Working Paper

PARAMETRIZED MULTISTATE
POPULATION DYNAMICS

Andrei Rogers

November 1982
WP-82-125

Paper prepared for presentation at the Task Force Meeting on Multistate Life-History Analysis, held on November 15-18, 1982, at the International Institute for Applied Systems Analysis, Laxenburg, Austria.

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PREFACE

Low fertility levels in IIASA countries are creating aging populations whose demands for health care and income maintenance (social security) will increase to unprecedented levels, thereby calling forth policies that will seek to promote increased family care and worklife flexibility. The new Population Project will examine current patterns of population aging and changing life-styles in IIASA countries, project the needs for health and income support that such patterns are likely to generate during the next several decades, and consider alternative family and employment policies that might reduce the social costs of meeting these needs.

The Project is seeking to develop a better understanding of how low fertility and mortality combine to create aging populations with high demands for health and income maintenance and reduced family support systems that can provide it. The research will produce analyses of current demographic patterns in IIASA countries together with an assessment of their probable future societal consequences and impacts on the aging. It will consider the position of the elderly within changing family structures, review national policies that seek to promote an enlarged role for family care, and examine the costs and benefits of alternative systems for promoting worklife flexibility by transferring income between different periods of life.

This paper outlines a methodological framework that will play an important role in future population research at IIASA. It describes and advocates a modeling perspective, *the multistate approach*, that focuses on gross flows between demographic states. This perspective directs attention to the evolutionary dynamics of a system of multiple interacting populations and thereby avoids some of the pitfalls that can arise from the use of the conventional unistate perspective of classical demography.

A list of related IIASA publications appears at the end of this paper.

Andrei Rogers
Chairman
Human Settlements
and Services Area

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CONTENTS

PARAMETRIZED MULTISTATE POPULATION DYNAMICS

1. INTRODUCTION

Demography is concerned with the description, analysis, and projection of human populations disaggregated by age, sex, and a number of states of existence. The principal topics of demographic study are changes in population size, composition, and distribution over space (Thompson 1953). Moreover, the focus is not only on this configuration at a particular moment in time but also on its evolution over time. Finally, demographers seek scientific explanations of why particular population conditions arise and why they change in the ways that they do.

Demography also has been characterized as the quantitative study of fundamental demographic processes, such as mortality, fertility, migration, and marriage (Bogue 1968). These processes may be viewed as transitions that individuals experience during the course of their life cycle. Individuals are born, age with the passage of time, enroll in school, enter the labor force, get married, reproduce, migrate from one region to another, retire, and ultimately die. These transitions contribute to changes in various population stocks through simple accounting identities. For example, the number of married people at the end of each year is equal to the number at the beginning of the year plus new marriages and arrivals of married migrants less divorces, deaths, widowings, and the outmigration of part of the married population.

The study of transition patterns generally begins with the collection of data and the estimation of missing observations, continues with the calculation of the appropriate rates and corresponding probabilities, and often ends with the generation of simple projections of the future conditions that would arise were these probabilities to remain unchanged.

Recent work in formal multistate demography has produced a generalization of classical demographic techniques that unifies most of the existing purely demographic methods for dealing with transitions between multiple states of existence. For example, it is now clear that multiple decrement mortality tables, tables of working life, nuptiality tables, tables of educational life, and multiregional life tables, all are members of a general class of increment-decrement life tables called *multistate life tables*. It is also now recognized that projections of populations classified by multiple states of existence can be carried out using a common methodology of *multistate projection*, in which the core model of population dynamics is a multidimensional generalization of either the continuous-age-time model of Lotka or of the discrete-age-time Leslie model.*

Another approach for analyzing changes in the distributions of populations across statuses such as married, divorced, employed, and unemployed is the perspective of causal analysis, developed largely within sociology, which emphasizes the effects of population heterogeneity on short-run multistate transitions. This approach, referred to as *life-history* or *event-history analysis*, combines behavioral hypotheses about the effects of heterogeneity on rates with stochastic process models. It focuses on the impacts of observed and unobserved heterogeneity, the effects of duration

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*For overviews of multistate mathematical demography, see Keyfitz (1979), Rogers (1980), and Land and Rogers (1982).

in a state of rates of exit from that state, the reasonableness of assumptions postulating homogeneity over time, and the influence of previous experiences on current and prospective patterns of behavior. Until very recently, however, event-history analysis has accorded little attention to age variation in rates and to long-run projections.*

The development of multistate life tables and projection models has brought the demographic tradition much closer to the sociological one, and a marriage between the two perspectives seems possible and desirable. An important consequence of such a merger could be the development of a micro and a macro branch of formal demography. The prospect for such a merger, however, is not the topic of this paper, which considers only macrodemographic multistate dynamics. The aim of this essay is to inform and perhaps convince demographers of the desirability of modifying the currently predominant perspective in population analysis: that of a single population whose interactions with other populations are expressed only by means of ratios or net changes in stocks. In place of this perspective we advocate one that focuses simultaneously on *several* populations whose interactions with each other are expressed in terms of *gross* flows. We shall refer to these two competing perspectives as the unistate and the multistate approaches, respectively.

Another firmly established bias within demography is the reverence accorded to observed data. Reared in the empirical traditions of census statisticians and actuarial calculations, most demographers turn to hypothetical parametrized schedules only as a last resort, usually in Third World country settings, which lack adequate population data. Yet the quality of population projections developed with the aid of parametrized schedules is high and constantly improving, and the traditional concern among demographers for calculations carried to many significant

*For an overview of life-history or event-history analysis, see Coleman (1981), Hannan (1982), and Tuma and Hannan (forthcoming).

digits is surely misplaced and often spurious. Thus a second goal of this paper is to convince demographers of the feasibility and desirability of adopting parametrized schedules instead of observed schedules in studies of multistate population dynamics. The transparency of the parametrized models used to describe such dynamics, and the consistency and convenience that such an exercise allows, more than compensate for what little might be lost in the way of accuracy.

The rest of this paper focuses on the three topics touched on in this Introduction: the multistate approach, the use of parametrized schedules, and the advantages of a multistate perspective. In short, it deals with the three questions: what is it? how should it be implemented? why is it important?

2. MULTISTATE MACRODEMOGRAPHY

The current distribution of a population across its constituent states, or subcategories, and the age compositions of its state-specific subpopulations reflect interactions between past patterns of mortality, fertility, and interstate transition. National statistical agencies all over the world deal with the influences of such interactions by relying on essentially standard unistate methods. A typical example is offered by current U.S. Census Bureau projections of mortality, fertility, immigration, school enrollment, educational attainment, family and household totals and composition, and the income distribution of households. In all of these, exogenously projected rates are applied to the same age-sex-race-specific national population distribution, and no attempt has as yet been made to produce an internal consistency among the various rates used (Long 1981).

Unistate models focus only on a single population. All rates, therefore, must of necessity use this population in the denominator. Thus the conventional methodology for analyzing labor force patterns, for example, employs labor force participation rates and that for generating regional population projections uses net migration rates. A multistate model introduces multiple populations and thereby permits the association of *gross flows* with the *populations at risk* of experiencing these flows. Labor force participation rates are dropped in favor of accession and separation rates and net migration rates are replaced by origin-destination-specific gross migration rates.*

This fundamental difference between the single-dimensional and the multidimensional approaches to population analysis may be illuminated by the illustration set out in Figure 1. Imagine a barrel containing a continuously fluctuating level of water. At any given moment the water level is changing as a consequence of losses due to two outflows, identified by the labels *deaths* and *outmigration*, respectively, and gains introduced by two inflows labeled *births* and *inmigration*, respectively.

If it is assumed that each barrel's migration outflow and its migration inflow, during a unit period of time, vary in direct proportion to the average water level in the barrel at that time, then the two flows may be consolidated into a single *net* flow (which may be positive or negative), and the ratio of this net flow to the average water level defines the appropriate rate of net inmigration. Such a perspective of the problem reflects a unistate approach.

Now imagine an interconnected system of three barrels, say, where each barrel is linked to the other two by a network of flows, as in Figure 1B. In this system the migration outflows from two barrels define the migration inflow of the third. A unistate analysis of the evolution of water levels in this three-barrel system would focus on the changes in the outflows and inflows in each barrel, one at a time. A multistate perspective, on the other hand, would regard the three barrels as a system of three interacting bodies of water, with a pattern of outflows and inflows to be examined as a simultaneous system of

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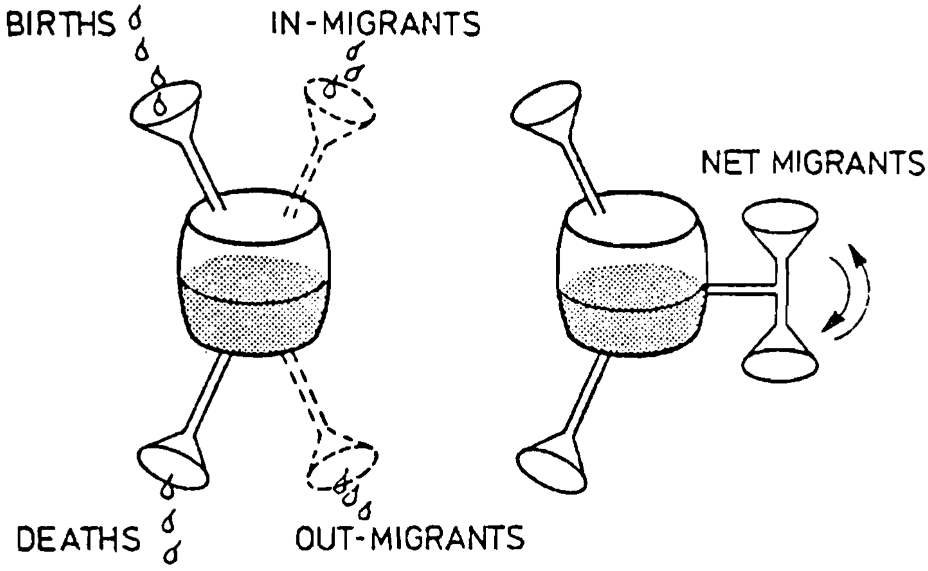
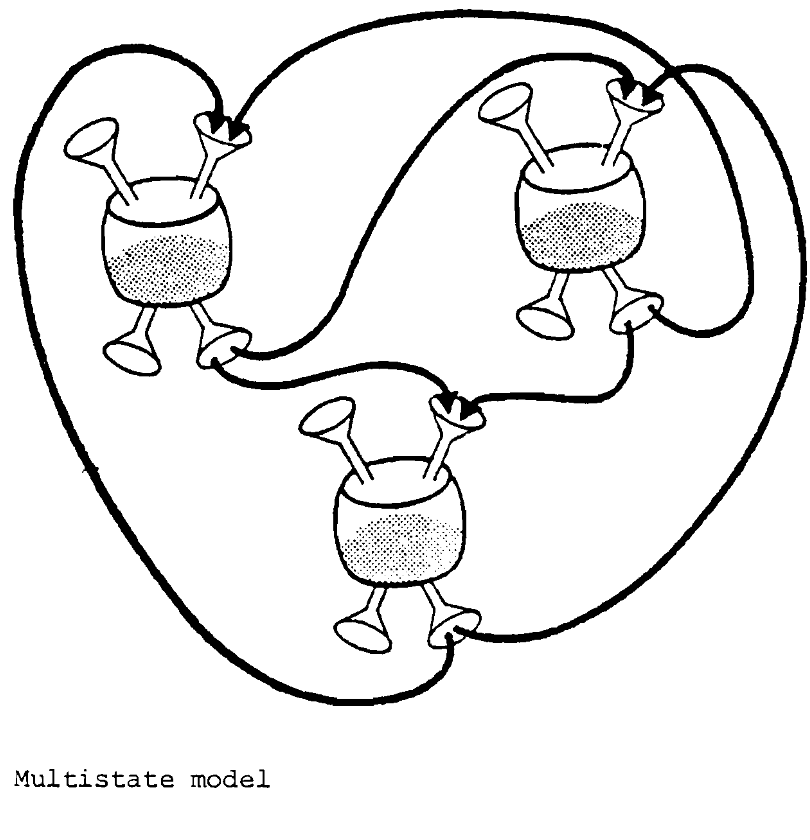
*Curiously, although the "participation" rate approach has been used in producing urban population projections (United Nations 1980), the "net migration" rate approach apparently has not been used to study labor force patterns.



A. Unistate model



B. Multistate model

Figure 1. Contrasting the unistate and the multistate perspectives.

relationships. Moreover, the multistate approach would focus on migration outflows; hence the associated migration rates would always be positive, and they would refer to the appropriate population exposed to the possibility of outmigrating.

Two fundamental features, then, distinguish the multistate from the unistate perspective: the population being examined and the definition of rates of flow. The multistate approach considers the entire population as an interacting system; the unistate approach examines each subpopulation one at a time. Moreover, the multistate approach employs rates of flow that always refer to the appropriate at-risk populations; the unistate approach, by relying on *net* rates, cannot do that.

2.1 Rates

The basic initial measure for most demographic analysis is a central rate, defined for a population in a given state during a particular time span. Its numerator is a count of occurrences of an event; its denominator describes person-years of exposure (number of people times the duration of their exposure to the event in question). In the demographic literature such rates have been called *occurrence/exposure* rates.

In mortality studies, for example, occurrence/exposure rates associate the number of deaths during a given interval with the number of person-years of exposure to death experienced by the population at risk. In labor force studies they take the form of accession and separation rates that relate entrances and exits to the at-risk inactive and active populations, respectively. Analogous principles apply to the construction of fertility rates, nuptiality rates, and divorce rates.

Empirical schedules of age-specific occurrence/exposure rates exhibit remarkably persistent regularities in age pattern. Mortality schedules, for example, normally show a moderately high death rate immediately after birth, after which the rates drop to a minimum between ages 10 to 15, then increase slowly until about age 50, and thereafter rise at an increasing pace until the last years of life. Fertility rates generally start to take on nonzero values at about age 15 and attain a maximum somewhere between ages 20 and 30; the curve is unimodal and declines to zero once again at some age close to 50. Similar unimodal profiles may be found in schedules of first marriage, divorce, and remarriage. The most prominent regularity in age-specific schedules of migration is the high concentration of migration among young adults; rates of migration also are high among children, starting with a peak during the first year of life, dropping to a low point at about age 16, turning sharply upward to a peak near ages 20 to 22, and declining regularly thereafter except for a possible slight hump or upward slope at the onset of the principal ages of retirement. Although data on rates of labor force entry and exit are very scarce, the few published studies that are available indicate that regularities in age pattern also may be found in such schedules. Figure 2 illustrates a number of typical age profiles exhibited by occurrence/exposure rates in multistate demography.

The shape or *profile* of a schedule of age-specific occurrence/exposure rates is a feature that may be usefully examined independently of its intensity or *level*. This is because there are considerable empirical data showing that although the latter tends to vary significantly from place to place, the former remains remarkably similar. Some evidence on this point will be presented in the course of the discussion of parametrized model schedules in section 4.

The level at which occurrences of an event or a flow take place in a multistate population system may be represented by the area under the curve of the particular schedule of rates. In fertility studies, for example, this area is called the gross reproduction rate if the rates refer to parents and babies of a single sex. By analogy, therefore, we shall refer to areas under all schedules of occurrence/exposure rates as *gross production rates*, inserting the appropriate modifier when dealing with a particular event or flow, for example, gross mortality production rate and gross accession production rate. The term

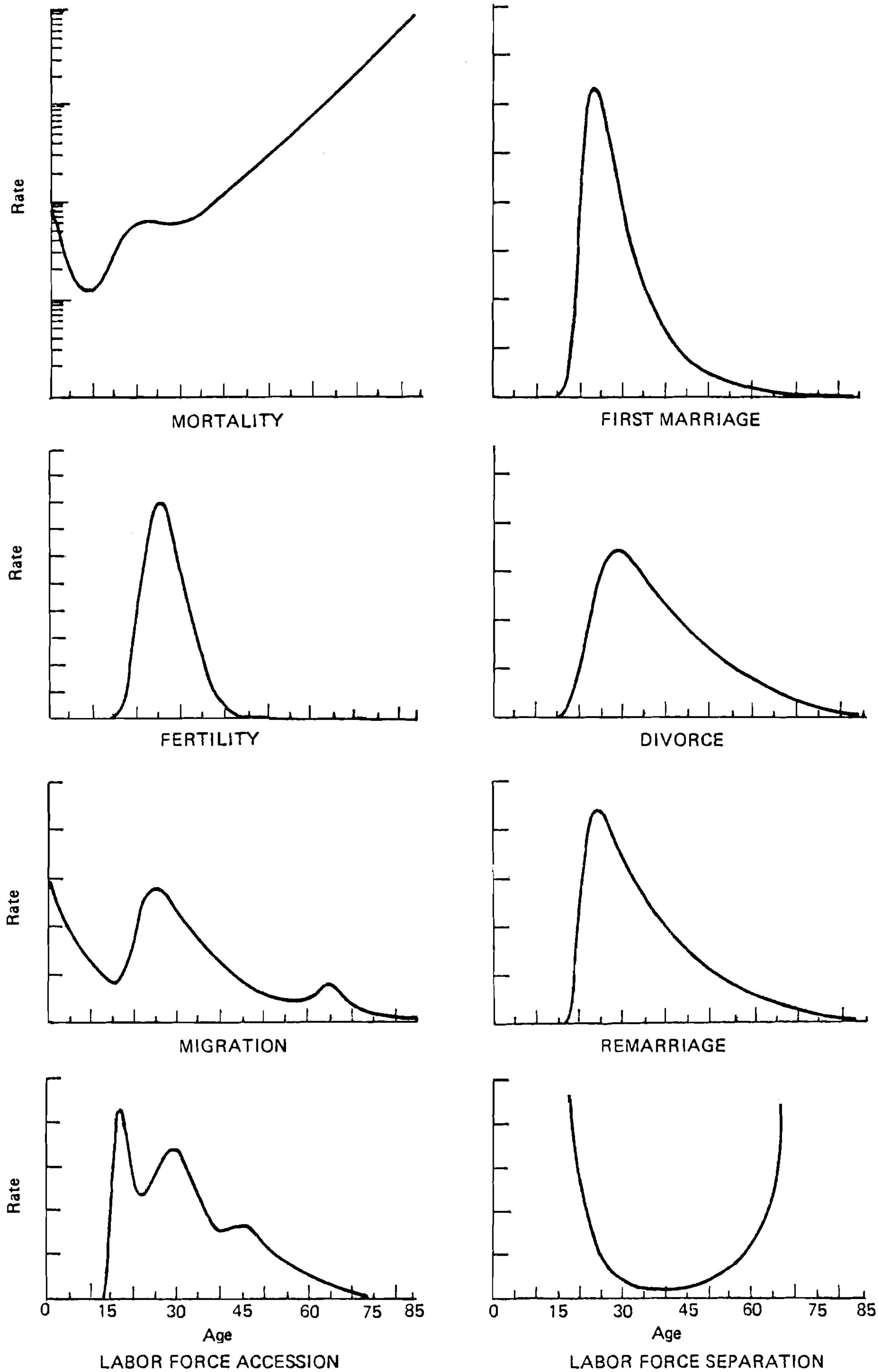


Figure 2. Multistate schedules.

"production" is retained throughout in order to distinguish this aggregate measure of level from the other more common gross rates used in demography, such as the directional gross (instead of net) rate of migration.

The gross production rate measures the intensity of particular events within a state population or of flows between two or more state populations during a given interval of time. The index, therefore, is a cross-sectional measure and should not be confused with the *net* production rate (such as the net reproduction rate), which is a cohort-related index that measures the intensity of such events or flows over a lifetime. Moreover, in a multistate framework, where return flows such as remarriages, play an important role, the gross rate and the net rate can give widely differing indications of interstate movement intensities.

2.2 Probabilities

Vital statistics, other demographic registers, surveys, and censuses provide the necessary data for the computation of occurrence/exposure rates. They may be used to answer questions such as: what is the current rate at which 50-year-old males are dying from heart disease or at which 30-year-old women are bearing their second child? But many of the more interesting questions regarding mortality and fertility patterns are phrased in terms of probabilities, for example: what is the current probability that a man aged 50 will outlive his 45-year-old wife, or that she will bear her third child before she is 40?

Demographers normally estimate probabilities from observed rates by developing a life table. Such tables describe the evolution of a hypothetical cohort of babies born at a given moment and exposed to an unchanging age-specific schedule of vital rates. For this cohort of babies, they exhibit a number of probabilities for changes of state, such as dying, and develop the corresponding expectations of years of life spent in different states at various ages.

The simplest life tables recognize only one class of decrement, e.g., death, and their construction is normally initiated by estimating a set of age-specific probabilities of leaving the population, e.g., dying, within each interval of age, q(x) say, from observed data on age-specific exit rates, M(x) say. The conventional calculation that is made for an age interval five years wide is

$$q(x) = 5M(x) \;/\; [1 + \frac{5}{2} M(x)]$$

or alternatively,

$$q(x) = 1 - q(x) = [1 + \frac{5}{2} M(x)]^{-1} [1 - \frac{5}{2} M(x)] \qquad (1)$$

where p(x) is the age-specific probability of remaining in the population, e.g., of surviving, between exact ages x to x+5.

Simple life tables, generalized to recognize several modes of exit from the population are known as *multiple-decrement life tables* (Keyfitz 1968, p. 333). A further generalization of the life table concept arises with the recognition of entries as well as exits. Such *increment-decrement life tables* (Schoen 1975) allow for multiple movements between several states, for example, transitions between marital statuses and death (married, divorced, widowed, dead), or between labor force statuses and death (employed, unemployed, retired, dead).

Multiple-radix increment-decrement life tables that recognize several regional populations, each with a region-specific schedule of mortality, and several destination-specific schedules of internal migration are called *multiregional life tables* (Rogers 1973a, b). They represent the most general class of life tables and were originally developed for the study of interregional migration between interacting multiple regional populations. Their construction is initiated by estimating a matrix of age-specific probabilities of surviving and migrating $\underset{\sim}{P}(x)$ from data on age-specific death and migration rates, $\underset{\sim}{M}(x)$. Rogers and

Ledent (1976) show that the equation for this estimation may be expressed as the matrix analog of Equation 1*,

$$\underset{\sim}{P}(x) = [\underset{\sim}{I} + \frac{5}{2}\underset{\sim}{M}(x)]^{-1}\ [\underset{\sim}{I} - \frac{5}{2}\underset{\sim}{M}(x)] \qquad (2)$$

One of the most useful statistics provided by a life table is the average expectation of life beyond age x, e(x) say, calculated by applying the probabilities of survival p(x) to a hypothetical cohort of babies and then observing at each age their average length of remaining life in each state:

$$\underset{\sim}{e}(x) = \underset{\sim}{T}(x)\underset{\sim}{\ell}^{-1}(x) = \left[\int_x^{\omega} \underset{\sim}{\ell}(a)\,da\right]\underset{\sim}{\ell}^{-1}(x)$$

where $\underset{\sim}{\ell}(a)$ is the matrix of survival probabilities to exact age a.

An example is set out in Table 1, calculated using Australian data for males in 1975/76. It shows a total remaining life expectancy of 53.09 years for a 20-year-old married man, with 48.58 years of this expected lifetime to be spent in the married state, 3.45 years in the divorced state, and 1.06 in the widowed state.

The transition probabilities in $\underset{\sim}{P}(x)$ refer to persons who are at exact age x. For population projections, however, it is useful to derive the corresponding survivorship proportions, $\underset{\sim}{S}(x)$, that refer to individuals in age group x to x+h at the start of the projection. Here again it is a simple matter to show that the multistate analog of the conventional unistate expression is

$$\underset{\sim}{S}(x) = [\underset{\sim}{I} + \underset{\sim}{P}(x+h)]\ \underset{\sim}{P}(x)\ [\underset{\sim}{I} + \underset{\sim}{P}(x)]^{-1} \qquad (3)$$

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*This formula is applicable only when migration is viewed as a move, i.e., an event. If the data treat migration as a transition, i.e., a transfer during a specified unit time interval, then Equation 2 yields only an approximation. See Ledent (1980).

Table 1. Expectancies of remaining lifetime in each marital state of 20-year-old Australian males: 1975/76 data.

Status at age 20	Remaining life expectancy: Never married	Married	Divorced	Widowed	Total
Never married	12.82	35.66	2.62	0.90	52.00
Married	0	48.58	3.45	1.06	53.09
Divorced	0	44.71	7.27	1.06	53.04
Widowed	0	36.63	2.74	13.36	52.73

Data Source: Brown and Hall (1978).

which yields the recursive expression

$$\underset{\sim}{P}(x) = [\underset{\sim}{I} + \underset{\sim}{P}(x+h) - \underset{\sim}{S}(x)]^{-1} \underset{\sim}{S}(x) \qquad (4)$$

All life-table functions originate from a set of transition probabilities, defined for all ages. The first question in constructing such tables, therefore, is how to transform observed age-specific death and migration rates, $\underset{\sim}{M}(x)$, or survivorship proportions, $\underset{\sim}{S}(x)$, into the age-specific transition matrices, $\underset{\sim}{P}(x)$. Equations 2 and 4 suggest two alternative procedures. The first focuses on observed rates, the second on observed proportions surviving. In Rogers (1975) these two estimation methods were called the "Option 1" and "Option 2" methods, respectively. As Hoem and Jensen (1982) point out, however, a number of other options are possible depending on the observational plan that underlies a particular data collection effort.

2.3 Projections

Multistate generalizations of the classical models of mathematical demography project the numerical consequences, to an initial (single-sex) multistate population, of a particular set of assumptions regarding future fertility, mortality, and interstate transfers. The mechanics of such projections typically revolve around three basic steps. The first ascertains the starting age-by-state distributions and the age-state-specific schedules of fertility, mortality, and interstate flows to which the multistate population has been subject during a past period; the second adopts a set of assumptions regarding the future behavior of such schedules; and the third derives the consequences of applying these schedules to the initial population.

The discrete model of multistate demographic growth expresses the population projection process by means of a matrix operation in which a multistate population, set out as a vector, is multiplied by a growth matrix that survives that population forward through time. The projection calculates the state- and age-specific survivors of a multistate population of a given sex and adds to this total the new births that survive to the end of the unit time interval. This process may be described by the matrix expression:

$$\{\underset{\sim}{K}(t+1)\} = \underset{\sim}{G}\{\underset{\sim}{K}(t)\} \tag{5}$$

where the vector $\{\underset{\sim}{K}(t)\}$ sets out the multistate population disaggregated by age and state, and the matrix $\underset{\sim}{G}$ is composed of zeros and elements that represent the various age-specific and state-specific components of population change.

Given appropriate data, survivorship proportions can be calculated using Equation 3 and applied to the initial male or female population, $\{\underset{\sim}{K}(t)\}$, as shown in Equation 5. For example, it is possible to simultaneously determine the projected male or female population and its age/marital status/regional distribution from the observed age/marital status/region-specific flows of marital status changes, regional migrant inflows and outflows, deaths, and fertility. The $\{\underset{\sim}{K}(t+1)\}$ so derived must then be augmented by the numbers of international migrant arrivals and departures (disaggregated by age, marital status, and region of arrival or departure) to give the projection of male or female population by age, marital status, and region of residence.

The asymptotic properties of the projection in Equation 5 have been extensively studied in mathematical demography. This body of theory draws on the properties of matrices with non-negative elements and, in particular, on the Perron-Frobenius theorem (Gantmacher 1959). Its application to Equation 5 establishes the existence of a unique, real, positive, dominant characteristic root, λ_1 say, and an associated positive characteristic vector, $\{\underset{\sim}{K}_1\}$ say. Inasmuch as λ_1 is greater in absolute value than any other λ_j, the effects of all components beyond the first ultimately disappear as the population converges to the stable distribution defined by $\{\underset{\sim}{K}_1\}$.

Since the sum of the elements of $\{\underset{\sim}{K}_1\}$ is unity, the total population added over all ages and states is $\lambda_1^t c_1$, say, for a large t and a constant projection matrix. This permits us to call c_1 the *stable equivalent population*. It is the total which, if distributed according to the stable vector $\{\underset{\sim}{K}_1\}$, would ultimately grow at the same rate as the observed $\{\underset{\sim}{K}(0)\}$ projected by the growth matrix.

Tables 2 and 3 present some numerical illustrations. Table 2 shows expectations of remaining life at age 20 for Australian females, disaggregated by marital status and two regions of residence. Table 3 presents the corresponding single-sex multistate population projection that arises under the assumption of fixed coefficients.

The fundamental dimensions of the growth dynamics of empirical populations are often obscured by the influences that particular initial conditions have on future population size and composition. Moreover, the vast quantities of data and parameters that go into a description of such empirical dynamics make it somewhat difficult to maintain a focus on the broad general outlines of the underlying demographic process, and instead often encourage a consideration of its more peculiar details. Finally, studies of empirical growth dynamics are constrained in scope to population dynamics that have been experienced and recorded; they cannot be readily extended to studies of population dynamics that have been experienced but not recorded or that have not

Table 2. Expectations of remaining lifetime in each marital state, by region[a], for 20 year-old Australian females: 1975/76 data.

Status at age 20	Remaining life expectancy								
	Region A				Region B				
	Never married	Married	Divorced	Widowed	Never married	Married	Divorced	Widowed	Total
Region A									
Never married	6.99	16.87	2.19	4.81	4.00	17.15	2.57	6.38	60.96
Married	0	24.50	2.91	5.84	0	18.80	2.85	6.83	61.73
Divorced	0	21.67	5.05	5.66	0	19.03	3.32	6.93	61.65
Widowed	0	21.53	2.85	18.25	0	12.01	1.84	4.93	61.42
Region B									
Never married	0.82	5.19	0.71	1.99	9.22	29.38	4.18	9.30	60.78
Married	0	6.24	0.85	2.28	0	36.84	4.96	10.31	61.47
Divorced	0	5.69	0.82	2.16	0	34.78	7.59	10.36	61.39
Widowed	0	4.05	0.55	1.72	0	29.20	4.18	21.26	60.97

[a]Regions A and B are defined in Rogers and Williams (1982).

Data Source: Brown and Hall (1978) and migration data provided by Pamela Williams.

Table 3. Multistate population projections by marital status and region[a]: for Australian females: 1975/76 data applied to 1975 base year.

Year and population measures	Region A				Region B				
	Never married	Married	Divorced	Widowed	Never married	Married	Divorced	Widowed	Total
1975									
Population	2,269,371	2,544,851	94,967	438,031	654,293	733,729	27,358	126,274	6,888,874
Mean age	14.56	42.13	46.46	68.64	14.56	42.13	46.45	68.65	32.68
Share	0.329	0.369	0.014	0.064	0.095	0.107	0.004	0.018	1.000
1985									
Population	1,570,848	2,170,038	179,435	585,752	1,140,219	1,450,486	96,999	186,073	7,379,850
Mean age	16.88	45.73	50.93	71.83	16.19	39.45	44.14	69.84	36.58
Share	0.213	0.294	0.024	0.079	0.155	0.197	0.013	0.025	1.000
Growth rate	-0.039	-0.016	0.024	0.008	0.029	0.051	0.078	0.027	0.004
1990									
Population	1,273,250	2,017,753	195,974	619,550	1,279,688	1,790,384	134,131	220,521	7,531,250
Mean age	17.99	46.85	53.98	72.58	16.13	39.75	45.10	69.34	37.99
Share	0.169	0.268	0.026	0.082	0.170	0.238	0.018	0.029	1.000
Growth rate	-0.044	-0.014	0.015	0.010	0.018	0.037	0.057	0.035	0.004

[a] Regions A and B are defined in Rogers and Williams (1982).

been experienced at all. In consequence, demographers occasionally have resorted to examinations of the dynamics exhibited by *model* populations that have been exposed to *model* schedules of growth and change (e.g., Coale and Demeny 1966; Rogers 1975).

As with most population projection models in the demographic literature, the multistate projection model deals only with a single sex at a time. However, the separate projection of the evolution of the male and female populations generally leads to inconsistencies, such as the number of married males not coinciding with the number of married females for a given year, the total number of new widows during a year not coinciding with the total number of deaths among married men that year, and so on. Thus it is not realistic to project the transitions among individuals of one sex without taking into account parallel transitions among individuals of the other sex. Methods for coping with this inconsistency and incorporating it into a multistate projection algorithm are discussed in Sams (1981b) and Sanderson (1981).

3. PARAMETRIZED MODEL SCHEDULES

The use of mathematical functions, expressed in terms of a small set of parameters, to smooth and describe parsimoniously schedules of age-specific rates is a common practice in demography. A large number of such functions have been proposed and fitted to mortality and fertility data, for example, and the results have been widely applied to data smoothing, interpolation, comparative analysis, data inference, and forecasting. The relevant literature is vast and entry into it can be made from such representative publications as Brass (1971), Coale and Demeny (1966), Coale and Trussell (1974), Heligman and Pollard (1979), Hoem et al. (1981), and United Nations (1967).

More recently, the range of parametrized schedules has been expanded to include interstate transfers such as migration (Rogers, Raquillet, and Castro 1978; Rogers and Castro 1981) and changes in marital status other than first marriage (Williams 1981). Thus it is now possible to define a model (hypothetical) multistate dynamics that describes the evolution of a single-sex population exposed to parametrized schedules of mortality, fertility, migration, and several forms of marital status change (that is, first marriage, divorce, and remarriage). The evolution of a two-sex population can also be examined by making use of procedures that strive to ensure some consistency between the transitions experienced by each sex (Sams 1981b and Sanderson 1981).

3.1 Advantages

The role of model schedules in parametrized multistate population dynamics is two-fold. *First*, model schedules allow one to condense an enormous amount of information about transitions between states of existence or the occurrences of vital events in each year into a few parameters. *Second*, model schedules provide a manageable number of interpretable descriptive statistics, for each demographic transition or vital event in each year, the time series of which can be the basis for econometric estimation. The criteria for the selection of appropriate model schedules should emphasize the interpretability of the parameters, their success in characterizing the important features of demographic behavior, and the goodness-of-fit of the parametrized schedules to available data.

A parametrized multistate population projection model should be designed to provide projections that are disaggregated and internally consistent and are useful for policy analysis.

Disaggregation. Since population projections are required by a wide variety of users, with vastly different applications in mind, a high level of disaggregation allows greater flexibility in the use of the projection. For example, disaggregation by region is of interest to urban planners; disaggregation by sex and marital status is important in studies of labor force participation and fertility; and disaggregation by age allows the producers and users of population projections to take explicit account of the age dependence of demographic events.

Disaggregation is important for another reason; it often leads to a detection of more homogeneous categories and thus to a greater consistency in trends. Indeed some demographers have argued that population forecasting is an exercise in identifying those demographic variables whose behavior over time is sufficiently regular that trend extrapolation becomes a plausible projection methodology (Brass 1974).

Consistency. Highly disaggregated population projections are of limited use if there is no consistency among the various outputs. The projections should satisfy standard demographic accounting identities; they should incorporate the impacts of preceding demographic events via the use of the changing age profile of the population; and they should ensure that the various disaggregated outputs are in harmony with one another (for example, the number of married males must coincide with the number of married females, and so on). This is not the current practice in most national statistical agencies:

> Perhaps the most striking results of this overview of projections methodologies are the lack of a mechanism for assuring consistency among projected variables and the apparent arbitrariness of many of the assumptions used to project (or more appropriately, to extrapolate) the proportions and ratios applied to the projected population base. Our interest at this point is to identify the most fruitful areas of research that may lead to specifying linkages between variables in the system, to estimating the parameters specified, and to devising a system or model for projecting these parameters. (Long 1981:317-318)

Policy Relevance. Forecasters generally make population projections on the basis of implicit and explicit assumptions regarding future levels of the demographic and economic variables that are thought to affect population change. Although such projections are of interest for specific purposes, they often are of little value for policy analysis. To enable analysis of the consequences of economic and population policies, projections should incorporate explicitly the assumptions that are made regarding the demographic and economic (demoeconomic) environment underlying the projections and should clearly and consistently specify the relationships that exist between this environment and population change. Operationally this suggests the need to *condense* to a manageable level the amount of information to be specified as assumptions, to express this condensed information in a *language* and in *variables* that are readily understood by the users of the projections, and to relate the variables to one another as well as to variables describing the *demoeconomic* environment that underlies the population projection.

A set of 85 death rates by single years of age is not very informative; nor is a collection of 30 age-specific fertility rates. Yet the meaning of the expectation of life at birth and the gross (or net) reproduction rate implied by these two sets of rates is readily grasped, and unrealistic values for these variables indicate possible sources of error in the data. A theoretical or empirical association of these variables to the demoeconomic setting in which they are expected to prevail provides further illumination and another check on the reasonableness of the assumptions adopted. Certainly the relationships between economic variables and demographic behavior cannot be ignored. There is an expanding body of economic theory that attempts to explain marriage, divorce, fertility, and labor force participation behavior on the basis of a systematic analysis of family choices. Migration flows, particularly between regions, can also be related to economic conditions in the source and destination regions.

Thus a framework for population projection, *first*, should incorporate a projection procedure that enables disaggregation and consistent projections to be made, *second*, should use model schedules which reduce the information load, and, *third*, should use an economic model to determine important demographic variables. An illustration of such a framework for the consistent projection of a population disaggregated by age, sex, marital status, and region of residence is given in Rogers and Williams (1982).

3.2 Examples

3.2.1 Fertility

Among the relatively large number of different parametric functions that have been recently proposed for representing schedules of age-specific fertility, the formula put forward by Coale and Trussell (1974) has assumed a certain pre-eminence. This formula can be viewed as the product of two component schedules: a model nuptiality schedule and a model marital fertility schedule. The former adopts the double-exponential first marriage function of Coale and McNeil (1972):

$$g(x) = \frac{0.19465}{k} e^{-\frac{0.174}{k}(x - x_0 - 6.06k) - e^{-\frac{0.2881}{k}(x - x_0 - 6.06k)}} \qquad (6)$$

where x_0 is the age at which a consequential number of first marriages begin to occur, and k is the number of years in the observed population into which one year of marriage in the standard population is transformed. Integrating, one finds

$$G(x) = \int_0^x g(a)da$$

which when multiplied by the proportion who will ever marry, represents the proportion married at each age.

Coale argues that marital fertility either follows a pattern that Henry (1961) called *natural* fertility or deviates from it in a regular manner that increases with age, such that the ratio of marital fertility to natural fertility can be expressed by

$$\frac{r(x)}{n(x)} = M\ e^{mv(x)}$$

where M is a scaling factor that sets the ratio $r(x)/n(x)$ equal to unity at some fixed age, and m indicates the degree of control of marital fertility. The values of $v(x)$ and $n(x)$ are specified by Coale, and they are assumed to remain invariant across populations and over time.

Multiplying the two-parameter model schedule of proportions ever married at each age by the one-parameter model schedule of marital fertility, Coale and Trussell (1974) generated an extensive set of model fertility schedules that have been shown to describe empirical schedules with surprising accuracy. Their representation

$$f(x) = G(x) \cdot r(x) = G(x)n(x)e^{mv(x)} \tag{7}$$

allows one to obtain the age profiles (but not the levels) of fertility, which depend only on the fixed single-year values of the functions $n(x)$ and $v(x)$, and on estimates for x_0, k, and m. Hoem et al. (1981) report the results of fitting a number of mathematical functions to the age-specific fertility rates of Denmark, for the ages 15 to 46 inclusive, during the years 1962 to 1971. Figure 3 illustrates the curves that were obtained.

If the populations to be projected are already disaggregated by marital status, such that the proportions married, never married, and previously married at each age are known, appropriate model schedules for the age-specific fertility rates of women of each marital status may be derived. This has the advantage of allowing one to consider separately marital and non-marital fertility, each of which may be influenced by different demographic and economic factors. In the illustrative projection given later in this paper, a double-exponential function [similar to that used by Coale and McNeil (1972) for first marriages] was used to describe, separately for women of each marital status in each region, fertility rates at age x:

$$f(x) = gae^{-\alpha(x-\mu) - e^{-\lambda(x-\mu)}} \tag{8}$$

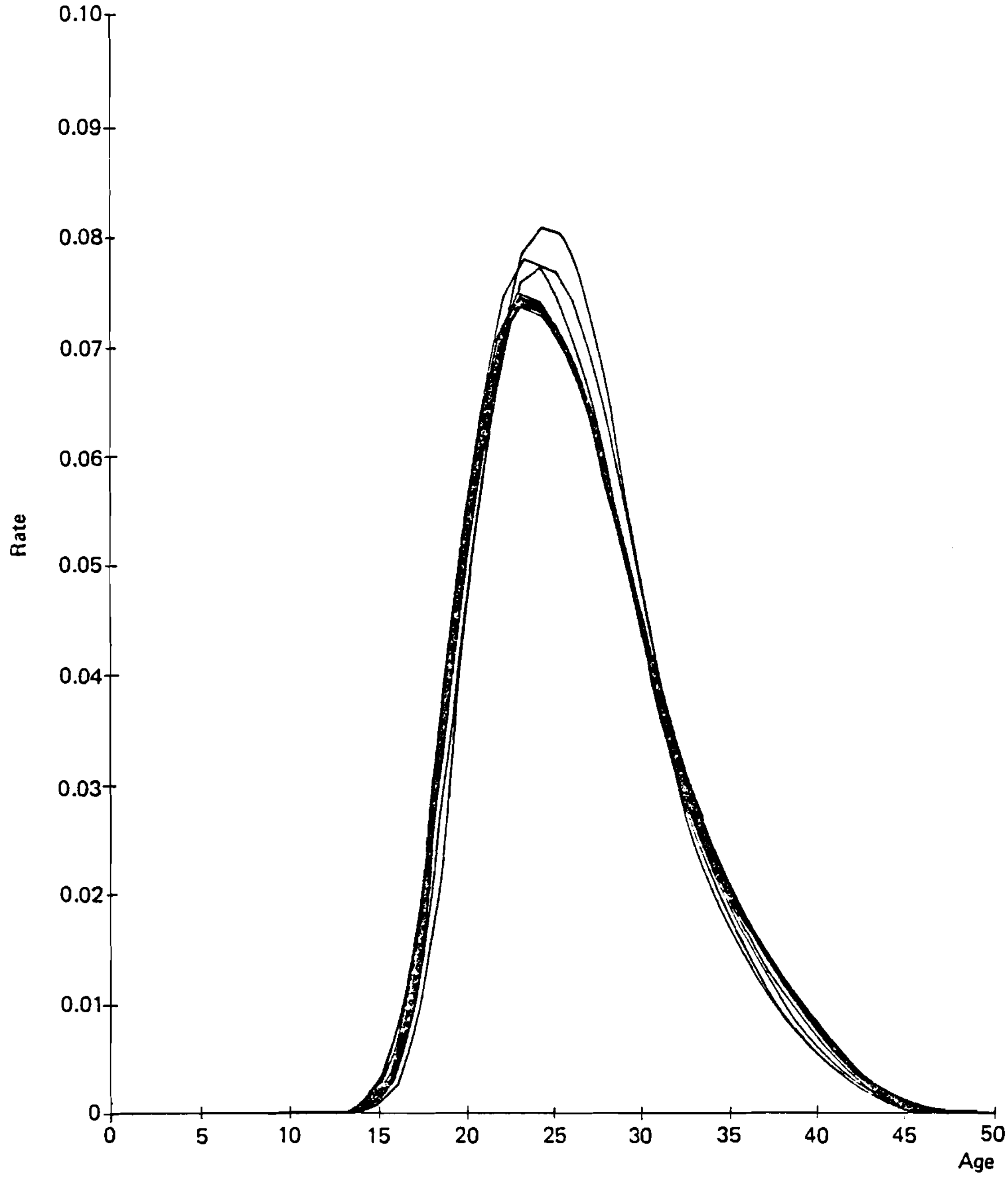


Figure 3. The Coale-Trussell model fertility schedule: Denmark, 1962-71.

Data Source: Hoem et al. (1981).

where the shape of the curve is defined by the three parameters, α, μ, and λ, and the level of the curve is defined by the scaling parameter a, and g, the gross fertility rate, which is the sum of all the age-specific fertility rates. Although these parameters (apart from g) are not easily interpretable, it is possible to derive the propensity, mean, variance, and mode of the double-exponential function in terms of them (Coale and McNeil 1972; Rogers and Castro 1981; and Sams 1981a).

3.2.2 Marital Status

Coale and McNeil's (1972) double-exponential model schedule of first marriages, discussed above, was introduced a decade ago. Parametrized schedules of other changes in marital status, however, seem to have been first used only recently, in a study carried out by the IMPACT Project in Australia (Powell 1977). Working with a detailed demographic data bank produced by Brown and Hall (1978), Williams (1981) fitted gamma distributions to Australian rates of first marriage, divorce, remarriage of divorcees, and remarriage of widows, for each year from 1921 to 1976. These model schedules provided adequate descriptions of Australian marital status changes, although some difficulties arose with age distributions that exhibited steep rises in early ages; in particular, the age distributions of first marriages. This difficulty was overcome by the addition of a second time-invariant gamma distribution.

Functions based on the Coale-McNeil double-exponential distribution, given in Equation 8, seem better able to cope with the problem of steeply rising age distributions than the gamma distribution. Figure 4 illustrates the goodness-of-fit of the double-exponential distribution for data on Norwegian males in 1977-78. Although the parameters of both functions can be expressed in terms of a propensity, mean age and variance in age, the double-exponential function requires a further parameter —the modal age—whose movements over time may be more difficult to model and project.

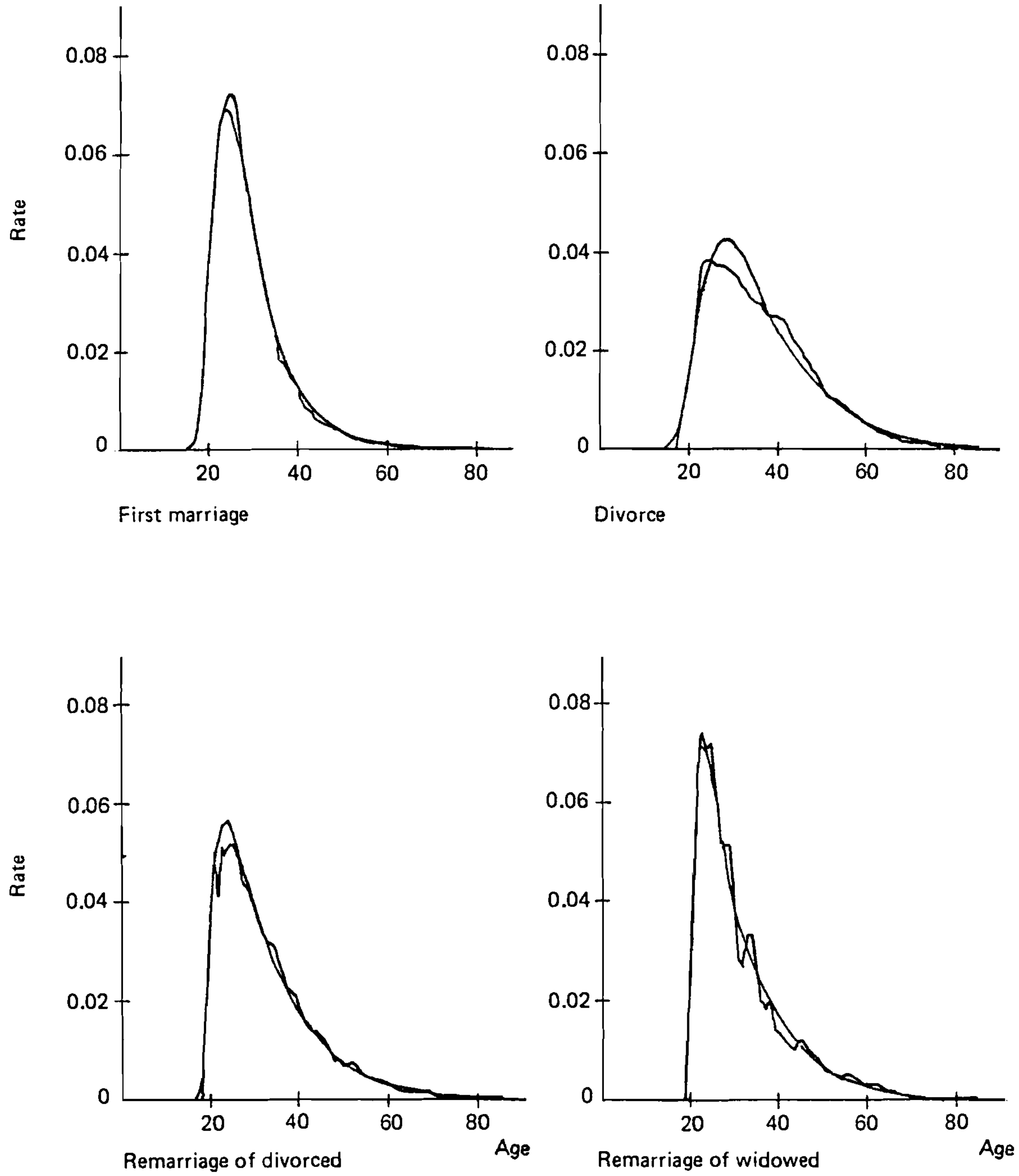


Figure 4. Model schedules of marital status change: Norwegian males 1977-1978.

Data Source: Brunborg, Monnesland, and Selmer (1981).

3.2.3 Mortality

Three principal approaches have been advanced for summarizing age patterns of mortality: *functional descriptions* in the form of mathematical expressions with a few parameters (Benjamin and Pollard 1980), *numerical tabulations* generated from statistical summaries of large data sets (Coale and Demeny 1966), and *relational procedures* associating observed patterns with those found in a standard schedule (Brass 1971).

The search for a "mathematical law" of mortality, has, until very recently, produced mathematical functions that were successful in capturing empirical regularities in only parts of the age range, and numerical tabulations have proven to be somewhat cumbersome and inflexible for applied analysis. Consequently, the relational methods first proposed by William Brass have become widely adopted. With two parameters and a standard life table, it has become possible to describe and analyze a large variety of mortality regimes parsimoniously.

In 1979 Heligman and Pollard published a paper setting out several mathematical functions that appear to provide satisfactory representations of a wide range of age patterns of mortality. Their function describes the variable q(x), the probability of dying within one year for an individual at age x. We have found it more useful to focus instead on d(x), the annual death rate at age x. Thus we adopt, in the illustrative projection given in this paper, the slightly modified Heligman and Pollard formula suggested by Brooks et al. (1980) of the IMPACT Project:

$$d(x) = d_I(x) + d_A(x) + d_S(x) \qquad \text{for } x = 0,1,\ldots,100+ \qquad (9)$$

where

$$d_I(x) = \begin{cases} Q_0 & \text{for } x = 0 \\ Q_1^{x^\gamma} & \text{for } x > 0 \end{cases}$$

$$d_A(x) = Q_A\, e^{-\left(\frac{\ln x - \ln X_A}{\sigma}\right)^2} \qquad \text{for } x \geq 0$$

and

$$d_S(x) = Q_S\, \frac{e^{x/X_S}}{1 + Q_S K e^{x/X_S}} \qquad \text{for } x \geq 0$$

Heligman and Pollard interpreted the three terms in their formula as representing infant and childhood mortality (I), mortality due to accidents (A), and a senescent mortality (S) component which reflects mortality due to aging. Figure 5 exhibits those three components and their sum, drawing on Australian data for 1950.

Death rates can be shown to differ markedly not only between ages, but also between sexes, between marital states and, perhaps, between regions. At the IMPACT Project, model schedules based on Equation 9 have been successfully fitted to Australian age-specific data for the death rates of persons of each sex and marital status. Not all components of the Heligman-Pollard curve were used, with the first component being omitted for married males and females and divorced and widowed females, and both the first and second components being omitted for divorced and widowed males. Given availability of data, such model schedules could be fitted in each region of a multiregional system. Movements over time in the parameters of such schedules could then be analyzed and used for projection of future mortality by age, sex, marital status, and region.

3.2.4 Migration

A recent study of age patterns in migration schedules (Rogers and Castro 1981) has shown that such patterns exhibit an age profile that can be adequately described by the mathematical expression:

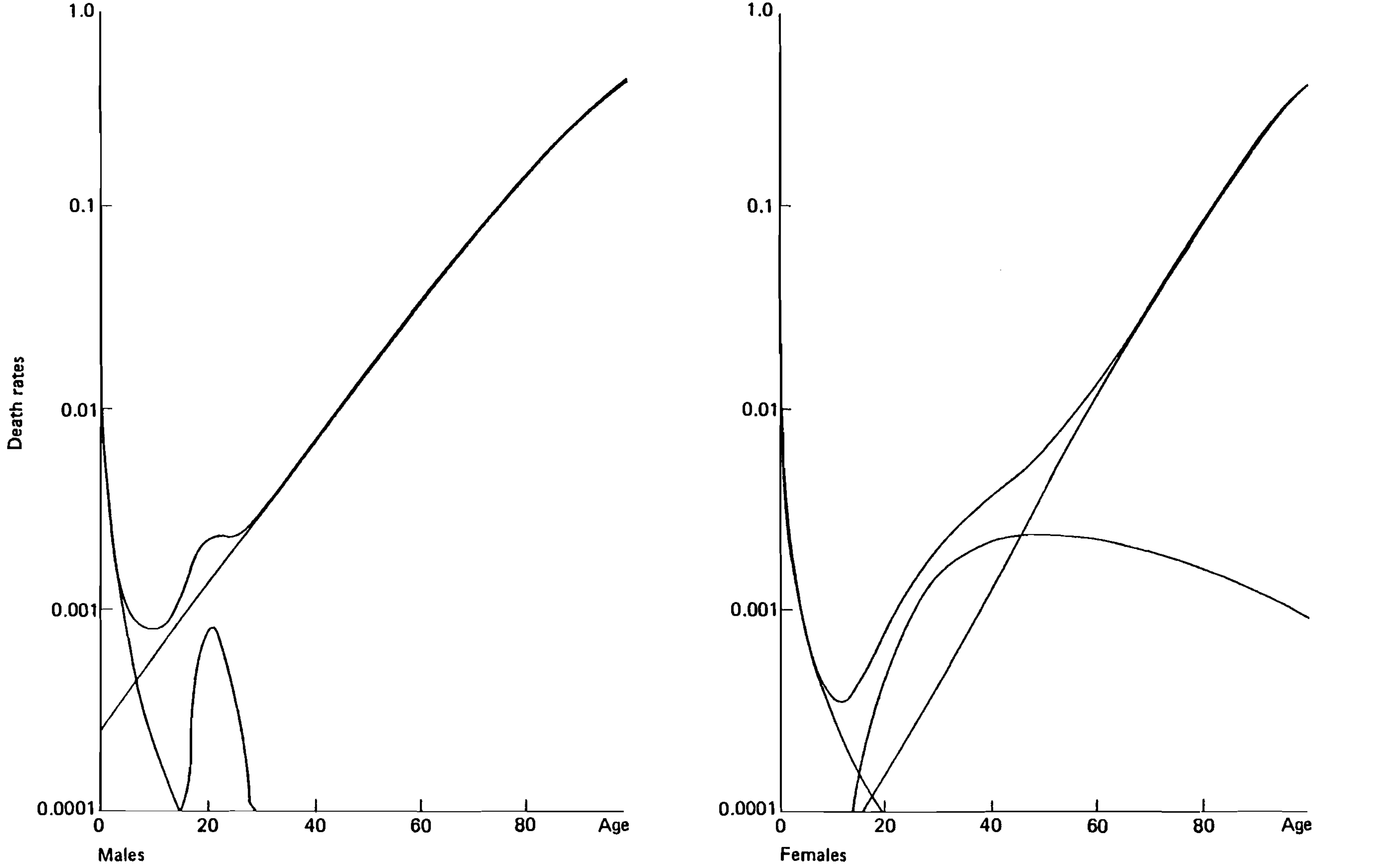


Figure 5. The components and total death rates for the Heligman-Pollard functions for never married Australian males and females for 1950.

Source: Brooks et al. (1980).

$$m(x) = a_1 e^{-\alpha_1 x} + a_2 e^{-\alpha_2 (x-\mu_2) - e^{-\lambda_2 (x-\mu_2)}} + R + c \qquad (10)$$

where

$$R = a_3 e^{-\alpha_3 (x-\mu_3) - e^{-\lambda_3 (x-\mu_3)}}$$

if the curve has a retirement peak,

$$R = a_3 e^{\alpha_3 x}$$

if the curve has an upward retirement slope, and

$$R = 0$$

if the curve has neither and is approximately horizontal at the post-labor force ages. The migration rate, $m(x)$, therefore, depends on values taken on by 11, 9, or 7 parameters, respectively.

The shape of the second term, the labor force component of the curve, is the double exponential formula put forward by Coale and McNeil (1972). The first term, a simple negative exponential curve, describes the migration age profile of children and adolescents. Finally, the post-labor force component is a constant, another double-exponential, or an upward sloping positive exponential. Figure 6 illustrates the fit of the nine parameter model schedule to intercommunal migration in the Netherlands.

In addition to the parameters and derived variables defined previously in the discussion of model double-exponential fertility schedules, we now introduce three additional measures useful for the study of migration age profiles: the index of *child dependency*

$$\delta_{12} = \frac{a_1}{a_2}$$

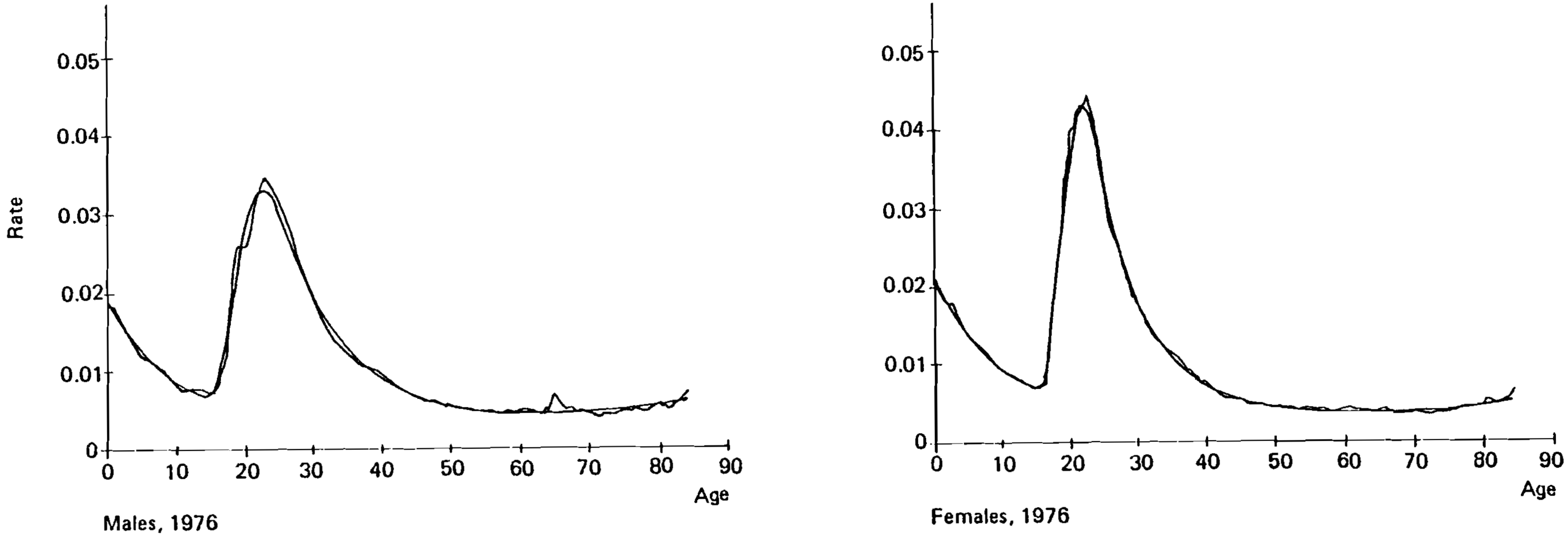


Figure 6. Model migration schedules for the Netherlands.

Data Source: Drewe (1980) and data provided by Drewe.

the index of *parental-shift regularity*

$$\beta_{12} = \frac{\alpha_1}{\alpha_2}$$

and the low point x_{ℓ}. The first measures the pace at which children migrate with their parents, the second indicates the degree to which the pattern of the migration rates of children mirrors that of their parents, and the third identifies the age at which the lowest migration rate occurs among teenagers.

3.2.5 Other Transitions

The notion of model schedules may be used to describe a wide range of demographic transitions. We have considered mortality, fertility, migration, marriage, divorce, and remarriage. We could as easily have focused on flows between different states of, for example, income, education, health, and labor force activity.

Consider, for example, the flows between active and inactive statuses in studies of labor force participation. Rates of entry into the labor force, called accession rates, exhibit an age profile that can be described as the sum of three double exponential distributions. Rates of exit from the labor force, called separation rates, may be described by a U-shaped curve defined as:

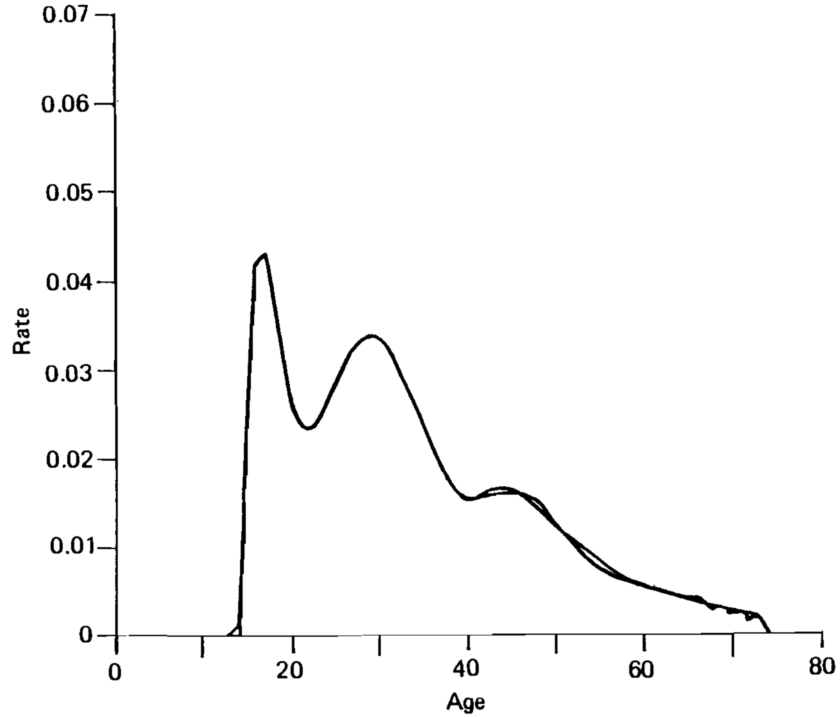
$$h(x) = a_1 e^{-\alpha_1 x} + a_3 e^{\alpha_3 x} + c \qquad (11)$$

Figure 7 illustrates the fit of these two curves to accession and separation rates, respectively, of Danish males in 1972-74 (Hoem and Fong 1976). The parameters for the former are:

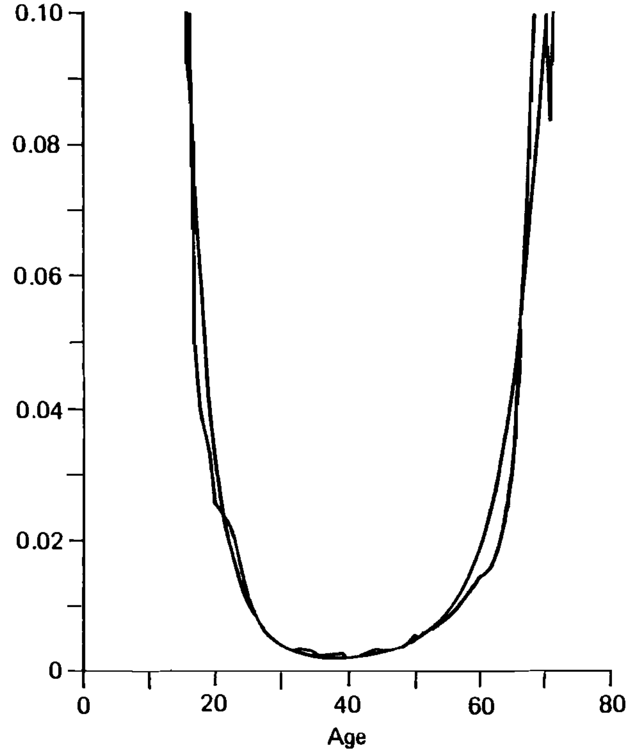
$a_1 = 0.072$, $\mu_1 = 15.65$, $\alpha_1 = 0.227$, $\lambda_1 = 1.240$
$a_2 = 0.082$, $\mu_2 = 28.81$, $\alpha_2 = 0.150$, $\lambda_2 = 0.189$
$a_3 = 0.017$, $\mu_3 = 42.51$, $\alpha_3 = 0.070$, $\lambda_3 = 0.381$

and those for the latter are:

$a_1 = 5.99$, $\alpha_1 = 0.254$, $a_3 = 0.000001$, $\alpha_3 = 0.159$, $c = 0.001$



A. Model labor force accession schedule



B. Model labor force separation schedule

Figure 7. Model schedules of male labor force accession and separation rates: Denmark 1972-1974.

Data Source: Hoem and Fong (1976).

4. UNISTATE VERSUS MULTISTATE PERSPECTIVES

A multistate perspective in demographic analysis focuses simultaneously on several interdependent population *stocks*, on the *events* that alter the levels of such stocks, and on the gross *flows* that connect these stocks to form a system of interacting populations. The perspective deals with rates that refer to true populations at risk, and it considers the dynamics of multiple populations exposed to multidimensional growth regimes defined by such rates. All of these attributes are absent in a unistate perspective of growth and change in multiple interacting populations.

To deal with the interlinkages that connect one population's dynamics to another's, the unistate perspective generally must resort to the use of ad hoc procedures and unsatisfactory concepts. For example, to study the evolution of a population that experiences the effects of migration, the unistate perspective has created the statistical fiction of the invisible net migrant. To examine the dynamics of labor force participation, it has defined the labor force participation rate, a measure that is a proportion and not a rate.

But does it really matter? What are the drawbacks of a view that ignores gross flows in favor of a focus on net changes in stocks? In what respects is a multistate perspective superior to a unistate one?

A focus on gross flows more clearly identifies the regularities, illuminates the dynamics, and enhances the understanding of demographic processes that occur within multiple interacting populations. Distinguishing between flows and changes in stocks reveals regularities that otherwise may be obscured; focusing on flows into and out of a state-specific stock exposes dynamics that otherwise may be hidden; and linking explanatory variables to disaggregated gross flows permits a more appropriately specified causal analysis.

4.1 Flows and Changes in Stocks: Problems of Measurement

Net rates express differences between arrivals and departures as a fraction of the single population experiencing both. Within demography they often are viewed as crude indices that reflect differences in propensities of movement, for example, by individuals at different locations, disaggregated by age, sex, and other classifications thought to be relevant for explaining migration.

But net rates also reflect sizes of population stocks. For example, if the gross rates of migration between urban and rural areas of a nation are held constant, the net migration rate will change over time with shifts in the relative population totals in each area. Accordingly, one's inferences about changes in net migration patterns over time will confound the impacts of migration propensities with those of changing population stocks.

To illustrate the above discussion consider, for example, the migration exchange between two neighboring regions of a multiregion system, regions i and j say, that initially contain populations of equal size, $P_i = P_j$ say. Assume that the gross migraproduction rates are equal to unity in both directions, and that the age profiles of both flows are those of the Rogers-Castro standard (Rogers and Castro 1981) defined in Appendix IV. Under these conditions the net migration rate into region i is zero at all ages, as shown by the dotted line in Figure 8. At each age, the number of migrants from region j to region i exactly equals the number in the reverse direction, and the equality also holds for the corresponding rates.

Now imagine that the population in region j grows more rapidly than that of region i, such that it becomes twice as large as its neighbor, that is, $P_j = 2P_i$. Assume that the propensities to migrate in both directions and the associated age profiles remain the same as before [that is, $M_{ij}(x) = M_{ji}(x)$ for all ages x]. Then the resulting net migration rate schedule of region i becomes that of the solid line in Figure 8, that is, the Rogers-Castro standard with a gross migraproduction rate of unity, for

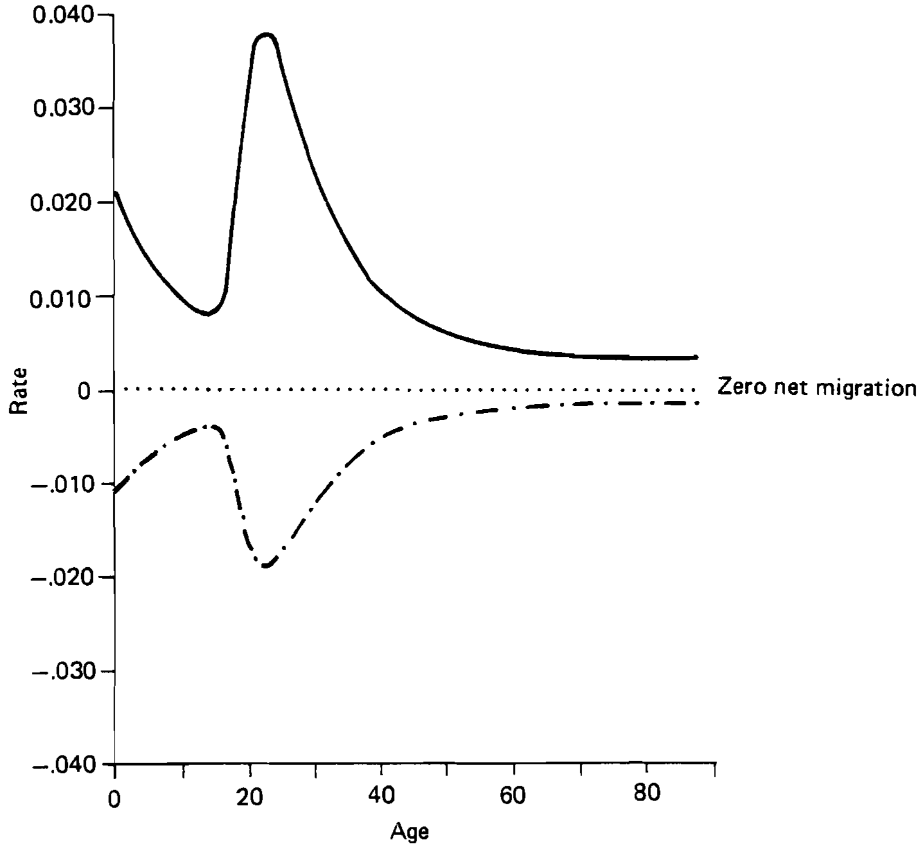


Figure 8. Net migration schedule.

we also include the corresponding net migration rate schedule when $P_j = P_i/2$.

The three net migration schedules in Figure 8 all reflect the same pair of gross migration schedules. In each instance the propensity to migrate in the two directions is the same, and so is the age profile. Yet the net migration rate for region i, say, varies directly with the relative sizes of the two populations, that is, with the ratio P_j/P_i. The net rate is zero at all ages when the ratio is unity, positive at all ages when the ratio exceeds unity, and negative at all ages when the ratio falls short of unity, in the latter two instances following the age profile of the Rogers-Castro standard. Thus, in this illustration, net migration clearly depends on relative population sizes; the effects of flows are confounded with the effects of changes in stocks.

Figure 9 illustrates how changes in levels can affect the age profile of labor force participation rates. In this example, the parametrized model schedules of accession and separation rates for Danish males set out earlier in Figure 7 were kept fixed, but their levels were set equal to those exhibited by Danish females in that year. The resulting new labor force participation rates are shown by the dotted line in Figure 9. A comparison of that schedule with the schedule defined by the solid line identifies the impacts of changing levels or, alternatively, of changing population stocks in the active and inactive states. Once again flows may be seen to be confounded with changes in stocks.

Because net rates confound flows with changes in stocks, they hide regularities that seem to prevail among gross flows. Although the latter tend always to follow the conventional age profile, defined in Section 3.4, the former exhibit a surprisingly wide variety of shapes. For instance, Figure 10 illustrates net migration flows into the Paris Region and the Paris Basin, respectively. Although the two sets of age patterns are distinctly different, the underlying gross flows in each case follow the conventional age profile. This can be seen in Figure 11, for example, which shows the gross rates for the Paris Region.

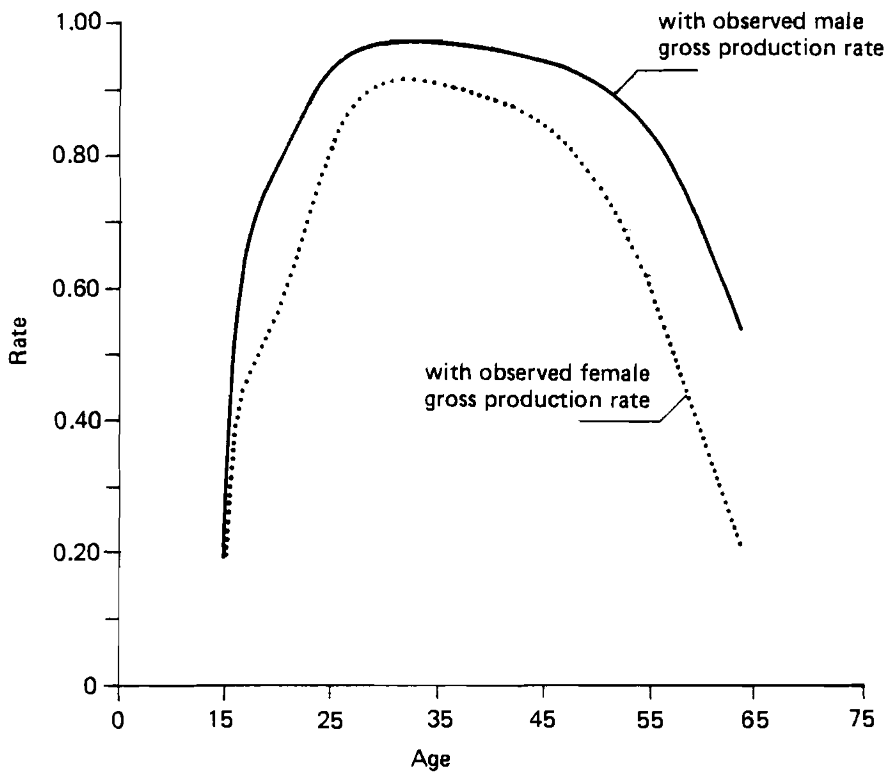


Figure 9. Labor force participation schedules for Danish males.

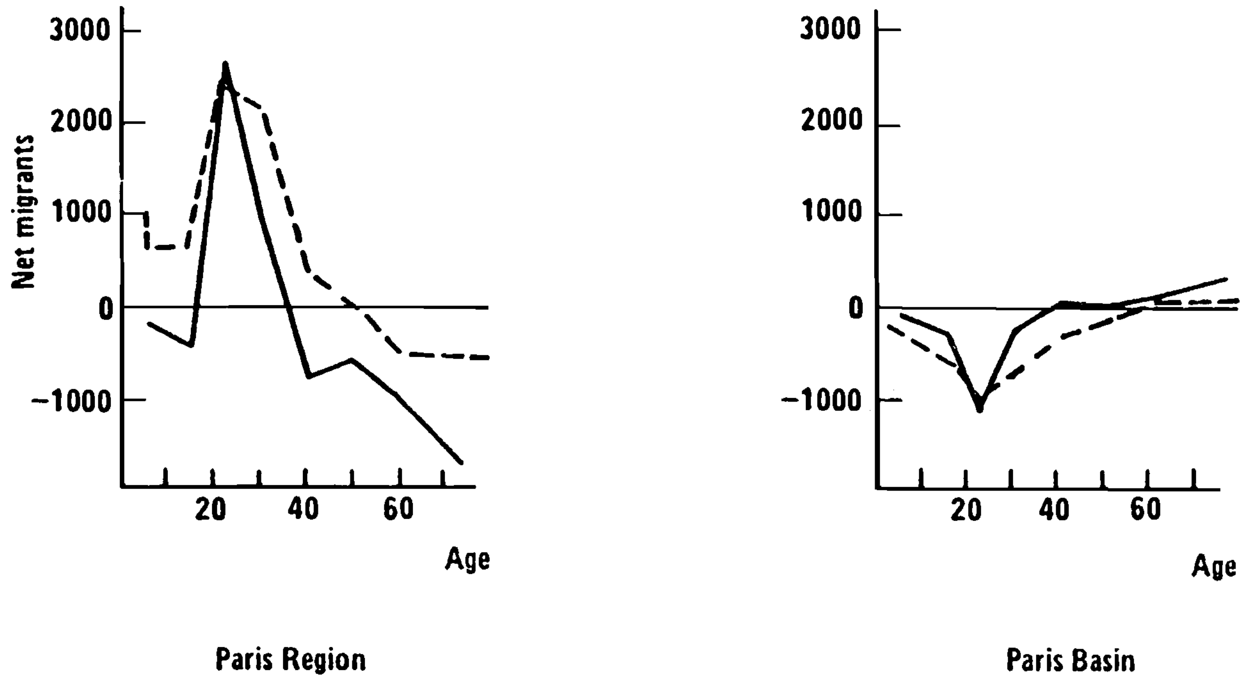


Figure 10. Net migration into the Paris Region and the Paris Basin: 1954-62 (---) and 1968-75 (——).

Source: Ledent with Courgeau (1982).

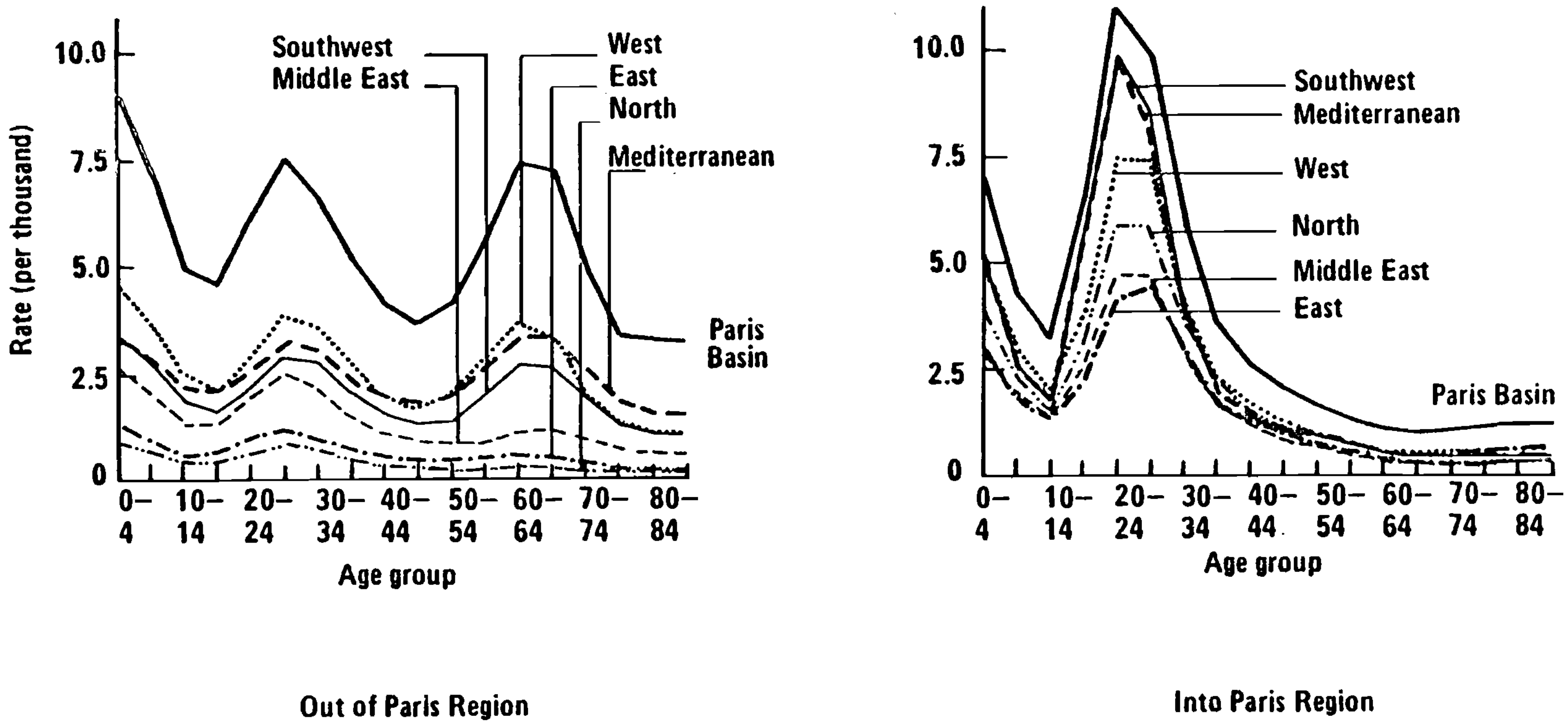


Figure 11. Gross migration schedules out of and into the Paris Region.

Source: Ledent with Courgeau (1982).

4.2 Hidden Dynamics: Problems of Description

Published monthly labor market data are "snapshots" that show the number of individuals in different labor force states at a particular moment in time. Known as the "stock" perspective, this categorization forms the basis of conventional analyses of the unemployment problem. It distinguishes population subgroups by means of differences in stock-based measures, such as the labor force participation rate and the unemployment rate. But these subpopulations also can be differentiated in terms of their gross flow rates, such as their propensities of moving each month, for example, from employment to unemployment or from employment to not in the labor force.

During the past decade, economists have come to realize that a fuller understanding of labor market dynamics can be gained by focusing on the gross flows between labor market states over a period of time—the so-called turnover view of unemployment. Labor market studies using gross flow data have particularly illuminated the ways in which such flows account for labor market behavior generating unemployment and the factors that affect the short-run cyclical behavior of labor supply (for example, the "discouraged" and the "added" worker effects).

Monthly changes in the number of people in each labor market state mask the dynamics underlying these changes. For example, a monthly change of several hundred thousand people in the number of unemployed may be the net result of flows of several million. The net changes in the stock figures are dwarfed by the gross flows that produce them. A recent picture of the Canadian experience during January and February 1981 conveys the message.

> Between these two months, the number of unemployed decreased from 945,000 to 928,000—a drop of only 17,000. Behind this net change were large flows, however: some 367,000 joined the pool of the unemployed (152,000 who were formerly employed and 215,000 who were not in the labor force), while 399,000 persons 'flowed out' (225,000 who became employed and 174,000 who left the labour force). (Economic Council of Canada 1982:47)

Similarly hidden dynamics have been identified in studies of marital status change and in analyses of internal migration. For example, net changes in the stock of married individuals are the consequence of much larger gross flows between those married and those never married, divorced, and widowed. The net contribution to a regional population by migration is also generated by large gross flows in both directions.

> Generally, net migratory gains or losses are only the surface ripples of powerful crosscurrents ... Between 1965 and 1970, for example, metropolitan San Jose, California, gained 75,000 new residents through migration, but 395,000 people moved either in or out. (Morrison 1975:243)

Gross flow statistics shed light on labor market patterns and unemployment dynamics. Particularly important information is provided by time series data that identify the timing of changes in gross flow levels. For example, it has been demonstrated that in the beginning stages of an economic turndown, as the unemployment rate starts to rise, an increasing number of people enter the labor force. At first no concomitant changes in withdrawals from the labor force are observed, but as the recession continues withdrawals increase until their level matches the flow of new entrants. Such patterns cannot be identified with an analysis that focuses on net changes in stocks.

Finally, gross flow data permit the construction of improved population projection models, the principal tool of the demographer's analytical apparatus. It can be demonstrated that multistate projection models based on gross flow statistics are superior to unistate models in at least three respects. First, unistate models can introduce a bias into the projections, and they can produce inconsistent results in long-term prognoses. Second, the impacts of changes in age compositions on movement patterns can be very important, yet a unistate perspective fixes these impacts at the start of a projection and thereby can introduce a potentially serious bias into the projection. Third, multistate projection models have a decisive advantage over unistate models in that they alone can follow subpopulations over time. Thus they can produce disaggregated projections that are impossible to obtain with unistate models.

Consider, for example, how projections of urbanization might be carried out with unistate and multistate models. In a unistate model, the urban population is the central focus of interest and all rural-to-urban migration flows are assessed only with respect to the population in the region of destination, that is, the urban population. Changes in the population at the region of origin are totally ignored, with potentially serious consequences. For example, the rural population ultimately may be reduced to nearly zero, but a fixed and positive net migration into urban areas will nevertheless continue to be generated by the unistate model.

Bias and inconsistency may result from a view of bistate (and, by extension, multistate) population systems through a unistate perspective. For example, expressing migration's contribution to regional population growth solely in terms of the population in the region of destination, can lead to over- or underprojection and introduce inconsistencies in long-run projections. To see this more clearly, consider how the migration specification is altered when a bistate model of urban and rural population growth is transformed into a unistate model. Let urban population growth be described by the equation

$$P_u(t + 1) = (1 + b_u - d_u - o_u)P_u(t) + o_r P_r(t) \qquad (12)$$

Equation 12 states that next year's urban population total may be calculated by adding to this year's urban population (1) the increment due to urban natural increase, (2) the decrement due to urban outmigration to rural areas, and (3) the increment due to rural-to-urban migration.

Now multiplying the last term in Equation 12 by unity expressed as $P_u(t)/P_u(t)$ transforms that equation into its unistate counterpart:

$$P_u(t+1) = (1 + b_u - d_u - o_u)P_u(t) + \left[o_r \frac{P_r(t)}{P_u(t)}\right] P_u(t)$$

$$= (1 + b_u - d_u - o_u + i_u)P_u(t)$$

$$= (1 + b_u - d_u + m_u)P_u(t) = (1 + r_u)P_u(t)$$

where

$$i_u = o_r \frac{P_r(t)}{P_u(t)} = o_r\left[\frac{1 - U(t)}{U(t)}\right]$$

$$m_u = i_u - o_u$$

and $U(t)$ is the fraction of the total national population that is urban at time t. If all annual rates are assumed to be fixed in the bistate projection, then in the unistate model i_u, and therefore also m_u, depend on $U(t)$, which varies in the course of the projection and thereby creates a bias. The dependence of the urban net migration rate m_u on the level of urbanization at time t, $U(t)$, means that m_u must decrease as the level of urbanization increases.

4.3 Heterogeneity, Nonstationarity, and Temporal Dependence: Problems of Explanation

Recent dramatic increases in the divorce rates of a number of populations in the more developed countries have directed attention to alternative explanations of the factors responsible for marital dissolution. The rise of unemployment in many of these same countries has led scholars to examine why the durations of unemployment spells have increased. And policies, aimed at promoting the growth of lagging regional economies, have fostered studies dealing with the migration behavior of subnational populations with various socioeconomic characteristics.

The causal explanations brought forth by studies of marital dissolution, labor market behavior, and population redistribution all too often have been founded on unistate models of population dynamics that reflect inadequate statistical perspectives. For example, no reliable inferences about divorce behavior can be made on the basis of cross-sectional tabulations of changing fractions of a population in the divorced state. Nor can "commitments" to the married state, the employed state, or the current place of residence be identified from such unistate information on changes in stocks. Data on gross flows is essential, and increasingly it is being recognized that such data must be available in disaggregated form.

Simple (Markovian based) multistate demographic models generally adopt three assumptions that are violated by the empirical process generating the data:

1. The population is not a homogeneous one; the same parameter values do not hold for all members of the population.
2. The observed parameter values do not remain constant over time.
3. An individual's propensity to leave a particular state of existence is not independent of his or her past transition patterns and also may not be independent of the transition patterns of others.

In other words the process is *inhomogeneous*, *nonstationary*, and *temporally dependent*. And tests for the empirical validity of each of these assumptions must either establish that the other two assumptions are valid or that controls for their effects are incorporated into the tests (Pickles, Davies, and Crouchley 1982).

As Goldstein (1954), Morrison (1975) and others have noted, observed migration rates tend to reflect the repeated movements of the same subgroup of individuals, *chronic* migrants, and not the single moves of a large number of individuals. The same pattern has also been found in divorce rates and job separation rates. Heterogeneity is the apparent cause.

An alternative explanation is one that associates the probability of a current move with a history of previous moves, positing a "learning" effect. To test for the latter "true contagion", without controlling for the effects of the former can lead to the identification of "spurious contagion" (Feller 1967:121) and to the adoption of higher-order models of temporal dependence than are appropriate.

If the experience of one event raises the probability of experiencing another, then it is equally plausible that non-experience of the event lowers the same probability over time, producing a form of "cumulative inertia":

> The probability of remaining in any state of nature increases as a strict monotonic function of duration of prior residence in that state (McGinnis 1968: 716)

Although early tests of the hypothesis of cumulative inertia may have identified the existence of duration-of-stay effects, they did not establish the existence of cumulative inertia because they did not control for heterogeneity and nonstationarity. The effects of all three factors are indistinguishable if one focuses only on the association between the aggregate rate of mobility and duration of residence. All may predict the same behavior for the total population but do so for different reasons at the individual level. If the population is inhomogeneous, for example, then the observed mobility rate will decline with increasing duration of residence simply because proportionately fewer representatives of the more mobile subpopulations will remain to be classed as nonmovers.

Duration-of-stay effects, and the role of heterogeneity in producing them have important policy implications. Ignoring the effects of heterogeneity leads to prescriptions that reduce the welfare significance of unemployment (Feldstein 1973). Average unemployment is seen to be the consequence of many individuals entering and leaving the pool of unemployed fairly often and not the result of a relatively few workers being without jobs for long periods of time. The implication drawn is that the burden of unemployment is minor and is widely shared and that the appropriate remedy is a policy facilitating job search instead of one of job creation.

The turnover school's view of unemployment has been challenged by those who argue that only a small fraction of total unemployment is accounted for by persons who find a job after a brief spell of unemployment. On this view, most unemployment is attributable to individuals with long spells of joblessness, with repeated spells of unemployment being separated by brief periods outside the labor force.

5. CONCLUSION

A number of pressing national and regional population issues arise as a consequence of unanticipated patterns of change in the age composition, spatial distribution, and group status of population *stocks*. These changes generally evolve slowly, but their effects are widely felt, and the problems they bring in their wake typically are lasting and complex. Public awareness and public action are slow in coming, and all too often both are stimulated by an inadequate understanding of the *processes* generating the patterns of change among stocks.

Demographers have addressed these issues and have sought to understand the associated underlying processes, but their analytical apparatus has been inadequate. A particular shortcoming of this apparatus has been its central focus on the evolution of a single population as it develops over time, while being exposed to sex- and age-specific rates of events, such as births and deaths. Such a unistate perspective of population growth and change is ill-equipped to examine the evolution of a system of interacting populations that are linked by gross flows between various states of existence.

This paper has argued for the adoption of a multistate analysis of population growth and change. In addition to events such as births and deaths, the multistate perspective focuses on gross flows and on multiple interacting populations. It uses these as numerators and denominators, respectively, to define rates of occurrence that refer to populations exposed to the risk of such occurrences, that is, occurrence/exposure rates.

Two important consequences follow. First, the multistate approach avoids potential inconsistencies arising from inappropriately defined rates. Second, it allows one to follow individuals across several changes of states of existence, thereby permitting the disaggregation of current or future population stocks and flows by previous states of existence.

A focus on occurrences of events and transfers and their association with the populations that are exposed to the risk of experiencing them enhances our understanding of, for example, patterns of fertility, mortality, and migration. By not permitting such an association, the unistate approach can produce undesirable biases. For example, urban populations growing rapidly as a consequence of large net rates of urban inmigration cannot be expected to continue to receive a steady stream of such inmigrants as the rural areas empty out. Yet since the latter populations never enter the calculation of net urban inmigration rates, fixed coefficient projections using such rates may produce inconsistencies in the long run.

Heterogeneous populations contain subgroups whose demographic behavior is diverse. To the extent that their differing propensities to experience events and movements can be incorporated in the formal analysis, illumination of the aggregate patterns of behavior is enhanced. For instance, our understanding of marital dissolution is enriched by information on the degree to which divorces occur among those previously divorced. In generating such information, a multistate analysis can identify, for example, how much of the current increase in levels of divorce in many countries can be attributed to "repeaters" as opposed to "first-timers".

Multistate demography is a young branch of formal demography, and its potential contributions are only now coming to be recognized. Further progress in the field will depend to a large extent on the availability of the necessary disaggregated data for carrying out the analyses and projections that should promote its further development and acceptance. In the meantime, parametrized model schedules, such as the ones set out in this paper, can be utilized to demonstrate the approach in problem settings lacking adequate data.

APPENDIX: A SAMPLE OF PARAMETRIZED MODEL SCHEDULES

Contents

IV. MIGRATION

Notation

I. Fertility

R = Total fertility rate

a_0, k, m = Parameters of the Coale-Trussell model fertility schedule

a, μ, α, λ = Parameters of the double exponential model fertility schedule with mean $\bar{x}$, mode x_h, and $\sigma = \lambda/\alpha$

II. Marital Status Change

GR = Gross production rate

a, μ, α, λ = Parameters of the double exponential model marital status change schedule with mean $\bar{x}$, mode x_h, and $\sigma = \lambda/\alpha$

III. Mortality

GR = Gross production rate

$Q_1, \gamma, Q_A, X_A, \sigma, Q_S, X_S$ = Parameters of the Heligman-Pollard model mortality schedule

IV. Migration

GR = Gross production rate

$a_1, \alpha_1, a_2, \mu_2, \alpha_2, \lambda_2, c$ = Parameters of the Rogers-Castro 7-parameter model migration schedule with mean $\bar{x}$, low point x_ℓ, mode x_h, $\sigma_2 = \lambda_2/\alpha_2$, $\delta_{12} = a_1/a_2$, and $\beta_{12} = \alpha_1/\alpha_2$

a_3, α_3 = Additional parameters for the 9-parameter schedule

$a_3, \mu_3, \alpha_3, \lambda_3$ = Additional parameters for the 11-parameter schedule

APPENDIX I: FERTILITY

Data Sources

Denmark: Hoem et al. (1981)

Sweden: Andersson and Holmberg (1980), and a data tape provided for that study by the Swedish Central Bureau of Statistics

Table I.1 Parameter values for Danish Coale-Trussell model fertility schedules.*

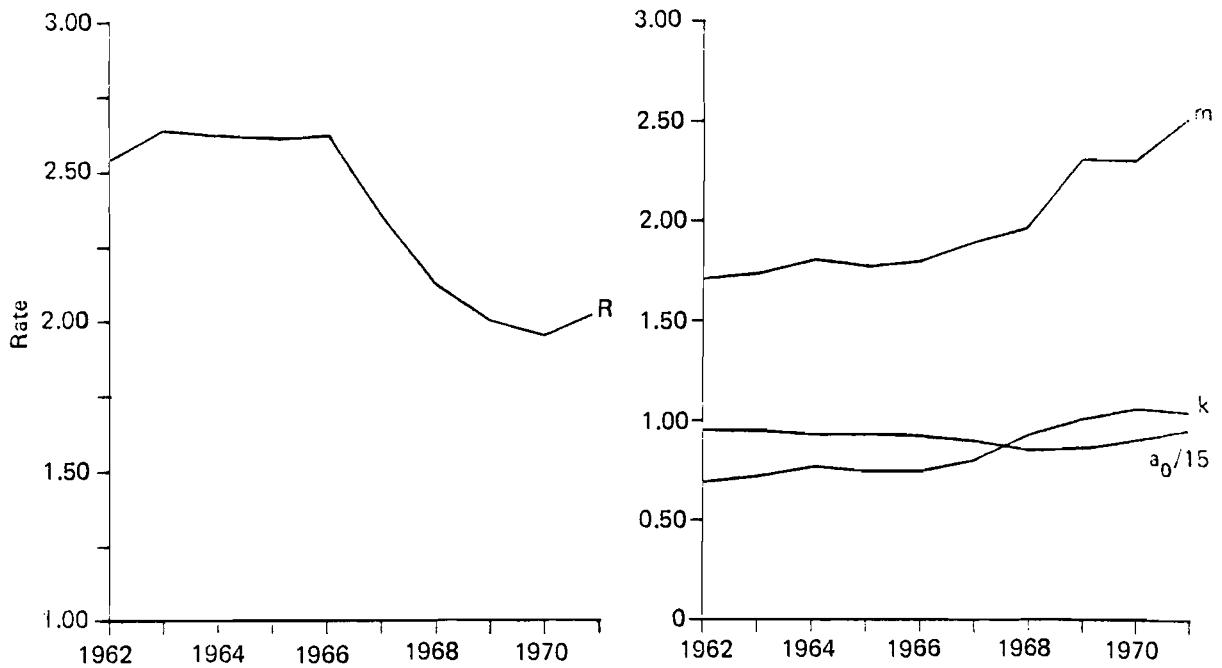
Year	Ŕ	a_0	k	m
1962	2.54	14.29	0.689	1.709
1963	2.64	14.20	0.720	1.734
1964	2.62	13.90	0.768	1.802
1965	2.61	13.95	0.744	1.768
1966	2.62	13.86	0.746	1.796
1967	2.35	13.46	0.805	1.895
1968	2.12	12.81	0.934	1.970
1969	2.00	13.03	1.018	2.112
1970	1.95	13.66	1.068	2.318
1971	2.03	14.38	1.050	2.536

*Add half a year to a_0 to get the "real" age.

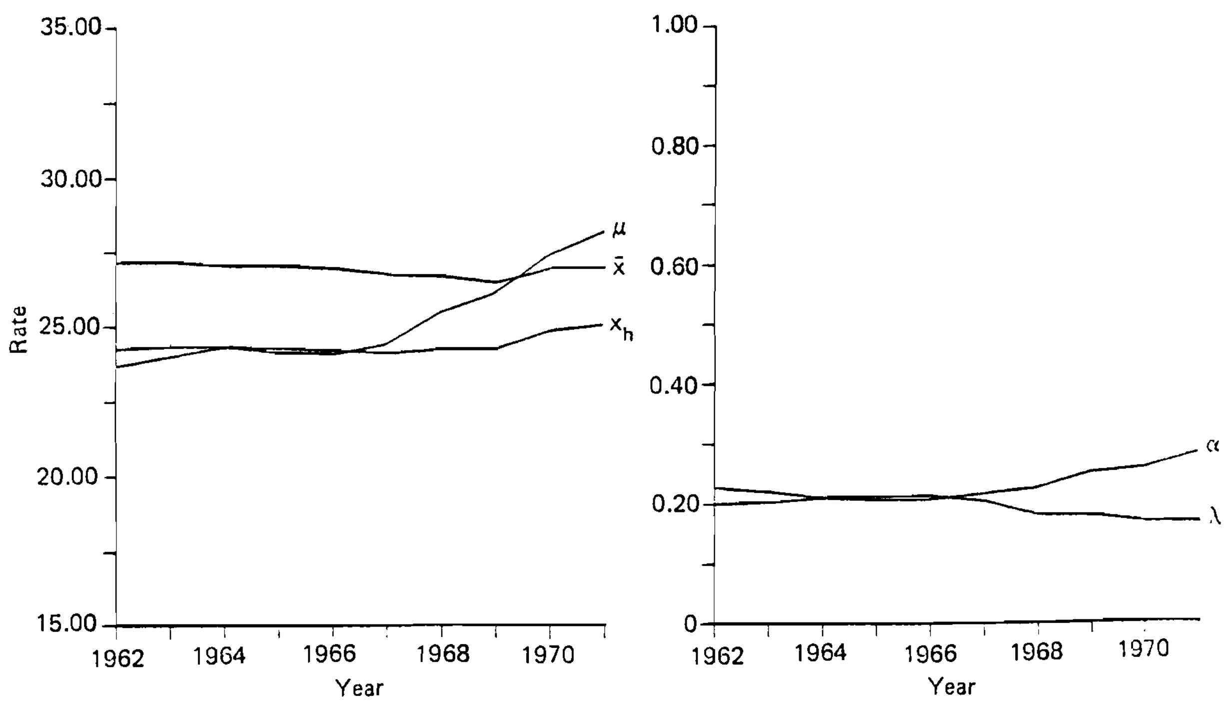
Source: Hoem et al. (1981)

Table I.2 Parameter values for Danish double exponential model fertility schedules (GR = 1).

Year	a	μ	α	λ	σ	$\bar{x}$	x_h
1962	0.212	23.71	0.201	0.226	1.13	27.17	24.26
1963	0.212	24.00	0.204	0.220	1.08	27.19	24.35
1964	0.212	24.37	0.211	0.209	0.99	27.04	24.33
1965	0.212	24.14	0.208	0.213	1.03	27.05	24.27
1966	0.213	24.08	0.209	0.213	1.02	26.95	24.19
1967	0.213	24.41	0.217	0.204	0.94	26.74	24.11
1968	0.203	25.49	0.229	0.182	0.79	26.68	24.23
1969	0.204	26.13	0.254	0.180	0.71	26.44	24.25
1970	0.193	27.39	0.263	0.170	0.65	26.94	24.85
1971	0.186	28.16	0.286	0.168	0.59	26.94	25.03



A. Evolution of the total fertility rate and of the Coale-Trussell model fertility schedule parameters.



B. Evolution of the double exponential model fertility schedule parameters.

Figure I.1 Evolution of model fertility schedule parameters: Denmark, 1962-1971.

Table I.3 Swedish fertility data, 1974.

Parameters and variables	Stockholm Male babies	Stockholm Female babies	Rest of Sweden Male babies	Rest of Sweden Female babies
GR	0.89	0.82	0.92	0.93
a	0.102	0.052	0.144	0.145
μ	32.54	35.89	30.32	30.21
α	0.329	0.361	0.302	0.303
λ	0.133	0.115	0.147	0.148
σ	0.40	0.32	0.49	0.49
$\bar{x}$	27.34	27.48	27.18	27.14
x_h	25.73	26.03	25.41	25.36

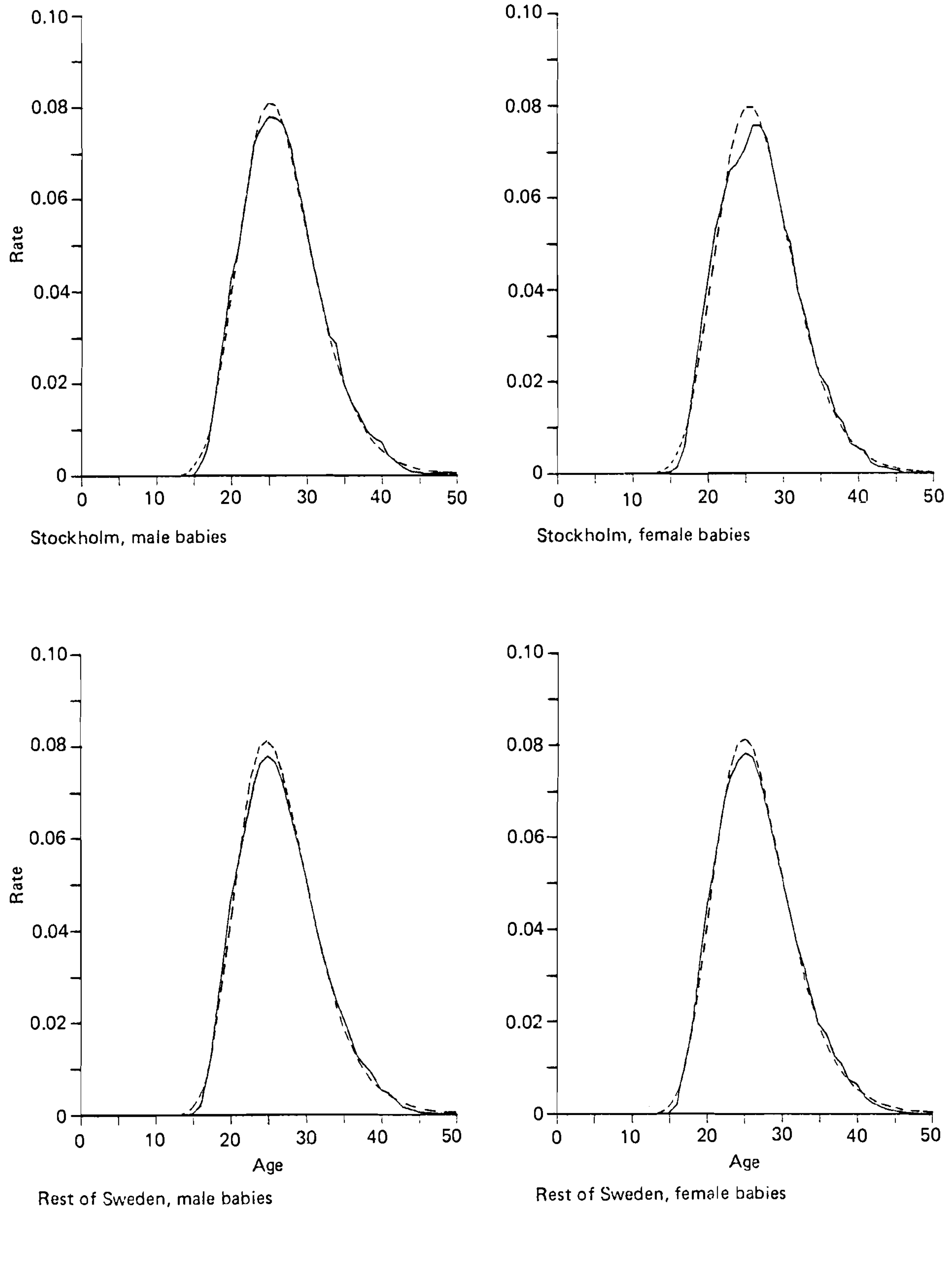


Figure I.2 Double exponential model fertility schedules for Swedish fertility data (--- model schedule, —— observed data), 1974.

APPENDIX II: MARITAL STATUS CHANGE

Data Sources

Australia: Brown and Hall (1978) and a data tape provided by Pamela Williams of the IMPACT Project

Belgium: Willekens (1979)

England and Wales: Schoen and Nelson (1974)

Netherlands: Koesoebjono (1981)

Norway: Brunborg, Monnesland, and Selmer (1981)

U.S. (1960): Schoen and Nelson (1974)

U.S. (1970): Krishnamoorthy (1979)

Table II.1 Norwegian marital status data, 1977-1978.

Parameters and variables	First marriage		Divorce		Remarriage of divorced		Remarriage of widowed	
	Males	Females	Males	Females	Males	Females	Males	Females
GR	1.70	2.06	0.31	0.31	3.32	3.18	4.34	1.56
a	0.145	0.157	0.086	0.081	0.091	0.084	0.098	0.129
μ	21.93	20.29	23.45	20.72	20.63	17.23	21.10	20.64
α	0.128	0.140	0.075	0.072	0.083	0.080	0.093	0.115
λ	0.353	0.343	0.227	0.224	0.485	1.000	0.999	0.333
σ	2.76	2.46	3.02	3.12	5.86	12.47	10.78	2.88
$\bar{x}$	30.03	27.62	36.98	35.02	33.20	30.05	32.24	29.65
x_h	24.82	22.91	28.33	25.79	24.28	19.76	23.49	23.82

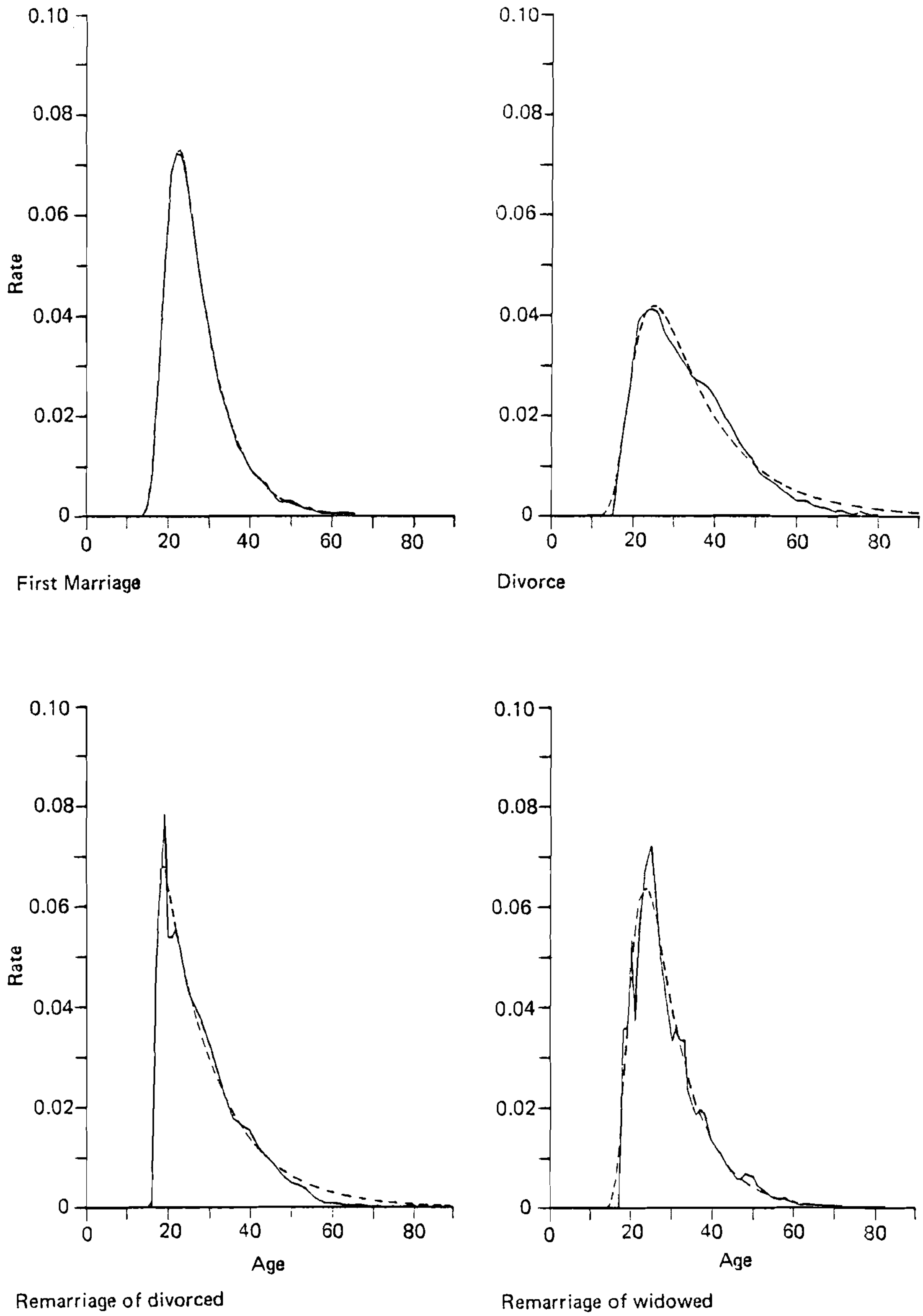


Figure II.1 Model schedules of marital status change: Norwegian females (--- model schedules, —— observed data), 1977-1978.

Table II.2 Model schedules of marital status change: Australian females 1921, 1950, 1976.

Parameters and variables	First marriage			Divorce			Remarriage of divorced			Remarriage of widowed		
	1921	1950	1976	1921	1950	1976	1921	1950	1976	1921	1950	1976
GR	2.17	3.33	2.73	0.05	0.16	0.29	5.74	10.39	6.48	3.57	4.77	2.83
a	0.147	0.133	0.119	0.094	0.108	0.060	0.091	0.086	0.079	0.109	0.103	0.082
μ	21.32	19.33	18.26	23.67	32.79	21.83	20.31	17.40	16.02	18.44	18.37	17.62
α	0.130	0.119	0.110	0.081	0.113	0.052	0.084	0.080	0.071	0.101	0.095	0.076
λ	0.282	0.395	0.528	0.196	0.100	0.319	1	1	1	1	1	1
σ	2.17	3.32	4.85	2.42	0.88	6.11	11.97	12.51	14.09	9.90	10.48	13.14
$\bar{x}$	29.00	28.15	27.95	36.03	36.50	40.31	32.58	30.21	30.28	28.73	29.23	31.00
x_h	24.07	22.38	21.25	28.19	31.52	27.52	22.80	19.94	18.68	20.74	20.73	20.21

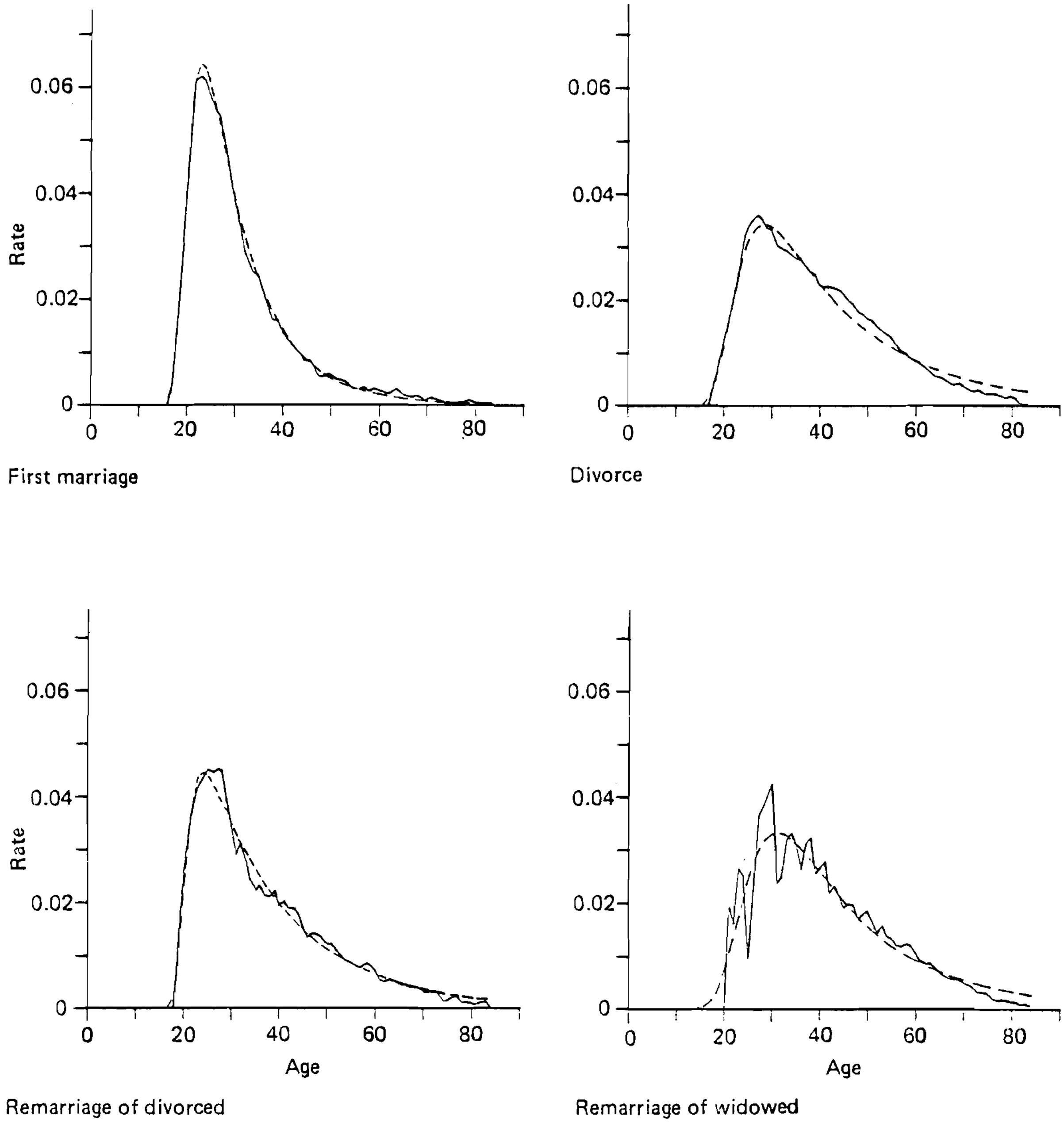


Figure II.2 Double exponential model schedules of marital status change (--- model schedule, —— observed data): Australian males 1976.

Table II.3 First-marriage rates of the Australian single population: 1921-1976.

Parameters and variables	MALES Mean value	MALES Range	FEMALES Mean value	FEMALES Range
GR	2.44	(1.56 - 3.01)	2.83	(1.57 - 3.56)
a	0.116	(0.101 - 0.128)	0.138	(0.114 - 0.151)
μ	21.88	(19.90 - 23.89)	19.88	(18.26 - 21.32)
α	0.104	(0.093 - 0.113)	0.123	(0.104 - 0.133)
λ	0.361	(0.253 - 0.532)	0.367	(0.273 - 0.528)
σ	3.50	(2.26 - 5.38)	3.02	(2.09 - 4.85)
$\bar{x}$	31.91	(30.12 - 33.36)	28.33	(27.46 - 29.06)
x_h	25.28	(23.03 - 27.09)	22.82	(21.25 - 24.08)

Table II.4 Divorce rates of the Australian married population: 1921-1976.

Parameters and variables	MALES Mean value	MALES Range	FEMALES Mean value	FEMALES Range
GR	0.14	(0.06 - 0.58)	0.14	(0.05 - 0.56)
a	0.087	(0.067 - 0.106)	0.095	(0.060 - 0.114)
μ	26.85	(23.73 - 31.80)	25.07	(21.04 - 32.79)
α	0.075	(0.058 - 0.095)	0.083	(0.052 - 0.113)
λ	0.189	(0.118 - 0.277)	0.185	(0.100 - 0.319)
σ	2.60	(1.25 - 4.22)	2.36	(0.88 - 6.11)
$\bar{x}$	39.85	(37.13 - 43.57)	36.73	(33.89 - 40.54)
x_h	31.46	(29.06 - 34.03)	29.02	(26.93 - 31.52)

Table II.5 Remarriage rates of the Australian divorced population: 1921-1976.

Parameters and variables	MALES Mean value	Range	FEMALES Mean value	Range
GR	7.99	(4.17 - 11.16)	8.08	(4.69 - 10.64)
a	0.073	(0.060 - 0.088)	0.091	(0.079 - 0.101)
μ	20.49	(17.89 - 23.58)	18.47	(16.02 - 20.31)
α	0.067	(0.054 - 0.081)	0.084	(0.071 - 0.093)
λ	1	1	1	1
σ	15.05	(12.33 - 18.45)	11.91	(10.71 - 14.09)
$\bar{x}$	35.43	(32.91 - 39.37)	30.69	(29.28 - 32.61)
x_h	23.21	(20.63 - 26.37)	20.96	(18.68 - 22.80)

Table II.6 Remarriage rates of the Australian widowed population: 1921-1976.

Parameters and variables	MALES Mean value	Range	FEMALES Mean value	Range
GR	4.47	(3.32 - 6.22)	3.80	(2.38 - 5.99)
a	0.058	(0.045 - 0.069)	0.105	(0.080 - 0.131)
μ	20.31	(17.38 - 22.29)	17.02	(15.34 - 18.89)
α	0.053	(0.038 - 0.063)	0.095	(0.073 - 0.114)
λ	1	1	1	1
σ	19.38	(15.92 - 26.17)	10.71	(8.77 - 13.67)
$\bar{x}$	38.41	(33.51 - 42.53)	28.09	(25.19 - 31.22)
x_h	23.27	(20.18 - 25.42)	19.39	(17.81 - 21.20)

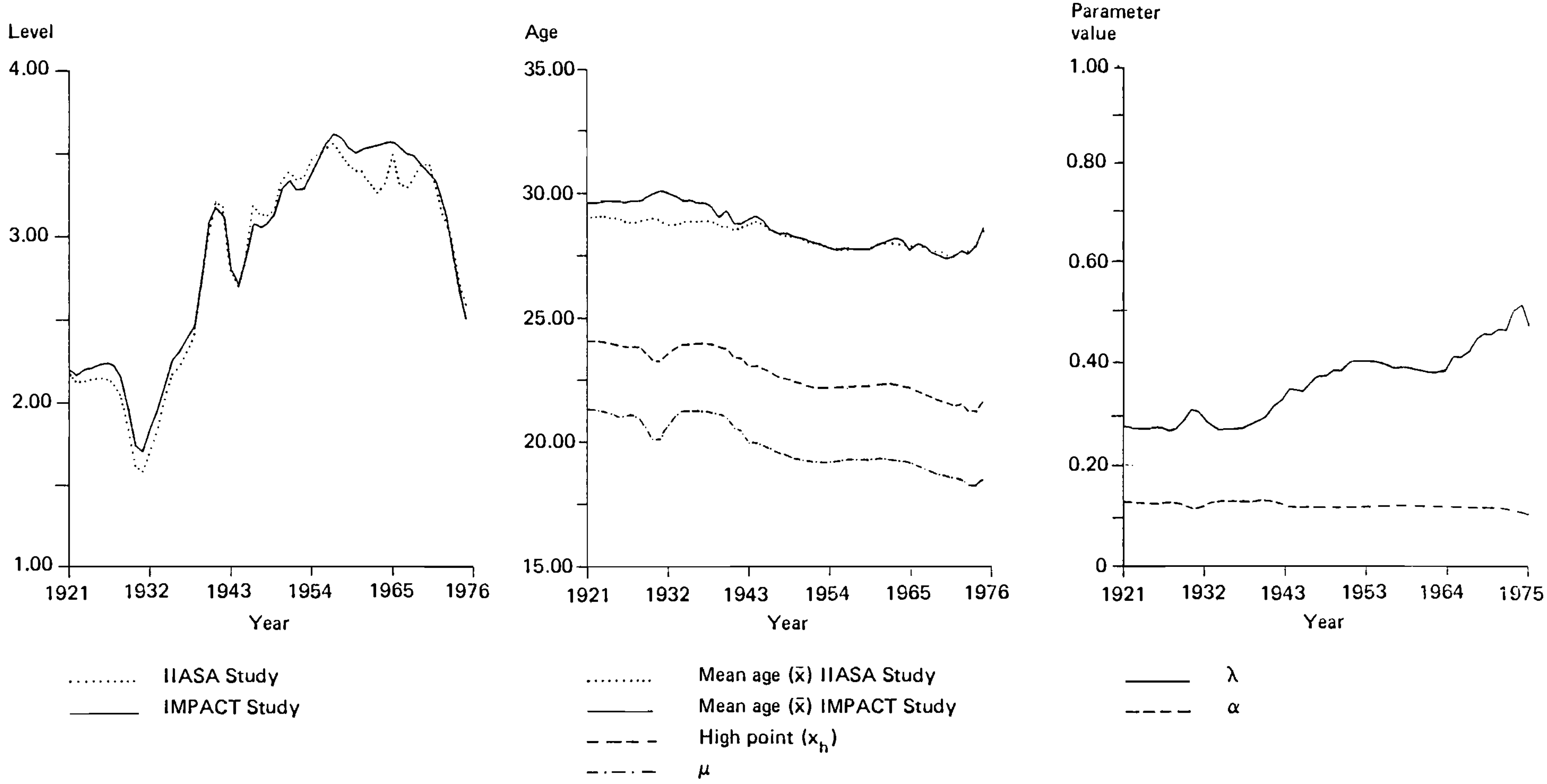


Figure II.3 The parameters and variables of the model schedules of first marriage for never married Australian females, 1921-1976.

Source: For IMPACT Study, Williams (1981).

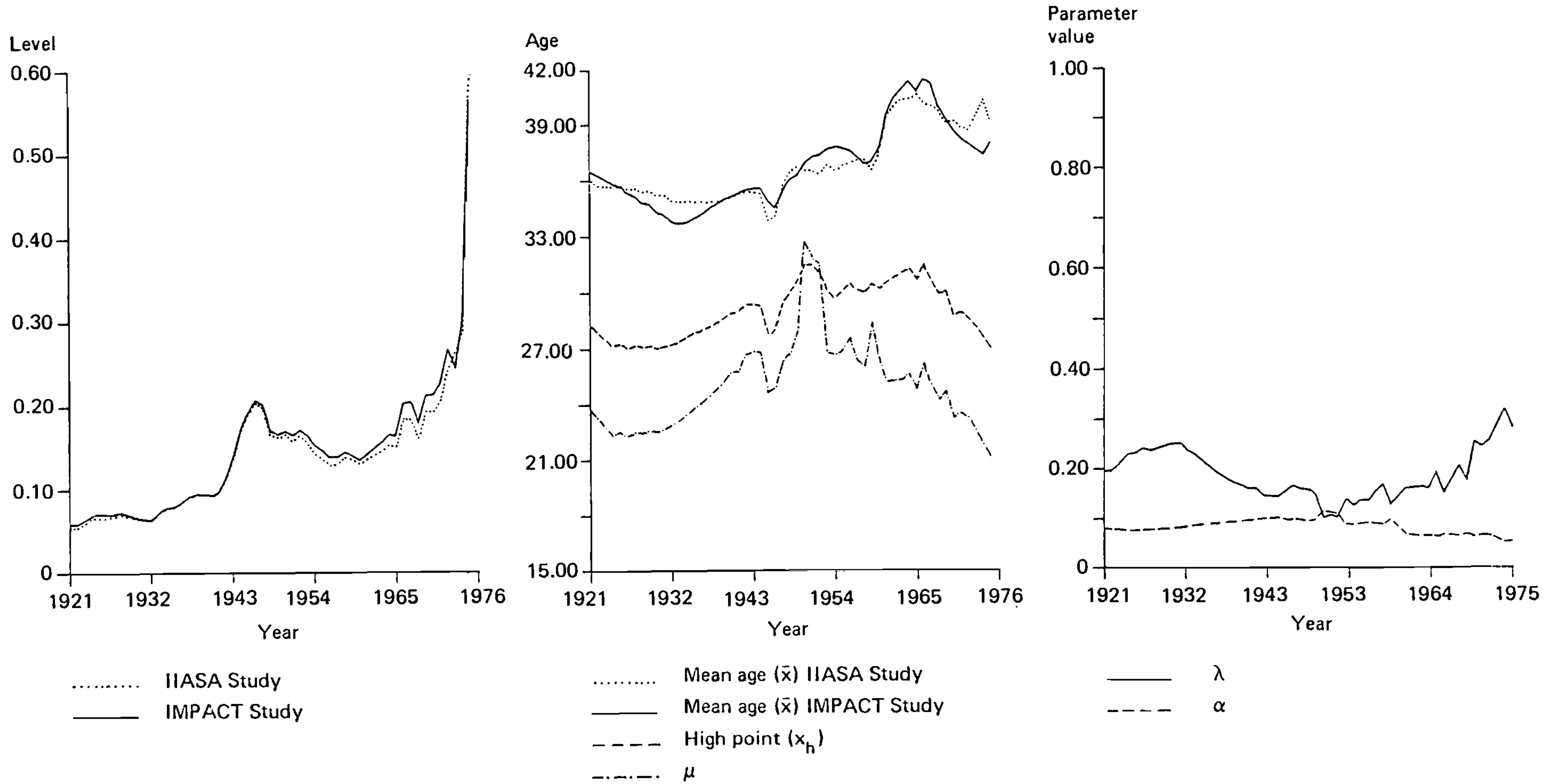


Figure II.4 The parameters and variables of the model schedules of divorce for married Australian females, 1921-1976.

Source: For IMPACT Study, Williams (1981).

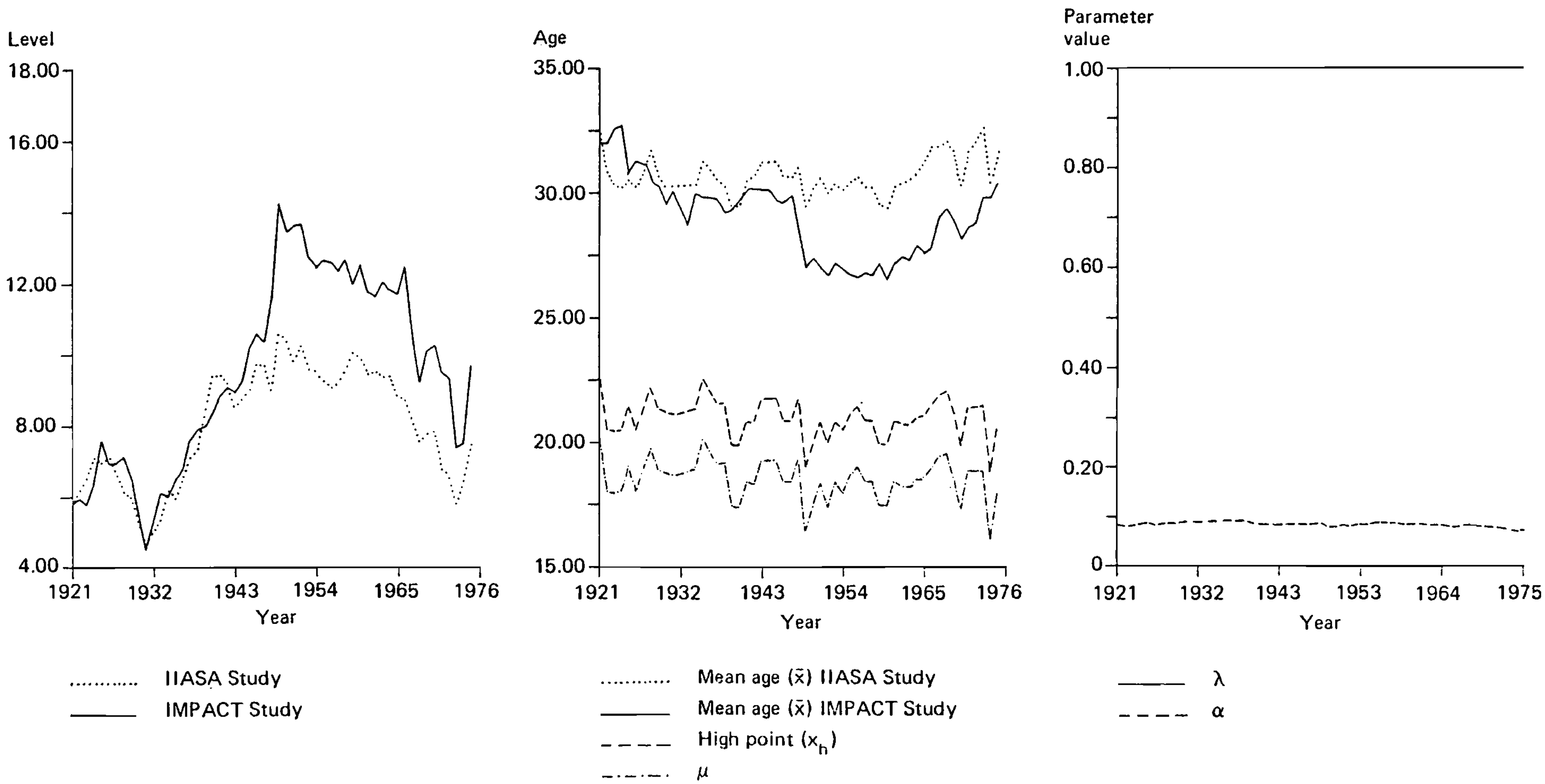


Figure II.5 The parameters and variables of the model schedules of remarriage for divorced Australian females, 1921-1976.

Source: For IMPACT Study, Williams (1981).

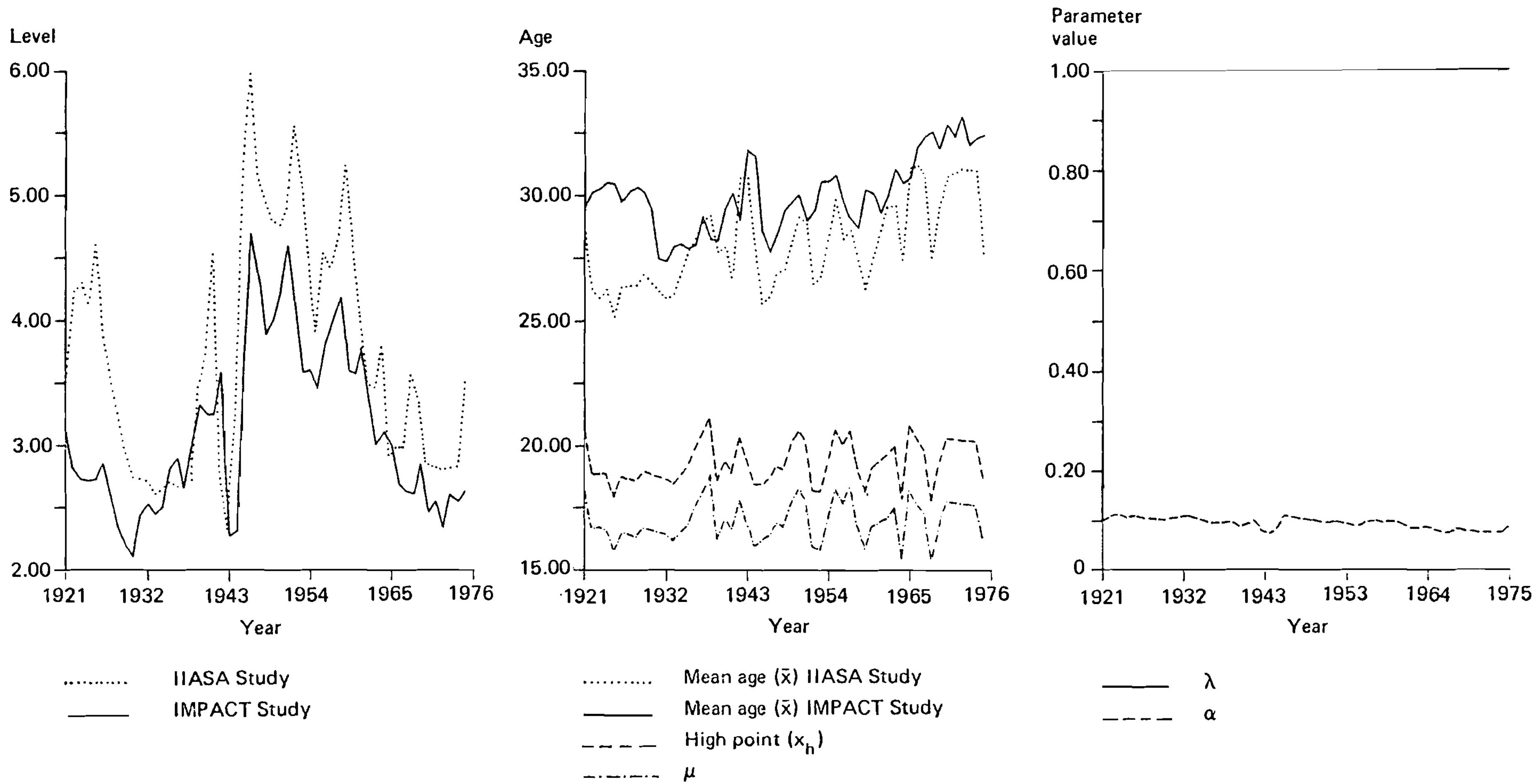


Figure II.6 The parameters and variables of the model schedules of remarriage for widowed Australian females, 1921-1976.

Source: For IMPACT Study, Williams (1981).

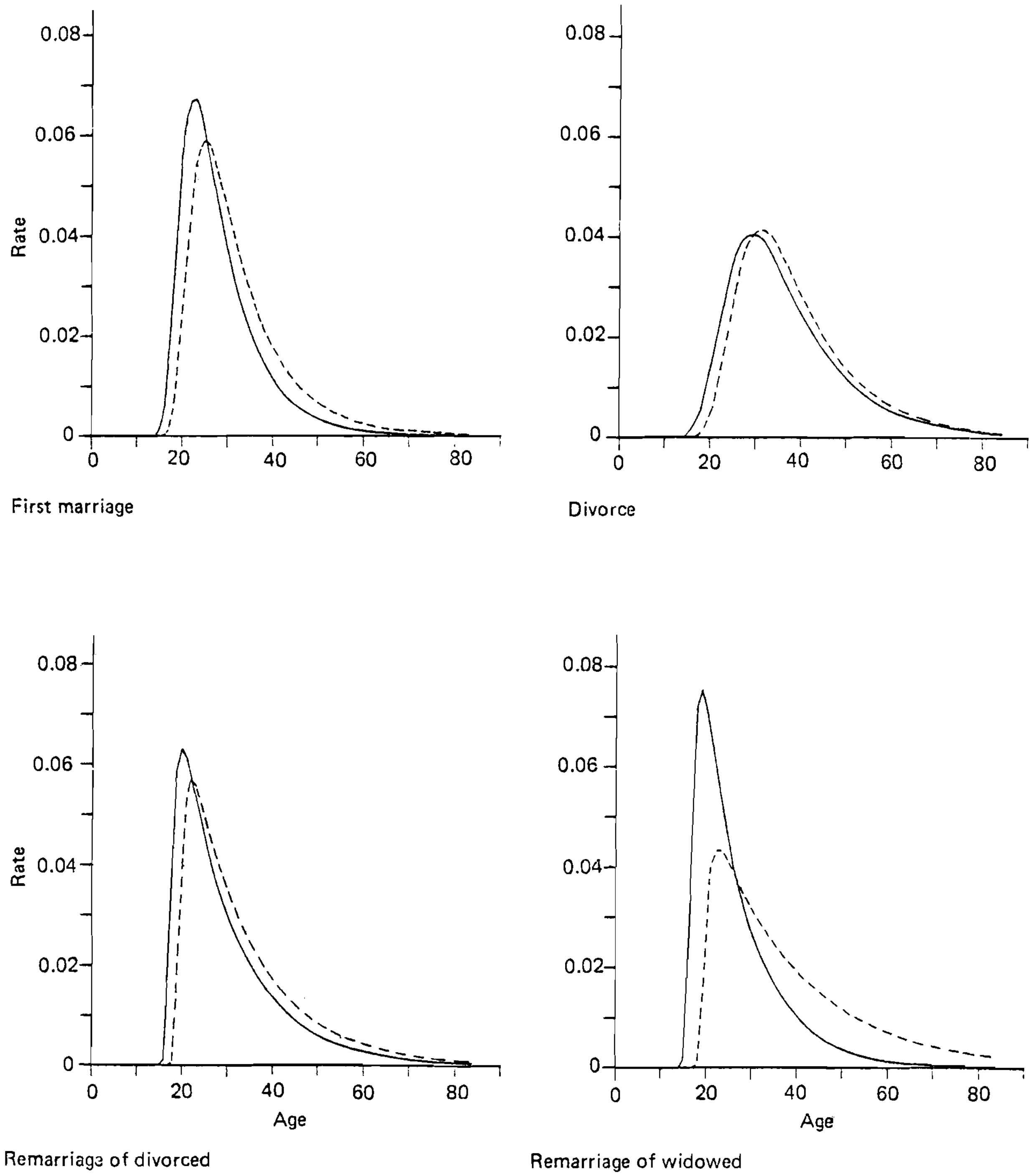


Figure II.7 Model schedules of marital status change (--- males, —— females): the Australian standard.

Table II.7 First-marriage rates of national female single populations.

Parameters and variables	Austra-lian standard	Australia 1975	Belgium 1971	England and Wales 1961	Nether-lands 1978	U.S. 1960	U.S. 1970
GR	2.83	2.73	3.34	3.29	2.11	3.72	3.42
a	0.14	0.119	0.168	0.142	0.157	0.131	0.125
μ	20	18.26	19.84	19.21	20.05	18.43	18.75
α	0.12	0.110	0.147	0.128	0.136	0.117	0.112
λ	0.37	0.528	0.475	0.516	0.389	0.455	0.429
σ	3.0	4.85	3.23	4.04	2.86	3.88	3.83
$\bar{x}$	28.3	27.95	26.99	27.47	27.70	27.44	28.18
x_h	22.8	21.25	22.33	21.92	22.76	21.43	21.90

Table II.8 Divorce rates of national female married populations.

Parameters and variables	Austra-lian standard	Australia 1975	Belgium 1971	England and Wales 1961	Nether-lands 1978	U.S. 1960	U.S. 1970
GR	0.14	0.29	0.14	0.10	0.33	0.47	0.76
a	0.10	0.060	0.111	0.092	0.066	0.073	0.066
μ	25	21.83	26.46	24.37	19.30	18.22	21.00
α	0.08	0.052	0.098	0.081	0.058	0.064	0.059
λ	0.18	0.319	0.149	0.242	0.211	0.301	0.273
σ	2.4	6.11	1.51	2.98	3.67	4.70	4.65
$\bar{x}$	36.7	40.31	35.34	36.97	36.56	34.13	37.96
x_h	29.0	27.52	29.24	28.90	25.47	23.37	26.63

Table II.9 Remarriage rates of national female divorced populations.

Parameters and variables	Australian standard	Australia 1975	Belgium 1971	England and Wales 1961	Netherlands 1978	U.S. 1960	U.S. 1970
GR	8.08	6.48	4.79	10.01	3.76	10.81	9.34
a	0.09	0.079	0.083	0.093	0.078	0.082	0.080
μ	18	16.02	19.41	16.50	15.51	16.16	15.94
α	0.08	0.071	0.076	0.086	0.075	0.077	0.076
λ	1	1	0.400	0.590	1.751	0.803	0.984
σ	11.9	14.09	5.29	6.90	23.37	10.41	12.95
$\bar{x}$	30.7	30.28	33.12	28.71	28.93	29.47	29.37
x_h	21.0	18.68	23.58	19.79	17.32	19.09	18.55

Table II.10 Remarriage rates of national female widowed populations.

Parameters and variables	Australian standard	Australia 1975	Belgium 1971	England and Wales 1961	Netherlands 1978	U.S. 1960	U.S. 1970
GR	3.80	2.83	1.92	3.41	1.62	3.80	2.37
a	0.10	0.082	0.109	0.084	0.122	0.078	0.075
μ	17	17.62	19.07	16.56	17.20	16.15	21.21
α	0.10	0.076	0.097	0.077	0.108	0.073	0.065
λ	1	1	0.343	0.530	0.890	0.805	0.279
σ	10.7	13.14	3.54	6.90	8.23	11.01	4.29
$\bar{x}$	28.1	31.00	29.90	30.05	28.87	30.12	36.80
x_h	19.4	20.21	22.77	20.21	19.58	19.14	26.44

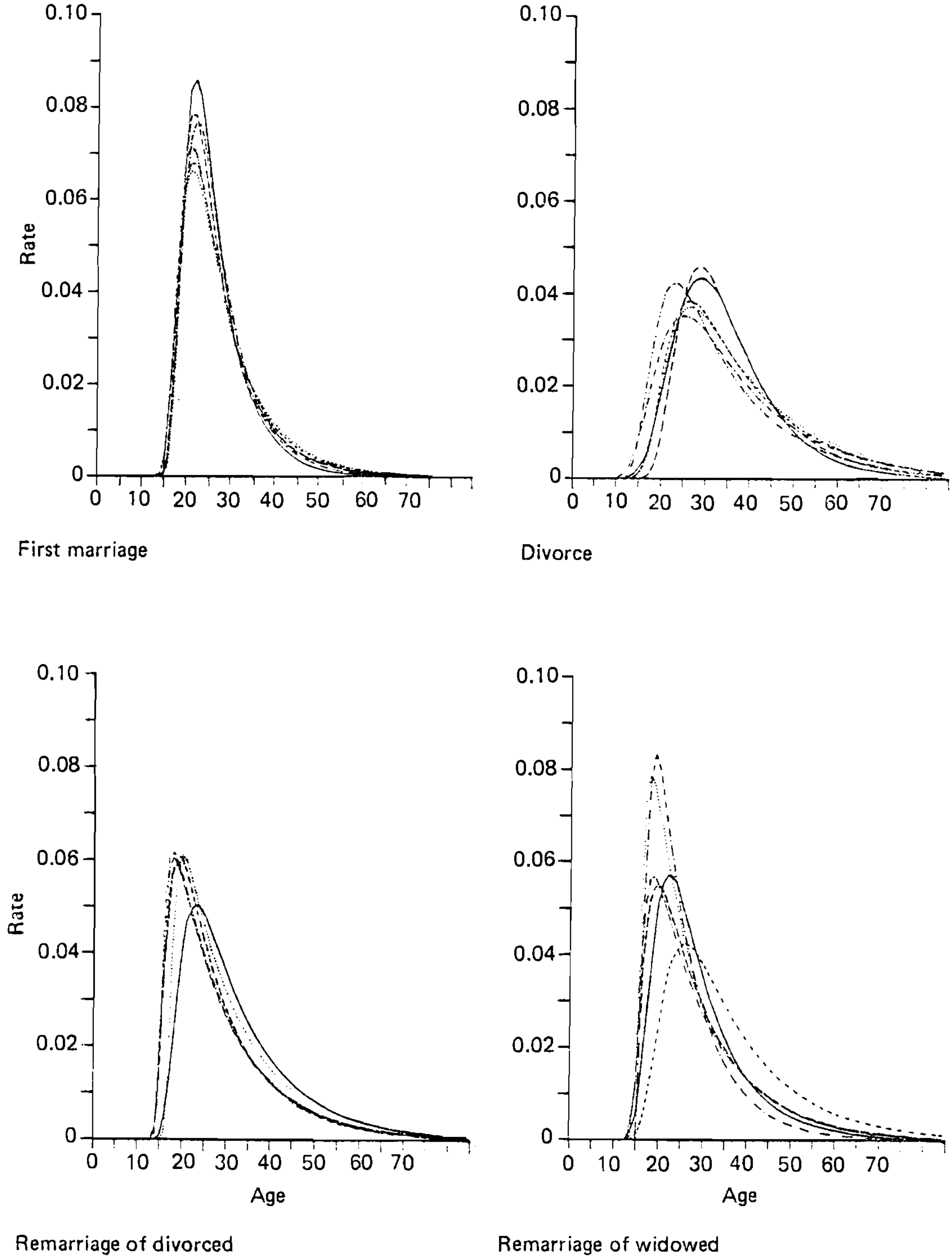


Figure II.8 Six national model schedules of marital status change.

APPENDIX III: MORTALITY

Data Sources

Australia: Brooks et al. (1980)

Sweden: Andersson and Holmberg (1980) and a data tape provided for that study by the Swedish Central Bureau of Statistics

Table III.1 Parameter values for never married Australian females, 1950-1975.

Year	Q_1	γ	Q_A	Q_S	X_S
1950	0.00569	0.192	0.003326	0.000031	8.8
1951	0.00566	0.188	0.003193	0.000010	8.7
1952	0.00538	0.184	0.003134	0.000008	8.6
1953	0.00505	0.183	0.003037	0.000009	8.6
1954	0.00491	0.187	0.002739	0.000011	8.9
1955	0.00471	0.183	0.002683	0.000010	8.8
1956	0.00482	0.194	0.002926	0.000009	8.7
1957	0.00454	0.189	0.002775	0.000011	9.0
1958	0.00424	0.186	0.002680	0.000012	9.0
1959	0.00466	0.197	0.003003	0.000009	8.7
1960	0.00405	0.179	0.002890	0.000007	8.6
1961	0.00396	0.188	0.002824	0.000008	8.7
1962	0.00408	0.196	0.003098	0.000008	8.7
1963	0.00389	0.188	0.002955	0.000010	8.9
1964	0.00375	0.183	0.002959	0.000011	8.9
1965	0.00357	0.187	0.003065	0.000010	8.9
1966	0.00417	0.213	0.003755	0.000006	8.4
1967	0.00405	0.210	0.003286	0.000012	9.1
1968	0.00425	0.208	0.003336	0.000008	8.7
1969	0.00381	0.198	0.003643	0.000011	9.0
1970	0.00419	0.223	0.003277	0.000017	9.4
1971	0.00394	0.212	0.003333	0.000014	9.4
1972	0.00336	0.194	0.003576	0.000011	9.1
1973	0.00347	0.209	0.002917	0.000015	9.5
1974	0.00260	0.165	0.003001	0.000010	9.0
1975	0.00282	0.191	0.002849	0.000011	9.1

*The parameters X_A and σ were held fixed and set equal to 50.0 and 0.7071, respectively.

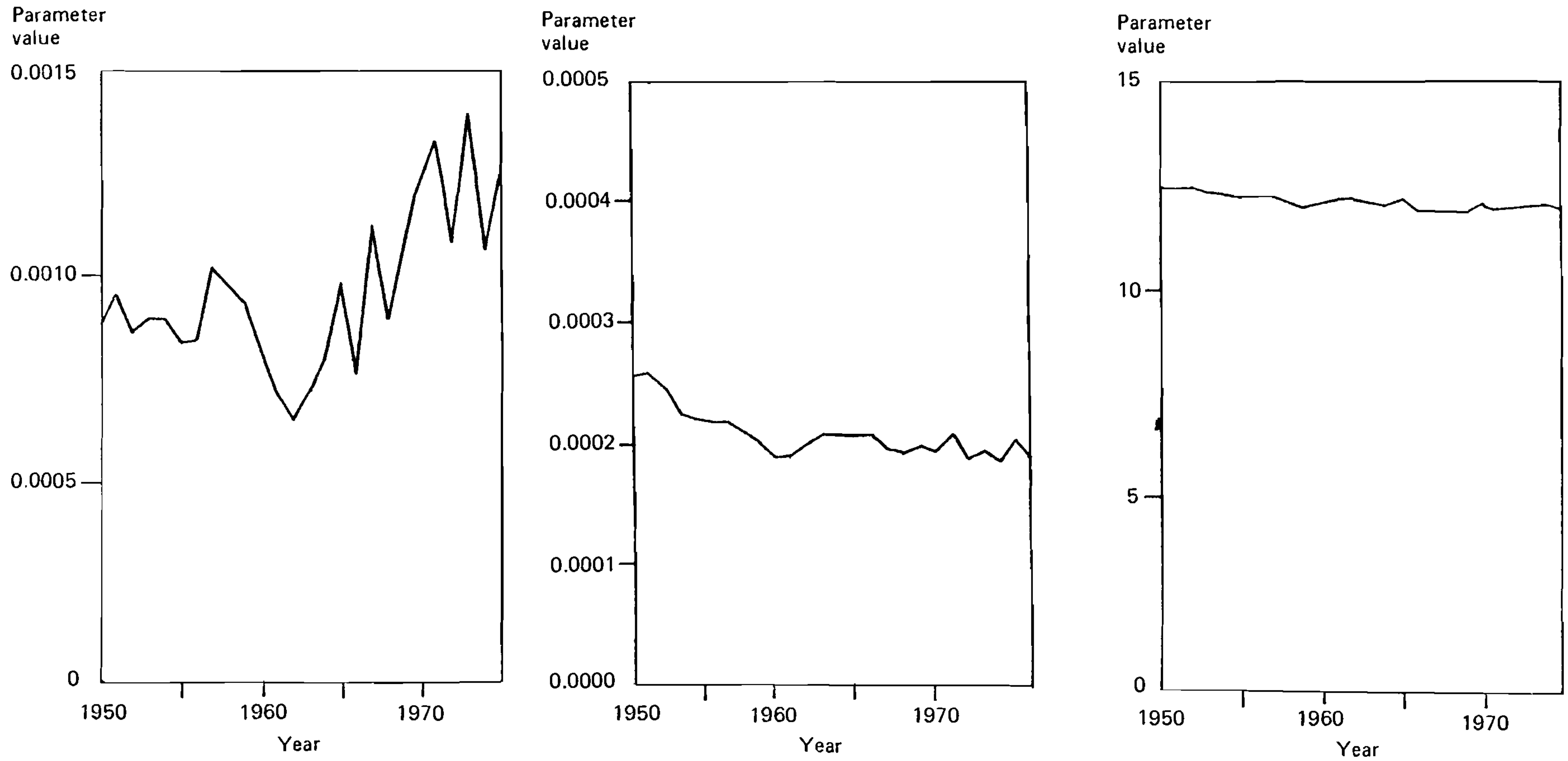
Source: Brooks et al. (1980).

Table III.2 Parameter values for never married Australian males, 1950-1975.

Year	Q_1	γ	Q_A	Q_S	X_S
1950	0.00675	0.226	0.000864	0.000255	12.4
1951	0.00658	0.228	0.000954	0.000259	12.4
1952	0.00644	0.230	0.000866	0.000248	12.4
1953	0.00614	0.221	0.000899	0.000226	12.3
1954	0.00605	0.226	0.000895	0.000221	12.2
1955	0.00596	0.239	0.000841	0.000218	12.2
1956	0.00576	0.241	0.000846	0.000218	12.2
1957	0.00571	0.241	0.001024	0.000210	12.2
1958	0.00549	0.242	0.000984	0.000202	12.1
1959	0.00551	0.243	0.000936	0.000190	11.9
1960	0.00520	0.247	0.000821	0.000192	12.0
1961	0.00492	0.249	0.000721	0.000201	12.1
1962	0.00505	0.267	0.000653	0.000208	12.2
1963	0.00506	0.273	0.000718	0.000207	12.1
1964	0.00481	0.259	0.000800	0.000206	12.0
1965	0.00483	0.271	0.000976	0.000207	12.1
1966	0.00532	0.281	0.000768	0.000196	11.9
1967	0.00446	0.273	0.001124	0.000193	11.9
1968	0.00469	0.279	0.000897	0.000198	11.9
1969	0.00455	0.266	0.001094	0.000194	11.8
1970	0.00463	0.278	0.001229	0.000207	12.0
1971	0.00505	0.295	0.001326	0.000189	11.9
1972	0.00433	0.263	0.001088	0.000195	12.0
1973	0.00445	0.280	0.001386	0.000187	12.0
1974	0.00386	0.259	0.001064	0.000203	12.0
1975	0.00356	0.277	0.001252	0.000186	11.9

*The parameters X_A and σ were held fixed and set equal to 21.0 and 0.2, respectively.

Source: Brooks et al. (1980).



The value of the parameter Q_A for never married males for the years 1950–1975

The values of the parameter Q_S for never married males for 1950–1975

The values of the parameter X_S for never married males for 1950–1975

Figure III.1 The values of the parameters for never married Australian males for the years 1950-1975.

Source: Brooks et al. (1980).

Table III.3 The Brass standard mortality schedule.

x	ℓ(x)	m(x)	x	ℓ(x)	m(x)	x	ℓ(x)	m(x)	x	ℓ(x)	m(x)
1	0.8499	0.05178	26	0.6764	0.00891	51	0.5010	0.01975	76	0.1358	0.12353
2	0.8070	0.02433	27	0.6704	0.00914	52	0.4912	0.02119	77	0.1200	0.13333
3	0.7876	0.01458	28	0.6643	0.00892	53	0.4809	0.02271	78	0.1050	0.14395
4	0.7762	0.00919	29	0.6584	0.00900	54	0.4701	0.02389	79	0.0909	0.15786
5	0.7691	0.00639	30	0.6525	0.00908	55	0.4590	0.02560	80	0.0776	0.17063
6	0.7642	0.00538	31	0.6466	0.00932	56	0.4474	0.02719	81	0.0654	0.18546
7	0.7601	0.00438	32	0.6406	0.00941	57	0.4354	0.02913	82	0.0543	0.20061
8	0.7564	0.00424	33	0.6346	0.00982	58	0.4229	0.03122	83	0.0444	0.22000
9	0.7532	0.00399	34	0.6284	0.00975	59	0.4099	0.03323	84	0.0356	0.23548
10	0.7502	0.00334	35	0.6223	0.01018	60	0.3965	0.03594	85	0.0281	0.25703
11	0.7477	0.00335	36	0.6160	0.01028	61	0.3825	0.03837	86	0.0217	0.28421
12	0.7452	0.00363	37	0.6097	0.01072	62	0.3681	0.04131	87	0.0163	0.30389
13	0.7425	0.00391	38	0.6032	0.01100	63	0.3532	0.04428	88	0.0120	0.33010
14	0.7396	0.00461	39	0.5966	0.01146	64	0.3379	0.04788	89	0.0086	0.35616
15	0.7362	0.00477	40	0.5898	0.01177	65	0.3221	0.05159	90	0.0060	0.40000
16	0.7327	0.00547	41	0.5829	0.01208	66	0.3059	0.05578	91	0.0040	0.42424
17	0.7287	0.00633	42	0.5759	0.01276	67	0.2893	0.06017	92	0.0026	0.47619
18	0.7241	0.00721	43	0.5686	0.01328	68	0.2724	0.06481	93	0.0016	0.46154
19	0.7189	0.00824	44	0.5611	0.01382	69	0.2553	0.07014	94	0.0010	0.50000
20	0.7130	0.00859	45	0.5534	0.01456	70	0.2380	0.07538	95	0.0006	0.66667
21	0.7069	0.00909	46	0.5454	0.01515	71	0.2206	0.08211	96	0.0003	0.40000
22	0.7005	0.00875	47	0.5372	0.01595	72	0.2032	0.08892	97	0.0002	0.66667
23	0.6944	0.00868	48	0.5287	0.01698	73	0.1859	0.09642	98	0.0001	2.00000
24	0.6884	0.00846	49	0.5198	0.01786	74	0.1688	0.10408	99	0.	0.
25	0.6826	0.00912	50	0.5106	0.01898	75	0.1521	0.11323	100	0.	0.

Source: Zaba (1979)

Table III.4 Parameter estimates of the Heligman-Pollard model mortality schedule.

Variables	Brass standard	Swedish males, 1974
GR	5.34	2.86
Q_1	0.006077	0.000872
γ	0.183204	0.160647
Q_A	0.001261	0.000238
X_A	27.5	22.8
σ	0.72	0.42
Q_S	0.000044	0.000013
X_S	12.018772	10.059491

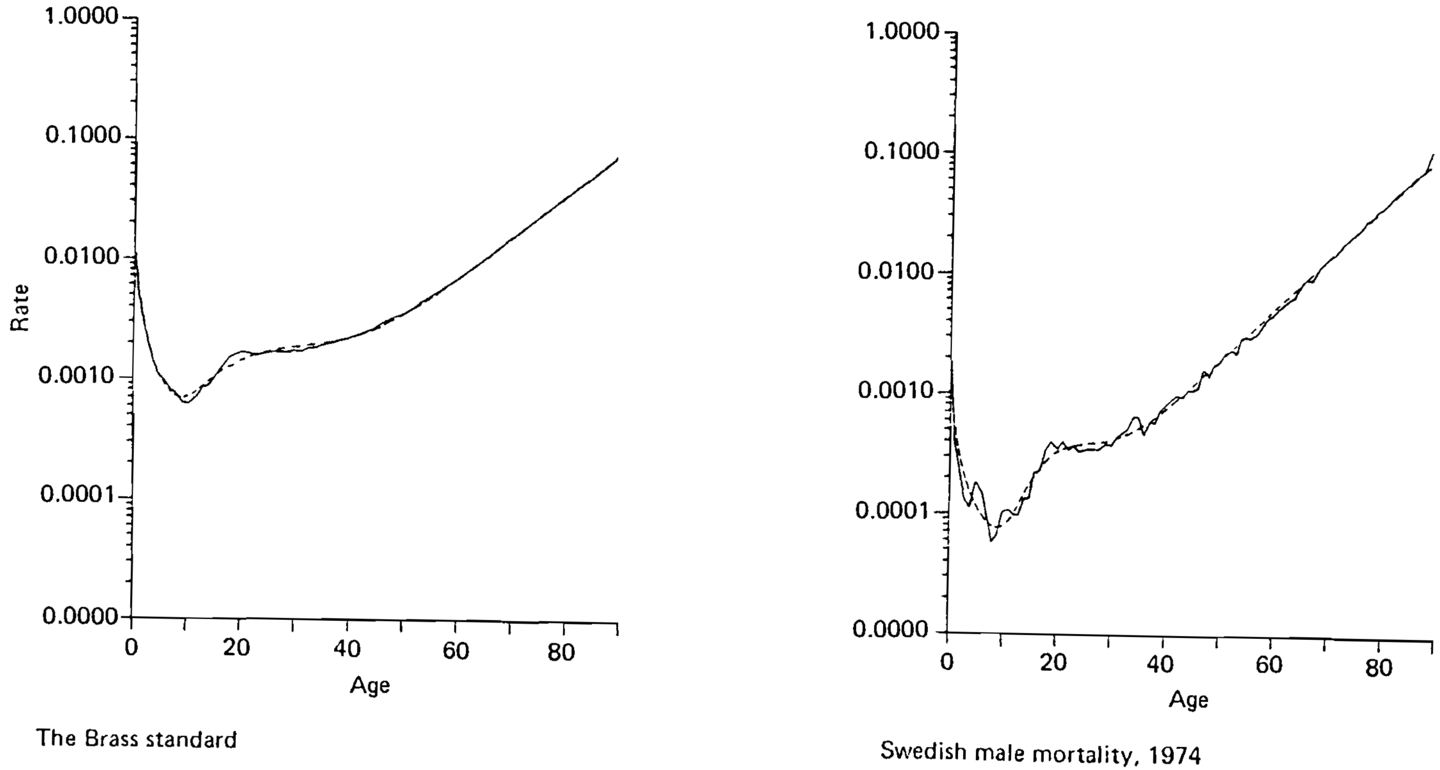


Figure III.2 The fit of the Heligman-Pollard model schedule (---) to the Brass standard (——) and to Swedish male mortality (——) in 1974.

Table III.5 Swedish mortality data, 1974.

	Stockholm		Rest of Sweden	
Parameters	Males	Females	Males	Females
GR	1.93	1.09	1.75	1.14
Q_1	0.001048	0.001576	0.001674	0.001927
γ	0.151943	0.177856	0.165989	0.170058
Q_A	0.000106	0.000442	0.000170	0.000258
X_A	22.0	22.0	22.0	22.0
σ	0.4	0.4	0.4	0.4
Q_S	0.000026	0.000019	0.000022	0.000011
X_S	10.358441	9.933713	10.216264	9.242013

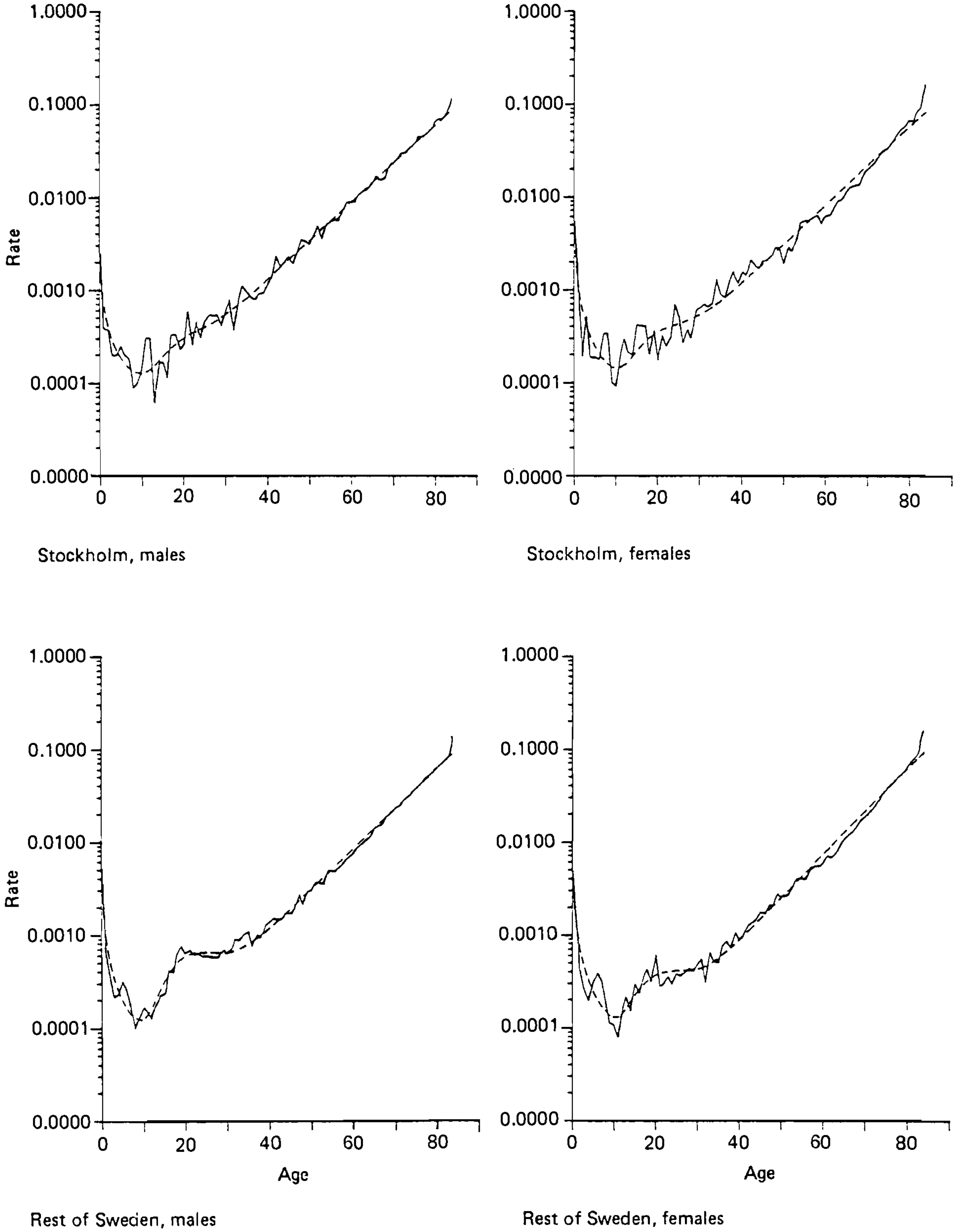


Figure III.3 The fit of the Heligman-Pollard model schedule (---) to Swedish regional mortality (——), 1974.

APPENDIX IV: MIGRATION

Data Sources

Great Britain: Rees (1979)

Japan: Nanjo, Kawashima, and Kuroda (1982)

Netherlands: Drewe (1980) and data provided by Drewe

Sweden: Andersson and Holmberg (1980) and a data tape provided for that study by the Swedish Central Bureau of Statistics

Table IV.1 Parameters and variables defining observed model migration schedules: the Netherlands, 1972-1976.

Parameters and variables	Males					Females				
	1972	1973	1974	1975	1976	1972	1973	1974	1975	1976
GR	4.06	4.17	4.12	3.84	3.70	4.16	4.23	4.18	3.93	3.74
a_1	0.018	0.019	0.018	0.018	0.018	0.017	0.018	0.018	0.018	0.018
α_1	0.095	0.098	0.095	0.100	0.098	0.132	0.136	0.116	0.110	0.110
a_2	0.056	0.055	0.055	0.056	0.055	0.068	0.064	0.064	0.063	0.062
μ_2	19.90	20.16	20.06	20.08	19.78	18.71	18.72	18.60	18.48	18.43
α_2	0.101	0.101	0.107	0.112	0.110	0.140	0.135	0.135	0.132	0.130
λ_2	0.305	0.305	0.329	0.333	0.377	0.493	0.516	0.574	0.621	0.726
a_3	0.0004	0.0004	0.0005	0.0005	0.0004	0.0003	0.0001	0.0006	0.0010	0.0002
α_3	0.033	0.032	0.028	0.026	0.029	0.034	0.044	0.027	0.021	0.034
c	0.001	0.001	0.001	0.002	0.002	0.003	0.003	0.002	0.001	0.002
σ_2	3.01	3.02	3.07	2.96	3.11	3.52	3.81	4.25	4.72	5.57
$\bar{x}$	37.17	37.00	37.02	36.47	36.23	37.27	36.97	36.95	36.55	35.93
x_h	23.34	23.62	23.31	23.21	22.91	21.22	21.27	21.08	20.93	20.75
δ_{12}	0.32	0.35	0.32	0.32	0.32	0.25	0.28	0.27	0.29	0.28
β_{12}	0.93	0.98	0.88	0.89	0.88	0.94	1.01	0.86	0.84	0.84
x_ℓ	14.04	14.33	14.54	14.58	14.89	14.73	14.93	15.20	15.33	15.71

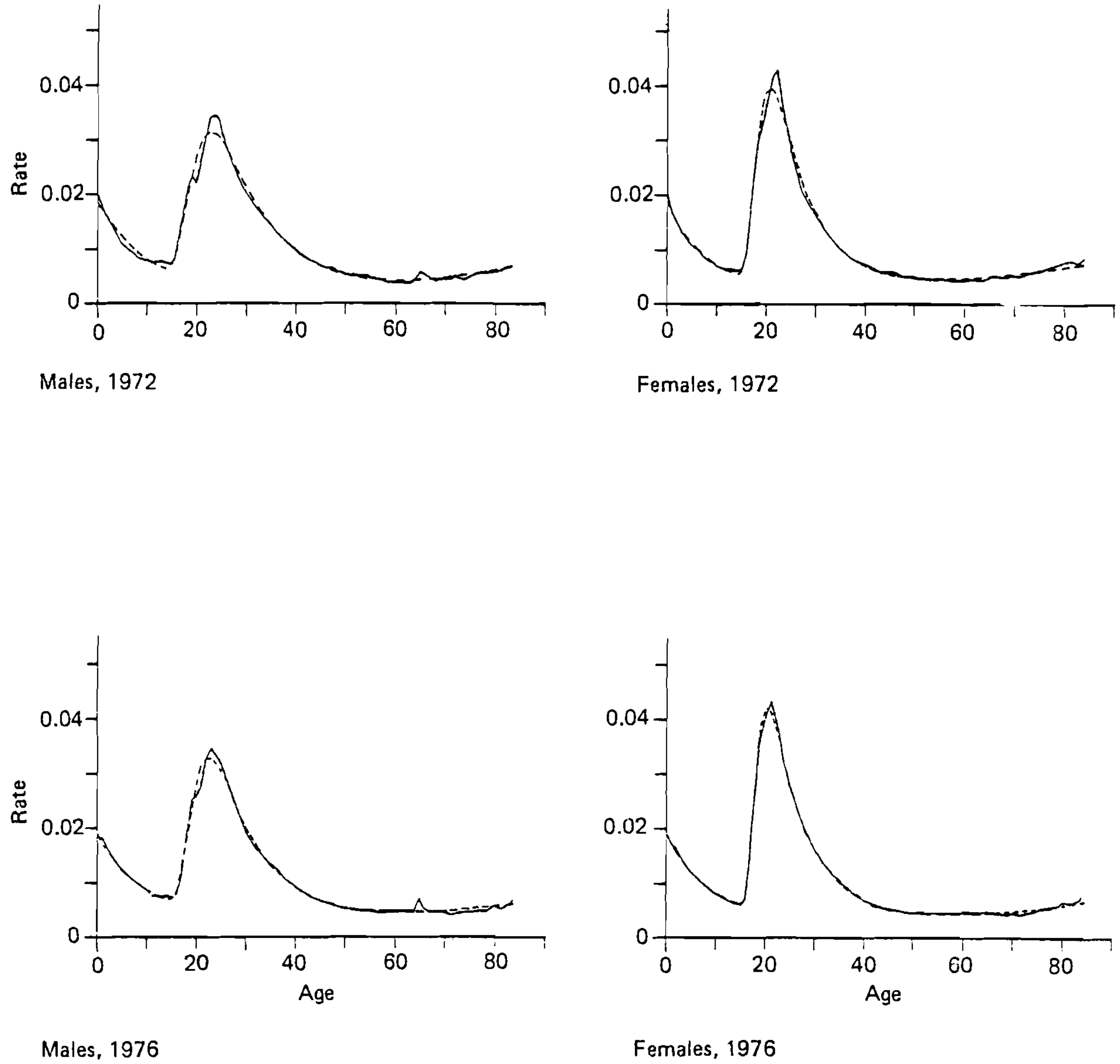


Figure IV.1 Model migration schedules for the Netherlands (--- model schedule, —— observed data).

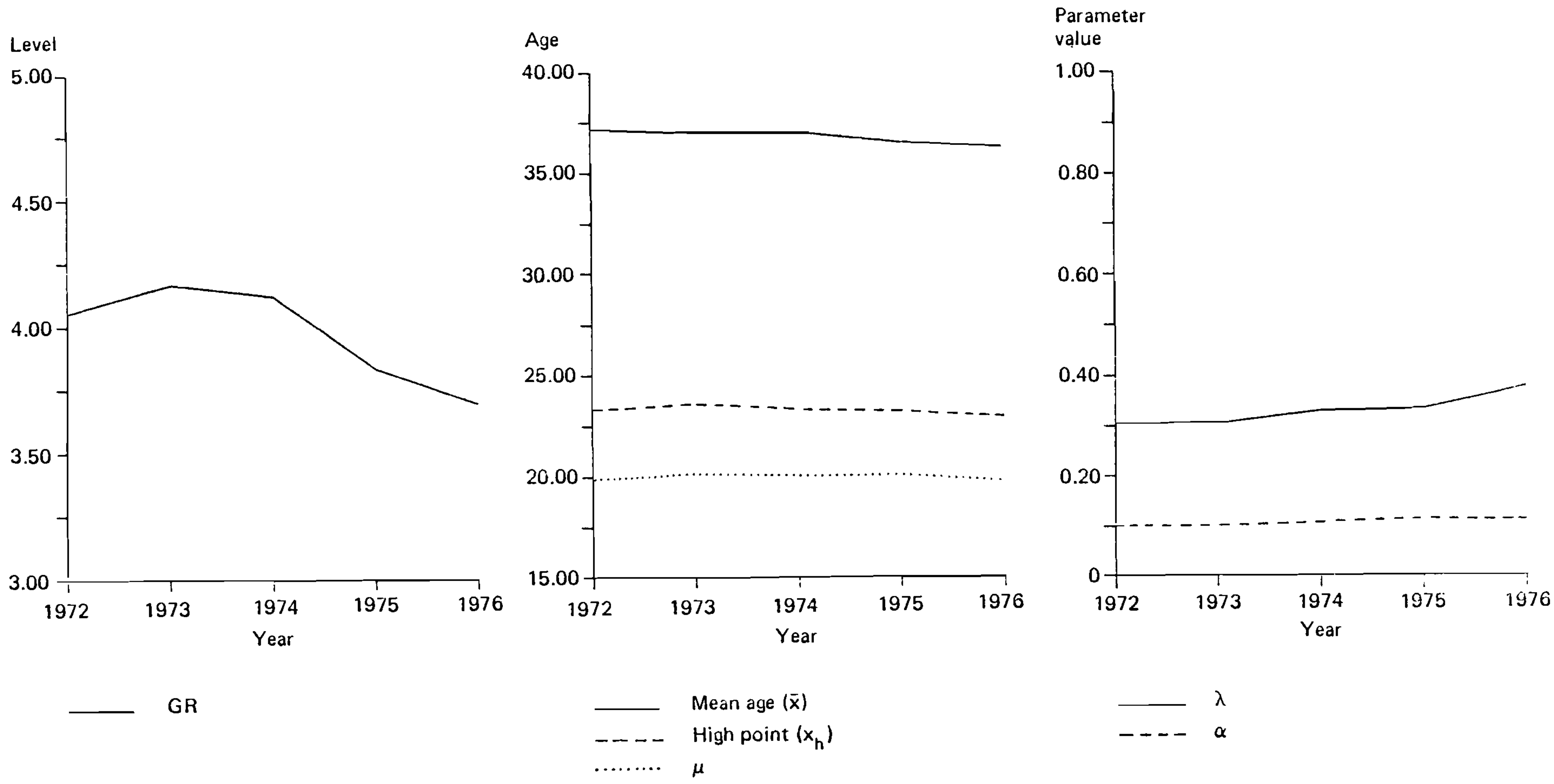


Figure IV.2 Evolution of model migration schedule parameters: Netherlands, males, 1972-1976.

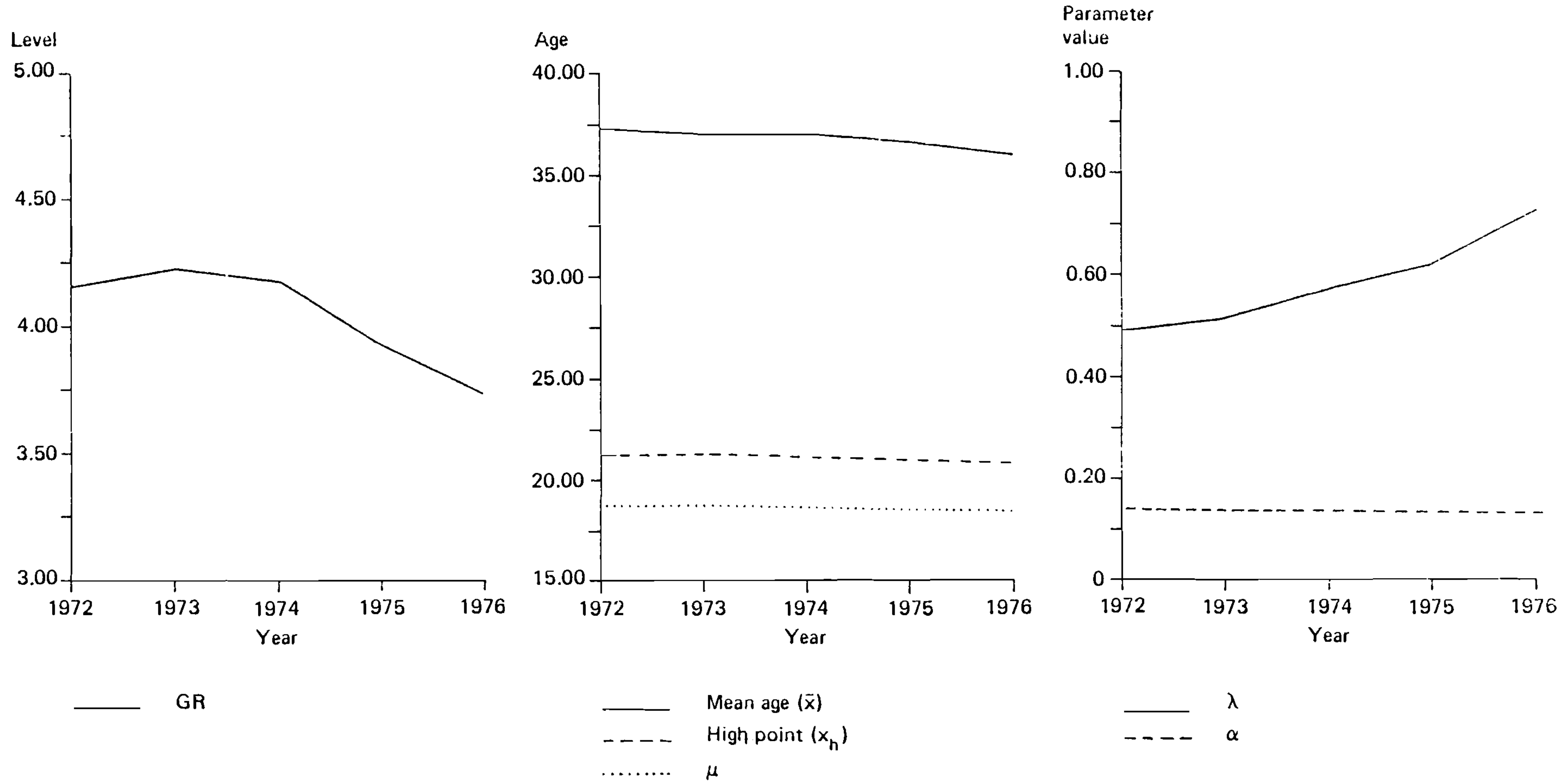
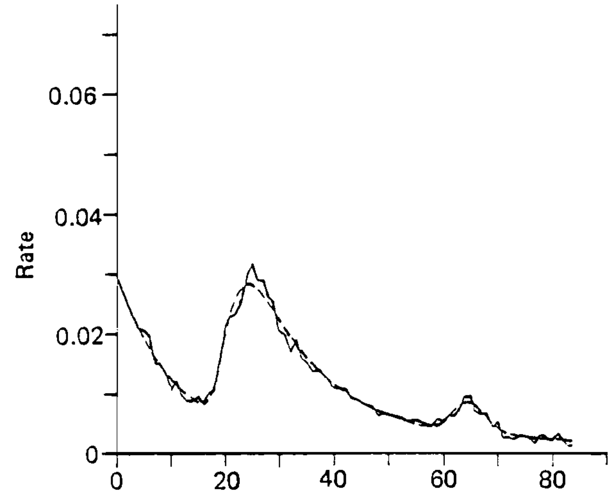


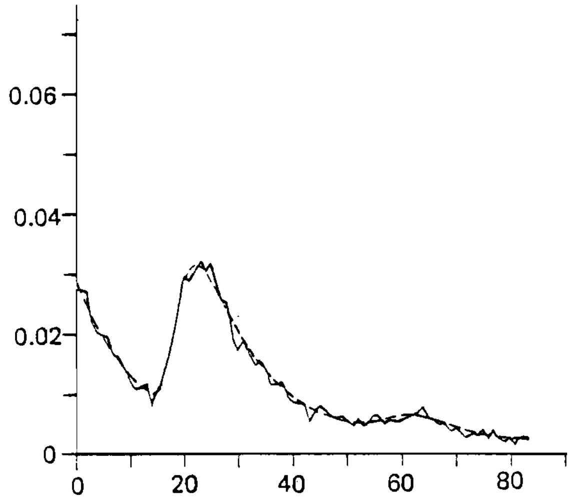
Figure IV.3 Evolution of model migration schedule parameters: Netherlands, females, 1972-1976.

Table IV.2 Parameters and variables defining regional model migration schedules: Stockholm and the rest of Sweden, 1974.

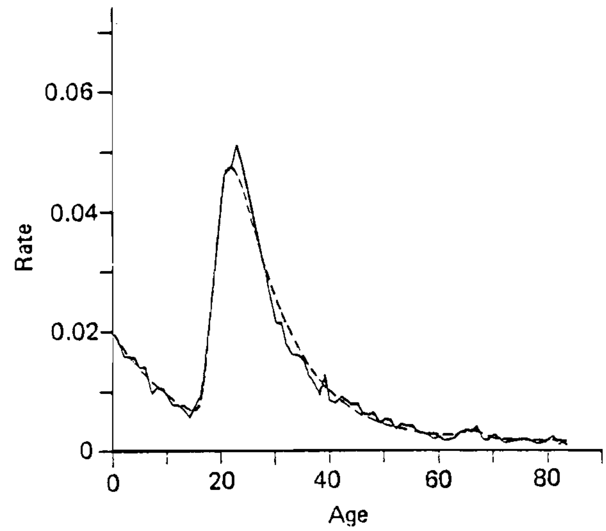
Parameters and variables	Stockholm		Rest of Sweden	
	Males	Females	Males	Females
GR	1.45	1.43	0.27	0.29
a_1	0.029	0.028	0.019	0.018
α_1	0.097	0.092	0.084	0.104
a_2	0.041	0.047	0.074	0.092
μ_2	20.80	19.32	19.38	17.92
α_2	0.077	0.094	0.106	0.140
λ_2	0.374	0.369	0.502	0.662
a_3	0.00008	0.00008	0.00002	---
μ_3	77.33	85.84	73.79	---
α_3	0.799	0.392	1.347	---
λ_3	0.140	0.072	0.233	---
c	0.002	0.002	0.002	0.003
σ_2	4.86	3.94	4.72	4.72
$\bar{x}$	31.09	29.72	28.37	28.52
x_h	24.68	22.70	22.37	20.23
δ_{12}	0.72	0.60	0.26	0.19
β_{12}	1.26	0.98	0.79	0.74
x_ℓ	16.39	14.81	15.59	14.90



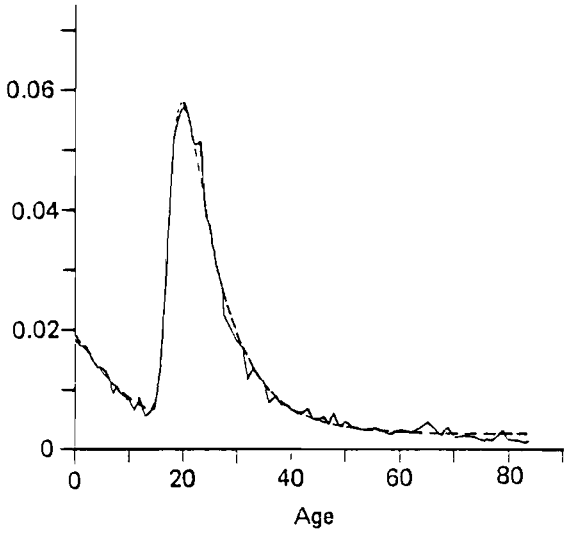
Stockholm, males



Stockholm, females



Rest of Sweden, males



Rest of Sweden, females

Figure IV.4 Model migration schedules for Swedish migration data (--- model schedule, —— observed data), 1974.

Table IV.3 Parameters and variables defining the Rogers-Castro standard and selected observed model migration schedules.*

Parameters and variables	Rogers-Castro standard	Swedish standard		British standard		Japanese standard	
		Males	Females	Males	Females	Males	Females
GR	1	1	1	1	1	1	1
a_1	0.02	0.029	0.026	0.021	0.021	0.014	0.021
α_1	0.10	0.124	0.108	0.099	0.097	0.095	0.117
a_2	0.06	0.067	0.076	0.059	0.063	0.075	0.085
μ_2	20	20.50	19.09	22.00	21.35	17.63	21.32
α_2	0.10	0.104	0.127	0.127	0.151	0.102	0.152
λ_2	0.40	0.448	0.537	0.259	0.327	0.480	0.350
c	0.003	0.003	0.003	0.003	0.003	0.002	0.004

*Source: Rogers and Castro (1981), Tables 5.4, 3.3, 3.5, and 3.6, respectively.

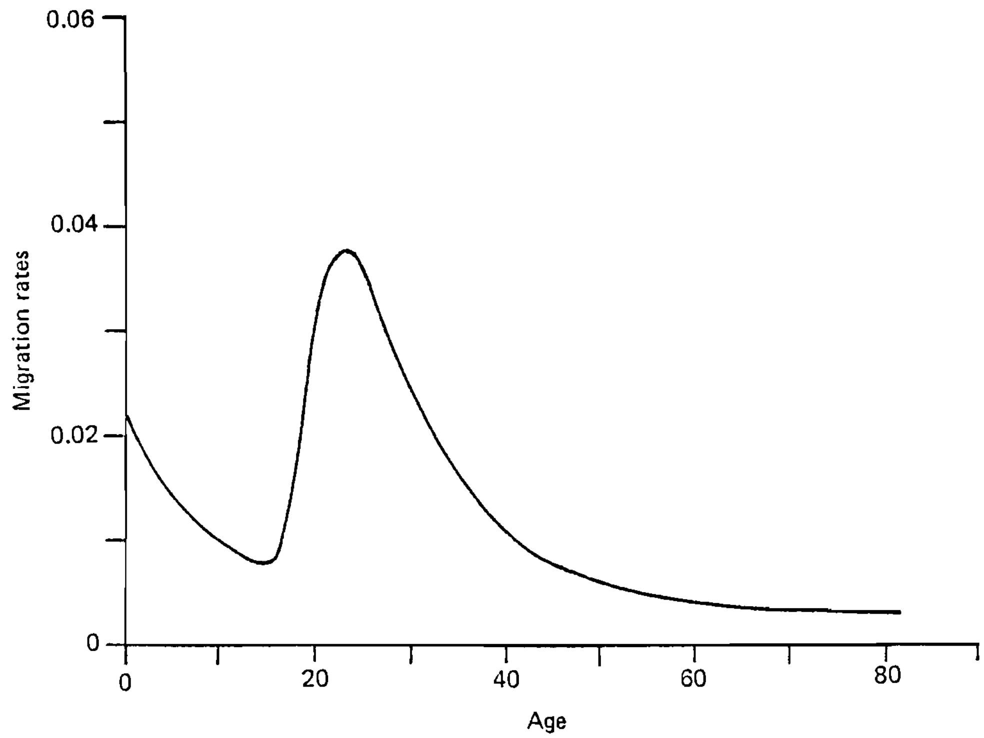


Figure IV.5 The Rogers-Castro standard migration schedule.

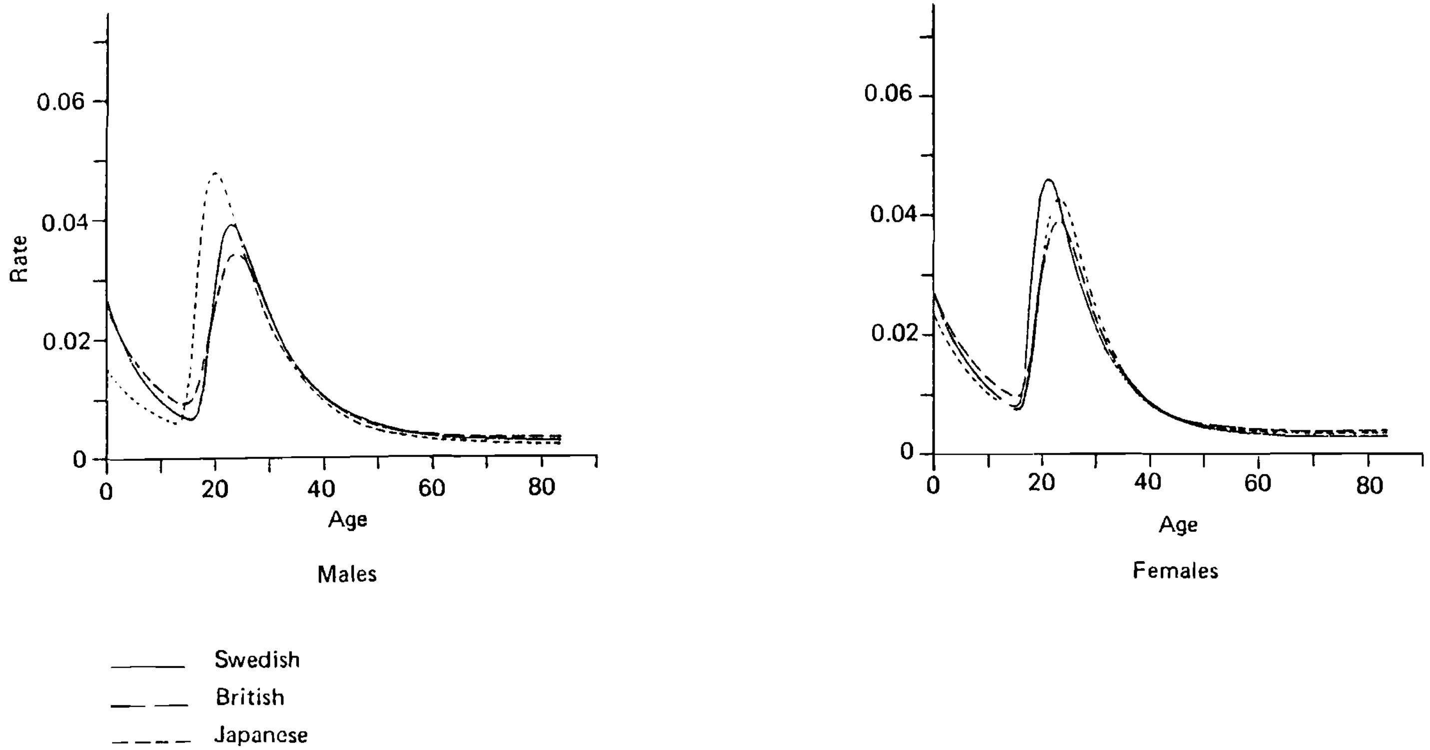


Figure IV.6 Standard model migration schedules: Swedish, British, and Japanese.

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