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Reputation or warranty, what is more effective against planned obsolescence?

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ABSTRACT
In determining the durability of its product a firm faces a trade off. Performing a policy of planned obsolescence by making their products less durable implies that the consumer needs to replace them earlier, which thus enhances demand. However, a lower quality of the product will result in a lower reputation, which in turn will affect demand negatively. In many cases, the government protects the consumer by implementing a warranty period. Our paper studies how a firm should optimally deal with this trade off and react to government policy. We obtain the following results. First, we find that the length of the warranty period has an inverted U-shaped effect on the product life time. Second, if more consumers are aware of the existence of a warranty period and ask for a free product replacement, this will increase the product life time. Third, increasing uncertainty about the breakdown of the product also has an inverted U-shaped effect on the product lifetime.

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1. Introduction

The downside of durable goods is already included in the name: they are durable and hence the consumer will not repurchase when he has already bought one item. Various strategies can be observed of how firms try to create a continuous stream of income from their consumers, even though the basic product is durable. We first describe three of them and point to the one we study in this paper. One recent approach is related to the sharing economy, i.e. consumers are motivated or even forced to rent the durable good rather than buying it. This is becoming popular, e.g. with cars (various car sharing systems often run by car producers, e.g. SHARENOW by BMW and Mercedes-Benz) or software (e.g. MS Office 365). Another strategy is to give away the basic durable good almost for free and to charge high prices for the related consumption materials. This has been popularised, e.g. in the area of ink jet printers or coffee capsule machines (e.g. Nespresso). However, this only works well if the producer can prevent that other producers also offer these consumption materials at lower prices. This will not work with appliances like washing machines or products like light bulbs, where the consumption material, i.e. washing powder or electrical energy, is not under control of the producer of the durable good. Yet another strategy, being the focus of the present paper, is the so-called planned obsolescence strategy, which exists already for decades. With this strategy, producers artificially and intentionally restrict the lifetime of a product to stimulate repeat purchases. While naively one might think that the life span of a product typically depends on wear and tear of its materials plus some random influence, it may also be the result of entrepreneurial choice. Planned obsolescence means that companies design their products in such a way that they do not last long. Typically, while most of the components are very durable, a single important component (or few components) has a short life cycle and replacing or repairing them is made uneconomical. This makes the life cycle of the whole product short. Bradley and Guerrero (2009) call this Life-Cycle Mismatch. The implication is that consumers have to buy new commodities more often so that product demand increases. In some sense consumer durables are made non-durable.

A classical example is the light bulb, which originally lasted rather long, thus cropping profits of this industry. This incentivised the founding of the so-called Phoebus cartel in 1924. The cartel more or less covered the total world production of light bulbs and its members agreed that the life span of light bulbs must not exceed 1000 hours. This was done by introducing some predetermined breaking point to reduce lifetime, which meant a reduction of the typical life span by more than 50 %; see,

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e.g. Krajewski (2014). Another classic case is women's tights made from nylon. The 'laddering' of stockings implies that consumers have to buy new ones. The first models had a rather high longevity. Only after inventing a new recipe that made the material less UV resistant, the laddering effect would occur more frequently and producers could enjoy more repeat purchases.

While the term 'bsolescence' dates back to the 1820s, it became well known in the 1930s through the book of London (1932). This book sees planned obsolescence as a way to overcome the great depression and the high unemployment at that time. The economic literature on planned obsolescence started more or less in the 1980s by the seminal paper by Bulow (1986) showing that planned obsolescence makes sense for producers under certain conditions. Planned obsolescence has also been recognised as an important topic within operations management. The handbook of operations management, Hill (2012), defines planned obsolescence as 'A strategy of designing products to become obsolete or non-functional after a period of time also known as built-in obsolescence.' Recent literature has focused on the detrimental effects of planned obsolescence. Considerations of sustainability with respect to climate change, shortage of energy and raw materials, and social consciousness made planned obsolescence inappropriate in the public opinion. Consumer organisations, media and even legislators have paid more attention to planned obsolescence, and this negatively affected the economic performance of producers that used planned obsolescence excessively. A prominent example is Apple that revised down its earnings expectations in the fourth quarter of 2018 (The Economist Espresso, 4 January 2019). Besides signs of economic weaknesses in China, this was largely because of lower iPhone sales. One of the main causes of the latter was longer battery lives. Originally, batteries lasted 18 months and could not be replaced, so that one had to buy a whole new smartphone once the battery stopped working. However, after some court cases and media pressure, Apple had to offer a battery repair service and a two years warranty period.

As mentioned, from the view-point of the firm, indeed the disadvantage of a durable good that does not break down is that the consumer only needs to buy it once. Hence, durable goods breaking down after some time will induce consumers to buy a new one, which stimulates demand for this product. However, on the other hand, if products break down too often, the firm will get a bad reputation, which consequently will lower sales. Kuppelwieser et al. (2019) show that a planned obsolescence strategy harms consumers' value perception and ultimately their willingness to pay.

We argue that the role of reputation requires the analysis to be dynamic, giving the decision maker the opportunity to build up reputation over time. This paper therefore develops a dynamic model of the firm that captures this trade off. The model contains a demand function being positively dependent on the stock of potential consumers and the product's reputation. Here the stock of potential consumers is positively affected by breakdowns, whereas the latter negatively affects the reputation of the product. For every vintage, the firm has to determine the (average) product lifetime and at every point in time, the firm has to decide about the quantity that is put for sale. To distinguish between vintages, we develop a model such that, besides time, also product age is reflected. A crucial question is to which extent a firm can steer the product lifetime in order to be able to carry out a meaningful planned obsolescence policy. The opinions are ambiguous and probably depend on the type of the product. According to Kreiss (2015), the lifetime can be planned and controlled very precisely by engineers. On the other hand, according to Reischauer (2011), Eduard Sailer (former CEO at Miele) questions whether this is possible. Therefore, our model takes the time to breakdown to be uncertain, where we will investigate how the results are affected by the level of the variance.

Our research questions are the following. *First*, we consider the effect of the length of the warranty period on optimal firm behavior. The social planner, for instance the EU, implements a warranty period with the aim to protect consumers. If a product breaks down during the warranty period, firms have the obligation to replace it by a new one for free.[1] Our analysis shows that the length of the warranty period has an inverted U-shaped effect on the product life time. If the warranty period is short enough, an increase will have a positive effect on product lifetime. The reason is that the firm wants to keep the probability that products break down in the warranty period low, so that no products have to be replaced for free. However, if the warranty period is too long, it will be too costly for the firm to increase the product quality so much that the probability that the product breaks down in the warranty period remains low. As a consequence, the firm then gives up to keep this probability low. Since this probability then is high anyhow, the firm puts less efforts in raising the product life time, which explains that it decreases with the length of the warranty period if the latter is large. The implication of this inverted U-shaped effect is that one should be careful in setting a warranty period length with the aim to increase the product life time. Overshooting would lead to an increased breakdown probability and thus a lower product lifetime.

Second, as it works in practice, not all consumers are aware that they are entitled to receive a free item when their product breaks down during the warranty period. Other consumers might lose the receipt or decide not to keep it, e.g. if the value of the product is not too high. In all these cases, the firm can escape the obligation to replace a defective item. The parameter α in our model specifies the fraction of consumers going for a free replacement once they have the right to do so. We find that the product lifetime is increasing if this fraction goes up. The firm reduces the risk that the product breaks down during the warranty period, since the risk is higher that the corresponding consumer will ask for a free replacement. We conclude that a way to let the firm increase product lifetimes is to advertise this existence of warranty periods such that the fraction of consumers asking for a free replacement increases.

Third, as we already pointed out, it is not always clear to which extent the firm can steer the time to breakdown, or the product lifetime. Therefore, it is meaningful to find out how the firm reacts to different levels of uncertainty in designing the average breakdown time of its product. We find that, given that in the deterministic case the optimal product lifetime is longer than the length of the warranty period, increasing the uncertainty has an inverted U-shaped effect on the (average) product lifetime. When the variance is small and the average product lifetime remains the same, an increase of the variance enlarges the probability mass of the probability distribution governing the time to breakdown in the warranty period. This gives the firm an incentive to increase the average product lifetime. However, for a large variance the effect is opposite. This is because the firm cannot really steer the time to breakdown if the variance is large, as the outcome is very uncertain. Therefore, the firm is reluctant to increase the expenses on raising the average time to breakdown.

The paper is organised as follows. Section 2 reviews the literature on planned obsolescence. The model is developed in Section 3. Section 4 analyses the model, whereas Section 5 concludes.

2. Literature review

This section is organised as follows. We first treat *general* literature about planned obsolescence, which is followed by an overview of the *economics/operations management* literature. Then we focus on the *legal* part and we finish with *warranty*.

We start out with a *general* overview of the topic planned obsolescence. In addition to planned obsolescence in the narrow sense, where the product is constructed in a way that it in fact becomes unusable after a certain period, there are also some related strategies, which are nicely summarised in Mellal (2020). This paper distinguishes between various types of obsolescence:

- Technical or technological obsolescence: The devaluation of an item is due to technical progress so that it becomes economically preferable to use the new technology instead of the old one, even though the old product is still fully functional.
- Functional obsolescence: This means that some main function is degraded or insufficient without opportunity to be updated. In the case of mobile phones, this may be increasing storage or processor speed used by newer software that could not run on older models.
- Planned obsolescence: Manufacturers introduce obsolescence into their production policy, e.g. by designing parts so that in the next few years, they will break down forcing consumers to 'inevitably' replace the product.
- Style or psychological obsolescence: When a product is no longer attractive because it is out of fashion.
- Optional obsolescence: This means that technological improvements are not applied to a product, even if they could be. It is not uncommon (e.g. in the automotive industry) that a manufacturer develops a new feature for its range of products, but chooses not to implement it in the production of the cheaper items in its portfolio.

Style or psychological obsolescence is called 'Planned fashion obsolescence' by Philip, Aswath, and Raja (2020). It exploits the consumers' desire to stay in line with the current fashion trends in the market. By rapidly changing designs or colours supported by media coverage, fashion-conscious consumers are forced to buy new products in high frequency. In fact, this phenomenon is not new and not confined to fashion goods. When the car industry started, Henry Ford had a sense of mission to provide products of high quality and product longevity to consumers. According to Slade (2007) (page 32), he resisted all attempts of a shortened product lifespan or premature obsolescence in his automobiles. This strategy was successful for a long time giving him about 60% market share in the US market beginning of the 1920s. General Motors, the main competitor in the early 1920s followed a different strategy. GM invested heavily in new design and short product cycles. By intensive marketing, they were able to turn the so far 'means of transport' into a lifestyle product and within just a few years Ford's market share shrunk from over 60% to 30 %.

Our paper focusses on planned obsolescence in the narrow sense, even though some of our analysis could also be applied to other types of obsolescence.

Next we give an overview of the *economic and operations management* literature on planned obsolescence. Waldman (2003) provides an overview of the theory of planned obsolescence in durable goods markets. The seminal paper on planned obsolescence is Bulow (1986). Employing a straightforward two-period model he shows that a durable goods monopolist has an incentive to reduce the durability of its output to enhance future demand. We extend Bulow (1986) by analysing a fully dynamic model, in which the firm can build up reputation by longer product lives, and we take into account that a firm may not be able to perfectly steer the product life because of the presence of uncertainty.

Fethke and Jagannathan (2002) generalise the results developed by Bulow (1986) for the two-period case to a more general multi-period setting. In doing so, they develop a model where durability is given for the initial stock of products and one durability level can be chosen for future production. Our model is more extensive in that for every vintage the firm can choose a separate expected product lifetime. Moreover, we include reputation of the product and allow for uncertain breakdown times.

Grout and Park (2005) consider in a two-period model framework whether planned obsolescence could arise in a competitive market. They find that under moderate technological progress planned obsolescence will occur in equilibrium. This is not the case when technological progress is more rapid or slow. Our model is different because we abstract from technological progress. Instead, in our framework the decision maker is a monopolist and the model is fully dynamic, allowing for uncertainty in the product life.

Agrawal, Kavadias, and Toktay (2016) study markets where consumers seek for exclusivity associated with conspicuous consumption. In such a situation, a strategy of planned obsolescence does not pay off. Instead, a firm benefits from designing products with higher durability. This is accompanied by charging high prices and lower sales volumes granting the exclusivity consumers are asking for.

Bhaskaran and Gilbert (2015) analyse a two-period model where durable goods are sold through a retailer. They show that there may be weaker incentives to perform a policy of planned obsolescence in a decentralised channel with sufficiently low marginal production costs. Also the combined effects of shifting from centralised to decentralised channel structure and shifting from a leasing to a selling mode of operations can increase a firm's incentive to increase durability. Instead of a vertical channel structure, we focus on a setting where a monopolist firm sells products directly to end consumers in a full dynamic framework.

Koenigsberg, Kohli, and Montoya (2011) study the design of durable goods. They characterise a relationship between optimal price, cost elasticities and opportunity costs associated with relaxing upper bounds on usable and physical lives. As an extension to their work, Koenigsberg, Kohli, and Montoya (2011) argue that it is useful to allow for uncertainty in the product lives and to consider a dynamic model to examine the effects of a planned obsolescence strategy. This is exactly what is done in the present paper. The products get broken with respect to a stochastic failure rate, which can be influenced during the production process. However, if products get broken too frequently, the reputation is diminished implying a negative effect of the demand-price relation.

Kinokuni, Ohori, and Tomoda (2019) employ the Bulow (1986) two-period model to consider the question of how planned obsolescence influences the environment and welfare. The point is that a less durable product increases the quantity of waste and associated environmental damage. They find that introducing a disposal fee reduces planned obsolescence and increases product durability.

There is also a bulk of recent literature on *legal* issues connected to planned obsolescence. Maggiolino (2019) states that – compared to economic and environmental aspects – the legal classification of planned obsolescence is instead a less explored and debated issue. Current antitrust rules would be able to sanction extreme undertakings such as the historic Phoebus cartel. However, many other cases are not so clear. At present, only in France (Art. L213-4-1 introduced by the law 2015–992) a rule exists defining planned obsolescence as 'the set of techniques by which an issuer on the market aims to deliberately reduce the lifetime of a product in order to increase its replacement rate' and qualifies this as criminal. Only in 2017 the European Parliament adopted a resolution (2016/2272 (INI)) inviting the Commission to propose a definition of programmed obsolescence and to 'examine the possibility of establishing an independent system able to test and detect the obsolescence embedded in products' and to 'provide for better legal protection for so-called whistleblowers and appropriate dissuasive measures for producers'. The conclusion by Maggiolino (2019) is: 'to be more effective in the fight against planned obsolescence, legislators should conceive of tools that make it possible to punish planned obsolescence directly, that is, regardless the specific practices used to deliberately reduce the life span of a product '. La Rosa (2020) concludes that it is necessary to adopt a model of regulation that integrates soft law instruments, commercial law actions (class action) and administrative controls. Against planned obsolescence, the use of criminal law should be limited only to cases of failure of other measures. Since it is apparently still not possible to prevent planned obsolescence by legal measures, we will have to live with this phenomenon in the near future and it makes sense to investigate monetary or market mechanisms that mitigate or discourage planned obsolescence to some extent, such as warranty periods or reputation effects.

Like the regulations of the European Union who demand a 2-year *warranty* period for most appliances and electronic equipment, in our model we introduce a warranty period to protect consumers. We establish how the firm should react to that in determining the optimal expected product life time and the sales level in a monopoly setting. On the other hand, contributions exist in which the introduction of a warranty period is done by the firm itself as a means to distinguish oneself from its competitors. In a model of two firms , Balachander (2001) presents a signalling-based explanation for the empirical phenomenon that a longer warranty period may be offered together with a product of lower quality. This is opposite to Spence (1977) who argues that a product of high quality should go along with a longer warranty period to signal its quality to uninformed buyers.

We assume that after breakdown within the warranty period, the product is disposed and the firm has the obligation to replace it for free. Within the area of 'Extended Producer Responsibility' also other options are considered, such as repair or refurbishing; see, e.g. Pince, Ferguson, and Toktay (2016), or Giu et al. (2016). For simplicity, we do not consider this option here. We also do not consider legislation requiring producers to collect and dispose or refurbish products that break down after the warranty period. Such an obligation could make planned obsolescence less attractive.

3. The model

Consider a firm that wants to maximise profits by selling a durable product. At each moment in time it has to determine the quantity, $s(t)$, that is put on the market for sale. In the production process the firm also has to decide about the quality of the product, and thus about its (expected) lifetime. Denoting the building year, or the vintage, by v, the expected product lifetime is denoted by $b(v)$. If the age of the product is a, we get

$$a = t - v. \tag{1}$$

As already stated in the Introduction, it can either happen that the product lifetime can be precisely steered (Kreiss 2015) or that it is rather uncertain when a product will break down (Eduard Sailer (Miele) in Reischauer 2011). We introduce a cumulative distribution function $F(a; \Phi)$, where $F(a, \Phi)$ equals the probability that a product breaks down before reaching age a for a parameter set Φ. The corresponding probability density function is denoted by $f(a, \Phi)$. In what follows, we impose that the cumulative distribution function $F(a; \Phi)$ is a gamma distribution,[2] which depends on a shape and a scale parameter, $k > 0$ and $\Theta > 0$ (i.e. $\Phi = (k, \Theta)$), respectively. This implies that for the average product lifetime we have $b = k\Theta$, and that the variance equals $\sigma^2 = k\Theta^2$ (or equivalently $k = \frac{b^2}{\sigma^2}$ and $\Theta = \frac{\sigma^2}{b}$). Consequently, the hazard rate $\bar{h}(a; k, \Theta)$ satisfies

$$\bar{h}(a; k, \Theta) = \frac{f(a; k, \Theta)}{1 - F(a; k, \Theta)} = \frac{a^{\frac{b^2}{\sigma^2}-1} e^{-\frac{ab}{\sigma^2}} \left(\frac{\sigma^2}{b}\right)^{-\frac{b^2}{\sigma^2}}}{\Gamma\left(\frac{b^2}{\sigma^2}, \frac{ab}{\sigma^2}\right)}. \tag{2}$$

The firm controls the average breakdown time $b(v)$, while we assume that the variance σ^2 of the time to breakdown is determined by the underlying production technology and cannot be controlled by the firm. To ease notation we will therefore write $h(a, b(v)) := \bar{h}(a; k, \Theta)$ for the rest of the paper.

Denoting the number of products in use of age a at time t by $q(t, a)$, the number of breakdowns of age a at time t equals $h(a, b(t - a))q(t, a)$. By integrating over age a we obtain the total number of breakdowns at time t,[3]

$$B(t) = \int_0^\infty h(a, b(t - a))\, q(t, a)\; da. \tag{3}$$

Note that, by employing expression (1), $b(v)$ can also be written as $b(t - a)$.

To protect the consumers, the government introduces a warranty period. If the consumer owns a product that breaks down before the warranty period is over, this consumer has the right to receive a new item for free. Let ω be the length of the warranty period, then the number of breakdowns within the warranty period, $W(t)$, equals

$$W(t) = \int_0^\omega h(a, b(t - a))\, q(t, a)\; da. \tag{4}$$

Analogously to the expression for $B(t)$, also in the expression for $W(t)$ we add up the number of breakdowns for different ages. The difference is that we only take into account ages such that the breakdown falls into the warranty period, i.e. the maximal age is ω. Focusing on the consumers that own a product that breaks down during the warranty period, we denote the fraction of them that are aware of the fact that one can ask for a new product, by α. Since newly produced products are either sold or given for free to consumers demanding their warranty rights, it follows that

$$q(t, 0) = s(t) + \alpha W(t). \tag{5}$$

The number of products (build at one instant of time) in use decreases over time because of the breakdowns. After noting that time and age develop in the same way, we get

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\right) q(t,a) = -h(a, b(t-a))\, q(t,a). \quad (6)$$

This is the classical McKendrick–von Foerster linear first-order partial differential equation encountered e.g. in demography and mathematical biology, see Keyfitz and Keyfitz (1997). Consumers having products that break down after the warranty period has expired, are up to buy a new item. Assuming that each consumer owns maximally one product, this number of consumers is equal to $B(t)-\alpha W(t)$. Denoting the stock of potential consumers by $C(t)$, we get that

$$\frac{\mathrm{d}C(t)}{\mathrm{d}t} = B(t) - \alpha W(t) - s(t). \quad (7)$$

Demand for this product also depends on the reputation this product has, which is negatively affected by its ability to break down. Denoting the number of products in use by

$$Q(t) = \int_0^\infty q(t,a)\, \mathrm{d}a, \quad (8)$$

the fraction of the products that do not break down equals $(Q-B)/Q$. We assume that there is a reputation for the product that is related to the durability and quality of the product as it is perceived by the consumers. Defining the speed of adjustment of reputation in the market for this durable good by δ, we get that reputation, $R(t)$, develops over time as follows:

$$\frac{\mathrm{d}R(t)}{\mathrm{d}t} = \delta\left(\left(1-\frac{B(t)}{Q(t)}\right) - R(t)\right). \quad (9)$$

The speed of adjustment, δ, can e.g. be influenced by publications of consumer organisations on observed durability or, as suggested by Munten and Vanhamme (2019), by providing information on the reparability of the products.

Kuppelwieser et al. (2019) have shown that a planned obsolescence strategy harms consumers' value perception and ultimately their willingness to pay, via a reputation effect. In that sense, a firm should take into account that early breakdowns damage reputation and thus demand. Furthermore, if the stock of potential consumers is large, then this has a positive effect on demand. As usual sales will decrease, if the price is higher. Assuming linear relationships, we arrive at the following inverse demand function:

$$p(R(t), C(t), s(t)) = \theta_1 R(t) + \theta_2 C(t) - s(t), \quad (10)$$

in which p is the product price, and the parameters θ_1 and θ_2 stand for the effect of reputation and the stock of potential consumers on the output price, respectively.

The costs associated with the production process consist of production costs related to quantity, $c_1 q(t,0)$, and costs that are related to the quality of the product, $c_2 b$, with b the average time to break down of the products produced at time t. If the firm increases this quality, the expected product lifetime goes up, and we therefore impose that it is more costly to produce products with a longer product lifetime and thus with a lower breakdown probability.

The objective of the firm is to maximise the discounted value of the stream of cash flows, where r is the discount rate. The resulting dynamic model of the firm now becomes

$$\max_{s(t),b(t)} \int_0^\infty e^{-rt}\left[\left(p(R,Q,s)-c_1\right)s(t) - c_1\alpha W(t) - c_2 b\right] \mathrm{d}t \quad (11)$$

subject to

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}\right) q(t,a) = -h(a, b(t-a))\, q(t,a), \quad (12a)$$

$$q(t,0) = s(t) + \alpha W(t), \quad (12b)$$

$$Q(t) = \int_0^\infty q(t,a)\, \mathrm{d}a, \quad (12c)$$

$$B(t) = \int_0^\infty h(a, b(t-a))\, q(t,a)\, \mathrm{d}a, \quad (12d)$$

$$W(t) = \int_0^\omega h(a, b(t-a))\, q(t,a)\, \mathrm{d}a, \quad (12e)$$

$$\frac{\mathrm{d}C(t)}{\mathrm{d}t} = B(t) - \alpha W(t) - s(t) \quad (12f)$$

$$\frac{\mathrm{d}R(t)}{\mathrm{d}t} = \delta\left(\left(1-\frac{B(t)}{Q(t)}\right) - R(t)\right) \quad (12g)$$

with initial values

$$C(0) = C_0 > 0 \quad (13a)$$

$$R(0) = R_0 \in [0,1]. \quad (13b)$$

The model implicitly assumes that the stock of consumers is constant, i.e. a constant market potential of $C_{\max}$ that is split into potential consumers, $C(t)$, and current users, $Q(t)$. This is formalised in the following lemma.

Lemma 3.1: *The model implicitly assumes a constant stock of consumers, i.e.*

$$C(t) + Q(t) = C_{\max}. \quad (14)$$

The proof can be found in the appendix.

4. Results

This section consists of two parts. Motivated by the statement in Kreiss (2015) that product lifetime can be planned and controlled very precisely by engineers, we first consider a variant of the model in which the firm can be precise in its planned obsolescence policy by perfectly steering the time to breakdown of the underlying product, i.e. *the breakdown time is deterministic*. This implies that the variance of the breakdown time is zero and the resulting model is deterministic. We provide the analytical solution for the steady state variant of this model. Second, we study the model with *uncertain breakdown time*. In particular, we numerically determine the optimal solution and provide economic interpretations.

4.1. Deterministic breakdown time

The model to be analysed is in principle the full model (11)–(13b) of the previous section. However, due to the fact that the breakdown time can be exactly determined, Equation (12a) can be rewritten into

$$q(t,a) = \begin{cases} q(t-a,0) & \text{if } a < b(t-a) \\ 0 & \text{if } a \geq b(t-a) \end{cases}. \quad (15)$$

Also the expressions for the total breakdowns (12d) and the breakdowns during the warranty period (12e) can be simplified:

$$B(t) = \int_0^{\infty} q(t-a,0)\mathbb{I}_{[b(t-a)=a]}\,\mathrm{d}a, \quad (16a)$$

$$W(t) = \int_0^{\omega} q(t-a,0)\mathbb{I}_{[b(t-a)=a,a\leq\omega]}\,\mathrm{d}a. \quad (16b)$$

For $\mathbb{I}_{[\cdot]}$ being the indicator function, $\mathbb{I}_{[b(t-a)=a]}$ equals 1 if the products produced at $t{-}a$ (i.e. a years ago) break down exactly at t and zero otherwise. Multiplication with the size of the production $q(t-a,0)$ and aggregation over all possible vintages gives the total breakdowns $B(t)$ at t. $W(t)$ is constructed analogously. For this model, we are able to provide an analytical solution for the optimal steady state problem. This problem is especially relevant for mature firms.

Theorem 4.1: *Consider the steady state situation of a firm that can exactly determine the breakdown time. The firm's optimal breakdown time and the optimal sales level are given by*

$$b = \sqrt{\frac{\theta_1}{\theta_2 s + c_2\frac{1}{s}}} \quad (17a)$$

$$s = \frac{1}{2(\theta_2 b + 1)}\left(\theta_1\left(1 - \frac{1}{b}\right) + \theta_2 C_{\max} - c_1\right) \quad (17b)$$

if the warranty period is shorter than the time to breakdown, i.e. $\omega < b$, and

$$b_\alpha = \sqrt{\frac{\theta_1}{\frac{\theta_2 s_\alpha}{1-\alpha} + c_2\frac{1}{s_\alpha}}} \quad (18a)$$

$$s_\alpha = \frac{1}{2\left(\frac{\theta_2 b_\alpha}{1-\alpha} + 1\right)}\left(\theta_1\left(1 - \frac{1}{b_\alpha}\right) + \theta_2 C_{\max} - \frac{c_1}{1-\alpha}\right) \quad (18b)$$

otherwise.

The proof can be found in the appendix. From (12g) we obtain the following reputation in the steady state:

$$R = 1 - \frac{B}{Q} = 1 - \frac{1}{b}. \quad (19)$$

Now we conclude from Theorem 4.1 that the optimal sales level positively depends on the effect of reputation on the output price. Furthermore, the number of consumers, including the potential ones, $C_{\max}$, also has a positive effect on sales. It makes sense that the unit cost related to the quantity part of the production cost function, c_1, affects sales negatively. The expression for the optimal sales level (17b) nicely reflects the trade off of the product life time. The term $\theta_1(1 - \frac{1}{b})$ represents that a large b increases reputation and thus the output price, which enhances sales. On the other hand, the term $\theta_2 b$ in the denominator reduces sales. The interpretation is that a large b implies that existing consumers can use their current products for a longer time. And, as long as a consumer has his/her product in use, he/she is not a potential consumer needing a new item, and this reduces demand.

In the other case, when the optimal sales level is s_α satisfying (18b), products break down in the warranty period, i.e. $b < \omega$. This implies that a fraction α of the consumers is asking for a new item. It makes sense that this reduces the sales level. Note that, given the situation that the product lifetime b is such that all products break down before the warranty period is over, when all consumers demand their warranty rights, i.e. $\alpha = 1$, it makes no sense for the firm to produce, so that the optimal sales level equals zero.

Longer product lifetimes enhance reputation and thus demand, which explains why the optimal time to breakdown is increasing in θ_1. On the other hand, as explained above, a longer time to breakdown reduces the stock of potential consumers, which explains that b is decreasing when θ_2, the effect of the stock of potential consumers on the output price, is large. Furthermore, if c_2 is large, it means that it is expensive to extend the time to breakdown, and therefore b is low.

In case the breakdown time is such that products break down during the warranty period, and the sales level s is given, the time to breakdown decreases with α. If a larger fraction of consumers demand their warranty rights, the market becomes less profitable for the firm. Therefore, the firm does not want to invest so much in the quality of the product, implying a short time to breakdown.

It is interesting to consider the effect of installing a warranty period by the government. The aim of the government to do so is to stimulate firms to increase the product lifetime, so that consumers are protected against the firm's planned obsolescence policy. In this light we share two observations. *First*, as long as the length of the warranty period, ω, falls below the time to breakdown b in (17a), installing a warranty period does not affect the product lifetime. *Second*, if the government increases ω beyond b in (17a), the time to breakdown decreases from b to b_α. The intuition is that, if $b < \omega$, the products break down in the warranty period, implying that the firm has to give away products for free to consumers demanding their warranty rights. This reduces the profitability of this market, which makes that the firm will invest less in this market. This translates in reducing the cost of product quality, resulting in an earlier time to breakdown of this product. We conclude that in the scenario where firms can exactly steer the time to breakdown, installing a warranty period either has no effect, or results in the opposite of what it wants to achieve in the sense that it reduces product lifetime.

4.2. Uncertain breakdown time

As we stated in the Introduction, opinions on to which extent a firm can influence the breakdown time of their products, are mixed. Kreiss (2015) states that the product lifetime can be planned and controlled very precisely. In such a case the analysis of the previous subsection applies. On the other hand, Reischauer (2011), and in particular Eduard Sailer (former CEO at Miele), questions whether this is possible. For this reason we consider a stochastic breakdown time in this subsection, for which the model (11)–(13b) applies. We vary the variance (see Figure 3 later on) to keep our analysis relevant both for a Kreiss (2015) and a Reischauer (2011) scenario.

Since the model cannot be solved analytically we have to resort to a numerical analysis.[4] As a numerical example, we consider the parameters:

$$\theta_1 = \theta_2 = 0.3, \quad C_{\max} = 1, \quad c_1 = 0.01, \quad c_2 = 0.001,$$
$$\delta = 0.1, \quad \omega = 2,$$
$$\alpha = 0.5, \quad \text{and} \quad \sigma^2 = 6. \tag{20}$$

These parameter values will be used through-out this section in all the illustrations that follow, where some key parameters will be varied to analyse different scenarios.

Figure 1 depicts optimal time paths of the relevant variables for varying levels of the length of the warranty period. Panel (a) shows that sales are lower the longer the warranty period is. This makes sense because a longer warranty period implies that there will be more breakdowns in the warranty period, which makes it less attractive to have a large sales level. Initially the sales level could

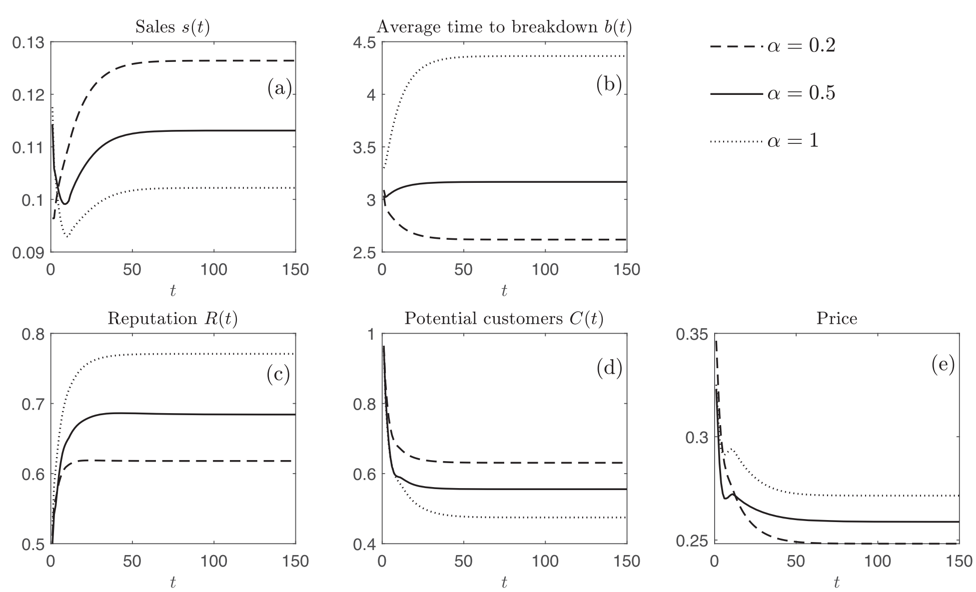

Figure 1. Optimal time paths for different levels of the length of the warranty period, ω.

Figure showing the sales $s(t)$, the average time to breakdown $b(t)$, the reputation $R(t)$, the potential customers $c(t)$, and price over time for three different choices of the warranty period, i.e. ω.

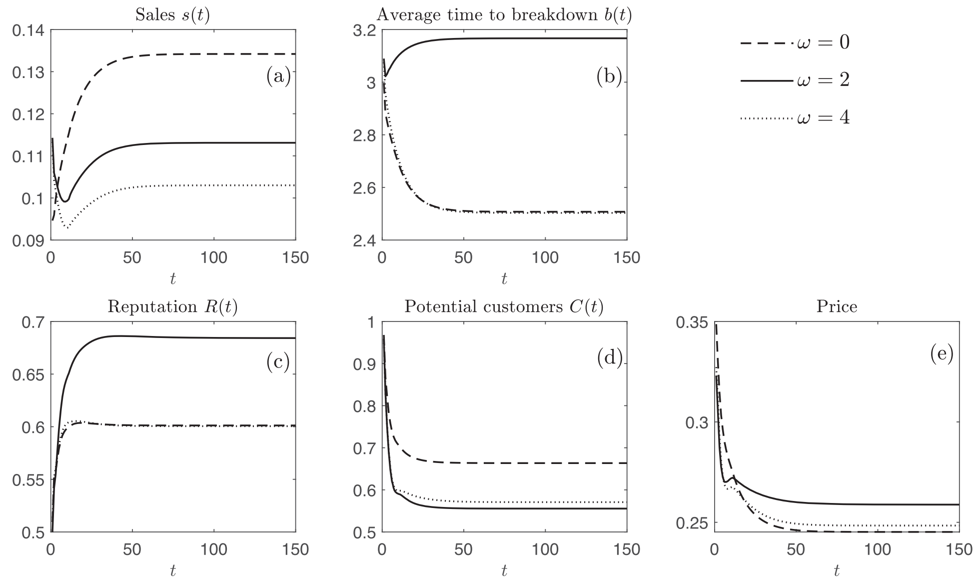

Figure 2. The optimal solution for different levels of the fraction of consumers, α, asking for a new product when their products break down in the warranty period.

Figure showing the sales $s(t)$, the average time to breakdown $b(t)$, the reputation $R(t)$, the potential customers $c(t)$, and price over time for three different choices of the fractions of customers, i.e. α, asking for a new product when their products break down during the warranty period.

be high because no consumer has bought the product yet. Then the stock of potential consumers is large (panel (d)), which has a positive effect on the output price (panel (e)). After that, sales are relatively low, but then they increase over time, while the firm builds up reputation (panel (c)).

Panel (b) shows that if there is no warranty period, i.e. $\omega = 0$, the firm starts out with setting a long expected product lifetime, because it wants to build up reputation for its product. Once reputation has been build up (panel (c)), it reduces product lifetime so that products break down more often, generating consumers that are up to buy a new item.

In case the warranty period has a length of two years, the product lifetime remains high. This is because the firm wants to keep the probability low that a product breaks down in the warranty period so that the owner can ask for a new one. This stands in contrast to an increase of the warranty period to four years. Then it becomes too expensive for the firm to keep this probability low and the result is that it is optimal to do not spend too much on the quality of the product. Under such a long warranty period, the profitability is too low to invest a lot in this market.

The implication is that varying the length of the warranty period has a non-monotonic effect on the average product lifetime. If the length is short, in the long run the firm sets an early (average) time to break down, because the probability a product breaks down in the warranty period is still low. If the length of the warranty period is long the probability that it breaks down in the warranty period is high anyhow, so the firm does not even try to get it lower by lengthening the product lifetime. This would simply be too expensive. The conclusion is that if the government wants to introduce a warranty period with the aim to protect the consumers by forcing firms to implement a long product lifetime, the length of such a period should not be too short but also not be too long. We summarise the effect of the length of the warranty period in Result 4.1.

Result 4.1: The average time to breakdown first increa ses and then decreases with the length of the warranty period. Sales are decreasing with the length of the warranty period.

In Figure 2, we have the time paths for different levels of α, being the fraction of consumers asking for a new item when their products break down during the warranty period. The effect on the (average) time to breakdown is as expected. If α is low, only a few consumers ask for a free item upon a breakdown in the warranty period. Then the firm does not have a high incentive to increase the product lifetime to avoid this, implying that they set a lower average product lifetime. A government wanting to mitigate the latter effect should advertise the existence of a warranty period. Then more consumers will ask for a free item when they are entitled to do so. This raises α and thus the average product lifetime.

However, according to panel (a) the effect on sales is exactly the opposite. If the fraction of consumers asking for a new product is low, the costs of having breakdowns

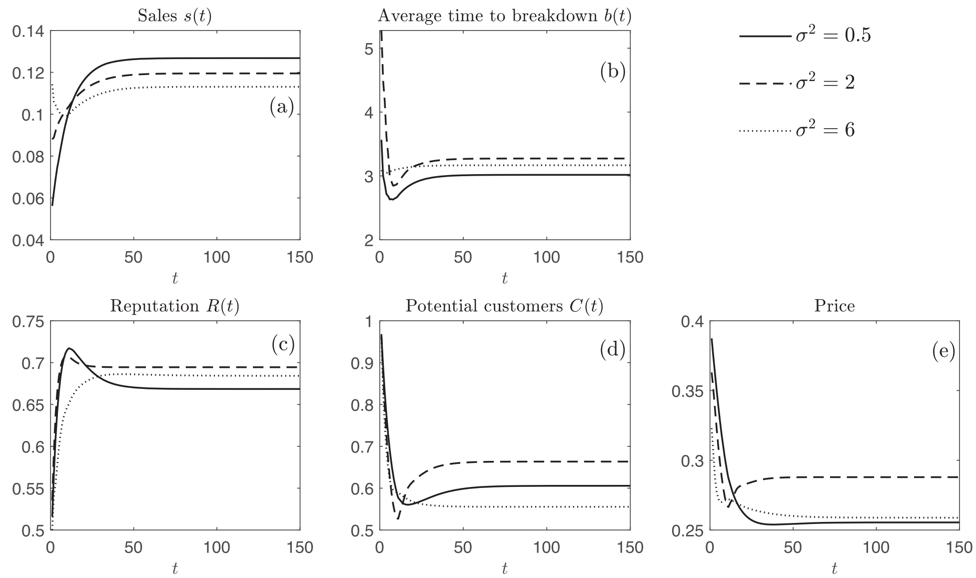

Figure 3. Optimal time paths for different levels of the variance of the product lifetime, σ^2.

Figure showing the sales $s(t)$, the average time to breakdown $b(t)$, the reputation $R(t)$, the potential customers $c(t)$, and price over time for three different choices of the variance of the product lifetime, i.e. σ^2.

during the warranty period is low as well. This makes this market more profitable, which stimulates sales. This is despite the fact that the product's reputation, which has a negative effect on the output price, is low (see panels (c) and (e)). We summarise the effect of α in Result 4.2.

Result 4.2: The average time to breakdown is increasing with α, while sales are decreasing with α.

In Figure 3, we vary the variance of the product lifetime to determine what the effect is on the optimal time paths of the different variables. First consider the long term effect of the variance on the average time to breakdown. Increasing the variance from $\sigma^2 = 0.5$ to $\sigma^2 = 2$ raises the product lifetime. The reason is that if, for a fixed b being larger than the length of the warranty period, the variance of the time to breakdown goes up, the probability mass related to breakdowns during the warranty period goes up as well. To reduce it the firm invests more to increase the average product lifetime.[5]

However, if the variance increases further from $\sigma^2 = 2$ to $\sigma^2 = 6$, the average time to breakdown decreases again. The intuition is that if there is a lot of uncertainty, the firm can hardly influence the breakdown realisation time. Then the firm is reluctant to invest a lot to increase the average time to breakdown, leading to a lower value of b. The overall result is non-monotonic where we have a firm that invests the most to increase the quality of the product and thus to lengthen the average product lifetime, for intermediate uncertainty levels.

Sales are at their highest level the more the firm can steer the breakdown time, thus the lower the variance is. A higher variance would make this market less profitable, because either the firm incurs more costs due to the fact that it has to increase the average product lifetime to keep the number of breakdowns in the warranty period limited, or because the uncertainty is so large that the outcome of the breakdowns can hardly be controlled. We summarise the effect of uncertainty in Result 4.3.

Result 4.3: The average time to breakdown first increa ses and then decreases with the variance of the breakdown time. Sales are decreasing with this variance.

To check robustness of Results 4.1–4.3, an extensive comparative statics analysis is provided in Appendix 2. Here we vary all parameter values with about $+/-20\%$ and obtain the three main results in all cases.

5. Conclusion

This paper considers the problem of planned obsolescence. In producing durable goods, a firm typically faces the problem that if it produces a good of high quality, consumers buy it once and use the good 'forever'. Producing a good of lower quality implies that the product breaks down at some point, so that the consumer has to look for a replacement. This enhances the demand for this product and thus the firm's payoff. The authority, being well aware of firm incentives to let their products break down relatively early, frequently installs a warranty period to protect the consumers.

This paper analyses the problem from the point of view of the firm. It basically considers two scenarios: one where the firm can perfectly steer the breakdown time of the product and one where the breakdown time is stochastic. In the first scenario, we find that installing a warranty period does not incentivise a firm to extend a breakdown time. Moreover, it could even be the other way around: installing a warranty period give consumers the right to ask for a free item once their product breaks down before the warranty period is over. This reduces the firm's profit and therefore it wants to invest less to increase the quality of the product, which reduces the product lifetime.

In the second scenario, thus when the time to breakdown is uncertain, the average time to breakdown of the underlying product depends on the length of the warranty period in an inverted U-shaped manner. This implies that the authority should be careful in fixing the warranty period length, because making this period too large has an adverse effect on the average time to breakdown. A way to give the firm the right incentive to invest in product quality is to make the existence of warranty rights more widely known to consumers. Once the firm knows that a considerable fraction of consumers will ask for a new product once their old one breaks down in the warranty period, it will increase product quality resulting in a longer average time to breakdown.

In the literature, different opinions can be found on whether a firm has the ability to exactly fix the time to breakdown. Varying the variance corresponding to the breakdown time learns that, if the variance is low, the average time to breakdown of the product will be longer when the variance increases. However, this effect is opposite when the variance is large. In such a situation, the firm feels it can only influence the breakdown time to a limited extent. This implies that it will invest less in the quality of the product when the variance gets larger.

An interesting topic for future research is to analyse the problem in a competing scenario. We could think of modelling two producers with different prices and product life spans.

Notes

1. Or, alternatively, repaired for free, which is also costly for the firm. Throughout the paper we assume that costs of repair and cost of providing a new item are similar.
2. Alternatively, also other distributions such as the Weibull distribution can be used. However, results will be comparable.
3. The hazard rate $h(a, b(t-a))$ denotes the probability of a product of age a and expected lifetime $b(t-a)$, breaking down at time t, given that it did not break down before time t.
4. Model (11)–(13b) is an age-structured optimal control model, for which standard boundary value solvers cannot be used. Thus we apply a gradient type based optimisation algorithm as described in Veliov 2003.
5. We analyse a mean preserving spread, in order to disentangle the effect of a larger uncertainty, while keeping the mean constant. This is possible because we have two parameters. As already mentioned on page 7, $\sigma^2 = k\Theta^2$ and $b = k\Theta$ holds, such that $\sigma^2 = b\Theta$. Then σ can be increased by increasing Θ. Consequently, b remains constant by decreasing k in a way that $b = k\Theta$ keeps the same value.

Data availability statement

Data sharing is not applicable to this article as no new data were created or analysed in this study.

Disclosure statement

No potential conflict of interest was reported by the authors.

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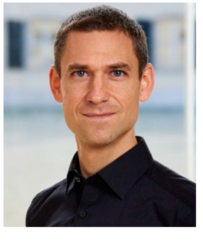

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References

Agrawal, V. V., S. Kavadias, and L. B. Toktay. 2016. "The Limits of Planned Obsolescence for Conspicuous Durable Goods." *Manufacturing & Service Operations Management* 18 (2): 216–226.

Balachander, S. 2001. "Warranty Signalling and Reputation." *Management Science* 47 (9): 1282–1289.

Bhaskaran, S., and S. Gilbert. 2015. "Implications of Channel Structure and Operational Mode Upon a Manufacturer's Durability Choice." *Production and Operations Management* 24 (7): 1071–1085.

Bradley, J., and H. H. Guerrero. 2009. "Product Design for Lifecycle Mismatch." *Production and Operations Management* 17 (5): 497–512.

Bulow, J. 1986. "An Economic Theory of Planned Obsolescence." *The Quarterly Journal of Economics*101 (4): 729–750.

Fethke, G., and R. Jagannathan. 2002. "Monopoly with Endogenous Durability." *Journal of Economic Dynamics & Control* 26 (6): 1009–1027.

Giu, L., A. Atasu, O. Ergun, and L. B. Toktay. 2016. "Fair and Efficient Implementation of Extended Producer Responsibility Legislation." *Management Science* 62 (4): 1098–1123.

Grout, P. A., and I.-U. Park. 2005. "Competitive Planned Obsolescence." *The RAND Journal of Economics* 36 (3): 596–612.

Hill, A. V. 2012. *The Encyclopedia of Operations Management: A Field Manual and Glossary of Operations Management Terms and Concepts*. New Jersey: Pearson Education.

Keyfitz, B. L., and N. Keyfitz. 1997. "The McKendrick Partial Differential Equation and Its Uses in Epidemiology and Population Study." *Mathematical and Computer Modelling* 26 (6): 1–9.

Kinokuni, H., S. Ohori, and Y. Tomoda. 2019. "Optimal Waste Disposal Fees when Product Durabilityc is Endogenous: Accounting for Planned Obsolescence." *Environmental Resource Economics* 73 (1): 33–50.

Koenigsberg, O., R. Kohli, and R. Montoya. 2011. "The Design of Durable Goods." *Marketing Science*30 (1): 111–122.

Krajewski, M. 2014, September 24. "The Great Lightbulb Conspiracy." *IEEE Spectrum*.

Kreiss, C. 2015. "Geplanter Verschleiß." *WiSt – Wirtschaftswissenschaftliches Studium* 44 (2): 97–99.

Kuppelwieser, V. G., P. Klaus, A. Manthiou, and O. Boujena. 2019. "Consumer Responses to Planned Obsolescence." *Journal of Retailing and Consumer Services* 47 (2): 157–165.

La Rosa, E. 2020. "Planned Obsolescence and Criminal Law: A Problematic Relationship?" In *Sustainability and Law*, edited by V. Mauerhofer, D. Rupo, L. Tarquinio. Springer.

London, B. 1932. *Ending the Depression Through Planned Obsolescence*. New York: Public Domain, Goolge-digitized.

Maggiolino, M. 2019. "Planned Obsolescence: A Strategy in Search of Legal Rules." *IIC-International Review of Intellectual Property and Competition Law* 50 (4): 405–407.

Mellal, M. A. 2020. "Obsolescence – A Review of the Literature." *Technology in Society* 63: 101347.

Munten, P., and J. Vanhamme. 2019. "Fighting Planned Obsolescence Practices and Excessive Consumption: The Positive Effect of Pre-Purchase Product Reparability Communication on Consumers' attitude." In *NA – Advances in Consumer Research*, Vol. 47, edited by R. Bagchi, L. Block, L. Lee, 776–777. Duluth, MN: Association for Consumer Research.

Philip, R. S., A. M. Aswath, and M. A. S. Raja. 2020. "Planned Fashion Obsolescence in the Light of Supply Chain Uncertainty." *Academy of Strategic Management Journal* 19 (1): 1–17.

Pince, C., M. A. Ferguson, and L. B. Toktay. 2016. "Extracting Maximum Value From Consumer Returns: Allocating Between Remarketing and Refurbishing for Warranty Claims." *Manufacturing and Service Operations Management* 18 (4): 475–492.

Reischauer, C. 2011. "Vermarkten Für Den Müll?." *Absatzwirt schaft* 12: 18–25.

Slade, G. 2007. *Made to Break – Technology and Obsolescence in America*. Cambridge and London: Harvard University Press.

Spence, A. M. 1977. "Consumer Misperceptions, Product Failure and Producer Liability." *Review of Economic Studies* 44 (3): 561–572.

Veliov, V. M. 2003. "Newton's Method for Problems of Optimal Control of Heterogeneous Systems." *Optimization Methods and Software* 18 (6): 689–703.

Waldman, M. 2003. "Durable Goods Theory for Real World Markets." *Journal of Economic Perspectives*17 (1): 131–154.

Appendices

A Proofs

A.1 Proof of Lemma 3.1

In order to verify that $C(t) + Q(t) = C_{\max}$ holds for all t, we have to prove that

$$\dot{C}(t) + \dot{Q}(t) = 0. \tag{A1}$$

Indeed, (12f), (12b) and (12c) yield

$$\dot{C}(t) + \dot{Q}(t) = B(t) - q\,(t,0) + \frac{\partial}{\partial t}\int_0^\infty q(t,a)\,\mathrm{d}a \tag{A2}$$

Inserting B from (12d) and interchanging the order of integration and differentiation in the last term, we obtain

$$\dot{C}(t) + \dot{Q}(t) = \int_0^\infty h(t,a,b(t-a))q(t,a)\,\mathrm{d}a - q\,(t,0) + \int_0^\infty \frac{\partial}{\partial t}q(t,a)\,\mathrm{d}a \tag{A3}$$

From (12a), it follows that

$$\dot{C}(t) + \dot{Q}(t) = -q\,(t,0) - \int_0^\infty \frac{\partial}{\partial a} q(t,a)\,\mathrm{d}a = 0 \qquad \text{(A4)}$$

because all products will broken down in the end, i.e. $q(t,\infty) = 0$.

A.2 Proof of Theorem 4.1

To solve the problem for the steady state, it is sufficient to consider the instantaneous profit, which can be obtained from (11):

$$\max_{s,b} \left[\left((\theta_1 R + \theta_2 C - s) - c_1 \right) s - c_1 \alpha W - c_2 b \right]. \qquad \text{(A5)}$$

In steady state the number of products in use is constant, which implies that the newly produced products are equal to the number of breakdowns:

$$q\,(0) = B. \qquad \text{(A6)}$$

Due to the deterministic nature of the breakdown time, we obtain

$$Q = bq(0), \qquad \text{(A7a)}$$

$$W = \begin{cases} q(0) & \text{if } \omega > b \\ 0 & \text{if } \omega \le b \end{cases}. \qquad \text{(A7b)}$$

By combining (12b) and (A7b), we get

$$q(0) = \begin{cases} s + \alpha q(0) & \text{if } \omega > b \\ s & \text{if } \omega \le b \end{cases}$$

$$= \begin{cases} \frac{s}{1-\alpha} & \text{if } \omega > b \\ s & \text{if } \omega \le b \end{cases}. \qquad \text{(A8)}$$

This implies, via (14) and (A7a), that

$$C = \begin{cases} C_{\max} - b\frac{s}{1-\alpha} & \text{if } \omega > b \\ C_{\max} - bs & \text{if } \omega \le b \end{cases}. \qquad \text{(A9)}$$

Since also reputation is constant in steady state, we obtain from (12g) that it holds that

$$R = 1 - \frac{B}{Q}, \qquad \text{(A10)}$$

which, due to (A6) and (A7a), is equivalent with

$$R = 1 - \frac{1}{b}. \qquad \text{(A11)}$$

From the above we get that the objective function satisfies

$$\omega < b : \max_{s,b} \left[\left(\left(\theta_1 \left(1 - \frac{1}{b} \right) + \theta_2 \left(C_{\max} - bs \right) - s \right) - c_1 \right) s - c_2 b \right], \qquad \text{(A12a)}$$

$$\omega \ge b : \max_{s,b} \left[\left(\left(\theta_1 \left(1 - \frac{1}{b} \right) + \theta_2 \left(C_{\max} - \frac{bs}{1-\alpha} \right) - s \right) - c_1 \right) s - c_1 s \frac{\alpha}{1-\alpha} - c_2 b \right]. \qquad \text{(A12b)}$$

Straightforward optimisation of expressions (A12a) and (A12b) yield (b,s) in (17a)–(17b) and (b_α, s_α) in (18a)–(18b), respectively.

The last point in the proof is to determine which solution is the optimal one depending on ω. Since $b_\alpha \le b$ (with equality only if $\alpha = 0$), we can distinguish the following three cases (ignoring the case of $\alpha = 0$ for which $(b,s) = (b_\alpha, s_\alpha)$ holds):

$$b_\alpha < b < \omega : (b_\alpha, s_\alpha) \text{ is optimal,} \qquad \text{(A13a)}$$

$$b_\alpha \le \omega \le b : \text{compare (A12a) and (A12b),} \qquad \text{(A3a)}$$

$$\omega < b_\alpha < b : (b,s) \text{ is optimal.} \qquad \text{(A13c)}$$

After inserting the corresponding optimal values of the controls into (A12a) and (A12b), we get that solution (b,s) is optimal in the second case.

Appendix 2. Sensitivity analysis

This appendix contains an extensive comparative statics analysis in which we show that the three main results listed in the Abstract, namely the inverse U-shaped dependence of the product lifetime on the length of the warranty period and the uncertainty parameter, and the fact that the product lifetime is increasing in consumer awareness of the warranty period, are robust. In particular, here we vary all parameter values with about +/ − 20 % and obtain the three main results in all cases, as reflected in Figures A1–A3 .

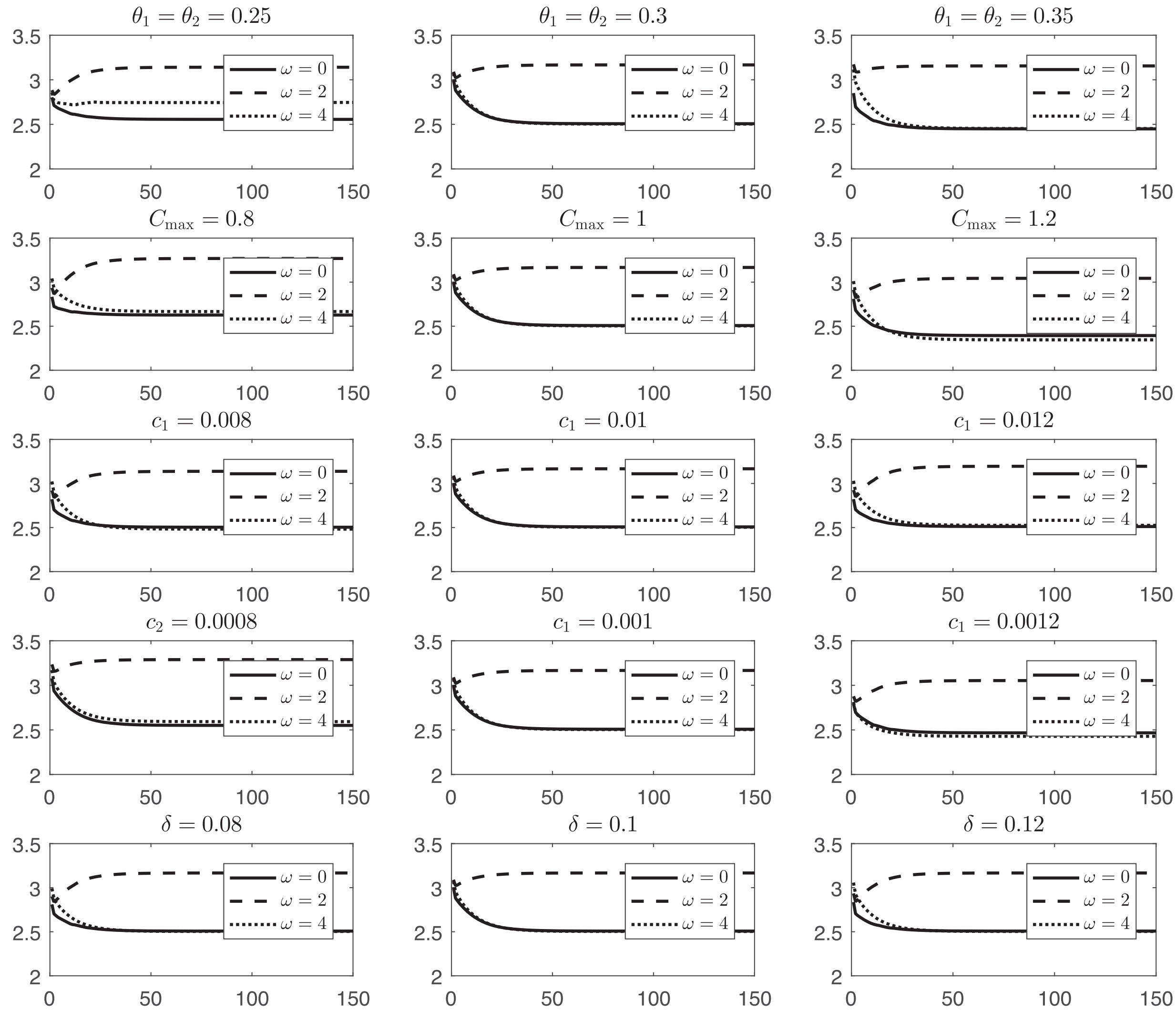

Figure A1. Sensitivity analysis performed with different ω.

Time paths of the average time to breakdown $b(t)$ for about ± 20 % of the model parameters $\theta_1 = \theta_2$, $C_{\max}$, c_1, c_2, δ for three different choices of the warranty period, i.e. ω.

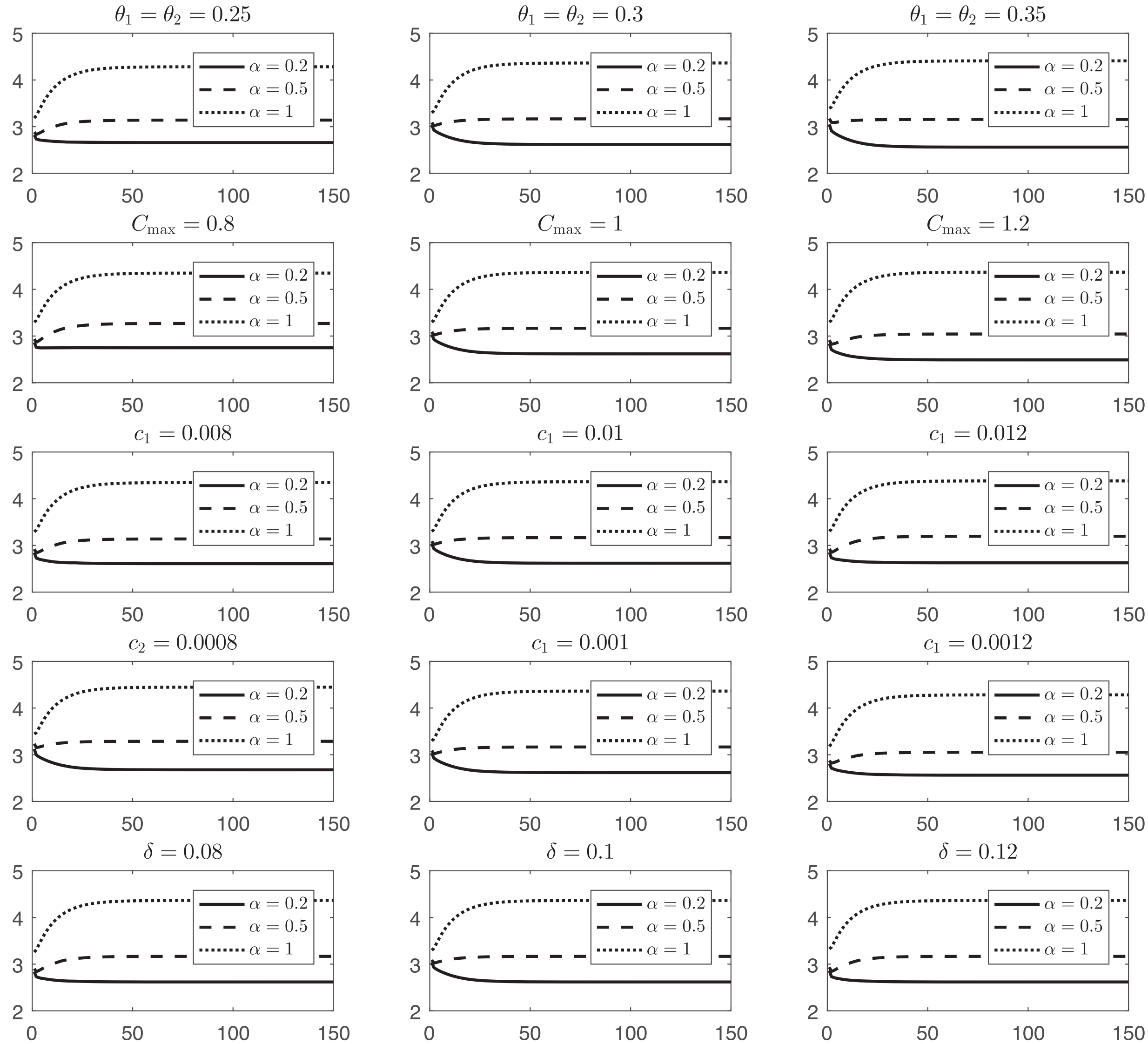

Figure A2. Sensitivity analysis performed with different α.

Time paths of the average time to breakdown $b(t)$ for about ± 20 % of the model parameters $\theta_1 = \theta_2$, $C_{\max}$, c_1, c_2, δ for three different choices of the fractions of customers, i.e. α, asking for a new product when their products break down during the warranty period.

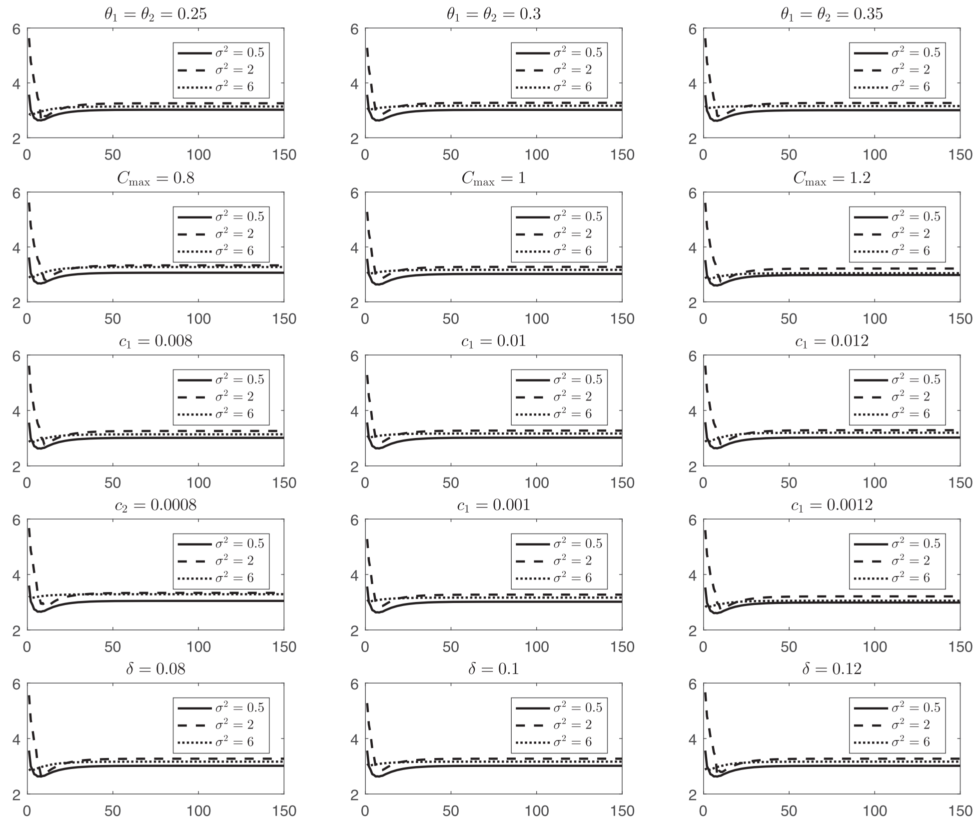

Figure A3. Sensitivity analysis performed with different σ^2.

Time paths of the average time to breakdown $b(t)$ for about ± 20 % of the model parameters $\theta_1 = \theta_2$, $C_{\max}$, c_1, c_2, δ for three different choices of the variance of the product lifetime, i.e. σ^2.