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A CASE STUDY IN HIERARCHICAL CONTROL - THE UPPER VISTULA MULTIRESERVOIR SYSTEM

Kazimierz A. Salewicz
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PREFACE

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis including economic, social and environmental evaluation of water resources development alternatives aided by application of mathematical modeling techniques, to generate inputs for planning, design, and operational decisions.

This paper is part of the comparative studies on operational decisionmaking in the multiple reservoir water resource systems initiated in 1979 by the "Regional Water Management" Research Task of the Resources and Environment Area of IIASA.

Following introduction to some basic concepts of a hierarchical approach to the control of complex systems, the model of the Upper Vistula System in Poland is presented and the results of preliminary computations are discussed.

The research presented in this paper has been carried out by the Institute of Automatic Control of the Technical University of Warsaw, Poland, and the Institute of Meteorology and Water Management, Warsaw, Poland, in collaboration with IIASA.

Janusz Kindler
Chairman
Resources & Environment Area

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CONTENTS

A CASE STUDY IN HIERARCHICAL CONTROL - THE UPPER VISTULA MULTIRESERVOIR SYSTEM

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1. INTRODUCTION

The purpose of this paper is to provide a general description of the research which is carried out jointly by the Institute of Automatic Control, the Technical University of Warsaw, and the Institute of Meteorology and Water Management in collaboration with the International Institute for Applied Systems Analysis, in the field of the application of the hierarchical control methods to multireservoir system operation. The research was undertaken in 1977, and from the beginning, the theoretical problems of the hierarchical control were formulated on the basis of analysis of problems which occur in reality, when the operation of multiple reservoir systems is considered. Simultaneously, the multireservoir system of Upper Vistula was chosen as a case study and the appropriate model of this system was formulated. The aim of continued and still expanding research is to provide a methodology and a suitable set of models which can be used in the future, when the operational centre of the Upper Vistula System will be established.

In this paper, basic concepts of a hierarchical approach to the multireservoir system operation are discussed. The model of the case system is described and results of some computations are included.

2. HIERARCHICAL CONTROL STRUCTURE AND ITS PROPERTIES

The theory of hierarchical control has been extensively investigated for many years and many authors have given a relevant contribution in this field, for instance Mesarovic et al., [1970], Findeisen [1974] and many others. Various aspects of hierarchical control of dynamical systems are discussed by Findeisen [1978], Malinowski and Findeisen [1978], and Malinowski [1978]. At the same time, some concepts of the hierarchical approach have been applied for water management (see Haimes [1978]), and operation of multiple reservoir systems (Malinowski, Salewicz, and Terlikowski [1979]). In the latter paper, some results concerning the application of the discrete feedback control method to water management systems are reported.

2.1 General Concepts of the Hierarchical Approach

Usually, control structure involves a system S with state variables x, manipulated (decision or control) variables m, u, disturbances (external influences) z and observations v; and the Control Unit which is responsible for realization of goals of control expressed in terms of performance index J and respective constraints. The scheme of control structure is shown in Figure 1. Only in simple cases can the Control Unit be designed physically in one place and its decision-making mechanism has a homogenous form. This means that the interventions of the Control Unit cannot be distinguished with respect

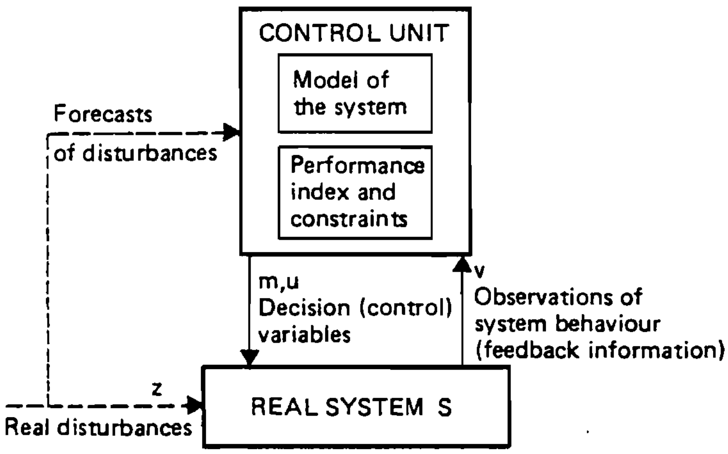

CONTROL UNIT
Model of the system
Performance index and constraints
Forecasts
of disturbances
m,u
Decision (control) variables
v
Observations of system behaviour (feedback information)
z
Real disturbances
REAL SYSTEM S

Figure 1.

to the way in which decision (control) variables are adjusted. Many complex control problems can be more effectively solved by designing hierarchical Control Units. There are two fundamental concepts of the hierarchical approach to designing a Control Unit, namely vertical and horizontal decomposition of control tasks. The vertical decomposition is equivalent to separation of actions which are performed by a Control Unit with respect to different frequencies of decision-making. Therefore, the vertical decomposition results from partitioning the control activity into several subproblems which are solved independently and on different time scales. Thus, the Control Unit consists of several layers generating the decisions influencing the behaviour of the controlled system with different frequencies. The highest layer of the Control Unit is making its decisions with the lowest frequency, but over the longest time horizon T. At the same time, the lowest layer is making its decisions with the highest frequency, but over the shortest time horizon. Decisions undertaken by a specified layer of the Control Unit may be influenced by the higher layers only, but not overridden. It is worthwhile to notice that the higher the layer of the structure is, the more aggregated model of the controlled system it uses, and its state variables, decision variables, and disturbances incorporated into the model are more aggregated. The principle of vertical decomposition may be illustrated as in Figure 2, where the two-layer structure of the Control Unit is shown. Horizontal decomposition of the control task is associated with the partition of information and competence of decision-making among several simpler subproblems. This kind of decomposition of the control task is very closely related to

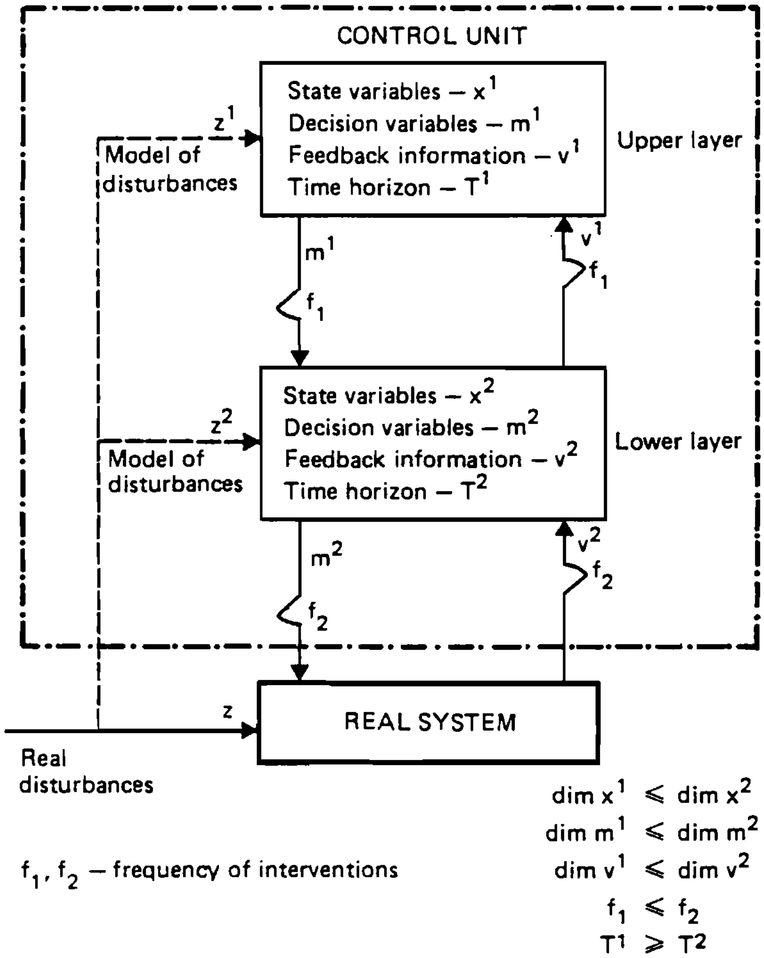

CONTROL UNIT
State variables – x1
Decision variables – m1
Feedback information – v1
Time horizon – T1
Upper layer
z1
Model of disturbances
m1
f1
v1
f1
State variables – x2
Decision variables – m2
Feedback information – v2
Time horizon – T2
Lower layer
z2
Model of disturbances
m2
f2
v2
f2
z
REAL SYSTEM
Real disturbances
f1, f2 – frequency of interventions
dim x1 ⩽ dim x2
dim m1 ⩽ dim m2
dim v1 ⩽ dim v2
f1 ⩽ f2
T1 ⩾ T2

Figure 2.

spatial decomposition of the control unit when control of vast system is considered, and it is possible to distinguish parts of the Control Unit, called Local Decision Units (LDU), which may be associated with specific parts of the controlled system, and so-called subsystems. If Local Decision Units act independently, then the control structure is fully decentralized. In an opposite case only one decision unit may be distinguished, and in such a case, there is fully centralized control structure. However, it is interesting to consider a partially decentralized control structure when Local Decision Units are influenced by a special unit called coordinator, which influences decisions of LDUs using chosen incentives. In Figure 3, an example of a partially decentralized control unit is shown with a coordinator which influences Local Decision Units associated with respective subsystems of a controlled real system.

2.2 Goals of Control in a Water Management System

The major objectives of the water management system in an industrial region are to secure water supply for the industrial and municipal water-users. At the same time, concentration of pollutants in the rivers should be maintained at the levels compatible with water quality requirements. The mutual relationships among processes occurring in a water management system can be described by means of the reservoir balance equations, the flow balance equations formulated for the selected cross-sections, and the pollutants balance equations. All variables describing the phenomena taking place in the system can be segregated into three groups:

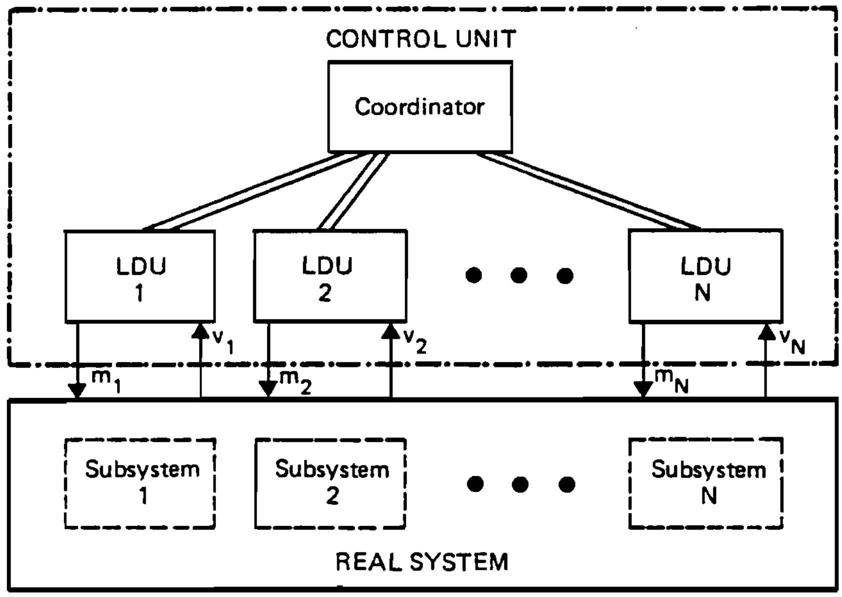

CONTROL UNIT
Coordinator
LDU 1
LDU 2
LDU N
v1
v2
vN
m1
m2
mN
Subsystem 1
Subsystem 2
Subsystem N
REAL SYSTEM

Figure 3.

- state variables w referred to as the volumes of water stored in the reservoirs belonging to the system;
- decision (control) variables referred to as releases from the reservoirs u and flow rates in conduits delivering water to specified users and denoted by m;
- disturbances or exogenous variables which are equivalent to natural inflows, d, to the system, water demands of users denoted by z, and pollutants loads denoted by S.

Therefore, discretized equations describing mutual relationships among control variables, disturbances, and state variables over the time horizon of N discretization intervals (let us say the length of these intervals is equal to 1 week), can be written in the following form:

$$w^{i+1} = w^i + D \cdot d^i - U \cdot u^i - M \cdot m^i \qquad (1)$$

where

i = number of discrete time interval, $i=1,2,\ldots,N$;

w^i = vector of values of state variables at the end of i-th time interval and dim w = number of reservoirs in the system considered;

d^i = vector of forecasted inflows to the reservoirs at time interval i;

m^i = vector of flow rates of water withdrawn at i-th time interval by specified users;

u^i = vector of releases from the reservoirs at i-th time interval;

D,M,U = matrices indicating dependence of the state of respective reservoir on coordinates of vectors d, m and u respectively;

w^o = vector of initial values of state variables.

Objectives of the system operation may be expressed in a mathematical form of the performance index J, which is used to measure the effects of the system operation. These effects may be evaluated from two points of view. The first of them is related to effects of short-term system operation, while the second one is related to long-term system operation. Therefore, the performance index of the system is composed of two parts: the first is associated with the effects of short-term operation while the second part of the performance index is related to long-term operation of the system. Effects of short-term operation of the system may be easily expressed in terms of functions which depend on water deficits affecting specified water-users and departures from the desired water quality standards expressed in terms of admissible pollutant concentrations c. Thus, the first part of the performance index J may be formulated as follows:

$$J_I = \sum_{i=1}^{N} \{[\sum_{\gamma\varepsilon\Gamma} f_\gamma^i(z_\gamma^i, m_\gamma^i)] + [\sum_{\alpha\varepsilon\Delta} g_\alpha^i(s_\alpha^i, u_\alpha^i, c_\alpha)]\} \qquad (2)$$

where

Γ = set of specified water-users withdrawing water from the system, while γ denotes respective users from this set;

Δ = set of control cross-sections on rivers belonging to the system, where water quality is controlled. The elements of this set, respective cross-sections, are denoted by α;

$i=1,2,\ldots,N$ is the number of discrete time interval, while N, the number of time intervals, is equivalent to the length of the time horizon on which operation of the system is considered;

z^i_γ = water demands of user γ at time interval i;

m^i_γ = water supply of user γ at time interval i (decision variable);

$f^i_\gamma(.,.)$=function evaluating losses of user γ at time interval i associated with water deficit (if such exists);

s^i_α = pollution load in cross-section α at time interval i;

The second part of the system's performance index is associated with the system operation over a time horizon longer than N discretization intervals. When we consider planning the system operation over the time horizon of N intervals, we have to take into account that the system will be operated also in the future, which will follow the N-th interval. Therefore, we are interested in the determination of such conditions in the system that will provide for satisfactory results of current (short-term) operation and may also assure (at the end of N intervals long time horizon), proper operation of the system in the future. It means that trajectories of the system's state variables should reach, with some accuracy, the desired range of values at the end of N-th time interval. The current operation of the system should follow up with some accuracy the predetermined, long-term operation trajectories. This accuracy may result from the compromise between realization of the current goals of the system operation, and the necessity of assuring proper operation of the system in the future. Therefore, we may introduce a second part of the system's performance index which evaluates the effects of departures of state trajectories from the predetermined long-term operation trajectories denoted by vector $\bar{w}$, and the respective part J_{II} of the performance index is formulated as:

$$J_{II} = \sum_{i=1}^{N} \{ \sum_{k=1}^{\dim w} f_k^i \; (w_k^i, \bar{w}_k^i) \} \tag{3}$$

where:

k = is the index of the particular reservoir in the system, while the number of reservoirs in the system is equal to dim w = dim $\bar{w}$;

$f_k^i(.,.)$ = express the losses caused by the departure of state trajectory w_k of reservoir k from the desired value $\bar{w}_k$ at the end of i-th time interval.

The total performance index J, which comprises two different goals of the system operation is given, therefore, as:

$$J = J_I + J_{II} \tag{4}$$

The operation of the system cannot violate any of the important constraints such as constraints on decision variables and state variables expressed in terms of inequality-type constraints. At the same time respective flow-balance equations and pollutant balance equations formulated for considered cross-sections in the rivers of the system should be satisfied.

As was mentioned, operation of the multireservoir system involves two kinds of activities concerning long-term and short-term operation. Consequently, the structure of the Control Unit for such a system should be constructed with respect to these two aspects of system operation. Assuming existence of two basic types of activities of the Control Unit in a water management system, a two-layer control structure is proposed, involving:

- Long-term operation planning at the higher layer which is equivalent to determination of the storage plan in the system over a long time horizon (let us say, 6 months or 1 year), using suitable long-term forecasts and aggregated information about the system as a whole;
- Short-term operation or current dispatching of the water resources, performed at the lower layer of the Control Unit. The elements of solution obtained at the higher layer of the Control Unit will be used for proper short-term system operation together with respective short-term forecasts and other updated information concerning the system.

In the following section, some details concerning these two layers of the Control Unit are discussed.

2.3 The Upper Layer - Long-Term Operation Planning

At the upper layer of the Control Unit, a long-term operation plan for the whole system is determined, over the time horizon of N time periods ahead. There are numerous methods and approaches which may be applied for the solution of this problem (see for instance, Prekopa et al., [1978], Gal [1979], however, in this paper, our attention will be focused on the so-called price coordination method (see Lasdon [1970], Mesarovic et al., [1970], or Malinowski [1975]), or Interaction Balance Method (IBM). Therefore, a long-term operation plan is determined using optimization techniques with a completely deterministic formulation of the optimization problem. In such a case, the uncertainty of the long-term forecasts influences the credibility of the results of optimization; however, the negative effects of errors in the forecast can be decreased by repetition of the long-term operation plan determination using updated forecasts (see Nowosad [1978]). Thus, the long-term storage policy w over N-time interval horizon may be obtained as the result of optimization of the performance index (4) with respect to decision variables m and u, subject to constraints given in the form of state equations (1); respective inequality-type constraints on decision variables m, u denoted symbolically as m, u ε *MU* where set *MU* is well defined, and constraints on state variables w. Let us now introduce a vector of auxiliary decision variables defined as:

$$a^i = D \cdot d^i - U \cdot u^i - M \cdot m^i \tag{5}$$

and therefore state equation (1) takes the following form (see (1):

$$w^{i+1} = w^i + a^i \tag{6}$$

where i = 1,2,...,N, w^o - given initial value. Consequently, the upper-layer optimization problem of:

$$\min_{(m,u)} \quad J(m,u,w) \tag{7}$$

subject to:

- state equation (1);
- inequality-type constraints on decision variables m, u ε *MU* and state variables;
- flow-balance and pollutant load balance equations formulated for a specified control cross-section in the system,

can be reformulated to a modified form of the upper-layer optimization problem:

OP:

$$\min_{(m,u,a)} \quad J(m,u,w\ (a)) \triangleq J_{*}(m,u,a) \qquad (8)$$

subject to:

- balance equations (5) and flow balance and pollutant load balance equations formulated for a specified control cross-section in the system;
- inequality-type constraints on decision variables $m,u \in MU$ and state variables;

where w(a) is determined by equation (6). The optimization problem OP is solved by using the so-called price coordination method (or IBM). Following introduction of the price vector p (vector of the Lagrangian multipliers), whose elements are time-dependent, the Lagrangian function of the OP can be formulated as:

$$\begin{aligned} L(m,u,a,p) &= J_{*}\ (m,u,a) + \sum_{i=1}^{N} \{<p^{i}, a^{i} - D \cdot d^{i} + U \cdot u^{i} + M \cdot m^{i}>\} = \\ &\sum_{i=1}^{N} \{[\sum_{\gamma \varepsilon \Gamma} f_{\gamma}^{i}(z_{\gamma}^{i}, m_{\gamma}^{i})] + [\sum_{\alpha \varepsilon \Delta} g_{\alpha}^{i}(S_{\alpha}^{i}, u_{\alpha}^{i}, c_{\alpha})] + \\ &+ <p^{i}, U \cdot u^{i} + M \cdot m^{i} - D \cdot d^{i}>\} + \qquad (9) \\ &+ \sum_{i=1}^{N} \{[\sum_{k=1}^{\dim w} f_{k}^{i}(w_{k}^{i}(a), \bar{w}_{k}^{i})] + <p^{i}, a^{i}>\} \quad , \end{aligned}$$

where p^{i} denotes the value of the price vector at i-th time interval. It is clear that dim w = dim p.

As was mentioned, the optimization problem OP is solved using a two-level price coordination method, and the structure of the algorithm consists of a coordinator and a lower level.

At the lower level, the task is to minimize, for the given price vector p, the Lagrangian function (9) with respect to the decision variables m,u,a, subject to respective constraints; or more formally, we can define the lower level task as the Infimal Problem:

<u>IP</u>:

for a given sequence of values of price vector p at time intervals $i = 1,2,\ldots,N$

$$\min_{(m,u,a)} L(m,u,a,p)$$

subject to inequality-type constraints on decision variables

$m,u \in MU$.

Therefore, the dual function $\varphi(p)$ is defined as:

$$\varphi(p) = \arg\min_{(m,u,a)} \{L(m,u,a,p)\} \quad (10)$$

The upper level problem (Supremal Problem or Coordinator Problem) is consequently defined as:

<u>CP</u>:

$$\max_{p = (p^1,p^2,\ldots,p^i,\ldots,p^N)} \varphi(p) \quad . \quad (11)$$

As a result of optimization performed by the upper layer of the control structure, the following elements are obtained:

- optimal values $\hat{m}$ and $\hat{u}$ of decision variables;
- optimal planned trajectories of state variables $\hat{w}$, and
- coordinating prices $\hat{p}$ defined as:

$$\hat{p} = \arg\max_{p} \varphi(p) \quad . \quad (12)$$

Looking at the Lagrangian function (9), one can easily notice that it may be decomposed (divided) into several independent, so-called, local, problems. Such an opportunity results from the additive form of the Lagrangian function and the separability of inequality-type constraints on decision (control) variables. This decomposition property of the Lagrangian function will be explored when the lower layer activity of the control structure will be discussed.

2.4 The Lower Layer - Current Water Dispatching

The task of the lower layer of a control structure is to make direct, current decisions (i.e. to determine the direct control variables m,u). This is done in such a way as to rationalize the realization of current goals, subject to constraints resulting from the long-term storage policy. At each lower layer intervention, a short time horizon is taken into account. Therefore, only a short-term storage plan (resulting from the applied storage policy) is needed for decision-making.

There are two main features of the presented structure of the lower layer: decentralization in making direct decisions, and application of a price mechanism for influencing these decisions. The first feature appears when process of direct control is partitioned between $\mathcal{N}$ local decision units (LDUs, see 2.1). The second one appears in establishing the coordinator, which influences the LDUs decisions with the aid of prices. Thus we obtain the two-level structure of the lower layer:

- the lower level consists of $\mathcal{N}$ independent LDUs : each of them optimizes its local current goals taking into account prices set up by the upper level;

- the upper level is a coordinator, which sets the prices for LDUs in such a way, that storage of the reservoirs in real controlled system is consistent with an accepted storage plan.

The scheme of the lower layer with a price mechanism is presented in Figure 4.

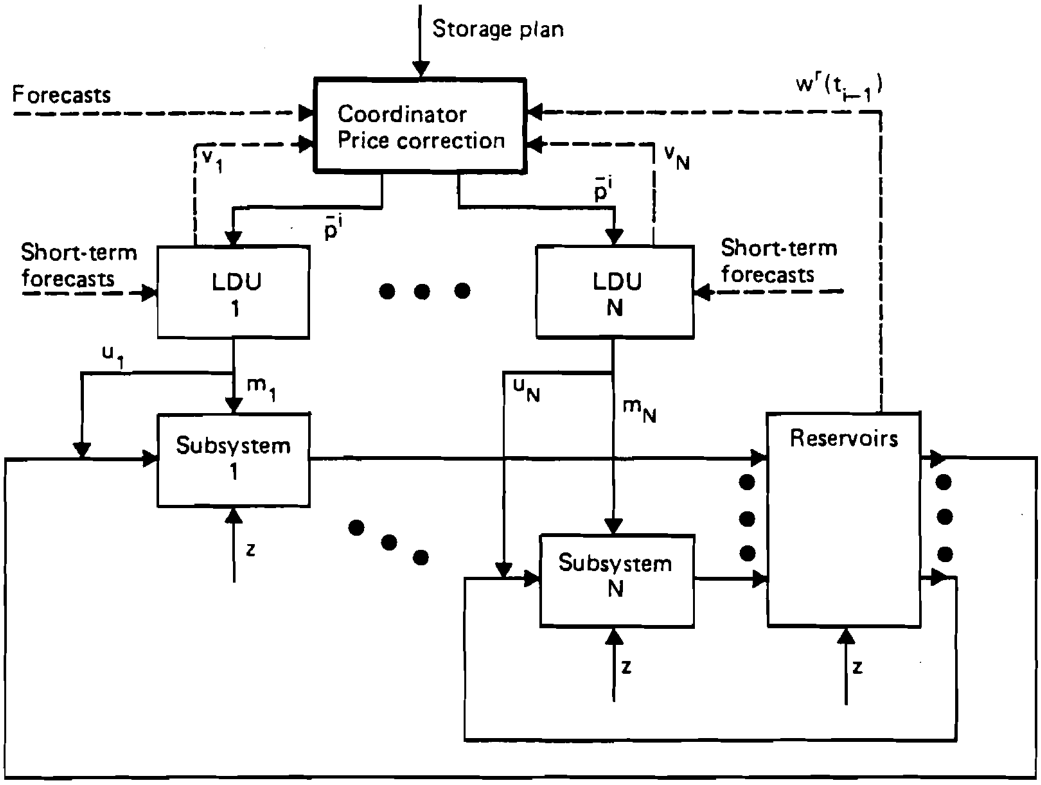
The activity of the lower level is expressed as the minimization of a properly defined Lagrangian function, and it may be interpreted as the optimization of current goals with regard to the prices of water. If we denote the short time horizon (by Δ_i) over which all LDUs act independently, and prices fixed by the coordinator over this horizon (by $\bar{p}^i$) then the direct decisions m^i, u^i are defined over Δ_i as a solution of the following optimization problem:

$$\min_{(m^i,u^i)\in MU} \{[\sum_{\gamma\in\Gamma} f^i_\gamma(\bar{z}^i_\gamma, m^i_\gamma)] + [\sum_{\alpha\in\Delta} g^i_\alpha(\bar{S}^i_\alpha, u^i_\alpha, c_\alpha)] + + \langle p^i, U \cdot u^i + M \cdot m^i - D \cdot \bar{d}^i \rangle\} \quad . \tag{13}$$

Let us assume that (13) may be decomposed into $\mathcal{N}$ independent decision problems having the following form (such a decomposition can be done for the water system, which is shown in section 4):

$$\min_{(m^i_j,u^i_j)\,\varepsilon\, MU_j} f^i_{oj}(m^i_j, u^i_j, \bar{z}^i_j, \bar{S}^i_j, \bar{d}^i_j, p^i) \tag{14}$$

$j = 1,\ldots,\mathcal{N}$. The components f^i_{oj} result from (13) and m^i_j, u^i_j, etc., are the respective subvectors of m^i, u^i, etc. In the above decision problems, $\bar{z}^i$, $\bar{d}^i$ and $\bar{S}^i$ are respectively the short-term forecasts of water demands, natural inflows and uncontrolled pollutant load discharges used by LDUs over Δ_i.



$V_1,\ldots,V_N$ - feedback information from LDU's.

z - real disturbances.

Figure 4. Lower layer using the price mechanism.

Each problem (14) expresses the performance of a LDU concerned with a respective part of the system. It is clear that local decisions are not overridden by the central unit; they are decided upon by local decision-makers who take into account the prices of water. For example, the amount of water withdrawn by an independent water-user is determined by himself, who takes into account only his actual demands of water and actual prices, $\bar{p}^i$, of water.

Successively, the coordinator adjusts the prices $\bar{p}^i$ in such a way, that direct control of the system, affected by $\bar{p}^i$, result in the desired balance of reservoirs over the time period Δ_i. For this purpose, the central unit (coordinator) has to use some model of the system controlled by LDUs , and--applying some algorithm for adjusting the prices $\bar{p}^i$--determines $\bar{p}^i$ in such a way as to obtain the desired effects. The model is expressed (in every Δ_i) by some vector (function $G_i(\cdot)$), depending on price (vector p). $G_i(p)$ defines--to the best of the coordinator's knowledge--the expected value of imbalance of reservoirs at the end of period Δ_i; i.e., a difference between the state of the reservoirs, for a given price p, and the desired state, resulting from the storage plan over Δ_i. The task of the coordinator is then to choose $\bar{p}^i$, at which $G_i(\bar{p}^i)$ is satisfactorily close to zero. This requirement is called the <u>coordination condition</u>, and the performance of the coordinator in finding the price $\bar{p}^i$--<u>price correction</u>.

The concrete realization of the whole lower layer proceeds thus: the way of the LDU's action (e.g., forecasts in (14)), the form of the coordination condition, the form of the function $G_i(\cdot)$, and the algorithm for price correction--all depend on the concrete possibilities and needs. One of the main features of any concrete realization of the lower layer is its information structure, i.e., the range and way of using current observations and forecasts by LDUs as well as by the coordinator. The LDUs are able and should use the actual short-term forecasts over Δ_i; especially those concerning their respective water demands. This assumption concerning the behaviour of LDUs seems to be the most realistic. On the other hand, the coordinator has the possibility of correctly modifying his model (expressed, for example, by function $G_i(\cdot)$), in very different ways. The basic information feedback from the real system to the coordinator is the real value of the reservoir's state $w^r(t_{i-1})$, measured at the beginning of each time period Δ_i. This feedback is an indispensable condition for correctness and efficiency of the coordinator's performance. At the same time, the coordinator has the possibility of adjusting further his model of the controlled system either by a direct use of the actual short-term forecasts, or by communication with the LDUs during the price correction process. Detailed analysis of these problems is given by Terlikowski [1979]. It is to be indicated that only the proposed structure for current control (i.e., the lower layer) is flexible with respect to admissible information structure of the system. It is possible to use the current information in various ranges or forms; in particular, it can be partitioned into separate areas (decentralized into subsystems) without the necessity of centralization. This means, for example, that LDUs may use their local, more precise, information and at the same time, the coordinator does not need to know this information in its entire primary form, but only in an aggregated form, obtained during communication with the LDUs, while computing the value of function $G_i(p)$.

In the basic version of the lower layer, it is assumed that the storage policy for the coordinator is described by trajectory $\hat{w}$, defined as a solution of the upper layer problem (2.3). This means that the short-term storage plan over each Δ_i results from $\hat{w}$; i.e., function $G_i(\cdot)$ has a form:

$$G_i(p) = \tilde{w}(t_i) - \hat{w}(t_i) \quad , \qquad (15)$$

where $\tilde{w}(t_i)$ is the value of the state variable, expected by the coordinator at the end of period Δ_i. Obviously, $\tilde{w}(t_i)$ depends on p, $w^r(t_{i-1})$, and on the forecasts used by LDUs and by the coordinator. The coordination condition is the following:

$$|G_i(\bar{p}^i)| < \alpha \cdot |G_i(\hat{p}^i)|, \quad \alpha < 1 \quad , \qquad (16)$$

where $\hat{p}^i$ is the price obtained from the long-term operation planning, determined by the upper layer (see 2.3). For this coordination condition, the following finite algorithm of price correction has been proposed by Malinowski and Terlikowski [1978]:

$$p^{i(k)} = p^{i(k-1)} - \varepsilon_i^k \cdot A_i^k \cdot G_i(p^{i(k-1)}) \qquad (17)$$
$$p^{i(0)} = \hat{p}^i, \quad p^{i(no)} = \bar{p}^i \quad .$$

The analysis of the properties of such algorithms (based on the theory of contraction algorithms) was developed by Malinowski [1978], and the efficiency analysis of the lower layer activity with such algorithms by Malinowski and Terlikowski [1978], and also by Terlikowski [1979]. The whole scheme of the considered control structure is given in Figure 5. The possibility of applying some finite and simple algorithms for the coordinator's performance is an advantageous feature of a control structure. It allows making entire use of the decentralized informational structure of the system.

General analysis of the thus defined lower layer (see Findeisen et al., [1979]), as well as the results of the computational experiments (see section 4 of this paper), indicate that the coordinator's operation (potentially) assures the desired balance of the real system (i.e. properly matched parameters, for example, $\alpha, |\Delta_i|$), according to the long-term storage plan. On the other hand, application of the price mechanism implies that current water dispatching is performed in the most rational way. This means formally, that, if LDUs, for example, use the accurate (i.e. consistent with occurring reality) short-term forecasts, then all controls determined by them are strictly optimal for the current goals J_I, subject to this storage which is realized (i.e. trajectory w^r):

$$(m,u) = \arg \min_{(m,u) \in MU} J_I$$

s.t. state balance constraints: (18)

$$w(t_i) - w(t_{i-1}) = w^r(t_i) - w^r(t_{i-1}) \quad .$$

Thus, no matter how the coordinator operates, and what the storage policy is, the price mechanism assures a rational current allocation of resources. In this sense, the proposed control structure is very rational and brings a real improvement in comparison to

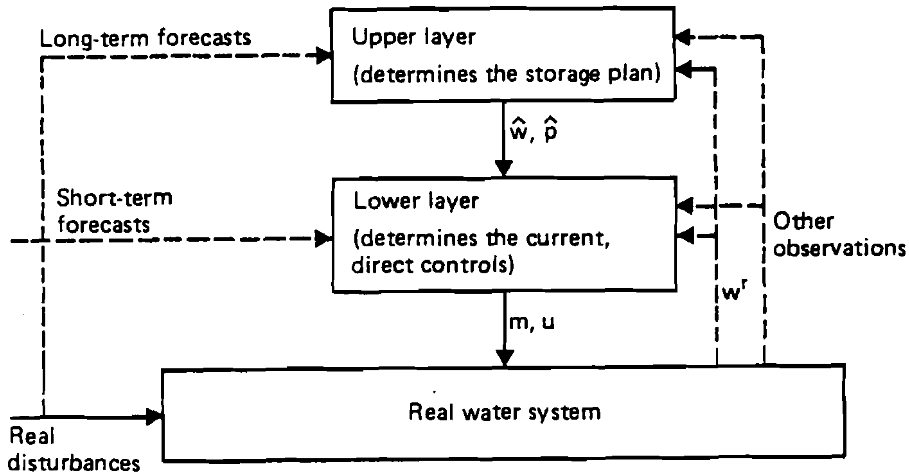


Figure 5. The whole control structure and the role of its layers.

other methods of current control, e.g., by the so-called stiff decision rules (see Kindler et al., [1979]).

In conclusion, the lower layer of the proposed control structure is flexible with respect to the information--competence structure of the system as well as to the structure of actual preferences, and enables an effective and simple realization of balancing storage reservoirs. Simultaneously, the concept of using a price mechanism, which is separated from any particular storage policy, always assures the rational current dispatching of water resources. Notice, that in the primary, simplest approach to control structure, as presented above, the problem of storage policy is entirely included in the task of the upper layer. However, it can be easily seen that the basic concept of the lower layer is adaptable, in a simple way, to another situation, while the storage plan is changed more frequently, or obtained by different methods other than solving the long-term operation planning problem. Moreover, we can imagine, that the lower layer changes the storage plan by itself, introducing its own elements into the whole storage policy. This problem, which is still not sufficiently theoretically analyzed, seems to be very important, if not the most important, for the practical realization of the lower layer. The proper weighting of current goals compared to "dynamic" goals concerned with reservoirs storage during the current control, is a very difficult problem, and still needs much investigation. The same refers to determination of the basic storage policy; i.e., the way of determining the storage plan by the upper layer. All the basic elements of control structure, presented in this paper, form only a general framework of the decision-making structure, which may be useful for aiding the dispatcher's decisions.

3. DESCRIPTION OF THE CASE SYSTEM MODEL

A general layout of the Upper Vistula system is shown in Figure 6. The system includes four storage reservoirs:

- the Goczałkowice reservoir (referred to as G) located on the small Vistula River;
- Tresna, Porabka and Czaniec reservoirs located on the Soła River. The Tresna reservoir is referred to as T, while Porabka and Czaniec are jointly referred to as C.

The major objectives of the system are to secure the water supply for the industrial and municipal water-users, namely Katowice and Bielsko; to supply the steel works "Katowice" via the Dziećkowice reservoir, and to supply water to the chemical plant Oświecim and fish farms around the town of Kety. At the same time, concentration of pollutants which are discharged mainly to the Vistula River downstream of the outlet of the Przemsza River should be maintained at the levels compatible with water quality requirements.

A model of the case system was formulated by Salewicz [1978], and its short description can be found in Kaczmarek et al., [1979], or in Malinowski et al., [1979], but for the completeness of the paper, it will be presented here.

3.1 Formulation of the Upper Vistula System Model

As it was stated above, the case system consists of four storage reservoirs, but for modeling purposes, only three of them are distinguished: G, T and C. Because the purpose of the model is to describe relationships between flow rates in the rivers and in the conduits delivering water to users over a long

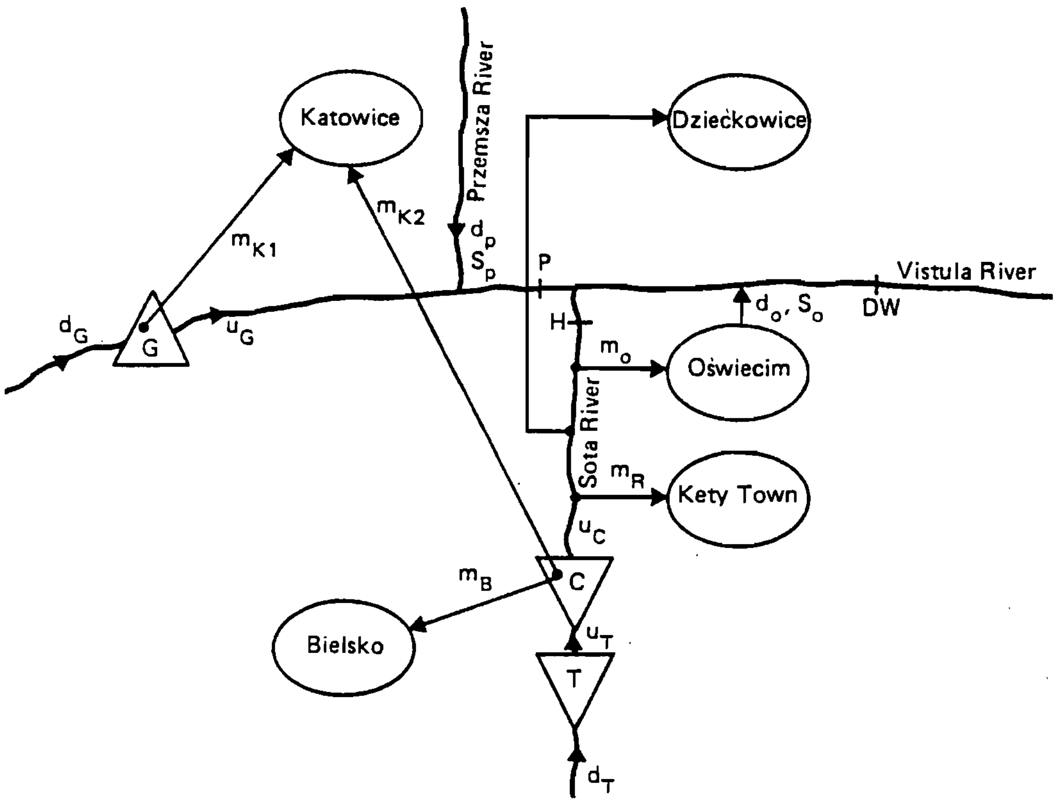

Katowice
Przemsza River
Dzieckowice
mK2
mK1
dp
Sp
P
Vistula River
dG
G
uG
H
do, So
DW
mo
Oświecim
Sota River
mR
Kety Town
uC
mB
C
Bielsko
uT
T
dT

Figure 6.

time horizon (let us say 6 months), only the dynamics of the storage reservoir will therefore be considered, while effects of dynamics of flow in the river channels are neglected because it does not influence the dynamics of reservoirs over the considered time horizon. In such a case, one can distinguish three state variables w_G, w_T, and w_C, which are referred to as volumes of water stored in the reservoirs Goczałkowice, Tresna and Czaniec, respectively. For brevity, the following subscripts are introduced:

B	- refers to Bielsko;
K1 and K2	- refers to Katowice;
R	- refers to fish farms;
O	- refers to Oswiecim, and
D	- refers to Dzieckowice,

while d_G and d_T denote inflows to the reservoirs; outflow from the Przemsza River is denoted by d_p, water demands of the users are denoted by z, with the respective subscript and pollutant load discharges denoted by S_p and S_o. The variables listed above are considered in the model as the external variables and are called <u>disturbances</u>. All "disturbance" variables are handled in the model as the long-term forecasts and are understood as the most probable realizations of natural phenomena. According to the introduced notation, we are able to write state equations for the system reservoirs and flow-balance equations formulated for the selected cross-sections (denoted respectively by P, H and DW). State equations of reservoirs are the following:

$$w_G^{i+1} = w_G^i + d_G^i - m_{K1}^i - u_G^i \tag{19}$$

$$w_T^{i+1} = w_T^i + d_T^i - u_T^i \tag{20}$$

$$w_C^{i+1} = w_C^i + u_T^i - m_{K2}^i - u_C^i \tag{21}$$

where

i = 1,2,...,N and N is the length of the optimization time horizon (e.g., N = 26 weeks);

w_G^o, w_T^o, w_C^o = initial values of state variables.

The flow-balance equations for the considered cross-section are as follows:

$$u_P^i = u_G^i + d_P^i + \alpha_V d_G^i \tag{22}$$

$$u_H^i = u_C^i - m_R^i - m_D^i - m_o^i + \alpha_S d_T^i \tag{23}$$

$$u_{DW}^i = u_P^i + u_H^i + d_o^i \quad , \tag{24}$$

while the pollutant load balance equation at the cross-section DW is the following:

$$s_{DW}^i = s_P^i + s_o^i \quad . \tag{25}$$

All these equations are valid for i=1,2,...,N. In equations (22) and (23), there are terms $\alpha_V d_G$ and $\alpha_S d_T$ which are used to evaluate the additional inflow to the river downstream of the reservoir because it was assumed that the additional inflow is correlated with the inflow to the reservoir located on the same river; of course $\alpha_V, \alpha_S \geq 0$.

3.2 Performance Index for the Upper Vistula System Model

For the case system, the first part of the performance index J, which is responsible for evaluating effects of the current (short-term) operation of the system, is formulated as:

$$
\begin{aligned}
J_I = \sum_{i=1}^{N} \{ & v_B (z_B^i - m_B^i)_+^2 + v_D (z_D^i - m_D^i)_+^2 + \\
& + v_o (z_o^i - m_o^i)_+^2 + v_K (z_K^i - m_{K1}^i - m_{K2}^i)_+^2 + \\
& + v_R^i (z_R^i - m_R^i)_+^2 + v_P \left(\frac{S_P^i}{c_{KP}} - u_G^i - d_P^i - \alpha_v d_G^i\right)_+^2 + \\
& + v_{DW} \left(\frac{S_{DW}^i}{c_{KD}} - u_G^i - d_P^i - \alpha_v d_G^i - u_H^i - d_o^i\right)_+^2 + \\
& + c_T (u_T^i - u_{TM})^2 \} \quad .
\end{aligned}
\tag{26}
$$

Function $(z-m)^2$ is defined as follows:

$$
(z-m)^2 = \begin{cases} 0 & \Leftrightarrow \; z \leq m \\ (z-m)^2 & \Leftrightarrow \; > m \quad . \end{cases}
\tag{27}
$$

In equation (26), symbols v with respective subscripts denote weighting coefficients, while c_{KP} and c_{KD} denote values of pollutant concentrations which should not be exceeded at cross-sections P and DW. Therefore terms:

$$
v_P \left(\frac{S_P}{c_{KP}} - u_G - d_P - \alpha_v d_G\right)_+^2 \quad \text{and}
$$

$$
v_{DW} \left(\frac{S_{DW}}{c_{KD}} - u_G - d_P - \alpha_v d_G - u_H - d_o\right)_+^2 \quad ,
$$

express "losses" associated with exceeding the desirable concentration c_{KP} or c_{KD} of pollution indices at the control cross-section P and DW. The term $c_T(u_T - u_{TA})^2$ was introduced to the performance index in order to obtain the convexity of

J_I with respect to u_T, and the weighting coefficient $c_T << 1$, while u_{TA} is the desired release from the Tresna reservoir. It is worthwhile to notice that the weighting coefficient v_R depends on time, but it results from the fact that the relative importance of such a water-user like fish farms depends strongly on the stage of fish growth.

The second part of the performance index J_{II} expressing "costs" of the system operation over time horizon longer than N, is formulated as follows:

$$J_{II} = \sum_{i=1}^{N} \{ r_G^i (\bar{w}_G^i - w_G^i)^2 + r_T^i (\bar{w}_T^i - w_T^i)^2 + r_C^i (\bar{w}_C^i - w_C^i)^2 \} \qquad (28)$$

where:

r_g, r_T and r_C are time-dependent weighting coefficients. The total performance index J, which comprises two different goals of the system operation, is therefore given as:

$$J = J_I + J_{II} \quad . \qquad (29)$$

To complete the description of the model, constraints on the state variables and control variables shall be introduced. The lower bounds on the state variables can be relatively easily determined; however, the upper bounds should be determined with respect to the flood phenomena, especially during the spring or summer period. For the modeling purposes, it was assumed that constraints on state variables are given in the following form:

$$W_{k\,min} \leq W_k^i \leq W_{k\,max}^i \qquad (30)$$

where:

k = G,T,C = the reservoir's index;

i = 1,2,...,N = index of the time interval;

W min = lower bound;

W^i_{max} = time-dependent upper bound.

The constraints on decision (control) variables m and u are determined in the following way. The amount of water delivered to each of the specified users except Katowice, is constrained by the upper bound which is equal to water demand of the user and the lower bound:

$$m_{min} \le m^i \le z^i, \quad i = 1,2,\dots,N \quad . \tag{31}$$

Flow rates in two conduits delivering water to Katowice are constrained by the minimal and maximal capacities of the pumping facilities:

$$\begin{aligned} m_{K1m} &\le m^i_{K1} \le m_{K1M} \\ m_{K2m} &\le m^i_{K2} \le m_{K2M} \quad . \end{aligned} \tag{32}$$

Releases from reservoirs are constrained by minimum acceptable flows to downstream reservoirs, or at cross-section H:

$$\begin{aligned} u_{Gmin} &\le u^i_G \\ u_{Tmin} &\le u^i_T \\ u_{Hmin} &\le u^i_H, \; i=1,2,\dots,N \quad . \end{aligned} \tag{33}$$

Following the equation (5), we define vector a of net inflow to reservoirs as:

$$a_G^i = d_G^i - m_{K1}^i - u_G^i \tag{34}$$

$$a_T^i = d_T^i - u_T^i \tag{35}$$

$$a_C^i = u_T^i - m_B^i - m_{K2}^i - u_C^i \tag{36}$$

for all i = 1,2,...,N, while the state equations of reservoirs are:

$$w_G^{i+1} = w_G^i + a_G^i \tag{37}$$

$$w_T^{i+1} = w_T^i + a_T^i \tag{38}$$

$$w_C^{i+1} = w_C^i + a_C^i \quad . \tag{39}$$

Following introduction of the price vector $p = (p_G, p_T, p_C)$, the Lagrangian function of the system is given as (see also (9)):

$$\begin{aligned} L(m,u,a,p) = \sum_{i=1}^{N} \{ & v_B(z_B^i - m_B^i)_+^2 + \\ & + v_D(z_D^i - m_D^i)_+^2 + v_O(z_O^i - m_O^i)_+^2 + v_R^i(z_R^i - m_R^i)_+^2 + \\ & + v_K(z_K^i - m_{K1}^i - m_{K2}^i)_+^2 + c_T(u_T^i - u_{TA})_+^2 + \\ & + v_P\left(\frac{S_P}{c_{KP}} - u_G^i - d_P^i - \alpha_V d_G^i\right)_+^2 + \\ & + v_{DW}\left(\frac{S_P^i + S_O^i}{c_{KD}} - u_G^i - d_P^i - \alpha_V d_G^i - u_C^i + m_R^i + m_O^i \right. \\ & \left. + m_D^i + \alpha_S d_T^i - d_O^i\right)_+^2 + r_T^i(\bar{w}_T^i - w_T^i)^2 + r_G^i(\bar{w}_G^i - w_G^i)^2 + \\ & + r_C^i(\bar{w}_C^i - w_C^i)^2 + p_T^i[a_T^i - (d_T^i - u_T^i)] + \\ & + p_G^i[a_G^i - (d_G^i - m_{K1}^i - u_G^i)] + p_C^i[a_C^i - (u_T^i - m_B^i - m_{K2}^i - u_C^i)]\} \end{aligned} \tag{40}$$

Therefore, for the price method, the Infimal Problem (IP) (see 2.3) is given as:

$$\min_{(m,u,a)} \quad L(m,u,a,p) \quad , \tag{41}$$

subject to inequality constraints (31)÷(33) and state equations (35)÷(37), while the Supremal Problem (SP) is given by (11) as it was indicated in section 2.3. The Lagrangian function (40) may be decomposed into seven independent Local Problems, because this function has the additive form and the respective constraints (31)÷(33) are separable with respect to decision (control) variables.

These local problems are therefore formulated for all $i = 1,2,\ldots,N$ in the following manner:

(1)

$$\min_{(m^i_{K1},\, m^i_{K2})} \quad [v_K(z^i_K - m^i_{K1} - m^i_{K2})^2_+ + p^i_G \cdot m^i_{K1} + p^i_C \cdot m^i_{K2}] \tag{42}$$

subject to

$$m_{K1m} \leq m^i_{K1} \leq m_{K1M}$$

$$m_{K2m} \leq m^i_{K2} \leq m_{K2M} \quad .$$

(2)

$$\min_{(m^i_B)} \; [v_B(z^i_B - m^i_B)^2_+ + p^i_C \cdot m^i_B] \tag{43}$$

$$\text{s.t.} \quad m_{Bm} \leq m^i_B \leq z^i_B \quad .$$

(3)

$$\min_{(m^i_O)} \; [v_O(z^i_O - m^i_O)^2_+ + p^i_C \cdot m^i_O] \tag{44}$$

$$\text{s.t.} \quad m_{Om} \leq m^i_O \leq z^i_O \quad .$$

(4) $$\min_{(m_D^i)} [v_D(z_D^i - m_D^i)^2 + p_C^i \cdot m_D^i] \tag{45}$$

$$\text{s.t. } m_{Dm} \leq m_D^i \leq z_D^i$$

(5) $$\min_{(m_R^i)} [v_R^i(z_R^i - m_R^i)^2 + p_C^i \cdot m_R^i] \tag{46}$$

$$\text{s.t. } m_{Rm} \leq m_R^i \leq z_R^i$$

(6) $$\min_{(u_T^i)} [c_T(u_T^i - u_{TA})^2 + (p_T^i - p_C^i) \cdot u_T^i] \tag{47}$$

$$\text{s.t. } u_{Tmin} \leq u_T^i$$

(7) $$\min_{(u_G^i, u_H^i)} [v_P(\frac{S_P^i}{c_{KP}} - \alpha_v d_G^i - u_G^i - d_P^i)_+^2 + $$

$$+ v_{DW}(\frac{S_{DW}^i}{c_{KD}} - \alpha_v d_G^i - d_P^i - d_o^i - u_G^i - u_H^i)_+^2 +$$

$$+ p_G^i \cdot u_G^i + p_C^i \cdot u_H^i] \tag{48}$$

$$\text{s.t. } u_{Gmin} \leq u_G^i$$

$$u_{Hmin} \leq u_H^i \quad .$$

Each of the Local Problems (42) - (48) expresses the performance of LDU concerned with a respective part of the system.

4. DESCRIPTION OF THE NUMERICAL EXAMPLE AND THE COMPUTATIONAL RESULTS

This section of the paper is devoted to the description of computations which have been performed for the Upper Vistula System. First, the results obtained from the long-term optimization of the systems performance index are described; that is, the results which are generated by the upper layer of the proposed control structure. Next, the activity of the lower layer of the control structure is simulated and the results of these computations are presented together with their discussions and interpretations.

4.1 Upper Layer Optimization - Long-Term Storage Planning

The task of the upper layer of the control structure, as well as the method of its solution, was described in Section 2.3 of this paper. Solution of this problem can be obtained for the following data:

- the time horizon of optimization - N weeks;
- forecasts of uncontrolled inflows to the system, namely d_T, d_G, d_P, d_O;
- forecasts of water demands z_W, z_B, z_D, z_O, z_R;
- forecasts of pollution load discharges S_P, S_O;
- weighting coefficients in a performance index;
- intial values of state variables w^O;
- desired trajectories of state variables $\bar{w}$;
- respective bounds for specified constraints.

The following data were used for the illustrative computations:

- N = 12 weeks; however it is possible to expand this value up to 26 weeks;
- forecasts of uncontrolled inflows:

d_G:	10.0	12.0	15.0	12.0	14.0	13.5	
	11.0	9.0	9.5	10.0	11.0	12.5	(m^3/s);
d_T:	13.0	13.0	16.5	15.0	13.0	15.0	
	12.0	11.0	10.2	9.8	8.0	10.0	(m^3/s);

d_P:	4.0	4.5	5.0	7.0	6.0	6.5	
	4.8	3.8	3.5	3.5	3.8	4.0	(m^3/s);
d_O:	2.8	2.7	2.7	2.5	2.7	2.7	
	2.5	2.7	2.7	2.7	2.7	2.7	(m^3/s);

- forecasts of inflows to the reservoirs during a six-week period are shown in Figures 10 and 11;
- forecasts of pollution load discharges:

S_P:	0.35	0.35	0.35	0.36	0.37	0.35	
	0.35	0.35	0.36	0.37	0.38	0.37	(kg/s);
S_O:	0.04	0.05	0.06	0.06	0.03	0.07	
	0.10	0.05	0.06	0.02	0.02	0.03	(kg/s);

- forecasts of water demands:

z_K:	9.0	9.0	9.0	9.0	9.0	9.0	
	10.0	10.0	10.0	10.0	10.0	10.0	(m^3/s);
z_B:	3.0	3.0	3.0	3.0	3.0	3.0	
	3.5	3.5	3.5	3.5	4.0	4.0	(m^3/s);

$z_D = 2(m^3/s) = \text{const.}$ for all $i = 1,\dots,12$

$z_O = 3(m^3/s) = \text{const.}$ for all $i = 1,\dots,12$

z_R:	1.0	1.0	2.0	5.0	6.0	6.0	
	7.0	7.0	7.0	7.0	6.0	2.0	(m^3/s).

- weighting coefficients in the performance index:

$v_B = 10.0$

$v_K = 10.0$

$v_O = 4.0$

$v_D = 3.0$

$v_P = 5.0$

$v_{DW} = 5.0$

$C_T = 0.01$

v_R^i vary between 1 and 10,

weighting coefficients of terms associated with following the desired trajectory of reservoirs vary between 10^{-5} and 10^{-7},

- initial values of state variables:

$$w_G^o = 100.0$$
$$w_T^o = 80.0$$
$$w_i^o = 25.0 \ (\text{mln } m^3);$$

- constraints on state and decision variables:

$$45 \le w_G^i \le 150$$
$$30 \le w_T^i \le 100$$
$$14 \le w_O^i \le 30 \ (\text{mln } m^3)$$
$$3.5 \le m_{K1}^i \le 5.5$$
$$3.5 \le m_{K2}^i \le 5.5$$
$$2.0 \le m_B^i \le z_B^i$$
$$0.5 \le m_O^i \le z_O^i$$
$$1.0 \le m_D^i \le z_D^i$$
$$1.0 \le m_R^i \le z_R^i$$
$$2.0 \le u_T^i \le 100.0$$
$$2.0 \le u_G^i$$
$$5.0 \le u_H^i \qquad (m^3/s)$$

for all i = 1,2,...,12.

The following values of additional coefficients appearing in the performance index (see (29)) of flow-balance equations (see (22)÷(24)) have been fixed:

$$\alpha_V = 0.3$$
$$\alpha_S = 0.25$$
$$C_{KP} = 16 \quad (\text{mg/l})$$
$$C_{KD} = 14 \quad (\text{mg/l})$$

As a result of optimization performed using the price-coordination method, the following outcomes have been obtained:

- coordinating prices $\hat{p}_T$, $\hat{p}_G$, $\hat{p}_C$ are shown in Table 1:
- optimal trajectories of reservoirs T, G and C which are shown in Figures 7, 8 and 9, respectively.

Other results of optimization, such as releases from the reservoirs, and values of users' supply, are not shown because they are not used by the lower layer of the control structure, which is responsible for short-term (direct) control of the system. The only results of the upper layer to be used during the simulations of control are optimal trajectories of reservoirs shown in Figures 7, 8, and 9 and coordinating prices $\hat{p}_r$, $\hat{p}_G$ and $\hat{p}_C$. However, results of optimization indicate that there is a forecasted scarcity of water in the system and there is a necessity to limit water supply to the users (because prices are positive).

Table 1.

Number of week	$\hat{p}_T$	$\hat{p}_G$	$\hat{p}_C$
1	2.050	1.667	2.031
2	2.048	1.628	2.049
3	2.039	1.532	2.034
4	2.035	1.511	2.074
5	2.014	1.507	2.080
6	1.956	1.581	2.036
7	1.866	1.729	1.984
8	1.716	1.903	1.832
9	1.514	1.853	1.645
10	1.258	1.620	1.397
11	0.938	1.206	1.121
12	0.537	0.620	0.680

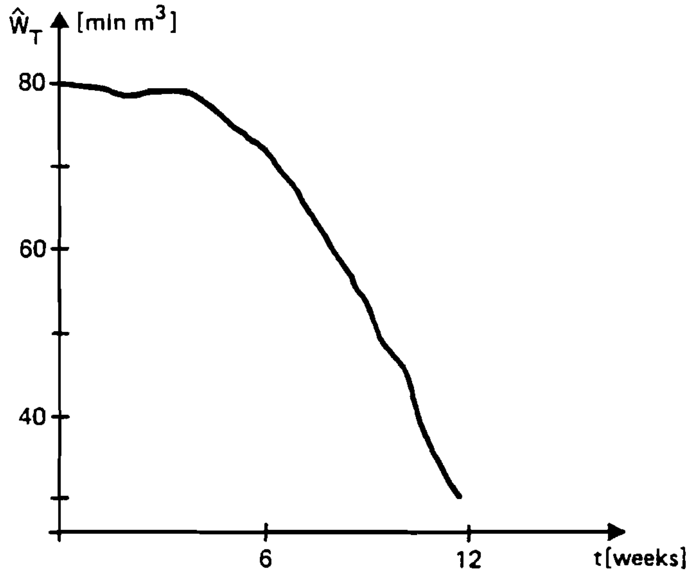


Figure 7. Optimal trajectory of Tresna reservoir.

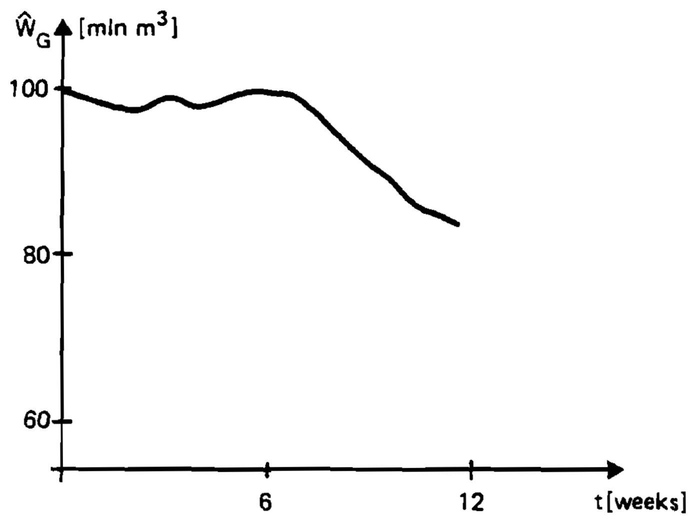


Figure 8. Optimal trajectory of Goczalkowice reservoir.

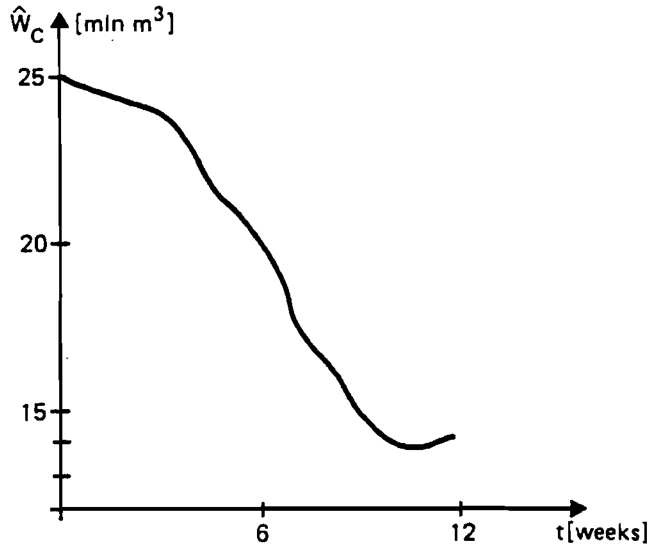


Figure 9. Optimal trajectory of Czaniec reservoir.

4.2 Simulation of the Lower Layer Activity

The following assumptions concerning the informational structure of the control process were made:

- Local Decision Units solve their optimization problem (see (42)-(48)) formulated for real values of water demands, natural inflows forecasted over a short-term horizon, and prices established by the coordinator;
- at the same time, the coordinator, on the basis of the measured values of state variables and values of decision variables determined by LDUs, establishes modified values of prices using formula (17) in a simplified form:

$$p^{i(k)} = p^{i(k-1)} - A \cdot G_i(p^{i(k-1)}) \qquad (49)$$

where:

$$p^{i(o)} = \hat{p}^i, \; p^{i(n_o)} = \bar{p}^i$$

n_o = maximal number of algorithm iterations assumed to be 6;

A is a constant 3 x 3 matrix.

Simulation of the control process was performed over a six-week long time period, involving 6 interventions of the coordinator. The price correction algorithm (49) was tested for three cases of matrix A:

(1) A ≡ [O] - which is equivalent to open-loop control in the system. This case is referred to as O.

(2) - the case referred to as I; matrix A was assumed as:

$$A = \begin{bmatrix} 0.00001 & 0. & 0. \\ 0. & 0.0001 & 0. \\ 0. & 0. & 0.00001 \end{bmatrix}$$

(3) - the case referred to as II, and matrix A was given as:

$$A = \begin{bmatrix} 0.00002 & 0. & 0. \\ 0. & 0.0002 & 0. \\ 0. & 0. & 0.00002 \end{bmatrix}$$

The values of the coefficients in matrix A result from the application of a contraction algorithm based on Newton's method, for solving the equation $G_i(p) = 0$ (see Malinowski and Terlikowski [1978]).

The data for simulation and its results are shown respectively:

- inflow to the Tresna Reservoir in Figure 10;
- inflows to the Goczałkowice Reservoir in Figure 11;
- resulting trajectories of the reservoirs - see Figures 12, 13 and 14;
- real water demands and respective values of decision variables associated with specified users in Figures 15, 16, 17, 18, 19;
- values of pollution indices at cross-sections P and DW in Figures 20 and 21;
- outflows from the reservoirs in Figures 22, 23 and 24.

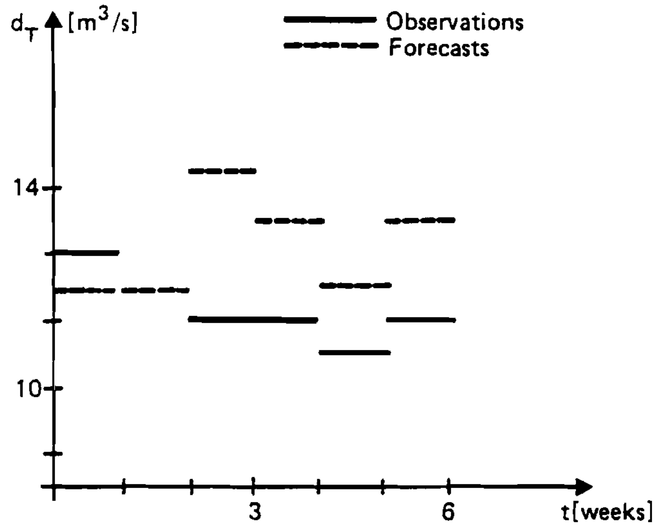


Figure 10. Forecasted and observed inflows to Tresna reservoir.

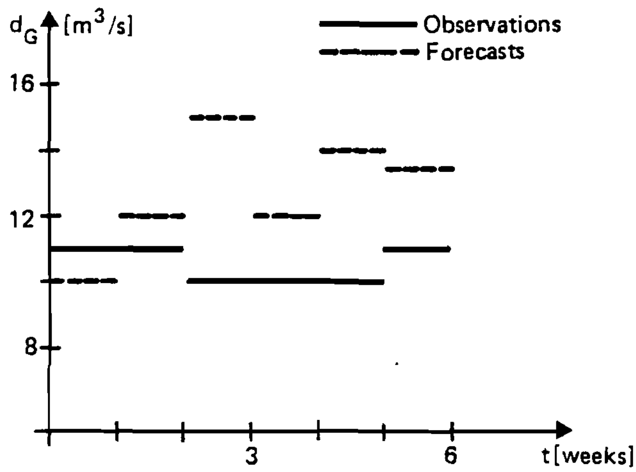


Figure 11. Forecasted and observed inflows to Goczalkowice reservoir.

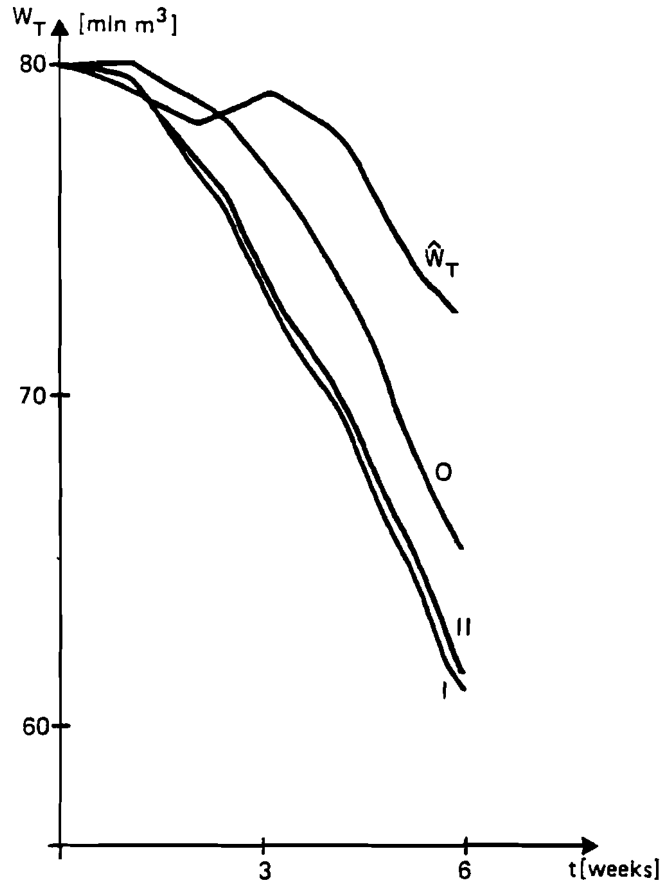


Figure 12. Trajectories of Tresna reservoir.

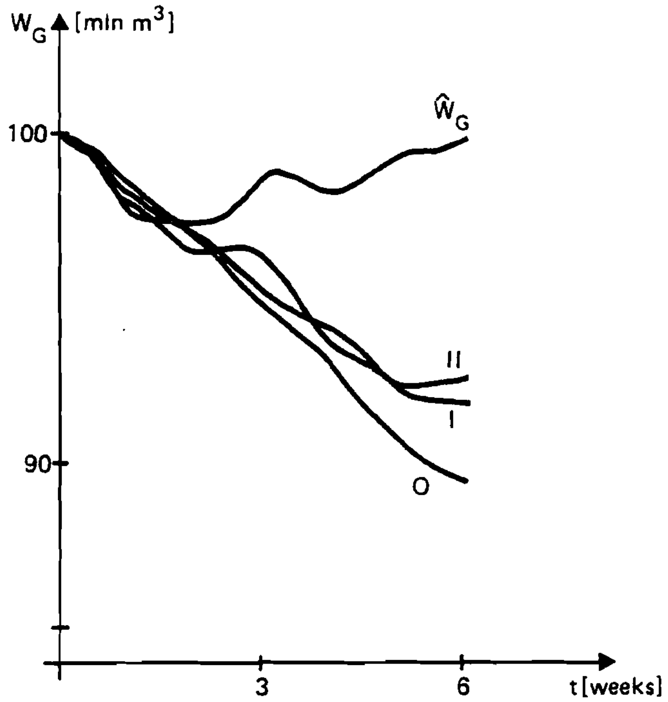


Figure 13. Trajectories of Goczalkowice reservoir.

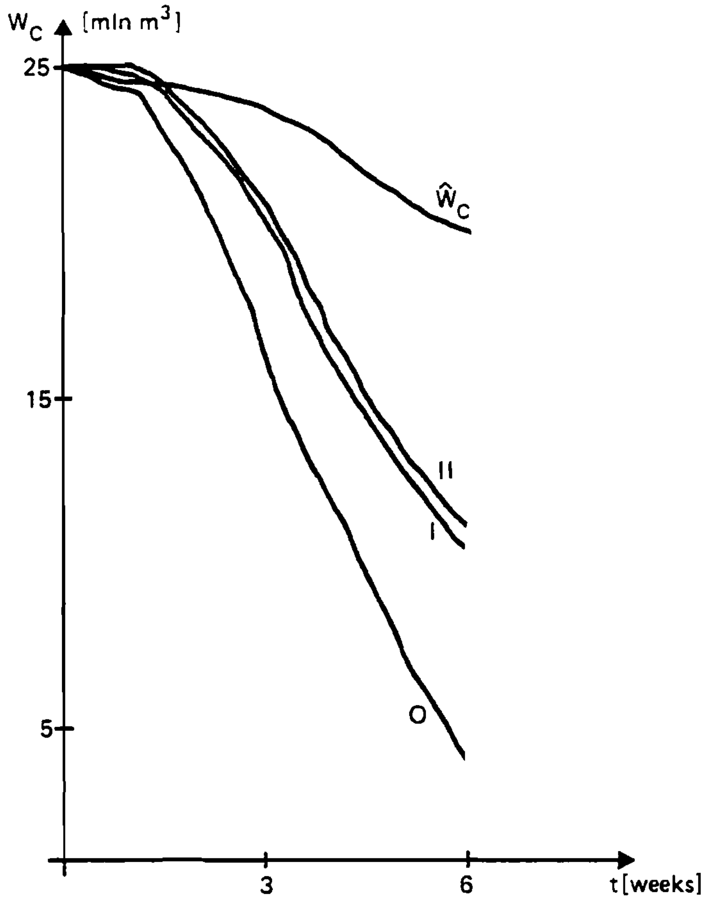


Figure 14. Trajectories of Czaniec reservoir.

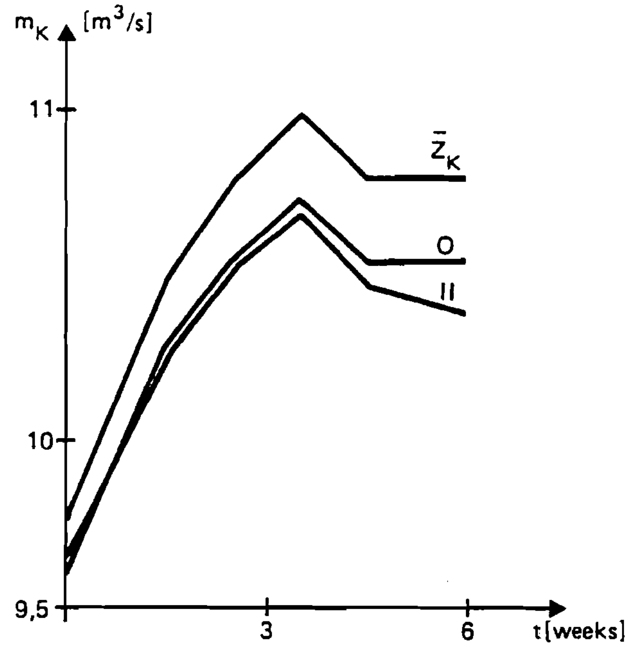


Figure 15. Demands ($\bar{Z}_K$) and supplies to Katowice.

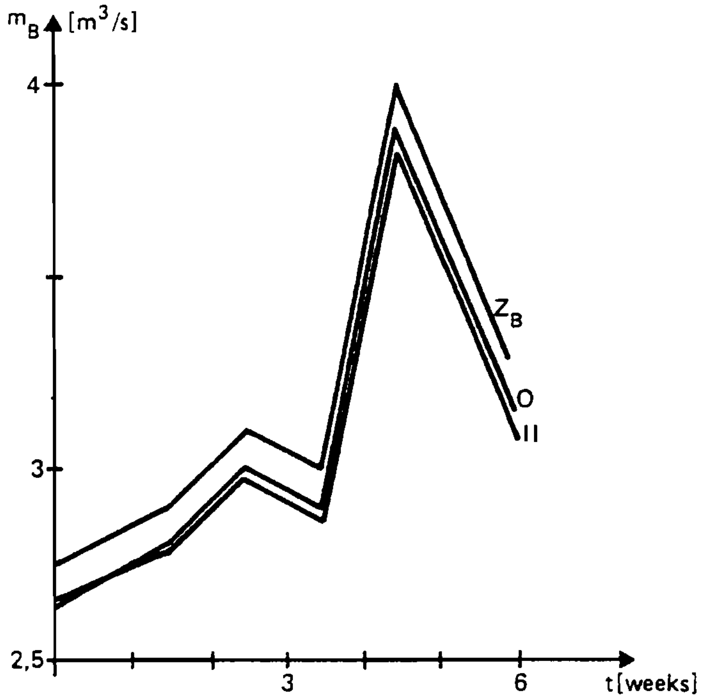


Figure 16. Demands ($\bar{Z}_B$) and supplies to Bielsko.

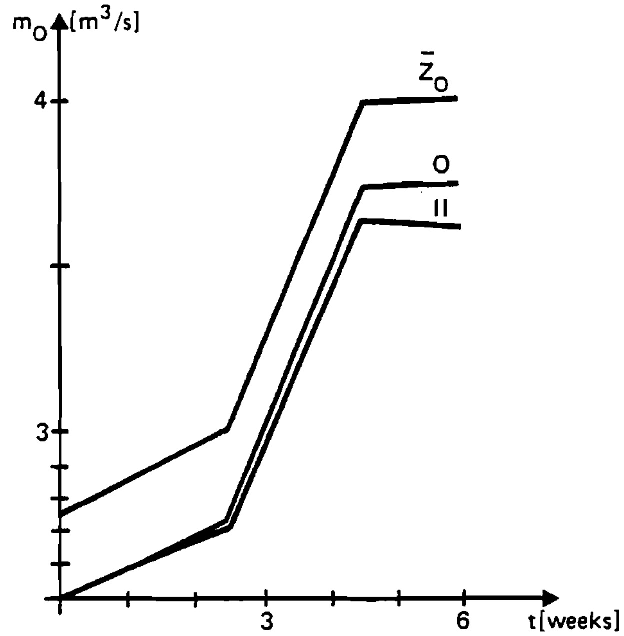


Figure 17. Demands ($\bar{z}_O$) to Oswiecim.

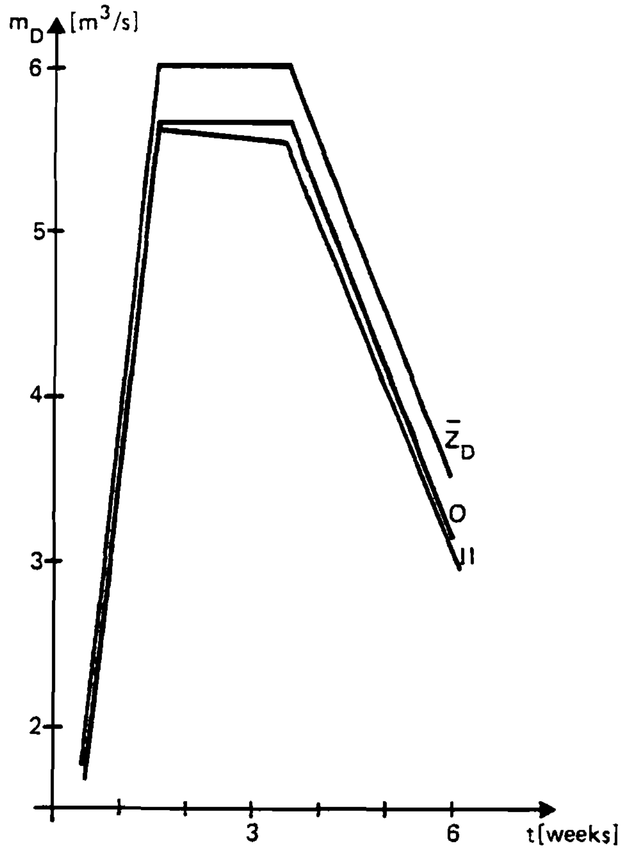


Figure 18. Demands ($\bar{Z}_D$) and supplies to Dzieckowice.

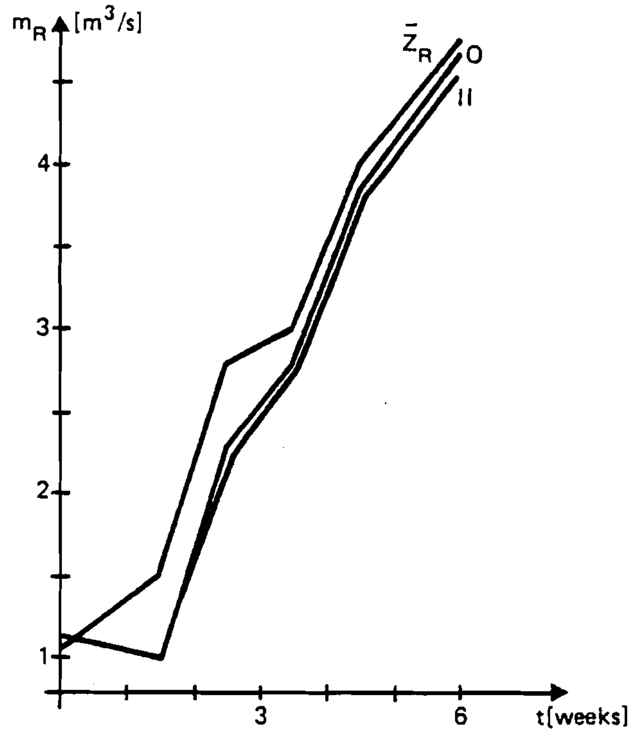


Figure 19. Demands ($\bar{Z}_R$) and supplies to fish farms (Kety Town).

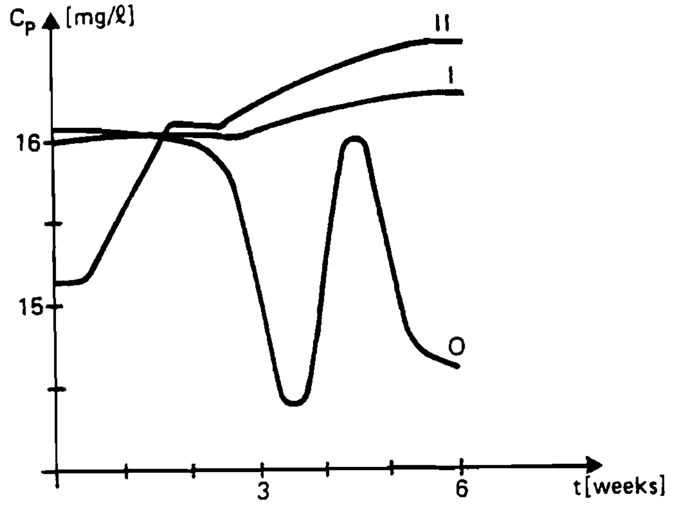


Figure 20. Values of pollution index at cross section P.

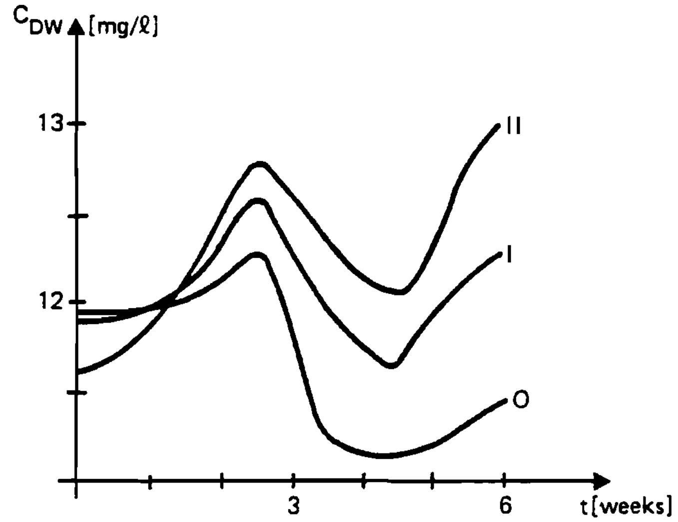


Figure 21. Values of pollution index at cross section DW.

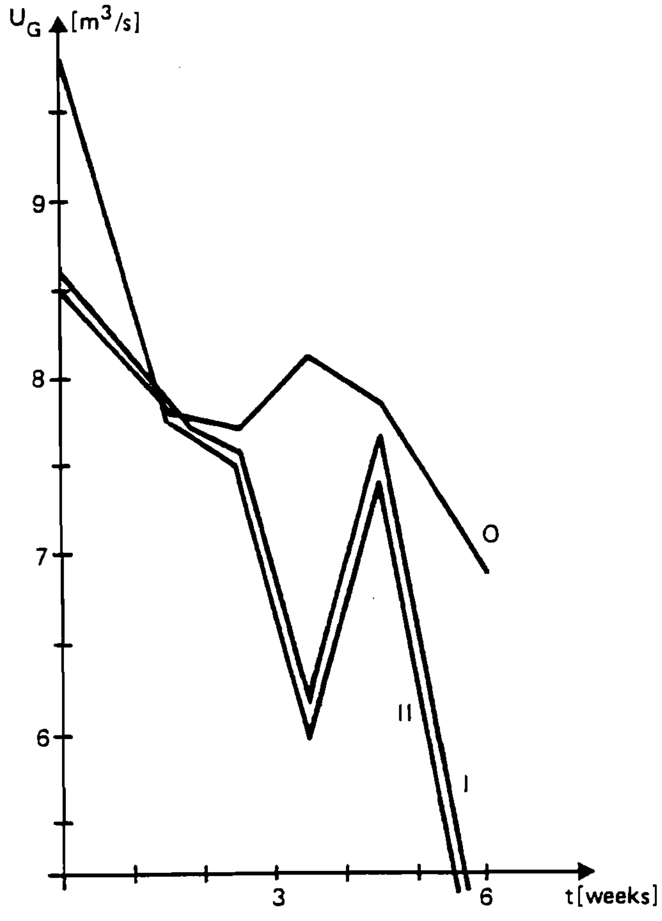


Figure 22. Outflow from Goczalkowice reservoir.

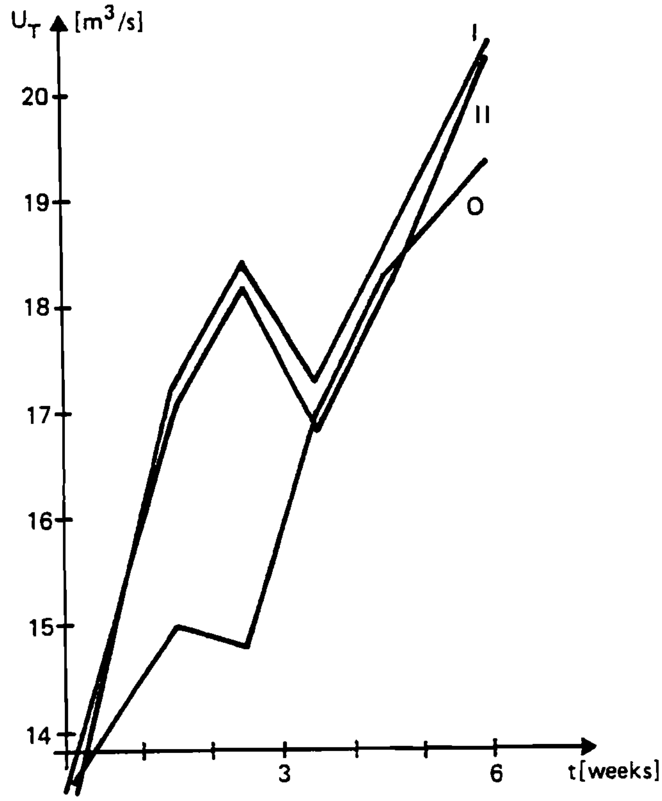


Figure 23. Outflow from Tresna reservoir.

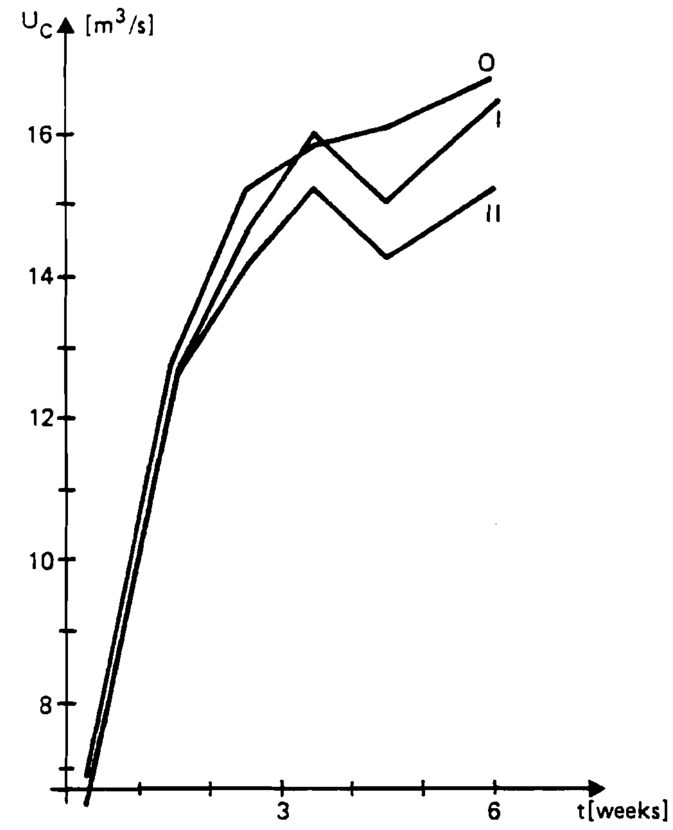


Figure 24. Outflow from Czaniec reservoir.

At the same time, coordinating prices and their values modified by the coordinator are shown in Table 2.

Table 2.

p_T			p_G			p_C		
$\hat{p}_T$	Case I	Case II	$\hat{p}_G$	Case I	Case II	$\hat{p}_C$	Case I	Case II
2.05	2.033	2.028	1.667	1.35	1.164	2.031	2.026	2.021
2.049	2.063	2.073	1.629	1.536	1.714	2.049	2.108	2.114
2.039	2.172	2.329	1.532	1.932	2.827	2.034	2.241	2.393
2.035	2.347	2.632	1.511	3.005	4.929	2.074	2.391	2.669
2.015	2.427	2.789	1.507	3.83	6.428	2.08	2.496	2.85
1.956	2.448	2.881	1.581	4.77	7.77	2.036	2.543	2.973

5. CONCLUSIONS

Conclusions which arise from the presented results can be briefly summarized as follows:

- Activity of the coordinator (see cases I and II) counteracts the excessive emptying of the reservoirs. The negative effects of water deficit are evident when the coordinator does not act; i.e., in case 0 (Figures 13 and 14).
- The coordinator enables proper collaboration between the Tresna and Czaniec reservoirs which are located on the same river and supply water to 5 users. This collaboration is accomplished in such a way that the Tresna reservoir (which does not supply water directly to any user) supplements water deficits occurring in

Czaniec. When coordination is used (cases I, II) contrary to case 0 (see Figure 12), this results in the emptying of the Tresna reservoir which has a good practical motive.

- An active collaboration between the Goczałkowice and Czaniec reservoirs is also possible. These reservoirs are located on different rivers, but they are connected by some common goals (water supply to Katowice, maintaining desired water quality at cross-section P and DW). This collaboration is illustrated in Figures 20 and 21. In cases I and II, the control structure, faced with water deficits, obviously yields less water quality in cross-sections P and DW, but collaboration of the whole system results in a more stable behaviour of the pollution index functions in time. This results partially from the relatively high importance of this goal; i.e., the relatively high values of weighting coefficients associated with the respective part of performance index J_I (see (24)).
- Compensation of current water deficits (i.e., deficits in relation to long-term forecasts) is performed by modification of all controls (decision variables), but with respect to established proportions of priorities:

  water supply to the users (Figures 15-19) is limited in a less rigid way, while the outflows from the reservoirs (Figures 22-24) are more intensively controlled.

These results illustrate the most obvious features of control of the water system under deficits, which may be obtained if the lower layer is applied with a proper coordination mechanism. They may be briefly summarized as follows: the water reservoirs are not emptied as much as in the open-loop control case (i.e., without coordination case 0) but the current goals (supply to water-users, protecting the water quality standards) are not fully satisfied. At the same time, the benefits resulting from application of the price mechanism (mostly active collaboration of the whole system) are obtained as was indicated above. However, these results are not fully complete, because the simulation of the control process is not performed over a long-time horizon including several repetitions of the upper layer interventions. When the simulation is performed only once, like in the case presented, the contradiction between water storage goal and current goals certainly appears (compare case 0 with I and II). Yet, after simulation of the lower layer operation over some six-week long period (covering, e.g., half or one year), with repetition of the long-term operating planning (i.e., intervention of the upper layer) at the beginning of each period, we shall obtain the consistency of these two goals. Then the value of the performance index J_I (concerned with current goals), integrated in the whole time horizon will be an adequate measure of the quality of control. Such a "full" simulation is necessary to show the real profits which may be gained by the application of the control structure presented. Finally, it is obvious that more research is needed before these methods for control of some real water management systems can be applied. These methods are still being extensively investigated.

6. GLOSSARY OF TERMS

Direct control variable (direct decision)

- a real, physical quantity, which value is to be decided and may be directly applied to a real system.

Intervention (of a decision unit)

- making a new decision = establishing a new control.

Storage Plan

- a set of values (or balances) of the reservoir's state given for some moments (or periods) of time and accepted by a decision unit.

Storage Policy - A Method of determining the actual Storage Plan:

- long-term storage policy
  - -- a method used by a control unit which takes into account a long time horizon (e.g., the upper layer in this paper), to determine the storage plan over the long horizon,

- short-term storage policy
  -- a method used by a control unit regarding a short time horizon (e.g., the lower layer) to determine the storage plan over the short horizon.

REFERENCES

Findeisen, W., (1974). Multilevel Control Systems, PWN, Warszawa (in Polish), German edition 1977.

Findeisen, W., (1978). Hierarchical Control System - An Introduction, IIASA Professional Paper PP-78-1, IIASA, Laxenburg.

Findeisen, W., and K. Malinowski (1979). Two Level Control and Coordination for Dynamical Systems. Archiwum Automatyki i Telemechaniki, Vol. XXIV z. 1.

Gal, S., (1979). Optimal Management of a Multi-Reservoir Water Supply System, Water Resources Research, Vol. 15, No. 4.

Haimes, Y., (1978). Hierarchical Analyses of Water Resources Systems, McGraw Hill, New York.

Kaczmarek, Z., K. Malinowski, and K.A. Salewicz (1979). Long-Term Resources Allocation for Control Purposes in a Water Management System, IAHS Bulletin 24, 3, 9.

Kindler, J., K.A. Salewicz, H. Słota, and T. Terlikowski (1979). Operation of Multiple Reservoir Systems: A Case Study of the Upper Vistula System (An Introduction); IIASA Collaborative Paper, CP-79-17, Laxenburg.

Lasdon, L.S., (1970). Optimization Theory for Large Systems, McMillan Company.

Malinowski, K., (1975). Properties of Two Balance Methods of Coordination, Bull. Pol. Acad. Sci., Vol. XXIII, No. 9.

Malinowski, K., (1978). Multilayer-Multiechelon Control Structures for Dynamical Systems. Editions of the Technical University of Warsaw, Elektronika, No. 38 (in Polish).

Malinowski, K., K.A. Salewicz, and T. Terlikowski (1979). Two-Layer Hierarchical Control Structure for Multireservoir Water Systems, edited by W. Findeisen. Studies in Control Methodology for Water Resources Systems. Technical Report, Institute of Automatic Control, Technical University of Warsaw.

Malinowski, K., and T. Terlikowski (1978). Coordination of Nonstationary Systems with Dynamic Inventory Couplings, Report, Institute of Automatic Control, Technical University of Warsaw.

Mesarovic, M.D., D. Macko, and Y. Takahara (1970). Theory of Hierarchical, Multilevel Systems. Academic Press.

Nowosad, K., (1978). Properties of Repetitive Control Structure for High Frequency of Interventions (in Polish), Archiwum Automatyki i Telemechaniki, t. XXIII, No. 3.

Prekopa, A., Rapczak, T., and I. Zsuffa (1978). Serially Linked Reservoir System Design Using Stochastic Programming, Water Resources Research, Vol. 14, No. 4.

Salewicz, K.A., (1978). Optimization Model of the Water Economic System of the Upper Silesia Industrial Region (in Polish). Technical Report, Institute of Meteorology and Water Management.

Terlikowski, T., (1979). Hierarchical Optimizing Control Structures for Dynamical Systems (in Polish). Ph.D. Dissertation Thesis in Institute of Automatic Control, Technical University of Warsaw.