SOME REMARKS ON PERIODIC STOCHASTIC LINEAR RESERVOIRS

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PREFACE

Analysis concerned with problems of the rational use of natural resources almost invariably deals with uncertainties with regard to the future behaviour of the system in question and with multiple objectives reflecting conflicting goals of the users of the resources. Uncertainty means that the information available is not sufficient to unambiguously predict the future of the system, and the multiplicity of the objectives, on the other hand, calls for establishing rational trade-offs among them. The rationality of the trade-offs is quite often of subjective nature and cannot be formally incorporated into mathematical models supporting the analysis, and the information with regard to the future may vary with time. Then the challenge to the analyst is to elaborate a mathematical and computer implemented system that can be used to perform the analysis recognizing both the above aspects of real world problems.

These were the issues addressed during the summer study "Real-Time Forecast versus Real-Time Management of Hydrosystems," organized by the Resources and Environment Area of IIASA in 1981. The general line of research was the elaboration of new approaches to analyzing reservoir regulation problems and to estimating the value of the information reducing the uncertainties. Computationally, the research was based on the hydrosystem of Lake Como, Northern Italy. This paper is concerned with the properties of periodic stochastic linear reservoirs which can be of considerable help in understanding the behaviour of regulated lakes or storage reservoirs. The analysis demonstrating the approach was carried out for lakes Maggiore, Como, and Iseo in Northern Italy.

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ABSTRACT

Very simple properties of stochastic linear reservoirs are derived for the case of cyclostationary stochastic inflows and seasonally varying operating rules. Although real reservoirs are fairly non-linear, these properties have proved to be helpful in understanding the seasonal pattern of releases and the long-term variations occurring in some of the regulated lakes of Northern Italy.

CONTENTS

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Sergio Rinaldi

1. INTRODUCTION

The theory of linear stochastic reservoirs has long since been developed and has been proved to be quite powerful for interpreting the real behavior of complex hydrosystems. Important contributions can be found in Moran (1959), and in numerous technical papers, (see for example, Kaczmarek 1963; Lloyd 1963,1977; Klemes 1974; Phatarfod 1976; Troutman 1978; and Anis et al., 1979). Some of these contributions consider the case of periodic (seasonally varying) stochastic inflows, but do not deal (with the exception of Lloyd 1977) with the most general case characterized by periodically varying operating rules. On the contrary, in this paper we explicitly consider stochastic periodic linear reservoirs and derive a few and very simple properties of such reservoirs. Some of these properties are later used for interpreting the seasonal and long-term variations of the operating rules of three regulated lakes in Northern Italy.

2. THE LINEAR PERIODIC RESERVOIR

Let us consider a linear reservoir described by the following difference equation

$$s_{t+1} = s_t + i_t - r_t \quad , \tag{1}$$

where s_t is storage at the beginning of period t, and i_t and r_t are inflow and release during period t. Moreover, let the inflow be a cyclostationary stochastic process with given mean μ_t^i, variance C_t^i, and lag-1 correlation ρ_t^i, i.e.,

$$E[i_t] = \mu_t^i \quad ,$$

$$E[(i_t-\mu_t^i)^2] = C_t^i \quad ,$$

$$E[(i_t-\mu_t^i)(i_{t-1}-\mu_{t-1}^i)] = \rho_t^i C_t^i \quad ,$$

and assume that higher order auto-correlations are negligible. The process is called cyclostationary since

$$\mu_t^i = \mu_{t+T}^i \quad , \quad C_t^i = C_{t+T}^i \quad , \quad \rho_t^i = \rho_{t+T}^i \quad , \quad \forall\, t \quad ,$$

where T is the number of considered periods in one year.

Finally, let us suppose that the operating rule is linear and periodic, i.e.,

$$r_t = \alpha_t \, s_t \quad , \tag{2}$$

with

$$0 < \alpha_t < 1 \quad , \tag{3}$$

and α_t periodic. Thus, from equations (1) and (2), we obtain the following difference equation for the storage capacity s_t

$$s_{t+1} = (1-\alpha_t)s_t + i_t \quad , \tag{4}$$

and from equation (3) it follows that $s_o \geq 0$ and $i_t \geq 0$, $t = 0,1,\ldots$ imply $s_t \geq 0$, $t = 1,2,\ldots$.
Equation (4) can be re-written in terms of mean values as

$$\mu^s_{t+1} = (1-\alpha_t)\mu^s_t + \mu^i_t \quad , \tag{5}$$

so that

$$\begin{aligned} C^s_{t+1} &= E[(s_{t+1}-\mu^s_{t+1})^2] = E[\{(1-\alpha_t)(s_t-\mu^s_t) + (i_t-\mu^i_t)\}^2] \\ &= (1-\alpha_t)^2 E[(s_t-\mu^s_t)^2] + E[(i_t-\mu^i_t)^2] + 2(1-\alpha_t)E[(s_t-\mu^s_t)(i_t-\mu^i_t)] \\ &= (1-\alpha_t)^2 C^s_t + C^i_t + 2(1-\alpha_t)E[(s_t-\mu^s_t)(i_t-\mu^i_t)] \quad , \end{aligned}$$

but

$$\begin{aligned} E[(s_t-\mu^s_t)(i_t-\mu^i_t)] &= E[(1-\alpha_{t-1})(s_{t-1}-\mu^s_{t-1})(i_t-\mu^i_t) + (i_{t-1}-\mu^i_{t-1})(i_t-\mu^i_t)] \\ &= (1-\alpha_{t-1})E[(s_{t-1}-\mu^s_{t-1})(i_t-\mu^i_t)] + \rho^i_t C^i_t \quad , \end{aligned}$$

and the first term is zero because s_{t-1} depends upon i_{t-2}, and the lag-2 auto-correlation of the inflows has been assumed to be negligible. Thus, in conclusion

$$C^s_{t+1} = (1-\alpha_t)^2 C^s_t + [1+2(1-\alpha_t)\rho^i_t]C^i_t \quad . \tag{6}$$

3. SOME SIMPLE REMARKS

Equations (5) and (6) show that the mean and variance of the stochastic process $\{s_t\}$ can easily be computed by solving two linear difference equations with periodic parameters and periodic forcing terms. The most interesting solution of equations

(5), and (6) is, of course, the periodic solution, because it is the one needed to describe the storage and the release of the reservoir as cyclostationary stochastic processes. Closed-form expressions of the periodic solution of equations (5) and (6) are not too simple, but numerical solutions can easily be obtained. In fact, the eigenvalues of equations (5) and (6) [$(1-\alpha_t)$ and $(1-\alpha_t)^2$ respectively] are both smaller than one, and this implies that the periodic solution is unique and that all solutions of equations (5) and (6) asymptotically tend (for $t\to\infty$) to the periodic one. This fact allows one to simply integrate equations (5) and (6), (note that the two equations are decoupled), starting from any initial condition, until the convergence to the periodic solution has been obtained. Obviously, this procedure for finding μ_t^s and C_t^s is much more simple and effective than any Monte-Carlo approach.

Equations (5) and (6) also constitute the basis for transforming many stochastic optimal management reservoir problems into an "equivalent" deterministic problem. To illustrate this, assume that the objective function J of the stochastic optimal control problem is specified in terms of mean and variance of storage and/or release during the year, i.e.,

$$J = \sum_{t=1}^{T} \varphi_t(\mu_t^s,\ \mu_t^r,\ C_t^s,\ C_t^r) \quad , \tag{7}$$

and suppose that one is interested in finding the operating rules, (i.e., the values of α_t, $t = 0,1,\ldots,T-1$), which minimize the objective J under the constraint that the distributions of the storage capacity at the beginning and at the end of the year are the same. This implies that the optimal solution must satisfy equations (5) and (6) with the periodicity constraint

$$\mu_0^s = \mu_T^s \quad , \quad C_0^s = C_T^s \quad . \tag{8}$$

Thus, the problem is reduced to a "deterministic" periodic optimal control problem: equations (5) and (6) are the state equations; equation (7) with $\mu_t^r = \alpha_t \mu_t^s$ and $C_t^r = \alpha_t^2 C_t^s$ is the objective function; α_t, $t = 0,1,\ldots,T-1$, are the control variables subject to constraint (3) (or any other equivalent constraint); and equation (8) is the periodicity constraint imposed on the state variables. Such a problem can be solved by the Maximum Principle or by any other equivalent technique, such as Dynamic Programming; it is worthwhile to note that a certain number of specific results are available for periodic optimal control problems (see Guardabassi et al., 1974 for a survey).

From equations (5) and (6) one can easily derive lower and upper bounds for the mean and variance of the storage s_t. In fact, by recursively using equation (5), one obtains

$$\mu_t^s = \mu_{t-1}^i + (1-\alpha_{t-1})\mu_{t-2}^i + (1-\alpha_{t-1})(1-\alpha_{t-2})\mu_{t-3}^i + \ldots \quad ,$$

so that

$$\mu_t^s \le \mu_{MAX}^i[1+(1-\alpha_{min}) + (1-\alpha_{min})^2 + \ldots] = \frac{\mu_{MAX}^i}{\alpha_{min}} \quad ,$$

$$\mu_t^s \ge \mu_{min}^i[1+(1-\alpha_{MAX}) + (1-\alpha_{MAX})^2 + \ldots] = \frac{\mu_{min}^i}{\alpha_{MAX}} \quad ,$$

where

$$\mu_{min}^i = \min_{1\le t\le T} \mu_t^i \qquad \mu_{MAX}^i = \max_{1\le t\le T} \mu_t^i$$

$$\alpha_{min} = \min_{1\le t\le T} \alpha_t \qquad \alpha_{MAX} = \max_{1\le t\le T} \alpha_t \quad .$$

Similarly, equation (6) gives rise to the expression

$$C_t^s = [1+2(1-\alpha_{t-1})\rho_{t-1}^i]C_{t-1}^i + (1-\alpha_{t-1})^2[1+2(1-\alpha_{t-2})\rho_{t-2}^i]C_{t-2}^i$$

$$+ (1-\alpha_{t-1})^2(1-\alpha_{t-2})^2[1+2(1-\alpha_{t-3})\rho_{t-3}^i]C_{t-3}^i + \ldots \quad ,$$

from which it follows that

$$C_t^s \le C_{MAX}^i[1+2(1-\alpha_{min})\rho_{MAX}^i][1+(1-\alpha_{min})^2 + (1-\alpha_{min})^4 + \ldots]$$

$$= \frac{C_{MAX}^i}{\alpha_{min}^2} \frac{\alpha_{min}}{2-\alpha_{min}}[1+2\rho_{MAX}^i(1-\alpha_{min})]$$

$$C_t^s \ge C_{min}^i[1+2(1-\alpha_{MAX})\rho_{min}^i][1+(1-\alpha_{MAX})^2 + (1-\alpha_{MAX})^4 + \ldots]$$

$$= \frac{C_{min}^i}{\alpha_{MAX}^2} \frac{\alpha_{MAX}}{2-\alpha_{MAX}}[1+2\rho_{min}^i(1-\alpha_{MAX})] \quad ,$$

where

$$C_{min}^i = \min_{1\le t\le T} C_t^i \qquad C_{MAX}^i = \max_{1\le t\le T} C_t^i$$

$$\rho_{min}^i = \min_{1\le t\le T} \rho_t^i \qquad \rho_{MAX}^i = \max_{1\le t\le T} \rho_t^i \quad .$$

By introducing the function

$$\psi(\alpha,\rho) = \frac{\alpha}{2-\alpha}[1+2\rho(1-\alpha)] \quad , \tag{9}$$

the preceding results can be summarized as follows

$$\frac{\mu_{min}^i}{\alpha_{MAX}} \le \mu_t^s \le \frac{\mu_{MAX}^i}{\alpha_{min}} \quad , \tag{10}$$

$$\frac{\psi(\alpha_{MAX}, \rho_{min}^i)}{\alpha_{MAX}^2} C_{min}^i \le C_t^s \le \frac{\psi(\alpha_{min}, \rho_{MAX}^i)}{\alpha_{min}^2} C_{MAX}^i \quad . \tag{11}$$

In the case of stationary stochastic inflows, ($\mu^i_{min} = \mu^i_{MAX} = \mu^i$, $C^i_{min} = C^i_{MAX} = C^i$, $\rho^i_{min} = \rho^i_{MAX} = \rho^i$), and constant operating rules, ($\alpha_{min} = \alpha_{MAX} = \alpha$), equations (10) and (11) degenerate to the well-known expressions

$$\mu^s_t = \mu^s = \frac{\mu^i}{\alpha} \quad , \tag{12}$$

$$C^s_t = C^s = \frac{\psi(\alpha,\rho^i)}{\alpha^2} C^i \quad , \tag{13}$$

which show that higher values of α imply lower values of μ^s and C^s (the function $\psi(\alpha,\rho^i)/\alpha^2$ decreases with α [see equation (9)]).

It might be worthwhile noticing that equations (10) and (11) can be given a particularly meaningful interpretation. In fact, the lower bounds of μ^s_t and C^s_t are the values of μ^s and C^s of a stochastic reservoir characterized by stationary inflows with $\mu^i = \mu^i_{min}$, $C^i = C^i_{min}$, $\rho^i = \rho^i_{min}$, and by a constant linear operating rule with $\alpha = \alpha_{MAX}$, while, on the contrary, the upper bounds are the values of μ^s and C^s of another reservoir with $\mu^i = \mu^i_{MAX}$, $C^i = C^i_{MAX}$, $\rho^i = \rho^i_{MAX}$, and $\alpha = \alpha_{min}$.
In other words, the behavior of any periodic stochastic linear reservoir can be bound by the behavior of two time-invariant stochastic linear reservoirs; one obtained by freezing mean, variance, and correlation of the inflows at their minimum seasonal values and discharging at maximum rate, and the other by freezing mean, variance, and correlation of the inflows at their maximum seasonal values and discharging at minimum rate.

Expressions similar to equations (10) and (11) can be derived for mean (μ^r_t) and variance (C^r_t) of the release r_t from the reservoir. In fact, since:

$$\alpha_{min}\mu_t^s \le \mu_t^r = \alpha_t\mu_t^s \le \alpha_{MAX}\mu_t^s \quad ,$$

$$\alpha_{min}^2 C_t^s \le C_t^r = \alpha_t^2 C_t^s \le \alpha_{MAX}^2 C_t^s \quad ,$$

from equations (10) and (11), we immediately obtain

$$\frac{\alpha_{min}}{\alpha_{MAX}}\, \mu_{min}^i \le \mu_t^r \le \frac{\alpha_{MAX}}{\alpha_{min}}\, \mu_{MAX}^i \quad , \tag{14}$$

$$\left(\frac{\alpha_{min}}{\alpha_{MAX}}\right)^2 \psi(\alpha_{MAX}, \rho_{min}^i) C_{min}^i \le C_t^r \le \left(\frac{\alpha_{MAX}}{\alpha_{min}}\right)^2 \psi(\alpha_{min}, \rho_{MAX}^i) C_{MAX}^i \quad . \tag{15}$$

Equations (14) and (15) clearly point out that large seasonal variations of the operating rule (i.e., high values of $\alpha_{MAX}/\alpha_{min}$) could generate large variations in the seasonal pattern of releases. Obviously, this might happen particularly when the variations of α_t are suitably tuned with the variations of μ_t^i, C_t^i, and ρ_t^i. Conversely, one might use the possibility of varying α_t to reduce overly large seasonal variations of μ_t^s and/or C_t^s as pointed out in the following.

Let us now discuss the sensitivity of the periodic solution of equations (5) and (6) to a change of the operating rule during any time period t, say t=0. Obviously, this can be done by direct comparison of the solutions of equations (5) and (6) for the nominal values $\{\bar{\alpha}_0, \bar{\alpha}_1, \ldots, \bar{\alpha}_{T-1}\}$ of the parameters and for the perturbed values $\{\alpha_0, \bar{\alpha}_1, \ldots, \bar{\alpha}_{T-1}\}$ with $\alpha_0 \neq \bar{\alpha}_0$. Nevertheless, general and simple conclusions can be drawn if the analysis is carried out for small variations of the parameters. In such a case, we are indeed only interested in determining the derivatives $\frac{d\mu_t^s}{d\alpha_0}$ and $\frac{dC_t^s}{d\alpha_0}$ of the periodic solution of equations (5) and (6).

For this, express $\mu_t^s = \mu_t^s(\alpha_0)$ and $C_t^s = C_t^s(\alpha_0)$ and let

$$\overline{\mu}_t^s = \mu_t^s(\overline{\alpha}_0) \quad , \quad \overline{C}_t^s = C_t^s(\overline{\alpha}_0) \quad ,$$

be the periodic solution of equations (5) and (6), corresponding to the nominal value $(\overline{\alpha}_0)$ of the parameter. Thus, from equation (5), it follows that

$$\left.\frac{d\mu_1^s}{d\alpha_0}\right|_{\alpha_0=\overline{\alpha}_0} = (1-\overline{\alpha}_0) \left.\frac{d\mu_0^s}{d\alpha_0}\right|_{\alpha_0=\overline{\alpha}_0} - \overline{\mu}_0^s \quad , \tag{16}$$

$$\left.\frac{d\mu_{t+1}^s}{d\alpha_0}\right|_{\alpha_0=\overline{\alpha}_0} = (1-\overline{\alpha}_t) \left.\frac{d\mu_t^s}{d\alpha_0}\right|_{\alpha_0=\overline{\alpha}_0} \qquad t = 1,2,\ldots,T-1 \quad . \tag{17}$$

Since these equations are linear difference equations of the form

$$y_{t+1} = \beta_t \, y_t - u_t \tag{18}$$

with $0 < \beta_t < 1$ and $u_t \geq 0$, the periodic solution is non-positive, i.e.,

$$\left.\frac{d\mu_t^s}{d\alpha_0}\right|_{\alpha_0=\overline{\alpha}_0} \leq 0 \qquad t = 1,2,\ldots,T \quad .$$

Moreover, from equation (17) it follows that

$$\left\| \left.\frac{d\mu_t^s}{d\alpha_0}\right|_{\alpha_0=\overline{\alpha}_0} \right\|$$

is non-increasing with t.

In conclusion, a small positive perturbation of the parameter α_0 generates a decrease of the mean value (μ_t^s) of the storage in all the periods of the year, and this decrease is higher for the periods immediately after the time at which the perturbation occurred.

Similarly, from equation (6), we obtain

$$\left.\frac{dC_1^s}{d\alpha_0}\right|_{\alpha_0=\bar{\alpha}_0} = (1-\bar{\alpha}_0)^2 \left.\frac{dC_0^s}{d\alpha_0}\right|_{\alpha_0=\bar{\alpha}_0} - 2[(1-\bar{\alpha}_0)\bar{C}_0^s+\rho_0 C_0^i] \quad ,$$

$$\left.\frac{dC_{t+1}^s}{d\alpha_0}\right|_{\alpha_0=\bar{\alpha}_0} = (1-\bar{\alpha}_t)^2 \left.\frac{dC_t^s}{d\alpha_0}\right|_{\alpha_0=\bar{\alpha}_0} \qquad t = 1,2,\ldots,T-1 \quad ,$$

which are again of the form (18), with $0 < \beta_t < 1$ and $u_t \geq 0$. Therefore, the same conclusion as above can be drawn for $\frac{dC_t^s}{d\alpha_0}$, which is indeed negative for all values of t, with t in absolute value.

4. THE ANALYSIS OF THREE REGULATED LAKES

The above sensitivity properties are quite important for understanding some of the basic features of real-time reservoir operation. In real-world terms, the operating rule is often seasonally varied in order to satisfy periodically varying demands and reduce the potential of floods; indeed, in many cases the "gain" α_t of the operating rule is particularly high during the flood season or, more precisely, during those periods characterized by high mean and variance of the inflows.

A detailed analysis has been carried out on three lakes in Northern Italy, (Maggiore, Como, and Iseo), and is now briefly reported. Daily data for the three lakes were available for different periods (at least 15 years), and it was possible to verify that the "total decadic inflows" were cyclostationary weekly correlated stochastic processes (see for example, Ambrosino et al., 1979).

The major discrepancy with the preceding analysis is that the operating rule of such lakes is non-linear. In fact, an agreement between all counterparts interested in the operation of the lake states that the manager of the lake can fix the discharge only when the storage is between a minimum value $\underline{s}$ and a maximum value $\overline{s}$. On the contrary, if $s_t = \underline{s}$ the release must be equal or smaller than the inflow (so that $s_{t+1} \geq \underline{s}$), and if $s_t \geq \overline{s}$ the release must be equal to the maximum possible release, i.e., $r_t = N(s_t)$ where N(s) represents the "open gates" stage-discharge function. In practice, the manager uses a periodically varying operating rule

$$r_t = R(t, s_t) \quad , \tag{19}$$

which can be identified by suitably fitting in the space (s, r) the daily pairs (s_t, r_t) of all the days t of the same decade. Figure 1 shows how the pairs (s_t, r_t) display for a particular decade: for $\underline{s} < s_t \leq s'$ and for $s_t \geq \overline{s}$, the points are on the open-gates stage-discharge curve, for $s' < s_t \leq s''$ the release is approximately constant and equal to the agricultural demand d characterizing that decade, while for $s'' < s_t \leq \overline{s}$, the points clearly indicate that the operating rule is increasing and convex with respect to s_t. For all the three lakes, the operating policy (19) has been identified among the class of piecewise linear functions by a special best-fitting technique, and the results are in good agreement with the analysis carried out in the preceding section. The seasonal variations of the slope of the operating rule just before the maximum storage $\overline{s}$ have been found particularly significant since they are perfectly in phase with the seasonal variations of μ_t^i and σ_t^i. Figure 2 shows

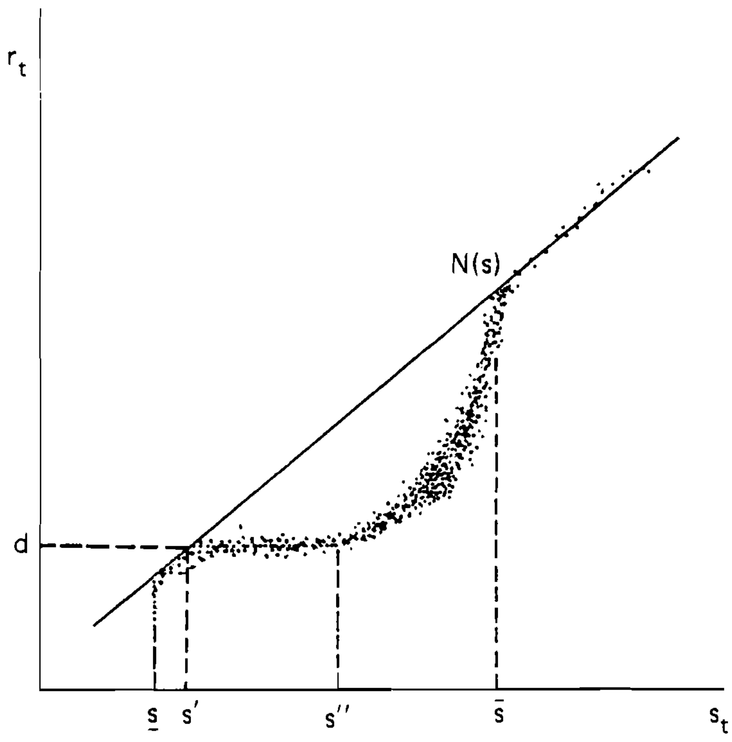

Figure 1. Daily values of storage (s_t) and release (r_t) for a given decade and for a certain number of years.

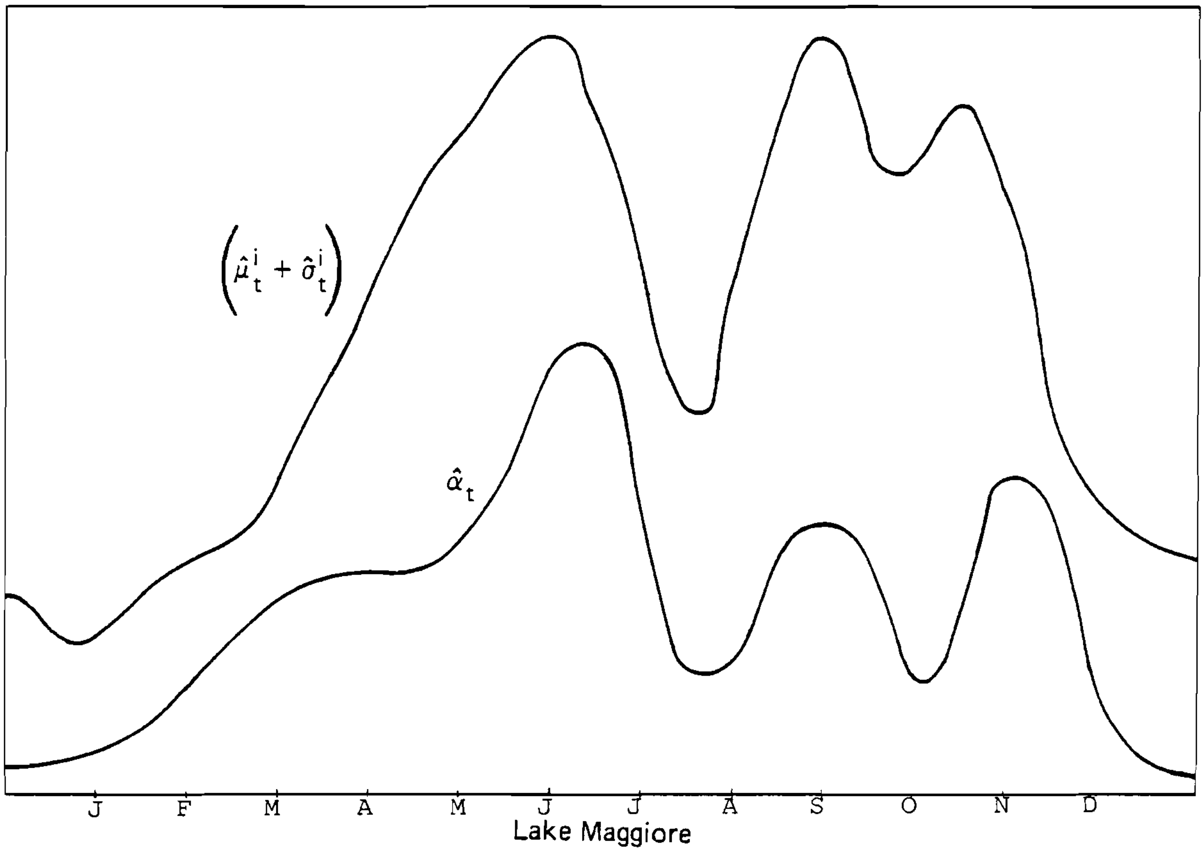

Figure 2a. The plots of $(\hat{\mu}_t^i + \hat{\sigma}_t^i)$ and $\hat{\alpha}_t$ for the three lakes.

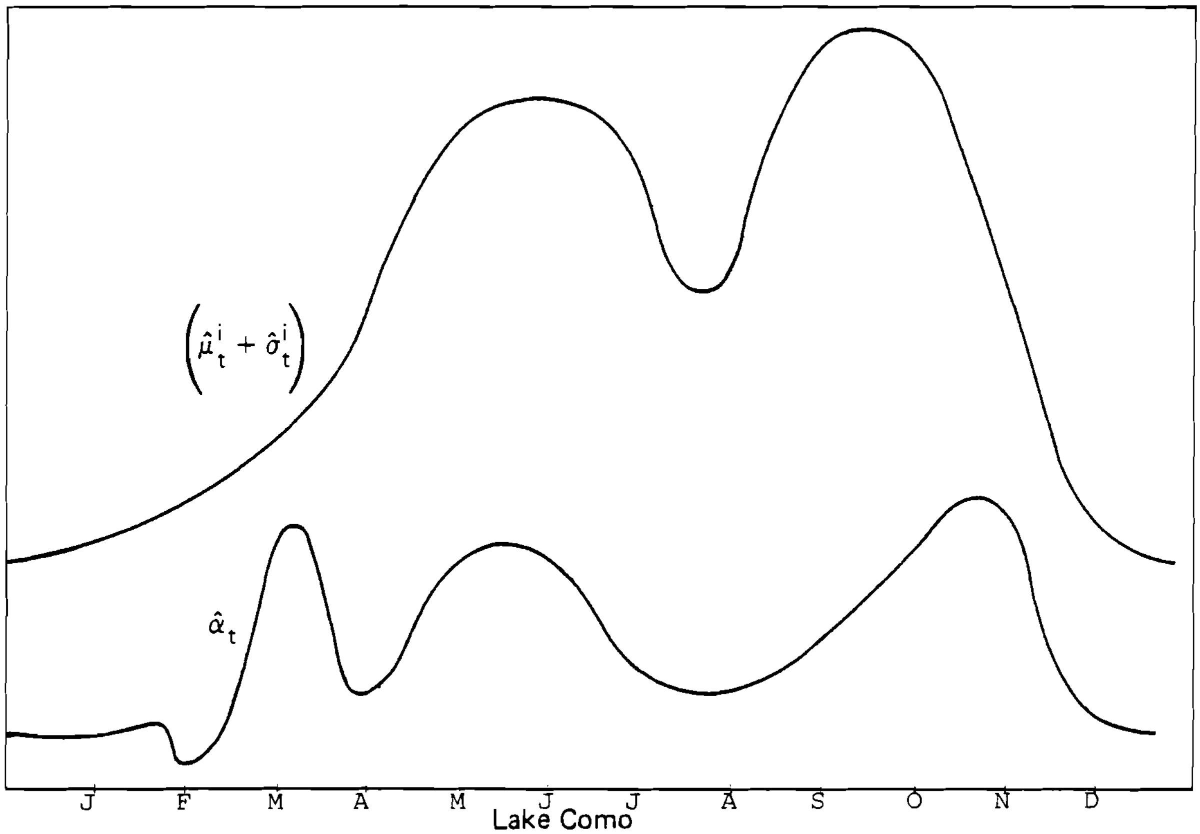

Figure 2b. The plots of $(\hat{\mu}_t^i + \hat{\sigma}_t^i)$ and $\hat{\alpha}_t$ for the three lakes.

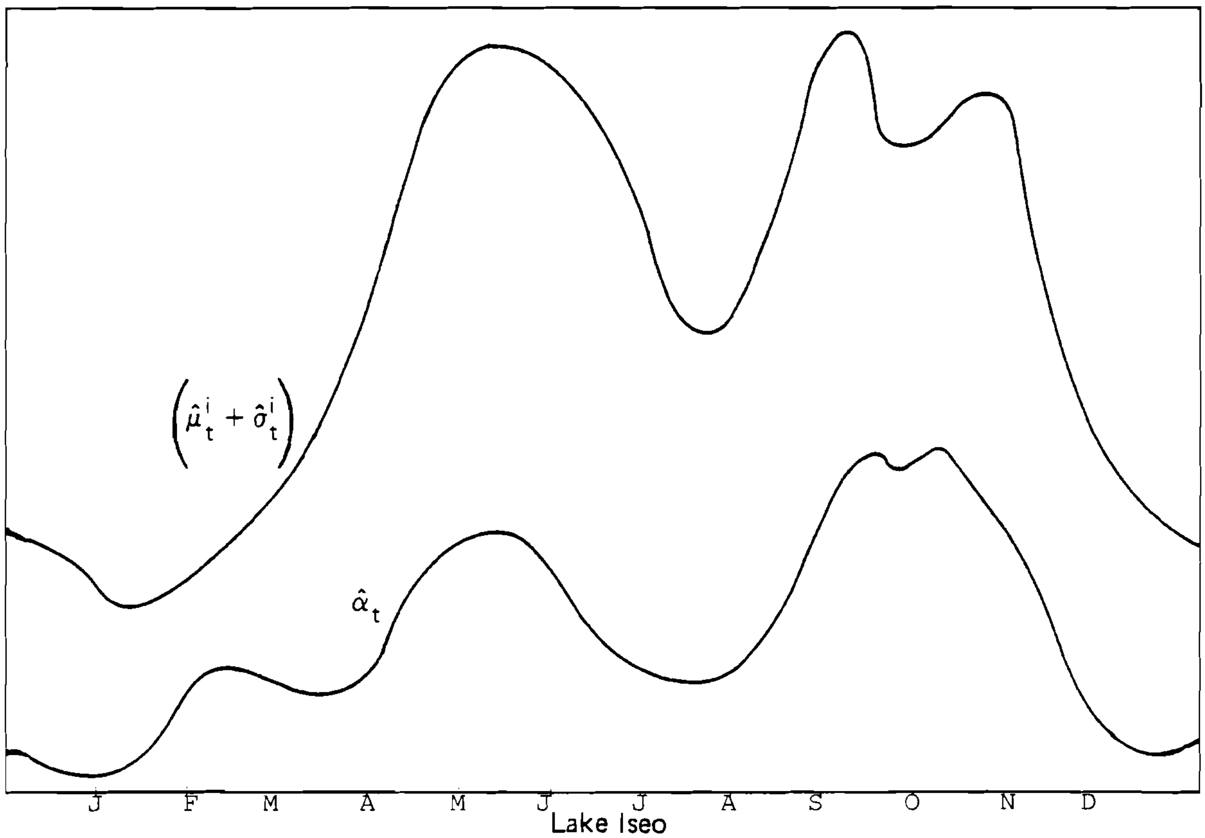

Figure 2c. The plots of $(\hat{\mu}_t^i + \hat{\sigma}_t^i)$ and $\hat{\alpha}_t$ for the three lakes.

(in suitable units), the yearly graph of that slope (called $\hat{\alpha}_t$) and the estimates, $(\hat{\mu}_t^i + \hat{\sigma}_t^i)$ (mean plus standard deviations) or the inflows for the three lakes. The correlation between the two graphs is 0.70 for Lake Maggiore, 0.65 for Lake Como, and 0.89 for Lake Iseo.

Moreover, a trend analysis was carried out for Lake Como to detect the reaction of the manager to the progressive sinking of the town, which, since 1965, has been the cause of higher and higher flood damages. From the sensitivity analysis carried out in the preceding section, one should expect that progressively higher values of $\hat{\alpha}_t$ might have compensated the sinking process by lowering the mean and variance of the storage. From the same analysis, one should also expect that the long-term trends of $\hat{\alpha}_t$ should be more detectable during the flooding season. Indeed, the identification of the operating policy, $R(t, s_t)$ and of the slope $\hat{\alpha}_t$ carried out for different periods allowed an a posteriori conclusion that the manager did exactly what the theory suggests. Figure 3 shows the graph of $\hat{\alpha}_t$ (measured in suitable units) during the snowmelt flooding season for the three periods indicated in the figure. The increase of $\hat{\alpha}_t$ is very relevant but not surprising if one takes into account that the main square of Como has sunk almost one meter during the last 15-20 years.

5. CONCLUDING REMARKS

The results obtained in this paper show that the analysis of the very simple linear periodic reservoir can be of help in understanding the behavior of real reservoirs or regulated lakes. The remarks developed in the paper are based on the assumption that high order correlations of the inflows are negligible, and that

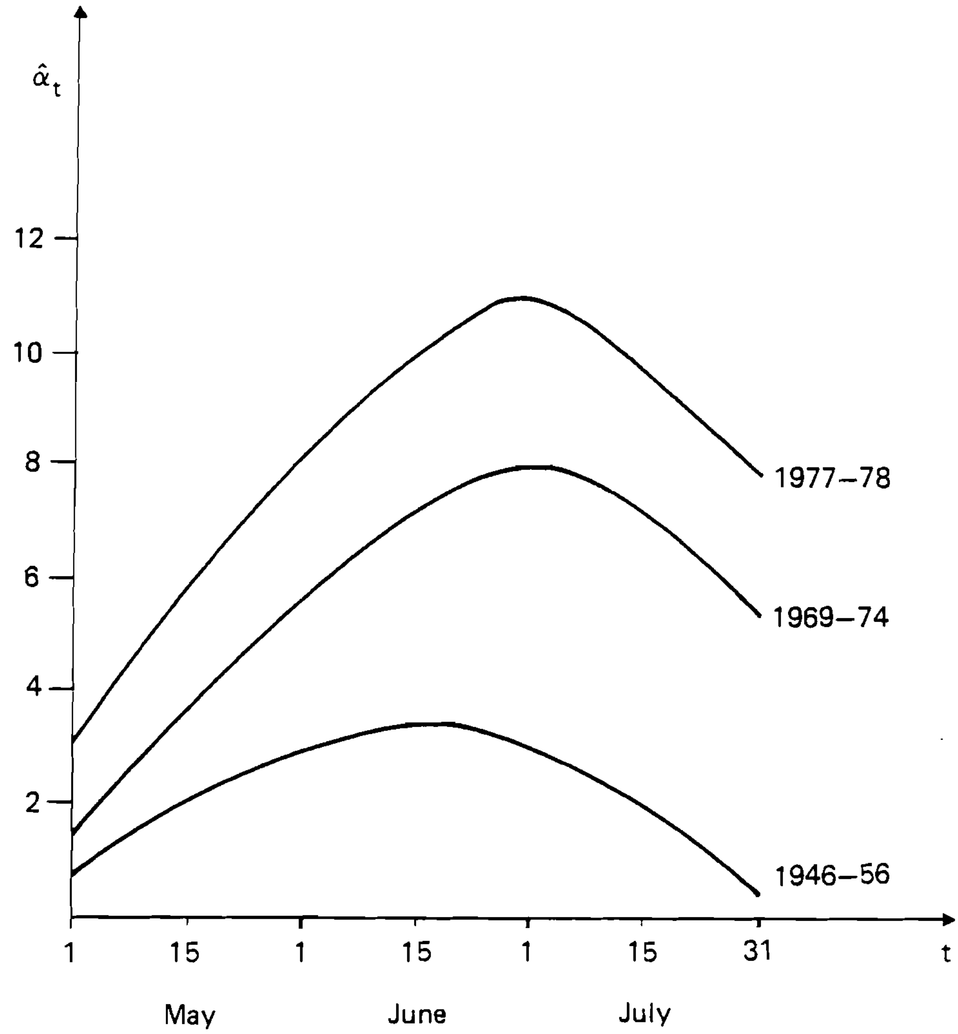

Figure 3. The long-term variations of $\hat{\alpha}_t$ in Lake Como.

the operating rule is linear. The main properties of such a stochastic reservoir are the following. First, mean and variance of the storage capacity satisfy linear difference equations which have a unique periodic solution. Second, this periodic solution can easily be obtained numerically, because the difference equations are asymptotically stable. Third, almost any stochastic optimal formulation of real-time management problems of such reservoirs can be transformed into an equivalent deterministic formulation which calls for the periodic optimal control theory. Fourth, the mean and variance of the storage of the reservoir can be given very simple lower and upper bounds which can be interpreted as mean and variance of two time-invariant linear reservoirs. Finally, a simple sensitivity analysis shows that reductions of mean and variance of the storage in any period of the year can be achieved by strenthening the operating rule in the preceding periods. Practitioners are probably aware of all, or at least some, of these facts, which are anyway simple extensions of well-known results of time-invariant linear stochastic reservoir theory.

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