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MODEL OF INTERINDUSTRY INTERACTIONS AS A SYSTEM OF SIMULTANEOUS EQUATIONS

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ABSTRACT

This paper considers the techniques used to estimate the parameters of the large-scale model of interindustry interactions. Use of the ordinary least squares technique sometimes leads to overestimation of model validity. These questions are of great importance in forecasting if the set of parameters is obtained by econometric techniques. Three examples illustrate the problem and show that the model should be represented as a system of simultaneous equations and therefore appropriate techniques have to be used for parameter estimation.

The original Model of Interindustry Interactions (*MII*) was developed in the USSR some years ago, and has been used since 1974 to forecast the structure of the USSR economy 5-7 years ahead. The *MII* is based upon 18-sector input-output tables in constant prices dating back to 1950. The preparation of these tables and the final demand components are described (in Russian) in [2,3,4] and in [1] given in English. An almost complete set of the equations and the estimates of the parameters are given in [1,2].

This aggregated model consists of about 120 econometric equations together with the balance equalities. The essence of the model is to replace the traditional input-output equation of the form:

$$X(i,j)_t = a(i,j)_t * X(j)_t$$

by

$$X(i,j)_t = \beta_0 + \beta_1 * X(j)_t + \beta_2 * X(i)_t + \beta_3 * X(k,l)_t + \varphi(t)$$

where all of the β 's depend on i and j. Here the term $\beta_2 * X(i)_t$ reflects the availability of product i; the $X(i)_t$ may be either set exogenously, or set equal to the demand for product i obtained by summing all intermediate flows and final demand components.

The term $\beta_3 * X(k,l)_t$ represents the influence of other important interindustry flows on $X(i,j)_t$, and $\varphi(t)$ reflects the impact of other factors on $X(i,j)_t$ over time.

An attempt is now being made to develop the same type of econometric model for 80 products. All of the post-1950 data used in the disaggregated model are also available in constant prices and in physical units and have been widely used in the development of the aggregated input-output tables. The time-series now cover 30 years and can therefore be used toestimate regressions with 4-5 parameters.

Each product is distributed among 18 industries and among all components of final demand as in the aggregated version of *MII*. This means that we have a system of about 300 equations and 100 equalities which we can use to estimate the output of each of the 18 industries. These equalities are expressed in terms of regression equations rather than as exact equalities. The system may also be used to estimate the production volume of each of the 80 products studied.

This system contains equations of three main types, which in their simplest form can be written as follows:

$$X(i_s,j)_t = F\,[\,X(i_s)_t\ ,X(j)_t, X(l_k,p)_t, t\,] \tag{1}$$

$$Y(i_s,\alpha)_t = F\,[\,X(i_s)_t\ ,\ Y(\alpha)_t, X(l_k,p)_t, t\,] \tag{1a}$$

$$X(i_s)_t = F\,[\,\sum_j X(i_s,j)_t, \sum_\alpha\ Y(i_s,\alpha)_t\,] \tag{2}$$

$$X(j)_t = F\,[\,\sum_s X(j_s)_t\,] \tag{3}$$

where:

t denotes time and is also used as an explanatory variable to absorb the influence of factors omitted from the regression;

i and j represent the aggregated industries and may take values from 1 to 18;

s denotes the product of aggregated industry i or j; thus $X(i_s)$ can be measured either in value or in physical terms;

α is used as an index for the ten components of final demand i.e.,

$Y(\alpha)_t = \sum_i Y(i,\alpha)_t$ or $Y(\alpha)_t = \sum_i \sum_s Y(i_s,\alpha)_t$

and thus the value of the final demand Y_t is given by the equality:

$$Y_t = \sum_\alpha Y(\alpha)_t$$

Thus the first set of equations is used to explain changes over time in both the interindustry flows and the components of final demand. There are about 300 of these equations, which are designed to reflect the following features:

1. The structure of the USSR economy moves in the same general direction for long periods of time. Each stage of economic development may be associated with a given relation between levels and rates of growth in different industries. There is therefore a connection between the levels of input-output coefficients and the rates of growth in a set of industries.

2. The supply of a product influences the input-output coefficients $a(i,j)_t$ in the matrix row corresponding to that product (industry), and the output of a product can serve as a proxy for its supply. Moreover, the necessary priority given to one industry in the distribution of a product will reduce the availability of that product and certain other goods to the rest of industry. This system of priorities is different in each stage of economic development.

3. The interaction between intermediate flows (and components of final demand) usually reflects the level of their substitutability - for example, there is considerable interaction between the different types of materials used in machine building or between energy sources used for electricity generation.

Substitution can be achieved in two ways:

(a) New products replace traditional products for various economic and technological reasons; in this case, the flows of new materials are on the right-hand side of our equation. In other words, the dynamics of these new products determine the changes in the growth of traditional products in our model.

(b) Sometimes there is a shortage of traditional goods in the economy and the rate of growth of new goods is stimulated by the total demand for materials according to the elasticity of substitution. In this case, a variation in the dynamics of traditional products generates changes in the rates of growth of new goods. For example, the amount of electricity used in the transportation sector must be considered in the equation for the coal and oil required in this sector of the economy.

Supply factors influence not only intermediate deliveries but also final demands, such as personal consumption, investment, exports, and imports.

The second set of equations may be used to determine the demand for each of the 80 products; there are therefore 80 equations in this group.

The third set of equations is used to estimate the aggregated output of the 18 industries.

We use regressions (2) and (3) instead of precise balance equalities because certain small flows are not described by the traditional equations with $a(i,j)$ coefficients.

Note that it is easy to solve system (1)-(3) if all equations are linear because all of the equations in the first set are expressed in terms of outputs.

When, in forecasting, the value of $X(i)_T$ is given exogenously, say as $X^*(i)_T$, we may obtain the difference between the derived demand and this value $X^*(i)_T$. However, the value $X^*(i)_T$ is used in all equations (1) and (1a) as a supply constraint. If we start from a set of final demands and a projection of $X^*(i)_T$, and find that $X(i)_T$ is greater than $X^*(i)_T$, then a plan with this level of final demand and this level of capacity expansion in product i will result in excessive output of i.

The interdependence between the equations clearly causes problems in estimating parameters of (1) and (2) in both aggregated and disaggregated models. These parameters are usually estimated using the time-series data under the assumption that each equation for flows $X(i,j)$ and $Y(i,\alpha)$ contains errors $\varepsilon(i,j)_t$, $\varepsilon(i,\alpha)_t$:

$$X(i,j)_t = F[\,X(i)_t,\, X(j)_t,\, X(k,l)_t, t\,] + \varepsilon(i,j)_t \tag{4}$$

or, in the linear form:

$$X(i,j)_t = \beta_i * X(i)_t + \beta_j * X(j)_t + \beta_{k,l} * X(k,l)_t + \delta * t + \varepsilon(i,j)_t \tag{4a}$$

Note that the intercept is omitted here and later for simplicity.

When these equations are combined into a set (model), there seems to be some implicit interdependence between some or all errors $\varepsilon(i,j)_t$ and hence between flows $X(i,j)_t$ and equations in the model yet each of equations contain only few variables on the right-hand side. This means that the estimates of the parameters in a separate equation of form (4) will depend on an error specification for all equations. Examples of this type of situation are given in many textbooks on econometrics. It is also clear that if $X(i,j)_t$ is explained by some $X(k,l)_t$ which includes an error term $\varepsilon(k,l)_t$, then the slope coefficient $\beta_{k,l}$ in equation (4a) will also be affected by this value. We should therefore rewrite equations (1)-(3) in the following form:

$$B*Y + G*X = \varepsilon \qquad (5)$$

i.e., as a system of simultaneous equations, with M dependent and K predetermined variables based on a sample of T observations.

Here Y is a $T*M$ matrix of dependent variables, and X is a $T*K$ matrix of predetermined variables that do not appear as dependent variables in this system.

B is an $M*M$ matrix of parameters for the dependent variables and G is an $M*K$ matrix of parameters for the predetermined variables.

The error term included in each equation leads to an $M*K$ matrix $\varepsilon(i,j)$ in time space.

We can therefore describe *MII* as a structural model; as this term is sometimes used without a formal definition we shall adopt the definition used in econometrics.

An equation from the first set can be rewritten in the form:

$$y(1)_t + \beta_{1,2}*y(2)_t + .. + \gamma_{1,1}*x(1)_t + \gamma_{1,2}*x(2)_t + .. = \varepsilon(1)_t \qquad (6)$$

i.e., replacing $X(i,j)$ by $y(i)$; for example, deliveries of ferrous metals for construction purposes $X(1,14)$ might be explained by the output of the ferrous metals industry $X(1)$, the output of the construction industry X(14), and the flow of ferrous metals for machine building $X(1,7)$:

$$X(1,14)_t + \beta_{1,2}*X(1,7)_t + \gamma_{1,1}*X(1)_t + \gamma_{1,2}*X(14)_t = \varepsilon(1,14)_t$$

Note that here $X(1)$ and $X(14)$ are considered as predetermined variables.

While specifying the model, we usually include some *a priori* restrictions on the values and signs of the parameters, e.g., we do not include the outputs of other industries as predetermined demand variables in a model for $X(1,14)$ and we would like $\beta_{1,2}$ to be negative. We also know that some $\beta(i,j)$ should be equal to zero, or to one, or to minus one for equalities.

Much work has been devoted to the problem of estimating the parameters in such systems. It is well-known that applying an ordinary least squares technique to a single equation of type (5) will lead to estimates of the parameters that are both biased and inconsistent. The presence of only one dependent variable from system (5) as an explanatory variable in equation (6) means that its error is included in all of the other explanatory variables.

For simplicity, we will assume that there is only one exogenous variable $x(1)_t$ in (5) and that the system consists of three equations. Let us write two of these equations, assuming that $y(1)_t$ depends only on $x(1)_t$:

$$y(1)_t + \gamma(1,1)^* x(1)_t = \varepsilon(1)_t \tag{7}$$

$$y(2)_t + \beta(2,1)^* y(1)_t + \gamma(2,1)^* x(1)_t = \varepsilon(2)_t \tag{8}$$

Therefore $y(1)_t$ can be represented by $\hat{y}(1)_t + \varepsilon(1)_t$, where $\hat{y}(1)_t$ does not contain the error term. Assuming that terms $\hat{y}(1)_t$ and $\varepsilon(1)_t$ have an equal influence on $y(2)_t$, equation (8) can be rewritten in the form:

$$y(2)_t + \beta(2,1)^* \hat{y}(1)_t + \gamma(2,1)^* x(1)_t = \varepsilon(2)_t - \beta(2,1)^* \varepsilon(1)_t \tag{8a}$$

To obtain an unbiased estimate of $\beta(2,1)$ we should minimize the sum of squared residuals for this equation with respect to the unknown level of the estimate to be found, namely $\beta(2,1)$, and with respect to values of $\varepsilon(1)_t$. In this case the estimate of the parameter $\beta(2,1)$ measures the overall effect of $\hat{y}(1)_t$ and its error $\varepsilon(1)_t$ on $y(2)_t$.

If we assume that the error $\varepsilon(1)_t$ does not influence $y(2)_t$, then the last term should be omitted from (8), which then becomes:

$$y(2)_t + \beta(2,1)^* \hat{y}(1)_t + \gamma(2,1)^* x(1)_t = \varepsilon(2)_t \tag{8b}$$

and we will obtain an unbiased and consistent estimate of $\beta(2,1)$ by minimizing the sum of squared residuals for (8b). The main problem here is that we do not always know $\hat{y}(1)_t$ and $\varepsilon(1)_t$ *a priori*. Different approaches are used to estimate $\beta(2,1)$ under these conditions.

Note that if there are no errors in the variables on the right-hand side of the equation, it is very easy to find unbiased estimates of β. For example, in the simplest case

$$Y_t = \beta * X_t + \varepsilon_t$$

if we use only the two points representing the minimum and maximum values of X_t, then the estimate

$$\hat{\beta} = \frac{y(x_{max}) - y(x_{min})}{x_{max} - x_{min}} \tag{9}$$

will be unbiased, i.e.,

$$E[\hat{\beta}] = \beta$$

where E is the mathematical expectation, and $\hat{\beta}$ is an estimate of the parameter β.

Assuming that the value of $\gamma(2,1)$ is known to be $\hat{\gamma}(2,1)$, equation (8) leads to:

$$\hat{\beta}(2,1) = \frac{\sum[y(2)_t - \hat{\gamma}(2,1) * x(1)_t] * y(1)_t}{\sum y(1)_t^2} \tag{10}$$

and the expectation of the estimate obtained, if $\hat{\gamma}(2,1) = 1.0$, is:

$$E[\hat{\beta}(2,1)] = \beta(2,1) + E\left[\frac{\sigma^2_{\varepsilon(1)}}{\sigma^2_{\hat{y}(1)} + \sigma^2_{\varepsilon(1)}}\right] \tag{11}$$

where $\sigma^2_{\hat{y}(1)}$ is the variance of $\hat{y}(1)_t$ and $\sigma^2_{\varepsilon(1)}$ is the variance of $\varepsilon(1)_t$. Obviously the estimate of $\beta(2,1)$ obtained using (10) does not converge to $\beta(2,1)$, yet T increases to infinity.

The well-known method of instrumental variables used when errors are present in explanatory variables yields consistent but biased estimates of parameters. Firstly, $\gamma(2,1)$ is estimated from (7), fitting $y(1)_t$ by the nonstochastic variable $x(1)_t$, and then using these fitted values $\hat{y}(1)_t$ as the new explanatory variable in (8) to obtain a consistent estimate of $\beta(2,1)$ through the expression:

$$\hat{\beta}(2,1) = \frac{\sum[y(2)_t - \gamma(2,1)*x(1)_t]*\hat{y}(1)_t}{\sum\hat{y}(1)_t*y(1)_t} \tag{12}$$

The expectation of $\hat{\beta}_{2,1}$ is:

$$E\,[\hat{\beta}(2,1)] = \beta(2,1)*[\,1 - E\,[\,\frac{\sum\varepsilon(1)_t*\hat{y}(1)_t}{\sum\hat{y}(1)^2_t + \sum\varepsilon(1)_t*\hat{y}(1)_t}]]$$

It is obvious that this type of procedure with independent $\varepsilon(1)_t$ and $\hat{y}(1)_t$ does not yield an unbiased estimate of the parameter because:

$$E[\frac{\sum\varepsilon(1)_t*\hat{y}(1)_t}{\sum\hat{y}(1)^2_t + \sum\varepsilon(1)_t*\hat{y}(1)_t}]$$

might not be equal to zero in every case, see [5], section 5.4.

We have given this example for the simplest recursive model to stress that the arrangement of the equations as a recursive model does not avoid the problem of errors in variables, and use of the ordinary least squares technique is not acceptable. It should be emphasized that the problems arising from errors in explanatory variables and the interdependence of errors within a set of equations are of great importance.

Note that a model is sometimes considered to be a recursive when this is not strictly justified. This can be illustrated using the following simple set of three equations:

$$\begin{aligned} y(1)_t + \qquad\qquad\qquad\qquad \gamma_{1,1}*x(1)_t &= \varepsilon(1)_t \\ \beta_{2,1}*y(1)_t + y(2)_t + \qquad \gamma_{2,1}*x(1)_t &= \varepsilon(2)_t \\ \beta_{3,2}*y(2)_t + y(3)_t + \gamma_{3,1}*x(1)_t &= \varepsilon(3)_t \end{aligned} \tag{13}$$

It is a pure simultaneous system and it could not be considered as the recursive one because $y(1)_t$ is omitted from the third equation. To be recursive, the system should fulfill the following conditions:

$$\sum_t \varepsilon(1)_t*\varepsilon(2)_t = 0 \tag{14}$$

$$\sum_t \varepsilon(1)_t*\varepsilon(3)_t = 0 \tag{15}$$

$$\sum_t \varepsilon(2)_t*\varepsilon(3)_t = 0 \tag{16}$$

In this case, it is possible to estimate the parameters for each one equation in sequence by using fitted values of the dependent explanatory variables or primary data. In system (13) exists the errors are independent; condition (14) is fulfilled but not necessarily both (15) and (16). Instead, we obtain the condition:

$$\sum_t [\ \gamma_{2,1} * \varepsilon(1)_t + \varepsilon(2)_t\ ] * \varepsilon(3)_t = 0 \tag{17}$$

For example, if we have the following errors $\varepsilon(i)_t$ for system (13):

$\varepsilon(1)_t$	-2	1	1	.	-2	1	1	.	.
$\varepsilon(2)_t$	0	-1	1	.	0	-1	1	.	.
$\varepsilon(3)_t$	1	-2	1	.	1	-2	1	.	.

and assuming $\gamma_{2,1} = 1$, it can be shown that both $\varepsilon(1)_t$ and $\varepsilon(2)_t$ are highly correlated with $\varepsilon(3)_t$, and yet

$$\sum_t [\varepsilon(1)_t + \varepsilon(2)_t] * \varepsilon(3)_t = 0$$
$$r\ [\ \varepsilon(1)_t * \varepsilon(3)_t] = -0.5$$
$$r\ [\ \varepsilon(2)_t * \varepsilon(3)_t] = 0.85$$

where $r[i,j]$ is the coefficient of correlation between the variables i and j.

The model of interindustry interactions given in the form of a set of simultaneous equations. It would be possible to rewrite it in reduced form but economists prefer to work with a structural form in which judgments can be made using the signs and values of the parameter estimates rather than the reduced form (18) in which dependent variables are expressed only as functions of the predetermined variables. However, a reduced form is really used for forecasting. It is therefore necessary to estimate the parameters of the following set of equations:

$$Y = \Pi * X + \eta \tag{18}$$

where η is the error term corresponding to the ε term in the structural form. As there are only nonstochastic variables on the right-hand side of (18), the estimates of Π must be unbiased if the ordinary least-squares technique is used. These estimates cannot be interpreted by economists but they are used in forecasting because only the predetermined values of variables X are known before the model is applied. However, these estimates can also be found using the estimates of B and Γ through the following equation:

$$\hat{\Pi} = -\hat{B}^{-1} * \hat{\Gamma}$$

It is sometimes impossible to estimate B and Γ from known estimates of Π but we will not consider this problem any further. Some research workers prefer to estimate Π from the last equation given above because all *a priori* restrictions on the structural form are used in estimating B and Γ - the elements $\hat{\pi}_{i,k}$ obtained in this way should be more effective than those obtained by applying the ordinary least-squares technique directly to (18).

All of this means that there is no unique way of obtaining the best estimates of π, β, and γ. In every case the main problem is connected with the errors in the explanatory variables, and with their interdependence.

We shall now consider some examples taken from the model of interindustry interactions.

Our first example is related to the distribution of the output of the "ferrous metals" industry (represented by index 1 in the following equations) between two major metal -consuming industries, namely "machine building" (index 7 used below) and "construction" (index 14). The system of equations is as follows (omitting the time index for simplicity):

$$x(1,7)=\beta_{1,1}*x(1) \qquad + \gamma_{1,1}x(7)+\gamma_{1,2}*x(2,7)+\varepsilon(1,7) \qquad (19)$$

$$x(1,14)=\beta_{2,1}*x(1) + \beta_{2,2}*x(1,7) \qquad +\gamma_{2,3}*x(14)+\varepsilon(1,14) \qquad (20)$$

$$x(1)= \qquad x(1,7) \;+x(1,14) \;+x(1,\textstyle\sum) \qquad +\varepsilon(1) \qquad (21)$$

$$x(1,\textstyle\sum)= \qquad \gamma_{4,4}*x(\textstyle\sum) \qquad +\varepsilon(1,\textstyle\sum) \qquad (22)$$

where $x(2,7)$ is the intermediate flow of nonferrous metals used by the "machine building" industry and $x(1,\sum)$ represents deliveries of ferrous metals to all industries except "machine building" ($x(1,7)$) and "construction" ($x(1,14)$). $x(\sum)$ is used as an explanatory variable for these other deliveries.

Here only $x(7)$, $x(14)$, $x(2,7)$, and $x(\sum)$ should be considered as predetermined variables. If the flow $x(1,\sum)$ is expressed using (21) instead of (22):

$$x(1,\sum) = x(1) - x(1,7) - x(1,14) + \varepsilon(1,\sum) \qquad (21a)$$

then $x(1)$, not $x(\sum)$ will be a predetermined variable. The system should then be reformulated, replacing some β for γ and *vice versa*. The goodness of fit of $x(1,\sum)$ should be estimated in both systems. Thus the system of equations (19), (20), and (21a) represents a simultaneous model of a recursive form because $x(1,7)$ is not influenced by $x(1,14)$ and $x(1,\sum)$, and $x(1,14)$ is a function of $x(1,7)$, and $x(1,\sum)$ absorbs errors $\varepsilon(1,7)$ and $\varepsilon(1,14)$. The ordinary least-squares technique *OLS* would give a biased estimate of the very important parameter $\beta_{2,2}$ which reflects the influence of supply constraints on changes in the intermediate flows. This recursive system may therefore be summarized as follows:

$$\begin{aligned} x(1,7) &= \gamma_{1,1}{}^*x(1) + \gamma_{1,2}{}^*x(7) + \gamma_{1,4}{}^*x(2,7) + \varepsilon(1,7) \\ x(1,14) &= \beta_{2,1}{}^*x(1,7) + \gamma_{2,1}{}^*x(1) + \gamma_{2,3}{}^*x(14) + \varepsilon(1,14) \\ x(1,\sum) &= -x(1,7) - x(1,14) + x(1) + \varepsilon(1,\sum) \end{aligned} \qquad (24)$$

This system does not contain any feedback between variables (or errors) and $\beta_{2,1}$ could be estimated by two techniques: firstly, by assuming that the errors in $x(1,7)_t$ are insignificant, and secondly, by dividing $x(1,7)_t$ into two parts according to (8). Assuming that the errors are insignificant, we have estimated $\beta_{2,1}$ by *OLS* and found it to be quite small (-0.5) and insignificant (standard error of the estimate is 0.35). The second technique described above is called the two-stage least-squares method (2*SLS*) and yields $\hat{\beta}_{2,1} = -2.0$ with the same standard error. The results of fitting are given in Table 1, where the numbers in parentheses are the standard errors of the estimates.

Table 1. The parameter estimates and the goodness of fit for model (24). R^2 is the coefficient of determination, and DW is the Durbin-Watson statistic.

Technique	$\beta_{2,1}$	$\gamma_{2,1}$	$\gamma_{2,3}$	R^2	DW
OLS	-0.4856 (0.346)	0.1759 (0.108)	0.3157 (0.125)	0.990	1.31
$2SLS$	-1.9935 (0.342)	0.7852 (0.128)	0.1798 (0.058)	0.997	1.73

The table shows that $2SLS$ estimates are more efficient (they have higher t-statistics) than those obtained by using OLS. All of the estimates change significantly when we remove the so-called errors from $x(1,7)_t$:

$$e(1,7)_t = x(1,7)_t - \hat{\gamma}_{1,1}{}^*x(1)_t - \hat{\gamma}_{1,2}x(7)_t - \hat{\gamma}_{(1,4)}{}^*x(2,7)_t$$

where β and γ are estimated by using OLS. Note that one important assumption was made: $x(1)_t$ does not include any error but there may be a correlation between $\varepsilon(1)_t$ and both $\varepsilon(1,7)_t$ and $\varepsilon(1,14)_t$ over the sample. If there is a positive correlation between $\varepsilon(1)_t$ and the other errors, then large errors will be obtained for $x(1,\sum)_t$ on applying $2SLS$ to the last equation of system (24). Moreover, $\varepsilon(1,7)_t$ will also affect the estimate of $\gamma_{2,1}$, leading to a large increase in the estimate on the second step (the $2SLS$ estimate is very high compared to the OLS estimate, as shown in the table). Note that the steady increase of $x(1,7)$ over time is fitted quite well, $R^2 = 0.998$ and DW statistic $= 1.6$. We can safely assume that the small changes in the annual steady increase of $x(1,7)$ do not influence either $\hat{x}(1,14)_t$ or its variance, represented by $\varepsilon(1,14)_t$. The estimate of $\beta_{2,1}$ can be written as follows:

$$-0.4856 = -1.9935{}^*\lambda + [1.0 - \lambda]{}^*\hat{\beta}_\varepsilon$$

where $\hat{\beta}_\varepsilon$ is a slope coefficient representing the impact of $\varepsilon(1,7)_t$ on $x(1,14)_t$. If $\lambda = 0.5$, then the estimate of β_ε should be positive (+1.0). Because in $2SLS$ $\varepsilon(1,7)_t$ acts only through $x(1)_t$, which is positively correlated with $x(1,14)_t$, the estimate of $\gamma_{2,1}$ is a mixture of both impacts $\hat{x}(1)_t$ and of the error term $\varepsilon(1)$.

The impact of small changes in $x(1)_t$ around the trend is possibly larger than the effect of a steady increase in the supply of ferrous metals.

When forecasting, we should use the estimates obtained by $2SLS$ if there is no *a priori* assumption that the rate of growth of $x(1)_t$ will differ significantly from the past trends.

Our second example is related to the set of equations describing the growth of the "transportation" industry. In the model of interindustry interactions, output of this industry is estimated by using the following equation:

$$x(16)_t = \alpha + \gamma * \sum_j x(16,j)_t + \delta * t + \varepsilon(16)_t \tag{25}$$

instead of an equality because not all flows are included in the sum. Here each flow $x(16,j)$ can be fitted by the regressions for separate flows in physical units $x(16,j_s)$; these flows are then aggregated using the equation:

$$x(16,j)_t = \sum_{s=1}^{S_j} a(16,j)_{1972} * x(16,j_s)_t \tag{26}$$

where $a(16,j)_{1972}$ are values taken from the 1972 input-output table to transform the values of intermediate transportation flows from physical units to monetary terms. Because only the main intermediate flows are estimated by regressions and the values of $a(16,j)$ change over time, regression (25) is used to fit the output of this industry rather than using the equality:

$$x(16)_t = \sum_{j=1}^{18} x(16,j)_t$$

We have fitted all of the flows $x(16,j)$ quite accurately, the variance of the residuals of the regressions being less than 5% in every case. We can therefore use (25) with the appropriate goodness of fit:

$$\hat{x}(16)_t = \hat{\alpha} + 1.18 * \sum_j x(16,j)_t \tag{25a}$$

with R^2= 0.999 and DW = 1.0. Obviously, the 18% underestimation of the output of the transportation sector ($\hat{\gamma}$=1.18) is not very large, and so this equation could be used in forecasting in the disaggregated version of *MII*. However, we have fitted $x(16)_t$ using the actual values of the flows instead of their fitted values $\hat{x}(16,j)_t$. In forecasting we can find only the fitted values from the corresponding equations for $x(16,j_s)_t$, and so it is necessary to derive the estimates of (25) with the fitted values $\hat{x}(16,j)_t$ from the reduced form:

$$x(16)_t = \alpha + \gamma * \sum_j \sum_{s=1}^{S_j} a(16,j)_{1972} * \hat{x}(16,j_s)_t + \delta * t + \varepsilon(16)_t$$

This is basically the same as estimating R^2 for the equality using a reduced form in a system of simultaneous equations. We have not imposed any restrictions on the interrelationship between errors in the regressions for $x(16,j_s)$, but there are about 15 equations for aggregated flows $x(16,j)_t$ and so the sum of the errors ($\sum_{j=1}^{15} \sum_{s=1}^{S_j} \varepsilon(16,j_s)_t$) should not be very large and we should be able to rely on the goodness of fit obtained. However, it sometimes happens that the parameter estimates in (25a) are unreasonable:

$$\hat{x}(16)_t = \alpha + 1.4 * \sum_j \sum_{s=1}^{S_j} a(16,j)_{1972} * \hat{x}(16,j_s)_t \qquad (25b)$$

with $R^2 = 0.858$ and $DW = 0.7$. It seems strange that the estimate of the slope coefficient γ increases by 20% with an unexpected decline in R^2. The average error of regression (as a percentage of the mean value of $x(16)$) has also risen, from 1.5% to 15%. This means that the residuals in the set of equations are very highly correlated and that their sum is far from zero for each value of t, despite the fact that $x(16,j)_t$ were fitted so well by their own explanatory variables.

This situation is illustrated in Figure 1.

Our final example is concerned with the allocation of the electricity industry output, as treated in the disaggregated model. There is some question of whether aggregated or disaggregated flows should be used to fit the total output of the electricity generating industry (represented by index 8). We divided electricity consumption within each industry into three parts: $x(8,j_1)$, $x(8,j_2)$, and $x(8,j_3)$. Here the first term represents the electricity used for driving mechanisms, the second, the electricity used for technological processes, and the third, the electricity used for lighting and for other purposes. We therefore have the equality for each industry j:

$$x(8,j)_t = \sum_{k=1}^{3} x(8,j_k)_t \tag{27}$$

and for each component we could find appropriate linear equations with explanatory variables, and fit their values quite well. Each of the electricity flows depends on certain characteristic variables, e.g., heating and lighting is connected with the length of time worked by the industry, the electricity used for driving mechanisms depends on the equipment used and its efficiency, and technological demand is quite closely related to the output of the industry, or to the amount of primary materials consumed. We can then obtain the total demand of each industry by summing the three flows, as before. We need to estimate the goodness of fit only for $x(8,j)$, not for each flow separately; problems similar to those discussed in the example on "transportation" can also arise. However, on testing the equations we found that the equalities are fulfilled quite adequately. For example, the values of R^2 for the oil-refining industry are 0.993, 0.998 and 0.989 for the three flows, respectively, and R^2 for $x(8,j)$ is 0.998. This is true for all industries, and so these sets of equations can be used in the model directly.

Sometimes dividing the flows yields better results for an aggregated flow than estimating an equation for the aggregated flow directly. As the model of interindustry interactions consists of some weakly separable blocks then appropriate techniques can be chosen for each case, according to the quality of the data and the associated restrictions.

Some examples in which the $2SLS$ technique is used to obtain estimates for the ferrous metals industry are given in [6].

For forecasting, we definitely need only the reduced form if the model is based on the standard input-output technique: i.e., is designed to calculate the demand for industries according to the exogenous estimates of total final demand and its components $Y(i,\alpha)_T$. This model can also be used to solve another problem - to find out whether the exogenous estimates of industrial outputs balance with one another. In this case $X^*(i)_T$ is given and equalities (2) and (3) are needed only to estimate the difference between the derived demand $X(i)_T$ and those estimates $X^*(i)_T$. We replace $X(i)_T$ in each equation of (1) by the exogenous values

$$X(i,j)_T = \beta_0 + \beta_1 * X(j)_T + \beta_2 * X^*(i)_T + \beta_3 * X(k,l)_T + \varphi(T)$$

and the impact will appear in the derived demand on solving the system both with and without given $Y(\alpha)_T$. Thus we need estimates of goodness of fit for all of the regressions (1)-(3) in a reduced form. Unfortunately, not all standard procedures can be used to obtain R^2 values for equalities, and this can lead to the overestimation of model quality in forecasting.

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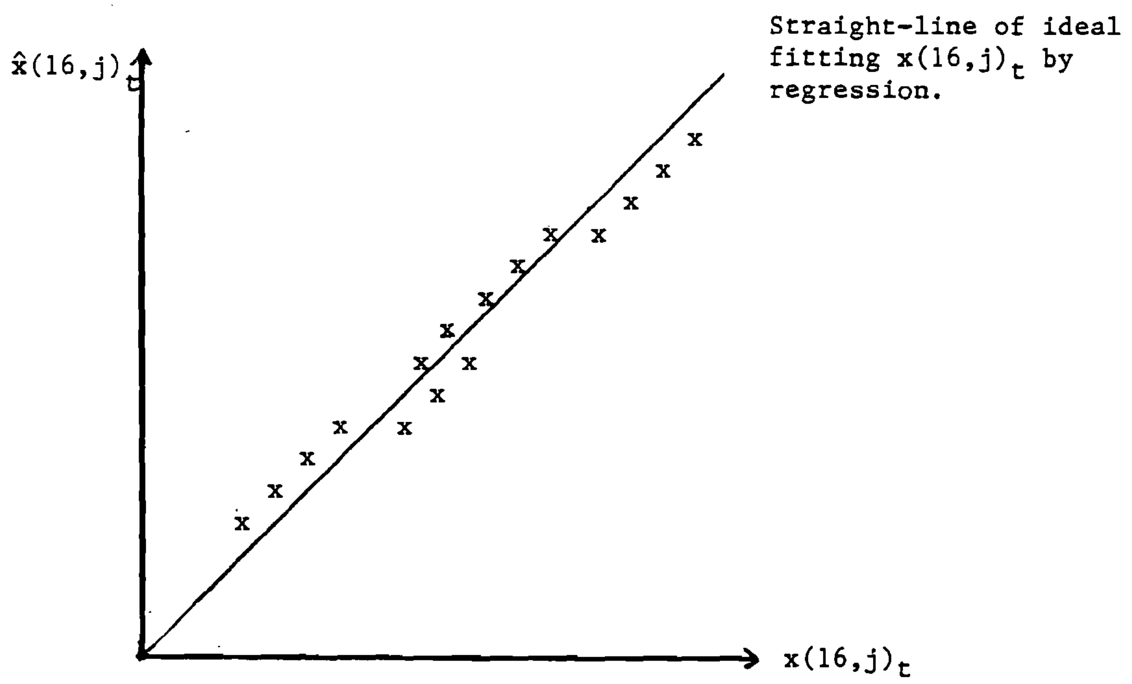
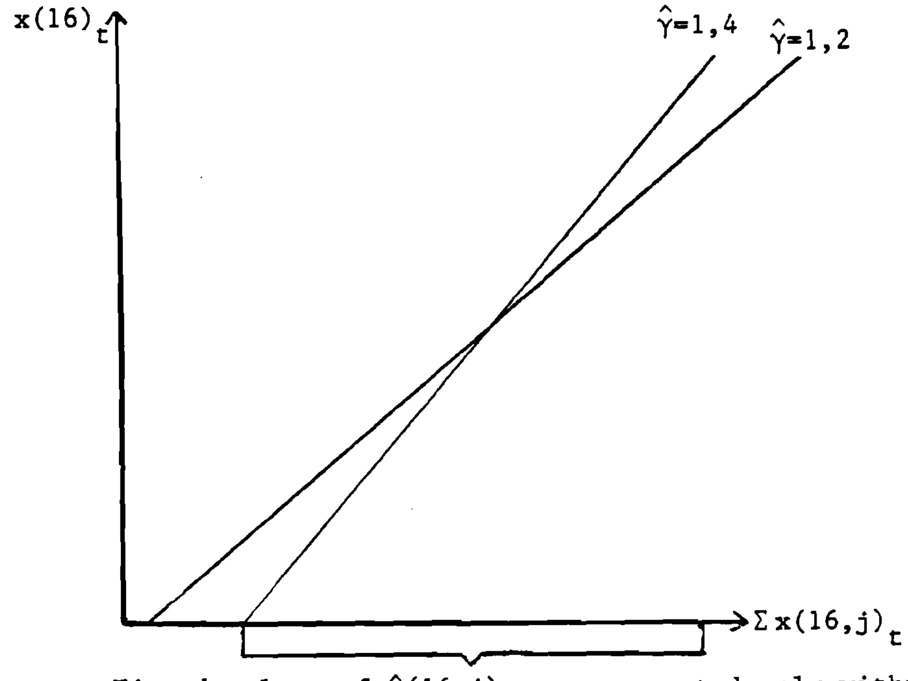
Figure 1. Scatter diagrams for sums of the real and fitted values and for the output values of the transportation sector.



x = actual results of fitting $x(16,j)_t$ as a typical case.



Fitted values of $\hat{x}(16,j)_t$ are presented only within this interval.