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Quantification of systemic risk from overlapping portfolios in the financial system

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Abstract

Financial markets create endogenous systemic risk, the risk that a substantial fraction of the system ceases to function and collapses. Systemic risk can propagate through different mechanisms and channels of contagion. One important form of financial contagion arises from indirect interconnections between financial institutions mediated by financial markets. This indirect interconnection occurs when financial institutions invest in common assets and is referred to as *overlapping portfolios*. In this work we quantify systemic risk from indirect interconnections between financial institutions. Complete information of security holdings of major Mexican financial intermediaries and the ability to uniquely identify securities in their portfolios, allows us to represent the Mexican financial system as a bipartite network of securities and financial institutions. This makes it possible to quantify systemic risk arising from overlapping portfolios. We show that focusing only on direct interbank exposures underestimates total systemic risk levels by up to 50% under the assumptions of the model. By representing the financial system as a multi-layer network of direct interbank exposures (default contagion) and indirect external exposures (overlapping portfolios) we estimate the mutual influence of different channels of contagion. The method presented here is the first quantification of systemic risk on national scales that includes overlapping portfolios.

Keywords: systemic risk, overlapping portfolios, financial networks, financial regulation, multi-layer networks

JEL: D85, G01, G18, G21

1. Introduction

Systemic risk (SR) in financial markets is the risk that a significant fraction of the financial system can no longer perform its function as a credit provider and collapses. In a more narrow sense, SR is the notion of contagion or impact that starts from the failure of a financial institution (or a group of institutions)

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and propagates through the financial system and potentially also to the real economy (De Bandt and Hartmann, 2000; Bank for International Settlements, 2010). In a broader sense SR also includes system-wide shocks that affect many financial institutions or markets at the same time (De Bandt and Hartmann, 2000).

SR arises through the probability of default propagating through many different mechanisms and channels of contagion. In addition to default contagion, financial SR arises from asset price shocks and funding liquidity shocks (Haldane and May, 2011). Losses from asset price shocks can result in contagious failures. *Marking to market*, an accounting practice of valuing assets according to current market prices, can induce a further round of assets sales, depressing prices further and lead to "fire sales" (Cifuentes et al., 2005). Liquidity hoarding in interbank funding markets can cascade through a financial network with severe consequences (Gai and Kapadia, 2010; Gai et al., 2011). In Brunnermeier and Pedersen (2009) it is shown that funding of traders affects, and is affected by, market liquidity in a non-trivial way. Market liquidity and funding liquidity shocks are therefore mutually reinforcing and can lead to liquidity spirals.

An important form of financial contagion arises from "indirect" links between financial institutions mediated by financial markets. When financial institutions invest in the same assets, their portfolios are said to overlap. This indirect connection is referred to as overlapping portfolios. Contagion can occur because of shocks that cause common assets to be devalued. Devaluations can cause further asset sales and devaluations leading to fire sales. During "the Great Moderation", a period starting in the mid-1980s until 2007, portfolios of financial institutions became increasingly similar (Haldane, 2013). For example in 2007, before the financial crisis, many large banks around the world held mortgage-backed securities (MBS) and collateralized debt obligations (CDO) in their portfolios. During the U.S. subprime mortgage crisis these banks faced write-downs and losses on the value of these investments. Together the losses on MBS and CDOs due to the subprime mortgage crisis totaled more than $500 billion (Bloomberg, 2012).

Asset commonalities between banks have been considered by Elsinger et al. (2006), who showed how losses due to common exposures dominate those due to direct contagion. In their model, common assets are however only affected by exogenous shocks, while the contagion dynamic is restricted to direct exposures. Cifuentes et al. (2005) are among the firsts to consider a dynamic model of losses due to indirect contagion. They study a model where banks interact through mutual exposures, modeling contagion through the Eisenberg-Noe algorithm, and indirectly due to the presence of an illiquid asset common to all banks. A more general setting is the one considered in Huang et al. (2013), who present a model of cascades on bipartite networks of banks and assets, which they apply as a case study to US commercial banks during the subprime crisis. Caccioli et al. (2014) consider cascades on bipartite networks and show how the amplification of distress can be described in terms of a branching process, which can be used to characterize the stability of the banking system through the analysis of the largest eigenvalue of its transition matrix. Corsi et al. (2016) introduce a stochastic model to study the effect of leverage targeting and financial innovation on the stability of networks of overlapping portfolios. Leverage targeting is also at the basis of the stress testing framework proposed by Greenwood et al. (2015). More recently, Cont and Schaanning (2017) present a framework to quantify indirect exposures due to deleveraging, and they show how these can be computed from the matrix of liquidity-weighted overlaps between portfolios of banks.

Huang et al. (2013); Caccioli et al. (2014); Corsi et al. (2016); Greenwood et al. (2015); Cont and Schaanning (2017) focus on fire-sales and contagion due to overlapping portfolios. Here we add to this previous work by considering the effect of fire-sales and overlapping portfolios in combination with contagion due to direct exposures, with the aim of quantifying the relative importance of the two contagion channels. From the theoretical point of view, the combination of common assets holdings and direct contagion was considered for instance in Cifuentes et al. (2005), Gai and Kapadia (2010) and May and Arinaminpathy (2009). Our analysis complements these papers because it is carried out on a high resolution regulatory data.

Financial contagion can be studied in empirical data through the availability of high-precision financial network data. Driven by data availability, research on financial networks has mainly focused on default contagion: mostly, on direct lending networks between financial institutions (Upper and Worms, 2002; Boss et al., 2004, 2005; Soramäki et al., 2007; Iori et al., 2008; Cajueiro et al., 2009; Bech and Atalay, 2010; Fricke and Lux, 2014; Iori et al., 2015), but also on the network of derivative exposures (Markose et al., 2012; Markose, 2012). Research on financial *multi-layer* networks, that considers multiple channels of contagion, has only appeared recently. Poledna et al. (2015) and León et al. (2014) study the interactions of financial institutions on different financial markets in Mexico and Colombia, respectively. Due to the lack of empirical data, research on networks of overlapping portfolios is practically nonexistent up to now. Caccioli et al. (2015) study two channels of contagion by combining empirical data on direct lending with a stylized model on overlapping portfolios. Greenwood et al. (2015) and Duarte and Eisenbach (2015) analyze networks of common asset exposures in the EU and the US with aggregated data on asset classes.

In this context several network-based SR-measures have been proposed recently (Markose et al., 2012; Billio et al., 2012; Thurner and Poledna, 2013). These approaches bear the notion of the *systemic importance* of a financial institution within a financial network and rely on *network centrality measures* or on closely related measures. A serious disadvantage of centrality measures is that the corresponding value for a particular institution has no clear interpretation as a measure for expected losses. A breakthrough that solves this problem is the so-called DebtRank, a recursive method suggested by Battiston et al. (2012) that quantifies the systemic importance of financial institutions in terms of losses that they would contribute to the total loss in a crisis. Since data on asset-liability networks is hard to obtain outside Central Banks and is not publicly available, there have been several attempts to quantify systemic importance of institutions without the explicit knowledge of the underlying networks (Cooley et al., 2009; Adrian and Brunnermeier, 2011; Acharya et al., 2012, 2013).

In this work we develop a novel method to quantify SR from overlapping portfolios. First, we extend the notion of systemic importance in financial networks to bipartite networks of financial institutions and securities. This makes it possible to assess SR from overlapping portfolios. Second, we compare SR from direct interbank exposures (default contagion) and indirect external exposures (overlapping portfolios). Using the methodology developed in Poledna et al. (2015), we represent the financial system as a multi-layer network and assess SR contributions from direct and indirect exposures.

This work is based on a unique data set containing various types of daily exposures between the major Mexican financial intermediaries (banks) over the period 2004-2013 (for this work we use data from 2009-2013). The data includes detailed information of security holdings of Mexican financial intermediaries by containing the International Securities Identification Number (ISIN) that uniquely identifies every security. The data further contains the capitalization of banks at every month and the market data (pirces) for the various securities. Data were collected and are owned by the Banco de México. Various aspects of the data have been studied before (Martínez-Jaramillo et al., 2010; López-Castañón et al., 2012; Martínez-Jaramillo et al., 2014; Molina-Borboa et al., 2015; Poledna et al., 2015). In this work we focus on the SR arising from overlapping portfolios. With the complete information of security holdings of major financial intermediaries at hand and the ability to uniquely identify securities in the portfolios, we represent the Mexican financial system as a bipartite network of securities and financial institutions. With this data we quantify the SR contributions of direct interbank exposures (default contagion) and indirect external exposures (overlapping portfolios) and estimate the mutual influence of different channels of contagion.

Our paper is structured in the following way. In section 2 we explain the methodology to quantify SR in bipartite and multi-layer networks. In section 3 we describe the data set used for this study. In section 4 we present the results and visualize overlapping portfolios in the Mexican financial system as a bipartite network. Finally, in section 5 we discuss the results.

2. Quantification of systemic risk from overlapping portfolios

2.1. Quantification of systemic risk in bipartite networks

We represent the financial system as a bipartite network S_{ia} of banks and assets. A link connects bank i to asset a if a is in i's investment portfolio. Contagion can occur in the system whenever the same asset is shared by more than one bank. In this case a bank selling off its portfolio can cause losses to other banks with overlapping portfolios. This occurs because of market-impact, that is the tendency of prices to move in response to trading activity. In particular, the price of an asset is expected to drop by an amount that depends on the size of the position that is liquidated. It is then clear that banks with similar portfolios are mutually exposed even if there are no direct linkages between them (in form of e.g. interbank lending). A correct assessment of SR should account for the stress imposed on the system by assets liquidation.

We address this problem by considering a modification of DebtRank for bipartite networks, a methodology recently introduced to identify systemically important banks in a network of mutual exposures (Battiston et al., 2012). It is a quantity that measures the fraction of the total economic value V in the network that is potentially affected by the distress of an individual node (bank) i, or by a set of nodes S. The DebtRank of a set of nodes S that is initially in distress is denoted by R_S. In those cases where only one node i is initially under distress (the set S contains only one node i) we denote the DebtRank of that node by R_i. In the following we use the generalised formulation of DebtRank developed in Bardoscia et al. (2015), for details see Appendix A.

DebtRanks can be calculated from any financial network representing financial interdependencies (Poledna et al., 2015). Although different financial interdependencies are associated with different types of financial risk, the interdependencies can be represented by an *exposure network*. We use the following notation for different exposure types: the size of every exposure of type α of institution i to institution j at time t is given by the matrix element $X_{ij}^{\alpha}(t)$. $\alpha \in \{\text{direct}, \text{OP}\}$ labels the layers *"direct exposures,"* and *"indirect exposures from overlapping portfolios (OP)"* respectively. DebtRank $R_i^{\alpha}(t)$ of layer α is a function $R_i^{\alpha}(X_{ij}^{\alpha}(t), C_i(t), v_i^{\alpha}(t), t)$, where $v_i^{\alpha}(t)$ is the respective economic value of layer α at time t. The links between nodes have the same meaning for all financial interdependencies: it is the total loss that might arise for a bank as the consequence of the default of another. The concept and dimension (dollars) of exposure is the same for all links: it is the total loss that one institution would suffer if a given counterparty defaulted.

The original DebtRank algorithm (Battiston et al., 2012) focuses on direct exposures between banks. In this case, the loss that can arise for bank i because of the default of bank j is simply the value of the direct exposure between i and j. In the case of indirect exposures, like those due to overlapping portfolios, the total loss that bank i can experience because of the default of bank j is related to the devaluation of its assets that can be caused by the liquidation of j's portfolios. To compute this potential loss, we need to compute the impact of j on the value of each asset a, and then the importance of asset a for bank i: Let us consider a network of b banks and m assets, and let us denote its equity by C_i, the number of shares of asset a owned by bank i by S_{ia}, the total number of outstanding shares of asset a by N_a, and the price of asset a by p_a respectively. As a measure of the direct impact of banks on assets we define the matrix

$$W'_{ja} = \frac{p_a S_{ja}}{N_a}, \tag{1}$$

i.e. we assume the impact of bank j on asset a is proportional to the fraction of shares owned by the bank.[1] The underlying assumption here is that of a linear market impact associated with the bank liquidating its position on the asset: Should the bank liquidate its entire position, the price would shift from p_a to $p_a(1 - S_{ja}/N_a)$. The importance of asset a for bank i is simply given by the number of shares i owns of asset a. Therefore, we define the indirect exposure of bank i to bank j from overlapping portfolios as (Guo et al., 2016; Schaanning, 2017)

$$X_{ij}^{\text{OP}} = \sum_a W'_{ja} S_{ia} = \sum_a \frac{p_a S_{ia} S_{ja}}{N_a} \quad . \tag{2}$$

Note that X_{ij}^{OP} is the appropriately weighted bank projection of the weighted bipartite network of banks and assets S_{ia}, so that the dynamic above is equivalent to the standard DebtRank on the projected network of overlapping portfolios. The matrix X_{ij}^{OP} is symmetric, and its diagonal elements are non-zero even though the bipartite network itself has, by definition, no self-loops. Diagonal elements represent the self-inflicted loss of a bank from (rapidly) liquidating its portfolio (market impact). This loss will be high if bank i holds a large fraction of asset a in its portfolio, and is negligible if i holds only a small fraction of asset a.

[1] For a collection of all assumptions see Appendix B.

For direct exposures, it was shown that the DebtRank dynamic can be derived from the balance sheet identity assuming that losses propagate linearly from borrowers to lenders (Battiston et al., 2016; Bardoscia et al., 2015). Similarly, for the case of overlapping portfolios we assume that a bank liquidates a fraction of its portfolio proportional to its relative loss of equity. This is a first assumption that we make on the dynamic of portfolio liquidation. Alternative rules have been considered in the literature, ranging from threshold dynamics (Huang et al., 2013) to leverage targeting (Greenwood et al., 2015). Leverage targeting in the context of DebtRank has also been considered in (Battiston et al., 2016). Our choice of proportional liquidation is a simplifying assumption that provides the smallest departure from the DebtRank algorithm, and allows us to use the DebtRank algorithm on the projected network of overlapping portfolios. Furthermore, and in line with the assumption of DebtRank algorithm, we assume an implicit 0 % recovery rate. This implies that our measure of SR is more conservative with respect to one that would be obtained by considering a non-zero recovery rate.

With respect to the case of leverage targeting - which is consistent with the optimal strategy of a risk-neutral investor subject to capital requirement, and for which Adrian and Shin (2010) provide some empirical evidence - our choice implies a less aggressive deleveraging of banks. This in turn implies that the contribution to SR due to overlapping portfolios that we estimate is a lower bound with respect to the one that would be obtained by considering a leverage targeting dynamics. A second simplifying assumption we make in relation to the liquidation process is that banks do not change the composition of their portfolios as they liquidate. This is a common assumption in the literature on fire-sales (Huang et al., 2013; Greenwood et al., 2015; Cont and Schaanning, 2017), and it has recently been shown (Schaanning, 2017) to be a good approximation of the behavior of large banks. A further assumption we make is that each bank knows the value of the equity of its counterparties at each step of the dynamic. This is required because DebtRank assumes banks to compute the value of their interbank assets using an ex-ante mark-to-market valuation, according to which the value of an interbank asset depends on the value of the equity of the borrower (Battiston et al., 2012; Bardoscia et al., 2015; Barucca et al., 2016; Roncoroni et al., 2019).

To consider contagion from asset liquidation we calculate the DebtRank of the indirect exposure network X_{ij}^{OP},

$$R_i^{\mathrm{OP}} \equiv R_i^{\mathrm{OP}}(X_{ij}^{\mathrm{OP}}, C_i, v_i^{\mathrm{OP}}) \tag{3}$$

where C_i is i's equity and v_i^{OP} i's economic value. Given the current value of assets a in i's investment portfolio, we define its economic value as

$$v_i^{\mathrm{OP}} = \frac{\sum_a p_a S_{ia}}{\sum_j \sum_a p_a S_{ja}} \quad , \tag{4}$$

i.e. the fraction of i's investment portfolio from the total investment portfolios of all banks. Note that if $\sum_i S_{ia} = N_a$, the definition of the economic value in eq. (4) is equal to the definition in eq. (A.1). R_i^{OP} measures the fraction of the total economic value ($V^{\mathrm{OP}} = \sum_i \sum_a p_a S_{ia}$) that is affected by the distress of a bank i from indirect exposure, i.e. from overlapping portfolios.

2.2. Quantification of systemic risk in multi-layer networks

We consider direct exposures and indirect exposures in the framework of a multi-layer network of exposure networks. In our case we study a multi-layer network consisting of two layers: direct exposures and indirect exposures. Direct exposures represent financial exposures from holding “deposits & loans” (DL), “derivatives” (deri), “securities” (secu), and “foreign exchange” (FX). Indirect exposures result from overlapping portfolios as defined in eq. (2).

DebtRank values can also be computed for multi-layer networks (Poledna et al., 2015). Specifically, DebtRank values can be computed for each layer of a multi-layer network separately, or for all layers combined (from the combined exposure network $X_{ij}^{\mathrm{comb}} = \sum_\alpha X_{ij}^{\alpha}$). We refer to the DebtRank of the combined exposure network as R_i^{comb} and the total economic value of the combined exposure network is given by $V^{\mathrm{comb}} = \sum_\alpha V^\alpha = V^{\mathrm{direct}} + V^{\mathrm{OP}}$.

To allow a comparison of R_i^α between different layers, R_i^α must be shown as a percentage of the combined total economic value V^{comb}. The normalized DebtRank for layer α is therefore defined in (Poledna et al., 2015) as

$$\hat{R}_i^\alpha = \frac{V^\alpha}{V^{\mathrm{comb}}} R_i^\alpha \quad , \tag{5}$$

where V^α is the respective total economic value of layer α.

2.3. Quantification of systemic risk at the country level

Using the methodology developed in Poledna et al. (2015), we estimate SR at the country level. The *SR-profile* of a country is defined as the rank-ordered normalized DebtRank $\hat{R}_i^\alpha$ for all financial institutions in a country. The SR-profile shows the distribution of systemic importance across institutions throughout a country. The institution with the highest systemic importance is shown to the very left. The average DebtRank is used to capture the SR of the entire economy (with b institutions),

$$\bar{R}^\alpha = \frac{1}{b} \sum_{i=1}^{b} \hat{R}_i^\alpha \quad . \tag{6}$$

For the combined network, $\hat{R}_i^\alpha$ is replaced by R_i^{comb}, and we write $\bar{R}^{\mathrm{comb}}$ for the combined average DebtRank. Note that $\bar{R}^\alpha$ depends on the network topology of the various layers (or the combined network) only and is independent of default probabilities, recovery rates, or other variables.

3. Data

The data used for this work is derived from a database on exposures at the Mexican Central Bank, built and operated with the specific purpose of studying contagion and SR. This database is maintained by the statistics unit at the financial stability general directorate at this institution. The statistics unit under the financial stability general directorate at Banco de Mexico gathers information and cross-validates it by using daily, weekly and monthly regulatory reports, which are used for regulatory and supervisory purposes. An illustrative and important example is the case of the daily regulatory reports known as “operaciones de captación e interbancarias en moneda nacional y udis” (OCIMN), and “operaciones de captación e interbancarias en moneda extranjera” (OCIME). These reports contain every single funding transaction on a daily basis in local and foreign currency, which are used to compute the daily funding costs for each bank. From these two regulatory reports it is possible to compute the exact daily unsecured exposures between banks, as well as more broadly, those between financial institutions like investment banks, brokerage houses, mutual funds and pension funds. In Solorzano-Margain et al. (2013), a stress testing study was carried out using an (extended set) of these exposures. Given the confidential nature of these transactions, data is kept under strict access control and can only be used for regulatory, supervisory and financial stability purposes. The present work is based on transaction data that is converted to bilateral exposures. Direct and indirect exposures are obtained in the following way.

3.1. Direct exposures

This paper uses data owned by the central bank which comprises exposures arising from different markets and transactions. From this data it is possible to compute gross exposures. The risks considered for building the exposures database include: issuer risk, counterparty risk, credit risk and settlement risk. To compute direct exposures related to issuer risk, information on securities holdings is used. The same information is also used to compute indirect exposures as will be explained later. To compute counterparty risk information on net positions from repos and derivatives is used. The credit risk component is computed by using data on unsecured deposits and loans. Finally, the settlement risk component is computed by using data on foreign exchange transactions. In most previous works, the only type of exposures which were considered was on the unsecured interbank market; nevertheless, this has changed and more jurisdictions are extending the information on the types of exposures in order to perform contagion risk studies. In the Mexican case, the most important type of exposures in terms of amount and SR are exposures that are related to issuer risk, i.e., exposures related to the cross holdings of securities among banks.

3.2. Indirect exposures

The data used for the indirect exposures comes from a database that contains the daily position on securities of each bank and brokerage house. For each day it is possible to determine exactly which securities are held by each bank at the level of individual security and the exact number of them. The number of outstanding securities can be computed; the market price is also obtained from the database.

Banks hold several types of securities: securities issued by non-bank companies, securities issued by commercial and development banks. Given the unique security ID, which is part of the database, we can obtain information on the currency in which the security was issued, the issuer and the number of securities issued. Further information on the specific characteristics of each security, like maturity, coupon type, coupon payments, etc. is available. With this information we represent the financial system as the bipartite network $S_{ia}(t)$ of banks and assets at a particular day t. The dimensions of the bipartite network vary depending on the number of of outstanding securities held by banks on a particular day.

The vector of prices, $p_a(t)$, for the securities is obtained form a database at the Mexican central bank which includes also other relevant aspects for each security. This database is used to compute the number securities issued for each asset ID. Finally, balance sheet data on 43 Mexican banks under study is also available, in particular the capitalization, measured on a monthly scale.

4. Results

4.1. Overlapping portfolios in the Mexican banking system

In fig. 1 we visualize overlapping portfolios in the Mexican financial system as a bipartite network. Nodes represent banks (blue) and assets (red). Links between an asset and a bank exist if a bank holds the asset in its portfolio. Two important aspects of an overlapping portfolio network arise: first, there are some banks that have very independent portfolios and some are even completely isolated form the rest of the banks; second, there is an important degree of overlapping, these are the red nodes at the center of the plot and it is noticeable that many banks are exposed to the same securities.

Figure 2 shows different layers of exposure of the Mexican financial system. The network of direct exposures from derivatives, foreign exchange exposures, and deposits & loans is seen in the top layer (fig. 2(a)). Nodes are shown at the same position in all layers. Node size represents the size of banks in terms of total assets. Nodes i are colored according to their systemic importance, as measured by DebtRank, $R_i^\alpha(t)$, in the respective layer (see section 2.2). Systemically important banks are shown in red; unimportant ones are green. The width of links represents the size of the exposures in the respective layer; link color is the same as the counterparty's node color (DebtRank).

Figure 2(b) shows the network of indirect exposures from overlapping portfolios $X_{ij}^{\mathrm{OP}}(t)$ visualized as the bank-projection of the bipartite network $S_{ia}(t)$ of banks and assets shown in fig. 1. Diagonal elements represent the loss for a bank itself from liquidating its portfolio and are typically larger than the indirect exposure to other banks with similar portfolios, as can be seen in fig. 2(b). The total exposure from overlapping portfolios is about three times larger ($\sum_{i,j} X_{ij}^{\mathrm{OP}}(t) \approx 1 \times 10^{12}$ Mex\$) than the direct exposures ($\sum_{i,j} X_{ij}^{\mathrm{direct}}(t) \approx 3.3 \times 10^{11}$ Mex\$). In fig. 2(c) we show the combined exposures, $X_{ij}^{\mathrm{comb}}(t) = X_{ij}^{\mathrm{direct}}(t) + X_{ij}^{\mathrm{OP}}(t)$. The different layers of exposure of the Mexican financial system are rather dense. The density of the direct exposures layer (fig. 2(a)) is 0.23, while densities of the indirect exposures layer from overlapping portfolios (fig. 2(a)) and for all layers combined (fig. 2(c)) is 0.43 and 0.49, respectively.

Figure 3(a) shows the SR-profile for the combined exposures, R_i^{comb}, (line) and stacked for different layers $\hat{R}_i^\alpha$ (colored bars). The stack bars show the SR contribution due to overlapping portfolios (red) and due to direct exposures (blue) for each bank. The DebtRank of the combined layers (line) is always larger for every bank than the sum of the layers separately, $R_i^{\mathrm{comb}} > \sum_\alpha \hat{R}_i^\alpha$. The reason for this behavior is that there is a contribution to SR due to the interaction between the two layers: The loss of equity experienced by a bank because of its interactions in the first layer are transmitted also to the second layer, and vice versa. These cross-layer contributions are not accounted for by simply summing the individual DebtRanks associated with the two layers.

Figure 3(b) shows the daily *average DebtRank*, $\bar{R}$, from 31 July 2008 to 30 September 2013 for the different layers (stacked) and from the combined network (line). The DebtRank of the combined layers is always larger than the combination of the layers separately and the SR contribution from direct exposures is smaller than the contributions from indirect exposures. The contributions of the exposure types are more or less constant over time. The ratio between SR contribution due to overlapping portfolios and due to direct exposures is on average 1.93 ± 0.97 and the ratio between contribution due to direct

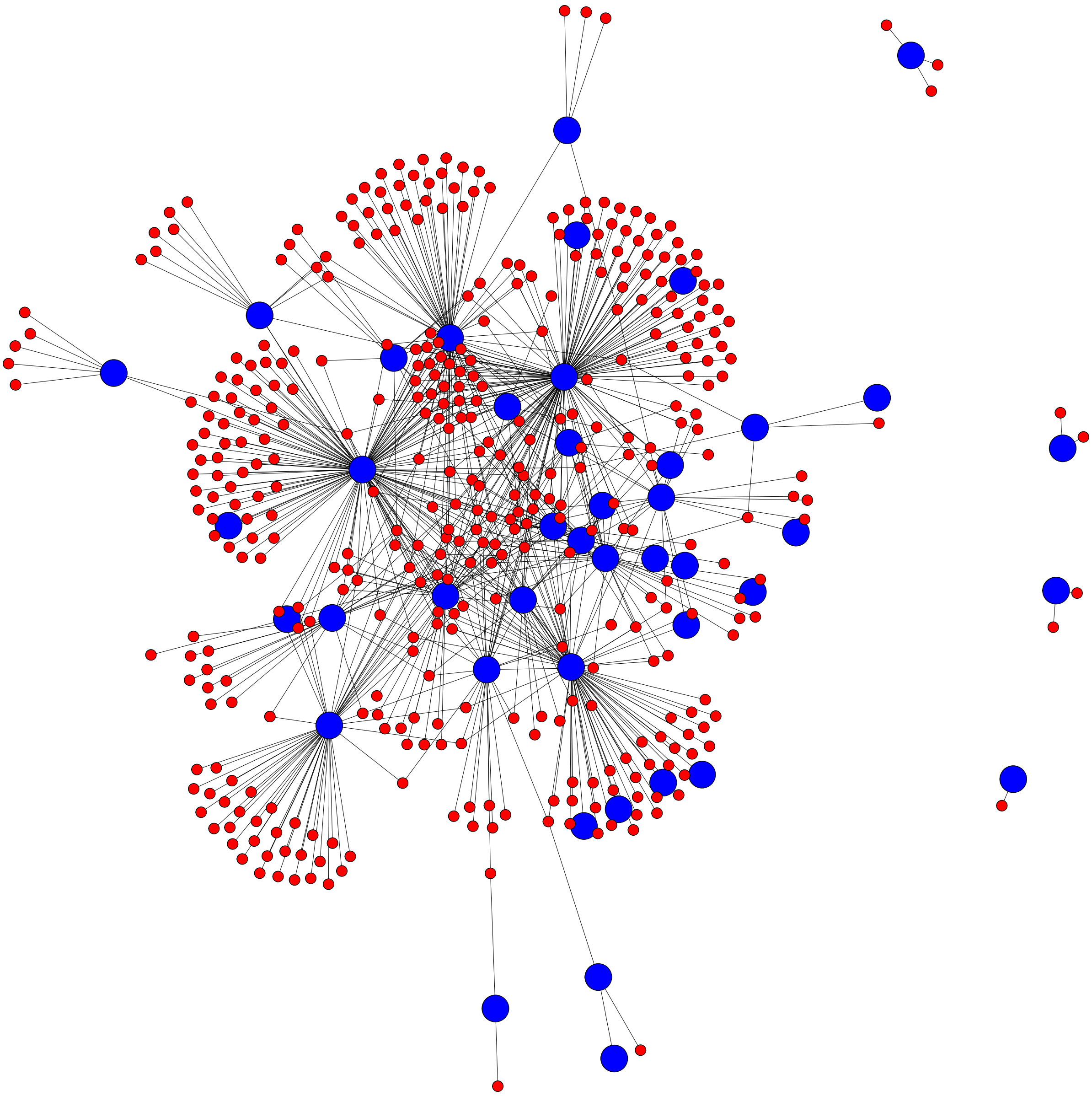

Figure 1: Bank-asset bipartite network of the Mexican financial system on a particular day. Nodes in the network represent banks (blue) and assets (red). Links between an asset and a bank exist if the bank holds the asset in its portfolio.

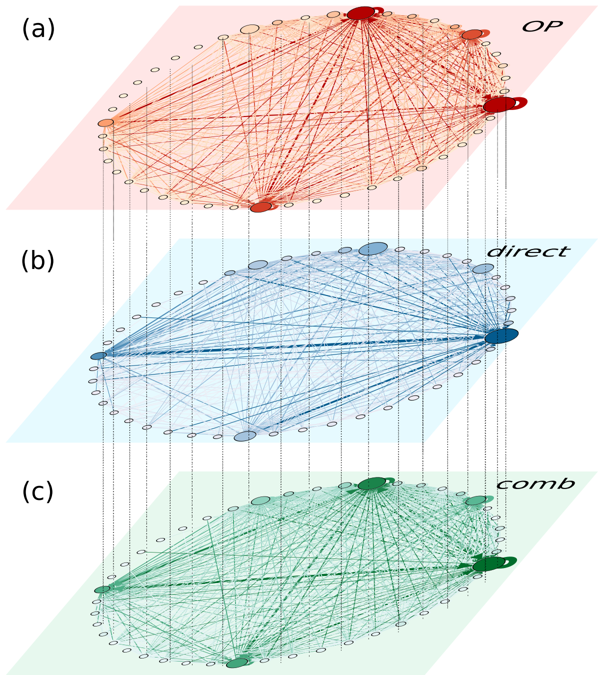

Figure 2: Multi-layer banking network of Mexico. (a) network of indirect external exposures from overlapping portfolios, $X_{ij}^{\mathrm{OP}}(t)$, (b) network of direct interbank exposures $X_{ij}^{\mathrm{direct}}(t)$, and (c) combined banking network $X_{ij}^{\mathrm{comb}}(t)$. The network of indirect exposures is the bank-projection of the bipartite network, $S_{ia}(t)$, of banks and assets. Nodes (banks) are colored according to their systemic importance, R_i^{α}, in the respective layer (see section 2.2): from systemically important banks (red) to systemically safe (green). Node size represents banks' total assets. Link width is the exposure size between banks (re-scaled), link color is taken from the counterparty.

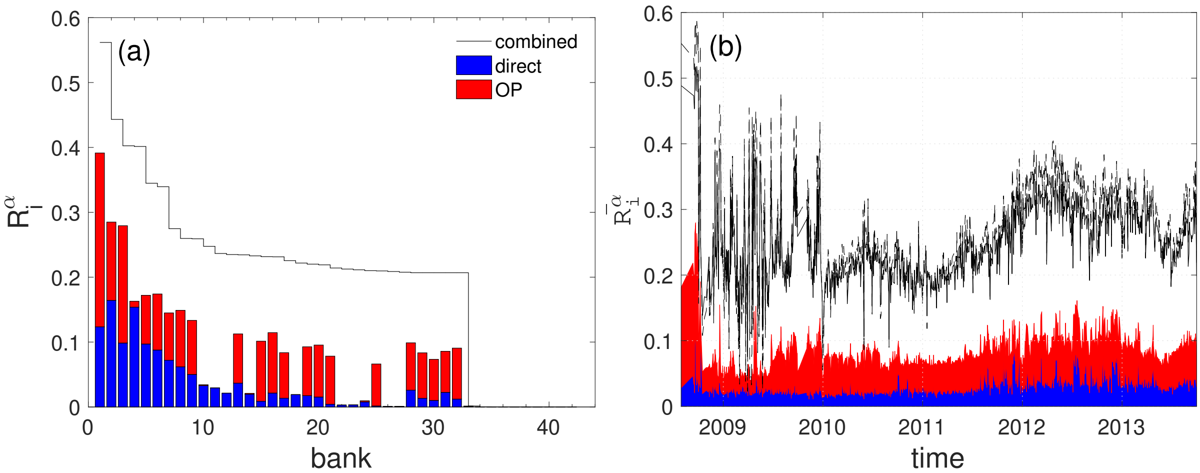

Figure 3: (a) SR profile for the different layers. Normalized DebtRank $\hat{R}_i^{\alpha}$ (see section 2.2) from different layers are stacked for each bank. SR contribution due to overlapping portfolios are shown in red and due to direct exposures in blue. Banks are ordered according to their DebtRank in the combined network from all layers (line). (b) Time series for the average DebtRank $\bar{R}^{\alpha}(t) = \frac{1}{b}\sum_{i=1}^{b} \hat{R}_i^{\alpha}(t)$ for all layers from 31 July 2008 to 30 September 2013. The black and the dashed lines show the average and the median DebtRank for all layers combined, respectively.

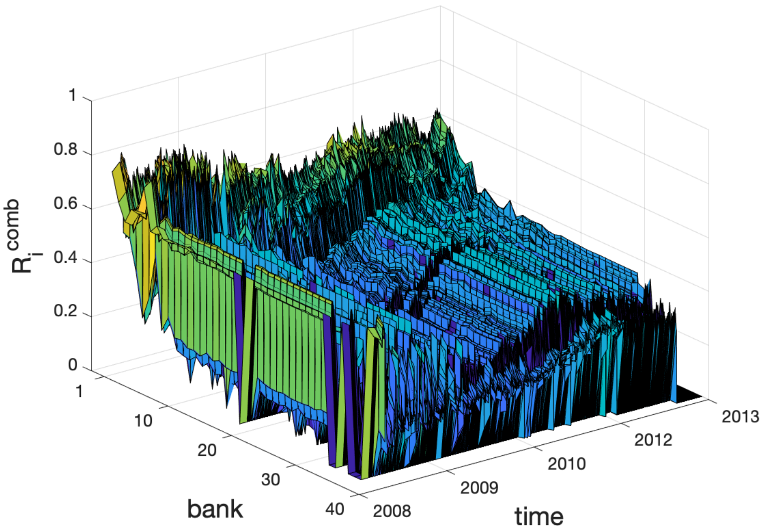

Figure 4: SR surface for the combined network from all layers. DebtRanks in the combined network from all layers ($\hat{R}_i^{\text{comb}}(t)$) are shown for each bank from 31 July 2008 to 30 September 2013. Banks are ordered according to their DebtRanks from 30 September 2013.

exposures and from the combined network is on average 8.77 ± 2.65. Table 1 shows descriptive statistics for four time series from 31 July 2008 to 30 September 2013 for the different layers: i) DebtRank from direct exposures ($\bar{R}^{\text{direct}}$); ii) DebtRank from overlapping portfolios ($\bar{R}^{\text{OP}}$); iii) DebtRank from direct exposures + DebtRank from overlapping portfolios ($\bar{R}^{\text{direct}} + \bar{R}^{\text{OP}}$) and iv) DebtRank of the combined network ($\bar{R}_i^{\text{comb}}$).

Table 1: Descriptive statistics for DebtRanks of the different layers and from the combined network

	$\bar{R}^{\text{direct}}$	$\bar{R}^{\text{OP}}$	$\bar{R}^{\text{direct}} + \bar{R}^{\text{OP}}$	$\bar{R}_i^{\text{comb}}$
Average	0.03	0.05	0.08	0.25
Std. Dev.	0.01	0.02	0.03	0.06
Median	0.03	0.05	0.08	0.24
Min	0.01	0.02	0.04	0.08
Max	0.12	0.22	0.28	0.52

Figure 4 shows the daily DebtRank values in the combined network from all layers ($\hat{R}_i^{\text{comb}}(t)$) for each bank from 31 July 2008 to 30 September 2013. It is noticeable that the most systemically important banks do not change too much over time. At the beginning of the measurement period, at the peak of the 2008 financial crisis, SR was higher for almost all banks. However, after the height of the financial crisis, there is a group of banks that are basically flat in terms of SR and over time. Moreover, there is a cliff between the most systemically important banks and the rest.

5. Conclusion

To a large extent SR arises from indirect interconnections between financial institutions mediated by financial markets. These indirect interconnections occur when financial institutions invest in common assets and are referred to as overlapping portfolios. This work provides, to our knowledge, the first empirical study that quantifies SR arising from overlapping portfolios with uniquely identified securities. By having complete information of security holdings of major Mexican financial intermediaries and the ability to uniquely identify securities in their portfolios we represent the Mexican financial system as a bipartite network of securities and financial institutions. In this study we consider overlapping portfolios from securities issued by non-bank companies and securities issued by commercial and development banks (government bonds are excluded).

By generalising DebtRank, a methodology recently introduced to identify systemically important banks in a network of mutual exposures (Battiston et al., 2012), for bipartite networks we extend the notion of systemic importance in financial networks to bipartite networks of financial institutions and securities. Similar to fire sales models we assume a linear price impact, which captures the spirit of most theoretical models of fire sales (Greenwood et al., 2015). Further, we assume that financial institutions liquidate their portfolios proportional to the relative loss of equity, which provides the smallest departure from the original DebtRank algorithm. Together with the assumption of mark-to-market valuation and an implicit 0 % recovery rate, we thus present a SR measure in order to evaluate the worst possible outcome.

Finally, by representing the financial system as a multi-layer network of direct interbank exposures (default contagion) and indirect external exposures (overlapping portfolios), we estimate the mutual influence of different channels of contagion. We show that focusing only on direct interbank exposures underestimates total SR levels by up to 50% under the assumptions of the model.

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Appendix A. DebtRank

DebtRank is a recursive method suggested in Battiston et al. (2012) to determine the systemic importance of nodes in financial networks. It is a number measuring the fraction of the total economic value in the network that is potentially affected by the distress of a node or a set of nodes. In the following we present the generalised formulation of DebtRank developed in Bardoscia et al. (2015). For the sake of simplicity let us think of the nodes in financial networks as banks. X_{ij} denotes the exposure network (loans of bank j to bank i), and C_i is the capital of bank i. Given the total outstanding interbank exposures of bank i, $X_i = \sum_j X_{ji}$, its economic value is defined as

$$v_i = X_i / \sum_j X_j \quad . \tag{A.1}$$

The impact of bank i on bank j can be defined in terms of the matrix of interbank leverage (Battiston et al., 2016)

$$W_{ij} = \frac{X_{ij}}{C_i} \quad . \tag{A.2}$$

We denote by $h_i(t) = (C_i(0) - C_i(t))/C_i(0)$ the relative loss of equity between iterations 0 and t of the dynamic. By iterating the balance sheet identity it is possible to write the set of equations

$$h_i(t) = \min \left[1, h_i(1) + \sum_j W_{ij} h_j(t-1) \right] \quad , \tag{A.3}$$

where $h_i(1)$ is the exogenous shock that causes the initial distress to the system. For instance for a financial networks with B nodes, the initial conditions with $S \subseteq B$ nodes initially in distress could be $h_i(1) = \Psi \,, \forall i \in S;\; h_i(1) = 0 \,, \forall i \notin S$ (where the parameter Ψ quantifies the initial level of distress: $\Psi \in [0, 1]$, with $\Psi = 1$ meaning default).

The DebtRank of the set S (set of nodes in distress at time 1), is $R'_S = \sum_j h^*_j v_j - \sum_j h_j(1) v_j$, where we denote by h^*_i the fixed point of equation (A.3). R'_S measures the distress in the system due to network effects, as it excludes the initial distress. If S is a single node, the DebtRank measures its systemic importance. The DebtRank of S containing only the single node i is

$$R'_i = \sum_j h^*_j v_j - h_i(1) v_i \quad . \tag{A.4}$$

The DebtRank, as defined in eq. (A.4), excludes the loss generated directly by the default of the node itself and measures only the impact on the rest of the system through default contagion. For some purposes, however, it is useful to include the direct loss of a default of i as well. The total loss caused by the set of nodes S in distress at time 1, including the initial distress is

$$R_S = \sum_j h^*_j v_j \quad . \tag{A.5}$$

Appendix B. Assumptions

- We assume the impact of bank j on asset a is proportional to the fraction of shares owned by the bank. The underlying assumption here is that of a linear market impact associated with the bank liquidating its position on the asset: Should the bank liquidate its entire position, the price would shift from p_a to $p_a(1 - S_{ja}/N_a)$.

- We assume that a bank liquidates a fraction of its portfolio proportional to its relative loss of equity. This is a first assumption that we make on the dynamic of portfolio liquidation.
- A second simplifying assumption we make in relation to the liquidation process is that banks do not change the composition of their portfolios as they liquidate. This is a common assumption in the literature on fire-sales (Huang et al., 2013; Greenwood et al., 2015; Cont and Schaanning, 2017), and it has recently been shown (Schaanning, 2017) to be a good approximation of the behavior of large banks.
- We assume an implicit 0 % recovery rate. This implies that our measure of SR is more conservative with respect to one that would be obtained by considering a non-zero recovery rate.
- With respect to the case of leverage targeting - which is consistent with the optimal strategy of a risk-neutral investor subject to capital requirement, and for which Adrian and Shin (2010) provide some empirical evidence - our choice implies a less aggressive deleveraging of banks. This in turn implies that the contribution to SR due to overlapping portfolios that we estimate is a lower bound with respect to the one that would be obtained by considering a leverage targeting dynamics.
- A further assumption we make is that each bank knows the value of the equity of its counterparties at each step of the dynamic. This is required because DebtRank assumes banks to compute the value of their interbank assets using an ex-ante mark-to-market valuation, according to which the value of an interbank asset depends on the value of the equity of the borrower (Battiston et al., 2012; Bardoscia et al., 2015; Barucca et al., 2016; Roncoroni et al., 2019).