Working Paper

STRATEGIC ASPECTS OF IIASA'S
FOOD AND AGRICULTURE MODEL

W. GÜTH
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International Institute for Applied Systems Analysis
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PREFACE

The linkage model of IIASA's Food and Agriculture Program (FAP) can be described as an econometric world model which investigates the interaction of many national economies on a number of agricultural markets and one residual nonagricultural market. The model contains a number of policy parameters which can be determined by the national governments in order to improve the economic results. Given that one can specify a goal function for every country, the IIASA linkage model can be thus viewed as a strategic game with the governments of the various countries as players.

In the paper some of the strategic aspects of the IIASA linkage model are investigated. Reasons of analytic tractability made it advisable to study a radically simplified version with only two commodities, food and nonfood, and with rather simple assumptions on government objectives such as short-run minimization of the costs of agricultural policy.

One of the major results is that under apparently weak restrictions on the parameters of our model it is an optimal policy for all countries to supply as much food as possible, which seems to contradict commonly held intuitions. Of course, one still has to determine whether the same result would hold for more complex models and whether the conditions would really be satisfied by econometrically specified model parameters. The basic reason for this counter-intuitive result in our game model is that demand is specified by linear expenditure systems. We therefore suggest alternative demand specifications, which would avoid the difficulties of linear expenditure systems.

ACKNOWLEDGEMENTS

This research project was undertaken while one of the authors was a regular member of the IIASA staff, and the other came to IIASA twice as a consultant. Throughout this time, we often had the opportunity to discuss the problem with various members of IIASA's Food and Agriculture Project and also with Peyton Young of IIASA's methodology group (SDS). Our discussions with Günther Fischer, Michiel Keyzer, Kirit Parikh, Ferenc Rabar and Dennis Warner were especially helpful. The text was prepared by Bonnie Riley.

Strategic Aspects of IIASA's Food and Agriculture Model

One of the major research efforts of IIASA's Food and Agriculture Program is to develop an econometric world model which describes the interaction of many national economies on a number of agricultural markets and one residual nonagricultural market (see Rabar, 1979). The model has not yet been completed, but its basic structure has been fixed and several national subsystems have been estimated (see, for instance, Keyzer, 1980, Csaki, 1979, Parikh, 1977 and Fischer and Frohberg, 1980). The model contains a number of policy parameters which can be used in order to study the consequences of various forms of government intervention. Obviously, the interaction of many governments who pursue different goals causes a strategic problem which -- at least in principle – should be amenable to game theoretic analysis.

The authors of this paper, who did not participate in the development of IIASA's food and agriculture model, wanted to study its strategic structure. Reasons of analytic tractability make it advisable to investigate a radically simplified version with only two commodities, food and nonfood. This can be done without losing relevant structural properties. The analysis of strategic interaction makes it necessary to complement the model by governmental goal functions and constraints on governmental action. In a realistic model one might want to specify a different goal function for every country, depending on a large variety of goal variables. We feel that the time is not yet ripe to take this approach. It seems to be better to aim at basic strategic insights with the help of very simple assumptions on government objectives such as the short-run minimization of the costs of agricultural policy for each country.

The analysis of our simplified models yields the result that under apparently weak restrictions on the parameters it is an optimal policy for all countries to supply as much food as possible. This contradicts commonly held intuitions. Of course, one still has to determine whether the same result holds for more complex models and whether the conditions will be really satisfied by econometrically specified model parameters. The basic reason for our counterintuitive result can be seen in the fact that in our model demand is specified by linear expenditure systems. As we shall see, in our model linear expenditure systems have the consequence that in most cases an individual country's revenue obtained by production is always increased by an increase of supply. The same does not hold for the market as a whole, total revenue is generally decreased by an increase of total supply. However, if the supply of the other countries is kept constant, it pays to expand one's own supply as far as possible.

IIASA's food and agriculture model is colloquially referred to as a linkage model. We shall employ the same term for the two commodity versions investigated here. In order to exhibit the basic facts in the simplest possible way, we shall first look at a special case where food stocks are neglected and domestic prices are assumed to be equal to international prices. Later we shall abolish these restrictions. Finally, we shall suggest alternative demand specifications which avoid the difficulties of linear expenditure systems.

1. A Special Linkage Model

The models investigated here contain two markets, a food market and a nonfood market. Variables related to the food market will be denoted by small Latin characters, whereas corresponding capital Latin letters will be used for nonfood variables. We assume that there are n countries 1,...,n. Subindices refer to the number of the country. We employ the following notation:

p food price

P nonfood price

r food/nonfood price ratio (r=p/P)

d_i domestic food demand in country i

s_i domestic food supply in country i

D_i domestic nonfood demand in country i

S_i domestic nonfood supply in country i

Y_i monetary national income of country i

E_i monetary national expenditures of country i (national absorption)

In IIASA's food and agriculture model the nonfood supply S_i is predetermined by production decisions in the previous period and therefore can be taken as constant for the purpose of short-term analysis. We shall treat total food supply s_i as country i's policy variable which can be chosen within given lower and upper bounds:

$$\underline{s}_i \leq s_i \leq \bar{s}_i \quad \text{for } i = 1,\ldots,n \tag{1}$$

Excess demands z_i and Z_i for country i are defined as follows:

$$z_i = d_i - s_i \tag{2}$$

$$Z_i = D_i - S_i \tag{3}$$

The excess demands z_i and Z_i must satisfy the following market clearing conditions:

$$\sum_{i=1}^{n} z_i = 0 \tag{4}$$

$$\sum_{i=1}^{n} Z_i = 0 \tag{5}$$

In IIASA's food and agriculture model trade surpluses and deficits are determined exogenously by appropriate readjustment of planned trade surpluses and deficits. Accordingly, we assume that a vector of real trade deficits K_i expressed in terms of nonfood is exogenously given:

$$K_i = rz_i + Z_i \quad \text{for } i = 1,\ldots,n \tag{6}$$

In order to assure the consistency with (4) and (5) trade deficits must be fixed in such a way that the following equation holds:

$$\sum_{i=1}^{n} K_i = 0 \tag{7}$$

Moreover, we have to impose a further restriction on the vector of K_i's to make sure that we obtain a positive price ratio p/P and demands d_i and D_i within the region where the linear expenditure system to be introduced later makes sense:

$$K_i > D_i^0 - S_i \quad \text{for } i = 1,\ldots,n \tag{8}$$

The reason why we need this condition will become apparent in section 2. Since the right-hand side of (8) is negative in view of condition (15) to be introduced below, this means that (8) excludes surpluses which are excessively large. Since we permit trade surpluses and deficits, national income

$$Y_i = s_i p + S_i P \tag{9}$$

and total expenditures

$$E_i = d_i p + D_i P \tag{10}$$

are not equal to each other. The following relationship holds:

$$E_i = Y_i + K_i P \tag{11}$$

In our view it would be preferable to determine trade deficits endogenously and, as we shall see later, this can be done easily with the help of national absorption functions as they are used in monetary international trade theory. For the time being we consider the K_i's as exogenously fixed in order to conform to the IIASA model in its present form. Demand is described by the following linear expenditure system:

$$d_i p = d_i^0 p + \alpha_i (E_i - d_i^0 p - D_i^0 P) \tag{12}$$

$$D_i P = D_i^0 P + (1 - \alpha_i)(E_i - d_i^0 p - D_i^0 P) \tag{13}$$

for $i = 1,\dots,n$. The parameters d_i^0 and D_i^0 are assumed to be non-negative. They have the interpretation of minimum demands which are to be satisfied independent of prices and income. The constants α_i satisfy $0 < \alpha_i < 1$ for $i = 1,\dots,n$. In game models it is important to obtain reasonable values for the variables for all choices of the strategic parameters and not only in equilibrium. Therefore, we impose the following restrictions:

$$s_i > d_i^0 \quad \text{for } i = 1,\dots,n \tag{14}$$

$$S_i > D_i^0 \quad \text{for } i = 1,\dots,n \tag{15}$$

Conditions (14) and (15) exclude situations which make it impossible to buy the minimum demands d_i^0 and D_i^0. Equations (12) and (13) show that total monetary demand $d_i p + D_i P$ equals monetary national expenditures E_i.

We assume that the share of farm incomes F_i in total income Y_i is protected by a parity policy expressed by the following equation:

$$F_i = f_i Y_i \quad \text{for } i = 1,\dots,n \tag{16}$$

where the f_i's are positive constants smaller than one. In the context of our short-run strategic analysis we look at (16) as a constraint on government policy rather than a result of political decisions. For our purpose it is not important in which way the governments operate to satisfy this constraint. One may think of direct subsidies or subsidized domestic producers' prices. In any case, the difference between the farm incomes F_i and the value of domestic food production $s_i p$ has to be covered by government expenditures. We assume that governments want to minimize the costs of agricultural policy or, equivalently, to maximize the revenue of agricultural policy. In the framework of a model which is homogeneous of degree 0 in prices, it is natural to suppose that governments are concerned with real revenues rather than monetary revenues. Therefore, we shall express the gains H_i of agricultural policy in nonfood terms:

$$H_i = [ps_i - f_iY_i]/P \quad \text{for } i = 1,\dots,n \tag{17}$$

It is, of course, possible that for some countries H_i is positive. In this case agricultural policies are used in order to finance government expenditures for other purposes. The farm income parity constraint (16) makes sense here, too, since taxation of farm incomes must be limited somewhere by the political consideration of farm interests.

The model can now be looked upon as a game where the governments of the countries 1,...,n are the players. Player i's strategy is the food supply quantity s_i which must be chosen in the interval (1). For every strategy combination $s = (s_1,\dots,s_i)$ we can compute the market clearing price ratio $r=p/P$. Moreover, demand quantities, real national incomes Y_i/P and governments' payoffs H_i can be determined. In this way we receive a payoff vector

$$H(s) = (H_1(s),\dots,H_n(s)) \tag{18}$$

with the governments' payoffs $H_i(s)$ resulting from the strategy vector $s = (s_1,\dots,s_n)$ as components. Thereby, we have specified a game in normal form with the pure strategy sets (1) and the payoff function H. In the next section we shall show that the game has a uniquely determined equilibrium point at $s = \bar{s} = \bar{s}_1,\dots,\bar{s}_n)$ under reasonable additional restrictions on the parameters $\underline{s}_i$ and d_i^0.

2. Analysis of the Special Linkage Model

An immediate consequence of (12) and (13) is:

$$r = \frac{p}{P} = \frac{\alpha_i}{1-\alpha_i} \cdot \frac{D_i - D_i^0}{d_i - d_i^0} \quad \text{for } i = 1,\dots,n \tag{19}$$

(6) is equivalent to:

$$K_i = rz_i + Z_i \tag{20}$$

(2) and (3) together with (20) yield:

$$D_i = K_i + S_i - r(d_i - s_i) \tag{21}$$

$$D_i - D_i^0 = K_i + S_i - D_i^0 - r(d_i - s_i) \tag{22}$$

The right-hand side of (22) can be substituted for D_i - D_i^0 in (19):

$$r(d_i - d_i^0) = \frac{\alpha_i}{1-\alpha_i}[K_i + S_i - D_i^0 - r(d_i - s_i)] \tag{23}$$

$$r(\frac{d_i - \alpha_i s_i}{1-\alpha_i} - d_i^0) = \frac{\alpha_i}{1-\alpha_i}(K_i + S_i - D_i^0) \tag{24}$$

In view of $d_i = s_i + z_i$ we obtain

$$r(s_i + \frac{z_i}{1-\alpha_i} - d_i^0) = \frac{\alpha_i}{1-\alpha_i}(K_i + S_i - D_i^0) \tag{25}$$

$$r((1-\alpha_i)s_i + z_i - (1-\alpha_i)d_i^0) = \alpha_i(K_i + S_i - D_i^0) \tag{26}$$

With the help of the market clearing condition (4) the n equations (26) can be summed up as follows:

$$r[\sum_{i=1}^{n}(1-\alpha_i)(s_i - d_i^0)] = \sum_{i=1}^{n}\alpha_i(K_i + S_i - D_i^0) \tag{27}$$

Condition (8) on the trade deficits K_i has the consequence that the right-hand side of (27) is positive. The coefficient of r in (27) is positive in view of (14). Therefore, we obtain a positive price ratio r:

$$r = \frac{\sum_{i=1}^{n}\alpha_i(K_i + S_i - D_i^0)}{\sum_{i=1}^{n}(1-\alpha_i)(s_i - d_i^0)} \tag{28}$$

Equation (28) shows that an increase of country i's supply decreases the food/nonfood price ratio r if the supplies of the other countries are kept constant. The nominator does not depend on the food supplies and the denominator is a linear function of the s_i where the coefficient $1 - \alpha_i$ is positive. It can be seen that the influence of the trade deficits K_i on r would vanish if all α_i's were equal. This is due to (7). However, for unequal coefficients α_i the distribution of trade deficits and surpluses influences the price ratio r.

For our purposes it is not necessary to derive explicitly formulae for the demand quantities d_i and D_i. But it is worthwhile to show how conditions (8) secure that the demand quantities d_i and D_i surpass the corresponding minimum demands d_i^0 and D_i^0. Equation (11) can be rewritten as follows:

$$s_i r + S_i + K_i = d_i r + D_i \tag{29}$$

It is sufficient to show that the expenditures are high enough to cover the costs of the minimum demands d_i^0 and D_i^0. This means that the inequalities $d_i > d_i^0$ and $D_i > D_i^0$ are satisfied if we have

$$s_i r + S_i + K_i > d_i^0 r + D_i^0 \tag{30}$$

(1) and (14) have the consequence that $s_i > d_i^0$ holds. Therefore, condition (8) secures the validity of (30).

In view of

$$Y_i / P = s_i r + S_i \tag{31}$$

the payoff H_i can be written as follows

$$H_i(s) = (1 - f_i)s_i r - f_i S_i \tag{32}$$

We look at our model as a noncooperative game in normal form. The notion of an equilibrium point in pure strategies is a natural solution concept for noncooperative games. In our case an equilibrium point in pure strategies can be described as a strategy combination $s^* = (s_1^*,...,s_n^*)$ such that for i = 1,...,n the following condition is satisfied:

$$H_i(s^*) = \max_{\underline{s}_i \leq s_i \leq \bar{s}_i} H_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*) \tag{33}$$

If every country i expects that all other countries j choose their equilibrium supply quantities s_j^*, then it can not do anything better than to choose s_i^*.

As we shall see, for reasonable parameter constellations our model has a uniquely determined equilibrium point in pure strategies, namely $\bar{s} = (\bar{s}_1,...,\bar{s}_n)$. Moreover, in these cases the selection of the maximum supply $\bar{s}_i$ is a dominant strategy for every country i in the sense that the selection of $\bar{s}_i$ will be best whatever the other countries do. The additional requirement under which this result holds is the following one:

$$\sum_{j=1,\, j\neq i}^{n} (1 - \alpha_j)\underline{s}_j > \sum_{j=1}^{n} (1 - \alpha_j) d_j^0 \quad \text{for } i = 1,\ldots,n \tag{34}$$

In order to interpret condition (34), it is useful to look at the special case where all α_i's are equal. Then condition (34) assumes the following form:

$$\sum_{j=1,\, j\neq i}^{n} \underline{s}_j > \sum_{j=1}^{n} d_j^0 \quad \text{for } i = 1,\ldots,n \tag{35}$$

(35) requires that the minimum production $\underline{s}_i$ of any collection of n-1 countries is sufficient to cover the minimum demands of all n countries. In view of the fact that minimum demands probably should be thought of as relatively small compared to minimum supplies, this seems to be a mild restriction. Obviously, the same can be said for (34) in the case of unequal α_i's which are not too different from each other.

Theorem: Under condition (34) the game model described in section 1 has one and only one equilibrium point in pure strategies, namely $\bar{s} = (\bar{s}_1,\ldots,\bar{s}_n)$. Moreover, under this condition $\bar{s}_i$ is a dominant strategy in the sense that

$$H_i(s_1, \ldots, s_{i-1}, \bar{s}_i, s_{i+1}, \ldots, s_n) > H_i(s) \tag{36}$$

holds for every $s = (s_1,\ldots,s_n)$ with $s_i \neq \bar{s}_i$.

Proof: In order to show that $\bar{s}_i$ is dominant, we prove that the partial derivative of $H_i(s)$ with respect to s_i is positive everywhere. (32) shows that the sign of this derivative is the same as that of the partial derivative of $s_i r$ with respect to s_i. Define

$$\mu = \sum_{j=1}^{n} \alpha_j (K_j + S_j - D_j^0) \tag{37}$$

$$\gamma = \sum_{j=1}^{n} (1 - \alpha_j) d_j^0 \tag{38}$$

It follows by (28) that we have

$$s_i r = \frac{\mu s_i}{\sum_{j=1}^{n}(1 - \alpha_j)s_j - \gamma} \tag{39}$$

We obtain

$$\frac{\delta(s_i r)}{\delta s_i} = \mu \frac{\sum_{j=1}^{n}(1 - \alpha_j)s_j - \gamma - (1 - \alpha_i)s_i}{(\sum_{j=1}^{n}(1 - \alpha_j)s_j - \gamma)^2} \tag{40}$$

The right-hand side of (40) is positive for

$$\sum_{j=1,\, j\neq i}^{n} (1 - \alpha_j)s_j > \gamma \tag{41}$$

This is always true because of condition (34). Consequently, the partial derivative of $H_i(s)$ with respect to s_i is always positive and the maximal payoff is achieved at $\bar{s}_i$ regardless of the strategies of the other players.

Remark: Equation (40) shows that the sign of the partial derivative of $H_i(s)$ with respect to s_i does not depend on s_i. If condition (34) is not satisfied, the partial derivative may be positive, negative or zero. If a player i at an equilibrium point has a strategy s_i with $\underline{s}_i < s_i < \bar{s}_i$, then the partial derivative of his payoff with respect to his supply must be zero at the equilibrium point and his payoff obtained against the equilibrium strategies of the others does not depend on his own supply. It is clear that such equilibrium points must be expected to be very unstable. An equilibrium point $s = (s_1,\ldots,s_n)$ where every player $i = 1,\ldots,n$ has a positive incentive to choose his equilibrium supply requires either $s_i = \bar{s}_i$ or $s_i = \underline{s}_i$.

As we have seen, under condition (34) the real national food revenue $s_i r$ is an increasing function of s_i if the food supplies of the other countries are kept constant. This may be described by saying that demand for an individual country's food supply is elastic. A similar statement does not hold for the market as a whole. In order to see this, we look at the special case where the α_i's are all equal to the same constant α. Define

$$\hat{s} = \sum_{i=1}^{n} s_i \tag{42}$$

In the case of equal α_i's total real food revenue can be written as follows:

$$\hat{s}r = \frac{\mu\hat{s}}{(1-\alpha)\hat{s}-\gamma} \tag{43}$$

We obtain

$$\frac{\delta(\hat{s}r)}{\delta\hat{s}} = \frac{\mu((1-\alpha)\hat{s}-\gamma) - \mu(1-\alpha)\hat{s}}{((1-\alpha)\hat{s}-\gamma)^2} \tag{44}$$

$$\frac{\delta(\hat{s}r)}{\delta\hat{s}} = -\frac{\mu\gamma}{((1-\alpha)\hat{s}-\gamma)^2} \tag{45}$$

This shows that the total real food revenue is a decreasing function of total food supply $\hat{s}$.

3. A more general linkage model

The special linkage model can be generalized by the introduction of buffer stocks and of differences between domestic prices and world market prices and between producers' and consumers' prices. Price differences of this kind may be introduced by tariffs, taxes and subsidies. It is also possible to assume that a government directly controls domestic prices. For given policies of other countries it does not matter too much in which way domestic prices are controlled, provided the government has sufficient degrees of freedom in order to achieve given targets. In a game theoretic context, however, the specific mode of government intervention may be of great importance for the economic results achieved in equilibrium. Thus a world in which differences between domestic prices and world market prices are created by tariffs and a world in which such differences are due to directly controlled prices may lead to quite different equilibrium situations. This is due to the fact that for given foreign domestic prices one country's policies affect the other countries' imports and exports in another way than for given foreign tariffs. This poses interesting research questions concerning the comparison of various institutional schemes which can be explored in the framework of our models. This task will not be pursued within this paper, but it may be a fruitful subject of further investigation.

In order to create a framework which permits more institutional details, we introduce the following variables:

q_i domestic producers' food price in country i

Q_i domestic producers' nonfood price in country i

p_i domestic consumers' food price in country i

P_i domestic consumers' nonfood price in country i

B_i food buffer stock in country i

b_i change of country i's food buffer stock

One may want to look at an institutional setup where the differences of domestic and international prices and of producers' and consumers' prices are controlled by proportional tariffs, proportional indirect taxes and/or subsidies. This leads to relationships of the following kind:

$$p_i = a_i p \tag{46}$$

$$P_i = A_i P \tag{47}$$

$$q_i = g_i p_i \tag{48}$$

$$Q_i = G_i P_i \tag{49}$$

for $i = 1,\dots,n$. The a_i, A_i, g_i and G_i are positive policy parameters which may be restricted by upper and lower bounds. We shall not look at the question of how these parameters should be optimally selected. Instead of this we shall treat them as exogenously fixed in order to generalize the game model of section 1 to the enlarged framework. As before we shall look at the food supplies s_i as strategic variables, but in addition to this we shall also permit the buffer stock changes b_i to be selected strategically. This means that player i's strategy is a pair (s_i, b_i) where s_i is restricted to the interval (1) and the b_i has to be chosen in such a way that the following inequality holds:

$$0 \leq B_i + b_i \leq \bar{B}_i \quad \text{for } i = 1,\dots,n \tag{50}$$

The constant $\bar{B}_i$ may be interpreted as country i's storage capacity for food. In order to avoid situations where net supply after the reduction of storage change does not cover minimum demand, we strengthen condition (14) in the following way:

$$\underline{s}_i >> d_i^0 + \bar{B}_i \quad \text{for } i = 1,\dots,n \tag{51}$$

Wherever we do not say anything to the contrary, we shall continue to employ the definitions of section 1. Thus national income Y_i will be defined as in (9) in terms of international prices. However, in some respect the enlarged framework makes it necessary to adapt previous definitions. Excess demands z_i for food is redefined as follows:

$$z_i = d_i + b_i - s_i \quad \text{for } i = 1,\dots,n \tag{52}$$

Equations (3), (4) and (5) remain unchanged. As in section 1 the price ratio p/P is denoted by r. The definition of K_i and the restrictions on the vector of K_i's remain the same, too. However, total national expenditures E_i now must be defined in terms of domestic consumers' prices. Instead of (10) we obtain

$$E_i = p_i d_i + P_i D_i \tag{53}$$

In (11), however, E_i must be evaluated in terms of international prices. Therefore, (11) must now be expressed as follows:

$$pd_i + PD_i = Y_i + K_i P \tag{54}$$

The expenditure system (12) and (13) must be rewritten by inserting domestic consumers' prices p_i and P_i in (12) and (13):

$$d_i p_i = d_i^0 p_i + \alpha_i (E_i - d_i^0 p_i - D_i^0 P_i) \tag{55}$$

$$D_i P_i = D_i^0 P_i + (1 - \alpha_i)(E_i - d_i^0 p_i - D_i^0 P_i) \tag{56}$$

The parity rule (16) now assumes the following form:

$$F_i = f_i (q_i s_i + Q_i S_i) \tag{57}$$

It is reasonable to tie farm incomes to national income evaluated in producers' prices.

The introduction of buffer stocks poses a problem of inventory evaluation. In this respect we shall simply assume that the evaluation of B_i is just taken over from the past and that each unit of b_i is evaluated at a constant proportion λ_i of the world market price p. With this kind of inventory evaluation the gains obtained by agricultural policy can be written as follows:

$$H_i = [d_i p_i - z_i p + b_i \lambda_i p - F_i] / P \tag{58}$$

We may think of the government as buying all food production s_i at producers' price g_i and adjusting the difference between $g_i s_i$ and F_i by appropriate taxes or subsidies. Agricultural production s_i is then partially sold on the domestic food market which yields $d_i p_i$, and partially put on the buffer stock which yields an increase of inventory value of $b_i \lambda_i p$. Excess demand z_i has to be traded at the world market price p. In this way it can be seen that (58) correctly describes the gains from agricultural policy. Of course, it must be pointed out that indirect effects on gains from nonfood tariff policies are not considered in the payoff function. This is done intentionally, since we take the point of view that government activity is not completely integrated. Those authorities who fix agricultural policies are held responsible only for the gains or losses by agricultural policy and therefore they are not motivated by indirect effects. Of course, it would be possible to look at different payoff functions based on another view of government motivation. But this will not be done here.

In a long run strategic analysis inventory evaluation would not be introduced as a behavioral rule. Instead of this the way in which the inventory should be evaluated would be determined by the strategic analysis of future periods. Since we restrict our intention to short-run strategic analysis, we have to rely on some plausible inventory evaluation rule. Of course, other such rules could be considered as part of the payoff function and investigated in the same way.

Inventory evaluation rules can be thought of as limited rationality methods to take account of long run effects. On the other hand, one may take the point of view that the deviations from perfect rationality incorporated in the payoff function are a reflection of the way in which the political public evaluates the performance of agricultural policy. The myopic goal orientation may be more seen as related to a short-run bias of the political public than to a lack of insight in the dynamic structure as far as the authorities are concerned. If this is taken into account, it becomes doubtful whether a long run strategic analysis could really be expected to yield more realistic results than a short run strategic analysis.

In the next section we shall indicate how the more general linkage model can be analyzed in the same way as the model of section 1.

4. Analysis of the more general linkage model

We shall proceed in a similar way as in section 2. Define

$$r_i = \frac{p_i}{P_i} \quad \text{for } i = 1,\dots,n \tag{59}$$

With the help of (46) and (47) we obtain

$$r_i = \frac{a_i}{A_i} r \quad \text{for } i = 1,\dots,n \tag{60}$$

Analogously to (19) in section 2 we obtain

$$r = \frac{\alpha_i A_i}{(1-\alpha_i)a_i} \frac{D_i - D_i^0}{d_i - d_i^0} \tag{61}$$

The following equation corresponds to (22):

$$D_i - D_i^0 = K_i + S_i - D_i^0 - r(d_i + b_i - s_i) \tag{62}$$

After inserting the right-hand side of (62) for D_i - D_i^0 in (61) we receive an equation which can be rewritten in the following form:

$$r[z_i + \frac{1-\alpha_i}{1+\alpha_i(A_i/a_i - 1)}(s_i - b_i - d_i^0)] = \tag{63}$$
$$= \frac{\alpha_i A_i / a_i}{1+\alpha_i(A_i/a_i - 1)}[K_i + S_i - D_i^0] \quad \text{for } i = 1,\dots,n$$

With the help of the market clearing condition (4) we receive

$$r = \frac{\sum_{i=1}^{n} \frac{\alpha_i \frac{A_i}{a_i}}{1+\alpha_i(\frac{A_i}{a_i} - 1)}(K_i + S_i - D_i^0)}{\sum_{i=1}^{n} \frac{1-\alpha_i}{1+\alpha_i(\frac{A_i}{a_i} - 1)}(s_i - b_i - d_i^0)} \tag{64}$$

In the same way as in section 2 it can be seen that under our assumptions on the parameters r is always positive and that the demand d_i and D_i exceed the corresponding minimum demands d_i and D_i^0. The payoff function H_i can be written as follows:

$$H_i = d_i a_i r - z_i r + \lambda_i b_i r - f_i g_i a_i s_i r - f_i G_i A_i S_i \tag{65}$$

$$H_i = r[(a_i - 1)d_i + (1-\lambda_i)(s_i - b_i) + (\lambda_i - f_i g_i a_i)s_i] - f_i G_i A_i S_i \tag{66}$$

(61) and (62) yield:

$$d_i = \frac{[(1-\alpha_i)d_i^0 + \alpha_i \frac{A_i}{a_i}(s_i - b_i)]r + \alpha_i \frac{A_i}{a_i}(K_i + S_i - D_i^0)}{[1 + \alpha_i(\frac{A_i}{a_i} - 1)]r} \tag{67}$$

The discussion of the influence of s_i and b_i on H_i becomes easy if an assumption on the parameters is added to the model, which probably is not too unrealistic:

$$a_i > \lambda_i > f_i g_i a_i \quad \text{for } i = 1,\dots,n \tag{68}$$

Highly industrialized food exporters like the U.S.A. have relatively small values of f_i and it is reasonable to suppose that for them $\lambda_i > f_i g_i a_i$ is satisfied. Underdeveloped countries with relatively high values of f_i may also have high values of λ_i since for them the storage of food is important to secure future food supply. $a_i > \lambda_i$ holds unless domestic consumers' food prices are highly subsidized.

We first observe that r and d_i depend on the net supplies s_i - b_i only. If this net supply is kept fixed, an increase of s_i will always increase H_i if (68) is satisfied. It follows that under this condition, a country will produce as much as possible as long as its storage capacity is not yet exhausted.

Now suppose that s_i is kept fixed and b_i is increased. The influence of a change of this kind depends only on the partial derivative of $(s_i - b_i)r$ with respect to b_i. The coefficient of $(s_i - b_i)r$ in H_i is positive if we have

$$1 - \lambda_i > (1 - a_i)\frac{\alpha_i \frac{A_i}{a_i}}{1 - \alpha_i + \alpha_i \frac{A_i}{a_i}} \tag{69}$$

In (69) the expression of $1 - a_i$ is multiplied by a fraction smaller than 1. Therefore, (69) holds for $a_i > \lambda_i$. Consequently, the sign of the partial derivative of H_i with respect to b_i with s_i kept constant will be the same as that of $(s_i - b_i)r$ with respect to b_i for s_i kept constant if (68) holds.

The influence of b_i on $(s_i - b_i)r$ can be investigated in a way similar to the influence of s_i on $s_i r$ in section 1. It is clear that a condition similar to (34) can be written down which secures that $(s_i - b_i)r$ is increased, if $s_i - b_i$ is increased or conversely if b_i is decreased. In the special case $A_i/a_i = 1$ for $i = 1,\dots,n$ this condition reduces to an inequality which is very similar to (34). The only difference is that d_j^0 on the right-hand side is replaced by $d_j^0 + \bar{B}_j$. Obviously, the new condition can be interpreted in the same way as (34). If A_i/a_i is near to 1 for $i = 1,\dots,n$, it is also plausible to assume that $(s_i - b_i)r$ is increased if $s_i - b_i$ is increased. If this is the case and (68) holds, it will pay to produce as much as possible and to sell all the food stock. Thus it is fair to say that for reasonable parameter constellations essentially the same result as in the special linkage model is obtained here, too. $(\bar{s}_i, -\bar{B}_i)$ is a dominant strategy of any country $i = 1,\dots,n$. This means that it pays to produce as much as possible and to sell the whole buffer stock regardless of what the other countries do.

Of course, stronger restrictions on the parameters than in the special linkage model are needed here in order to produce the result. However, this does not really matter as long as those cases which are within the restrictions are the typical ones. Moreover, it is worth pointing out that producing as much as possible and selling everything is a dominant strategy for those players for whom (68) holds if the condition corresponding to (34) is satisfied even if (68) does not hold for some other players.

5. Endogenous determination of trade balances

As we have said already in section 1, in our view it would be preferable to determine trade deficits and surpluses endogenously. This can be done with the help of absorption functions as they are commonly used in monetary international trade theory. In order to explain the approach which we are going to propose, we want to rely on the simpler model of section 1, since it will be quite clear how the idea can be generalized to more complicated models. The absorption function for country i connects national expenditures E_i to national income Y_i:

$$E_i = \hat{E}_i + e_i Y_i \tag{70}$$

where $\hat{E}_i$ and e_i are positive constants with $e_i < 1$ for $i = 1,\ldots,n$. In the context of a dynamic model $\hat{E}_i$ may be determined by past national incomes and/or expenditures. Monetary trade deficits K_iP are determined endogenously by the system of absorption functions.

The linear expenditure system (12) and (13) together with (70) implies

$$d_i p = [(1 - \alpha_i)d_i^0 + \alpha_i e_i s_i]p + [\alpha_i e_i S_i - \alpha_i D_i^0]P + \alpha_i \hat{E}_i \tag{71}$$

$$D_i P = [(1 - \alpha_i)e_i s_i - (1 - \alpha_i)d_i^0]p + [\alpha_i D_i^0 + (1 - \alpha_i)e_i S_i]P + (1 - \alpha_i)\hat{E}_i \tag{72}$$

In view of (4) and (5) we have

$$\sum_{i=1}^{n} d_i = \sum_{i=1}^{n} s_i \tag{73}$$

$$\sum_{i=1}^{n} D_i = \sum_{i=1}^{n} S_i \tag{74}$$

(71), (72), (73) and (74) yield

$$p\sum_{i=1}^{n}[(1-\alpha_i e_i)s_i - (1-\alpha_i)d_i^0] - P\sum_{i=1}^{n}\alpha_i(e_iS_i - D_i^0) = \sum_{i=1}^{n}\alpha_i\hat{E}_i \tag{75}$$

$$-p\sum_{i=1}^{n}(1-\alpha_i)(e_is_i - d_i^0) + P\sum_{i=1}^{n}[(1-(1-\alpha_i)e_i)S_i - \alpha_iD_i^0] = \sum_{i=1}^{n}(1-\alpha_i)\hat{E}_i \tag{76}$$

For given strategies $s_1,\ldots,s_n$ equations (75) and (76) determine p and P. Actually at this point one would have to discuss the problem under which conditions the system (75) and (76) is solvable and yields positive values for p and P. We do not want to discuss this question here, since it is not our intention to explore the modified model of section 1 in greater detail.

With the help of p and P we can compute Y_i by (9) and then E_i by (70). Finally, we derive the trade deficits K_i with the help of (11).

Our investigation of the modified model which we shall not report on here has lead us to conclusions which are similar to those reached for the unmodified model of section 1. Therefore,we think that more changes than the introduction of endogenous trade balances by absorption functions are needed in order to design models whose strategic analysis yield less counterintuitive results.

The introduction of absorption functions (70) destroys the homogeneity of the system with respect to prices. Not only the price ratio $r=p/P$ but also the nominal prices p and P are now determined by the system. This is less objectionable than one may think, since the model will still be "homogeneous in the long run" if $\hat{E}_i$ depends homogeneously on past national incomes and/or expenditures. "Homogeneity in the long run" means that for all our periods together prices are only determined up to a constant (factor).

The determination of trade balances with the help of an absorption approach can be modeled in a much simpler way if one is willing to assume that the adjustment processes which achieve the equality of demand and supply are mainly determined by the nonfood sector. It is reasonable to suppose that the adjustments will be quicker there than in the agricultural sector. The idea of nonfood dominance in the absorption approach can be modeled by an absorption function of the following kind:

$$PD_i = \hat{D}_i + e_iS_iP \quad \text{for } i = 1,\ldots,n \tag{77}$$

This equation is analogous to (70) with the only difference that the absorption hypothesis is now restricted to the nonfood sector. Summation of equations (77) together with the market clearing condition (5) yields the following formula for P:

$$P = \frac{\sum_{i=1}^{n} \hat{D}_i}{\sum_{i=1}^{n} (1 - e_i) S_i} \tag{78}$$

With the help of (78) the demands D_i can be computed by (77). If we divide each of the equation (13) by $1 - \alpha_i$ and sum up over i, we obtain

$$P \sum_{i-1}^{n} \frac{D_i - D_i^0}{1 - \alpha_i} = \sum_{i=1}^{n} E_i - p \sum_{i=1}^{n} d_i^0 - P \sum_{i=1}^{N} D_i^0 \tag{79}$$

Since the sum of all E_i is nothing other than the sum of all Y_i, this yields

$$r = \frac{\sum_{i=1}^{n} \left(\frac{D_i - D_i^0}{1 - \alpha_i} - D_i^0 - S_i \right)}{\sum_{i=1}^{n} (s_i - d_i^0)} \tag{80}$$

Finally, the d_i's can be determined by the help of (12) and the K_i can be computed by (2), (3) and (6).

6. An alternative approach to demand

Linear expenditure systems have very good analytical properties and it is hard to find other specifications of demand which are as flexible and as easy to handle. Unfortunately, we have to look for something else if we want to produce a more meaningful strategic analysis.

Linear demand functions would be an obvious possibility. One could specify a food demand function where d_i depends linearly on $r=p/P$. Such a food demand function could be derived from utility functions which are additively composed of a quadratic utility from food and a linear utility from nonfood. On the one hand, one would obtain good analytic properties, but on the other hand one would have to accept the undesirable property that food expenditures do not depend on income at all, but only on relative prices. One may have, of course, an influence of past incomes on the parameters of the linear demand functions for food. Since time periods in the IIASA model are relatively long (1 year), this does not seem to be very recommendable.

One can retain some of the advantages of the linear demand functions for food if it is complemented by a linear dependence on nonfood demand D_i:

$$d_i = u_i - v_i r + w_i D_i \quad \text{for } i = 1,\ldots,n \tag{81}$$

The coefficients u_i, v_i and w_i are assumed to be positive. This demand relationship involves the following demand functions:

$$d_i = \frac{u_i}{1 + w_i r} - \frac{v_i r}{1 + w_i r} + \frac{w_i}{1 + w_i r} \cdot \frac{E_i}{P} \tag{82}$$

$$D_i = -\frac{u_i r}{1 + w_i r} + \frac{v_i r^2}{1 + w_i r} + \frac{1}{1 + w_i r} \cdot \frac{E_i}{P} \tag{83}$$

for i = 1,...,n. A utility interpretation can be given for such demand functions, but this shall not be done here. It can be seen immediately by (82) that real food expenditures $d_i r$ go to zero with decreasing r which is not the case for linear expenditure functions.

It is worth pointing out another difference to linear expenditure systems. Consider the share of food expenditures $d_i p / E_i$. For the linear expenditure system we obtain:

$$\frac{d_i p}{E_i} = \frac{d_i^0 (1 - \alpha_i) r - \alpha_i D_i^0}{E_i / P} + \alpha_i \tag{84}$$

For high relative prices of food the first term on the right-hand side of (84) is positive. This has the consequence that for high relative prices of food, the food expenditure share is a decreasing function of total expenditures. For sufficiently low relative prices of food, however, the food expenditure share is an increasing function of total expenditures. It would be much more plausible to have an opposite effect of low and high relative prices. For (82) the food expenditure share in total expenditures is as follows:

$$\frac{d_i p}{E_i} = \frac{u_i r - v_i r^2}{(1 + w_i r) \frac{E_i}{P}} + \frac{w_i r}{1 + w_i r} \tag{85}$$

It can be seen immediately that the first term which is influenced by E_i is positive for low relative food prices and negative for high relative food prices. For low relative prices of food the expenditure share of food is a decreasing function of E_i, the opposite relationship can only hold for sufficiently high relative food prices r.

The demand relationship (81) together with the simplified absorption approach (77) produces an analytically well behaved system in which the payoff functions considered in this paper are quadratic in the strategic variables s_i and b_i. The detailed exploration of this modified model cannot be presented here. We plan to do this in a separate report.

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