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A decision-theoretic approach towards modelling resilience

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Background:

- Well-known measures of resilience based on eco-systems modelling.
- Some socio-economic conceptualisations but few decision-theoretic formulations to date.

Objectives:

- To set out a (simple) model of renewable resource use and conceptualise resilience in a rigorous decision-theoretic way.
- To derive a model-based measure of resilience and apply it to assess resilience of resource use.

Model ingredients:

- (Optimal) behaviour leads to long-term sustenance of the resource stock if and only if the level of the stock is above a (Skiba-)threshold.
- Random shock may put resource stock below the threshold.
- Appropriate actions (e.g., pre-cautionary extraction) allow the decision-maker to increase the probability of remaining above the threshold.

Resource dynamics:

- Economy in which consumption $C(t)$ is harvested from a renewable resource stock $R(t)$ → **decision**
- Resource dynamics: $\dot{R}(t) = g(R(t)) - C(t)$ with $g(R(t)) = \frac{aR^2}{b+R^2}$ as replenishment → **state**
- Shock arrives at exogenous rate η and destroys $D(\tau) = (1-\epsilon)R(\tau)$ of the stock at random time τ.
- Two stages: 1 = before shock; 2 = after shock.

Decision problem:

(extension of Skiba 1978, Econometrica, by including shocks)

Discounted stream of consumption utility up until (random) τ — Discounted continuation value from τ

$$\max_{C(t)} \mathbb{E}_\tau\left[\int_0^\tau e^{-\rho t}C(t)^{0.5}dt + e^{-\rho\tau}V(R(\tau^+),\tau^+)\right]$$

with stage-2 value:

$$V(R(\tau^+),\tau^+) := \max_{C(t)} \int_{\tau^+}^{\infty} e^{-\rho t}C(t)^{0.5}dt$$

Subject to: $\dot{R}(t) = g(R(t)) - C(t),\ R(0) = R_0$

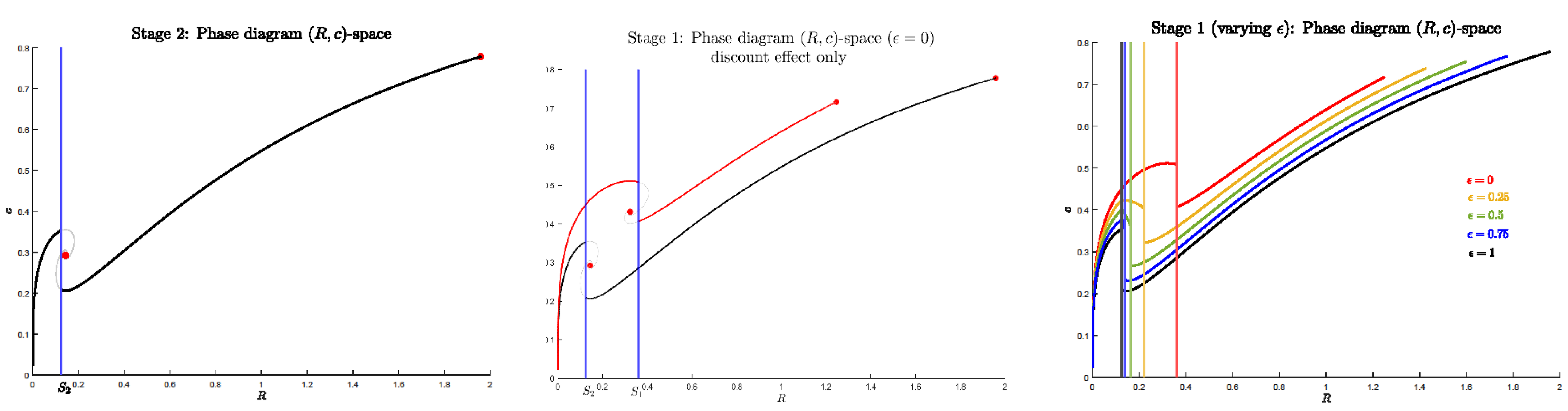
$R(\tau^+) = R(\tau^-) - D(\tau) = \epsilon R(\tau^-)$

Remaining resource stock following shock.

Optimal policies in (R, C)-space:

- **Panel 1:** Equilibrium structure (stage 2; and stage 1 for $\epsilon = 1$): stable/high (resilient) and unstable/low (non-resilient) equilibrium (red/dots), and a Skiba threshold (blue line).
- **Panel 2:** Stage-1 anticipation of a fully destructive shock ($\epsilon = 0$) shifts high equilibrium downward and low equilibrium and Skiba upward (red curve). Additional discounting compromises resilience.
- **Panel 3:** For $0 \le \epsilon \le 1$ intermediate outcomes with extraction policy turning more precautionary with increasing ϵ.

Panel 1 Panel 2 Panel 3



Resilience measure (adapted to this model):

$$\mathcal{R}(R(t),t) = \mathcal{R}_1(R(t),t) + \mathcal{R}_2(R(t),t)$$

- **Ex-ante resilience** (averting the shock)

$$\mathcal{R}_1(R(t),t) = \frac{\mathcal{L}(t)}{\mathcal{L}(t)+1}\mathbb{I}_{R(t)\ge R_1^S}$$

where $\mathcal{L}(t) = \eta^{-1}$= life-expectancy in stage 1 and where $\mathbb{I}_{R(t)\ge R_1^S}$ indicates long-run sustained resource use if and only if the resource level exceeds the Skiba-threshold R_1^S.

- Resilient: $\mathbb{I}_{R(t)\ge R_1^S} = 1$; Non-resilient: $\mathbb{I}_{R(t)< R_1^S} = 0$.
- **Ex-post resilience** (adapting to the shock)

$$\mathcal{R}_2(R(t),t) = \frac{1}{\mathcal{L}(t)+1}\int_t^\infty e^{-\eta s}\eta\mathcal{R}(s)ds$$

measures resilience for future shocks at $s \in [t, \infty[$

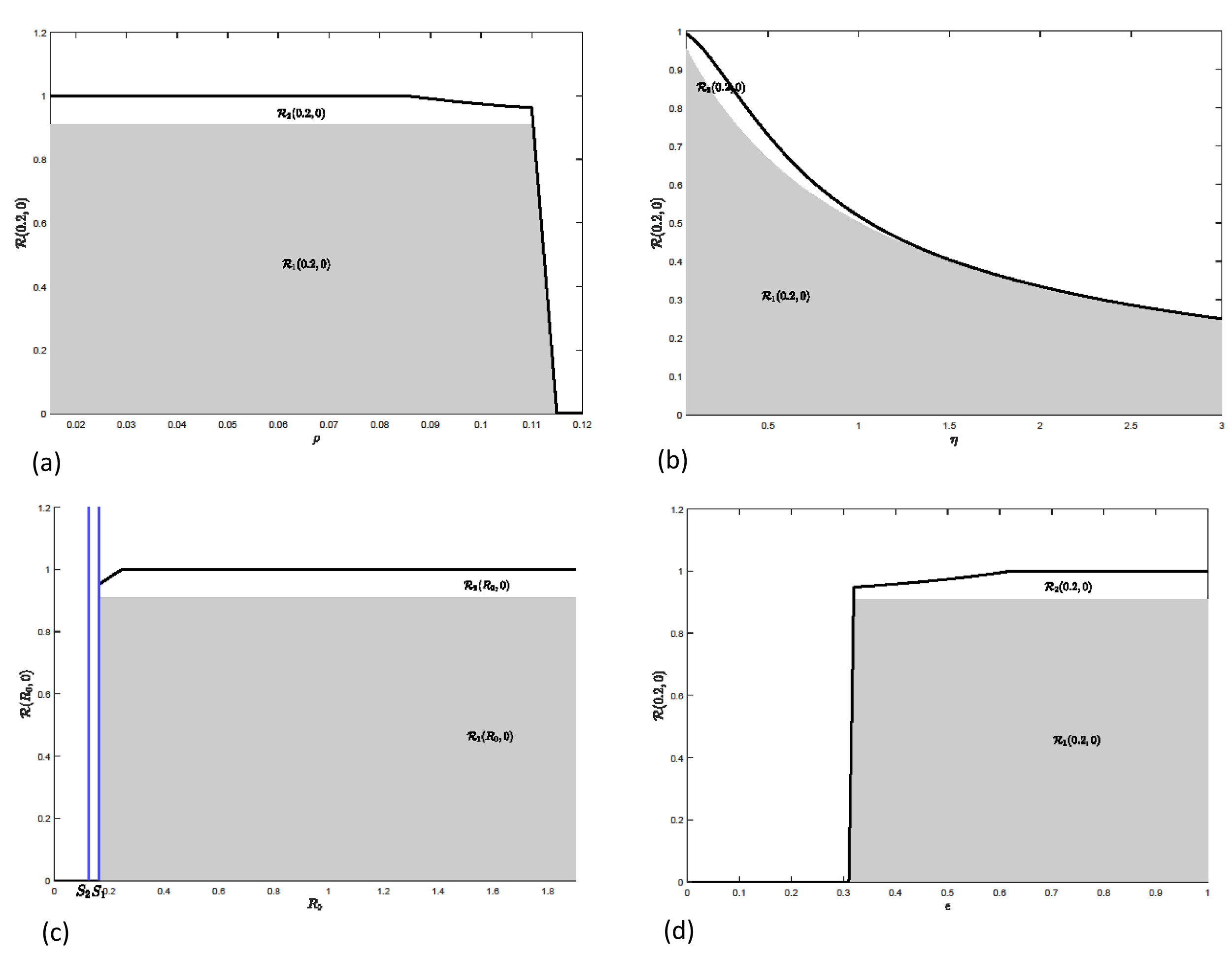
- **Value range:** $\mathcal{R}_i(R(t),t) \in [0,1]$

polar values: 1… full resilience

0… no resilience

Resilience of optimal policy:

- Benchmark scenario: $R_0 = 0.2$; $\rho = 0.1$; $\eta = 0.5$; $\epsilon = 0.5$
- Resilience diminishes in (a) discount rate ρ; (b) arrival rate of unavoidable (!) shock η (note that this extends to stage 2 due to reduction in precaution);
- Resilience increases in (c) initial resource stock $R(0)$ and (d) share of surviving resource stock ϵ



Work in Progress. For further details and updates contact: kuhn@iiasa.ac.at or wrzaczek@iiasa.ac.at