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A MODEL FOR THE OPTIMAL OPERATION OF A SINGLE RESERVOIR FOR USE OF IRRIGATION

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PREFACE

Water resource systems have been an important part of resources and environment related research at IIASA since its inception. As demands for water increase relative to supply, the intensity and efficiency of water resources management must be developed further. This in turn requires an increase in the degree of detail and sophistication of the analysis including economic, social, and environmental evaluation of water resources development alternatives aided by application of mathematical modeling techniques, to generate inputs for planning, design, and operational decisions.

This paper is part of comparative studies on operational decisionmaking in the multiple reservoir water resource systems initiated in 1979 by "Regional Water Management" Research Task of the Resources and Environment of IIASA.

Much research has been done world-wide on the operation of reservoir systems in the moderate climatic conditions. This paper is concerned with the operation of a reservoir supplying water to an irrigation system located in the arid zone. A set of mathematical models and computer programs was developed to determine the optimum operation rule for a storage/irrigation system.

The research presented in this paper has been carried out by the Author at IIASA within the framework of the 1979 Young Scientists Summer Program.

Janusz Kindler
Task Leader
Regional Water Management

ABSTRACT

A model is developed for the optimal operation of the releases from a reservoir which are used to irrigate an area in an arid environment. The operation is based on a forecast technique. The forecast consists of expected values based on the application of Dynamic Programming Technique as well as on the use of generated inflows. The generation model for rivers in arid areas consists of three components. The first determines if the following year will be dry or wet. The second generates a sequence of days in which a flow may occur, and the third calculates the magnitude of such a flow. The third component depends directly on the first. The optimization problem has two state variables and one decision variable, and is solved by Dynamic Programming. The state variables are the reservoir content and the soil moisture of the irrigation area. The decision variable is that quantity of water which should be applied to irrigation. Because of the rapidly changing inflow process the model has to provide "adequate" representation of the evaporation losses which are calculated on a daily basis. The optimal scheduling process is non-stationary because of the continuing reservoir sedimentation which gradually decreases the reservoir volume.

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INTRODUCTION

In the Near-East country a group of single storage reservoirs shall be built. The inflows in the reservoirs are produced by irregular rainstorms in a mountain region. But there is no contribution by precipitation to the irrigated area in the coastal plain downstreams (annual precipitation less than 100 mm). The purpose of this paper is to develop a set of models which permit to determine the optimal operation rule of such a reservoir. The optimal operation of a reservoir, which is used to irrigate an area in an arid or semi-arid region is as optimal as the forecasts are. Because of the irregular inflow process to a reservoir, the forecast consists of expected values based on the application of an optimization technique, on the use of generated inflows (only four years of measured data are available) and on the size of the area to be irrigated. At the beginning of a growing period, the area to be planted has to be decided. If either the whole developed area should be planted or only a smaller part, depends on the local conditions. The whole area to be developed may be determined by the proposed model of Plate and Treiber (1979). Once the area to be planted has been decided, the initial storage content together with the future inflow has to be allocated, so that at least no crop failure should occur. (That means no area should be abandoned.)

The main concepts of the simulation model is given in the above reference. Therefore, only a brief repetition will be given. However, the derivation of the forecasting model will be described in detail. The hydrologic simulation model including all the submodels are schematically given below. It is the model of a single purpose reservoir whose input is a mixture of water and sediment. The water is stored in, or released from the reservoir according to demand, water availability, and forecast. Because of the gradual filling up of the reservoir as a result of the continuous sediment deposit, the process of reservoir operation is a non-stationary one. The model as a whole consists of three main parts:

1. Input Model
   - 1.1 Generation model of daily discharges
   - 1.2 Rating curve for sediment deposit
2. Reservoir
   - 2.1 Evaporation model
   - 2.2 Stage-area and stage-volume curves for the reservoir
3. The Operation Rule
   - 3.1 Determining the irrigation area
   - 3.2 Forecast of the reservoir contents, based on the application of a dynamic programming technique, the use of generated daily discharges and the use of a multiple regression analysis for the results of the optimization model to determine the operation rule.

THE RESERVOIR MODEL

The reservoir operation is based on the following equations:

Continuity Equation (daily basis)

$$S_i = S_{i-1} + Q_{ini} - Q_{outi} - E_i - Spill_i \ , \qquad (1)$$

where

S_i = volume of water in the reservoir on day i (cbm);

Q_{ini} = flow volume flowing into reservoir during day i (cbm);

Q_{outi} = flow volume released from reservoir during day i (cbm);

E_i = volume lost due to evaporation during day i (cbm) calculated by the Penman method (1948) and by use of the stage-area and stage-volume curves;

$Spill_i$ = spill occurring on day i (cbm).

Maximum Reservoir Size

The volume S_i is restricted in the following equation:

$$o < S_i < S_{max} - S_{s,i} \quad ,$$

S_{max} = maximum storage capacity (cbm);

$S_{s,i}$ = volume of reservoir filled with sediment on day i (cbm).

Sediment Deposit (daily basis)

$$S_{s,i} = S_{s,i-1} + n_T \cdot C_{s,i}\beta_T/\rho_T \quad , \qquad (2)$$

where

n_T = trap efficiency of sediment (dimensionless);

ρ_T = density of sediment in reservoir (tons/cbm);

$C_{s,i}$ = sediment load carried into the reservoir during day i (tons);

β_T = development and bedload factor.

The sediment load is given in the following rating curve:

$$C_{s,i} = a \cdot Q_{ini}^{b} \quad . \qquad (3)$$

The parameters a, b, n_T, ρ_T and β_T are estimated from observations.

THE RUNOFF GENERATION MODEL

Since the measured data series of discharges were too short to be taken as representative for conditions during the lifetime of the reservoir, a generation model was designed for daily values. A model known from literature is the model from Yakowitz (1973), specially developed for arid and semi-arid areas. But it needs a considerable amount of measured data which were not available, for estimation of the model parameters, to build a data generation model. Based on investigations of available data, a schematical record of daily flows of a wadi is shown in Figure 1. The flow in larger wadis, consist of a continuous and low flow on all days (baseflow) plus a flood peak added to this baseflow on days with rainfall, and in smaller wadis, of a flood event only without baseflow. Therefore, it is possible to construct a model which consists of three parts:

1. A generation model for the sequence of wet and dry days (days with and without floods).
2. A generation model for the magnitude of floods on wet days.
3. A model for the magnitude of the baseflow for the larger wadis.

Furthermore, the model suffice the condition to reproduce the standard deviation of the total annual inflows.

WET OR DRY DAY DECISION MODEL

The probability on the occurrence of a wet day depends whether the day before was wet or dry. The probability that a flood occurs today is higher, if yesterday was a wet one and is smaller if yesterday was a dry one. This behaviour may be described by a Markov chain. The simplest probability model of this kind contains two parameters:

P_o = probability that a flood occurs if the day before was a dry day;

P_i = probability that a flood occurs if the day before was a wet day.

The parameters P_o and P_i are determined in such a manner that the mean number of pulses and the variance of the number of pulses in a month are preserved (Treiber and Plate, 1977). Then by means of a random number generator, dry and wet days are generated.

GENERATION OF FLOOD MAGNITUDES (PULSES)

Preliminary investigations of the available data shows that a high flood is usually followed by a high flood and low by a low one. To reproduce the availability of the date as well as the corrrelation between neighbouring wet days, the flood magnitudes were generated by a Markov-I model, which was used in two steps:

In the first step, it was assumed that a Markov-I process exists continuously, i.e. that a flood magnitude Y_i occurred on each day

$$Y_i = Y_{i-1} \cdot \rho_1 + Z_i \quad , \qquad (4)$$

where ρ_1 is the lag-one correlation coefficient of the succeeding flood events and Z_i a random variable taken from an appropriate distribution. In the second step all values Y_i, which occurs on dry days were eliminated.

The generation of random variable Z_i was done by the use of transformed exponential distribution using the following equation:

$$Z_i = [- \frac{1}{\lambda} \ln(1-t_i)^{\alpha}] \quad . \qquad (5)$$

In Equation (5), t_i is a random number, λ the parameter of the exponential distribution and α the parameter of the transformation. These parameters are determined in such a manner, that mean and variance of the flood magnitudes Y_i are preserved (Treiber and Plate, 1977).

GENERATION OF THE BASEFLOW

In wadis where a kind of baseflow exists, it is necessary to add a flow also on dry days. Therefore, for each month a maximum level Q_{BL} was set as shown in Figure 1. The flow below this level is called baseflow. Denoting QL_j the mean total flow below Q_{BL} during the month j and MQ_j the average total monthly flow, and under the assumption that the baseflow Q_{Bj} is constant within the month j, it can be calculated by the following equation:

$$Q_{Bj} = \frac{QL_j}{MQ_j - QL_j} \cdot \sum_{1=n}^{n} Y_{ij} \quad , \tag{6}$$

with Y_{ij} = Pulse at day i in the month j.

The Variability of the Total Annual Flows

With the model described above, a total record of 1000 years has been generated, whose statistics were compared with those of the measured data. A comparison of observed and calculated total annual flow volumes (= yearly sums) yields a difference of 33 percent between the standard deviations. Therefore, the model has to be extended for a component, to reproduce these standard deviations.

Preliminary investigations of the available data shows that a relationship exists only between the amount of the yearly sums and the pulses Y_i within a month, that suggests there exist a relation between the amount of the total annual flow and the means m_k and standard deviations s_k of the months, i.e. between the parameters α and λ.

To solve the problem, two assumptions have to be introduced. As only four complete years of runoff were measured, the first assumption was made that the total annual flows are lognormally distributed with the mean and standard deviation of the non-transformed values XMIT and σJ. To get generalized results, every annual flow has been divided by the mean XMIT, so that the resulting values have a mean of XMJ = 1.0 and a standard deviation $\sigma\mu$, with

$$\sigma\mu = XMJ \cdot \frac{\sigma J}{XMIT} \tag{7}$$

The same procedure, to divide every daily values through the corresponding monthly mean value has been done for the pulses, which yields mean values m_k of 1.0 and standard deviations S_k. (This has to be borne in mind when the generation of a data series has to be done.) The second assumption is, that the mean monthly values of the pulses m_k are lognormally distributed with the parameters of the yearly sums XMJ = 1.0 and $\sigma\mu$ after equation (7). The standard deviation of month S_k was at first looked upon as a constant value.

The simulation programme was modified in a way that at the start of a year, a lognormally distributed (XMJ = 1.0; $\sigma\mu$) sum XJSU was generated and the XJSU was used as the mean m_k for the month to calculate with the fixed S_k - values α and λ. The implementation of these assumptions in the generation programme yields a standard deviation of the yearly sums which was larger than the measured one (just the opposite effect). Therefore, the effect of the variability on the standard deviations of the month S_k has to be determined. This relationship may be demonstrated by the following steps:

- generate lognormally distributed values XJSU (yearly sums) with XMJ = 1.0 and $\sigma\mu$;
- each value XJSU is identical with the mean of each month m_k;
- by the use of XJSU and S_k, calculate the parameters α and λ for the generation of the flood magnitudes;
- by aid of the set of the α and λ values generate n series of data, which belongs to a shifted exponential distribution;
- the mean function of the n series yields a mean value μ_t which equals XMJ = 1.0 and a standard deviation $\sigma_t > S_k$.

To solve the problem of the increasing standard deviation is to answer the question, how to diminish S_k to get as a result of the generation programme S_k (simulated) equal S_k (measured). That means a functional relationship between σ_t, the resulting standard deviation of a month, and a lognormal distributed mean value XM and a fixed standardization S_k, has to be established. This can be expressed with the following equation:

$$\sigma_t = f(XM(XMJ, \sigma\mu),\ S_k) \quad . \qquad (8)$$

The solution was obtained by computer simulation for XMJ = 1.0 and for every $\sigma\mu$ = 0.0 until 2.0 one hundred lognormally distributed numbers were generated. For each of these values and for S_k = 0.2 until 2.0 (but fixed) 120 numbers were generated by the aid of the parameters α and λ of the transformed exponential distribution. For these 12,000 numbers the mean XM and the standard deviation σ_t were calculated. This procedure was repeated with 9 other sequences and the mean of these values of σ_t were plotted in Figure 2.

This graph may now be used, to diminish the standard deviations of the months S_k so that the standard deviation of the yearly sums σ_t will be reproduced by the generation model.

COMPARISON OF MEASURED AND GENERATED DATA

Ten data series of runoff, 100 years each, were generated, whose statistics are compared with those of the measured data. Because of relatively short length of the measured data only the measured data case serve as comparison. A comparison is made of observed and calculated annual flow yields, now that the mean annual flow is identical and the standard deviation differs less than 2 percent. Measured and generated daily flows are compared in Figures 3 and 4, measured and generated monthly flows in Figures 5 and 6. In these figures the measured value, the mean value of 10 hundred data series, the smallest and the greatest value of one of a hundred data series are plotted. Good agreement is considered to exist. For a detailed description of the data generation model, see Schmidt and Treiber (1979).

THE OPERATION RULE

Determination of the Irrigation Area

The number of hectares N_k for a crop in year k can be calculated either by

$$N_k = \text{Savail/DEMAND} \quad , \qquad (9)$$

where Savail is the total quantity of water which should be available for irrigation purpose and DEMAND is the effective water use for irrigation according to equation (10).

$$\text{DEMAND} = \sum_{j=1}^{n} \text{ETPOT}_j \cdot G \qquad (10)$$

where ETPOT is the potential evapotranspiration of a stage j and G is the assumed project efficiency, or is given as a fixed value as a result of the model of Plate and Treiber (1979). The value of Savail is determined by equation (11).

$$\text{Savail} = S_{ini} + Z_x \quad , \qquad (11)$$

where S_{ini} is initial storage content at the beginning of the irrigation period and Z_x is the sum of the expected total monthly inflows during the forecasting range. Investigations have shown that the values of Z_x are gamma-distributed and x is therefore the level of the chosen probability which should be exceeding x percent in all cases. A better alternative would be to calculate the area to be planted every year on the basis of a decision analysis, but such a calculation is beyond the scope of this paper.

THE FORECASTING METHOD

Once the size of the irrigation area has been decided, the question arise how to operate the reservoir to get the highest yield [tons] from the irrigation area. Because of the irregular nature of the inflows, no traditional forecasting technique can be used. Therefore the method described in this paper, approaches the problem in a different way consisting of the following three steps:

1. Development of a data generation model for arid rivers (see previous chapters).
2. Development of a mathematical model of a water resources system and simulation of its operation over a long trace of synthetic inflows (simulation coupled with a mathematical programming technique).
3. Statistical analysis of the results of the simulation-optimization computations and identification of the optimal operation rule for the reservoir system.

This implicit stochastic approach was used by Kindler (1977) to determine the operation rules for a system of storage reservoirs.

SIMULATION-OPTIMIZATION MODEL

Several researches have been done to develop the optimal irrigation strategy (Flinn and Musgrave, 1967; Hall and Butcher, 1968; Dudley, 1969). The main scope of the model, however, was on the irrigation management. Therefore, for example, the integration of the future inflows in the reservoirs was done by mean monthly inflows (Dudley, 1969). Furthermore, the yield model used by them is very rough. Therefore a simulation model should be used to obtain better results, which additionally accounts for reservoir sedimentation and reservoir evaporation (because of daily inflows), and which incorporate a model for calculating the soil moisture, and as a consequence a dated production function which pays attention to stress susceptibility of the plant during the growing period.

THE OPTIMIZATION MODEL

The optimization technique applied to solve the problem is backward Dynamic Programming. Dividing the irrigation season in n stages, which must not have all the same length, two state variables, reservoir content and available soil moisture at the beginning of each stage, and one decision variable, the quantity of irrigation water which should be applied, may be identified. That means it is necessary to describe the reservoir system and the soil regime.

Reservoir System

The state transition is given through the following equation:

$$S_j = S_{j-1} + Q_j - E_j - D_j - SPILL_j \quad , \qquad (12)$$

constraint to $0 < S_j < S_{max} - S_{s,p}$ where,

S_{j-1} = reservoir content at the beginning of stage j;
Q_j = inflow during the stage j;
E_j = evaporation losses of the reservoir during stage j (function of the reservoir sedimentation);
D_j = decision variable, depending on the available soil moisture of the irrigation area;
$SPILL_j$ = spill volume during stage j;
S_{max} = maximum storage capacity;
$S_{s,p-1}$ = volume of reservoir filled with sediment at the beginning of stage j (the assumption was made, that the reservoir sedimentation during one year is negligible and could be added as a whole at the end of the year);
$j=1,...,n$ = number of stages.

The Soil Regime

The state transition yields equation (13) (values per hectare irrigation area):

$$BF_j = BF_{j-1} + IR_j - Wa_j \quad , \qquad (13)$$

constraint to

$$PWP \leq BF_j \leq FC \quad ,$$

where

BF_{j-1} = available soil moisture at the beginning of stage j;
IR_j = irrigation quantity during stage j;
Wa_j = water use of the crop during stage j (calculated on a daily basis);
PWB = permanent wilting point;
FC = field capacity.

The relation between the two branches of the system is given through the following equation:

$$D_j = IR_j \cdot AREA \cdot G \quad , \qquad (14)$$

constraint to

$$IR_{min} < IR_j < (S_{j-1} - E_j - Spill_j + Q_j)/(AREA \cdot G) \quad ,$$

where

AREA = size of the irrigation area;

IR_{min} = irrigation quantity which should be given within a growth stage to avoid wilting.

The stage return is expressed through equation (15)

$$Y_j(S_j, BF_j) = \max(D_j \circ Y^*_{j-1}(S_{j-1}, BF_{j-1})) \quad , \qquad (15)$$

where

Y_j = return obtained from a j stage process when starting with specific values S_{j-1} and BF_{j-1};

o = mathematical operator (e.g. + or x).

The variable which is unknown until now is the soil moisture available in the root zone which knowledge allows the calculation of the irrigation quantity to be applied and furthermore, to calculate the actual daily evapotranspiration and hence to establish the resulting yield of the irrigation area.

THE SOIL MOISTURE MODEL

Much research has been done to explain the relationship between the atmospheric demand, plant, behaviour, available soil moisture and yield. The main problem seems to identify a relationship between actual evapotranspiration ETA and available soil moisture SM in the root zone. Because no measured data are available the following procedure is proposed.

The potential evapotranspiration ETPOT within one stage was calculated by the Blaney-Criddle (1950) method, as recommended by the agricultural consultants in the field. The appropriate values were used as daily values. To establish the relation between ETPOT, ETA and SM, a procedure proposed by Minhas, et al. (1975) was adopted. The functional relationship is given through equation (16)

$$\frac{ETA}{ETPOT} = f(SM) \quad , \qquad (16)$$

where SM is in the range between FC and PWP. There are several curves on the behaviour of SM presented in the literature. We selected one of those by the aid of agricultural consultants. After Minhas, et al. ETA is calculated as a function of ETPOT and the available soil moisture (SM_o,SM_1) at two different time points t_o,t_1 by solution of the following differential equation:

$$\frac{dSM}{dT} = ETPOT(t) \cdot f(SM) \quad , \qquad (17)$$

with the knowledge of ETA, the harvestable yield per unit water can be calculated, because there are two or more harvests possible, a grain yield model and a model which calculates the dry matter yield as a number for fodder crops have to be used.

THE GOAL FUNCTION

A number of attempts have been made to derive production functions for different crops. Broadly speaking, there is a clear line of evolution in the literature, from the simple concept of a fixed water requirement to complex production and allocation models. For the purpose of this study, however, it is essential to have a recorded production function capable of estimating the level of crop yields under any irrigation practice, given the magnitude of the deficits and the time of their occurrence within the growth cycle, divided into a number (m) of physiological meaningful stages. Jensen (1968) derived the following multiplicative expression which is well-suited to this type of analysis:

$$\frac{Y}{Y_o} = \prod_{j=1}^{m} [\frac{W_a}{Wo}]_j^{\lambda_j} \quad , \qquad (18)$$

where

y/yo = actual yield/maximum yield if the soil moisture is not limited;

λ_j = relative sensitivity of the crop to water stress during the j-th stage of growth;

Wa/Wo = net water use/use of water if soil moisture is not limited.

That means Wa is the sum of the daily values of ETA within the stage j.

This expression is for determining crops, such as grains, which have a specific period of flowering. Furthermore, it is valid in the case of limited soil moisture provided other factors, such as plant nutrients are not a constraint and yield is not otherwise reduced (e.g. by pests and diseases). It does, however, make the simplifying assumption that the water input can be considered in isolation from other inputs and constraint. The usefulness of the approach depends on the accuracy with which the λ_j values are determined. A technique was developed by Naiziri and Rydzewski (1977) to obtain these values for a number of crops from published experimental data.

For indeterminate crops, such as grasses, which have no specific flowering period, it is a common practice to use a summation-type relation. For the calculation of dry matter yield, the following relation was introduced which is based on the results given by the FAO (1975).

$$y/yo = \frac{1}{W_{max}} \cdot \sum_{j=1}^{\ell} W_{aj} \quad , \qquad (19)$$

where W_{max} is the cumulative potential evapotranspiration of the growing period, consisting of ℓ stages and y/yo is the actual, respectively, the maximum yield if the soil moisture is not limited. With these two models and by aid of the weighting factors P_k, which allows a comparison between grain and dry matter yield, the goal function for the two crops can be formulated.

$$\frac{Z}{Zo} = \prod_{j=1}^{m} P_1^{1/m} \left(\frac{Wa}{Wo}\right)_j^{\lambda_j} + \frac{1}{W_{max}} \cdot \sum_{j=m+1}^{n} P_2 \cdot Wa_j \quad , \qquad (20)$$

with P_1 and P_2 as the unit values for the net return on water for grain and dry matter yield.

With all the models described above it is possible to calculate the reservoir releases dynamically according to water demand and inflow situation. The resulting yield of the irrigation area is optimal per unit water, the limiting factor. If we do the optimization in a long-term run (r years) we get r optimal yields. As a second result of the optimization computations, a set of optimal control vectors S_j, final storage volumes, are obtained for each of these stages.

Establishing the Operational Rules of the System

In the method presented above the decision maker has a perfect knowledge of the future inflows, that means he has the perfect forecast. To receive a reliable forecast of this inflows, and furthermore to establish the operation rule of the reservoir, a linear regression analysis based on the optimal control vectors S_j was used. That means for each stage j of the annual growing period, a multiple regression was assumed to exist between the following parameters:

$$S_j = a_o + a_1 S_{j-1} + a_2 Q_j \quad , \tag{21}$$

where a_o, a_1 and a_2 are the regression coefficients. As Q_j normally is not predictable, the mean monthly inflows were replaced in the operation system. The set of regression equations constitute the operation that secure long-term optimality of the reservoir operation. With the known reservoir content at the beginning of a stage S_{j-1}, the regression coefficients and the mean monthly inflow, S_j can be calculated. The difference between S_j and S_{j-1} is the quantity of water which should be applied (or not) for irrigation purpose.

The Application of the Model

The method presented above could be applied to a single purpose reservoir in a near-east country. For purpose of demonstration, some assumption will be done in this paper. A more detailed description of the implementation of the model in a real world setting will be given in future. But nevertheless, for the aim of demonstration the application given below shows the practicability of the method itself. The assumptions are:

- The only investigated crop is grain sorghum.
- Two crops can be harvested (grain + fodder).
- The growing cycle consist of 6 stages (monthly basis, 3 months for grain producing, and 3 for fodder crop).
- The susceptibility factors for the grain yield were used with $\lambda = 0.5$, $\lambda = 1.5$, and $\lambda = 0.5$ according to the three-month period (in a later case of research work more accurate values will be used).

With these assumptions the following computer model can be established in Figure 7. There are two models for generating and calculating the input data. The model QSYE generate a sequence of daily runoff values with consideration of reproducing the variability of the yearly sums. The model SOPLAT estimates the soil moisture values within one stage, when the beginning soil moisture content at stage j has a fixed value and a stress of the plant (actual evapotranspiration is less than the potential one) under consideration of the principle that two shorter stresses lead to less yield reduction than a single longer one. The model DASSA computes for a given irrigation area the optimal operation of the reservoir. There are three main subroutines used in DASSA. The first one, SPUEBG, computes for one year the reservoir and the soil system, that means the transitions of the reservoir contents and the behaviour of soil moisture within this stage if the soil moisture content at the beginning of a stage has a fixed value , according to Dynamic Programming technique. The second one, DUNST, generates the evaporation losses of the reservoir as a consequence of the policy of SPUEBG, and with the subroutine YIELD, the resulting optimal yield for the two crops is calculated. If DASSA is run for a fixed area and with, e.g. 50 years generated data series, r reservoir contents for each of the n = 6 stages are received. For this set of r x n values, a multiple regression analysis has been done by aid of the model MULREG which yields the regression coefficients as a function of the size of area which are looked upon as forecasting values. These coefficients can be used for the actual reservoir irrigation system, which means a model which use the forecasting values to operate the reservoir and to calculate the behaviour of the irrigation area on a daily basis. For the purpose of demonstration the same 50 years of generated runoff data were used. This model, called SPSIM operates in the following way.

After reading in all the necessary data, the simulation runs are started. One run consists of two steps. In the first step the inflows in the reservoir are stored under consideration of the sedimentation process and evaporation losses. At the end of this period, by use of the reservoir content and with the expected monthly total inflows in the future, that means the level of the chosen probability, the size of the irrigation area is determined. With the knowledge of the size of area the forecast values can be joined, which means, the appropriate set of regression coefficients can be adopted. With these values the reservoir and irrigation system can be operated, by controlling the available soil moisture. When the irrigation season has finished, the resulting yield of the area will be calculated. Repeating this procedure for all the r years of expected project life, the total yield of the long-term run can be received.

For the purpose of demonstration this simulation model was run over 50 years. Every year the planted area was determined by the first method described in a previous chapter. The probability level x was chosen arbitrarily as 20 percent. With the same sequence of the estimated areas the optimization model DASSA was run. For both simulation runs the cumulative total yield was computed according to equation (22).

$$y = \sum_{p=1}^{r} (Z/Zo)_p \cdot AREA_p \quad , \qquad (22)$$

where $(Z/Zo)_p$ is the relative yield of a year and $AREA_p$ is the irrigated area of this year. For p = r and for the results of DASSA the maximum possible yield y_{max} is given. In a graphical comparison the relatively cumulative total yields y/y_{max} are plotted in Figure 8. Variant (1) is the relative yield if DASSA has been run. That means, the system operator has a perfect knowledge of inflows. Variant (2) shows the cumulative relative yield as a result of the simulation run by SPSIM, that means the system operator uses the case given above by the implicit stochastic approach.

CONCLUSION

A set of mathematical models and computer programs was developed to determine the optimum operation rule of a storage reservoir supplying an irrigated area. The implicit stochastic optimization by combining data generation of the inflows, simulation with optimization and regression analysis seems to be a valuable technique for the solution of such problems. Research should go on in estimating the annual cropping area based on a decision analysis, and costs and benefits of such a project should be integrated in the model. Furthermore, the effect of different project efficiencies, on the total yield should be investigated.

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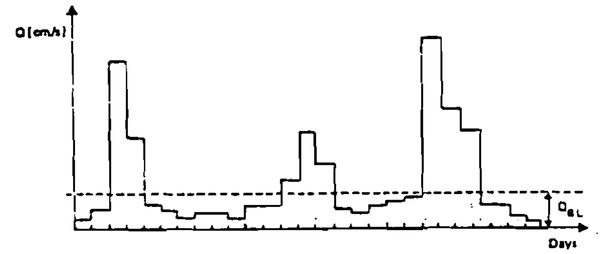

Figure 1. Schematical run-off record

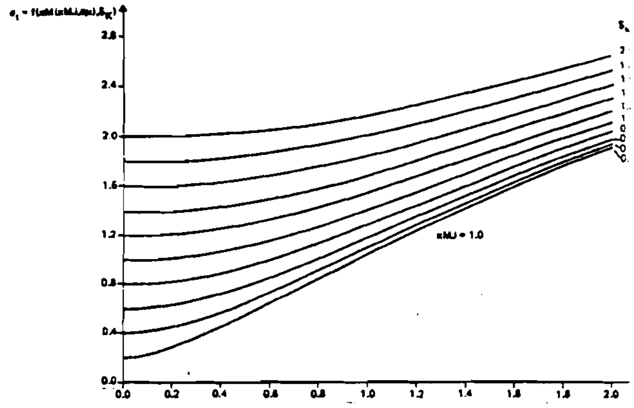

Figure 2. Functional behaviour of the increasing standard deviation

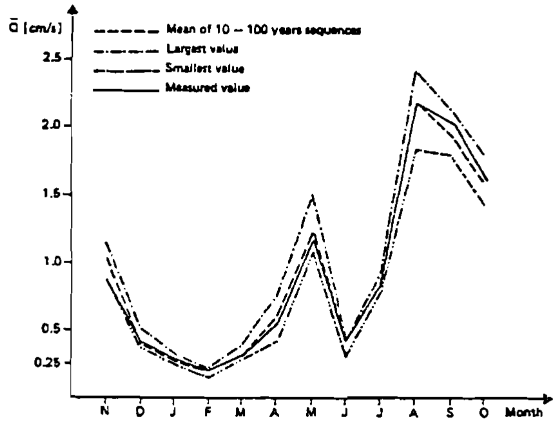

Figure 3. Means of daily flows

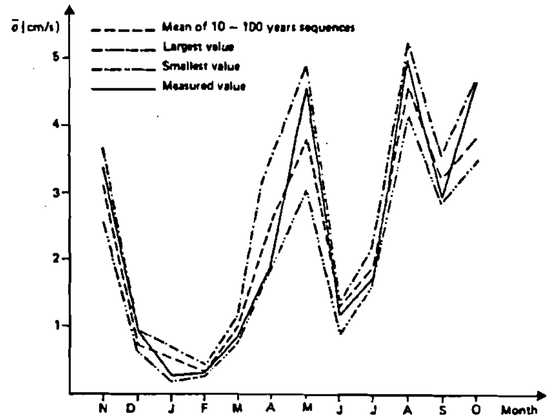

Figure 4. Standard deviations of daily flows

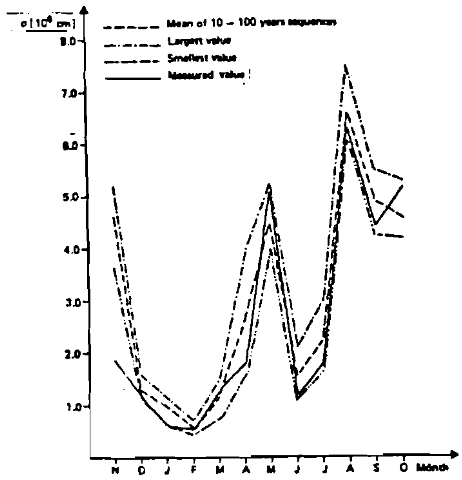

Figure 5. Means of monthly flows

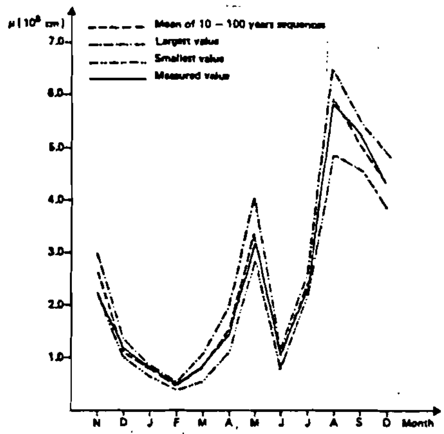

Figure 6. Standard deviations of monthly flows

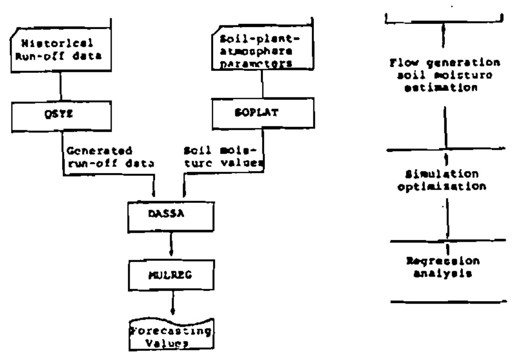

Figure 7. The computer models for the determination of the operation rules

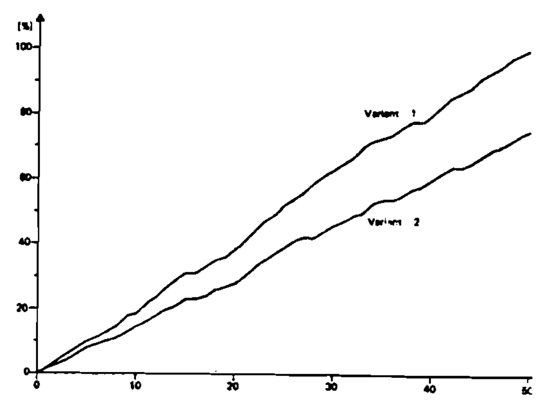

Figure 8. Cumulative relative total yield (simulation for 50 years)