Journal of Environmental Economics and Management 110 (2021) 102522

Contents lists available at ScienceDirect

Journal of Environmental Economics and Management

journal homepage: www.elsevier.com/locate/jeem

Optimal carbon pricing in general equilibrium: Temperature caps and stranded assets in an extended annual DSGE model[☆]

Frederick van der Ploeg[a,b], Armon Rezai[c,d,e,*]

[a] OXCARRE, Department of Economics, University of Oxford, Oxford, OX1 3 UQ, UK
[b] Faculty of Economics and Business, University Amsterdam, P.O. Box 15551, 1001, NB Amsterdam, the Netherlands
[c] Department of Socioeconomics, WU - Vienna University of Economics and Business, Welthandelspl. 1, 1020 Vienna, Austria
[d] IIASA, Schloßpl. 1, 2361 Laxenburg, Austria
[e] WIIW - Vienna Institute for International Economic Studies, Rahlgasse 3, 1060 Vienna, Austria

ARTICLE INFO

JEL classification:
H21
Q51
Q54

Keywords:
Carbon price
Tractable rule
General equilibrium
Utility and growth damages
Technical progress
Population growth
Logarithmic depreciation
Differential discount rates
Temperature cap
Stranded oil and gas reserves

ABSTRACT

The general equilibrium model developed by Golosov et al. (2014), GHKT for short, is modified to allow for additional negative impacts of global warming on utility and productivity growth, mean reversion in the ratio of climate damages to production, labour-augmenting technical progress, and population growth. We also replace the GHKT assumption of full depreciation of capital each decade by annual logarithmic depreciation. Furthermore, we allow the government to use a lower discount rate than the private sector. We derive a tractable rule for the optimal carbon price for each of these extensions. We then simplify the GHKT model by making temperature a linear function of cumulative emissions and making the proportion of output lost due to global warming a linear function of temperature. Finally, we consider how the rule for the optimal carbon price must be modified to allow for a temperature cap, and what this implies for stranded oil and gas reserves. We illustrate our analytical results with a range of optimal policy simulations.

1. Introduction

Carbon emissions are at the root of the most important global externality (Stern, 2007). The first-best response is to price carbon, either via a carbon tax or an emissions market, at a uniform price throughout the globe and rebate the revenue as lump-sum rebates. The tractable and influential general equilibrium model of growth and climate change developed by Golosov et al. (2014), denoted

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[☆] We have benefited from the incisive comments of Lint Barrage, Elisa Belfiori, David von Below, Christian Traeger, and Cees Withagen on this and an earlier version of this paper entitled “Stranded assets, the social cost of carbon, and directed technical change: Macroeconomic dynamic of optimal climate policy”, which has been extended to have a yearly time scale and allow a more general production function, logarithmic depreciation and differential discounting for public and private sector. We are also grateful to the helpful comments and advice of the editor, Andreas Lange, and two anonymous referees. The Mathematica source code containing our computations is available under https://github.com/armonrezai/Optimal_carbon_pricing_in_GE.

[*] Corresponding author. Department of Socioeconomics, WU - Vienna University of Economics and Business, Welthandelsplatz 1/D5/level 3, 1020, Vienna, Austria.

E-mail addresses: rick.vanderploeg@economics.ox.ac.uk (F. van der Ploeg), armon.rezai@wu.ac.at (A. Rezai).

https://doi.org/10.1016/j.jeem.2021.102522
Received 8 January 2021; Received in revised form 19 May 2021; Accepted 12 August 2021

GHKT from hereon, presents a simple and intuitive rule for the optimal pricing of carbon in general equilibrium: the price of emissions should be proportional to aggregate output and thus grow in line with trend growth. The proportionality factor depends on the discount rate, the severity of damages, and the dynamics of atmospheric carbon. The GHKT model has become very popular due to its analytical versatility. Its rule for the optimal carbon price is intuitive and easy to calculate but can be criticized for the strong assumptions necessary to derive it. Our aim is therefore to offer various extensions of the GHKT model, some more important than others, which still yield an analytically tractable rule for the optimal carbon price. We also offer a simplification of the model where temperature is driven by cumulative emissions and damages are calibrated to Burke et al. (2015) rather than to Nordhaus (2014). We thus take stock of the merits, uses, and limitations of the GHKT model, and believe that the extended GHKT model is more realistic and can be used in teaching.

We first extend the GHKT model to allow for logarithmic depreciation. Using theoretical and empirical arguments, Anderson and Brock (2021) demonstrate that logarithmic depreciation is better able to fit the aggregate data than geometric depreciation and they suggests its tractability lends itself for tractable expressions for optimal policy. Full depreciation each decade, as is in GHKT, is a special case of logarithmic depreciation. We adopt their depreciation model and find a tractable expression for the optimal carbon price. This allows us to use finer time resolutions (yearly instead of decadal), opening the model to the inclusion of business-cycle interactions.[1]

Global warming damages in the GHKT model are proportional to aggregate production. We also allow for disutility from global warming and climate damages to the growth of total factor productivity. These extensions also lead to a tractable expression for the optimal carbon price, allowing any combination of damages to consumption and utility. Using recent calibration studies, we show that the optimal carbon price in 2019 rises from \$64 to \$456 per ton of carbon if damages affect productivity growth permanently.

Exogenous growth in population (cf. Kelly and Kolstad, 2001) and productivity largely determine the severity of climate change. We, therefore, include positive rates of population growth and labour-augmenting technical progress in the GHKT model. This has important implications for the rates of growth and interest, while still allowing us to obtain a generalized tractable expression for the optimal carbon price. A one percent growth rate in population increases the optimal carbon price per ton of carbon from \$64 to \$164 per ton of carbon (and to \$2039 per ton of carbon if damages affect productivity growth). We also allow for mean reversion in the process of total factor productivity growth and show how this affects the rule for the optimal carbon price.

Following Belfiori (2017) and Barrage (2018) we allow the government to have a lower discount rate than the private sector. This implies that the carbon pricing policy must be complemented with a subsidy for natural and man-made capital to offset the relative myopia of the private sector. In this case, the social cost of carbon increases as the government places greater weight on future generations. Given that a handout to the owners of capital and, more importantly, oil reserves is politically unlikely, we show by how much the carbon price would be increased to offset the missing subsidy. This increase is a second-best way to compensate future generations for the lower material wealth left to them in the form of capital stocks and fossil fuel reserves. We derive the tractable rules for each of these policy instruments in general equilibrium and illustrate why such an equilibrium is time consistent. In our numerical simulations the second-best carbon price is up to 10% above its first-best level.

We also apply two simplifications of the GHKT model based on recent developments in atmospheric science and economics, which lead to a general-equilibrium extension of the framework set out in van der Ploeg (2020). First, recent atmospheric science insights suggest that temperature is driven by cumulative emissions (Allen et al., 2009; Matthews et al., 2009; Anderson et al., 2019; van der Ploeg, 2018; Dietz and Venmans, 2019; Dietz et al., 2021). While the carbon cycle of GHKT is fairly accurate on a decadal scale, this is not the case for shorter time periods (Dietz et al., 2021). Second, the GHKT model models the ratio of damages to output as a negative exponential function of the atmospheric carbon stock and calibrates the function to the DICE-13 model of Nordhaus (2008). We have calibrated our damages to the GHKT model. Since these damages are very modest compared to those found econometrically in Burke et al. (2015), we also calibrated damages to these much higher estimates. Both simplifications increase the realism of the GHKT further and bring it up to date with the most recent atmospheric insights and empirical evidence on global warming damages. They yield a simpler expression for the optimal carbon price and a much higher optimal carbon price of \$1507 instead of \$64 per ton of carbon.

Finally, we add realism to our welfare analysis by adding a ceiling on temperature or equivalent a cap on cumulative emissions. This captures the 2015 Paris Climate Accord where countries agreed to limit global warming to 2 °C while aiming to keep warming at or below 1.5 °C from pre-industrial levels. We show that, if the temperature cap bites, this requires adding a Hotelling term that rises at a rate equal to the market rate of interest to the welfare-maximizing carbon price. The size of this term increases for tighter temperature caps with the initial carbon prices \$281 and \$1152 for the 2 °C and 1.5 °C targets. We also the effects on stranded oil and gas reserves and the fossil fuel mix.

Section 2 presents the original GHKT model and our extensions. Section 3 derives the social optimum and decentralized equilibrium. Sections 4 discusses the case when the government discounts the future at a lower rate than private agents. Section 5 simplifies the GHKT model by letting temperature be driven by cumulative emissions and damages be calibrated on Burke et al. (2015). Section 6 explores how our results are modified by a temperature cap. Section 7 calibrates our annual version of the GHKT model and discusses the baseline optimal policy and business as usual results. Section 8 demonstrates the numerical sensitivity of the optimal policy simulation with respect to each of our extensions of the GHKT model. Section 9 takes stock and discusses the merits and versatility, uses, and limitations of the GHKT model. Section 10 concludes.

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[1] Cai et al. (2012) already used an annual time step for their version of the DICE model but they were not concerned with tractable expressions for the optimal social cost of carbon.

2. Five extensions of the GHKT model

We build on the familiar Brock and Mirman (1972) and Golosov et al. (2014) assumptions: logarithmic utility, Cobb-Douglas production function, full depreciation of capital within a decade, exponential climate damages in production, fossil fuel extraction and production of renewable energy requiring no capital, and a two-box carbon cycle with a part of carbon staying up permanently in the atmosphere and another part that gradually decays and is returned to the surface of the earth and oceans. Energy is derived from coal, oil/gas, and renewable sources.

Replacing the assumption of manmade capital depreciating fully in every decade, we allow for logarithmic depreciation as in Anderson and Brock (2021). Here, the case of full depreciation corresponds to a special case but importantly we can vary temporal resolution to yearly or even quarterly. We extend the GHKT model in four other directions by allowing for a direct negative effect of climate change on household utility and on productivity growth including allowing for mean reversion in total factor productivity, population growth, and growth in labour-augmenting technical progress. We assume that the public and private discount rates are the same but in section 4 we will investigate differential discount rates.

The social planner maximizes utilitarian social welfare, which consists of utility derived from per capita consumption, $\ln(C_t/N_t)$. Climate change lowers output via production damage and, in a first extension of the GHKT model, via instantaneous per capita welfare loss due to climate change, ψE_t where $\psi \geq 0$,

$$\sum_{t=0}^{\infty} \beta^t N_t [\ln(C_t / N_t) - \psi E_t] \tag{1}$$

Subscripts denote periods of time $t = 0, 1, 2, \ldots$. The time impatience factor is constant and denoted by $0 < \beta < 1$. Population at time t is N_t and, in a second minor but quantitatively important extension to the GHKT model, its constant gross growth factor equals $N_{t+1}/N_t = \gamma$.[2] The stock of atmospheric carbon is E_t.

Production in the GHKT framework occurs in energy and final goods sectors and is given by a nested production function. Final goods output, Y_t, is produced by combining labour, L_t, capital, K_t, and energy from a Cobb-Douglas production function with a unit elasticity of substitution, while energy output is described by a CES production function and produced from (i) a finite stock of fossil fuel S_t which can be extracted without cost, but subject to a Hotelling rent, (ii) an infinite stock of fossil fuel (i.e. coal), and (iii) renewable energy sources, with the latter two requiring only labour input, L_{2t} and L_{3t}, but no capital. A_{1t} and A_{2t} are the corresponding exogenous labour productivities. Production is thus given by $Y_t = A_t K_t^{\alpha} [\kappa_{1t} F_t{}^{\rho} + \kappa_{2t}(A_{2t}L_{2t})^{\rho} + \kappa_{3t}(A_{3t}L_{3t})^{\rho}]^{\upsilon/\rho} L_t{}^{1-\alpha-\upsilon}$, where F_t denotes fossil fuel use in period t. The elasticity of substitution between different energy types is constant and given by $1/\ (1-\rho)$. The share parameters in the energy sub-production function are positive and satisfy. $\kappa_{1t} + \kappa_{2t} + \kappa_{3t} = 1$.

In a third minor extension of the GHKT model, we allow for Harrod-neutral technical progress and assume a positive constant gross growth in labour productivity, denoted by $\omega \geq 1$, and define the number of (exogenous) efficiency units in the economy as $M_t \equiv \omega^t N_t$. Labour (in efficiency units) is allocated either to final goods production L_t or to fossil or renewable energy production, L_{2t}andL_{3t}, respectively, so that. $L_t + L_{2t} + L_{3t} = M_t$.

Our fourth extension is logarithmic depreciation of capital. This can be formulated as $K_{t+1} = \iota I_t{}^{\kappa} K_t^{1-\kappa}$, where $\iota > 0$ and $0 \leq \kappa \leq 1$. While the case of geometric depreciation is commonly used in growth theory due to its linearity, nonlinear specifications have often been applied, e.g. Uzawa (1969), Lucas and Prescott (1971), Hayashi (1982), and Abel and Blanchard (1983). Anderson and Brock (2021) compare logarithmic depreciation to the more conventional assumption of geometric depreciation of capital. Using theoretical and empirical arguments, they show that logarithmic depreciation is superior in explaining aggregate data and recommend it, therefore, in the formulation of economic policy. Using material balance, $C_t + I_t = Y_t$, we can write the dynamics of capital as

$$K_{t+1} = \iota \left(A_t K_t^{\alpha} [\kappa_{1t} F_t{}^{\rho} + \kappa_{2t}(A_{2t}L_{2t})^{\rho} + \kappa_{3t}(A_{3t}L_{3t})^{\rho}]^{\upsilon/\rho} (M_t - L_{2t} - L_{3t})^{1-\alpha-\upsilon} - C_t \right)^{\kappa} K_t^{1-\kappa}. \tag{2}$$

The formulation in (2) captures GHKT model as a special case, since with $\iota = \kappa = 1$ it boils down to full depreciation,$K_{t+1} = Y_t - C_t$. The dynamics of fossil fuel depletion are

$$S_{t+1} = S_t - F_t, \quad \sum_{t=0}^{\infty} F_t \leq S_0, \quad S_0 \ \text{given}. \tag{3}$$

The carbon dynamics of the GHKT model are as follows. Burning fossil fuel leads to carbon emissions of which a fraction stays permanently in the atmosphere, $0 < \phi_L < 1$. Of the transitory emissions, a fraction $0 < \phi_0 < 1$ is still there at the end of the period (i.e. a decade in the GHKT model), and a fraction $1 - \phi_0$ is absorbed by carbon sinks within each period. The decay factor of the stock of atmospheric carbon is $0 < \varepsilon < 1$. With $E_t^p$and E_t^t denoting the permanent and transient stocks of atmospheric carbon (the sum of which

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[2] We assume that the discount rate corrected for population growth is positive, i.e..

is E_t), the dynamics of the stock of carbon in the atmosphere are[3]

$$E_t^p = E_{t-1}^p + \phi_L(F_t + A_{2t}L_{2t}), \quad E_t^t = \varepsilon E_{t-1}^t + \phi_0(1-\phi_L)(F_t + A_{2t}L_{2t}), \quad E_t = E_t^p + E_t^t \tag{4}$$

In our numerical simulations, we annualize the decadal calibration of (4) in GHKT or replace it by a climate model solely based on cumulative emissions.

Total factor productivity in final goods production, A_t, falls as the stock of atmospheric carbon increases. The instantaneous damage of the stock of atmospheric carbon to total factor productivity is denoted by $\chi > 0$. In our fifth extension of the GHKT model we allow for mean reversion around an exogenous trend growth path for total factor productivity, $\overline{A}_t$.With mean reversion in the log of total factor productivity denoted by $0 \le 1-\delta \le 1$, the development of total factor productivity is given by

$$\ln(A_t) = \delta \ln(A_{t-1}) + (1-\delta)\ln\left(\overline{A}_t\right) - \chi\left(E_t - \overline{E}\right). \tag{5}$$

The GHKT model corresponds to $\delta = 0$ in which case climate change (i.e. atmospheric carbon concentrations in excess of pre-industrial level $\overline{E}$)lowers the *level* of total factor productivity in that period. However, if $\delta = 1$, climate change affects *growth* of total factor productivity permanently. Intermediate values of δ allow for transitory effects, i.e. mean reversion in the effects of climate change on total factor productivity. Since Harrod-neutral and Hicks-neutral technical progress are equivalent for Cobb-Douglas production functions, we capture the former by $\omega > 0$ and abstract from the latter and thus assume $\overline{A}_t$ is constant. In addition, we allow for labour-augmenting (Harrod-neutral) technical progress in the energy sector.

3. The social optimum

The social optimum maximizes utilitarian social welfare, eqn. (1), subject to equations (2)–(5) and the non-negativity constraints $F_t \ge 0,\ L_t, L_{2t}, L_{3t} \ge 0$, and $S_t \ge 0$for all$t \ge 0$,and satisfies the properties stated in the following three propositions.

Proposition 1: *The socially optimal saving and consumption functions are*

$$K_{t+1} = \iota\left(\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)^{\kappa} Y_t^{\kappa} K_t^{1-\kappa} \quad \text{and} \quad C_t = c(\beta)\ Y_t \quad \text{with} \quad c(\beta) \equiv \left(1 - \frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right). \tag{6}$$

Proof: see Appendix A.

The saving and consumption functions (6) follow from the Euler equation and the capital accumulation equation (2) These are modified versions of those presented in Brock and Mirman (1972) and Anderson and Brock (2021) to allow for population growth and logarithmic depreciation. They indicate that a higher capital share, more patience, higher population growth, and lower depreciation boost incentives for aggregate investment and, hence, curb the propensity to consume. If capital depreciates fully and population growth is absent (i.e. and $\gamma = 0$), the consumption share equals $(1-\alpha\beta)$ and the equations in (6) reduce to that in the GHKT model. As in the GHKT model, the propensity to consume, c, and the marginal propensity to save are constant and independent of assumptions on the climate and its interactions with the economy.

Proposition 2: *Demand for the three energy types follow from the efficiency conditions*

$$\left.\begin{array}{c} \dfrac{\partial Y_t}{\partial F_t} \le h_t + \tau_t \\ F_t \ge 0 \end{array}\right\}\text{c.s.}, \qquad \left.\begin{array}{c} \dfrac{\partial Y_t}{\partial A_2 L_{2t}} \le \dfrac{w_t}{A_2} + \tau_t \\ L_{2t} \ge 0 \end{array}\right\}\text{c.s.}, \qquad \left.\begin{array}{c} \dfrac{\partial Y_t}{\partial A_3 L_{3t}} \le \dfrac{w_t}{A_3} \\ L_{3t} \ge 0 \end{array}\right\}\text{c.s.}, \tag{7}$$

where $h_t,\ \tau_t, w_t \equiv (1-\alpha-\upsilon)Y_t/L_t$, and b_t are the scarcity rent of fossil fuel, the social cost of carbon (SCC) and the social wage (i.e. the marginal product of labour), respectively, all in units of final goods. The scarcity rents on fossil fuel follow from

$$h_t = j_t h_{t-1}, \tag{8}$$

where $j_t = \frac{1}{\beta\gamma}\frac{Y_t}{Y_{t-1}}$. In case of full depreciation, $\iota = \kappa = 1$, this gross growth rate of the carbon price equals the marginal product of capital, $j_t = \alpha Y_t / K_t$.

Proof: see Appendix A.

Equation (7) indicate that an energy good is not used in the production of final goods if its marginal product is less than its social marginal cost. For the scarce fossil fuel type (oil and gas), this cost consists of the scarcity rent plus the SCC. If fossil fuel use is used in production, its marginal product exactly equals its social marginal cost. As fossil fuel reserves are fully depleted (asymptotically), the marginal product of fossil fuel must rise indefinitely. Similarly, the abundant fossil fuel type (coal) is only used if its marginal product

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[3] The carbon cycle can be extended to include arbitrary many boxes. The model of Gerlagh and Liski (2017a) has 3 boxes. Joos et al. (2013) and Aengenheyster et al. (2018) have, respectively, a deterministic and a stochastic carbon dynamics model with 4 boxes. Temperature is modelled indirectly, assuming an equilibrium relationship between carbon stocks and global mean temperature $T_t = ECS\,\ln(E_t/596.4)/\ln(2)$, with the equilibrium climate sensitivity ECS equal to 3. This gives temperature in 2010 as 1.3 °C, with the GHKT calibration of $E_{2010} = 802$.Rezai and van der Ploeg (2016) introduce a lagged response of temperature to emissions and derive a tractable expression for the optimal SCC which is akin to the one given in (8) of Proposition 1. In the GHKT notation $\varepsilon = 1 - \phi$.

equals unit labour cost (i.e., the wage divided by sector-specific labour productivity) plus the SCC. The marginal social cost of renewable energy consists of the wage divided by (potentially) endogenous labour productivity. If the marginal product of renewable energy is less than its marginal social cost, it is not used in production.

Equation (8) is the Hotelling rule. It states that the growth rate of the scarcity rent must equal the social rate of interest as only then will society be indifferent between depleting an extra unit of oil or gas and getting a return equal to the social rate of interest and leaving this unit in the ground and getting the social capital gains. To see this, define growth of output and consumption as $g_t \equiv (Y_t - Y_{t-1})/Y_{t-1}$, the rate of time impatience as $\rho \equiv (1-\beta)/\beta$, the population growth rate as $n \equiv \gamma - 1$, and the social interest rate as $r_t \equiv j_t - 1$. We then have $j_t = 1 + r_t = \frac{(1+\rho)(1+g_t)}{(1+n)}$ or $r_t \cong \rho - n + g_t$, where the approximation becomes exact for infinitely short time units. The social rate of interest thus equals the rate of growth of the scarcity rent and equals the rate of time impatience minus the rate of population growth plus the rate of consumption growth. This follows from the Keynes-Ramsey rule with a unit elasticity of intertemporal substitution, which states that the optimal growth rate in per-capita consumption equals the difference between the social rate of interest and the rate of time impatience, $g_t - n = r_t - \rho$.

Proposition 3: *The first-best optimal SCC is*

$$\tau_t = \phi\left(\beta\right) Y_t \ \text{ with } \ \phi\left(\beta\right) \equiv \left(\frac{\phi_L}{1-\beta\gamma} + \frac{\phi_0(1-\phi_L)}{1-\beta\gamma\varepsilon}\right)\left[\psi\left(1 - \frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right) + \frac{\chi}{1-\beta\gamma\delta}\right] \geq 0. \tag{9}$$

Proof: see Appendix A.

Equation (9) implies that the optimal SCC is a constant proportion of aggregate output and this proportion is bigger if the damage parameters χ and ψ are large and carbon resides in the atmosphere for longer (i.e. if the fraction staying permanently in the atmosphere, ϕ_L, is large or the fraction absorbed by carbon sinks within each period, $1-\phi_0$, and the decay factor, ε, are low). The component of the SCC due to production damages increases if society has more patience (i.e., has a large discount factor β), population growth γ is high, and the mean reversion in the process of total factor productivity is slow (δ is high). Deviations from full depreciation of capital, i.e. $\kappa < 1$, depress the effect of utility damages on the SCC. As in the original GHKT model, technical assumptions about substitutability of fossil energy sources influences the SCC only indirectly via their effect on the level of output.

Corollary 1: *If the utility damage parameter,* ψ*, is calibrated to a given amount of utility lost in today's dollar terms, i.e.* $\psi^{\$} \equiv \psi / C_0$*, the proportionality factor in the rule for the optimal carbon price is.* $\phi\left(\beta\right) = \left(\frac{\phi_L}{1-\beta\gamma} + \frac{\phi_0(1-\phi_L)}{1-\beta\gamma\varepsilon}\right)\left[\frac{\psi^{\$}}{Y_0} + \frac{\chi}{1-\beta\gamma\delta}\right]$.

A negative effect of global warming on utility ($\psi > 0$) pushes up the optimal SCC. This effect can increase or decrease since shifts in parameters discussed above induce a simultaneous revaluation of marginal utility, potentially leading to offsetting effects. However, if we calibrate these damages in dollars, this component of the SCC increases if the utility discount factor is small (i.e. society has more patience and population grows more rapidly, higher $\beta\gamma$) and if carbon resides in the atmosphere for a longer period (higher ϕ_L, lower ε).

Corollary 2: *The GHKT model has a constant population, a negative effect of global warming on the* ***level*** *of total factor productivity but not on utility* ($\gamma = 1$, $\delta = \psi = 0$)*. Its optimal SCC is* $\tau_t = \chi\left(\frac{\phi_L}{1-\beta} + \frac{\phi_0(1-\phi_L)}{1-\beta\varepsilon}\right) Y_t$*. If global warming affects* ***growth*** *of total factor productivity* ($\gamma = \delta = 1$, $\psi = 0$), $\tau_t = \frac{\chi}{1-\beta}\left(\frac{\phi_L}{1-\beta} + \frac{\phi_0(1-\phi_L)}{1-\beta\varepsilon}\right) Y_t$.

If the GHKT model is only extended to allow for population growth and mean reversion in effects of global warming on total factor production, the optimal SCC is

$$\tau_t = \phi\left(\beta\right) Y_t \text{ with } \phi\left(\beta\right) \equiv \left(\frac{\phi_L}{1-\beta^{GHKT}} + \frac{\phi_0(1-\phi_L)}{1-\beta^{GHKT}\varepsilon}\right)\left(\frac{\chi}{1-\beta^{GHKT}\delta}\right). \tag{9'}$$

where $\beta^{GHKT} \equiv \beta\gamma$.[4] This expression boils down to the GHKT expression if global warming affects only current total factor productivity, $\delta = 0$. If global warming also affects future total factor productivity, $0 < \delta < 1$, the optimal SCC is higher. Note that, if utility damages are absent, the optimal SCC does not depend on the depreciation scheme.

With growth rather than level damages the effect on the optimal SCC is $(1-\beta)^{-1}$ times bigger. The elasticity of the SCC with respect to the productivity damage parameter δ is $\frac{\partial \tau_t}{\partial \delta} / \frac{\tau_t}{\delta} = (1-\beta\gamma\delta)^{-1} - 1 > 0$ which increases in β, γ, and δ. If the growth in population is higher (larger γ), equation (8) indicates that the SCC increases for production damages but falls for utility damages. The former is intuitive, the latter is the result of two opposing effects: higher population growth increases the attractiveness of capital accumulation due to higher consumption possibilities in the future but also increases damage created by carbon. The utility cost of damages to utility is constant in our formulation. This leaves the effect on the attractiveness of capital accumulation and, hence, the SCC decreases as population growth increases. All other parameters affect the SCC as in the GHKT model. As discussed above, the SCC is higher under a lower discount rate and a longer residence time of carbon emission in the atmosphere, either because of a larger fraction staying permanently or a lower dissipation rate of the transient fraction (higher β, ϕ_L, ϕ_0, ε).

Note that the coefficients of the production functions for final goods and for energy outputs and the coefficients driving the process of logarithmic depreciation (2) do not affect the expression for the optimal SCC (9) directly. They do affect energy use and world GDP,

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[4] Population growth is absent from the expression for the optimal SCC in the GHKT model but can be accommodated for by simply replacing β by the discount factor adjusted for population growth $\beta\gamma$.

and thus only affect the expression for the optimal SCC indirectly.

Under the assumption of the model, all components of the efficiency conditions in equation (7) scale to output and, thus, the energy system decouples from the rest of the economy.[5] To see this, note that Cobb-Douglas technology in the final goods sector implies proportionality of the marginal products of energy and labour to GDP. Further, the combination of logarithmic utility and logarithmic depreciation ensures that the inverse of marginal utility equals GDP times the constant consumption share. Given the equilibrium outcomes of the energy sector, the evolution of climate change is fully determined. Aggregate sector variables, such as total factor productivity, output, and capital stock, follow from those. This sequence of equilibria depends on the fact that the energy system is solely using labour as an input. We summarize this general finding of the model in the following corollary.

Corollary 3: *The evolution of the energy sector only depends on the stock of fossil fuel reserves and determines the evolution of temperature and of the permanent and atmospheric stocks of carbon. The evolution of capital and total factor productivity follows from fossil fuel reserves and the stocks of atmospheric carbon.*

Proof: see Appendix A.

Proposition 1–3 and Corollary 3 characterize the command optimum. Proposition B.1 in Appendix B shows that this command optimum can be sustained in the decentralized market economy if carbon is priced at a level equal to the SCC, via levying a carbon tax or setting up a competitive market for carbon permits, and the revenue is rebated as lump sums to the private sector. These correspond to the first-best climate policies. Decentralization of the command optimum is only feasible if all other externalities and market failures are appropriate dealt with. If not, it is necessary to consider second-best climate policies (e.g. Bovenberg and van der Ploeg, 1994; Bovenberg and Goulder, 2002; Kalkuhl et al., 2013; van der Ploeg, 2016; Rezai and van der Ploeg, 2017; Barrage, 2019).

4. Different discount rates for the private and public sector

There has been a debate on what the appropriate choice of discount rate for designing climate change policies is. On the one hand, Nordhaus (2008) adopts a utility discount rate of 1.5% per annum ($\beta = 0.985$). With trend growth of 2% per annum and an elasticity of intertemporal substitution (EIS) of 1/1.45, this gives a consumption discount rate of 4.4% per annum. This approach calibrates the consumption discount rate to match market returns on assets. Stern (2007) takes the stance that it is unethical to discount the welfare of future generations and therefore chooses EIS = 1 and very small utility discount rate of 0.1% per annum (corresponding to the risk of a meteorite ending the earth as we know it). This reflects ethical preferences, which lead to a much higher SCC. Rather than trying to reconcile the “descriptive” and the “normative” approaches, it seems more realistic to use a high discount rate for the private sector and a low one for the government. Lower public discount rates are equivalent to greater Pareto weights on future generations (Farhi and Werning, 2007; Belfiori, 2018).

Under differential discounting, the first-best solution for the optimal carbon prices requires a capital income subsidy alongside as an additional instrument (von Below, 2012; Belfiori, 2017; Barrage, 2018).[6] The reason is that lower social discount rates lead to socially insufficient saving by private agents with relatively high private discount rates, even in the absence of climate change, and can warrant action to overcome the excessive consumption bias as an additional policy goal. We thus assume that the government is more patient than the private sector who has discount factor $\beta^P > 0$, hence we assume $\beta > \beta^P$. In Proposition 4 we extend the findings of von Below (2012) and Belfiori (2018) and state the optimal capital income subsidy, denoted by σ, and carbon tax for our model.

Proposition 4: *The social optimum is replicated in the decentralized market economy with a government that is more patient than private agents,* $\beta > \beta^P$, *if the capital subsidy is* $\sigma^{FB} = (\beta - \beta^P)/\beta^P > 0$ *and the carbon tax* ${\tau_t}^{FB} = \phi(\beta) Y_t$ *is given by* equation (9), *where the superscript FB denotes first-best policies. The first-best optimal capital income subsidy depends on the gap between the public and private discount factors. The first-best optimal social cost of carbon increases in the public patience.*

Proof: see Appendix B.

The interpretation of Proposition 4 is as follows. A constant capital income subsidy curbs the interest rate to its socially optimal level. The optimal capital income subsidy is independent of the carbon tax, and of the capital income tax when measured in utility units. Since households own all assets in the economy, i.e. capital and fossil fuel reserves, this amounts to an identical subsidy on capital and fossil fuel reserves.[7]

Proposition 5: *If the capital subsidy cannot be positive,* $\sigma = \overline{\sigma} \le 0$, *the second-best optimal carbon tax is* ${\tau_t}^{SB} = \phi^{SB} Y_t > {\tau_t}^{FB}$ *with*$\phi^{SB} \equiv \phi(\beta) c^{SB}(\beta^P, \overline{\sigma}) / c(\beta) > \phi(\beta)$, *where revenue of the taxes on carbon and capital income is rebated as lump-sum transfers. This policy is time consistent.*

Proof: see Appendix B.

The constrained-optimal or second-best carbon tax is relevant when the capital subsidy is fixed at a too low level. It decreases as the capital subsidy is raised to the socially optimal level. Society wants to compensate future generations, who are relatively worse off due to the suboptimal capital subsidy, by improving the climate and lowering damages from global warming. If the capital subsidy is set too high, the planner would set the carbon tax below the Pigouvian rate to take some of the pressure off current generations. Hence, we can determine second-best policies if governments are constrained in the choice of their instruments.

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[5] The labour allocation across production of final goods, coal and renewable energy requires that the marginal products of labour net of the carbon tax must all equal the market wage.

[6] Von Below (2012) and Belfiori (2017, 2018) allow for scarce fossil fuel reserves and Hotelling price dynamics, while Barrage (2018) does not.

[7] The first-best policy in von Below (2012) and Belfiori (2017, 2018) includes separate subsidies for capital and fossil fuel assets. In assuming that households own both assets, we simplify the policy mix.

Our rule for the second-best optimal carbon tax is unaffected by re-optimization and is therefore time consistent. If each period capital fully depreciates as in the GHKT model ($\iota = \kappa = 1$), the equilibrium consumption share simplifies to $c^{SB}(\beta^P, \overline{\sigma}) = 1 - \alpha\beta^P\gamma(1 + \overline{\sigma})$ and the second-best consumption share as fraction of the first-best consumption share is $c^{SB}(\beta^P, \overline{\sigma}) / c(\beta) = [1 - \alpha\beta^P\gamma(1 + \overline{\sigma})] / (1 - \alpha\beta\gamma) > 1$. We thus see that, if the subsidy for capital and fossil fuel reserves cannot be given, the second-best carbon tax has to be set higher than the first-best carbon tax to compensate. This holds also for the case of partial depreciation. If the government were to re-optimize, it does not have an incentive to deviate from its second-best optimal carbon tax path. The second-best policy is thus time consistent. This is quite a special result, which is due to the assumptions of the GHKT model muting all strategic intertemporal considerations.

There are various other ways of implementing policies with different discount rates for the government and private sector. One is to subsidize the fossil fuel owners and the final goods producers instead of subsidizing the households. Another one is if to have a carbon tax that rises faster than the rate of growth of GDP as this would also force the economy to deplete less quickly (von Below, 2012; Belfiori, 2018).

5. Two simplifications of the GHKT model: global warming and damages

Here we give two science-based simplifications of the GHKT model. First, we use recent atmospheric science insights that suggest that temperature is driven by cumulative emissions (Allen et al., 2009; Matthews et al., 2009). This has been already applied in various economic applications (e.g. Anderson et al., 2019; van der Ploeg, 2018; Dietz and Venmans, 2019) and captures the climate dynamics better than most economic models, especially at grids that are finer than a decadal time grid (Dietz et al., 2021). Hence, we replace the two difference equations for the temporary and permanent component of temperature (4) by an equation linking temperature to cumulative emission and one difference equation for the stock of cumulative emissions:

$$T_t = T_0 + \zeta_1 E_t, \quad E_{t+1} = E_t + F_t, \tag{4′}$$

where $E_t$denotes the stock of cumulative emissions, not the stock of atmospheric carbon, T_0 denotes initial temperature, and ζ_1 the transient climate response to cumulative emissions (TCRE). We set the former to 1.3 °C and the TCRE to 2 °C/TtC.

Second, we calibrate our damages to Burke et al. (2015) instead of taking them from the GHKT model. The GHKT model models the ratio of damages to output as a negative exponential function of the atmospheric carbon stock. Note that the GHKT model like the DICE-2013 and the most recent DICE-2016 models have very small damages compared to those found econometrically in Burke et al. (2015), which are almost linear in temperature. We replace equation (5) by $\ln(A_t) = \delta \ln(A_{t-1}) + (1 - \delta)\ln(\overline{A}_t) - \zeta_2 T_t$. and substitute for temperature from (4′) to obtain

$$\ln(A_t) = \delta \ln(A_{t-1}) + (1 - \delta)\ln\left(\overline{A}_t\right) - \chi E_t - \zeta_2 T_0, \tag{5′}$$

where $\chi \equiv \zeta_1\zeta_2 > 0$. Both extensions simplify the GHKT model and make it more realistic and up to date with the most recent atmospheric insights and empirical evidence on global warming damages.

Proposition 6: *With temperature given by (4′) and total factor productivity by (5′), the expression for the optimal carbon price is*

$$\tau_t = \phi\left(\beta\right) Y_t \quad \text{with} \quad \phi\left(\beta\right) \equiv \frac{1}{1 - \beta\gamma}\left[\psi\left(1 - \frac{\alpha\beta\gamma\kappa}{1 - \beta\gamma(1 - \kappa)}\right) + \frac{\chi}{1 - \beta\gamma\delta}\right] \geq 0. \tag{9″}$$

Proof: analogous to proof of Proposition 3. See also Appendix C.

The optimal SCC thus still grows in line with the level of aggregate output. Without utility damages from global warming and no mean reversion in total factor productivity (i.e. $\psi = 0$ and $\delta = 0$), this simplifies to. $\tau_t = \chi Y_t / (1 - \beta\gamma)$.

6. Implications of a cap on temperature or cumulative emissions

Using the finding that temperature is driven by cumulative emissions and given that climate policy is often articulated in terms of temperature targets, we now suppose that there is a cap on cumulative emissions, $\overline{E}$, so that $E(t) \leq \overline{E}, \ \forall t \geq 0$, in addition to the marginal damages of climate change on production and utility. Such caps can be formulated on a global level or may result from the nationally determined contributions to emissions reductions as allocated in the Paris Accord. Alternatively, it corresponds to a temperature cap which from (4′) implies a cap on cumulative emissions. E.g. a cap of 2 °C implies a global cap on cumulative emissions of $\overline{E} = (2 - T_0)/\zeta_1$. With $T_0 = 1.3$ °C and $\zeta_1 = 2$ °C/TtC, the carbon budget would be $\overline{E} = 350$ GtC. Proposition 5 states how the carbon price needs to be modified to ensure that the temperature cap or cap on cumulative emissions is satisfied. An extra term is needed emissions are too high if the carbon tax if set according to equation (8′). In that case, the cap of cumulative emissions bites and the carbon tax must be adjusted upwards.

Proposition 7:. *With temperature given by (4′) and total factor productivity by (5′), the expression for the optimal carbon price under a cap on cumulative emissions is given by*

$$\tau_t = \phi(\beta) Y_t + (\beta\gamma)^{-(t-1)} \Delta Y_t, \quad \forall t \geq 0, \tag{10}$$

where $\phi(\beta)$is defined in (9′) and $\Delta = \varpi\left(1 - \frac{\alpha\beta\gamma\kappa}{1 - \beta\gamma(1 - \kappa)}\right)$. The constant ϖ is zero$\tau_{t+1}/\tau_t = j_{t+1}$. if the optimal trajectory does not fully exhaust the carbon budget, $\lim_{t\to\infty} E_t < \overline{E}$. And $\varpi > 0$ if the optimal trajectory fully exhausts the carbon budget, $\lim_{t\to\infty} E_t = \overline{E}$. The constant ϖ then has to be chosen so that the carbon budget is exactly exhausted. **Proof:** see Appendix C.

Equation (10) shows that the optimal carbon price consists of two terms. The first term is the usual term corresponding to the optimum without a (biting) temperature cap as in equation (9′) of Proposition 4. This term rises at a rate equal to the rate of growth of aggregate output. The second term is only present if the cumulative emissions constraint or the temperature cap bites. It corresponds to a Hotelling path along which this term rises at a rate equal to the rate of interest (i.e. the rate of time impatience plus the per-capita growth rate of the economy according to the Keynes-Ramsey rule with a unit elasticity of intertemporal substitution or $j_t = 1 + r_t$ as in Proposition 2), since the stock of carbon that can be emitted is now finite. If there are no damages of global warming to utility or total factor productivity, then equation (10) for the optimal carbon price reduces to the Hotelling rule $\tau_t = (\beta\gamma)^{-(t-1)}\Delta Y_t,\ \ \forall t \geq 0$, and its growth rate to the rate of interest, If the temperature cap bites, then tightening the temperature or cumulative emissions cap requires the Hotelling path for the second term to be lifted (i.e. Δ increases as $\overline{E}$ is cut).

The first two conditions of (7) give efficient use for coal and gas, $\frac{\partial Y_t}{\partial F_t} / \frac{\partial Y_t}{\partial A_2 L_{2t}} = \frac{\kappa_{1t} {F_t}^{\rho-1}}{\kappa_{2t} {A_{2t}}^{\rho} {L_{2t}}^{\rho-1}} = \frac{h_t + \tau_t}{w_t / A_2 + \tau_t}$, so that carbon emissions from coal relative to those of fossil fuel are $\frac{A_{2t} L_{2t}}{F_t} = \left(\frac{\kappa_{2t} A_{2t}}{\kappa_{1t}} \frac{h_t + \tau_t}{w_t / A_2 + \tau_t} \right)^{\varepsilon}$. The price of the oil-gas aggregate is currently less than that of coal, so that a higher carbon tax curbs at least initially emissions from coal relative to that of oil and gas. As time increases, the scarcity rent on oil and gas increases exponentially and thus eventually a carbon tax boosts emissions from coal relative to that of oil and gas.

As time goes to infinity, the carbon tax and wage rise in line with aggregate output which is at a smaller rate than the growth rate of the scarcity rent on oil and gas which equals the interest rate. It follows that as time goes to infinity and with asymptotic depletion of oil and gas, the ratio of emissions of coal to that of fossil fuel goes to infinity. However, as time goes to infinity, coal use and oil/gas use tend to zero, for else the cap on cumulative emissions will be violated, while renewable energy grows to a positive asymptote and grows forever if the economy grows. It is not immediately clear yet how the cap on cumulative emissions, $\lim_{t\to\infty} \left(\sum_{t=1}^{t} (F_t + A_2 L_{2t}) \right) = \overline{E} > 0$, is met by depleting oil/gas so that $S_t \to \overline{S} \geq 0$ as $t \to \infty$ with $S_0 - \overline{S} \leq \overline{E}$, or by curbing the amount of coal use, cumulative emissions approach $\overline{E}$only asymptotically (but see section 8.5).

7. Matching the GHKT model: logarithmic depreciation on an annual time scale

Our model is an extension of the original version presented in GHKT. We mean by this that if capital depreciates fully, population and TFP growth are absent, damages only impact production rather than utility or productivity growth, private and public discount rates are the same, and time is on a decadal scale, our extension reproduces the findings of GHKT exactly. However, since our analytical solution allows for logarithmic depreciation, we can abandon the restrictive assumption of a decadal time scale. In this section, we show that our model reproduces the numerical findings of GHKT even on an annual scale with partial logarithmic depreciation. We then illustrate how the equilibrium trajectories under business-as-usual and optimal policy change if we allow for the various extensions of our model and estimates for depreciation in Anderson and Brock (2021). In our baseline numerical simulations, we adopt the following parameter values from the calibration of GHKT given in Table 1.

This implies a capital share of 30% and an energy share of 4% of value added and a rate of time impatience of 1.5% per annum. The carbon cycle is calibrated to the following points: 20% of carbon emissions stay up forever in the atmosphere, of the remainder 60% is absorbed by oceans and the surface of the earth within a year and the rest has a mean life of 300 years. Half of a carbon emissions impulse is removed from the atmosphere after thirty years. Production damages from global warming are 2.35% of global GDP for each trillion ton of excess carbon in the atmosphere. Initial output is calibrated to match \$700 trillion per decade.[8] Using these parameters and a decadal time scale, setting population and TFP constant, depreciation of capital to 100% ($\iota = \kappa = 1$), and with only production damages ($\gamma = 1$, $\psi = \delta = 0$), the optimal carbon price directly follows from expression (9) and equals \$56.5 per ton of carbon (tC) or \$15.4 per ton of CO_2 in 2010.

Population is constant and normalized to unity, $N_t = 1$. GHKT set the elasticity of substitution between energy types to $0.945 < 1$ (i. e. $\rho = -0.058$). This implies that all energy factors are essential to production and can never be phased out completely. Climate policy, therefore, solely aims at depressing fossil energy use rather than a transition to a carbon-free economy where emissions are zero. Relative prices and demand of different energy types (in GtC) and extraction costs of coal are used to calibrate the energy share parameter $\kappa_1 = 0.5008$, $\kappa_2 = 0.08916$ and $\kappa_3 = 0.41004$, and the initial labour-efficiency parameters for coal, $A_{2,0} = 7683$ and renewables, $A_{3,0} = 1311$. The efficiency of labour in coal and renewables is assumed to grow at 2% per annum (i.e. $A_{2,t+1}/A_{2,t} = A_{3,t+1}/A_{3,t} = 1.02^{10}$). The finite stock of oil is set to 300 Gt of oil which converts to 253 GtC. It is assumed that there is no productivity increase in the aggregate goods sector, $\omega = 1$. Together with the initial conditions for the atmospheric stocks of carbon, $E_0^p = 684, E_0^t = 118$, the equilibrium trajectories of energy use and climate change can be computed.[9] Using energy inputs, total factor productivity is calibrated to reproduce initial output Y_0 with $K_0 = \$128{,}922$ billion from Barrage (2014). This gives $A_0 = 18{,}298$.

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[8] In gauging the effect of parameter changes on the initial carbon price, we follow GHKT in assuming that Y_0 is given. This is, however, only a first-order approximation as Y_0 will be affected by climate policy. Since energy is only a small share in value added, this is not a bad approximation.

[9] Note that in the supplementary material to GHKT Barrage (2014) reports initial values of 699GtC for permanent and 103 GtC for transitory atmospheric carbon and $\kappa_1 = 0.5429$ and $\kappa_2 = 0.1015$.

Table 1
Benchmark - GHKT calibration of the model.

Decadal GHKT model	
Final goods production function	Share of capital $= \alpha = 0.3$, share of energy $= \upsilon = 0.04$, Initial world GDP per decade $= Y_0 = 700$ T$
Energy production function	$\kappa_1 = 0.5008$, $\kappa_2 = 0.08916$, $\kappa_3 = 0.41004$, $\rho = -0.058$
Process of logarithmic depreciation	$\iota = \kappa = 1$
Population growth and technical progress	$N_t = 1$, $A_{2,0} = 7683$, $A_{3,0} = 1311$, $A_{2,t+1}/A_{2,t} = A_{3,t+1}/A_{3,t} = 1.02^{10}$
Gross growth labour productivity	$\omega = 1$
Dynamics of atmospheric carbon	$\varphi_L = 0.2$, $\varphi_0 = 0.393$, $\varepsilon = 0.0228$, $S_0 = 253.8$ GtC, $E_0^p = 684$ GtC, $E_0^t = 118$ GtC
Global warming damages and TFP	$\chi = 2.379\ 10^{-5}$, $\delta = 0$
Time impatience and utility damages	$\beta = 0.985^{10}$, $\psi = 0$
Gross population growth	$\gamma = 1$
Annual model with log depreciation	
Recalibration to annual time scale and matching of above decadal model	$Y_0 = 70$ T$, $\iota = 1.26$, $\kappa = 0.1$ $N_t = 1$, $A_{2,0} = 768.3$, $A_{3,0} = 131.1$, $A_{2,t+1}/A_{2,t} = A_{3,t+1}/A_{3,t} = 1.02$ $\omega = 1$, $\varphi_L = 0.2$, $\varphi_0 = 0.401$, $\varepsilon = 0.0023078$, $S_0 = 253.8$ GtC $\chi = 2.379\ 10^{-5}$, $\delta = 0$, $\beta = 0.985$, $\psi = 0$, $\gamma = 1$

The equations of Propositions 1–3 can readily be solved numerically using standard routines.[10] Fig. 1 reproduces the findings of GHKT for the optimal policy (orange) and business-as-usual (blue) cases. The decadal time scale is visible by the stepwise increments. We compare these with annual version of our model (smooth solid lines). Adjustment of all time-dependent parameters to the annual scale is reported in Table 1 (see also Appendix D). We change depreciation from 100% in each decade to annual partial (logarithmic) depreciation with $\iota = 1.26$ and $\kappa = 0.1$ to match the output and capital dynamics of the decadal GHKT model with 100% depreciation.

We base our calibration of income-per-capita damages on the detailed empirical estimates in Burke et al. (2015). The blue line in Fig. 5(d) of this study is the “differentiated response, lung-run effect”, i.e. a middle-range estimate, and suggests that for every increase in temperature by 1 °C, the global warming damage in terms of lost GDP per capita increases by 12.5% of global economic activity.[11] This is much higher than the damages in the DICE models (e.g. 2.35% of global GDP for each trillion ton of excess carbon in the atmosphere; see section 6). We thus calibrate $\zeta_2 = -ln(1 - 0.125) = 0.1333$ and. $\chi = \zeta_1 \zeta_2 = 0.002 \times 0.133 = 2.66 \times 10^{-4}$.

The top two panels of Fig. 1 show the effects of pricing carbon on global equilibrium temperature and coal use. As described in detail in GHKT, pricing carbon effectively avoids the worst effects of climate change, virtually exclusively through a reduction in coal use as this is the most carbon-intensive fossil fuel. Temperature increases beyond 2 °C at the end of this century and 3 °C at the end of next century even under carbon pricing. Since oil can be used without cost, its stock of reserves will always be fully depleted (asymptotically) in this model. Policy intervention hardly affects the time profiles of oil use and renewable energy output, but coal use falls significantly albeit it takes a century or so for this to occur.

Capital stock and output (net of damages) are reported in the bottom panels of Fig. 1. Both increase rapidly within the first decades but due to the absence of any growth engines, both converge to their steady state level around 2050. This steady-state level is falling over time due to the continued emissions of carbon and increases in damages. Given that carbon-based energy inputs are essential in production, this is an unavoidable feature of the GHKT model. Our calibrated annual version of the model can match the dynamics of the decadal version very well. The initial carbon price in 2010 is slightly lower at $52.5/tC but taking account of output growth over the whole decade, the carbon price averages at $56.3/tC which matches the decadal number of $56.5/tC closely.

Thus far, our annualized model was calibrated to reproduce the dynamics of the baseline of GHKT which features a decadal time scale and full depreciation. Anderson and Brock (2021) compare empirically the cases of geometric and logarithmic depreciation and present econometric estimates from a world panel with $\iota = 1.17$ and $\kappa = 0.05$. Adopting these two parameters affects the consumption and investment decisions, as lower depreciation (relative to the complete case) increases the return to capital. The trajectories of capital and output change, while all other variables, most notably those of the energy sector, are unaffected.

Key: Optimal policy (orange) and business-as-usual (blue) simulations in the decadal GHKT model with full depreciation (step-bars) every decade and in our annual model with partial logarithmic depreciation (smooth lines), where the latter model is matched to the GHKT model.

Fig. 2 shows that reducing κ from 0.1 to 0.05 increases the consumption share given in equation (6) in addition to lowering depreciation. With the latter effect dominating, this leads to a substantially higher accumulation of capital and a prolonged period of output growth. With no exogenous growth drivers, output growth starts at 1.6% per annum and falls below 1% by 2025. In the optimal policy scenario, positive economic growth is maintained until the mid of next century while growth turns negative by 2115 in the business-as-usual scenario. Capital accumulation is the only growth engine in the GHKT model (together with exogenous progress in the coal and renewable sectors which affects output growth by less than a twentieth of a percentage point at most).

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[10] The source code containing our solution routines is available upon request.

[11] The empirical results of Burke et al. (2015) also suggest that the change in GDP per capita is smallest at an annual temperature of about 13 °C; it drops off rapidly if temperature is either lower or higher than that (see their Fig. 2).

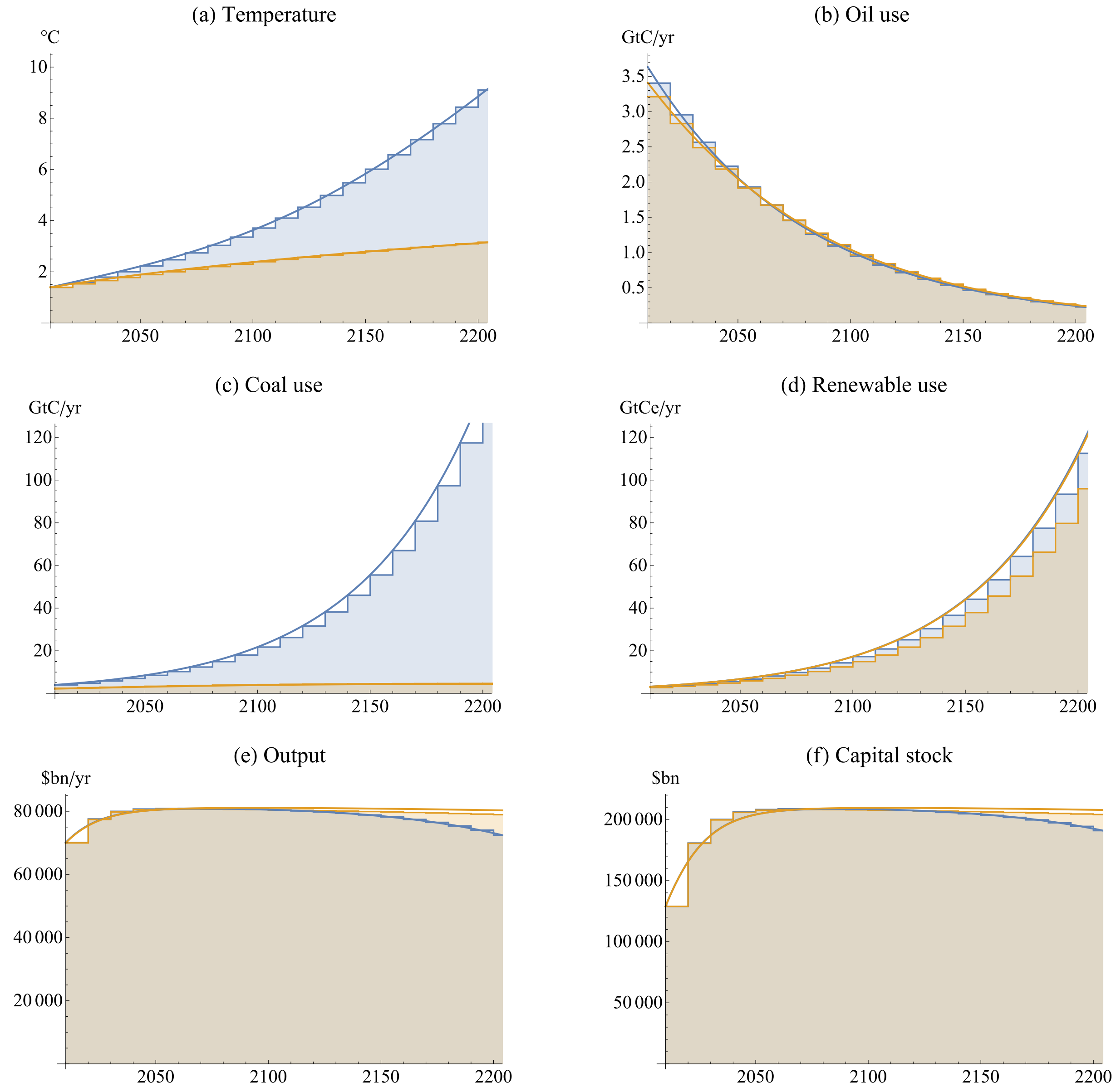

Fig. 1. Comparing business-as-usual and optimal outcomes in the decadal GHKT model and our annual model.
Key: Optimal policy (orange) and business-as-usual (blue) simulations in the decadal GHKT model with full depreciation (step-bars) every decade and in our annual model with partial logarithmic depreciation (smooth lines), where the latter model is matched to the GHKT model.

8. Pricing carbon in our extended GHKT model

We use our annualized version of GHKT model as a baseline to compare how extensions regarding long-run growth in population and productivity and damages to productivity and utility change the model's predictions. Table 2 presents key environmental variables for these scenarios. The reported social cost of carbon are updated to 2019's GDP level of $85 trillion in constant 2010 US$ (World Bank, 2020). This increases the baseline carbon tax from $56 to $64 and increases to grow at the rate of (real) GDP. The sensitivity of the optimal carbon tax to differences in public and private discounting, to different climate and damage formulations, and caps of temperature is presented in Table 3.

8.1. Population growth and technological progress

The introduction of population growth lowers the population-adjusted discount rate and increases the saving rate and the social cost of carbon as the current generations are more willing to provide for a larger population in the future and to make sacrifices to curb

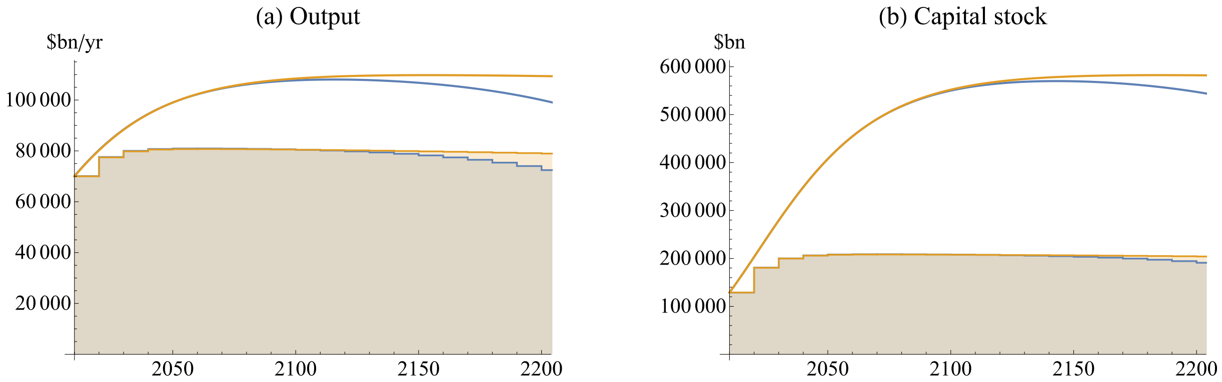

Fig. 2. Logarithmic depreciation (using Anderson and Brock (2021) estimates). **Key:** Calibrated annual model (lines) versus decadal GHKT (bars) with optimal policy (orange) and business-as-usual (blue) simulations. None of the other variables are affected by this process of logarithmic depreciation (see Corollary 1). Bars are same as in Fig. 1. Lines correspond to the outcomes with the Anderson and Brock (2021) estimates of logarithmic depreciation.

Table 2
Sensitivity of optimal policy simulations.

	SCC in 2019 ($2010/tC)	Temperature in 2100 (°C)	Temperature max (°C)
Baseline annualized GHKT model	$64	2.4	3.7
(1) Population grows 1% p.a. ($\gamma = 1.01$)	$164	1.9	2.7
(2) TFP growth of 1% p.a.	$64	2.5	3.8
(3) Damages to utility	$63	2.4	3.8
(4) Damage effects on TFP growth	$100	2.2	3.1
(1) & (4): population growth and TFP growth damages	$259	1.8	2.4
(2) & (4): TFP growth and TFP growth damages	$100	2.2	3.2
(3) & (1): Utility damages and population growth	$162	1.9	2.8
(3) & (2): Utility damages and TFP growth	$63	2.4	3.8
(3) & (4): Utility damages and TFP growth damages	$90	2.2	3.2
(1), (2), (3), and (4)	$232	1.8	2.5

Table 3
Further sensitivity of optimal policy simulations.

	SCC in 2019 ($2010/tC)	Temperature in 2100 (°C)	Temperature max (°C)
Baseline annualized GHKT model	$64	2.4	3.7
(1) social discounting 0.1%: first-best	$601	1.6	2.0
(2) public 0.1% private 1.5%: no subsidy	$650	1.8	2.0
(3) social discounting 1%: first-best	$93	2.2	3.3
(4) public 1% private 1.5%: no subsidy	$95	2.2	3.2
(5) Cumulative emissions	$135	2.9 (2.0)	4.6 (3.1)
(5) & Damages from Burke et al. (2015)	$1507	2.0 (1.6)	2.8 (2.0)
(6) Cap on temperature, 2 °C	$281	1.8	1.9
(7) Cap on temperature, 1.5 °C	$1152	1.5	1.5

Note: Temperature follows from radiative forcing (see in fn. 2); values in (.) follow from equation (4′).

future global warming. With annual population growth of 1% ($\gamma = 1.01$), the optimal carbon price more than doubles to $164/tC in 2019 in Table 2 and temperature at the end of the century falls by half a degree to 1.9 °C. Peak warming falls from 3.7 to 2.7 °C. In contrast, TFP growth of 1% per annum does not affect the initial carbon price. However, over time the economy grows at a faster pace and thus the optimal price of carbon grows at a faster pace too. As a result, energy output increases slightly and temperature by 0.1 °C. Bohn and Stuart (2015) consider endogenous population size, calculate the externality from an extra birth on climate change and find it to be large. This requires policy to be less pronatalist and may require a sizeable Pigouvian tax on having children. We abstract from these issues here.

8.2. Utility damages and damages to productivity growth

To allow for utility damages we follow the calibration of Barrage (2018) and assign 74% of damages at 2.5 °C to production and 26% of damages to utility ($\chi = 1.806 \times 10^{-5}$ and $\psi = 7.376 \times 10^{-6}$ but $\delta = 0$).[12] The optimal carbon price falls only slightly to $63/tC, which causes peak temperature to edge up by 0.1 °C in Table 2.

In the GHKT formulation damages only affect the current level of TFP but not its growth rate. Dell et al. (2012) find that a temperature increase of 1 °C lowers per capita income growth of an economy on a balanced growth path by 1.171%-points in poor and 0.152%-points in rich countries (although the latter result is not statistically significant) which requires setting $\delta = 0.367$ in our model. The resulting optimal carbon price increases significantly from $64 to $100/tC. Pricing emissions at this level is still insufficient to be consistent with the ambitions of the Paris Agreement to keep the temperature increase by the end of the century well below 2 °C, because temperature increases at the end of the century to 2.2 °C and peak temperature is 3.1 °C.

Table 2 also reports combinations of the effects discussed so far. Most notably the effects of population growth (1) and damages to productivity growth (4) compound to an initial carbon price of $259/tC and temperature of 1.8 °C in 2100 and a peak temperature of 2.4 °C. Adding the effects of TFP growth (2) to the effects of damages to productivity growth (4) does not change the optimal carbon price: it stays at $100/tC.

Allowing for damages to utility (3) does not alter scenarios (1) or (2) much, except when combined with damages affecting economic growth (4). Here it lessens the lasting effect of productivity damages as these only affect the damage component affecting production. When utility damages (3) are added to damages to the growth rate (4), the optimal SCC falls from $100/tC to $90/tC. Finally, adding effects (1), (2), (3) and (4) leads to an optimal SCC of $232/tC and a temperature increase of 1.8 °C at the end of the century.

8.3. Different discount rates for the private and public sector

If the public sector applies a lower discount rate, the policymakers can reproduce the social optimum if they price carbon and simultaneously subsidize saving. If the government discounts the future at 0.1% per year while the private sector maintains the baseline rate of 1.5% per year, the SCC increases to $601/tC in Table 3 and, following proposition 2, the required capital income subsidy is 1.4%. If the government cannot subsidize capital income, the second-best policy defined in proposition 3 is to increase the carbon tax by 9% to $650/tC to compensate the future for the inefficiently low savings rate. Note that the missing capital subsidy also makes saving in the form of fossil fuel less attractive. Oil use is brought forward. Temperature at the end of the century increases by 0.2 °C as a result, while peak temperature remains unchanged. If the government uses a discount rate of 1%, the optimal carbon tax is $93/tC, the capital subsidy 0.5%, and the second-best carbon tax $96/tC.

8.4. Temperature driven by cumulative emissions and damages of Burke et al. (2015)

We can simplify the climate model of GHKT by assuming that cumulative emissions drive temperature, as in section 5. This is equivalent to assuming that $\phi_L = 1$. If carbon remains in the atmosphere permanently, the social cost of emitting increases to $135/tC in Table 3 (see also the discussion of equation (9)). Temperature based on this model increases significantly compared to 2.9 °C at the end of the century and 4.6 °C at the end of the simulation period or, if temperature is derived from the cumulative emissions approach, by 2.0 °C and 3.1 °C. Adopting the estimates of Burke et al. (2015) increases the damage parameter to $\chi = 2.66 \times 10^{-4}$ and the SCC tenfold to $1507/tC. Temperature in increases by 2.0 °C by 2100 and at most 2.8 °C using the radiative-forcing-based temperature formula in footnote 2 or 1.6 °C in 2100 and at most 2.0 °C if formula (4') based on cumulative emissions is used. This huge difference with the optimal carbon price under GHKT or Nordhaus damages suggests that much more econometric work is needed on the effects of climate change on economic damages.

8.5. Cap on temperature or on cumulative emissions and stranded oil and gas reserves

The finding that temperature is driven by cumulative emissions has given rise to temperature targets being expressed in carbon budgets. Using budgets presented in Table 2.2 of the IPCC's Fifth Assessment Report, we can add temperature targets as discussed in section 6 and quantify the 1.5 °C and 2 °C as a cap on cumulative emissions of 150 GtC and 355 GtC, respectively. This is consistent with our calibration on cumulative emissions for the 2 °C target. These constraints on cumulative emissions are based on the baseline GHKT rather than the model of cumulative emissions to make the figures reported below consistent with those displayed in Figs. 1 and 2.

The introduction of emission caps increases the optimal carbon tax to $281 and $1152 for the more stringent target but also augments the carbon tax in that a fraction grows at the interest rate, and therefore each year 1.5% (using $(\beta\gamma)^{-1} = 1.015$) faster than GDP. This permits a low temperature peak around 2100 after which temperature falls. Panel (a) in Fig. 3 plots the time profiles of the carbon taxes for temperature caps and compares them to the baseline case of no cap. Since in the baseline calibration, all exogenous growth engines are turned off, the optimal carbon tax is fairly flat in the case of no cap. Following equation (9), however, the carbon

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[12] The scientific uncertainty about damage forms and functional forms is large, partly due to the weak signal at low historical temperature variations. More empirical work is needed on these issues.

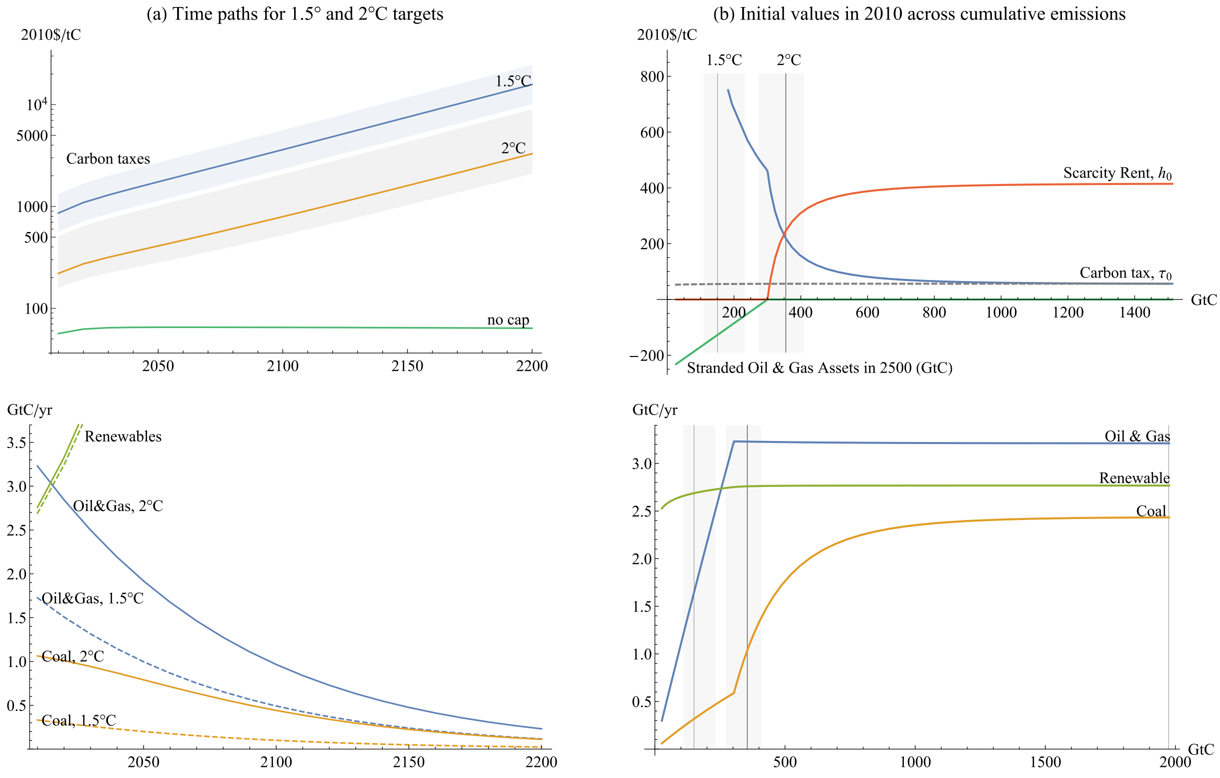

Fig. 3. Caps on cumulative emissions and temperature.

taxes increase exponentially if the cap is binding (note the log-scale in the plot). Under the budgets used, 50% of the models used by the IPCC remain within the respective temperature target. The IPCC also reports carbon budgets for the 33% and 66% mark, which are captured by the shaded areas in Fig. 3.

Coal use is most affected by carbon taxation. As shown in Fig. 1, the use of renewable sources and oil and gas only decrease by 0.3% and 6%, while coal use nearly halves from 4 GtC to 2 GtC per annum initially. Given that oil and gas are available without cost (save the intertemporal scarcity rent), all reserves are generally used up eventually, slashing coal use is the primary function of the carbon tax. Similar results hold for carbon taxation under a 2 °C cap. The lower panel in Fig. 3(a) shows that renewable use and oil and gas extraction barely change, while initial coal use drops to 1 GtC. The more forceful pricing of carbon under the 1.5 °C target has a similar effect, again reducing coal to 10% of its business-as-usual level and less than half its use under the 2 °C target. While renewable use falls only marginally, pricing emissions at $1152 per tC also hits oil and gas, cutting emissions from its use in half.

The qualitative difference in oil and gas use between the 1.5 °C and 2 °C caps is illustrated in Fig. 3b, which plots initial carbon taxes and scarcity rents and long-run stranded oil and gas assets as a function of the carbon budget. In the baseline case of no cap, where carbon is only taxed to avoid marginal damages, cumulative emissions amount to roughly 2000 GtC. As the budget becomes more smaller, climate policy must become more stringent and the carbon tax gradually deviates from the baseline tax of $56/tC (gray dashed). As the budget approaches the level of 300 GtC the initial carbon tax rises rapidly to above $400/tC, after which it continues to increase at a lower rate. The behaviour of the carbon tax is mirrored by the scarcity rent on oil and gas reserves. Since a lower carbon budget implies less carbon-intensive use, the Hotelling rent declines as the carbon budget increases. Below 500 GtC of cumulative emissions, the decline accelerates before reaching zero at 300 GtC. This is the budget of cumulative emissions, which directly equates supply and use of oil and gas reserves. Lower levels of cumulative emissions keep total use below supply and some oil and gas assets will be abandoned underground (see green slope in Fig. 2(b)). The (natural resource) wealth effects of temperature caps can be measured in terms of write-off due to depressed prices or in the amount of unburnable fossil fuel. At the introduction of a 2 °C cap the value of oil and gas reserves drops from $105bn to $62bn. This is $43bn reduction is 128 times the $334mn drop when carbon prices based on baseline damages are introduced. The cap for 1.5 °C makes reserves worthless as half (or 128 GtC) will never be used and the scarcity rent is, therefore, zero.

The lower panel of Fig. 2(b) plots energy use across the carbon budget. Oil and gas use is flat as long as the scarcity rent is positive. From equation (7) this implies that the carbon tax is set to offset the decrease in the scarcity rent and only coal use is affected at carbon budgets above 300 GtC, which is consistent with the Herfindahl rule. At lower levels, coal and oil and gas use also drop linearly, although reduction in cumulative use are proportionally larger for oil and gas (due the greater slope) which contradicts the Herfindahl rule and stems from the fact that coal and oil and gas use are regulated by a uniform tax. Carbon taxation also lowers renewable energy use, since the GHKT calibration assumes that all energy types are cooperative factors of production. These reductions are small, reaching at most 10% at very high levels of carbon taxation.

9. Discussion: merits, uses and limitations of the GHKT model

9.1. Merits and versatility of the GHKT model

The GHKT model has very bold assumptions but nevertheless has become a reliable workhorse for many applications in climate economics. This literature has shown a multitude of interesting applications of the GHKT model that permit a tractable and convenient analysis. Li et al. (2016), Anderson et al. (2019) and Gerlagh and Liski (2017a) have used the model to allow for climate model uncertainty, Engström and Gars (2016) for abrupt climate change and tipping points, Gerlagh and Liski (2017b) and Iverson and Karp (2021) for hyperbolic discounting, and Hassler and Krusell (2011) for multi-country analysis.

We have extended the model to set of relevant policy questions by allowing for population growth and labour-augmenting technical progress in the final goods and energy sectors and for an annual instead of a decadal time scale by having logarithmic depreciation, while still obtaining an exact tractable expression for the optimal SCC. This is also the case if global warning negatively impacts utility or the growth rate of total factor productivity, changes in global warming only affect total factor productivity gradually, and the government uses a lower discount rate than the private sector. Tractable expressions of the optimal SCC can only be obtained if a more realistic model of temperature depending on cumulative emissions is used and damages are a linear function of cumulative emissions. Finally, we have shown that these rules for the optimal SCC can be easily modified to allow for temperature caps.

9.2. Extensions of the GHKT model with approximate rules for the optimal SCC

There are some important modifications of the GHKT model that break the exact simple rule for the optimal SCC but nevertheless yield approximate expressions for the optimal SCC that lead to outcomes that are close to the numerical optimum and yield negligible welfare losses (e.g. van den Bijgaart et al., 2016; Rezai and van der Ploeg, 2016).

First, logarithmic utility implies a unit coefficient of relative risk aversion, a unit coefficient of intertemporal substitution and a unit coefficient of intergenerational inequality aversion which is too restrictive from an ethical and preferences point of view. If we have a power utility function with a constant non-unitary elasticity of intertemporal substitution (*EIS*), we replace the growth-corrected discount factor $\beta\gamma \cong 1/(1+\rho-n)$ in expressions (9) or (9′) for the optimal carbon price by $1/(1+\rho-n+(EIS^{-1}-1)(g_t-n))$ to allow for a non-unitary EIS in the Keynes-Ramsey rule (cf. equation (2′) of Rezai and van der Ploeg (2016)), where g_t, n and ρ denote the long-run growth rate of the economy, the population growth rate, and the rate of time impatience, respectively. This leads to a good approximate rule for the optimal SCC. Provided $EIS < 1$, higher growth and higher intergenerational inequality aversion (i.e. lower *EIS*) increase the discount factor that must be applied to calculate the optimal SCC, since current generations are then less willing to make sacrifices to lower temperature for future generations. Consequently, the optimal SCC is lower than in the GHKT model with a unit *EIS*.

Second, if parameters or functional forms are changed that only affect the optimal carbon price via GDP, our optimal policy rules still give good outcomes that are not too far from the true optimum. This is so if, for example, the Cobb-Douglas production function for final goods is changed to a CES function or depreciation is not quite 100% per period. This is also so if the exhaustible energy type (oil) is replaced by an abundant energy type or if this energy type has strictly positive extraction costs.[13] All these changes will affect GDP but will have only a (minor) effect on the rule for the optimal SCC.

Third, if damages to total factor productivity are not proportional to aggregate output but additive, replacing current GDP (Y_t) by initial GDP (Y_0) and replacing $\beta\gamma \cong 1/(1+\rho-n)$ in expressions (9) or (9′) for the optimal carbon price by $1/(1+\rho+EIS^{-1}(g_t-n))$ to allow for a non-unitary EIS by gives a good approximation for the optimal SCC (cf. Result 1 in Rezai and van der Ploeg (2016)). The path of the optimal SCC is now flat rather than rising at the rate of economic growth and the initial level is lower than with multiplicative damages as the growth-corrected discount factor is bigger (Rezai et al., 2020).

Fourth, perturbation methods yield tractable rules for good approximations to the optimal SCC in dynamic stochastic general equilibrium models with uncertain shocks to the economy, temperature response, and damages, allowing for correlated shocks and for the coefficient of relative risk aversion to differ from the inverse of the elasticity of intertemporal substitution (van den Bremer and van der Ploeg, 2021). The optimal SCC then depends on precautionary, insurance, and hedging motives. Note that with the logarithmic utility of the GHKT model all the effects of uncertainty about the rate of economic growth and all hedging effects cancel out, albeit uncertainty about the temperature response and damages does affect the optimal SCC.

9.3. Limitations of the GHKT and the extended GHKT model

The model runs, however, into limits and fails to generate tractable closed-form rules for the optimal SCC when reasonable features are considered. For example, the GHKT model does not allow the cost of fossil fuel extraction to increase as reserves are depleted. This would allow for partial exhaustion of reserves, so that policy can affect how fast to deplete fossil fuel reserves and how much of the reserves to abandon, introducing considerations of economic obsolescence to the discussion of stranded assets (Rezai and van der

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[13] One can get a correct tractable expression for the optimal SCC if oil uses only labour input and no capital.

Ploeg, 2015). Moreover, even under stringent climate policy, scarcity rents could remain positive as depleting reserves is costly. Furthermore, as Golosov et al. (2014) point out, their model can be extended for learning by doing in production of renewable energy. The carbon price must then be complemented with a renewable energy subsidy, which is set equal to the present discounted value of all present and future benefits of using one unit of renewable energy for short or the social benefit of learning for short (van der Zwaan et al., 2002; Popp, 2004; Rezai and van der Ploeg, 2017). This leads to a spike in renewable energy subsidies and a gradual ramp in carbon prices (cf. Acemoglu et al. (2012) who show this in a context with directed technical change).[14]

Unfortunately, the GHKT model also cannot speak to relative price effects as the atmospheric carbon stock (or cumulative emissions) is separable from consumption of final goods in utility. It is infeasible to obtain tractable expressions for the optimal SCC if the utility of non-market environmental goods is non-separable from the utility of consumption of market goods. If that were the case and environmental goods grow more slowly than consumption goods, relative prices increase over time and damages becomes more costly over time (Hoel and Sterner, 2007; Sterner and Persson, 2008; Zhu et al., 2019; Drupp and Hänsel, 2021; Bastien-Olvera and Moore, 2021). This leads to higher mitigation rates and a declining term structure for the discount rate.

Finally, a major limitation of the GHKT model is that it is effectively static, i.e. income and intertemporal substitution effects in the consumption-saving decisions cancel out. Hence, issues of time inconsistency only play a limited role and that the GHKT model is of limited interest for the analysis of second-best policy issues. Similarly, strategic interactions between countries are severed as the pre-commitment and subgame-perfect outcomes coincide (e.g. Hambel et al., 2019). More realistic analysis of strategic issues thus requires a model which allows for more interesting dynamic interactions.

10. Conclusion

We have shown that the GHKT model still offers a tractable rule for the optimal SCC and the price of carbon emissions when we relax the underlying restrictive assumptions. First, we use logarithmic depreciation (Anderson and Brock, 2021) to allow for a finer time resolution with an annual time scale. Second, we allow global warming to negatively affect utility and the rate of growth of total factor productivity (as estimated in Dell et al. (2012) and studied in Dietz and Stern (2015)), and also allow more generally for mean reversion in total factor productivity. Third, we follow Von Below (2012), Belfiori (2017, 2018) and Barrage (2018, 2019) and allow policymakers to be more patient than the private sector. This requires the carbon tax to be complemented with a capital subsidy. We also consider the second-best optimal carbon tax in case a capital subsidy is infeasible and discuss its time consistency. We also show that the rule for the optimal SCC can easily be adapted to allow for positive long-run growth by introducing population growth and labour-augmenting technical progress.

We have also adopted the GHKT model to make it more realistic by adopting recent atmospheric insights that temperature is a linear function of cumulative emissions. If damages are then a function of cumulative emissions instead of the stock of atmospheric carbon, we get a tractable modified rule for the optimal SCC. We calibrated this to both the detailed econometric damage estimates of Burke et al. (2015) and of the GHKT model. We then use this modified model to derive a tractable expression for the optimal carbon price under a temperature cap. If the cap bites, this requires adding a term to the unconstrained optimal carbon price that rises at a rate equal to the rate of interest. Such a policy leads to stranded oil and gas reserves. We illustrate all our results with numerical simulations which can be done with a simple programme (available upon request).

If policymakers internalize global warming damages to total factor productivity, the optimal SCC will grow at the same rate as GDP. If damages are unrelated to aggregate output, the optimal SCC will be stationary. The initial SCC increases in the degree of patience of policymakers, the effect of global warming on utility, the effect of global warming on total factor productivity and the persistence of this effect, the sensitivity of temperature to emissions, and the rate of population growth. In case temperature is determined by the dynamics of atmospheric carbon as in the GHKT model, it decreases in the degree to which atmospheric carbon decays. However, if policymakers implement a temperature cap, the optimal SCC will grow at a rate equal to the rate of interest which is typically higher. The initial carbon price then increases in the level at which temperature is capped, and the final cost of decarbonizing the economy and decreases in the rate of growth of the carbon price (i.e. the interest rate).

Although we might have come to the limits of this popular and tractable general equilibrium model, we believe that the lasting contribution of the extended GHKT model is to offer analytical insights in key drivers of optimal carbon pricing that cannot be obtained through numerical optimal policy simulations, serve as a useful work horse for future applications, and be invaluable in teaching. We also think that these type of tractable rules for the optimal SCC might serve as good rules of thumb for the optimal trajectory of carbon pricing in more complicated, numerical large-scale integrated assessment models. Such rules have the added advantage that they are easier to communicate and to commit to by policymakers.

Acknowledgements

Financial support from the Schrödinger fellowship of Austrian Science Fund (FWF): J 3633 is gratefully acknowledged.

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[14] These extensions are discussed for a model where energy types are perfect substitutes in van der Ploeg and Rezai (2016).

Appendix A. Proof of Propositions 1, 2 and 3

The Lagrangian for this problem is

$$\sum_{t=0}^{\infty}\beta^t N_t\{\ln(C_t/N_t)-\lambda_t\left[K_{t+1}-\iota\left(\frac{A_tK_t^{\alpha}[\kappa_{1t}F_t^{\rho}+\kappa_{2t}(A_{2t}L_{2t})^{\rho}+\kappa_{3t}(A_{3t}L_{3t})^{\rho}]^{\upsilon/\rho}(M_t-L_{2t}-L_{3t})^{1-\alpha-\upsilon}-C_t}{K_t}\right)^{\kappa}K_t\right]$$
$$-\psi\left(E_t^p+E_t^t\right)-\mu_t(S_{t+1}-S_t+F_t)-\varphi_t\left[ln(A_t)-\delta ln(A_{t-1})-(1-\delta)ln\left(\overline{A}\right)+\chi E_t\right]-\left[\eta_t^p\left(E_{t+1}^p-E_t^p-\phi_LF_t\right)+\eta_t^t\left(E_{t+1}^t-\varepsilon E_t^t-\phi_0(1-\phi_L)F_t\right)\right]\},\tag{A1}$$

where λ_t, μ_t, and φ_t denote the shadow values of capital and fossil fuel reserves and the shadow value of the log of total factor productivity at time *t*, respectively, and the η_t^t and η_t^p denote the shadow disvalues of the transitory and permanent components of atmospheric carbon, respectively.

The first-order conditions for C_t and K_t are $\frac{1}{C_t}=\lambda_t\iota\kappa\left(\frac{Y_t-C_t}{K_t}\right)^{\kappa-1}\equiv x_t$ and $\iota\left((1-\kappa)\frac{Y_t-C_t}{K_t}+\alpha\kappa\frac{Y_t}{K_t}\right)\left(\frac{Y_t-C_t}{K_t}\right)^{\kappa-1}\lambda_t=\frac{\lambda_{t-1}}{\beta\gamma}$. In choosing consumption, the marginal benefit of using output for consumption today has to equal the benefit of transferring output into the future, the shadow price of capital. Under geometric depreciation this term is constant, but here depreciation depends on the level of investment and, therefore, consumption. Define $j_t\equiv\iota\left((1-\kappa)\frac{Y_t-C_t}{K_t}+\kappa\alpha\frac{Y_t}{K_t}\right)\left(\frac{Y_{t-1}-C_{t-1}}{K_{t-1}}\right)^{\kappa-1}$. We have $\frac{C_t}{C_{t-1}}=\beta\gamma j_t$ and thus the growth rate of per-capita consumption at time *t* must equal

$$\frac{C_t/N_t}{C_{t-1}/N_{t-1}}=\beta j_t,\quad\text{where}\quad j_t\equiv\iota\left((1-\kappa)\frac{Y_t-C_t}{K_t}+\kappa\alpha\frac{Y_t}{K_t}\right)\left(\frac{Y_{t-1}-C_{t-1}}{K_{t-1}}\right)^{\kappa-1}.\tag{A2}$$

In case $\kappa=1$, we have $j_t=\alpha\frac{Y_t}{K_t}$. The Euler equations (A2) and (2) form a difference equation in the consumption share which is saddle-point stable, since $\beta\gamma(\alpha+(1-\kappa))<1$. The stable manifold is given by $C_t=c(\beta)Y_t$ with $c\left(\beta\right)\equiv\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)$, which gives equation (6) in Proposition 1. Using this and (6) after some algebraic manipulation, it follows that j_t boils down to $j_t=\frac{1}{\beta\gamma}\frac{Y_t}{Y_{t-1}}$. This leads to equation (8) in Proposition 2.

The first-order optimality condition for the log of total factor productivity gives $x_tN_t\frac{Y_t}{A_t}-\frac{N_t\varphi_t}{A_t}+\beta\delta\frac{N_{t+1}\varphi_{t+1}}{A_t}=0$ or, using (6), $\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)^{-1}-\varphi_t+\beta\gamma\delta\varphi_{t+1}=0$. The only non-explosive solution of this difference equation gives a constant:

$$\varphi_t=\frac{1}{(1-\beta\gamma\delta)}\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)^{-1}.\tag{A3}$$

The first-order optimality conditions for the transient and permanent components of the atmospheric carbon stock give $N_t\eta_t^t-\beta\varepsilon N_{t+1}\eta_{t+1}^t-\psi N_t-\chi\varphi_tN_t=0$ and $N_t\eta_t^p-\beta N_{t+1}\eta_{t+1}^p-\psi N_t-\chi\varphi_tN_t=0$. Using (A3) this boils down to.

$\eta_{t+1}^t=\frac{1}{\beta\gamma\varepsilon}\eta_t^t-\frac{1}{\beta\gamma\varepsilon}\left(\psi+\chi\varphi_t\right)=\frac{1}{\beta\gamma\varepsilon}\eta_t^t-\frac{1}{\beta\gamma\varepsilon}\left[\psi+\frac{\chi}{(1-\beta\gamma\delta)}\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)^{-1}\right]$ and. $\eta_{t+1}^p=\frac{1}{\beta\gamma}\eta_t^p-\frac{1}{\beta\gamma}\left(\psi+\chi\varphi_t\right)=\frac{1}{\beta\gamma}\eta_t^p-\frac{1}{\beta\gamma}\left[\psi+\frac{\chi}{(1-\beta\gamma\delta)}\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)^{-1}\right]$.

Since $\beta\gamma<1$, this difference equation satisfies the saddle-point condition, so the only non-explosive solution equation are the following positive constants:

$$\begin{aligned}\eta_t^t&=\frac{1}{1-\beta\gamma\varepsilon}\left[\psi+\frac{\chi}{(1-\beta\gamma\delta)}\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)^{-1}\right]>0,\\ \eta_t^p&=\frac{1}{1-\beta\gamma}\left[\psi+\frac{\chi}{(1-\beta\gamma\delta)}\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)^{-1}\right]>0.\end{aligned}\tag{A4}$$

Hence, using (6) and $\tau_t\equiv\frac{\phi_L\eta_t^p+\phi_0(1-\phi_L)\eta_t^t}{x_t}$, we finally get equation (9) of Proposition 3. The first-order optimality conditions for fossil fuel and renewables give rise to the Kuhn-Tucker conditions stated in (7) in Proposition 2, where $w_t\equiv(1-\alpha-\upsilon)Y_t/L_t$ and $h_t\equiv\mu_t/x_t$. The first-order optimality condition for reserves is $N_{t-1}\mu_{t-1}=\beta N_t\mu_t$ and recalling that $h_t\equiv\mu_t/x_t$, it follows (using equation (A2)) that scarcity rents must satisfy equation (8) in Proposition 2. □

Appendix B. Implementing the social optimum in decentralized market economy

The discussion of the social optimum conceals the underlying market dynamics of our economy. In a competitive market economy households receive wage, capital, and resource income and government transfers, T_t, and choose consumption, investment, and natural resource use to maximize their utility, $\sum_{t=0}^{\infty}(\beta^P)^t N_t[\ln(C_t/N_t) - \psi E_t]$, subject to their household budget constraint $Z_{t+1} = (1+i_{t+1})(1+\sigma_t)Z_t + w_t L_t + (p_t - \tau_t)F_t - C_t + T_t$, and the depletion constraint $S_{t+1} = S_t - F_t$ or $\sum_{t=0}^{\infty} F_t \leq S_0$, where Z_t denotes household assets, w_t the wage rate, τ_t the carbon tax, i_{t+1} the interest rate on assets, and σ_t the capital income subsidy rate. Households take prices, tax and subsidy rates, and transfers as given. The Euler equation for the representative household (A2) becomes $\frac{1}{\beta}\frac{C_t/N_t}{C_{t-1}/N_{t-1}} = (1+i_t)(1+\sigma_t)$. Households provide natural resources according to $p_t - \tau_t = h_t$, i.e. households need to be indifferent between selling the resource and keeping it under the ground with the Hoteling rent evolving according to the standard Hoteling rule corrected for the subsidy on capital income, $h_t = (1+i_t)(1+\sigma_t)h_{t-1}$. The capital gain on the resource wealth has to match the return a household can make on the capital market.

Production occurs under perfect competition and firms choose production to maximize profits. In the final goods sector, firm hire labour and capital and use energy to maximize profits, $Y_t - w_t L_t - (i_{t+1} + \delta)K_t - p_{1t}F_t - p_{2t}A_{2t}L_{2t} - p_{3t}A_{3t}L_{3t}$, taking the wage rate w_t, the market interest rate i_{t+1}, the market prices for fossil and renewable energy p_{it} for $i = 1, 2, 3$, and level of productivity as given. Asset market and final good market equilibrium require $Z_t = K_t$. Capital accumulation follows from (2). The finite resource is provided by households. The firm using the abundant fossil fuel type maximizes its profits $\sum_{t=0}^{\infty}\Delta_t[(p_{2t} - \tau_t)A_{2t}L_{2t} - w_t L_{2t}]$ with $\Delta_t \equiv \prod_{s=0}^{t}(1+i_{s+1})^{-1}$, and renewable energy producers $\sum_{t=0}^{\infty}\Delta_t[p_{3t}A_{3t}L_{3t} - w_t L_{3t}]$, choosing energy output (i.e. labour employed in either sector) and taking the market prices of energy p_{it} (with $i = 2, 3$), the carbon tax τ_t, and the wage rate w_t as given. Since Ricardian debt neutrality holds, there is no loss of generality in assuming that the government balances its books in each period, $T_t = \tau_t(F_t + A_{2t}L_{2t}) - \sigma_t r_t K_t$.

Given that all agents have perfect information and all markets are complete, the first fundamental theorem of welfare economics applies. The first-best optimum for the command economy can thus be sustained in a market economy.

Proposition B.1: *The social optimum is replicated in the decentralized market economy if $\beta = \beta^P$ when $\sigma_t = 0$, and $\tau_t = \phi Y_t$ following from* (9).

Proof: The optimality conditions of the agents and the equilibrium conditions in the decentralized economy reduce to the social optimality conditions if the capital income subsidy is set to zero, the carbon tax is set to the SCC and the government's net revenue is distributed in lump-sum fashion.

The government might want to choose to discount the future at a lower rate than private agents, so $\beta > \beta^P$. The first best solution can only be implemented if a capital income subsidy, $\sigma_t > 0$, is introduced. As in the social optimum, the saving/consumption rate is constant, if the capital income subsidy is constant (which is the case for the socially optimal outcome as we show below), and equals

$$c^{SB}\left(\beta^P, \sigma\right) = \left(1 - \frac{\alpha\beta^P\gamma\kappa\left(1+\sigma\right)}{1 - \beta^P\gamma\left(1-\kappa\right)\left(1+\sigma\right)}\right). \tag{B1}$$

The government maximizes aggregate utility $\sum_{t=0}^{\infty}\beta^t N_t[\ln(C_t/N_t) - \psi E_t]$ subject to equations (2)-(5) and the decentralized equilibrium conditions above. With a capital income subsidy, the first-best optimum for the command economy can be sustained in a market economy.

Proof of Proposition 4: *The social optimum is replicated in the decentralized market economy even under differing discount rates if $\sigma^{FB} = (\beta - \beta^P)/\beta^P > 0$ and. $\tau_t^{FB} = \phi(\beta)Y_t$.*

Proof: The optimality conditions of the agents and the equilibrium conditions in the decentralized economy reduce to the social optimality conditions if the capital income subsidy is set to $(\beta - \beta^P)/\beta^P$ from equation (B1), the carbon tax is set to the SCC and the government's net revenue is distributed in lump-sum fashion.

Proof of Proposition 5: *If the capital subsidy cannot be positive, $\sigma = \overline{\sigma} \leq 0$, the second-best optimal carbon tax is $\tau_t^{SB} = \phi^{SB}Y_t > \tau_t^{FB}$ with $\phi^{SB} \equiv \phi(\beta)c^{SB}(\beta^P, \overline{\sigma})/c(\beta) > \phi(\beta)$, where revenue of the taxes on carbon and capital income is rebated as lump-sum transfers. This policy is time consistent.*

Proof: If capital income cannot be subsidies, e.g. due to political resistance, the second-best policy is derived under fixed subsidy rate $\overline{\sigma} \leq 0$ by applying consumption share $c^{SB}(\beta^P, \overline{\sigma})$ when converting equilibrium variables from utils into final good units. We have $c^{SB}(\beta^P, \overline{\sigma}) > c(\beta)$, since $\overline{\sigma} \leq 0$ and $\beta > \beta^P$. As shown in equation (A4), the SCC in utils is independent of consumption and income. The change in the consumption rule due to differing discount rates matters only when converting the SCC into units of the consumption good, i.e. $\tau_t^{SB} = \phi^{SB}Y_t$ with

$$\phi^{SB} \equiv \frac{c^{SB}\left(\beta^P, \overline{\sigma}\right)}{c(\beta)}\phi(\beta) = \left(\frac{\phi_L}{1-\beta\gamma} + \frac{\phi_0(1-\phi_L)}{1-\beta\gamma\varepsilon}\right)\left[\psi + \frac{\chi}{(1-\beta\gamma\delta)}\left(1 - \frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)\right]\left(1 - \frac{\alpha\beta^P\gamma\kappa(1+\sigma)}{1-\beta^P\gamma(1-\kappa)(1+\sigma)}\right).$$

Appendix C. Proofs of Propositions 6 and 7

The aim is to solve for the optimal SCC under a cap on cumulative emissions. Damages are based on Burke et al. (2015), total factor productivity follows (5′), and temperature depends on cumulative emissions E_t as in (4′). The Lagrangian for this problem is

$$\sum_{t=0}^{\infty}\beta^t N_t\{ln(C_t/N_t)-\omega_t\left(E_t-\overline{E}\right)-\lambda_t\left[K_{t+1}-\iota\left(\frac{A_tK_t^{\alpha}[\kappa_{1t}F_t^{\rho}+\kappa_{2t}(A_{2t}L_{2t})^{\rho}+\kappa_{3t}(A_{3t}L_{3t})^{\rho}]^{\upsilon/\rho}(M_t-L_{2t}-L_{3t})^{1-\alpha-\upsilon}-C_t}{K_t}\right)^{\kappa}K_t\right]$$
$$-\psi E_t-\mu_t(S_{t+1}-S_t+F_t)-\varphi_t\left[ln(A_t)-\delta\,ln(A_{t-1})-(1-\delta)ln\left(\overline{A}\right)+\chi E_t\right]-\eta_t(E_{t+1}-E_t-F_t-A_{2t}L_{2t})\Big\},\tag{C1}$$

where η_t denotes the shadow disvalue of cumulative emissions and ω_t the Kuhn-Tucker multiplier for the cap on cumulative emissions at time *t*. The first-order optimality conditions for $C_t,\ K_t$ and $\ln(A_t)$ are the same as in Appendix A. They give the Euler equation (A2), consumption $C_t=\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)Y_t$ as in (6), and equation (A3) for φ_t. The optimality conditions for fossil fuel and renewables give rise to the Kuhn-Tucker conditions stated in (7). The optimality condition for reserves is again $N_{t-1}\mu_{t-1}=\beta N_t\mu_t$, so scarcity rents must satisfy equation (8). The first-order condition for E_t is $N_t\eta_t-\beta N_{t+1}\eta_{t+1}-\psi N_t-\chi\varphi_t N_t-\omega_t N_t=0$. Using (A3) this condition boils down to

$$\begin{aligned}\eta_{t+1}&=\frac{1}{\beta\gamma}\eta_t-\frac{1}{\beta\gamma}\left(\psi+\chi\varphi_t+\omega_t\right)\\&=\frac{1}{\beta\gamma}\eta_t-\frac{1}{\beta\gamma}\left[\psi+\frac{\chi}{(1-\beta\gamma\delta)}\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)^{-1}+\omega_t\right].\end{aligned}\tag{C2}$$

The complementary slackness conditions are $E_t\le\overline{E}$ and $\omega_t\ge0$. It is optimal to asymptotically deplete fossil fuel reserves from $S_0>0$ down to $S_t\to\overline{S}\ge0$ as $t\to\infty$ with $S_0-\overline{S}\le\overline{E}$ and the remainder of the carbon budget used up by coal. The cumulative emissions constraint never bites, $E_t<\overline{E},\ \forall t\ge0$, but with $\lim_{t\to\infty}(\sum_{t=1}^{t}(F_t+A_2L_{2t}))=\overline{E}>0$. Hence, $\omega_t=0,\ \forall t\ge0$. Define $\overline{\eta}\equiv\frac{1}{1-\beta\gamma}\left[\psi+\frac{\chi}{1-\beta\gamma\delta}\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)^{-1}\right]\Big\rangle 0$ and write (C2) as $\beta\gamma\eta_{t+1}=\eta_t-(1-\beta\gamma)\overline{\eta}$. This gives $\eta_t=(\beta\gamma)^{-(t-1)}\eta_1-(1-\beta\gamma)\overline{\eta}\sum_{s=1}^{t-1}(\beta\gamma)^{-s}$. It can easily be verified that the general solution to (C2) equals

$$\eta_t=\overline{\eta}+\varpi(\beta\gamma)^{-(t-1)},\tag{C3}$$

where $\varpi\ge0$ is a constant that is to be determined and $\eta_t=\overline{\eta}$ is the saddle-point solution to (C2). Using (6) and $\tau_t=\eta_tC_t=\eta_t\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)Y_t$, we get (10) in Proposition 5 or

$$\tau_t=\phi(\beta)Y_t+\Delta(\beta\gamma)^{-(t-1)}Y_t,\quad\forall t\ge0,\tag{C4}$$

where $\phi\left(\beta\right)\equiv\frac{1}{1-\beta\gamma}\left[\psi\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)+\frac{\chi}{1-\beta\gamma\delta}\right]$ and $\Delta=\varpi\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)$. The constant Δ is zero if the optimal trajectory does not fully exhaust the carbon budget, $\lim_{t\to\infty}E_t<\overline{E}$. This case also pertains if there is no temperature cap and thus corresponds to Proposition 6. However, $\Delta>0$ if the optimal trajectory fully exhausts the carbon budget, $\lim_{t\to\infty}E_t=\overline{E}$. In that case, the constant Δ must be chosen so that the carbon budget is exactly exhausted. This completes the proof of Proposition 7.

Appendix D. Details of Calibration

Our model allows for damages to utility and productivity growth drawing on previous studies by Barrage (2017) and Dell et al. (2012). The adaptation to our model formulation is described below.

Annual time scale: Adjustment of the time steps from a decadal to an annual scale requires changes in the time-sensitive parameters β, ϕ_0, $\phi=1-\varepsilon$, and the levels and growth rates of A, A_2 and A_3. Converting β and the growth rates of A_2, and A_3 is easily achieved by taking decadal value to the power 1/10. Productivity levels are adjusted by dividing by 10. (Adjusting A is equivalent to adjusting Y_0). To recalibrate the parameters describing the carbon cycle, we use the two calibrations points used by GHKT, i.e. that the temporary component of atmospheric carbon has a mean lifetime of 300 years and that half of emissions is removed after 30 years, or $\frac{1}{2}=(1-\phi)^{30t}$ and $\frac{1}{2}=\phi_L+(1-\phi_L)\phi_0(1-\phi)^{3t-1}$ where $t=1$ on a decadal and $t=10$on an annual grid. With $t=10$,we have $\phi_0=0.401$and. $\phi=1-\varepsilon=0.0023078$.

Utility damages: We follow Barrage (2017) who carefully attributed the aggregate damage function of DICE (which was used for the calibration of damages in GHKT) to utility and production damages, finding that 26% of damages should be attributed to utility damages at 2.5 °C. To (re-)calibrate our production and utility damage parameters χ and ψ we assume that damages affect the level of productivity only (i.e. $\delta=0$). With $\varpi=0.76$, the share of GHKT damages attributable to production, we calibrate the new damage parameter for production damage $\widetilde{\chi}$ according to

$$\left(1-e^{-\widetilde{\chi}\left(E_{2.5^{\circ}C}-\overline{E}\right)}\right)=\varpi\left(1-e^{-\chi^{GHKT}\left(E_{2.5^{\circ}C}-\overline{E}\right)}\right)$$

Remaining damages $(1-\varpi)(1-e^{-\chi^{GHKT}E_{2.5^{\circ}C}})$ are damages to utility. So we have for utility damages, converted into dollars (by dividing by the marginal utility of consumption and expressed as a fraction of Y_t:

$$\frac{\psi(E_{2.5^\circ C}-E_{0^\circ C})}{u'(C)Y_t}=\psi\left(E_{2.5^\circ C}-E_{0^\circ C}\right)\left(1-\frac{\alpha\beta\gamma\kappa}{1-\beta\gamma(1-\kappa)}\right)=\left(1-\varpi\right)\left(1-e^{-\chi^{GHKT}E_{2.5^\circ C}}\right)$$

Using the standard parameter values and the stock level for the pre-industrial and 2.5 °C carbon stock (581 GtC and 1035 GtC, respectively), we solve both equations for $\chi = 1.806\ 10^{-5}$ and $\psi = 7.376\ 10^{-6}$.

Productivity damages: We calibrate the growth damage, the parameter δ, to the empirical findings of Dell et al. (2012) using their linear long-run relationship between income per capita growth and temperature changes (see equation A1.6 in their online appendix). If the economy is on a balanced growth path and temperature increases by 1 °C, they find that per capita income growth falls by 1.171 percentage points in poor and 0.152 pp. in rich countries (although the latter result is not statistically significant). To calibrate our parameter δ we derive an analogue of this long-run relationship in our model. If we assume that damages are time-invariant and that initial state is without damages, we have.

$A_t = e^{-\chi(E-\overline{E})}A_{t-1}^{\delta}\overline{A}^{1-\delta} = \overline{A}(e^{-\chi(E-\overline{E})})^{(1-\delta^t)/(1-\delta)}$ and. $A_\infty = \overline{A}(e^{-\chi(E-\overline{E})})^{1/(1-\delta)}$

Dell et al. (2012) use a linear relationship between income p.c. growth rate and temperature. To relate to this formulation, we need to convert a 1 °C increase at current concentration (with implies an equilibrium temperature increase of 1.3 °C) into an increase in concentration using the equilibrium relationship $T_{eq} = 3\ln(E/581)/\ln(2)$. A 1 °C increase corresponds to a concentration of atmospheric carbon of 1010 GtC. Figure D1 plots the difference in percentage losses in aggregate productivity in the long run, $100(1-A_\infty)$, as a function of δ.

Dell et al. (2012) find that in rich country, allowing for up to 10 annual lags, an increase of 1 °C leads to a long-run reduction in the annual income per capita growth rate of 0.152 percentage points. This coefficient is statistically insignificant. For poor countries, an increase of 1 °C leads to a statistically significant long-run reduction in the annual income per capita growth rate of 1.171 percentage points. We use this value under the assumption that most long-run growth in the future will result in today's poor countries. To convert from p.c. income into a growth rate of TFP, we multiply by the labour share which equals $1-\alpha-\nu = 0.66$. This gives a calibrated value of $\delta = 0.367$.

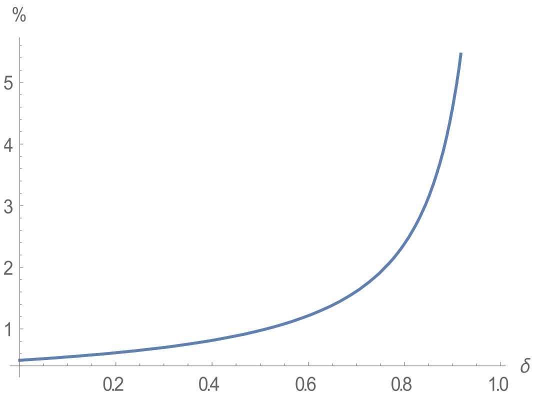

Fig. D.1. Losses in long-run productivity versus δ

Our framework assumes a mean-reverting process, implying that TFP growth is higher, the further TFP is from its trend (the case $0 < \delta < 1$). This allows for catching-up processes and some forms of adaptation to climate change. The frameworks used in Dietz and Stern (2015) and Moore and Diaz, 2015 trade the conventional assumption in the DICE model and in Golosov et al. (2014) of "level" effects (i.e. global warming negatively affects the level of TFP) which are one-off effects for "growth" effects (global warming negatively affect the growth rate of TFP) in which case losses can never be caught up. The mean-reverting process used in our paper captures these cases with $\delta = 0$ (level effects) and $\delta = 1$ (growth effects). Given that our calibrated case uses the value $\delta = 0.367$, the effects of climate change on TFP growth are lower (relative to GHKT's level effect specification) than those in other frameworks. This can be seen from Table 2, which indicates that if global warming affects the growth rate rather than the level of TFP the optimal carbon tax increase from $64 to $100/tC while they rise from $44 to $118/tC in Dietz and Stern (2015). In general, the higher the value of δ, the more persistent the effect of global warming on TFP and thus the higher the optimal carbon tax.

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