RELIABILITY, RESILIENCY, ROBUSTNESS, AND VULNERABILITY CRITERIA FOR WATER RESOURCE SYSTEMS

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FOREWORD

Early in IIASA's history, Professors C.S. Holling (now IIASA's Director) and M.B. Fiering began introducing some new concepts into the ecological and water resource literature, all of which had to do with the inherent uncertainties and risks of such systems. Since their work here at IIASA in the early 1970s, terms such as stability, safe-fail (as opposed to fail-safe), surprise, and those used in the titles of the following two articles are becoming increasingly discussed and used. These two articles continue that discussion. They also reflect the continuing interest at IIASA in the issues involving risk and uncertainty in water resource system design and operation.

The two articles reprinted here were among seven appearing in the same issue of *Water Resources Research*, which the editor grouped into a section he termed "Risk and Uncertainty in Water Resources Management". In his introduction to this section he wrote the following:

> We have, due to some excellent research and writing and fortunate timing, an extremely interesting collection of seven papers, grouped under the title of 'Risk and Uncertainty in Water Resources Management.' Although the title could be used to characterize much of the work reported in this journal over the last 17 years, it is nonetheless an accurate description of the seven papers collected here; these papers offer fresh and exciting ideas on a topic of traditional interest...
>
> Hashimoto, Stedinger, and Loucks have offered measures of system performance that seem to be capable of offering new insights to water managers faced with the ubiquitous problem of solving tomorrow's problems today. The notions of robustness, reliability, resiliency and vulnerability are quantitative (though not unique) indicators of the value of today's decisions in an uncertain future. And what should be of considerable interest to some researchers and to most practitioners are the conflicts among these criteria. A robust decision leads to a water system or policy that performs reasonably well in many situations, but that system or policy may lead to disastrous results in other situations (vulnerability) and take an unacceptably long time to recover after things do go wrong (lacking in resiliency).

These articles also illustrate the type of collaboration that often takes place among those here at IIASA and research institutions in other parts of the world.

JANUSZ KINDLER
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Resources & Environment Area

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Reliability, Resiliency, and Vulnerability Criteria For Water Resource System Performance Evaluation

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Three criteria for evaluating the possible performance of water resource systems are discussed. These measures describe how likely a system is to fail (reliability), how quickly it recovers from failure (resiliency), and how severe the consequences of failure may be (vulnerability). These criteria can be used to assist in the evaluation and selection of alternative design and operating policies for a wide variety of water resource projects. The performance of a water supply reservoir with a variety of operating policies illustrates their use.

Introduction

The ability of existing and proposed water resource systems to operate satisfactorily under the wide range of possible future demands and hydrologic conditions is an important system characteristic. The likely performance of water resource systems is often described by the mean and variance of benefits, pollutant concentrations, or some operating variable. This paper develops additional performance criteria that capture particular aspects of possible system performance which are especially important during periods of drought, peak demands, or extreme weather. The proposed criteria are called reliability, resiliency, and vulnerability. These performance measures should be useful in the selection of water resource system capacities, configurations, operating policies, and targets.

Bayesian methods are one natural and rigorous way of dealing with the uncertainty which arises in many planning studies. *Davis et al.* [1972] and *Benjamin and Cornell* [1970] review the basic methodology. When Bayesian analysis is combined with multiattribute utility theory [*Keeney and Raiffa*, 1976], the analysis can incorporate the variability in system performance and uncertainty in planning parameters with a single decision maker's attitudes toward risk. Examples of the use of multiattribute utility theory in water resources planning are given by *Keeney and Wood* [1977], *Goicoechea et al.* [1979] and *Krzysztofowicz and Duckstein* [1979].

Unfortunately, there are several drawbacks to this methodology. In particular the method requires the development of a utility function which incorporates a decision maker's or society's tradeoffs between competing system attributes and also their attitudes toward risk. Not only is such a function very difficult to construct for a single identified 'decision maker,' but such a function will probably not reflect the priorities of all groups having significant influence on the public decision-making process [*Loucks et al.*, 1981, pp. 137–138]. *Starr and Whipple* [1980] discuss the differences in risk preferences exhibited by society and by individuals.

The multiobjective multiple-decision–maker character of public decisions is widely recognized, and multiobjective planning algorithms have been developed [*Cohon*, 1978]. The value of a multiobjective framework in water resources planning is that the benefit and disbenefit bundle associated with alternative projects and proposals can be better identified. As a result, the public as well as different participating public agencies and interest groups can better evaluate proposed projects using their own unarticulated objectives.

Advocated here is the inclusion of special risk-related system performance criteria within the multiobjective analysis of alternatives. By adding these performance measures to those already used to describe the expected costs and benefits of projects, individuals and groups should be better able to understand how a project might perform in the uncertain future. If they better understand how water resource systems may operate and how unpleasant any periods of unsatisfactory performance may be, individuals will be prepared to make better decisions.

Of interest are system performance criteria which are suitable for characterizing the stochastic and dynamic performance of such water resource systems as wastewater treatment plants, multireservoir water supply systems, or flood-flow forecasting and control systems. Some recent work on the properties of ecological systems is relevant to this problem.

Holling [1973] used the concept of resilience to describe the ability of a dynamic multispecies ecological system to persist with the same basic structure when subjected to stress. Resilience is to be contrasted with stability, which pertains to the variability of species densities over time. Holling points out that some systems may appear to be unstable because population densities vary over wide ranges. However, such systems may be very resilient, for they can persist after severe shocks or during periods of stress because of their capacity to accommodate variability in individual species densities. Very stable systems may not be able to cope with large variations in population densities.

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They may disintegrate if they suffer large losses due to fire or disease, the introduction of a new pollutant, or a radically new management strategy.

Later work has extended this idea to environmental/ecosystem management [*Fiering and Holling*, 1974; *Holling*, 1978]. These authors question the wisdom of management strategies which force natural systems to be highly stable. Enforcing stability may result in changes in the structure of managed systems which could greatly reduce their resilience. For example, enhancement of salmon spawning should lead to more productive fisheries and, as a result, greater fishing pressure. However, this greater pressure is very likely to cause the less productive stocks to become extinct or nearly so. This would leave the fishing ecosystem precariously dependent on a few artificially enhanced species [*Larkin*, 1979].

Several individuals have applied similar ideas to water and land related resource systems management. *Haimes and Hall* [1977] introduce several criteria for describing the characteristics of system models and planning situations. *Fiering* [1976, 1977] has developed measures of resilience which can be useful in water resource planning. *Hashimoto* [1980*a*,*b*] and *Hashimoto et al.*, [this issue] have advanced the idea of system robustness, in which robustness describes the possible deviation between the actual costs of a proposed project and those of the least cost project design.

MEASURES OF SYSTEM PERFORMANCE

In many studies the operational status of a water resource system can be described as either satisfactory or unsatisfactory. The occurrence of unsatisfactory performance will be described in this paper as a failure. A failure could correspond to the actual structural failure of a dam from a catastrophic flood event or an earthquake [*Mark and Stuart-Alexander*, 1977]. The modes of failure of concern here are less severe and more common. A failure may be a 50-year or 200-year flood event which may cause extensive but not catastrophic flooding, moderate and severe droughts which make it impossible for reservoir systems to meet contractual obligations, or unexpected peaks in demand which tax water supply and wastewater treatment systems.

A number of indicators can be used to describe the possible performance of water resource systems. Simple and frequently used measures of system performance are the mean and variance of system outputs and performance indices. While the mean and variance of such quantities as project net benefits or DO concentrations in rivers are useful statistics, they are often not sufficient. In particular, the mean and variance describe the average level and average squared deviation from the mean of the parameters in question. These statistics provide a very vague description of just how poorly a system might behave in the infrequent situation when a failure does occur. The DO concentration in a river or the BOD removal rate in a wastewater treatment plant may be satisfactory 360 days a year. However, our primary concern may be the 5 days when things go wrong and aquatic communities might be seriously degraded (at least temporarily). For example, our attention should not be focused exclusively on the 10-year, 7-day low flow as things can be worse in critical parts of the river with the minimum 1-year, 7-day low flow due to the increased flow rates (*Loucks et al.* [1981], pp. 527–528, provide an example).

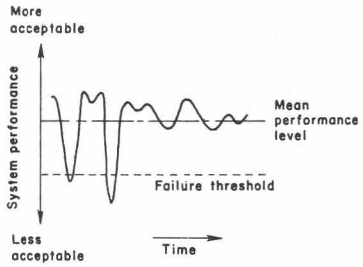

Fig. 1. Variable system performance with infrequent failures.

Figures 1 and 2 illustrate the inability of the mean and variance by themselves to define how severe and how frequent periods of poor performance may be. The figures contain a time history of the performance of two possible systems. The mean and variance of the performance parameter is the same in both cases over the time period shown. In fact, the curves are mirror images across their mean level. However, the performance history in Figure 1 displays two periods where performance clearly fell below the performance standard. This is never the case for the performance history in Figure 2.

When summarizing the values of performance parameters by their mean and variance, it is also difficult to determine if an improvement in the mean accompanied by an increase in the variance is an overall improvement. Theory addressing the relative tradeoff between the mean and variance of risky investments is well developed for small risks [*Pratt*, 1964]. However, if performance is highly variable or if the consequences of poor performance are severe, then it is appropriate and desirable to employ risk descriptors which (unlike the mean and variance of a parameter) describe in clear and meaningful terms what the character of failures might be.

Our analysis of system performance focuses on system failure, defined as any output value in violation of a performance threshold (such as a performance standard or a contractual obligation). System performance can be described from three different viewpoints: (1) how often the system fails (reliability), (2) how quickly the system returns to a satisfactory state once a failure has occurred (resiliency), and (3) how significant the likely consequences of failure may be (vulnerability). Descriptive as well as mathematical definitions of these criteria follow.

The definitions of these criteria are formulated assuming that the performance of the water resource system in ques-

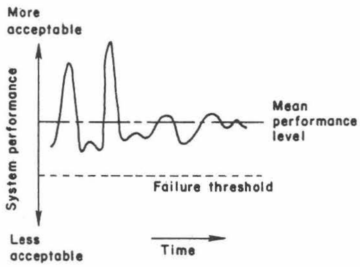

Fig. 2. Variable system peformance without failures.

tion can be described by a stationary stochastic process. That is, the probability distributions that describe the output time series do not change with time. Of course this is only an approximation of reality but it is often quite reasonable. For instance, the probability distribution of streamflows at a particular site may change over time due to climatic shifts or land use changes in the drainage area. Still, it is both convenient and satisfactory in many cases to assume that streamflows are a stationary process over typical planning horizons.

Reliability

Denote a system's output state or status by the random variable X_t at time t, where t takes on discrete values 1, 2, 3, $\cdots$. In general, the possible values of X_t can be partitioned into two sets: S, the set of all satisfactory outputs, and F, the set of all unsatisfactory (failure) outputs. At any time t the system output is assumed to be an element of one of these sets. The reliability of a system can be described by the frequency or probability α that a system is in a satisfactory state:

$$\alpha = \text{Prob}\,[X_t \in S] \tag{1}$$

An alternate definition of reliability not adopted here is that reliability is the probability that no failure occurs within a fixed period of time, often taken to be the planning period. If the planning period is a single period, then the two definitions are equivalent.

Reliability is a widely used concept in water resources planning. Reliability is sometimes taken to be the opposite of risk. That is, the risk or probability of failure is simply one minus the reliability α. Both reliability and this definition of risk do not describe the severity or likely consequences of a failure. The possible severity of failures can be described by other criteria, such as resiliency and vulnerability.

Resiliency

Resiliency will describe how quickly a system is likely to recover or bounce back from failure once failure has occurred. If failures are prolonged events and system recovery is slow, this may have serious implications for system design. One would like to design systems which can recover and return to a satisfactory state rapidly.

Resiliency may be given a mathematically precise definition. Let T_F be the length of time a system's output remains unsatisfactory after a failure. The resiliency of a system can be defined as the inverse of the expected value of T_F. To derive a mathematical expression for that expected value, let

$$Z_t = 1 \qquad X_t \in S$$
$$Z_t = 0 \qquad X_t \in F$$

Then $(1/n)\,\Sigma_{t=1}{}^{n}\,Z_t$ is the fraction of time from period $t = 1$ to $t = n$ that the system output or performance is satisfactory. In the long run this fraction approaches the probability of the performance being satisfactory, and hence equals system reliability:

$$\lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} Z_t = \alpha \tag{2}$$

Let W_t indicate a transition from a satisfactory to an unsatisfactory state:

$$W_t = 1 \qquad X_t \in S \qquad X_{t+1} \in F$$
$$W_t = 0 \qquad \text{otherwise}$$

In the long run the mean value of W_t will equal the probability ρ of the system being in the set S in some period t and going to the set F in the following period:

$$\rho = \text{Prob}\,\{X_t \in S,\, X_{t+1} \in F\} = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} W_t \tag{3}$$

The average sojourn time in the unsatisfactory or failure states during an n-period experiment is:

$$\bar{T}_F = \frac{A}{B} \tag{4}$$

where A is the total time in F and B is the number of times the process went into F. Hence

$$\bar{T}_F = \frac{1}{n} \sum_{t=1}^{n} (1 - Z_t) \left(\frac{1}{n} \sum_{t=1}^{n} W_t \right)^{-1} \tag{5}$$

As n approaches infinity, the average sojourn time $\bar{T}_F$ will approach its mean value $(1 - \alpha)/\rho$. Thus the expected length of time that the system's output or performance remains unsatisfactory once it becomes unsatisfactory equals

$$E[T_F] = \frac{1 - \alpha}{\rho} \tag{6}$$

This defines the average number of time periods a failure is expected to last once it has occurred. The inverse of this is the system's average recovery rate and is our measure of resiliency:

$$\gamma = \frac{\rho}{1 - \alpha} = \frac{\text{Prob}\,\{X_t \in S \text{ and } X_{t+1} \in F\}}{\text{Prob}\,\{X_t \in F\}} \tag{7}$$

In the long run, the number of transitions from satisfactory states in S to unsatisfactory states in F must equal the number of transitions in the reverse direction:

$$\text{Prob}\,\{X_t \in S \text{ and } X_{t+1} \in F\} = \text{Prob}\,\{X_t \in F \text{ and } X_{t+1} \in S\} \tag{8}$$

Hence γ is equivalent to the average probability of a recovery from the failure set in a single time step:

$$\gamma = \frac{\text{Prob}\,\{X_t \in F \text{ and } X_{t+1} \in S\}}{\text{Prob}\,\{X_t \in F\}}$$
$$= \text{Prob}\,\{X_{t+1} \in S \mid X_t \in F\} \tag{9}$$

Note that if the occurrence of a failure $X_t \in F$ and a subsequent success $X_{t+1} \in S$ are probabilistically independent events, then γ would reduce to Prob $\{X_{t+1} \in S\}$, which is our measure of reliability.

Vulnerability

Here vulnerability refers to the likely magnitude of a failure, if one occurs. Even when the probability of failure is small, attention should be paid to the possible consequences of failure. *Holling* [1978] discusses the idea of safe-fail as opposed to fail-safe. Attempts to maximize system reliability are attempts to make a system's operation failure-free. Still, few systems can be made so large or so redundant that failures are impossible. Even when it is possible to raise levees high enough or make water supply reservoirs large enough that failure is hard to imagine, it is often not economical to do so. After a point, effort is better expended making the consequences of failure less severe and more acceptable than in trying to eliminate the possibility of failure altogether. Early warning systems, flood insurance, and flood-proofing of structures are three approaches to decreasing the costs of flooding when floods do occur. Likewise, the exclusion of buildings from floodways and the use of flood-prone areas for parks, natural areas, and agriculture are other means of minimizing the costs of floods.

It is important to realize that efforts to maximize system efficiency and reliability can actually increase a system's vulnerability to costly failure should failure occur. Transformation of traditional agricultural systems to high yield single-species crops sets the stage for disaster should a new crop disease or pest develop. Likewise, flood control reservoirs and levees that control small floods create an image and sense of security; as a result, unwise development in partially protected areas can occur. This creates the potential for large losses should a large flood occur or a levee break. Replacement of small unreliable wastewater treatment plants by large well-managed regional facilities may decrease the frequency of plant failures, yet by concentrating the total treated wastewater flow in a single location, the impact and consequences of a breakdown in the biological oxidation process will be greatly magnified should the plant be overloaded or receive a slug of concentrated or toxic material [*Adams and Gemmell,* 1980].

The loss of a rear cargo door on the DC-10 aircraft due to improper latching provides an excellent illustration of fail-safe versus safe-fail design. The blow out of the cargo door at high altitudes causes a rapid decompression of the cabin and the severing of control cables by the collapse of the floor separating the cabin and lower storage area. Commercial airlines emphasized design modifications and safety procedures to prevent such mishaps. Unfortunately, a failure occurred and many died. In some military aircraft, holes were cut in the floor separating the two compartments, allowing rapid decompression of the cabin should the cargo door be lost. This prevented structural damage to the aircraft and made the planes 'safe in failure.'

It is important that decision makers be aware of the vulnerability of a system to severe failure should a failure occur. This should be an important criterion in water resource system design and selection. To construct a mathematical index of system vulnerability, assume that the system performance variable X_t can take discrete values $x_1, \cdots, x_n$. To construct a quantitative indicator of system vulnerability to severe failure should a failure occur, assign to each discrete failure state $x_j \in F$ a numerical indicator of the severity of that state, denoted s_j. Furthermore, let e_j be the probability that x_j, corresponding to s_j, is the most

TABLE 1. Characteristics of River Flows

	Winter	Summer	Annual
Mean flows, $\times 10^7$ m^3	4.0	2.5	6.5
Standard deviation, $\times 10^7$ m^3	1.5	1.0	2.3

Correlation of flows: winter with following summer, 0.65; summer with following winter, 0.60.

unsatisfactory and severe outcome that occurs in a sojourn into the set of unsatisfactory states F. Then e_j equals Prob $\{x_j$, corresponding to s_j, is the most severe outcome in a sojourn in $F\}$. One reasonable metric for overall system vulnerability would be the expected maximum severity of a sojourn into the set of unsatisfactory states:

$$\nu = \sum_{j \in F} s_j e_j \tag{10}$$

Here emphasis is placed not on how long failure persists (the inverse of resiliency) but on how bad things may become.

RELIABILITY, RESILIENCY, AND VULNERABILITY OF A WATER SUPPLY RESERVOIR

Use of the reliability, resiliency, and vulnerability concepts is illustrated with a reservoir operation problem. For a reservoir of given capacity the reservoir operating policy determines the reliability, resiliency, and vulnerability of a water supply system. *Kitson* [1979] emphasized the need in reservoir operating policy development to consider reductions, during drought periods, in the amount of water available. He stated that this need leads to 'the concept of expressing reliability in terms of the frequency, duration and intensity with which restrictions have to be placed on water consumption.' *Velikanov* [1979], referring to irrigation water use, pointed to the necessity of evaluating in probabilistic terms system performance under conditions of both excessive and deficient water availability.

The reservoir operation example presented by *Loucks et al.* [1981, pp. 138–152] is used here to illustrate the use of risk-related system performance criteria. In that example a small reservoir with capacity 4×10^7 m^3 was to provide 4.5×10^7 m^3 of water to meet summer irrigation needs. The logarithms of the inflows to the reservoir were modeled with a Thomas-Fiering model which reproduced the mean and variance of flows in each of two seasons and the season to

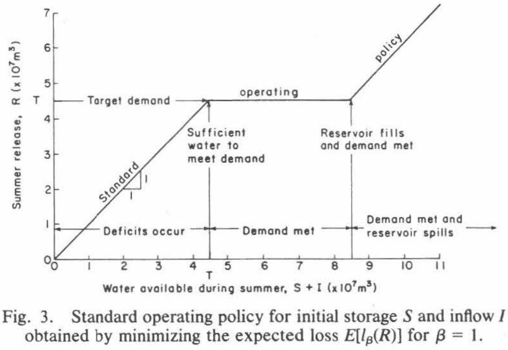

Fig. 3. Standard operating policy for initial storage S and inflow I obtained by minimizing the expected loss $E[l_\beta(R)]$ for $\beta = 1$.

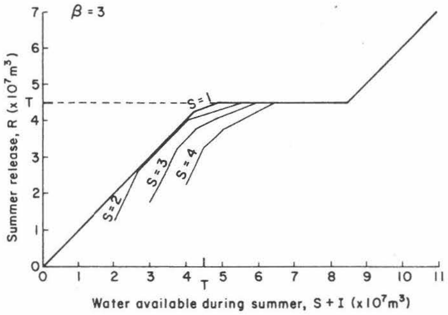

Fig. 4. Optimal summer release policy for $\beta = 3$. The lines show best value of release R as a function of initial storage S plus inflow I for specified value of S and release target T.

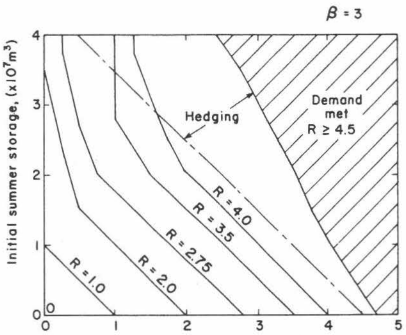

Fig. 5. Optimal reservoir summer release for $\beta = 3$ as a function of initial storage and total summer inflow.

season correlation of the flows (*Loucks et al.*, 1981, pp. 141, 283–284, 305–307). The values of the statistics describing the relevant hydrology are given in Table 1. It was also necessary to release 0.50×10^7 m^3 of water during the wet season to satisfy minimum flow requirements.

The steady state operation of this simple system was simulated with a range of summer season operating policies. The winter operating policy was always to release 0.50×10^7 m^3 of water if possible and to store as much of the excess water as the reservoir could hold. The summer season operating policies were derived by stochastic dynamic programing [e.g., *Loucks et al.*, 1981, pp., 324–331] with the objective of minimizing the expected or average long-run loss:

$$E[l_\beta(R)] \tag{11}$$

where

T target release of 4.5×10^7 m^3;
R summer season release;
$l_\beta(R) = 0$, when $R \geq T$;
$l_\beta(R) = [(T - R)/T]^\beta$, when $R < T$.

The exponent β defines the shape of the loss function $l_\beta(R)$. A range of β values between 0 and 7 were considered to provide a range of policies. In the optimization, inflows, and storage volumes in each season were discretized in units of 0.25×10^7 m^3. Optimal policies were a function of initial summer storage and the actual summer period inflow.

Note that the parameter β is an artificial device introduced to facilitate the generation of operating policies which reflect different tradeoffs between shortfall magnitudes and failure frequency and hence different tradeoffs among reliability, resiliency, and vulnerability.

For $\beta = 1$, one obtains the 'standard' operating policy shown in Figure 3. In the figure, I denotes the summer inflow. The standard policy meets as much of the demand target as possible.

For $\beta > 1$, operating policies exhibit 'hedging': they sometimes provide only a portion of the target release, when in fact all or at least more of the target volume could be provided. (*Klemeš* [1977] and *Stedinger* [1978] discuss this phenomena.) This saves water to protect against future deficits which could be even larger. This is illustrated by the policy in Figures 4 and 5, obtained with $\beta = 3$.

In Figure 4, several operating curves are discontinuous because they are defined over only a portion of the initial storage plus inflow ($S + I$) axis. For example, if the initial summer storage is $S = 3$, then the only legitimate values of $S + I$ are those greater than or equal to 3. As the two figures show, the optimal policy for $\beta = 3$ can result in large and unnecessary deficits when the current summer inflow is below normal levels. To incur such deficits is optimal for the specified loss function, for it minimizes the expected value of immediate and possible future losses which could occur if streamflows remain below normal.

For $\beta < 1$, a very different operating policy behavior results. In this case the marginal disutility of deficits is a decreasing function of the total deficit. As a result, optimal policies always meet the entire target if this is possible but sometimes fail to release any water at all when a modest failure is already unavoidable. Such a policy for $\beta = 0.50$ is displayed in Figures 6 and 7.

In the limit as β approaches zero, the loss function becomes

$$l_0(R) = 0 \qquad R \geq T$$

$$l_0(R) = 1 \qquad R < T$$

In this instance the optimal policy is to meet the summer release target $T = 4.5 \times 10^7$ m^3 if possible and to deliver as

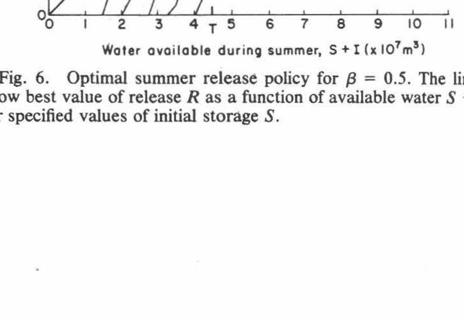

Fig. 6. Optimal summer release policy for $\beta = 0.5$. The lines show best value of release R as a function of available water $S + I$ for specified values of initial storage S.

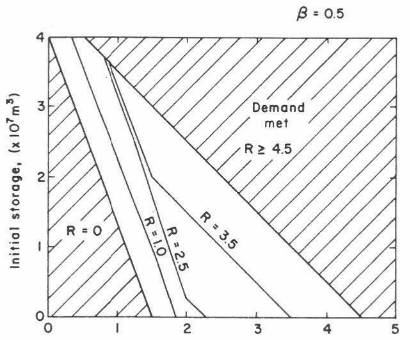

Fig. 7. Optimal reservoir summer release for $\beta = 0.5$ as a function of initial storage and total summer inflow.

little water as one can if a failure cannot be avoided. This maximizes system reliability by saving water to avoid possible future failures when a failure in the current period is already unavoidable.

With each policy the reservoir-irrigation system was simulated for 10,000 years to determine (1) the reliability α with which the summer irrigation target was met, (2) the resiliency γ of the system equal to the reciprocal of the average length of sequences of failure years, and (3) the vulnerablity ν of the system equal to the average of the maximum deficit that occurred in each sequence of failure years. A failure year occurred whenever the summer release R was less than the target release T, equal to 4.5×10^7 m^3.

Figure 8 illustrates the values of system reliability α, resiliency γ, and vulnerability ν as a function of β, the exponent in the loss function used to derive the various operating policies. As β increases, the penalty on large deficits becomes increasingly severe. As a result, as β increases, system reliability α decreases because the optimal policies incorporate a propensity to incur small deficits so as to minimize the expected loss from larger deficits at later times.

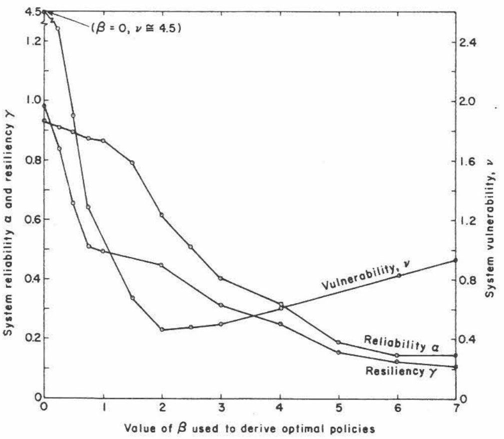

Fig. 8. System reliability, resiliency, and vulnerability as a function of parameter β used to derive operating policies.

TABLE 2. Reliability and Expected Losses Achieved With Operating Policies Derived with Different Values of β.

β Used to Derive Policy	Reliability of System Operation, α	Expected Value of Three Loss Functions		
		$E[l_1(R)]$	$E[l_2(R)]$	$E[l_5(R)]$
0	0.93	6.6	6.5	6.1
0.25	0.91	4.2	2.7	1.47
0.50	0.89	3.2	2.0	0.98
0.75	0.87	2.6	1.04	0.31
1.00	0.87	2.5*	0.76	0.062
1.50	0.79	2.6	0.70	0.051
2.00	0.62	3.5	0.67*	0.040
3.00	0.41	5.3	0.79	0.027
5.00	0.19	9.1	1.37	0.022*
7.00	0.15	12.4	2.2	0.029

*Note that minimum value of $E[l_k(R)]$ is achieved at $\beta = k$ because the policy derived with given β by construction minimizes $E[l_\beta(R)]$.

Resiliency generally shows the same trend as reliability. For $\beta = 0$, system resiliency is high and sequences of failure years are very short. Deficits are very severe, often equaling the entire target. For $\beta \geq 3$, resiliency is low because periods of failure can be very long, although deficits are often small.

The vulnerability trend is different from that obtained with the other risk-related performance criteria. It achieves its maximum at $\beta = 0$ when almost every failure is a complete failure. It then decreases with increasing β to achieve a minimum at $\beta = 2$. Above $\beta = 2$, vulnerability actually increases with increasing β. This occurs because operating policies derived with large β will frequently incur deficits much larger than is necessary. This saves water as a hedge against the possibility of even larger deficits in future periods. This tradeoff (for $\beta > 2$) decreases the reliability and resiliency as well as the vulnerability of the system's performance. Still, it is optimal with respect to each policy's loss function. This is shown by Table 2, which reports the value of the expected loss function $E[l_\beta(R)]$ for $\beta = 1$, 2, and 5.

The values of reliability, resiliency, and vulnerability in Figure 8 reveal some of the characteristics of reservoir system performance that can be obtained with reservoir policies that minimize the specified loss functions. Realistic policies probably correspond to β in the range of 1.0–2.0 and hence would have high reliability, modest resiliency, and close to minimal vulnerability. Figure 9 provides a more explicit description of the unavoidable tradeoff between vulnerability and reliability. One cannot have both the maximum possible reliability and minimum possible vulnerability.

CONCLUSIONS

In general, there exist tradeoffs among expected benefits, reliability, resiliency, and vulnerability. Use of the three risk criteria improves our ability to describe how often failures may occur, how long periods of unsatisfactory performance are likely to last, and just how severe failure might be. This was illustrated with a water supply reservoir example. There, high system reliability was accompanied by high system vulnerability. This information should be used to supplement other standard project evaluation criteria, including the distribution of project benefits and costs as well as various social and environmental impacts. By using

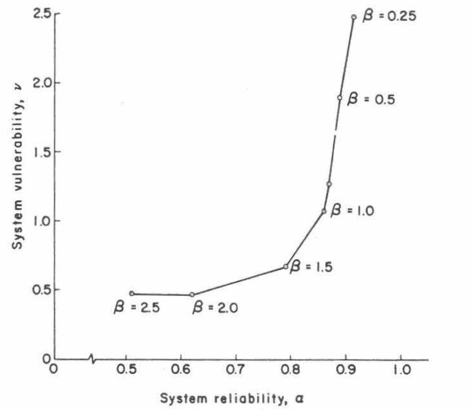

Fig. 9. Tradeoff between system reliability and vulnerability for β between 0.25 and 2.5.

improved descriptions of the possible nature of poor system performance, should it occur, individuals should be able to better understand the risks to which they are exposed by various project and no-project alternatives.

The particular mathematical definitions advanced here for resiliency and vulnerability should be viewed as illustrative examples. Every planning situation is in some way unique and calls for creativity in the definition of appropriate performance descriptors, such as resiliency, reliability, and vulnerability. It is unlikely that a single mathematical defintion of these concepts will be appropriate or useful in all situations. However, recognition and description of the possibility of low-probability but undesirable consequences of alternative plans should be an important component of the planning process. Hence engineers and planners need to develop appropriate quantitative risk criteria that describe the undesirable events that individuals may experience as a consequence of particular investment or operating policy decisions.

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Robustness of Water Resources Systems

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When water resource systems investments are made there is little assurance that the predicted performance will coincide with the actual performance. Robustness is proposed as a measure of the likelihood that the actual cost of a proposed project will not exceed some fraction of the minimum possible cost of a system designed for the actual conditions that occur in the future. The robustness criterion is illustrated by its application to the planning of water supply systems in southwestern Sweden.

INTRODUCTION

Risk and uncertainty are characteristic of most planning situations. Water resource investment planning is no exception. Water resource projects often are large and expensive and require long lead times. Once the facilities are in place they are often operated for decades. Dams, pipelines, water and waste treatment facilities, canals, hydroelectric power plants, and water and sewer distribution networks are examples of such expensive long-lived investment projects. The uncertainty as to the level of service these facilities will need to provide in 5, 10, 20, or 50 years from when they are planned and implemented makes the project evaluation and selection process difficult.

It is impossible to forecast the actual demand that a particular investment project will serve in the future. However, some project designs and operating policies may be sufficiently flexible to permit their adaptation to a wide range of possible demand conditions at little additional cost. Such systems can be called robust. This definition of robustness corresponds to Stigler's concept of economic flexibility [*Stigler*, 1939; *Hashimoto*, 1980*b*].

Others have used the term robustness in water resources planning to describe whether or not the optimal project design parameter values would remain essentially unchanged if the future demand conditions were to vary from those for which the project is designed [*Fiering*, 1976; *Matalas and Fiering*, 1977]. However, optimal design parameter values can be very sensitive to assumed future demand conditions, and this may not involve large economic opportunity costs [*Loucks et al.*, 1981, pp. 122–129]. Thus it is appropriate to define system robustness in terms of the sensitivity of total system cost rather than the sensitivity of system design.

In this paper, robustness measures describe the overall economic performance of a water resource project. As such, they complement the more traditinal benefit cost and cost effectiveness criteria used for project selection. Other criteria designed to measure the dynamic system performance of projects are described in a companion paper [*Hashimoto et al.*, this issue].

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MEASURES OF ROBUSTNESS

Water resource project planning is based on forecasted or assumed future supplies, flows, qualities, costs, and benefits. It is also based on some assumed demand for the services the project is to provide. These assumed demand conditions, together with the environmental impacts and constraints that must be met, determine to a large extent the particular design, and hence the cost, of a project. In this paper all assumed future conditions that properly determine the actual motivation for and design of a project will be termed the 'demand conditions.'

Suppose a project is planned with a forecast of future demand conditions. If the forecast is not correct and another set of demand conditions actually occurs, the original project design may be inferior to another design better suited to the demand conditions that actually occurred. The difference between what the actual project costs and the costs that would be incurred with a cost effective design for the actual demand conditions is called the opportunity cost or regret. This is the cost of not having perfect information about the future.

Some projects may have the ability to adjust their final configuration or operating policies to the actual conditions as they evolve in the future, so that the opportunity cost of an original incorrect assumption about future demand conditions is reduced. Robustness measures should include the benefits and costs of such adjustments. If such modifications are cost effective for a reasonable range of future demand conditions, a project may be considered more desirable than one that is cost effective only for the most likely demand condition.

To define this concept more clearly, let D denote a particular design and q a future demand condition (e.g., wastewater flow, municipal water demand, low flow augmentation requirement, or level of flood protection desired). The function $C(q \mid D)$ will be the cost of accommodating the demand condition q with the project design D. This cost includes the amortized construction, operation and maintenance costs, and the costs of any measures that need to be

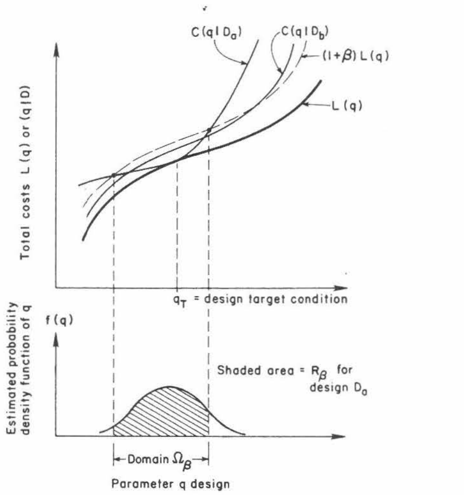

Fig. 1. For given β, robustness R_β of design D_a is the probability, given by shaded area in lower graph, that design D_a's cost $C(q \mid D_a)$ is not more than $(1 + \beta)L(q)$.

taken to satisfy the actual demand conditions with design D.

Of interest for any demand condition q is the minimum cost $L(q)$ of a design that can satisfy that assumed demand condition

$$L(q) = \min_{\text{all } D} C(q \mid D) \qquad (1)$$

For any demand condition q the opportunity cost of selecting design D is the difference between the actual cost $C(q \mid D)$ and the minimum cost $L(q)$ of a design that satisfies q.

When examining the merits of a particular design D, one might ask for what values of demand conditions q is the opportunity cost of D no greater than a fraction β of the minimum cost $L(q)$. If this set of q values includes all those values of q that could conceivably occur, then the cost of D will always be within $100\beta\%$ of the cost of the cost effective design no matter what the value of q. Thus attention is reasonably directed to those values of q for which

$$C(q \mid D) \le (1 + \beta)L(q) \qquad (2)$$

or

$$\frac{C(q \mid D) - L(q)}{L(q)} \equiv R(q \mid D) \le \beta \qquad (3)$$

for a given β and design D.

The opportunity cost ratio $R(q \mid D)$ defined in (3) is the opportunity cost or regret divided by the minimum cost. This ratio is a measure of the relative magnitude of the opportunity cost of design D. This ratio may be more meaningful to some than the opportunity cost itself.

It is likely, especially for relatively small values of β, that no system design alternative D will satisfy (2) or (3) for all conceivable future demand conditions q. This suggests that a probabilistic description of system robustness may be advantageous. Assume that one can assign probabilities to the likely future demand condition values of q. This defines the probability density function $f(q)$. Now possible system performance can be described, in part, by the expected opportunity cost.

$$E_q[C(q \mid D) - L(q)] = \int_{-\infty}^{+\infty} [C(q \mid D) - L(q)] f(q)\, dq \qquad (4)$$

or by the expected utility of system cost,

$$E_q[U(C(q \mid D))] = \int_{-\infty}^{+\infty} U(C(q \mid D)) f(q)\, dq \qquad (5)$$

[*Friedman and Savage*, 1948; *Raiffa*, 1968].

While utility theory provides an appropriate solution to the problem of design selection under risk or uncertainty, there are a number of reasons why its use has been limited in practice [*Loucks et al.*, 1981]. Use of expected opportunity costs or, equivalently, the use of expected costs, is reasonable and commonly done. However, these expected costs provide little insight into how confident one can be that a particular design D will be near or reasonably close to the least cost design. This need can be met by defining design robustness R_β as the likelihood or probability that (2) or (3) will be satisfied:

$$R_\beta = \text{Prob}\,[C(q \mid D) \le (1 + \beta)L(q)] \qquad (6)$$

Other measures of economic robustness have also been proposed [*Hashimoto*, 1980*b*].

The concept of robustness defined by (6) is illustrated in

TABLE 1. Cost of Each Design-Outcome Combination and Design Comparison Based Upon Cost

	Costs C_{ij} for Design D_j				Probability of Condition, P_i	Least Cost, L_i
	D_1	D_2	D_3	D_4		
Future demand condition						
q_1	60	90	110	75	0.10	60
q_2	55	30	35	50	0.20	30
q_3	50	30	20	35	0.50	20
q_4	55	35	35	25	0.20	25
Maximum cost*	60	90	110	75		
Expected cost†	53	37	35	40		
Variance of cost	11	316	675	200		

*Best design D_1.
†Best design D_3.

Figure 1 for a situation where q is a scalar quantity. Two alternative designs are considered, D_a and D_b. The alternative design D_a whose cost is represented by the cost function $C(q \mid D_a)$ is designed for a demand condition q_T. The design D_a may also result in the minimum cost at other demand conditions. However, design D_a incurs relatively large opportunity costs for demand conditions significantly different from q_T. An explicit consideration of robustness may result in the selection of an alternative design D_b which is robust at level β for a wider range of demand conditions, even though design D_b is not cost effective for any q.

The value of robustness R_β at the level β is simply the probability that the system's opportunity cost $C(q \mid D) - L(q)$ will not exceed β times the minimum total cost $L(q)$. It is the probability that the design parameter q will have a value within the domain Ω_β shown in Figure 1. In symbols,

$$R_\beta = \int_{\Omega_\beta} f(q)\, dq \qquad (7)$$

AN EXAMPLE

The usefulness of robustness measures can be illustrated by an example. Suppose that there are four design alternatives, D_j for $j = 1, \cdots, 4$, which have total costs C_{ij} for four possible future demand conditions q_i as shown in Table 1. Table 1 also gives the probabilities of each q_i and the cost L_i of the most cost effective alternative for each q_i. Alternative D_j is cost effective for future demand conditions q_i when $j = i$. Table 1 also reports the maximum cost that may be incurred with each design, the expected cost, and the variance of costs. These criteria can be used for decision-making [*Fabrycky and Thuesen,* 1980]. One can insure that costs do not exceed 60 by choice of design D_1 which has the minimum maximum cost. The table also shows that design D_3 achieves the minimum expected cost. However, D_2 has only a slightly higher expected cost while the variance of costs is much lower, so that a risk averse individual may very likely prefer D_2 to design D_3 [*Fabrycky and Thuesen,* 1980; *Pratt,* 1964]. Likewise, design D_4 has a larger expected cost than design D_2 but a smaller cost variance, so that one might prefer design D_4 over D_2.

Table 2 reports the regret $R_{ij} = C_{ij} - L_i$ associated with each design choice D_j and future demand condition q_i. Regret is another metric for comparing the cost effectiveness of competing design alternatives. In this particular example,

TABLE 2. Regret of Each Design-Outcome Combination and Design Comparison Based Upon Regret

	Regret R_{ij} for Design D_j				
	D_1	D_2	D_3	D_4	L_i
Future demand condition, q_i					
q_1	0	30	50	15	60
q_2	25	0	5	20	30
q_3	30	10	0	15	20
q_4	30	10	10	0	25
Maximum regret*	40	30	50	20	
Expected regret†	26	10	8	13	
Variance of regret	79	60	211	46	

*Best design D_4.
†Best design D_3.

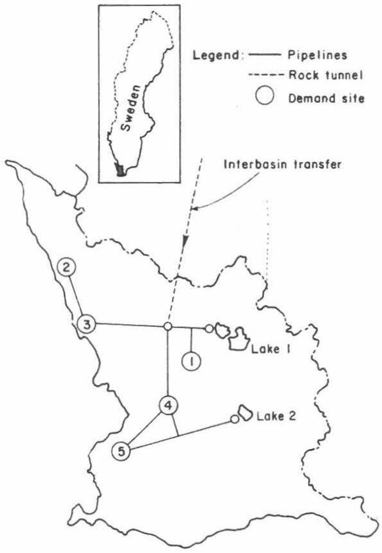

Fig. 2. Diagram of water supply system of southwestern Skane, Sweden, considered in this study.

design D_4 achieves the minimum maximum regret. Because design D_3 achieved the minimum expected cost, it also achieves the minimum expected regret [*Benjamin and Cornell,* 1970, pp. 585–586]. However, one may again want to trade off expected regret with the variance of regret reflecting a desire not to select a design whose performance will be too far from that of the most cost effective design. Hence design D_3 may be inferior to D_2, which in turn may be inferior to D_4.

Minimizing the maximum cost or regret, or minimizing the expected value of either project cost or regret, are all reasonable criteria for project selection. However, each has its drawbacks. The min/max criterion focuses only the worst possible outcome that can result from each design selection, regardless of the probability or likelihood of that event. The expected value criterion looks only at the averge return and ignores risk aversion. When supplemented with a measure of dispersion such as the variance of costs, the approach is improved, but one often does not know how to trade off increases in expected costs for decreased cost variance: design D_3 versus D_2 and D_2 versus D_4. Even then, as *Hashimoto et al.* [this issue] show, these two statistics need not be an adequate summary of the entire distribution of possible outcomes.

Table 3 reports the values of the R_β robustness criterion for several reasonable values of β. Suppose that one is concerned about design decisions within $\beta = 20\%$ of the cost effective alternative because one's cost estimates have that level of imprecision or because the public and other interested parties will be relatively unconcerned with such modest inefficiencies. Then design D_3 is very attractive because it has a 70% probability of achieving that level of cost efficiency. To use a less stringent standard, one could consider future demand conditions that result in opportunity costs in excess of 50% of the least cost design. Then design D_2 is

TABLE 3. Design Selection Based on System Robustness

Robustness Level, β	Design, D_j				Most Robust Design at Level β
	D_1	D_2	D_3	D_4	
20%	0.10	0.20	0.70	0.20	D_3
50%	0.10	1.00	0.90	0.30	D_2
70%	0.10	1.00	0.90	0.50	D_2
100%	0.30	1.00	1.00	1.00	D_2, D_3, D_4
200%	1.00	1.00	1.00	1.00	Indifferent

most attractive, for it appears to have a 100% probability ($R_{0.50} = 1.00$) of achieving this level of cost efficiency; design D_3 is a close second with an $R_{0.50}$ value of 0.90. Use of the R_β robustness criterion indicates that designs D_1 and D_4 are relatively unattractive, even though they achieve the minimum-maximum cost and minimum-maximum regret, respectively.

APPLICATION OF ROBUSTNESS ANALYSIS TO A SWEDISH WATER SUPPLY SYSTEM

The measure of robustness defined above has been applied to a specific regional water supply system planning problem in southwestern Skane, Sweden (Figure 2). In this area a large-scale interbasin water transfer project was proposed to meet projected water demands. Since the projected demands were uncertain, it was not clear just when and to what extent both local source supply capacity and/or the interbasin transfer should be increased or implemented.

At the time that this decision was made (1970), two local lakes were satisfying a major portion of the water demand of five municipalities. In addition, groundwater served each municipality, but substantial expansion of these sources was not possible. To meet increasing demands, lake water withdrawals could be increased and water could be imported through a tunnel, to be built, from a distant lake.

The interbasin water transfer project does not fit well into a stagewise development planning framework because of its indivisibility. Either the tunnel would be built or it would not. In such a situation it is relevant to ask how long the implementation of this major development should be deferred in expectation of obtaining more information about future demand [*Hall et al.*, 1972]. Two results follow immediately from deferment: (1) The present discounted cost of the major development will decrease, first directly from the deferment and second, possibly from reduction in scale of the major development and (2) the cost of the interim development of local supplies will increase because it must provide for the larger demand expected by the time when the major development, i.e., the tunnel, is implemented. Conventional practice is to pick the deferment time that minimizes the total (present discounted) cost of meeting the future demand. The problem is that the demand is uncertain.

Because of the uncertainty in demand, a number of different decisions could be made, each assuming a particular demand projection up to the planning horizon, which was set at the year 2000. Figure 3 illustrates several possible demand trajectories as seen in 1970 and the resulting uncertainty as to when the major development project should be implemented. Of course, possible decisions are not confined to timing or sizing. In the case of low demand, excess water may be diverted to other uses which yield additional benefits, thus reducing the opportunity cost of overdesign. In the case of high demand the demand itself might be reduced using appropriate pricing policies [*Kindler et al.*, 1980].

The range of possible demand trajectories shown in Figure 3 was approximated by seven discrete projections which are characterized by the eventual demand in 1975, denoted

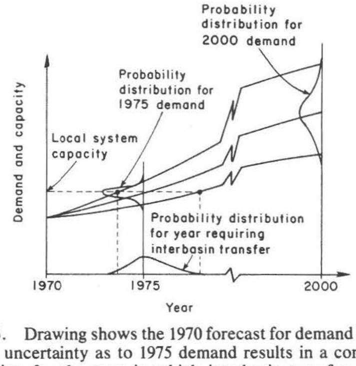

Fig. 3. Drawing shows the 1970 forecast for demand in 1975 and in 2000; uncertainty as to 1975 demand results in a corresponding distribution for the year in which interbasin transfer of water is required.

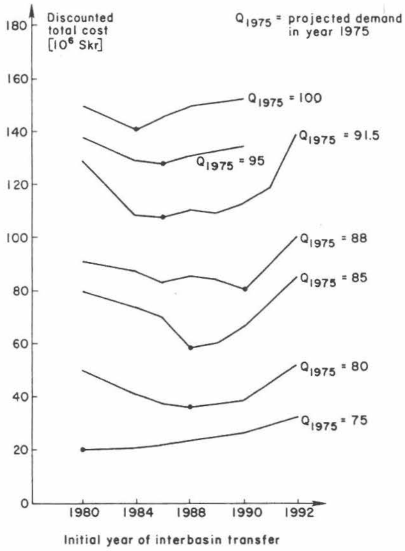

Fig. 4. Discounted total cost of alternative projects as a function of the time of the interbasin transfer under different demand projections for the year 1975.

Q_{1975}. Each discrete projection has associated with it a minimum cost decision, i.e., the extent of local source development and the timing and size of the tunnel project that are cost effective for that particular demand projection. If indeed a decision is made and the actual future demand is not what was assumed, the discounted total design and operating costs will be higher than expected. The cost functions are shown in Figure 4.

The minimum total costs for the various demand alternative designs define the minimum total cost curve in Figure 5. Also shown in Figure 5 are the cost functions of four alternative development plans, all designed to meet the forecasted demand in year 2000. Design D_2 is the cost effective plan for the expected value of future demand equivalent to 91.5 Mm^3/yr in 1975. If indeed the actual demand in year 2000 is as projected in 1970 and hence is equal to 91.5 Mm^3/yr in 1975, then the design D_2 will be the cost effective alternative. The total cost function for that alternative is tangent to the minimum cost function at a 1975 demand of 91.5 Mm^3/yr in Figure 5.

Values of robustness R_β can be computed for each alternative based on the cost functions shown in Figure 5. Once again, R_β is the probability that the project costs will be within $100\beta\%$ of the lowest possible cost of meeting the actual future demand. From Figure 5 one can estimate the R_β robustness values for the four alternative designs. Table 4 reports R_β values for three values of β.

To use the R_β robustness criterion effectively one must determine the β level at which the difference in cost between a particular design and the least cost design is relatively unimportant. It is certainly reasonable to expect that the error in future project construction and operating cost estimates may be ±15% of the actual costs; this suggests that β values of 0.20 or greater may be appropriate. Certainly the

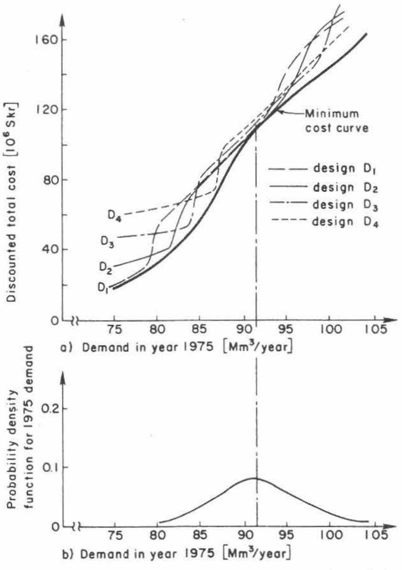

Fig. 5. (*a*) Discounted total cost functions for minimum-cost and alternative development plans and (*b*) estimated probability distribution of 1975 future demand.

TABLE 4. Robustness R_β of Four Alternative Design Implementation Years

Robustness Level β	Design D_1	D_2	D_3	D_4
0.10	0.50	0.50	0.60	0.60
0.20	0.80	0.85	0.80	0.60
0.50	0.95	0.98	0.90	0.80

public and public decision-makers would like to select the most cost effective design for the actual demand conditions that materialize. Unfortunately, this is not always possible given the uncertainty in future demand conditions. This being the case, one can at least discard designs that potentially may perform very poorly. In this instance, β defines a cost threshold for poor economic performance.

All of the designs listed in Table 4, except D_4, have at least an 80% probability of having their actual costs fall within β = 20% of the estimated minimum possible cost. However, at this β value, D_2 is the most robust design, with an R_β value of 0.85. To consider a case where possible system cost performance may be even less satisfactory, line 3 of the table shows that all but design D_4 have at least a 90% probability of having their costs fall within β = 50% of the estimated minimum possible costs. Again, at this β value, D_2 is the most robust design with $R_{0.50}$ = 98%. Thus with design alternative D_2 there is only a 2% probability of relatively very poor cost performance. In this case, both the robustness criterion and cost minimization with the expected 1975 demand point to selection of the same design alternative.

LIMITATIONS AND SUGGESTED FURTHER ANALYSIS

A simple example, but one based on an actual situation, has been presented to illustrate how the robustness measure may be used. This preliminary study has several limitations and suggests the need for further work. In particular:

1. In the above example, only different combinations of three water sources are considered as alternatives. Total cost curves may be more irregular if alternatives with different types of components are compared; for example, surface reservoirs, groundwater, desalinization, or reclaimed wastewater. Naturally, the robustness measures will be more useful in situations where design costs vary more widely among alternatives.

2. Only the uncertainty in future water demand has been integrated into the robustness measure. One could also include in the robustness measure variable energy costs, different interest rates, or more generally, project costs under alternative policies or scenarios [*Hashimoto*, 1980*a*].

3. Only physical adjustments (of timing and sizing of the projects) have been considered in the present study. Possible adjustments, however, are not confined to such physical adjustments. Suppose, for instance, that industrial water demand in the region turns out to be lower than originally expected and thus some opportunity cost for overdesign is incurred. This cost might be reduced if the excess water can be diverted to, say, supplementary irrigation which will yield additional benefits. On the other hand, suppose agricultural water demand increases. The cost of making water available to various other uses may increase if no adjustment is made in the face of such an event. Whether such adjustments are possible depends very much on institutional arrangements of

the region of concern. The flexibility and efficiency of decision-making processes and financing procedures can determine to a certain extent if redesign and reauthorization of the projects are possible in response to changes in the planning environment.

4. One of the most essential tasks for the analysis of robustness is to identify and to describe in appropriate ways those parameters which characterize system inputs. For the water supply system that has been analyzed here, a water demand study should be carefully carried out, taking account of possible changes in future policies. Water demand is as important as water supply when considering the robustness of the entire system. In this respect, the present study is incomplete.

SUMMARY

In this paper a robustness criterion R_β is introduced as the probability that the cost of a specific system will be no greater than $1 + \beta$ of the cost of the minimum cost design for the realized future demand condition. The difference between the cost of a project and the minimum cost that need be incurred for those particular future demand conditions provides a basis for comparing alternatives. The robustness measure is defined based on this opportunity cost and was applied to planning the expansion of a water supply system under demand uncertainty in southwestern Skane, Sweden.

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