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Working paper

Out of the black into the green? Modeling the pathways for regional coal transitions

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WP-25-002

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Date: 11 September 2025

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ZVR 524808900

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The authors gratefully acknowledge funding from IIASA and the Member Organizations that support the institute.

The authors gratefully acknowledge funding from IIASA along with The Austrian Academy of Sciences, and the other Member Organizations that support IIASA.

This work received financial support from FTI22-G-019 : Modellierung regionaler Auswirkungen auf die Wohlfahrt eines gerechten Kohleaustiegs

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Out of the black into the green? Modeling the pathways for regional coal transitions

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This work received financial support from FTI22-G-019 : Modellierung regionaler Auswirkungen auf die Wohlfahrt eines gerechten Kohleaustiegs

Keywords: coal phase-out, just transition, regional economy, welfare, dynamic optimization, bifurcation analysis, path dependency

Abstract

Coal is a major contributor to anthropogenic carbon emissions and climate change. Coal mining and combustion are also a leading cause of premature mortality due to local air pollution. On the other hand, coal is central to many regional economies that rely on its mining, power generation, industrial use and exports. With changing climate and rising pollution levels associated with coal, the urgency for coal phase-out has become more prominent in recent years. This has put pressure on coal-dependent regional economies to implement energy transitions in a time bound manner. This paper studies optimal pathways for a coal phase-out within a small, open, regional economy consisting of a coal extraction sector, an energy sector composed of both coal-based and renewable power generation and a final consumption sector that relies on coal and electricity. Taking the perspective of a social planner who maximizes regional welfare and employing optimal control theory, we study the conditions under which coal extraction and fossil power generation are phased out, depending on preferences, cost and price structures. We also provide a systematic analysis of the dynamic processes associated with the transition out of coal of (formerly) coal-based regional economies, including the scope for multiple equilibria that may reflect stalling transitions. Our results will be relevant for regional governments in undertaking a transition away from coal in way that safeguards regional welfare and at the same time contributes to the global climate goals.

1 Introduction

Coal, the most carbon-intensive fossil fuel, was responsible for over 40% of the global carbon dioxide emissions from fuel combustion in the past three decades (IEA, 2019). Around 165 partners including 70 countries are now part of the Powering Past the Coal Alliance (PPCA) which has committed to coal phase-out at the Glasgow conference of parties (COP26) in 2021 (PPCA, 2022). Apart from the climate impacts, coal mining, transport and combustion activities affect the local environment and health of people. The global premature mortality due to fossil fuel related air pollution was estimated to be around 8.7 million in 2018 (Vohra et al., 2021). These factors have made coal phase-out central to the debates surrounding climate change, local pollution and public health. However, coal is also key to many local economies that rely on its mining, transportation, energy production and exports. In many countries, these coal-based economies are concentrated in a few sub-national regions that would have to undergo enormous energy and economic transitions in case of a coal phase-out. Examples of such coal-based regional economies include the Ruhr region which produced over 80% of the coal in Germany during the country's peak mining phase in the 1950s (Dahlbeck et al., 2021). A current example is India where around 10 districts in eastern states like Jharkhand, Chattisgarh and West Bengal are responsible for 70% of the country's annual coal production (Agrawal et al., 2024). The calls for coal phase-out at national and international level have a direct impact on the economy of such coal-dependent regions. As a result, finding ways for coal phase-out and the associated restructuring of the regional economies without compromising regional welfare has become a major challenge of our times.

Past studies have mainly dealt with coal transitions using dynamic, integrated models of energy, climate and economy at a global or multi-regional level (Nordhaus, 1993; Nordhaus, Yang, 1996; Golosov et al., 2014). Multi-regional models such as RICE by Nordhaus, Yang (1996) deal with energy transitions at national level where the world is divided into 6 to 10 major regions that have the ability to influence the supply and demand of energy sources such as coal. In another approach Bonneuil, Boucekkine (2016) study the switch from fossil fuels to renewable energy in a social planner's problem with primary consideration for the timing and rate of transitions. The results are driven by rising pollution levels due to fossil fuels and availability of renewable resources. Pommeret, Schubert (2022) set up a dynamic model of optimal transitions from coal to intermittent sources like solar for electricity generation. In this case, the social planner is optimizing the net surplus of the economy when switching electricity generation from coal to intermittent renewables. Bigerna et al. (2019) use a real options approach to study a firm's problem to invest in renewables with the help of available subsidies under uncertainty. In summary, these models present important insights about how a social planner or firm can optimally transition away from fossil fuels. However, when dealing with small, open, coal-dependent economies like the Ruhr region in Germany or Jharkhand in India, some additional considerations are required. First, coal is not just a source of energy but also a source of income and employment for the regional economies. This makes welfare impacts an important element when modelling the regional coal transitions. Second, coal mining is a key industry in the region which may have led to the establishment of coal power plants and other coal-based industries like steel-making. As a result, phasing out coal from such regions would require an integrated approach to transition away from the entire coal supply chain. Third, small open economies are price takers in regard to internationally traded energy commodities such as coal and electricity. World market prices may, therefore, have an impact on the evolution of coal-based industries and renewable investments in the region. Finally, the option to trade allows the regional social planner to adjust the allocation flexibly and e.g. export surplus coal in the world market if the coal prices are attractive even if the regional economy switch from coal to renewables for electricity generation.

Evidence from the social and political science literature on past coal phase-outs suggest that regional coal-dependent economies faced many challenges during such transitions. A report on coal transitions in industrialized countries found that the decline of coal in these regions was mostly unplanned and led to social and economic losses to local economies that were dependent on coal for revenues and employment (Caldecott et al., 2017). The report also highlights non-climatic factors such as the fall of the Soviet Union which led to the decline of coal demand in the Czech Republic or the availability of cheaper alternatives like natural gas in the Netherlands and the US which were responsible for the coal phase-out in these regions. Diluiso et al. (2021), through a systematic review of coal transitions from 44 countries, also found supply-side transitions in most cases resulting from changing trade dynamics due to globalization. A case study of coal transition policies in the Latrobe Valley of Victoria, Australia, found that the compensation given by the government was inadequate to resolve the distributional and welfare concerns of local communities in the region (Weller, 2019). In the coal-dominant Ruhr region in Germany, when coal mining started declining in the 1950s due to international competition, the initial policy response was to secure mining jobs and the coal businesses. However, the Ruhr region eventually saw more structural reforms of the local economy starting in the 1980s (Dahlbeck et al., 2021). For the US, findings by Mayer (2018) and Snyder (2018) suggest that a targeted approach with welfare considerations of coal workers and other stakeholders affected by decarbonization policies could facilitate just transitions and gain support from local policymakers.

Past studies have focused on transitions in industrialized countries where coal consumption is already declining. Few studies consider the economic and welfare implications of coal transitions in low and middle-income countries where investments in new coal projects are underway (Diluiso et al., 2021). This highlights the need to systematically model the welfare impacts in coal-dependent regions that are under pressure from national and international climate and environment policies to phase out coal.

To address this gap, we propose a comprehensive framework, grounded in optimal control theory (cf., Grass et al. (2008)), to model the transition towards clean energy for coal-based regional economies with consideration for the welfare impacts. We describe the economy by a system of ordinary differential equations (describing the states of the system, specifically the capital invested into coal extraction, coal-based and renewable energy generation, and final goods production), that reacts on control variables (e.g., investments into the respective types of capital) set by a social planner who maximizes an inter-temporal welfare function subject to the capital dynamics, production technologies, a pollution function, and a budget constraint that embraces the investment costs, expenditures for private and public goods and the proceeds from net exports of coal, energy and final goods. To our knowledge, this is one of the first attempts to systematically model and analyze the welfare implications of coal phase-out at the level of a small, open, coal-dependent regional economy.

2 Model

Our analysis is based on optimal control theory to model the transitional process of the coal phase out. In this section, we motivate step-by-step the model of a regional economy with an energy system that initially is coal-based but allows for an endogenous transition towards regenerative energy sources. Figure 1 depicts a coal-dependent, regional (small) open economy with coal and renewables as domestic energy sources. The energy sources are available for electricity production. Electricity and coal can be further utilized in industrial production (such as steel-making) or they can be exported in the world market. In addition, the economy can import coal and electricity from the world market for producing industrial goods.

Essentially, our economy consists of four sectors: coal extraction, coal-based energy production, renewable energy production, and industrial production. All four sectors are based on a corresponding capital stock, modelled as state variable $K^i(t)$ and evolving according to the classical capital accumulation dynamics (t denotes time):

$$\dot{K}^i(t) \quad = \quad I^i(t) - \delta_i K^i(t). \tag{1}$$

The index $i \in \{C, R, N, Y\}$ stands for coal extraction (C), renewable energy (R), coal power generation (N), and industrial (final goods) production (Y). The stocks can be increased by (non-negative) sector i-specific investments $I^i(t)$ (control variables set by the decision maker) that are associated with convex costs in the planner's problem below. Capital stocks depreciate at rates $\delta_i > 0$, which may generally differ due to varying running times of the underlying technologies.

The four industry sectors produce three different kinds of outputs: coal, energy (i.e., electricity) which is produced both in the renewable and coal-power sectors, and final goods. As all of them are structured differently and employ distinct inputs, each of them is described by a specific production function built around the four types of capital.

Coal production $f^C(\cdot)$ uses $K^C(t)$ as single input with constant returns to scale. We furthermore assume that the entire coal extraction capital at t is used, i.e., that the policy maker is unable to phase out coal instantaneously by shutting down (parts of) the coal extraction industry and treating the unused capital as stranded asset. A coal phase-out is only possible by a smooth running-down of extraction capital.

Electricity $E(t)$ can be produced out of coal and renewables (solar, wind, etc.). Therefore, electricity is the additive output of the two corresponding production functions:

$$E(t) \quad = \quad f^N(K^N(t)) + f^R(K^R(t)). \tag{2}$$

Both functions $f^N(\cdot)$ and $f^R(\cdot)$ share the properties of the coal production function and are expressed as functions of the respective capital levels. For coal-based power generation, we make the additional two assumptions: (i) Similar to the case of coal extraction, the entire capital stock $K^N(t)$ is used, such that there is no room for stranded assets in coal-power generation either. (ii) One unit of coal-produced energy $f^N(\cdot)$ requires one unit of coal as input, implying that $f^N(\cdot)$ expresses the coal input into fossil power generation. Note that this assumption facilitates the exposition without loss of generality.

Industry output $Y(t)$ is structured differently. The production function features production capital $K^Y(t)$ and energy as inputs. While production capital is modeled as state variable $K^Y(t)$, coal and electricity are variable energy inputs that can be substituted (at least) to a certain degree. Hence, industry output reads

$$Y(K^Y(t), w^E(t), w^C(t)), \tag{3}$$

where $w^E(t)$ and $w^C(t)$ denote electricity and coal used for industrial production, respectively. The decision maker chooses the energy mix in production. Both $w^i(t)$ are therefore control variables in the optimization.

For an open economy, the optimal input quantities need not coincide with the regional production of electricity or coal. Any excess supply or demand is balanced on the world market for the respective energy input as follows. The coal balance is a simple static balance equation with coal production $f^C(K^C(t))$ on the supply side. Coal is used for energy production, generating a demand $f^N(K^N(t))$, and for industry production, generating a demand $w^C(t)$.[1] Hence,

$$B^C(t) \quad = \quad f^C(K^C(t)) - f^N(K^N(t)) - w^C(t), \tag{4}$$

gives the net export of coal (i.e., an export for $B^C(t) > 0$ and an import for $B^C(t) < 0$) to the world market at a constant price p_C. Note that the model allows the region to transform from a coal-importing

[1] The household sector naturally also consumes energy, both coal and electricity, but is suppressed as it does not lie at the core of the present analysis.

to an exporting economy and vice versa. This change in the positioning (possibly even repeatedly) on the world market may turn out to be optimal on the transitional path towards a long-term coal phase-out.

The electricity balance follows the same logic. Electricity can be produced by coal-power plants or renewables, generating a total supply of $E(t) = E(K^N(t), K^R(t))$, based on the corresponding capital stocks as inputs, and is used in final goods production, generating a demand of $w^E(t)$. Hence,

$$B^E(t) = E(t) - w^E(t). \tag{5}$$

Excess production ($B^E(t) > 0$) can be sold on the world market at (constant) price p_E and any shortfall ($B^E(t) < 0$) has to be imported at the same price.

In our setting, coal pollutes the environment through three channels: coal extraction, coal-based generation of electricity, and its use in industrial production. Assuming that the social-planner is pursuing a strictly regional objective, they are only interested in (localized and short-term) air pollution, including its adverse health effects, and treat the long-term global impacts of CO_2 emissions as exogenous. For this reason, we consider pollution as a flow rather than a stock. Specifically, the pollution flow associated with the local production and consumption of coal can be formulated as

$$P(t) = F(K^C(t), K^N(t), w^C(t)), \tag{6}$$

where the three arguments correspond to the aforementioned channels.

We consider a social planner who maximizes the present value of the flow of social welfare (over in infitite time horizon)

$$\int_0^\infty e^{-\rho t} S\left(G(t), P(t)\right) \mathrm{d}t, \tag{7}$$

with respect to inputs into industrial production, $w^C(t)$ and $w^E(t)$, and sector investments, $I^i(t)$ ($i \in \{C, R, N, Y\}$). The period social welfare $S(\cdot)$ increases in aggregate consumption $G(t)$, which bunches private and public consumption, including infrastructure, health care, transfers, etc., and decreases in the flow of pollution $P(t)$. ρ depicts the planner's discount rate.

Abstracting from regional credit or debt, aggregate consumption in the region (or rearranged the regional domestic product) is given by

$$\begin{aligned} G(t) = \; & Y(\cdot) + p_E B^E(t) + p_C B^C(t) \\ & - c_1 \left(I^C(t)\right)^2 - c_2(t) \left(I^R(t)\right)^2 \\ & - c_3 \left(I^N(t)\right)^2 - c_4 \left(I^Y(t)\right)^2, \end{aligned} \tag{8}$$

according to which aggregate consumption $G(t)$ is sustained from the domestic production $Y(\cdot)$ of final goods, the price of which is normalized to one, plus the revenue from the net export of electricity $p_E B^E(t)$, and the revenue from the net export of coal $p_C B^C(t)$, less the costs of investment into the four types of capital. The c_i ($i = 1, \dots, 4$) are coefficients of the convex (i.e., quadratic) costs of investing in the different types of capital. To acknowledge the secular decline in renewable energy investment costs, we assume $c_2(t)$ to be a non-autonomous function decreasing over time (i.e., $\dot{c}_2(t) < 0$), while the investment cost coefficients remain constant in respect to coal extraction, coal power generation and manufacturing. For a discussion on the form of $c_2(t)$ we refer to section 3.

The full (non-autonomous) optimal control model then reads:

$$
\begin{aligned}
&\max_{\substack{I^C(t), I^R(t), I^N(t), I^Y(t)\\ w^E(t), w^C(t)}} && \int_0^\infty e^{-\rho t} S\left(G(t), P(t)\right) \mathrm{d}t && \text{(9a)}\\
&\dot{K}^C(t) &=\;& I^C(t) - \delta_C K^C(t) && \text{(9b)}\\
&\dot{K}^R(t) &=\;& I^R(t) - \delta_R K^R(t) && \text{(9c)}\\
&\dot{K}^N(t) &=\;& I^N(t) - \delta_N K^N(t) && \text{(9d)}\\
&\dot{K}^Y(t) &=\;& I^Y(t) - \delta_Y K^Y(t) && \text{(9e)}\\
&\text{Electricity} &:\;& E(t) = f^N(K^N(t)) + f^R(K^R(t)) && \text{(9f)}\\
&\text{Production} &:\;& Y(K^Y(t), w^E(t), w^C(t)) && \text{(9g)}\\
&\text{Pollution} &:\;& P(t) = F(K^C(t), K^N(t), w^C(t)) && \text{(9h)}\\
&\text{Electricity balance} &:\;& B^E(t) = E(t) - w^E(t) && \text{(9i)}\\
&\text{Coal balance} &:\;& B^C(t) = f^C(K^C(t)) - f^N(K^N(t)) - w^C(t) && \text{(9j)}\\
&G(t) &=\;& Y(\cdot) + p_E B^E(t) + p_C B^C(t) && \\
& & & -c_1 \left(I^C(t)\right)^2 - c_2(t) \left(I^R(t)\right)^2 && \\
& & & -c_3 \left(I^N(t)\right)^2 - c_4 \left(I^Y(t)\right)^2 && \text{(9k)}\\
&\text{Control constraits} &:\;& I^i(t), w^E(t), w^C(t) \geq 0, \qquad i \in \{C, R, N, Y\} && \text{(9l)}
\end{aligned}
$$

Table 1 summarizes the main model ingredients (variables, functions and parameters) at a glance. The optimality conditions (i.e., the Maximum Principle) are given in Appendix A.

3 Numerical Analysis

To investigate the optimal energy transition we have to resort to numerical solutions of (9), as an analytic characterization of the solution pattern of the four dimensional optimal control model (9) is not possible. The (necessary) optimality conditions of the Maximum Principle are stated in appendix A along with an overview of the advanced numerical technique to derive the optimal solution.

Independent variables:	
time	t
Control variables (non-negative):	
Investments into extraction capital	$I^C(t)$
Investments into capital for renewable energy	$I^R(t)$
Investments into coal electricity plants	$I^N(t)$
Investments into industry capital	$I^Y(t)$
electricity used for industry production	$w^E(t)$
coal used for industry production	$w^C(t)$
State variables:	
extraction capital	$K^C(t)$
capital for renewable energy	$K^R(t)$
coal electricity plants	$K^N(t)$
industry capital	$K^Y(t)$
Functions and parameters:	
coal production	$f^C(\cdot)$
electricity production out of coal	$f^N(\cdot)$
electricity production by green resources	$f^R(\cdot)$
industry output	$Y(\cdot)$
pollution	$P(t) = F(\cdot)$
Electricity/coal balance	$B^i(\cdot),\ i \in \{E, C\}$
costs/revenues of WM for imports/exports of electricity or coal	$p_i(\cdot),\ i \in \{E, C\}$
governmental expenditures	$G(t)$
social welfare function	$S(\cdot)$
discount rate	ρ
depreciation rates	$\delta_i,\ i \in \{C, R, N, Y\}$
cost parameters	$c_i,\ i \in \{1, 2, 3, 4\}$

Table 1: Control and state variables, functions.

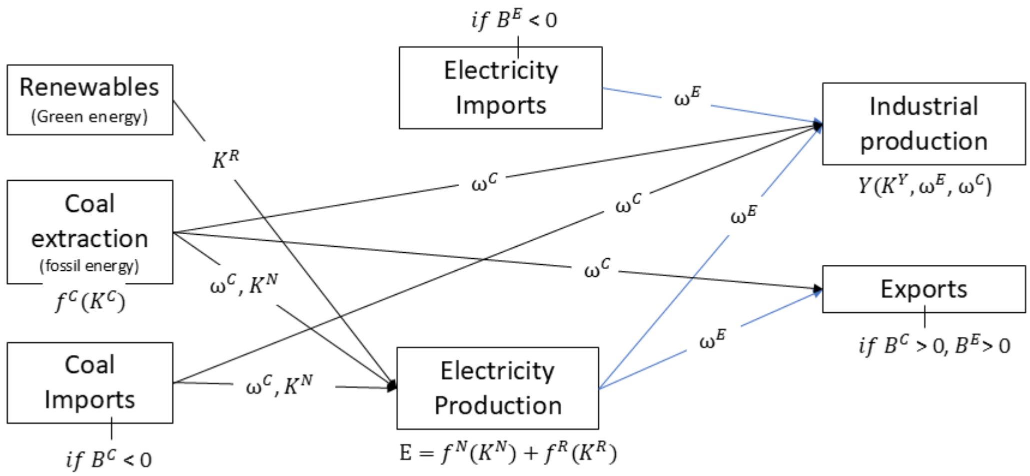


Figure 1: Representative energy system for a coal-based regional economy

3.1 Functional Forms and Parameter Settings

First, we have to specify the functional forms of the production, the pollution and the social welfare function. As mentioned in the previous section we assume constant returns to scale for producing coal, $f^N(\cdot)$, as well as for producing electricity, either from renewables $f^R(\cdot)$ or from coal $f^C(\cdot)$. W.l.o.g. we also assume the linear coefficient to be equal to one. Thus (suppressing t in the following),

$$f^N\left(K^N\right) = K^N, \quad f^R\left(K^R\right) = K^R, \quad f^C\left(K^C\right) = K^C. \tag{10}$$

The industry produces with diminishing returns to scale according to the standard Cobb Douglas (CD) function w.r.t. industry capital and a composite energy input with the latter being of a Constant Elasticity of Substitution (CES) form:

$$Y = \left(K^Y\right)^\alpha \left(\theta \left(w^E\right)^\beta + (1-\theta)\left(w^C\right)^\beta\right)^{\frac{1-\alpha}{\beta}}, \tag{11}$$

with $0 < \alpha, \beta < 1$. The CES function allows both electricity and coal to be used as (partial) substitutes in the production process.

The flow of pollution is assumed to be additively separable in its three components, i.e., coal extraction, electricity production and coal used for industry production:

$$P = \phi_C f^C(K^C) + \phi_N f^N(K^N) + w^C \tag{12}$$

with $\phi_C > 0$ and $\phi_N > 0$ being parameters converting total output into emissions. A corresponding parameter for w^C is used as numéraire (for pollution) and therefore omitted.

We assume that the social welfare function is linear in aggregate consumption but multiplicatively reduced by pollution damage. The latter is convex, meaning that higher pollution levels are associated with higher marginal damage. We then have

$$S = GP^{-\bar{\alpha}} \quad \text{with} \quad \bar{\alpha} > 1. \tag{13}$$

Table 2 summarizes the set of parameters for our numerical analysis. For this analysis, the parameters are not based on a specific context. However, we have used indicative values of these parameters based on their conversion efficiencies and costs to calibrate the model. For instance, industrial production (such as steel) uses electricity and coal as inputs whereas electricity is also generated using coal. Using conversion efficiencies to produce industrial goods from electricity and coal (or coal to electricity), along with the mining and plant costs, the investment costs for coal extraction (c_1), coal power plants (c_3) and industrial production (c_4) were selected for the calibration. For c_2 (investment in renewables), we assume a cost function based on the learning curve for solar photovoltaic technology. In the past 50 years, the cost has reduced by 20% for every doubling of solar capacity (Kavlak et al., 2018). Considering solar as well as other type of renewable technologies available in the market, we assume a learning curve with initial value of \$100 per unit of capacity which exponentially declines to a value of \$20 in about 50 years and eventually to \$10 in the long-run, as shown in Figure 2 marking the \$20 price by a black dot.

Coal combustion is a leading source of atmospheric pollutants, including sulfur dioxide, nitrogen oxides, carbon monoxide, particulates and heavy metals like mercury. In comparison, coal mining, transport and cleaning generates relatively lower levels of air pollutants. The level of pollution varies based on coal mining and cleaning techniques and coal combustion technologies. For our analysis, the pollution parameter for coal-based power generation (ϕ_N) is set as 1 and the corresponding coal mining pollution parameter (ϕ_N) was set to 0.4 using the USEPA's guidelines for air emission factors from stationary sources (USEPA, 2024). Other parameters, like discount rates and elasticities, are presented in Table 2 and can be adjusted based on a given case study of a regional economy.

The prices for coal and electricity, p_C and p_E, are not fixed but we are using them as bifurcation parameters. Using data from IEA and World bank databases (IEA, 2024; World Bank, 2024), we observe that for a normalized coal price of $p_C = 0.5$, the prices for electricity (p_E) vary between 0 and 1 in different country contexts and at different points in time, depending inter alia on the share of coal in the electricity mix, coal import dependence and country-specific energy policies.

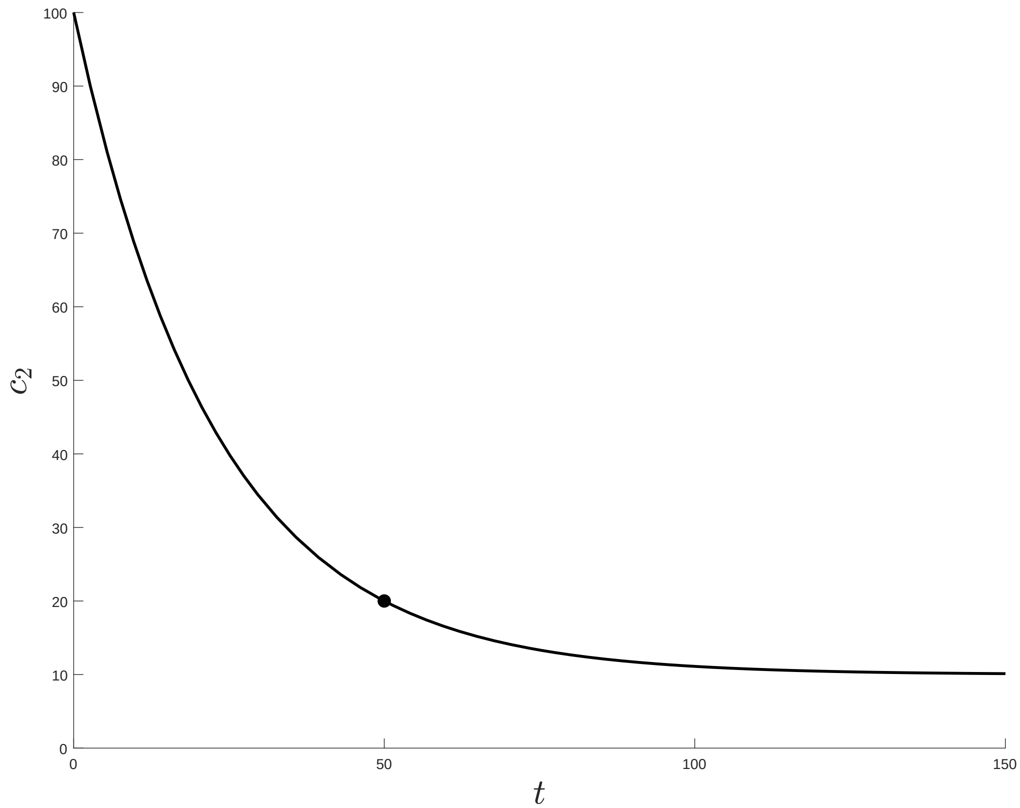


Figure 2: Development of investment costs $c_2(t)$ over time (base case). Black dot denotes the \$20 price after 50 years.
Definition: $c_2(t) = \frac{a}{b} + e^{-bt}\left(c_2^0 - \frac{a}{b}\right)$. Parameter values: $a = 0.4394$, $b = 0.04394$ and $c_2^0 = 100$.

Parameter	**Value**	**Description**
ρ	0.03	planner's discount rate
α	0.5	elasticities in industry production function
$\bar{\alpha}$	0.25	pollution exponent
β	0.5	elasticity of substitution between energy sources
θ	0.5	production input parameters among energy sources
δ_i	0.05	depreciation rates, $i \in \{C, R, N, Y\}$
c_1	10	cost coefficient for coal extraction
c_3	30	cost coefficient for coal power plants
c_4	50	cost coefficient for industrial production
ϕ_C	0.4	pollution parameter for coal mining
ϕ_N	1	pollution parameter for coal-based electricity power plants

Table 2: Base case parameter values.

3.2 Structure of Long-run Transition Outcomes

To explore how the qualitative nature of an optimal transition out of coal varies with the energy price structure, we conduct a dynamic sensitivity analysis with respect to the coal and electricity price, respectively, understood as bifurcation parameters. The results, presented in Figure 3, are depicted as a bifurcation diagram, highlighting distinct regions (separated by bifurcation curves) where the qualitative structure of the solution remains robust against changes in exogenous parameters plotted on the axes. Given that the information in Figure 3 is compact, we will walk through the key insights step by step. First, we explain the solid curves and the structure of the optimal solution above and below. Second, we focus on the dashed curves, motivate the specific model case they relate to, and argue how this contributes to the analysis of the general model.

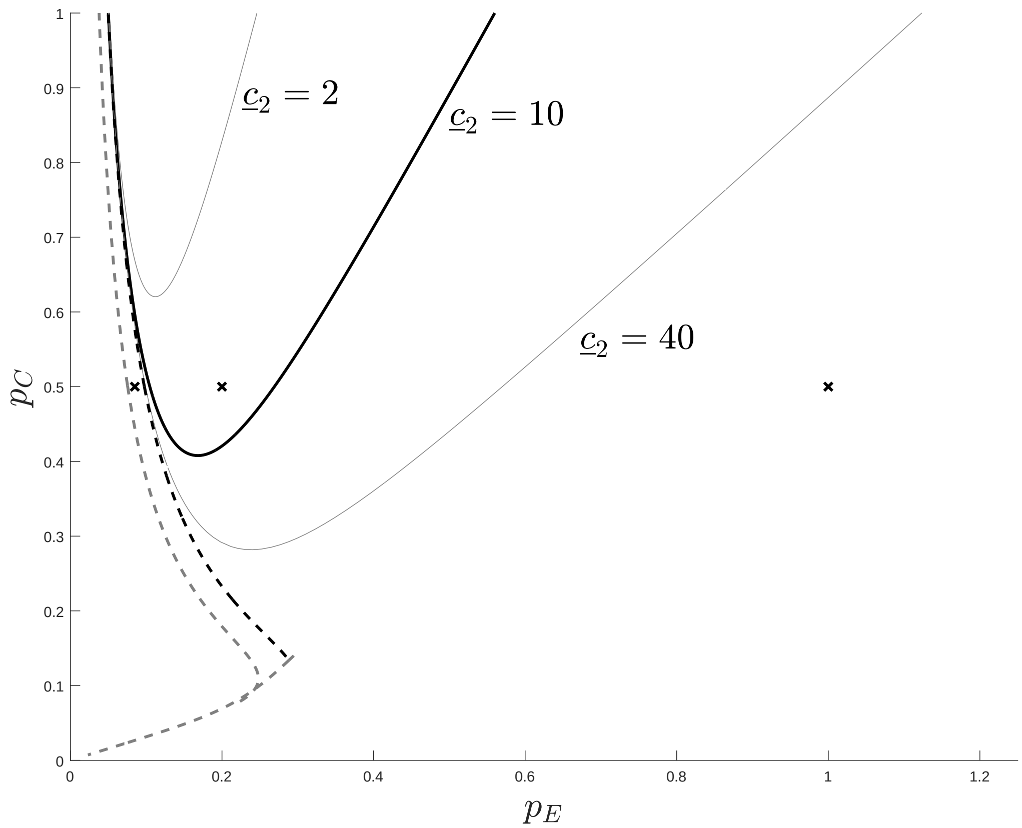


Figure 3: Bifurcation diagram in (p_E, p_C)-space. Solid lines correspond to the full model (9), dashed ones to the reduced model without the possibility of electricity production out of renewable resources.

The black solid curve is based on an asymptotic (long-run) value of the cost parameter for renewable energy of \$10 (corresponding to \$20 in about 50 years) used as label $\underline{c}_2 := \lim_{t\to\infty} c_2(t) = 10$, as plotted in Figure 2. It separates the (p_E, p_C)-parameter space into two regions, i.e., above and below, where the optimal solutions show different qualitative properties: (i) If energy prices (p_E, p_C) are below the black bifurcation curve, the optimal solution always tends to a unique steady state irrespective of the initial conditions. As will be observed in subsections 3.3.1 and 3.3.3, the economy has undergone a complete coal phase-out in this green steady state and relies only on electricity from renewable sources. The character of the regional economy, however, varies with the specific values of the coal and electricity prices. (ii) In contrast, if energy prices (p_E, p_C) lie above the black bifurcation curve, two steady states exist as optimal long-run solutions depending on the initial structure of the economy (i.e., depending on the initial conditions as discussed in subsection 3.3.2). Thus, there is history dependence, as characterized by a Skiba-surface.[2] In addition to the green steady state, that is characterized analogously to the steady state described under (i), the economy in the second steady state does not rely entirely on electricity. Although it is optimal to close all coal power plants because of intensive pollution, the economy continues to extract coal mainly to sell it on the world market under a comparatively high price. The Skiba surface then defines to which steady state the optimal solution will converge, i.e., it separates the regions of attraction. Starting exactly on the surface means that approaching either of the two steady states along an optimal path is associated with an identical value of the objective function (9a) such that the policy maker is indifferent among the two options.

The grey bifurcation curves are analogous to the black one, but based on different asymptotic values of the cost parameter for renewable energy. The labels are analogously defined as for the black curve. Obviously, a higher value of $\underline{c}_2$ makes coal extraction more attractive, as domestic investments in renewable energy are getting more expensive. Therefore, the bifurcation curve moves down, increasing the set of prices p_C and p_E for which an equilibrium without phase out coexists with the green equilibrium and may, indeed, be the preferred one. For a lower $\underline{c}_2$ value, an analogous argument implies that the bifurcation curve moves up and reduces the scope for equilibria without phase-out.

In a nutshell, the solid bifurcation curves (corresponding to different values $\underline{c}_2$) characterize the qualitative structure of the optimal solution of (9) and tell us whether the optimal solution tends to a unique steady state or whether it is history dependent. In contrast, the dashed bifurcation lines do not correspond to the full model (9) but to a reduced version, where it is assumed that renewable energy production is not possible.[3] This reduced version of the model, for which the path of $c_2(t)$ is irrelevant, serves as a basis for identifying a set of initial values for the three remaining capital stocks (coal extraction, coal-based power generation, industrial production) which provide plausible representations of the economic structure before the advent of renewable energy technologies.[4] The initial configuration of the economy (obtained by the

[2]As the model (9) is a four-dimensional optimal control model, the Skiba surface has dimension four as well and separates the regions of attraction of the two steady states.

[3]In mathematical terms this implies a reduction of (9) by one dimension, which has no bearing on the solution method.

[4]In the case of the reduced model, we abstain from presenting a time path or elaborating on the underlying economic intuition. The sole purpose of deriving this simplified framework is to establish well-defined initial conditions for the comprehensive model (9).

numerical solution of the reduced version including a complete bifurcation analysis) depends on the energy prices (p_E, p_C) and will constitute the starting points for the numerical exercises in subsections 3.3.1–3.3.3.

The dashed bifurcation lines separate the (p_E, p_C)-parameter space into three regions:[5] (i) To the right of the black dashed curve the long-run steady state is unique and the economy is heavily coal based. Coal is extracted and employed as an input to both the generation of electricity and industrial production. Whether it is exported depends on the world market price for coal. (ii) To the left of the grey dashed curve the solution tends to a different steady state. Here, electricity is available at low cost from the world market.[6] Therefore, the whole industry is based on electricity imports that feed industrial production. The coal sector (both extraction and electricity generation), which is accompanied by the positive effect of avoiding pollution, has been shut down completely. (iii) Between these two regions, i.e., between the grey and black dashed curves, a third region exists, where the optimal solution turns out to be history dependent. Again, the initial state of the pre-renewable economy determines whether a higher value of the objective function is associated with an adjustment to the coal-based steady state outlined previously in (i) or to the electricity-only steady state outlined in (ii). In subsection 3.3.3 a specific (p_E, p_C)-price constellation will be used to derive two initial states of the economy (between which the policy maker remains indifferent, as both lead to equivalent levels of social welfare) to elaborate the implications of different initial structures of the economy for the renewable transition in the full model.

3.3 Transition Scenarios

As illustrated by the bifurcation diagram (Figure 3), the price of energy and coal will determine the long-run extraction of coal and its use for energy production as well as the configuration of the pre-renewable economy that is tantamount to the initial sets of capital for our analysis of the energy transition. Notably, however, the transition pathways through which the long run state of a renewable economy is approached from its pre-renewable configuration feature distinctive representations, depending on the specific combination of the prices for energy and coal and the initial conditions. To gain insight into the transitional dynamics towards a renewable economy (and coal phase-out), we will now study the time paths of key variables of our model for three different values of the electricity price (high, medium, low) relative to a coal price that we fix at a value of 0.5, as marked by three crosses in Figure 3: Subsection 3.3.1 examines a high electricity price, demonstrating that the optimal transitional pathway (cf., Figure 4)

[5]Note, that the two regions of (9) and the three regions of the reduced and the full model do not overlap for our set of parameters, though this cannot be proven in general. However, drawing an economic interpretation is hardly possible as both models are different in the potential short- and long-run development of the economy.

[6]When excluding water from our definition of (modern) renewable technologies (solar, modern wind), low cost electricity may also be sourced from domestic water power in certain areas.

leads unequivocally to a green economy, the sole feasible long-run outcome under these conditions. In contrast, Subsection 3.3.2 explores a medium electricity price, where the energy transition exhibits history dependence, allowing for two distinct long-run outcomes: a green or a coal-dependent economy. This is illustrated by comparing transitional trajectories originating from the steady state of the reduced model with those commencing from an economy that has received an exogenous (initial) boost of renewable capacity. Subsection 3.3.3 finally investigates the implications of a low electricity price. Although the long-run outcome is a uniquely green economy, the price configuration within the Skiba region of the reduced model (between the two dashed bifurcation curves) permits two distinct initial economic states, both plausible outcomes of the reduced model.

3.3.1 High electricity price

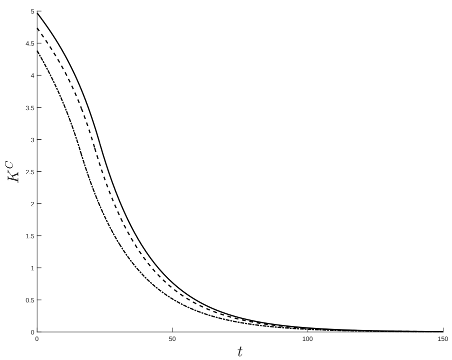
In this scenario we assume the electricity price to be twice the price of coal, $p_C = 0.5, p_E = 1$; all relevant time paths are included in Figure 4. As we can see from Figure 3, such a setting is associated with a unique post-transition equilibrium featuring a complete phase-out, whereas the initial equilibrium features both coal extraction and coal-based power generation. Considering that the latter is particularly polluting, it is of interest to understand the pace at which it is phased out. For this purpose, we plot all subsequent time paths for three different levels of the pollution parameter ϕ_N. Solid lines refer to $\phi_N = 1$, dashed lines to $\phi_N = 0.5$ and dotted-dashed lines to $\phi_N = 0.2$. Here, we note that variation in ϕ_N may reflect both differences in technology and in the location of the power plants, i.e., whether or not they are located within or close to densely populated areas.

The economy starts in a pre-transition equilibrium with high values of the coal extraction capital, K^C, and capital for coal power plants, K^N, while capital for renewable energy production, K^R, is not available initially (cf., Figure 4, panel (a), (b) and (c)). Higher values of ϕ_N are both associated with less coal-based power generation even before the transition and a faster phase-out of coal power plants in panel (b). Notably, the effect of ϕ_N on coal extraction capital and industry capital is the opposite, with capital investments in both of these sectors being higher before the transition (cf., panel (a) and (d)).[7]

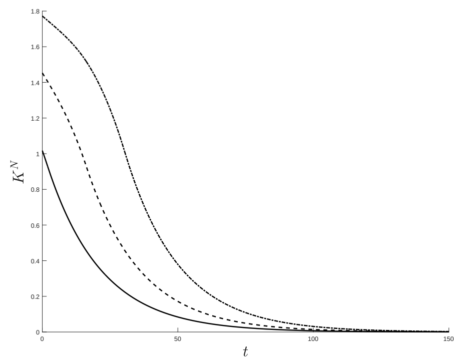
As indicated by the initial high value of the coal balance, B^C, the region starts out as a very dominant coal exporting region (panel (g)). Industrial output, Y, (panel (i)) is initially high as well and associated with high initial levels of coal, w^C, and electricity w^E employed in the production sector (panel (e) and (f)). Differences in the extent of coal-power generation translate into (modest) differences in industrial structure. With significantly less power generation taking place in the presence of strong pollution, the region relies to somewhat lesser extent on the export of electricity but rather on the export of coal and manufacturing, with the latter being powered by coal rather than electricity.

[7] Note, that the strict linearity of the pollution flow in its three components underlies these results.

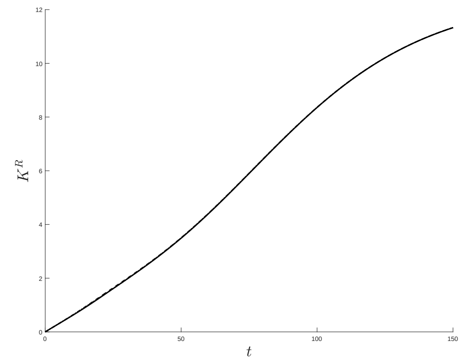
Over time, the price of renewable energy investment decreases and capital for renewable energy production, K^R, starts to increase. At the same time industrial output slows down. Finally the extraction of coal and coal electricity plants are shut down thereby reducing its pollution flow. However, high energy prices foster further investment into renewable energy production, which spurs electricity exports while reducing industrial output. The overall effect on welfare is positive since the gains from electricity exports together with lower levels of pollution outweigh the reduction in the industrial output.



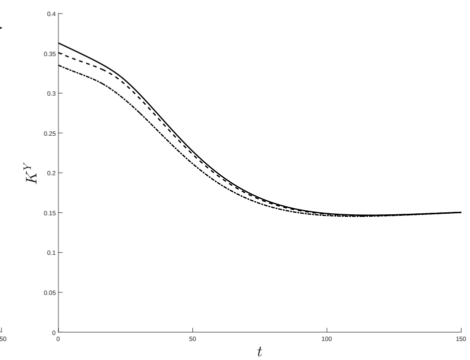
(a) Extraction capital.



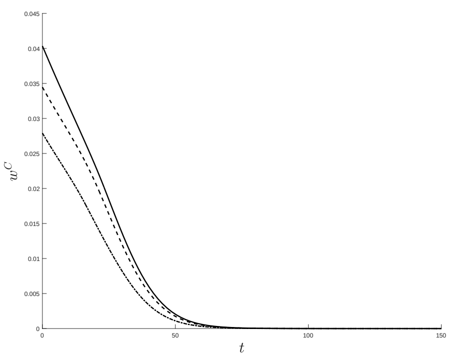
(b) Coal electricity plants.



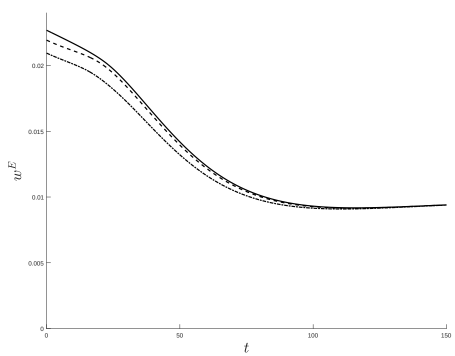
(c) Capital for renewable energy



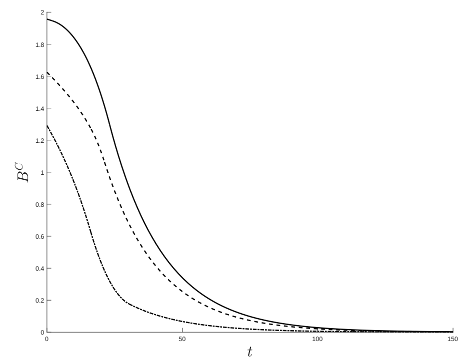
(d) Industry capital



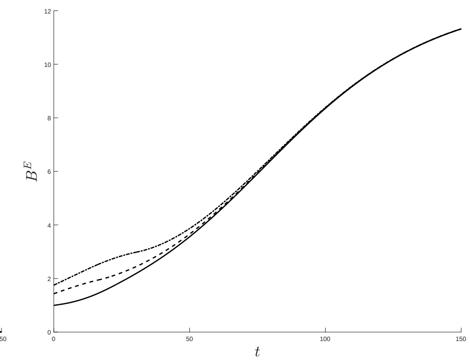
(e) Coal used for industry production.



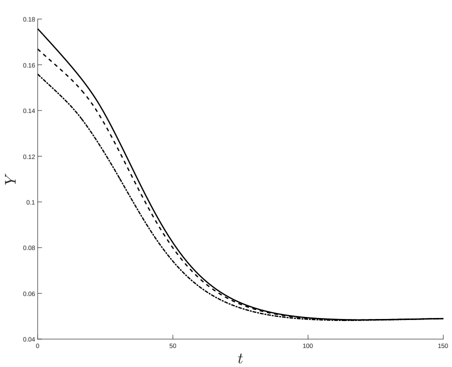
(f) Electricity used for industry production.



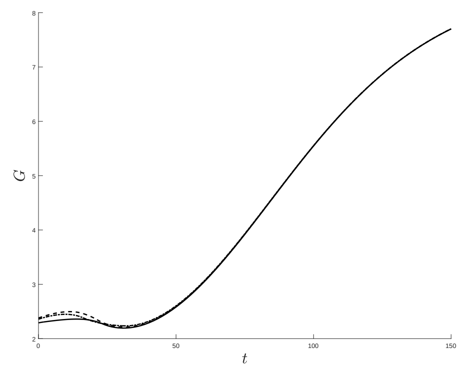
(g) Coal balance.



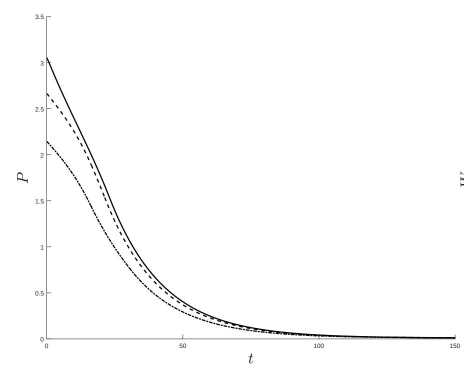
(h) Electricity balance.



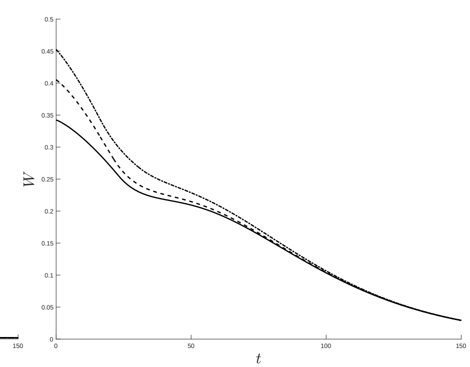
(i) Industry output



(j) Aggregate consumption



(k) Pollution



(l) Period social welfare

Figure 4: Time paths for high electricity price ($p_C = 0.5$, $p_E = 1$). Varying pollution parameter: $\phi_N = 1$ (solid lines), $\phi_N = 0.5$ (dashed lines), $\phi_N = 0.2$ (dashed-dotted lines).

3.3.2 Medium electricity price

The medium electricity price scenario, $p_C = 0.5$, $p_E = 0.2$, corresponds to the region above the black curve in the bifurcation diagram (Figure 3), where two (potentially optimal) steady states exist. It depends on the initial structure of the economy which one will be approached in the long-run; all relevant time paths are included in Figure 5. To highlight this history dependence we therefore illustrate two optimal transition paths starting from different initial conditions: the solid paths depict an economy A that starts from the pre-renewable steady state of the reduced version of the model with zero renewable capital stock; whereas the dashed paths represent an economy B that starts from an identical pre-renewable equilibrium in terms of extractive capacity, K^C, and manufacturing capacity, K^Y (see Figure 5, panels (a) and (d)), but is endowed with a positive initial capital stock, K^R, for the generation of renewable energy (panel (c)). The availability of initial capital in the renewable energy sector within economy B may represent a setting in which the central government, a development agency, or a financially strong corporate actor has sought to provide an initial stimulus for the energy transition through an upfront investment into renewable capital. The pre-renewable equilibrium does not feature coal-power generation such that $K^N = 0$ in both economies A and B (panel (b)).[8]

Figure 5 illustrates several aspects of the development of the two economies in the long-run as well as over the transitional period. Economy B moves toward a green steady state by phasing out coal over approximately 100 years, whereas economy A invests in renewable energy capacity as an additional source for the generation of electricity but remains heavily reliant on coal, both during the transition and in the long run. These differences are evident in two key panels. Panel (a) demonstrates that coal extraction persists at high levels along path A, in contrast to its complete shutdown along path B. Furthermore, panel (a) highlights that coal remains a significant input for industrial production along path A. In the greening economy B, coal continues to be utilized only during the transitional phase, reflecting an economic decision to allocate some of the resource to domestic production rather than fully export it.

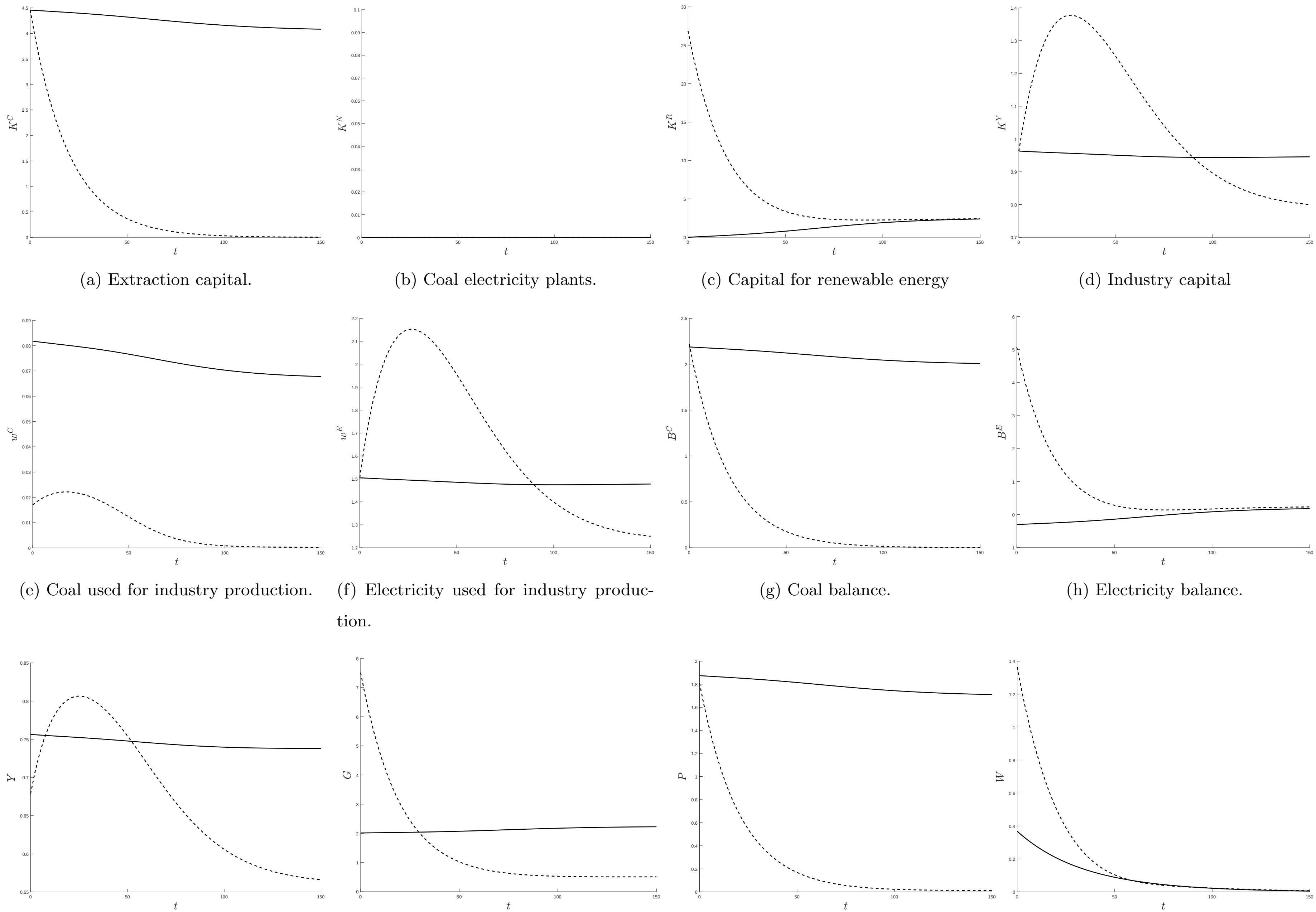
Despite these differences, both economies never invest into coal-based electricity plants and converge to the same long-term renewable energy capacity.

As shown in panel (g), economy A remains a coal-exporter and derives wealth from coal trade surpluses, it also maintains higher industrial output in the long run. In contrast, economy B achieves a long-run equilibrium with balanced coal and electricity accounts but sustains lower industrial production in the long-run, as depicted in panel (i). Overall, this translates into a sizable difference in the long-run level of aggregate consumption across the two economies. While the dirty economy A sustains a higher long-run

[8] Note that while we assume $\phi_N = 1$ for both economies throughout, the parametric choice is immaterial to the extent that investments in coal power generation remain zero throughout the full transition.

level of consumption, this is accompanied by significant pollution (see panel (k)). The green economy B achieves comparable long-run social welfare outcomes at a lower trade-surplus, lower industrial output and lower consumption, which trade off against much lower pollution. On the transitional path towards the (unique) long-run green outcome, economy B temporarily but significantly increases the industry capital and production (see panel (d) and (i)). Note, that the increase in industry production is driven by a higher electricity input (see panel (f)). This corresponds to the specific initial condition of economy B, which uses the optimal steady state values of the reduced model for extraction capital, coal electricity plants and industry capital (same values as economy A) but with an initial boost in renewable capital energy. The temporary increase is an adaptation to the availability of electricity from the renewable energy sector.

In summary, the two strikingly different pathways taken by economies A and B highlight the dependence of the long-run transition dynamics on the comparative prices of coal and electricity as well as on even minor differences in the initial structure of the economy. Our findings suggest for settings of balanced energy prices that whether or not coal is phased out depends on whether the economy has (low cost) access to early-on investments in renewable technologies (such as for economy B). If this is not the case, as for economy A, it may be better for the economy to forego a phase-out and continue as a coal exporting and heavily industrialized economy even at high levels of pollution. Notably, this is the case despite the build-up of an additional renewable energy sector.



(a) Extraction capital. (b) Coal electricity plants. (c) Capital for renewable energy (d) Industry capital

(e) Coal used for industry production. (f) Electricity used for industry production. (g) Coal balance. (h) Electricity balance.

(i) Industry output (j) Aggregate consumption (k) Pollution (l) Period social welfare

Figure 5: Time paths for medium electricity price ($p_C = 0.5$, $p_E = 0.2$). Varying initial setup of economy: Steady state of pre-renewable economy (economy A; solid lines), pre-renewable economy plus initial renewable push (economy B; dashed lines).

3.3.3 Low electricity price

The low electricity price scenario, $p_C = 0.5$, $p_E = 0.1$, corresponds to the left part of the bifurcation diagram (Figure 3) outside the parabola defined by the black bifurcation curve for the full economy; all relevant time paths are included in Figure 6. We find that the transition will ultimately lead into a renewable economy with a full phase-out of coal. However, the price set lies in the Skiba region of the pre-renewable economy. We therefore choose two differing sets of initial state values, both corresponding to optimal steady states in the reduced pre-renewable economy, and study their transitional paths towards the (unique) steady state of the renewable economy. We can infer from this to what extent transition strategies may vary and what implications there are for social welfare. The solid path corresponds to a coal-based pre-renewable economy A, whereas the dashed one to a green economy B. Notably, economy B has no initial capital stock in coal mining nor in power plants ($K^C(0) = K^N(0) = 0$) and imports electricity at a low world market price for industrial production.[9]

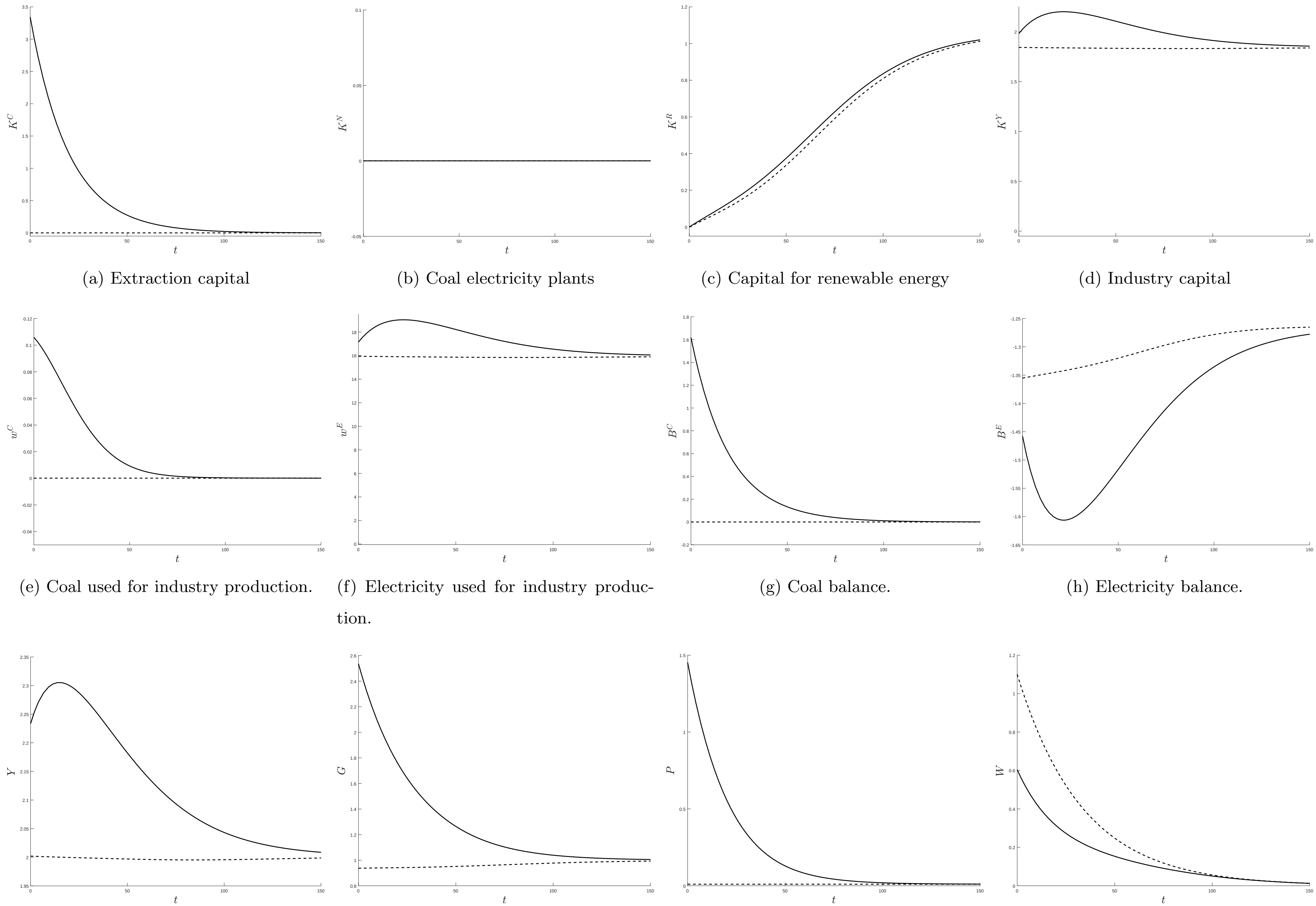
The pathway for economy B towards a renewable equilibrium does not see any capital formation in coal-based sectors and hence continues to avoid the damages resulting from coal-based pollution. At the same time, the economy builds renewable capacity, mostly to reduce the reliance on energy imports. The size of the manufacturing sector barely changes over the transition period. In contrast, the coal-based economy A sets out from a position in which it is strongly dependent on coal extraction as an input into industrial production and for selling it on the world market. Coal power plants do not play a role as electricity can be sourced at a low world market price. The gradual build-up of renewable capacity in economy A is accompanied by a gradual phase-out of coal. Notably, the transition process is accompanied by a temporary increase in industrial capacity, which is enabled by the additional availability of electricity from renewable sources and a concomitant increase in electricity imports, while coal is still abundantly available. Over time the economy then stabilizes to one with a slightly lower manufacturing capacity, while at the same time energy imports are much reduced due to the availability of domestic renewable energy.

Recalling that the price set is located outside the Skiba region for the full model in the bifurcation diagram (Figure 3), is is optimal for both economies A and B to eventually converge towards a unique green long-run steady state. We also see, however, that the transition is associated with significant differences in the sources of social welfare. While economy A can sustain a significantly higher consumption level that is financed from both the export in coal and a higher level of manufacturing, economy B features very low (in fact zero) pollution from coal-based activities. For our parameterization the social costs of pollution in economy A outweighs by far the value of additional consumption, such that social welfare is lower in

[9] As outlined in Footnote 6 one alternative way of reading the pre-renewable equilibrium would relate to a historically water-powered economy.

economy A over the entire transition process, in particular the early part.

In summary, this setting illustrates that history dependence (as implied by the presence of Skiba points) matters not only in respect to the scope for multiple endpoints of the transition process, as shown for intermediate levels of electricity prices, but also in respect to the scope for multiple economic configurations at the starting point of the transition.



(a) Extraction capital

(b) Coal electricity plants

(c) Capital for renewable energy

(d) Industry capital

(e) Coal used for industry production.

(f) Electricity used for industry production.

(g) Coal balance.

(h) Electricity balance.

(i) Industry output

(j) Aggregate consumption

(k) Pollution

(l) Period social welfare

Figure 6: Time paths for low electricity price ($p_C = 0.5$, $p_E = 0.1$). Varying initial setup of economy starting at two possible outcomes of pre-renewable economy (history dependence): dirty economy in the past (solid lines), green economy in the past (dashed lines).

4 Conclusion

We propose a modelling framework to comprehensively analyze the dynamics of a coal phase-out from a regional welfare point of view. To this end, we have set out an optimal control problem, involving the maximization of the intertemporal flow of regional welfare for a four-sector economy, including a coal extraction sector, a coal-based and a renewable energy sector, as well as a manufacturing sector. The social planner controls the investments into the four sectoral capital states and an array of input variables. The model can be applied to study how the regional coal transition is affected by national and international policies that bear on the price structure and/or prescribe certain transition thresholds.

Our numerical analysis shows that the transition follows non-trivial time paths and may, indeed, enter into an array of different long-run equilibria, corresponding to different degrees of phase-out, depending on the energy price structure. For a given long-run cost of renewable energy ($\underline{c}_2$), and depending on the energy prices (p_E, p_C), the optimal solution may either tend to a unique equilibrium state or be subject to bifurcation, based on the initial conditions.

In case of a high world market price for electricity (subsection 3.3.1), we observe a unique steady state. For a corresponding high enough level of the price for coal, the pre-transition equilibrium entails a strongly coal-dependent economy with high levels of coal extraction, K^C, and power generation capital, K^N. Increasing access to renewable technology then leads to a steady decine to zero of coal extraction, consumption and export over 50 years and a full phase-out in the long-run. While this may correspond to an "archetypical" phase-out scenario, we observe different equilibrium states and trajectories for lower levels of the electricity price.

For a medium level of the electricity price and a high enough level of the coal price (subsection 3.3.2) the transition outcome is history dependent and could entail both a coal phase-out and an economy that invests in renewables but retains a large coal-based sector. This latter outcome will arise, for instance, if the size of the coal sector before the transition is large. Our analysis shows that under such circumstances a phase-out can only be achieved under an (exogenous) initial boost to renewable energy capital.

Finally, a full phase-out is always attained in the low price scenario (subsection 3.3.3). Such a setting, however, may be associated with widely differing pre-transition settings of the economy, embracing both strongly coal-reliant economies with heavy industry and coal-free economies that sustain a smaller industry based on electricity that is sourced from outside or abundant local sources (such as water). Our analysis shows that these economies are prone to converge to the same post-coal equilibrium, following, however, very different transition pathways.

Our findings bear a number of important insights for policy-making. First, we note that the initial industrial structure of a region alone is not sufficient for predicting whether a phase-out will be in the region's interest and, therefore, materialize without further policy intervention at a national level. Notably, the development of national or international energy prices plays an important role for whether or not a transition will be triggered.

Second, the scope for history-dependent pathways needs to be borne in mind when it comes to stimulating and structuring a coal phase-out in any particular region. This is true in particular in those cases in which the initial structure of the economy (typically heavily coal-based) leads to a process where renewables are phased in but coal mining is retained for accommodating a heavy industry and exports. Recalling that this constitutes an optimal allocation on the part of a regional planner, we note a conflict of interest between the regional interests and national/international climate commitments. While the world market integration is advantageous from a regional welfare perspective in as far as it facilitates a flexible response to the energy transition, it may also stand in the way of a global or national coal phase-out, a situation that may be reflective of e.g the context in China. Our analysis shows that in such settings, a strong exogenous push towards the early-on adoption of renewable energy sources may be required to trigger the phase-out.

Third, we note that a coal phase-out and its impact on regional welfare depends on the extent to which the region benefits from the reduction in local air pollution. While our analysis shows that substantial welfare gains are feasible, in many scenarios this comes with a strong decline in aggregate consumption. On the one hand, this trade-off brings into focus the important role of the underlying preferences and local circumstances: for a lower weight of pollution in the social welfare function and/or for a lower impact of air pollution, due to local weather conditions or the siting of coal-based activities, a phase-out may not be optimal. On the other hand, the resolution of the trade-off towards a phase-out also illustrates the importance of technological progress, as embraced by the falling cost of renewable investments. By opening up the possibility to maintain electricity-based manufacturing activities, investments in renewable energy mitigate the trade-off and therefore allow for substantive reductions in pollution without giving up even greater quantities of consumption.

Finally, we note that the strong trade-off between consumption and pollution can be attributed to our assumption of a stationary economy. Productivity growth and the associated income gains would soften the trade-off between consumption and pollution such that a simultaneous increase in consumption and reduction in pollution may ultimately be feasible. The superiority of a coal phase-out would even be reinforced, as for increasing levels of consumption, pollution abatement becomes socially more valuable. Plausibly, productivity growth itself may be associated with structural change in manufacturing away from heavy industry to service and IT-based industries, with the latter relying on electricity as their prime energy source. This would further boost a coal phase-out.

The analysis presented has been conceived primarily with the objective of providing a conceptual framework for the comprehensive analysis of the complex dynamics behind a coal-phase out (as illustrated by the scope for multiple equilibria and history-dependence). While it provides plausible outcomes in terms of transition scenarios, further work is required to generate more practical policy insights, in particular as to how national policies can and should be used for the simulation of a coal phase-out. This requires careful calibration of the model, but as our analysis suggests, it may be necessary to do this for particular case studies rather than "across the board".

Our modeling comes with a number of limitations. First, we assume that capital stocks are fully utilized without accommodating the intentional shutdown of a sector or its components. As a result, the model does not account for the potential emergence of stranded assets, a critical concern during the transition away from fossil fuels, with significant implications for policy and economic planning. This simplification is necessary due to limited numerical feasibility.[10] The model could, nevertheless, be adapted to approximate the value of stranded assets (for capital stock C or N) as follows. In a first step, the model is solved under the assumption that all available capital is employed. The second step involves solving the model from time t, imposing the condition $K^j(t) = 0$ ($j \in \{C, N\}$) while retaining the levels of all other capital stocks as determined in the first step. Comparing the values of the objective function obtained from these two scenarios yields the value of the stranded asset. Another limitation of the model is its partial equilibrium setup which confines the analysis to a setting in which a small regional economy responds to given changes in international energy prices. While appropriate scenario assumptions would need to be made about the emergence of global prices, general equilibrium analysis may be required to understand the energy transition at a global level. Finally, a future version of the model may explore a detailed household sector to study the impacts of a coal phase-out on regional employment and wellbeing of coal-dependent communities and, thereby, inform policies about justice concerns in the design of the transition.

[10] A comprehensive bifurcation analysis of a four-dimensional optimal control model involving six control variables is already at the limit of feasibility at present

A Necessary conditions

The full model (9) is a non-autonomous optimal control model, which is transformed into a standard autonomous optimal control model (with infinite time horizon) by formulating $c_2(t)$ as state variable. The model can be solved by application of the Maximum Principle (see e.g., Grass et al. (2008)). The necessary optimality conditions are presented in the following.

The Hamiltonian reads

$$\begin{aligned}\mathcal{H} &= S(G,P) + \lambda^C \left(I^C - \delta_C K^C\right) + \lambda^R \left(I^R - \delta_R K^R\right) \\ &\quad + \lambda^N \left(I^N - \delta_N K^N\right) + \lambda^Y \left(I^Y - \delta_Y K^Y\right),\end{aligned} \tag{14}$$

where λ^i ($i \in \{C, R, N, Y\}$) denote the adjoint variables for the corresponding capital stocks.

Maximization with respect to the control variable implies the first order conditions:

$$\begin{aligned}\mathcal{H}_{I^C} &= S_G\left(-2c_1 I^C\right) + \lambda^C \leq 0 \\ \mathcal{H}_{I^R} &= S_G\left(-2c_2 I^R\right) + \lambda^R \leq 0 \\ \mathcal{H}_{I^N} &= S_G\left(-2c_3 I^N\right) + \lambda^N \leq 0 \\ \mathcal{H}_{I^Y} &= S_G\left(-2c_4 I^Y\right) + \lambda^Y \leq 0 \\ \mathcal{H}_{w^E} &= S_G\left(Y_{w^E} - p_E\right) \leq 0 \\ \mathcal{H}_{w^C} &= S_G\left(Y_{w^C} - p_C\right) + S_P P_{w^C} \leq 0.\end{aligned} \tag{15}$$

Note that for inner solutions, i.e., positive value of the control variable, the above equations hold with equality. Only at the boundary, i.e., when a control variable equals zero, the <-sign applies. For inner solutions the above conditions can be simplified to:

$$I^C = \frac{\lambda^C}{2c_1 S_G} \tag{16a}$$

$$I^R = \frac{\lambda^R}{2c_2 S_G} \tag{16b}$$

$$I^N = \frac{\lambda^N}{2c_3 S_G} \tag{16c}$$

$$I^Y = \frac{\lambda^Y}{2c_4 S_G} \tag{16d}$$

$$Y_{w^E} = p_E \quad \Longrightarrow \quad Y_{w^E} = p_E \tag{16e}$$

$$Y_{w^C} + \frac{S_P}{S_G} P_{w^C} = p_C \quad \Longrightarrow \quad Y_{w^C} = p_C - \frac{S_P}{S_G}. \tag{16f}$$

Here the standard economic interpretation of balancing cost with intertemporal benefit/utility applies. Optimal investments (equations (16a)-(16d)) are equal to the monetary value of the marginal (intertemporal) effect to the social welfare. In equation (16e) the marginal return to electricity in the production process is equalized to the world market price of electricity. Equation (16f) is very similar. Here the marginal return to coal in the production process is equalized with the full coal price, i.e., the world market price and the monetized disutility from pollution.

The adjoint equations follow the adjoint dynamics:

$$\begin{aligned}\dot{\lambda}^C &= (\rho + \delta_C)\,\lambda^C - S_G p_C - S_P P_{K^C} \\ \dot{\lambda}^R &= (\rho + \delta_R)\,\lambda^R - S_G p_E f^R_{K^R} \\ \dot{\lambda}^N &= (\rho + \delta_N)\,\lambda^N - S_G f^N_{K^N}\,(p_E - p_C) - S_P P_{K^N} \\ \dot{\lambda}^Y &= (\rho + \delta_Y)\,\lambda^Y - S_G Y_{K^Y}, \end{aligned} \tag{17}$$

which hold together with standard transversality conditions for infinite time horizon.

For the numerical calculations we introduced two additional parameters $\tau_e, \tau_c \ll 1$. These parameter values prevent possible singularities in the derivatives if $w^E = 0$ or $w^C = 0$. Another small parameter $\phi_0 \ll 1$ prevents that the pollution function P becomes zero and hence $P^{-\alpha_4} = \infty$. With these modifications the model can be solved numerically using a boundary value approach. The derivation of the bifurcation diagram applies advanced numerical skills on numerical continuation. For details see Grass et al. (2008).

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