the two variants of the water supply systems.

Table 11. Comparison of the Water Supply System Variants

Facility Variant	Determined by Modeling	Envisaged by the Institute for Water Projects	Units
Pumping Station 1	38.054	23.22	m^3/s
Reservoir 1	5.248	26.02	million m^3
Reservoir 2	2.454	26.03	million m^3
Reservoir 3	131.645	193.80	million m^3
Canal 1	38.054	23.22	m^3/s
Canal 2	15.813	12.10	m^3/s
Canal 3	21.344	12.40	m^3/s
Generalized Annual Cost	29.400 by model run 39.212 by hydraulic calculation	45.50	million lv/year

By analyzing Table 11, we can conclude that the main distinction of the modeled variant from the projected one consists in the decrease in the capacities of all reservoirs and, on the other hand, in the respective increase in the capacities of the main canals and pumping station. The analysis of the generalized annual costs shows that the variant of the irrigation system obtained by modeling is cheaper by 6-304 million lv/year (or about 15%) in comparison with the projected one.

The more detailed cost analysis, carried out by the Sofia Institute for Water Projects points out the additional advantages of the optimal variants of the system compared to those envisaged by the project. Specifically, for the variant of the irrigation system determined by modeling, the total capital investment and cost of submerged land is 32 million lv and 8.4 million lv respectively; less than for the variant envisaged by the project.

Thus, using the optimization model in choosing the optimal variant of the Silistra water supply system has resulted in a considerable budget saving. If there had not been a projected variant, the use of the model would have saved designers from the labor-consuming hydraulic calculations for a number of preliminary variants of the water supply system. The latter advantage of the mathematical modeling approach to a water supply problem is of great importance in planning the new irrigation systems.

6. CONCLUSIONS

This paper sums up the IIASA's work on the water supply modeling in the Silistra Case Study. The major results are:

1. Development of the general water supply model suitable for irrigation systems of rather arbitrary configurations. The main objective of modeling is to determine the least-cost variant of a water supply system.

2. Development and computer implementation of the Silistra
    irrigation water supply model based on the general one.
    For a wide range of input data, the developed model
    allows one to determine:

    o   capacities,

    o   geographically distributed marginal costs of
           water, and

    o   within-year operation regimes,

    for all the water supply facilities of the Silistra
    irrigation system.

3. The sensivitity analysis which results in that Silistra
    water supply system is rather sensitive to the varia-
    tions in water requirements, and fairly insensitive
    to the variations in prices of land and electric energy.

4. Practical application of the modeling results to the
    Silistra irrigation system points to the advisability
    of a considerable decrease in the capacities of all
    three reservoirs. The optimal variant obtained by
    modeling is about 40 million lv in capital investment,
    and 6.3 million lv in annual cost cheaper than that
    envisaged by the project.

5. The developed modeling approach is expected to be used
    by the water use designer as a universal tool to search
    for the least-cost alternatives for many irrigation
    systems in Bulgaria.

REFERENCES

Albegov, M., and V. Chernyatin. 1978. An Approach to the Con-
struction of the Regional Water Resource Model. RM-78-59.
Laxenburg, Austria: International Institute for Applied
Systems Analysis.

Chernyatin, V., and I. Gouevsky. 1981. Coordination of Water
Demand and Supply Models: Silistra Region Case Study.
WP-81-94. Laxenburg, Austria: International Institute
for Applied Systems Analysis.

Gouevsky, I., and D. Maidment. 1977. Agricultural Water
Demands: Preliminary Results of Silistra Case Study.
RM-77-44. Laxenburg, Austria: International Institute
for Applied Systems Analysis.

Kindler, J., and C.S. Russell, eds. Modelling of Water Demands.
To appear in the Wiley International Series on Applied
Systems Analysis. Laxenburg, Austria: International
Institute for Applied Systems Analysis.

APPENDIX: COMPUTER OUTPUT OF THE LP SILISTRA MODEL

The linear programming model of the Silistra Irrigation was implemented on the Pisa IBM 370/165 by Andras Por from the SDS Section of IIASA. The Sesame package was used for generating the model into a MPS 360 format and was also used for the solution. Below, the identifiers for constraints (2^1) - (5^1), (horizontal section), and decision variables (vertical section), and the listing of optimal solutions are presented.

Name of Row	Definition	Reference	Unit
DEMjk	Flow balance at node j in time period k	(2^1)	m^3/s
STORik	Mass balance in reservoir i in time period k	(3^1)	m^3
BALAi	Annual cycle condition for reservoir i	(3^1)	m^3
RELik	Upper bound on water release from reservoir i in period k	(4^1)	m^3
INFLOWk	Flow balance at node 5 in period k	(2^1)	m^3/s
CONSTRK	Physical constraint on flows at node 19	(4^1)	m^3/s
UPSTOik	Upper bound on water storage in reservoir i in period k	(4^1)	M^3
UPCNnk	Upper bound on water flow in canal n in period k	(4^1)	m^3/s
FUNC	Objective function	(5^1)	lv/year

The decision variable identifiers almost coincide with their notions in the mathematical model. The only difference is that the first identification number corresponds to the lower index of a variable and the second identifier number corresponds to the upper one.

AT 1H:3H:22 ON 02/19/80

CELESTIAL WATER SUPPLY

SOLUTION

OPTIMAL SOLUTION AT ITERATION NUMBER 332

...NAME...	...ACTIVITY...	DEFINED AS
FUNCTIONAL	29400014.	FUNC
RESTRAINTS		RHS

GILSON CALLE SUPPLY

Solution

BASIC SECTION

NUMBER	...ROWS...	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
1		BS	24400014.	-24400014.	NONE	NONE	1.0000000
2		EQ					-.00593000
3		EQ					-.00593000
4		EQ					-.02163347
5		EQ					-61482.240
6		EQ					-61482.240
7		EQ					-61482.240
8		EQ					-65204.352
9		EQ					-65204.352
10		EQ					-68128.128
11		EQ					-77874.048
12		EQ					-83317.248
13		EQ					-83317.248
14		EQ					-207160.78
15		EQ					-224295.78
16		BS					.00593000
17		EQ	-5748584.0	5748584.0	NONE	NONE	
18		OL			NONE		.00593000
19		BS			NONE		.02163347
20		EQ	-2454408.0	2454408.0	NONE		
21		BS	-10157930.	10157930.	NONE		
22		BS	-12148703	12148703	NONE		61482.240
23		BS	-10.891945	10.891945	NONE		
24		BS	-27.162295	27.162295	NONE		
25		BS	-15.813250	15.813250	NONE		
26		BS	-10.452045	10.452045	NONE		
27		BS	-7.6450455	7.6450455	NONE		
28		BS	-5.4900455	5.4900455	NONE		
29		BS	-4.1530455	4.1530455	NONE		
30		BS	-1.5420000	1.5420000	NONE		
31		UL	.51200000	.51200000	NONE		101779.69
32		BS			NONE		17115.000
33		UL	-38.640000	38.640000	NONE		
34		BS	-10.909645	10.909645	NONE		
35		EQ	-.24800000		.24800000	.24800000	-15370.560
36		EQ	.17900000		.17900000	.17900000	-15370.560
37		EQ	4.0150000		4.0150000	4.0150000	-15370.560
38		EQ	.38300000		.38300000	.38300000	-15370.560
39		EQ	.62600000		.62600000	.62600000	-16301.088
40		EQ	.41300000		.41300000	.41300000	-16301.088
41		EQ	.25700000		.25700000	.25700000	-17032.032
42		EQ	.48700000		.48700000	.48700000	-19468.512
43		EQ	1.0570000		1.0570000	1.0570000	-50552.985
44		EQ	.52500000		.52500000	.52500000	-50552.985
45		EQ	.41400000		.41400000	.41400000	-56073.945
46		EQ	7.4130000		7.4130000	7.4130000	-56073.945
47		EQ			.	.	.00593000

SELISTON WATER SUPPLY

AT 18:38:22 ON 02/19/80

SOLUTION

ROWS SECTION

NUMBER	...ROW..	AT	..ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
53		HS	.	.	NONE	.	.
54		HS	-5248584.0	5248584.0	NONE	.	.00593000
55		EQ	.	.	NONE	.	.
56		UL	.	.	NONE	.	.
57		HS	-2459808.0	2459808.0	NONE	.	.02163347
58		EQ	.	.	NONE	.	.
60		HS	-22930835.	22930835.	NONE	.	.
61		HS	-8559451.6	8559451.6	NONE	.	.
62		EQ	-14.616955	14.616955	NONE	.	15370.560
63		HS	-18.612295	18.612295	NONE	.	.
64		HS	-11.619250	11.619250	NONE	.	.
65		HS	-6.3440455	6.3440455	NONE	.	.
66		HS	-4.5460455	4.5460455	NONE	.	.
67		HS	-2.8040455	2.8040455	NONE	.	.
68		HS	-1.7240455	1.7240455	NONE	.	.
69		UL	.	.	NONE	.	29723.673
70		HS	-.02700000	.02700000	NONE	.	.
71		HS	-.04000000	.04000000	NONE	.	.
72		HS	-.45400000	.45400000	NONE	.	.
73		PL	.	.	NONE	.	.
74		HS	-7.1840455	7.1840455	NONE	.	.
75		EQ	.31700000	.	.31700000	.31700000	-10247.040
76		EQ	.22800000	.	.22800000	.22800000	-10247.040
77		EQ	5.1270000	.	5.1270000	5.1270000	-10247.040
78		EQ	.48800000	.	.48800000	.48800000	-10247.040
79		EQ	.80000000	.	.80000000	.80000000	-10867.392
80		EQ	.52800000	.	.52800000	.52800000	-10867.392
81		EQ	.32800000	.	.32800000	.32800000	-11354.688
82		EQ	1.1350000	.	1.1350000	1.1350000	-12979.008
83		EQ	1.3500000	.	1.3500000	1.3500000	-33701.990
84		EQ	.67100000	.	.67100000	.67100000	-33701.990
85		EQ	.53000000	.	.53000000	.53000000	-37382.630
86		EQ	9.4670000	.	9.4670000	9.4670000	-37382.630
87		EQ	.	.	NONE	.	-37382.630
88		UL	.	.	NONE	.	-.00593000
89		HS	-5248584.0	5248584.0	NONE	.	.00593000
90		EQ	.	.	NONE	.	.
91		UL	.	.	NONE	.	.
92		HS	-2459808.0	2459808.0	NONE	.	.02163347
93		EQ	.	.	NONE	.	.
94		HS	-64156477.	64156477.	NONE	.	.
95		HS	-71876.45	71876.45	NONE	.	.
96		HS	-15.224955	15.224955	NONE	.	.
97		EQ	.	.	NONE	.	10247.040
98		HS	-12.208420	12.208420	NONE	.	.
99		HS	-7.4208750	7.4208750	NONE	.	.
100		HS	-4.2075455	4.2075455	NONE	.	.
101		HS	-4.1120455	4.1120455	NONE	.	.
102		HS	-2.4450455	2.4450455	NONE	.	.
103		HS	-1.4760455	1.4760455	NONE	.	.
104		PL	.	.	NONE	.	19815.782

DISTRIBUTION WATER SUPPLY

SOLUTION

ROWS SECTION

...ROW...	AT	...ACTIVITY...	SLACK ACTIVITY	.LOWER LIMIT.	.UPPER LIMIT.	.DUAL ACTIVITY
	BS	-.32000000	.32000000	NONE	.	.
	BS	-.47900000	.47900000	NONE	.	.
	BS	-1.00900000	1.00900000	NONE	.	.
	UL	-6.5760455	6.5760455	NONE	.	.
	EQ	1.1910000	.	1.1910000	1.1910000	-25010.520
	EQ	.85900000	.	.85900000	.85900000	-25010.520
	EQ	19.248000	.	19.248000	19.248000	-25010.520
	EQ	1.8330000	.	1.8330000	1.8330000	-32605.020
	EQ	3.0030000	.	3.0030000	3.0030000	-32915.196
	EQ	1.9830000	.	1.9830000	1.9830000	-32915.196
	EQ	1.2290000	.	1.2290000	1.2290000	-33158.844
	EQ	4.2630000	.	4.2630000	4.2630000	-48192.595
	EQ	5.0690000	.	5.0690000	5.0690000	-48646.195
	EQ	2.5220000	.	2.5220000	2.5220000	-48646.195
	EQ	1.9870000	.	1.9870000	1.9870000	-50486.515
	EQ	35.551000	.	35.551000	35.551000	-50486.515
	EQ	.	.	.	.	.02894736
	UH	.	.	NONE	.	.02301736
	EQ	.	.	NONE	.	.03773729
	UL	-98260207.	98260207.	NONE	.	.
	UH	-18.413000	18.413000	NONE	.	.
	EQ	.	.	NONE	.	.03180729
	UL	.	.	NONE	.	.05843347
	BS	-.24400000	.24400000	NONE	.	.
	UL	-.24000000	.24000000	NONE	.	.03680000
	EQ	-.10400000	.10400000	NONE	.	.
	EQ	.	.	NONE	.	.
	BS	32605.020	.	NONE	.	32605.020
	BS	19887.000	.	NONE	.	19887.000
	UL	.	.	NONE	.	7594.5000
	BS	-1.65194545	1.65194545	NONE	.	.
	BS	-5.69095545	5.69095545	NONE	.	.
	BS	-7.70095545	7.70095545	NONE	.	14221.591
	UL	-9.68795545	9.68795545	NONE	.	.
	BS	-.54100000	.54100000	NONE	.	.
	EQ	.89700000	.	.89700000	.89700000	-26279.560
	EQ	.61000000	.	.61000000	.61000000	-75031.560
	EQ	13.674000	.	13.674000	13.674000	-75031.560
	EQ	1.3020000	.	1.3020000	1.3020000	-97815.060
	EQ	2.1330000	.	2.1330000	2.1330000	-98745.588
	EQ	1.4080000	.	1.4080000	1.4080000	-98745.588
	EQ	.87300000	.	.87300000	.87300000	-99476.532
	EQ	3.0270000	.	3.0270000	3.0270000	-144577.78
	EQ	3.6000000	.	3.6000000	3.6000000	-145938.58
	EQ	1.7910000	.	1.7910000	1.7910000	-145938.58
	EQ	1.4110000	.	1.4110000	1.4110000	-151459.54
	EQ	25.248000	.	25.248000	25.248000	-151459.54

SYSTEM WATER SUPPLY

SOLUTION

ROWS SECTION

AT 18:38:22 ON 02/19/80

NUMBER	..ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
157	STOR105	EQ	.	.	NONE	.	.02894736
158	DEL105	UL	.	.	NONE	.	.
159	OPSTO105	BS	-3963816.0	3963816.0	NONE	.	.
160	STOR205	EQ	.	.	NONE	.	.03773729
161	DEL205	UL	.	.	NONE	.	.
162	OPSTO205	BS	-1511136.0	1511136.0	NONE	.	.
163	DEL305	BS	.	.	NONE	.	.05843347
164	OPSTO305	BS	-1814478.5	1814478.5	NONE	.	.
165	OPSTO305	BS	-2967580R.	2967580R.	NONE	.	.
166	CONSTR405	BS	-19.093000	19.093000	NONE	.	.
167	DEL0905	EQ	.	.	NONE	.	97815.060
168	OPGR0105	UL	.	.	NONE	.	10909.000
169	OPGR0205	UL	.	.	NONE	.	48752.000
170	OPGR0305	BS	.	.	NONE	.	71535.500
171	OPGR0405	BS	-1.2110000	1.2110000	NONE	.	.
172	OPGR0505	BS	-.46400000	.46400000	NONE	.	.
173	OPGR0605	UL	.	.	NONE	.	42664.773
174	OPGR0705	BS	-.41545455	.41545455	NONE	.	.
175	OPGR0805	BS	-2.7859545	2.7859545	NONE	.	.
176	OPGR0905	BS	-4.2649545	4.2649545	NONE	.	.
177	OPGR1005	BS	-5.6759545	5.6759545	NONE	.	.
178	OPGR1105	BS	.	.	NONE	.	.
179	OPGR1205	BS	-2.3420000	2.3420000	NONE	.	.
180	DEL1006	EQ	1.2940000	.	1.2940000	1.2940000	-92387.520
181	OPGR1206	EQ	.96100000	.	.96100000	.96100000	-110531.52
182	OPGR1306	EQ	20.927000	.	20.927000	20.927000	-110531.52
183	DEL1206	EQ	1.9930000	.	1.9930000	1.9930000	-92387.520
184	OPGR1506	EQ	3.2640000	.	3.2640000	3.2640000	-137657.70
185	OPGR1606	EQ	2.1550000	.	2.1550000	2.1550000	-206557.70
186	OPGR0706	EQ	1.3370000	.	1.3370000	1.3370000	-334889.34
187	OPGR0806	EQ	4.6340000	.	4.6340000	4.6340000	-428233.14
188	OPGR0906	EQ	5.5100000	.	5.5100000	5.5100000	-428686.74
189	DEM1006	EQ	2.7410000	.	2.7410000	2.7410000	-428686.74
190	OPGR0606	EQ	2.1600000	.	2.1600000	2.1600000	-430527.06
191	DEM1806	EQ	38.640000	.	38.640000	38.640000	-118240.51
192	STOR106	BS	.	.	NONE	.	-.00593000
193	DEL106	UL	.	.	NONE	.	-.12200000
194	OPSTO106	UL	.	.	NONE	.	.09898264
195	STOR206	BS	.	.	NONE	.	-.00593000
196	DEL206	BS	.	.	NONE	.	-.10100000
197	OPSTO206	UL	.	.	NONE	.	-.06919271
198	STOR306	BS	.	.	NONE	.	.05843347
199	DEL306	BS	-16661455.	16661455.	NONE	.	92387.520
200	OPSTO306	BS	-81598752.	81598752.	NONE	.	87264.000
201	CONSTR406	EQ	-18.954000	18.954000	NONE	.	18144.000
202	DEL0906	EQ	.	.	NONE	.	.
203	OPGR0106	UL	.	.	NONE	.	68900.000
204	OPGR0206	UL	.	.	NONE	.	128088.00
205	OPGR0306	BS	-.39700000	.39700000	NONE	.	92531.636
206	OPGR0406	UL	.	.	NONE	.	.
207	OPGR0506	BS	.	.	NONE	.	.
208	OPGR0606	UL	.	.	NONE	.	.

SOUTHERN WATER SUPPLY

SOLUTION

ROWS SECTION

NUMBER	..ROW..	AT	..ACTIVITY..	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
209		BS	-2.0229545	2.0229545	NONE	.	67754.000
210		BS	-6.5029545	6.5029545	NONE	.	44960.000
211		BS	-8.7319545	8.7319545	NONE	.	-10247.040
212		UL	-10.891955	10.891955	NONE	.	-10247.040
213		UL	.	.	NONE	.	-10247.040
214		EQ	.58200000	.	.58200000	.58200000	-10247.040
215		EQ	.42000000	.	.42000000	.42000000	-10867.392
216		EQ	9.4190000	.	9.4190000	9.4190000	-10867.392
217		EQ	.89800000	.	.89800000	.89800000	-11354.688
218		EQ	1.4690000	.	1.4690000	1.4690000	-12979.008
219		EQ	.97000000	.	.97000000	.97000000	-97292.390
220		EQ	.60200000	.	.60200000	.60200000	-97292.390
221		EQ	2.0860000	.	2.0860000	2.0860000	-100973.03
222		EQ	2.4800000	.	2.4800000	2.4800000	-100973.03
223		EQ	1.2340000	.	1.2340000	1.2340000	.00593000
224		EQ	.97200000	.	.97200000	.97200000	
225		EQ	17.390000	.	17.390000	17.390000	.00593000
238		BS	-524584.0	524584.0	NONE	.	
239		UL	.	.	NONE	.	
240		EQ	.	.	.	.	
241		EQ	-2459808.0	2459808.0	NONE	.	.02163347
243		BS	.	.	NONE	.	.03680000
244		EQ	.	.	.	.	
245		EQ	-11484E03	11484E03	NONE	.	10247.040
246		EQ	-17.560955	17.560955	NONE	.	
247		EQ	-9.1742955	9.1742955	NONE	.	
248		BS	-5.9742500	5.9742500	NONE	.	
249		BS	-2.8850455	2.8850455	NONE	.	83406.182
250		BS	-2.4450455	2.4450455	NONE	.	
251		BS	-1.2600455	1.2600455	NONE	.	
252		BS	-.52504545	.52504545	NONE	.	
253		UL	-1.4500000	1.4500000	NONE	.	-15370.560
254		EQ	-2.1720000	2.1720000	NONE	.	-15370.560
255		EQ	-3.1440000	3.1440000	NONE	.	-15370.560
256		EQ	-28.997955	28.997955	NONE	.	-15370.560
257		EQ	-4.2400455	4.2400455	NONE	.	-16301.088
258		EQ	.24200000	.	.24200000	.24200000	-16301.088
259		EQ	.17400000	.	.17400000	.17400000	-17032.032
260		EQ	3.9140000	.	3.9140000	3.9140000	-19468.512
261		EQ	.37300000	.	.37300000	.37300000	-38524.676
262		EQ	.61000000	.	.61000000	.61000000	-47177.676
263		EQ	.40300000	.	.40300000	.40300000	-56073.945
264		EQ	.25000000	.	.25000000	.25000000	
265		EQ	.86700000	.	.86700000	.86700000	
266		EQ	1.0300000	.	1.0300000	1.0300000	
267		EQ	.51200000	.	.51200000	.51200000	
268		EQ	.40400000	.	.40400000	.40400000	

AT 18:38:22 ON 02/19/80

CITY [...] WATER SUPPLY

SOLUTION

ROWS SECTION

NUMBER	...ROW..	AT	...ACTIVITY...	SLACK ACTIVITY	..LOWER LIMIT.	..UPPER LIMIT.	.DUAL ACTIVITY
261	DEMAND	EQ	6.5690000	.	6.5690000	6.5690000	-56073.945
262	STORAGE	EQ	.	.		.	.00593000
263	RELIGB	BS	-5248584.0	5248584.0	NONE	.	.
264	DESTGIOT	BS	.	.	NONE	.	.00593000
265	REL/OR	UL	.	.	NONE	.	.
266	DESTGIOR	BS	-2459808.0	2459808.0	NONE	.	.
267	REL/IPR	EQ	-2459808.0	.	NONE	.	.02163347
268	REL/OR	BS	.	.	NONE	.	.
269	DEST/70308	BS	-131645E03	131645E03	NONE	.	.
270	DEST/80308	BS	-14.563955	14.563955	NONE	.	.
271	COST/80308	BS	.	.	NONE	.	.
272	DEST/10308	EQ	.	.	.	.	15370.560
273	DEST/70310	BS	-18.787295	18.787295	NONE	.	.
274	DEST/80302	BS	-11.725250	11.725250	NONE	.	.
275	DEST/80303	BS	-6.4070455	6.4070455	NONE	.	.
276	DEST/80304	BS	-4.5430455	4.5430455	NONE	.	.
277	DEST/80305	BS	-2.8310455	2.8310455	NONE	.	.
278	DEST/80306	BS	-1.7440455	1.7440455	NONE	.	.
279	DEST/80307	BS	.	.	NONE	.	17695.364
280	DEST/80407	UL	.	.	NONE	.	8653.0000
281	DEST/80408	UL	.	.	NONE	.	3375.3091
282	DEST/80308	BS	-.40400000	.40400000	NONE	.	.
283	DEST/81108	BS	-38.640000	38.640000	NONE	.	.
284	DEST/81208	BS	-7.2370455	7.2370455	NONE	.	.

SOLEISURA WATER SUPPLY

SOLUTION

COLUMNS SECTION

NUMBER	COLUMN	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
	701	BS	38.054250	118060.00	.	NONE	.
	702	BS	15.813250	66896.000	.	NONE	.
	703	BS	21.344000	79130.000	.	NONE	.
	704	BS	18.537000	64900.000		NONE	
	705	BS	16.382000	128088.00		NONE	
	706	BS	15.045000	14941.00		NONE	
	707	BS	12.433955	150641.00		NONE	
	708	BS	11.403955	8653.0000		NONE	
	709	BS	10.891955	105155.00		NONE	
	710	BS	10.891955	17115.000		NONE	
	711	BS	38.640000	67754.000		NONE	
	712	BS	52485x4.0	.12200000		NONE	
	713	BS	2459x08.0	.10100000		NONE	
	714	BS	131845E03	.03680000		NONE	
	715	BS	21.801000	44960.000		NONE	
	7140I	LL	.	.	.	NONE	.
	51401	BS	.			NONE	
	52401	BS	.			NONE	
	53401	BS	.			NONE	
	71501	BS	10157930.			NONE	
	71401	BS	10157930.			NONE	
	71501	LL	.			NONE	
	71601	BS	10.891955	3722.1120		NONE	
	71601	BS	10.891955			NONE	
	71801	LL	10.891955			NONE	
	70101	BS	10.891955	61442.240		NONE	
	70401	BS	10.891955			NONE	
	70401	BS	10.891955	2923.7760		NONE	
	70401	BS	10.891955	4745.9200		NONE	
	70701	BS	10.891955	4443.2000		NONE	
	70801	BS	10.891955			NONE	
	71001	BS	10.891955	22083.840		NONE	
	51701	LL	.			NONE	
	71801	BS				NONE	
	5302	LL	1230x06x03			NONE	
	71302	BS	4.01500000			NONE	
	71402	LL	.			NONE	
	71702	BS	14.616955	930.62400		NONE	
	71602	BS	14.616955			NONE	
	71102	BS	41.664955			NONE	
	70102	BS	38.664000			NONE	
	70202	BS	19.441955	15370.560		NONE	
	70202	BS	4.194000			NONE	

SOLUTION WATER SUPPLY

SOLUTION

COLUMNS SECTION

NUMBER	.COLUMN	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
337	Y0402	BS	14.999955	.	.	NONE	.
338	Y0402	BS	13.990055	.	.	NONE	.
339	Y0402	BS	13.577955	730.94400	.	NONE	.
340	Y0602	BS	13.320955	2436.4800	.	NONE	.
341	Y0702	BS	12.433955	1360.8000	.	NONE	.
342	Y0802	BS	11.376955	.	.	NONE	.
343	Y0902	BS	10.851955	5520.9600	.	NONE	.
344	Y1002	BS	10.437955	.	.	NONE	.
345	S103	LL	.	.	.	NONE	.
346	S203	LL	.	.	.	NONE	.
347	Y1703	BS	.	.	.	NONE	.
348	Y1802	BS	.	.	.	NONE	.
349	S303	BS	130426E03	.	.	NONE	.
350	Y1303	BS	8.1643750	.	.	NONE	.
351	Y1603	BS	3.0373750	.	.	NONE	.
352	Y1503	BS	.	.	.	NONE	.
353	Y1103	BS	16.648455	.	.	NONE	.
354	Y1203	BS	15.224955	620.35200	.	NONE	.
355	Y2003	BS	39.055955	.	.	NONE	.
356	Y1103	BS	38.640000	.	.	NONE	.
357	Y0103	BS	25.845830	10247.040	.	NONE	.
358	Y0203	BS	8.3923750	.	.	NONE	.
359	Y0303	BS	17.136455	.	.	NONE	.
360	Y0403	BS	14.424955	.	.	NONE	.
361	Y0503	BS	13.896955	487.29600	.	NONE	.
362	Y0603	BS	13.568955	1624.3200	.	NONE	.
363	Y0703	BS	12.433955	907.20000	.	NONE	.
364	Y0803	BS	11.083955	.	.	NONE	.
365	Y0903	BS	10.412955	3680.6400	.	NONE	.
366	Y1003	BS	9.8829545	.	.	NONE	.
367	S104	BS	52485840	.	.	NONE	.
368	S204	BS	2459808.0	.	.	NONE	.
369	Y1703	BS	1.4235000	.	.	NONE	.
370	Y1803	BS	.	.	.	NONE	.
371	S304	BS	131645E03	.	.	NONE	.
372	Y1304	BS	14.660250	.	.	NONE	.
373	Y1404	BS	1.4470000	.	.	NONE	.
374	Y1504	BS	6.0747500	.	.	NONE	.
375	Y1604	BS	19.511000	.	.	NONE	.
376	Y1204	BS	21.260000	310.17600	.	NONE	.
377	Y2004	BS	4.2930000	.	.	NONE	.
378	Y1104	BS	38.640000	.	.	NONE	.
379	Y0104	BS	34.054250	4123.5600	.	NONE	.
380	Y0204	BS	15.519250	.	.	NONE	.
381	Y0304	BS	21.344000	.	.	NONE	.
382	Y0404	BS	18.257000	243.64800	.	NONE	.
383	Y0504	BS	16.274000	812.16000	.	NONE	.
384	Y0604	BS	15.045000	453.60000	.	NONE	.
385	Y0704	BS	10.782000	.	.	NONE	.
386	Y0804	BS	5.7130000	.	.	NONE	.
387	Y0904	BS	3.1910000	1840.3200	.	NONE	.
388	Y1004	BS	1.2040000	.	.	NONE	.

CRITICAL WATER SUPPLY

SOLUTION.

COLUMNS SECTION.

NUMBER	.COLUMN	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
		HS	1284768.0			NONE	
		HS	948672.00			NONE	
		HS	1.0980000			NONE	
		HS	2.8470000			NONE	
		HS	101969E03			NONE	
		HS	15.203250			NONE	
		HS	2.0249167			NONE	
		US	.4956667			NONE	
		US	20.042000			NONE	
		HS	19.459000	930.52800		NONE	
		HS	18.608000			NONE	
		HS	38.640000			NONE	
		HS	38.054250	15370.560		NONE	
		HS	15.813250			NONE	
		HS	21.344000			NONE	
		HS	17.326000			NONE	
		HS	15.918000	730.94400		NONE	
		HS	15.045000	2436.4800		NONE	
		HS	12.018000	1360.8000		NONE	
		HS	8.418000			NONE	
		HS	6.627000	5520.9600		NONE	
		HS	5.216000			NONE	
		HS	5248684.0			NONE	
		HS	2459808.0			NONE	
		HS	.94900000			NONE	
		HS	.36600000			NONE	
		HS	50046415.			NONE	
		HS	14.852250			NONE	
		LL	6.0747500			NONE	
		HS	18.954000			NONE	
		HS	21.801000	310.17600		NONE	
		LL				NONE	
		HS	38.640000			NONE	
		HS	38.054250	5123.5200		NONE	
		HS	15.813250			NONE	
		HS	20.947000			NONE	
		HS	18.537000			NONE	
		HS	16.382000	243.64800		NONE	
		HS	15.045000	812.16000		NONE	
		HS	10.411000	453.60000		NONE	
		HS	4.9010000			NONE	
		LL	2.1600000	1840.3200		NONE	105408.00
		HS				NONE	67754.000
		LL				NONE	
		HS				NONE	
		LL	2.8470000			NONE	
		HS	16661455.			NONE	312286.55
		LL	9.4190000			NONE	87264.000
		HS				NONE	

C

SILISTRA WATER SUPPLY

AT 1H:3H:22 ON 02/19/80

SOLUTION

COLUMNS SECTION

NUMBER	COLUMN	AT	...ACTIVITY...	..INPUT COST..	..LOWER LIMIT.	..UPPER LIMIT.	.REDUCED COST.
441	Y1607	HS	17.560955	.	.	NONE	.
442	Y1207	HS	17.560955	620.35200	.	NONE	.
443	Y2007	LL	.	.	.	NONE	63590.400
444	Y1107	HS	9.6420455	.	.	NONE	.
445	Y0107	HS	28.879955	10247.040	.	NONE	.
446	Y0207	HS	9.8390000	.	.	NONE	.
447	Y0307	BS	18.458955	.	.	NONE	.
448	Y0407	HS	16.091955	.	.	NONE	.
449	Y0507	HS	15.121955	487.29600	.	NONE	.
450	Y0607	HS	14.519955	1624.3200	.	NONE	.
451	Y0707	HS	12.433955	907.20000	.	NONE	.
452	Y0807	HS	9.9539545	.	.	NONE	.
453	Y0907	BS	8.7199545	3680.6400	.	NONE	.
454	Y1007	HS	7.7479545	.	.	NONE	.
455	S107	HS	.	.	.	NONE	.
456	S207	HS	.	.	.	NONE	.
457	Y1707	LL	.	.	.	NONE	.
458	Y1807	HS	.	.	.	NONE	.
459	S307	BS	.	.	.	NONE	.
460	Y1308	HS	3.9140000	.	.	NONE	.
461	Y1408	HS	.	.	.	NONE	.
462	Y1508	LL	.	.	.	NONE	.
463	Y1608	BS	14.563955	930.52800	.	NONE	.
464	Y1708	HS	14.563955	.	.	NONE	.
465	Y2008	BS	3.9189545	.	.	NONE	.
466	Y1108	LL	.	.	.	NONE	.
467	Y0108	BS	19.266955	15370.560	.	NONE	.
468	Y0208	HS	4.0880000	.	.	NONE	.
469	Y0308	HS	14.936955	.	.	NONE	.
470	Y0408	HS	13.953955	.	.	NONE	.
471	Y0508	HS	13.550955	730.94400	.	NONE	.
472	Y0608	HS	13.300955	2436.4800	.	NONE	.
473	Y0708	HS	12.433955	1360.8000	.	NONE	.
474	Y0808	HS	11.403955	.	.	NONE	.
475	Y0908	HS	10.891955	5520.9600	.	NONE	.
476	Y1008	HS	10.487955	.	.	NONE	.
477	Y1708	LL	.	.	.	NONE	.
478	Y1808	HS	.	.	.	NONE	.