DYNAMIC LINEAR MODELS FOR THE STUDY OF AGRICULTURAL SYSTEMS

C.Csaki and A.Propoi
Editors

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Foreword

According to the Research Plan of the International Institute for Applied Systems Analysis (IIASA) for the years 1977 to 1979, a set of agricultural development models using dynamic linear programming techniques were to be developed at IIASA with the cooperation of other research institutes. As part of this task, a regional agro-industrial development model was elaborated by IIASA researchers and cooperation was established between IIASA and Bulgaria to carry out a methodological case study on agro-industrial development in the Silistra region of Bulgaria. Scientists from the United States and France also participated in the investigations. Though the study of dynamic linear models for agricultural systems has always remained a minor part of the research program of the Food and Agricultura Program (FAP) at IIASA, the experience gained from this study helped focus the two major tasks of FAP. These studies are of considerable methodological and substantive interest for the problems of studying agricultural development at regional and enterprise levels.

This volume, edited by C. Csaki and A. Propoi, presents five such studies.

Kirit S. Parikh
Program Leader
Food and Agriculture Program

Preface

This volume presents five studies for modeling agro-industrial development at a regional and enterprise level with the basic methodological framework of dynamic linear programming. The first two papers focus on methodological problems. Section 1, written by C. Csaki and A. Propoi, who were working at IIASA, describes a dynamic linear modeling framework for studying agro-industrial development. Other related IIASA publications are the papers by Csaki, (1977); Carter, Csaki, and Propoi, (1977); and Propoi, (1977).* The introduction of uncertainties in linear dynamic agricultural models is discussed by J.M. Boussard of the Institute National de la Recherche Agronomique, (Paris), in Section 2.

Concrete modeling projects using dynamic linear programming are reported in the next three papers. A farm level application of dynamic linear models is presented in Section 3 by E.O. Heady and R.C. Kay from the Iowa State University (United States). The research group of the Institute of Social Management in Sophia (Bulgaria) gives an account in Section 4 of the Silistra case study where the dynamic linear model elaborated at IIASA was used as the basic methodology for planning the region's agro-industrial development. Finally, the paper by I.V. Goueysky of the National Industrial Association (Sofia, Bulgaria) in Section 5, describes a dynamic linear water demand model.

For his continuous support we would especially like to thank the former leader of the FAP, Ferenc Rabar, who is currently at the Karl Marx University of Economic Sciences in Budapest. To Cynthia Enzlberger and Vivien Landauer who typed and prepared the final version of these papers, our warm appreciation. Many thanks are also due to Anne Morgan for her contribution to editing this volume.

C. Csaki and A. Propoi
Editors

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* C. Csaki, Dynamic Linear Programming Model for Agricultural Investment and Resource Utilization Policies, RM-77-36; H. Carter, C.Csaki, A. Propoi, Planning Long Range Agricultural Investment Projects: A Dynamic Linear Programming Approach, RM-77-38; and A. Propoi, Dynamic Linear Programming Models for Livestock Farms, RM-77-79.

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1. DYNAMIC LINEAR PROGRAMMING MODELS IN AGRICULTURE

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1. DYNAMIC LINEAR PROGRAMMING MODELS IN AGRICULTURE

C. Csaki and A. Propoi

1.1 Introduction

Agricultural production is one of the most complex and many-sided activities of mankind, involving the coordination of biological, technical, human, and economic factors. In recent years considerable effort has been devoted to the analysis and modeling of agricultural systems. Models describing these systems have been formulated emphasizing different aspects of agricultural production and using various computational techniques. They vary in the degree of detail and sophistication

In the early 1960's several versions of the linear programming models were developed (Agrawal and Heady 1972; Beneke and Winterboer 1973; Carter et al. 1977; Chien and Bradford 1976; Csaki 1977; Olson 1971, 1972). In recent years more advanced programming techniques (for example, quadratic and stochastic programming) have been applied, and a considerable effort has been made to analyze agricultural systems by simulation. The choice of technique depends on the objectives of the modeling effort. This paper deals with models for planning agricultural development projects. Due to the importance of the time dimension in such models, dynamic linear programming (DLP) seems to be one of the most appropriate techniques (see Propoi 1979a; Propoi 1979b) for this purpose.

In some cases, it may appear necessary to include nonlinearities. However, it may be preferable to run a linear programming (LP) model several times rather than to develop one large nonlinear model.

Another important aspect of agricultural systems is their stochastic nature. Here again, an alternative technique such as stochastic programming may be conceptually superior but operationally inferior (e.g., lack of sufficient data). It should also be noted that, in long-term aggregate studies, we deal with expectations, thus staying within a deterministic framework. Stochastic techniques can then be used for short-term studies.

For solving large-scale optimization problems in agriculture, LP and its extension, DLP, can be considered as basic techniques. This paper serves as the methodological introduction to this volume. We shall first describe different agricultural activities in individual submodels, and then discuss the linkage of these submodels in order to build a more complete agricultural system.

1.2. General Structure of the Model

In formulating the DLP problem it is useful to identify

(i) the *state equations* of the system, distinguishing between *state* (descriptive) and *control* (decision) variables

(ii) the *constraints* imposed on these variables

(iii) the *planning horizon T*, the *number* of time periods to be considered in the system and the *length* of each time period

(iv) the *objective function* (performance index) which quantifies the contribution of each variable to some performance measure or index (e.g., profit, net return, asset value)

State equations. State equations have the following form

$$x(t + 1) = A(t)x(t) + B(t)u(t) + s(t) \tag{1}$$

where the vector $x(t) = \{x_1(t),\ldots,x_n(t)\}$ defines the state of the system at stage t in the state space X; $u(t) = \{u_1(t),\ldots,u_r(t)\}$ specifies the controlling decisions at stage t; $s(t) = \{s_1(t),\ldots,s_n(t)\}$ is a vector defining the external effects on the system.

In coordinate form the equations can be written as

$$x_i(t + 1) = \sum_{j=1}^{n} a_{ij}(t)x_ju(t) + \sum_{k=1}^{r} b_{ik}(t)u_k(t) + s_i(t) \tag{2}$$

The *state* variables $x(t) = \{x_i(t)\}$ for $(i = 1,\ldots,n)$ are usually associated with the volume of production capacities or with the stock of commodities in the system at the beginning of each time period, while the control vector $u(t) = \{u_k(t)\}$ for $(k = 1,\ldots,r)$ represents activities used to develop these production capacities. In this case the matrix $A(t)$ and its elements $(a_{ij}(t))$ usually associated with the depletion or attrition of commodities, and the matrix $B(t) = (b_{ik}(t))$ shows how different activities influence available products during the current time period. Therefore, the state equation (1) represents a balance between commodities at the beginning and at the end of the current time period, or at the beginning of the new period. For example, the state variables may be the number of machines, animals, or trees at the beginning of a time period, or the level of production or the storage capacities for processing and for storing agricultural products.

The *control* variables are usually either selling and buying activities in agriculture, or harvesting and planting activities and the construction of new production capacities.

It is natural to assume that the initial state of the system is given by

$$x(0) = x^0 \text{ or } x_i(0) = x_i^0 \qquad (i = 1,\ldots,n) \tag{3}$$

Constraints. In rather general form constraints imposed on the state and control variables may be written as

$$G(t)x(t) + D(t)u(t) \leqq f(t) \tag{4}$$

with

$$x(t), u(t) \geq 0 \tag{5}$$

where $f(t) = \{f_1(t)\ldots,f_m(t)\}$ is the vector for all t, $(t = 0, 1,\ldots,T - 1)$ of constraints such as resource availability; and G and D are unit input coefficient matrices. The constraints can also be written in coordinate form

$$\sum_{i=1}^{m} g_{si}(t)x_i(t) + \sum_{k=1}^{k} d_{sk}(t)u_k(t) \leqq f_s(t) \qquad (s = 1,\ldots,m) \tag{6}$$

where $g_{si}(t)$ represents the unit inputs for production type i and for developing type k activities.

Usually, there are two groups of constraints. The first group consists of *resource availability* constraints. The maintenance of operational production capacities and the development of new ones requires resources (labor, money, arable or irrigated land, rtc.). The components of the vector $f(t)$ represent the amount of available resources, which are exogenous to the system. The left side of (4) expresses the amount of resources that are needed, which should not exceed the amount of available resources; therefore the inequality sign is usually used in (4).

The second group of constraints consists of *demand* constraints. For example, the output of the system should not sink below a required level. In this case the opposite inequality sign of (4) should be used (or G(t), D(t) and f(t) should have in (4) the opposite sign).

In addition, there are environmental constraints, and constraints on the development of the system (e.g., the number of cows at each subsequent time period should not be less than at the current period).

The *planning horizon T* is supposed to be fixed. Thus in equation (1) t varies from 0 to T - 1. The length of each time period may be the same over the whole planning horizon (say, a month, one year, 5 years), or different for each period. For example, in the Silistra model (see section 4 of this volume) the duration of the first 5 time periods is one year and the duration of the subsequent 5 time periods is two years. Therefore the planning horizon consists of 10 time periods which represent 15 years.

The total length of the planning horizon is also an important issue. On the one hand, large investments must be considered over a long time span in order to analyze the consequences of depreciation. On the other hand, a long planning horizon leads to uncertainty about the validity of some of the coefficients in the model (e.g., prices, technological coefficients) and to an increase in the number of dimensions in the model (e.g., a "simple" model with some 50 to 100 constraints at each time period has 1250 to 2500 constraints over a 25 year planning horizon). Therefore a reasonable compromise should be made. Some methodological questions concerning the influence of uncertainty on the length of the planning horizon are discussed in Section 2 of this volume.

Objective function (Performance Index). The general form of the objective function J(u) is

$$J(u) = a(T)x(T) + \sum_{t=0}^{T-1} [a(t)x(t) + b(t)u(t)] \qquad (7)$$

In vector form, the right vector is a column vector and the left vector is a row vector. In coordinate form, the objective function (7) is

$$J(u) = \sum_{i=1}^{n} a_i(T)x_i(T) + \sum_{t=0}^{T-1} \left[\sum_{i=1}^{n} a_i(t)x_i(t) + \sum_{k=1}^{r} b_k(t)u_k(t) \right] \qquad (8)$$

The choice of objectives in a dynamic system is determined by the type of models. Any real optimization model is of a multiobjective nature. Different objective functions are discussed in this paper. However, one should only note that an optimal solution obtained from a single run of the model is not of great practical value. Many numerical runs of the model using different objective functions and different assumptions about the parameters should be made, in order to select the most appropriate plan of development for the system.

Before formulating an optimization problem let us introduce some definitions. The sequence of vectors $u = \{u(0),\ldots,u(T-1)\}$ is a *control* of the system; the sequence of vectors $x = \{x(0),x(1),\ldots,x(T)\}$, which correponds to control u in equations (1) and (2), defines a *trajectory*. The pair $\{u,x\}$, which satisfies all the constraints (e.g., equations (4) - (6)) is a *feasible process*. A feasible process $\{u^*,x^*\}$ which maximizes the objective function (e.g., equation (7)) is called *optimal*. The DLP problem in its canonical form is formulated as follows

Problem 1. Find a control u^* and a trajectory x^*, satisfying the state equations (1) with the intitial state (3) and the constraints (4) and (5), which maximize the objective function (7).

In Problem 1 the vectors $x(0)$, $f(t)$, $a(t)$, and $b(t)$ and the matrices $A(t)$, $B(t)$, $G(t)$, and $D(t)$ are supposed to be known.

The choice of the canonical form is to some extent arbitrary and there are various possible versions and modifications of Problem 1. For example, state equations may include time lags; or constraints on the state and control variables may be considered separately in the form of equalities and inequalities; or the objective function may only be defined by the terminal state $x(T)$ of the system (Carter et al 1977; Chien and Bradford 1976). However, these modifications can either be reduced to Problem 1, or methods of solving Problem 1 can be extended easily to meet these modifications.

Note, that if $T = 1$, then Problem 1 becomes a conventional LP problem. Problem 1 itself can also be considered as a structured LP problem with a staircase constraint matrix.

Sometimes dynamic LP problems are formulated using only LP language, as for example, in the following problem (see also Cocks and Carter 1970; Csaki 1977; Csaki and Varga 1976; Dean et al 1973).

Problem 2. Find the vectors $\{x^*(1),\ldots,x^*(T)\}$, which maximize

$$\sum_{t=1}^{T} c(t)x(t)$$

subject to

$$A(1)x(1) = d(1)$$

$$B(t - 1)x(t - 1) + A(t)x(t) = d(t) \qquad (t = 2,\ldots,T)$$

$$x(t) \geq 0 \qquad (t = 1,\ldots,T)$$

One can also express the state variables x(t) in Problem 1 as an explicit function of control. This leads to the following LP problem with a block-triangular matrix.

Problem 3. Find the vectors $\{u^*(0),\ldots,u^*(T - 1)\}$, which maximize

$$\sum_{t=0}^{T-1} w(t)u(t)$$

with

$$D(0)u(0) = h(0)$$

$$W(t,0)u(0) + \ldots + W(t,t - 1)u(t - 1) + D(t)u(t) = h(t)$$

$$u(t) \geq 0 \qquad (t = 0,1,\ldots,T - 1)$$

where the vectors h(t), w(t) and the matrices $W(t,\tau)$ depend on the known parameters of Problem 1.

Problems 2 and 3 are typical examples of structured LP models and can be modified in the same way as Problem 1 (e.g., a block diagonal structure with coupling constraints and/or variables). But unlike Problem 1, such formulations do not explicitly introduce the dynamics of the system and therefore make it difficult for subsequent methodological analysis. We will use the formulation of DLP models as given in Problem 1.

Let us consider separate submodels describing agricultural activities. The following subsystems are selected

--the livestock subsystem

--the crop subsystem (annual and perennial crops)

--the product utilization subsystem of primary production activities

--the processing subsystem

--the utilization of purchased inputs

--the capacities subsystem

--the water supply subsystem

--the financial subsystem

The first three of them are related to production, the fourth describes food processing, the next three are related to non-agricultural inputs and resource utilization, and the last one accounts for the financial consequences of the planned structure of the system.

In Section 1.3 of the paper the above submodels are individually described. Several integrated models (cattle breeding and crop production, water supply and agricultural production) and a model for the whole agro-industrial system will be discussed in Section 1.4.

1.3 Separate Agricultural Submodels

1.3.1 Livestock Subsystem

We consider a livestock subsystem consisting of several types of livestock. All animals according to their type (dairy, beef, hogs, etc.) and their maturity or age classes are divided into I groups (Killen and Keane 1978; Krilatykh 1979; Len'kov 1979; Propoi 1979a).

Let

$x_i(t)\ (i = 1,\ldots,I)$	be the number of animals of type i (dairy calf, dairy heifer, dairy cow, sow, etc.) at year period t;
$u_i^+(t)$	be the number of animals of type i purchased at period t;
$u_i^-(t)$	be the number of animals of type i sold at period t; and
a_{ij}	be the coefficient, which shows what proportion of animals of type j will progress to type i in the succeeding period (i.e. attrition rate = $1 - a_{ij}$).

Then we can write the state equations for the livestock subsystem as

$$x_i(t + 1) = \sum_{j=1}^{I} a_{ij}x_j(t) + u_i^+(t) - u_i^-(t) \tag{8}$$

or in matrix form

$$x(t + 1) = Ax(t) + u^+(t) - u^-(t) \tag{9}$$

Here $x(t) = \{x_1(t),\ldots,x_I(t)\}$ is the vector of state variables; $u^+(t) = \{u_1^+(t),\ldots,u_I^+(t)\}$ and $u^-(t) = \{u_1^-(t),\ldots,u_I^-(t)\}$ are vectors of control variables.

In many cases, buying and selling activities are allowed only for specific types of animals (e.g., bulls). Let I^+ and I^- be the groups of animals for buying and selling, respectively. In this case, the equations (8) should be replaced by

$$x(t + 1) = Ax(t) + P^+u^+(t) - P^-u^-(t) \tag{10}$$

where

P^+ and P^- are matrices of dimension $I \times I^+$ and $I \times I^-$, with units only if the activity exists, and zeros otherwise.

The state equations (8) or (9) can be specified in more detail.

Let

$x_i^a(t)$ be the number of animals of type i and age group a at period t. $(i = 1,\ldots,n;\ a = 0,\ldots,N - 1;\ t = 0,1,\ldots,T - 1)$.

An animal belongs to age group a, if its age is τ and $a\,\Delta \le \tau \le (a + 1)\,\Delta$

where

Δ is the given time interval

Vector $x^a(t)$ defines the distribution of animals of one type in group a at period t

$$x^a(t) = \{x_1^a(t),\ldots,x_i^a(t),\ldots,x_n^a(t)\}$$

Let the reproductive age begin with the group a_1 and end with group a_2. Usually, $a_2 = N - 1$. Then the number of animals born (belonging to group 0) at year t + 1 is equal to

$$x^0(t + 1) = \sum_{a=a_1}^{N-1} B(a)x^a(t) \tag{11}$$

where

B(a) is a birth matrix of group a, and

$b_{ij}(a)$ is an element of B(a) showing the number of animals of type i "produced" (born) by one animal of type j and group a.

The transition of animals from group a to group a + 1 is described by equation

$$x^{a+1}(t + 1) = S(a)x^a(t) \tag{12}$$

where

S(a) is the survival matrix showing what proportion of group a progresses to group a + 1 in one time period.

If, for example, $\Delta = 1$ year, and the group a suffers an attrition rate each year of α_i^a $(0 \leq \alpha_i^a \leq 1)$, then the equation (12) can be written as

$$\begin{bmatrix} x_1^{a+1}(t+1) \\ \cdot \\ \cdot \\ \cdot \\ x_n^{a+1}(t+1) \end{bmatrix} = \begin{bmatrix} (1-\alpha_1^a) & & 0 \\ & \ddots & \\ 0 & & (1-\alpha_n^a) \end{bmatrix} \begin{bmatrix} x_1^a(t) \\ \cdot \\ \cdot \\ \cdot \\ x_n^a(t) \end{bmatrix}$$

Let us introduce a vector

$$x(t) = \{x_i^a(t)\} \qquad (i = 1,\dots,n;\ a = 0,1\dots,N-1)$$

Then the equations (11) and (12) can be combined

$$x(t+1) = Ax(t) \qquad (t = 0,1,\dots,T-1) \tag{13}$$

where

$$A = \begin{bmatrix} 0 & 0 & \dots & B(a_1) & \dots & B(N-1) \\ S(0) & 0 & \dots & 0 & \dots & 0 \\ 0 & S(1) & \ddots & & & \\ \vdots & & & \ddots & & \vdots \\ \vdots & & & S(a_1) & \ddots & \vdots \\ \vdots & & & & \ddots & \vdots \\ 0 & \dots & \dots & \dots & \dots & S(N-1) \end{bmatrix}$$

A is the growth matrix.

Let us introduce control vectors

$$u^+(t) = \{u_i^{a+}(t)\} \quad , \quad u^-(t) = \{u_i^{a-}(t)\}$$

with unit-zero matrices

$$P^+ = \{p_i^{a+}\} \quad , \quad P^- = \{p_i^{a-}\}$$

which specify buying and selling activities

$$p_i^{a+},\ p_i^{a-} = \begin{cases} 1, & \text{if this activity exists for animals of type } i \text{ and age group } a \\ 0, & \text{otherwise} \end{cases}$$

Again we have a state equation of the same general form as in (10)

$$x(t+1) = Ax(t) + P^+u^+(t) - P^-u^-(t) \tag{14}$$

One additional point should be made. The attrition rate a_{ii} is usually divided into two terms

$$a_{ii} = a_{ii}^{r} + a_{ii}^{b} \tag{15}$$

where

	a_{ii}^{r}	is the base attrition rate due to the death loss, and
the coefficient	a_{ii}^{b}	expresses the ratio of animals removed from the livestock subsystem for breeding or culling,
	a_{ii}^{b}	is a (decision) parameter of the system.

Another way of introducing a breeding or culling policy is to divide the control vector $u^{-}(t)$ into two parts

$$u_{i}^{-}(t) = u_{is}^{-}(t) + u_{ib}^{-}(t) \tag{16}$$

where

$u_{ib}^{-}(t)$ represents the number of animals of type i removed from the subsystem at period t for breeding or culling, and

$u_{is}^{-}(t)$ represents selling.

Let us consider some examples. The cattle subsystem is presented in the form of a chart (in Figure 1).

Example 1. Cattle subsystem (dual purpose dairy cattle)

The time period is equal to one year. The state and control variables are

$x_i(t)$ the number of cattle of group i at year t,

$u_{i}^{-}(t)$ the number of cattle of group i sold at year t, and

$u_{i}^{+}(t)$ the number of cattle of group i purchased at year t.

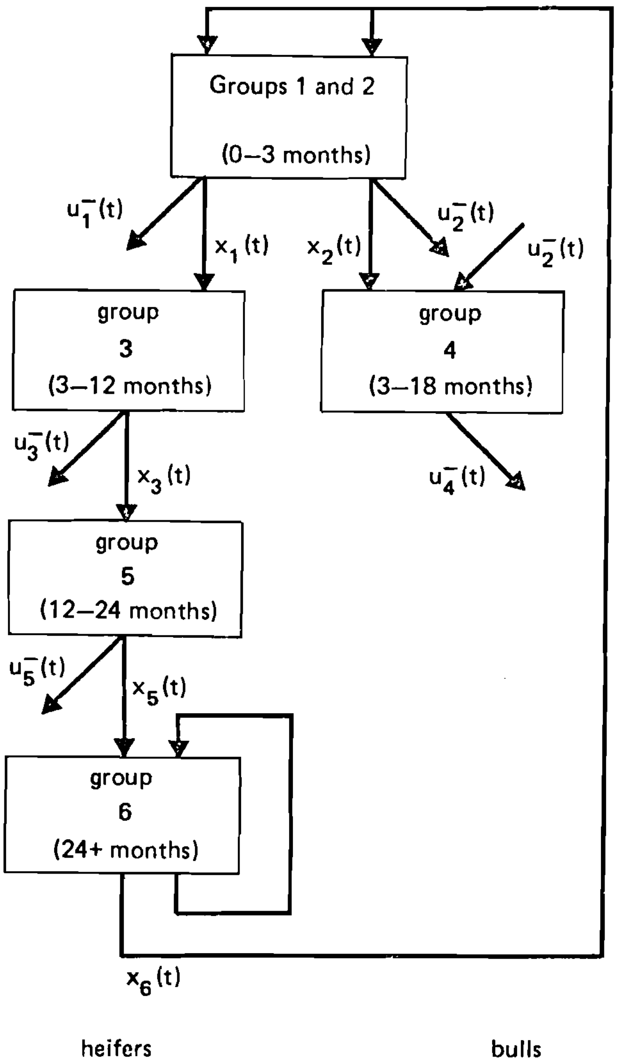


Figure 1. Dual Purpose Cattle

All animals are divided into six groups according to type (heifers, bulls) and age

$x_1(t)$ 0 - 3 month bulls;

$x_2(t)$ 0 - 3 month heifers;

$x_3(t)$ 3 - 12 month heifers;

$x_4(t)$ 3 - 18 month bulls;

$x_5(t)$ heifers aged 12 - 24 months, and

$x_6(t)$ heifers aged 24 months or more.

It is assumed that heifers are sold at all age groups, except those producing milk in $x_6(t)$, and that bulls are all sold after the age of 18 months. The state equations are then

$$a_6x_6(t) = 0.5x_1(t) + 0.5x_2(t)$$
$$x_3(t) = a_{31}x_1(t) - u_1^-(t)$$
$$x_4(t) = a_{42}x_2(t) - u_2^-(t) + u_2^+(t)$$
$$x_5(t+1) = a_{53}x_3(t) - u_3^-(t)$$
$$x_6(t+1) = a_{66}x_6(t) + a_{65}x_5(t) - u_5^-(t)$$
$$0 = a_4x_4(t) - u_4(t)$$

where a_{ij} are retention rates. The last equation should be considered as a constraint on the variables.

In this system we have six control activities at each time period

$$u_i^-(t), i=1,\dots,5 \text{ and } u_2^+(t)$$

and 6 state variables

$$x_i(t) \qquad (i=1,\dots,6)$$

Example 2. Dairy cattle subsystem

In this case, all bulls are sold after birth, and the flow diagram of the system is shown in Figure 2.

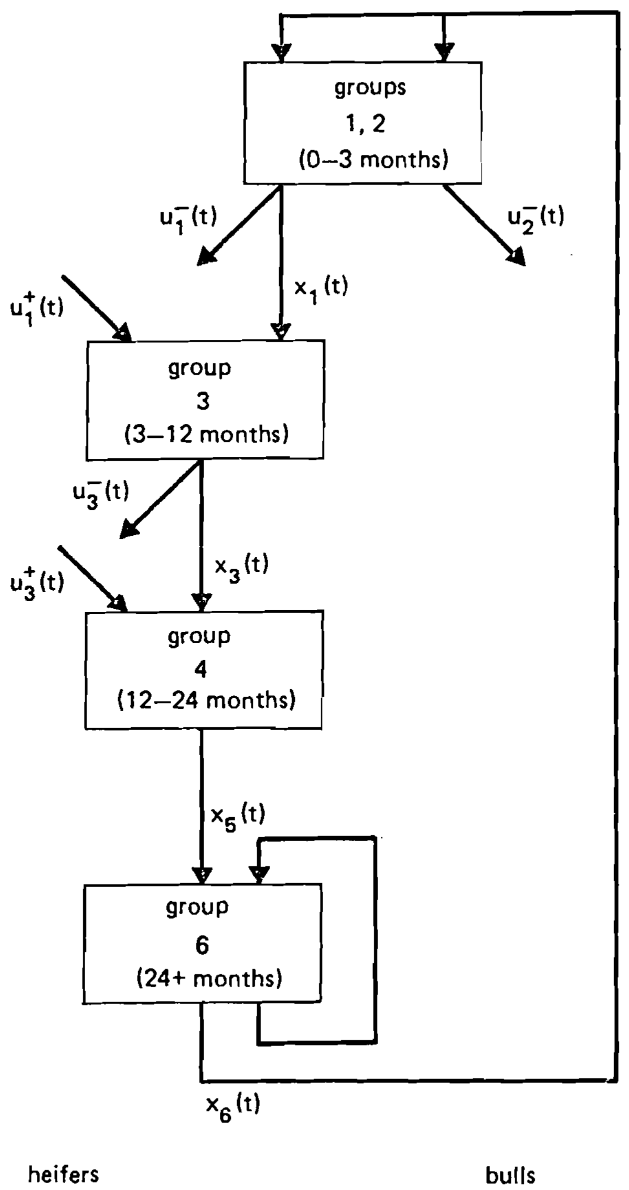


Figure 2. Dairy Cattle

For dairy subsystems we have the following equations (t = 1 year):

$$a_6 x_6(t) = 0.5x_1(t) + 0.5x_2(t)$$
$$x_3(t) = a_{31}x_1(t) - u_1^-(t) + u_1^+(t)$$
$$0 = a_{42}x_2(t) - u_2(t)$$
$$x_5(t+1) = a_{53}x_3(t) - u_3^+(t) + u_3^+(t)$$
$$x_6(t+1) = a_{66}x_6(t) + a_{65}x_5(t)$$

with five control variables

$$u_i^-(t) \text{ with } (i = 1,2,3) \qquad u_i^+(t) \text{ with } (i = 1,3)$$

and five state variables

$$x_1(t) \quad x_2(t) \quad x_3(t) \quad x_5(t) \quad x_6(t)$$

Example 3. Pig-breeding subsystem

When it is necessary to introduce dynamics explicitly in the pig-breeding system, then the use of shorter time periods, e.g., 3 months, is desirable. In this case, the flow diagram is shown in Figure 3 and the state equations are (t = 3 months)

$$a_{41}x_4(t) = x_1(t)$$
$$x_2(t+1) = a_{21}[x_1(t) - u_1^-(t)]\beta$$
$$x_3(t+1) = a_{31}[x_1(t) - u_1^-(t)](1 - \beta_2)$$
$$x_4(t+1) = a_{44}x_4(t) + a_{42}x_2(t) + u_2^+(t) - u_2^-(t)$$

We have three control variables

$$u_1^-(t) \quad u_2^-(t) \quad u_2^+(t)$$

and four state variables

$$x_i(t) \qquad (i = 1,\ldots,4)$$

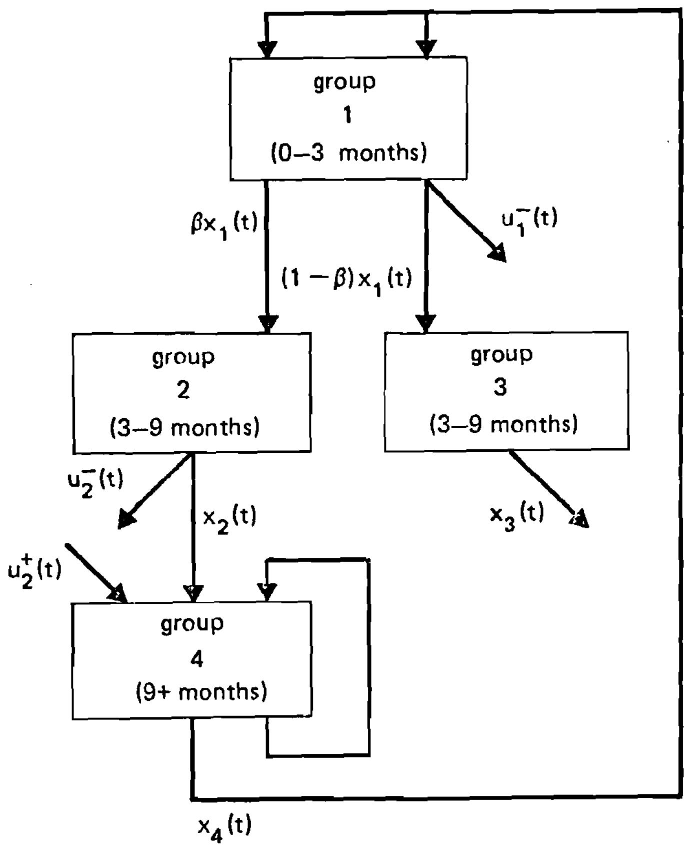


Figure 3. Pig-breeding System

If the cattle and pig-breeding subsystems are to be considered jointly on a rather long-term basis (say, for several years, with each time period being more than one year), then the pig-breeding subsystem can only be described by a static equation.

Even within separate livestock models, it should be possible to analyze different policies when the livestock structure is changed. Therefore, we should also define some constraints on the variables. In their simplest form they can be written as follows.

Apart from the obvious nonnegativity constraints on the variables, constraints associated with the care and feeding of animals should be taken into account. In a rather general form, these can be written as

$$R\,x(t) \leq r(t) \tag{17}$$

where

the k-th component of vector r(t) defines the available amount of the k-th resource (labor, storage capacity, forage, machinery, etc.); and

r^a_{ki} is an element of matrix R specifying the requirements of the k-th resource by animals of type i and age group a.

In addition, the constraints

$$F\,u^-(t) \leq f(t) \tag{18}$$

should be imposed on buying activities (money, transportation, etc.).

The simplest optimization problem can be formulated for the livestock subsystem. Let us specify the objective of the system so as to maximize the total profit for a given planning horizon. This can be expressed by

$$J = \sum_{t=0}^{T-1} \beta(t)\,[a(t)x(t) - b^+(t)u^+(t) + b^-(t)u^-(t)] \tag{19}$$

where

a(t) is per unit profit ($a(t) \geq 0$) or expenses ($a(t) \leq 0$);

$b^+(t)$ is the expense for the purchase of one animal;

$b^-(t)$ is the profit from selling one animal; and

$\beta(t)$ is a discount factor.

The problem is to find those controls

$u^+(t)$ and $u^-(t)$

which maximize (19) under the constraints (17) and (18). Even in this simple form, the model can help analyze different policies concerning a change in the livestock structure under different assumptions about resource availability.

Because this approach deals with the dynamics of a population, its range of application is large; it can be used for cattle, pig and sheep breeding, poultry farming, the optimal control of fish farming, etc. (for examples, see Polyektov 1974). Similar problems also arise when planning the migration of wild animals, or when controlling pests. However, livestock farms usually have their own forage production. Thus, we should introduce submodels which describe the development of forage production.

1.3.2 Crop Production Subsystem: Annual Crops

The crop production subsystem includes both annual and perennial crops. We shall first consider annual crops

In the long run dynamics are important for annual crops for many reasons; these include changes in production stocks over time, the rotation of crops, and the development of irrigation systems. First, we will consider the simplest case, i.e. the fixed potential yield of annual crops. Only the dynamics of stocks needed to be analyzed.

Let

$y_k(t)$ be the number of hectares for producing (planting) the k-th crop resource at time period t $(k = 1,...,K)$;

a_k be the potential or yield of one hectare for the k-th crop.

Then the crop production will be defined by the term

$$a_k y_k(t) \quad \text{or} \quad Ay(t) \tag{20}$$

where A is a diagonal matrix with the elements a_k in the main diagonal.

Sometimes several products k can be cropped from the same lot j (e.g. wheat and straw). So instead of equation (20) we have

$$\sum_{j=1}^{J} a_{kj} y_j(t) \tag{21}$$

which in matrix form will again be $Ay(t)$. Here a_{kj} denotes the capacity of lot j for producing crop k.

Let also

- $z_k(t)$ be the amount of the k-th crop (corn, hay, etc.) at the beginning of time period (year) t $(k = 1,\ldots,K)$;
- $w_k^+(t)$ be the amount of the k-th crop purchased during time period t;
- $w_k^-(t)$ be the amount of the k-th crop removed from the system at time period t (for selling or feeding animals, etc.).

Then the balance equations for storable products will be

$$z_k(t + 1) = z_k(t) + \sum_{j=1}^{J} a_{kj} y_j(t) + w_k^+(t) - w_k^-(t) \tag{22}$$

or in matrix form

$$z(t + 1) = z(t) + Ay(t) + P^+w^+(t) - P^-w^-(t) \tag{23}$$

where the matrices P^+ and P^- have the same meaning as in (14).

If feeding animals are presented in equation (23), then this equation is replaced by

$$z(t + 1) = z(t) + Ay(t) - Bx(t) + P^+w^+(t) - P^-w^-(t) \tag{24}$$

where

b_{ki}^{a} are elements at matrix B showing the per unit consumption of the k-th crop by animals of type i and age group a ; and

$w^{-}(t)$ represents in this case only the selling activity

In (24) the vector x(t), the age/type distribution of animals over time, is either given exogenously, or is computed from (14). In the latter case, Bx(t) defines a linkage between the livestock and crop production systems. There are other alternative ways of linking the systems, for example, with common resources, such as a building and labor.

The capacity of storage is limited

$$z_k(t) \leq \bar{z}_k(t) \tag{25}$$

where $\bar{z}_k(t)$ is given.

If there is no possibility (or necessity) of storing the k-th product, then the equation (22) is rewritten

$$\sum_j a_{kj} y_j(t) + w_k^{+}(t) - w_k^{-}(t) = 0 \tag{26}$$

There are also limitations on arable land. In their simplest form these constraints can be written as

$$\sum_{j=1}^{J} y_j(t) \leq Y \tag{27}$$

The equations (23) or (24) represent the state equations for the storage of crop products (they are, in fact, the same for perennial crops). In (23) we can single out the contol $[w^{+}(t)$ and $w^{-}(t)]$ and the state variables z(t). In addition to constraints (25)-(27) we have nonnegativity constraints on these variables.

Other constraints on resource availability (labor, machinery, etc.) should be included. These can be written in the same for as (17)

$$Ry(t) \leq r(t)$$

These equations, together with the equations describing the livestock subsystem, can be used for building an integrated model of livestock breeding and crop production.

A very important dynamic element of the system for annual crops is the influence of the previous crop on the yield of the crop in the following year.

$$\sum_k y_{jk}(t - 1) = \sum_j y_{aj}(t) \tag{29}$$

where

$Y_{jk}(t - 1)$ is the scale of production of crop j after the production of crop k in period t - 1; and

$Y_{aj}(t)$ is the scale of production of crop a after crop j in period t.

The initial cropping structure is given by

$$\sum_j y_{jk}(t_1) = c_k(0) \tag{30}$$

here $c_k(0)$ is the initial scale of production of crop k, and the available land is fixed as follows

$$\sum_j \sum_k y_{jk}(t - 1) = \sum_a \sum_j y_{aj}(t) = Y \tag{31}$$

where Y is the available land for annual crop production.

Technology and disease control may limit the production of various crops. For example, in most cases sugar beet can be sown on a given piece of land only after four years. These restrictions can be formulated as follows

If

$$y_{jk}(t) \geq 0 \quad (j \in P) \tag{32}$$

then

$$\begin{aligned} y_{uj}(t + 1) &= 0 \quad (u \in P) \\ Y_{uj}(t + 1) &\geq 0 \quad (u \notin P) \end{aligned} \tag{33}$$

and

$$Y_{zu}(t + 2) = 0 \quad (u \in P)$$
$$Y_{zu}(t + 2) \geq 0 \quad (u \notin P) \tag{34}$$

and

$$y_{yz}(t + 3) = 0 \quad (u \in P)$$
$$y_{yz}(t + 3) \geq 0 \quad (u \notin P) \tag{35}$$

where P is the group of crops being restricted.

Some crops may not be followed by others

$$y_{jk}(t) = 0 \tag{36}$$

if

$$k \in K$$
$$y_{jk}(t) \geq 0 \tag{37}$$

if

$$k \notin k$$

where K is the group of crops which can not be followed by crop j.

A stationary crop structure (the same crops grown each year) may also be required

$$\sum_k y_{jk}(t - 1) = \sum_s y_{js}(t) \tag{38}$$

1.3.3 Crop Production Subsystem: Perennial Crops

The perennial crop subsystem is very similar to the livestock subsystem.

Let

$y_j(t)$ $(j = 1,\ldots,J)$ be the number of hectares used for perennial crop j (grape, apricot, alfalfa, etc.) at period t;

$v_j^+(t)$ be the number of hectares used for new plantings of perennials of type j at year t;

$v_j^-(t)$ be the number of hectares of perennial of type j removed j at year t; and

b_{jk} be the proportion of land of type k (i.e. with trees of type k) which will progress to type j in one year.

The state equations are then defined as

$$Y_j(t + 1) = \sum_{k=1}^{J} b_{jk}y_k(t) + v_j^+(t) - v_j^-(t) \tag{39}$$

or in matrix form

$$y(t + 1) = By(t) + P^+ v(t) - P^- v(t) \tag{40}$$

where

$y(t) = \{y_1(t),\ldots,y_J(t)\}$ is the state vector; and

$v^+(t) = \{v_1^+(t),\ldots,v_J^+(t)\}$

and $v^-(t) = \{v_1^-(t),\ldots,v_J^-(t)\}$ are the control vectors.

We can illustrate the state equations for the perennial crop subsystem with an example of orchard planting (i.e. apple, plum, or apricot trees).

We divide all the trees into groups according to their age. Let $y_i(t)$ be the number of trees in age i at time period t $(i = 1,\ldots,N$; where N is the group of mature or producing trees); $y_i(t)$ may also be the number of hectares of trees in group i. Then the state equations which describe the system of planting fruit trees will be the following

$$
\begin{aligned}
y_1(t+1) &= b_{10}v_1^+(t) \\
y_2(t+1) &= b_{21}y_1(t) \\
\vdots \quad &\quad \vdots \quad \vdots \\
y_N(t+1) &= b_{NN}y_N(t) + b_{N,N-1}y_{N-1}(t)
\end{aligned} \tag{41}
$$

where

$\{y_1(t),\ldots,y_N(t)\}$ are the state variables;

$v_1^+(t)$ is the control variable; and

b_{ij} are coefficients of the attrition rate for transition from group j to group i.

The equations (41) have the matrix form (40) with

$$
B = \begin{bmatrix}
0 & 0 & & 0 & 0 \\
b_{21} & 0 & & 0 & 0 \\
0 & b_{32} & & & \\
0 & 0 & \ddots & & \\
\cdot & \cdot & \cdots & \cdot & \cdot \\
0 & 0 & \cdots & b_{N,N-1} & b_{NN}
\end{bmatrix}; \quad P^+ = \begin{bmatrix} b_{10} \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}; \quad P^- = 0
$$

If the information on premature groups of fruit trees ($j < N$) is not needed, then the state variables $y_j(t)$, for $j < N$, can be excluded.

$$
y_N(t+1) = b_{NN}y_N(t) + bv_1^+(t - N + 1) \tag{42}
$$

where

$$
b = b_{N,N-1}b_{N-1,N-2},\ldots,b_{32}b_{21}
$$

If one chooses a time period equal to N years, then the time delay can be eliminated.

$$y_N(t + 1) = \tilde{b}y_N(t) + \tilde{v}(t) \tag{43}$$

where $\tilde{v}(t)$ is the number of plantings during each period of N years.

Example 1. Apricot production subsystem

Let us divide all apricot trees according to their age into five groups

number of years	number of trees
0-1	$y_1(t)$
1-2	$y_2(t)$
2-3	$y_3(t)$
3-4	$y_4(t)$
4 and more (producing or mature trees)	$y_5(t)$

The state equations are

$$y_1(t + 1) = v_1^+(t)$$

$$y_2(t + 1) = b_{21}y_1(t)$$

$$y_3(t + 1) = b_{32}y_2(t)$$

$$y_4(t + 1) = b_{43}y_3(t)$$

$$y_5(t + 1) = b_{55}y_5(t) + b_{54}y_4(t)$$

Here we have five state variables and only one control variable. The above system of five state variables can be simplified by successive substitution

$$y_5(t + 1) = b_{55}y_5(t) + bv_1^+(t - 4)$$

where

$$b = b_{54}b_{43}b_{32}b_{21}$$

For perennial crop production as in the production of apricots, the constraints on the availability of resources (land, labor, machinery, etc.) are written in a similar form to (27) and (28).

The objective of running the model described in (40) might be to determine the distribution of planting and cutting activities over time, and the type of perennials, in order to obtain the maximal revenue under the given land, labor, water, and other resource limitations.

1.3.4 Product Utilization Subsystems of Primary Production Activities. Outputs of livestock and crop activities may be processed. We distinguish primary production activities (producing milk, apples, wheat, etc.) and secondary production activities (producing meat, canned fruit, etc.). The primary product subsystem is broken down into 3 further subsystems; utilization of outputs from livestock, perennial and annual crops.

These subsystems can be described either in a static way, which delineates the balance between inputs from livestock, perennial and annual crop production systems, and outputs from the primary processing system, or in a dynamic way, which delineates the changing stocks of primary products over time. The latter will be similar to the equations (22).

Let

$z_m^x(t)$ $(m = 1,\ldots,M_x)$ be the stock of the primary product of type m produced by the livestock subsystem (milk, meat, eggs, etc.);

$z_m^y(t)$ $(m = 1,\ldots,M_y)$ be the stock of the product of type m, produced by perennial crops (apples, plums, etc.);

$\tilde{z}_m^y(t)$ $(m = 1,\ldots,\tilde{M}_y)$ be the stock of the product of type m produced by annual crops (corn, wheat, vegetables, etc.); and

$z_m(t)$ $(m = 1,\ldots,M)$ be the stock of purchased inputs of type m (fertilizers, pesticides, etc.).

These are all state variables.

Similar to the other subsystems, we have buying and selling activities (or control variables) for the product utilization subsystem. These are

$$z_m^{x+}(t),\ z_m^{x-}(t),\ z_m^{y+}(t),\ z_m^{y-}(t),\ \tilde{z}_m^{y+}(t),\ \tilde{z}_m^{y-}(t),\ z_m^{+}(t)$$

In addition we have the variables x(t), y(t), and y(t), which represent the production from livestock, perennial and annual crops, and define their linkage to the primary processing subsystem.

Also let

g_{mi}^{x} define the output of the m-th primary product (e.g., milk) from a unit of livestock of type i (when the animals are kept at the system);

g_{mi}^{u} define the per unit output of the m-th primary product (e.g., meat) when animals are withdrawn from the system;

α_{mi}^{x} be the volume of livestock product m consumed by the unit of livestock i;

β_{mj}^{x}, β_{mj}^{x} be the volume of livestock product (e.g., manure) utilized per one hectare of crop (perennial or annual) of type j;

δ_{mk}^{x} be the utilization of livestock product m for producing one unit of the secondary product k; and

$q_{mk}^{x}(t)$ be the level of activity for processing the m-th livestock primary product (e.g., milk) into the k-th secondary product (e.g., butter) at period t.

Then the state equations, which describe the change in stock outputs from the primary processing of livestock, will have the form

$$\begin{aligned} z_m^{x}(t+1) = z_m^{x}(t) &+ \sum_i g_{mi}^{x} x_i(t) + \sum g_{mi}^{u}\, u_i^{-}(t) \\ &- \left[\sum_i \alpha_{mi}^{x} x_i(t) + \sum_j \beta_{mj}^{x} y_j(t) + \sum_j \tilde{\beta}_{mj}^{x} \tilde{y}_j(t) \right. \\ &\left. + \sum_k \delta_{mk}^{x}\, q_{mk}^{x}(t)\right] + z_m^{x+}(t) - z_m^{x-}(t) \qquad (44) \end{aligned}$$

If there are no stocks in the system, then $z_m^x(t) = z_m^x(t+1) = 0$ and these equations are then static. Many coefficients in (44) are zeros. In matrix form the equations (44) can be rewritten as

$$z^x(t+1) = z^x(t) + G^x x(t) + G^u u^-(t) - [\alpha^x x(t) + \beta^x y(t)$$
$$+ \tilde{\beta}^x \tilde{y}(t) + \Delta^x q^x(t)] + z^{x+}(t) - z^{x-}(t) \qquad (45)$$

with matrices

$$G^x = \{g^x_{mi}\}$$

$$G^u = \{g^u_{mi}\}$$

$$\alpha^x = \{\alpha^x_{mi}\}$$

$$\beta^x = \{\beta^x_{mj}\}$$

$$\tilde{\beta}^x = \{\tilde{\beta}^x_{mj}\}$$

$$\Delta^x = \{\delta^x_{mk}\}$$

$$q^x(t) = \{q^x_{mk}(t)\}$$

$\Delta^x q^x$ is the vector of the "row by row" product of the matrices Δ^x and q^x.

In the above equation it is assumed that all the animals $u_i^-(t)$ to be sold are processed before sale. Otherwise, it would be necessary to differentiate between animals to be sold with and without prior processing. We have similar equations for the utilization of outputs from the annual crop production subsystem

$$\tilde{z}^y_m(t+1) = \tilde{z}^y_m(t) + \sum_i \tilde{g}^y_{mj}\tilde{y}_j(t) - \left[\sum_i \tilde{\alpha}^y_{mi} x_i(t) + \sum_j \tilde{\beta}^y_{mj}\tilde{y}_j(t)\right.$$
$$\left. + \sum_k \tilde{\delta}^y_{mk}\; \tilde{q}^y_{mk}(t)\right] + \tilde{z}^{y+}_m(t) - \tilde{z}^{y-}_m(t) \qquad (46)$$

and from the perennial crop subsystem

$$z^y_m(t+1) = z^y_m(t) + \sum_j g^y_{mj} y_j(t) - \left[\sum_i \alpha^y_{mi} x_i(t) + \sum_j \beta^y_{mj} y_j(t)\right.$$
$$\left. + \sum_k \delta^y_{mk}\; q^y_{mk}(t)\right] + z^{y+}_m(t) - z^{y-}_m(t) \qquad (47)$$

Coefficients for the above are similar to those of (44). In matrix form the equations (46) and (47) are written as

$$\tilde{z}^y(t+1) = \tilde{z}^y(t) + \tilde{G}^y\tilde{y}(t) - [\tilde{\alpha}^y x(t) + \tilde{\beta}^y y(t) + \Delta^y q^y(t)] + \tilde{z}^{y+}(t) - \tilde{z}^{y-}(t) \qquad (48)$$

$$z^y(t+1) = z^y(t) + G^y y(t) - [\alpha^y x(t) + \beta^y y(t) + \Delta^y q^y(t)] + z^{y+}(t) - z^{y-}(t) \qquad (49)$$

Many coefficients in (46) and (47) are again zeros. The constraints on storage capacity and on the availability of production capacities for primary production activities should also be included in the description of the primary production subsystem.

1.3.5 Secondary Processing Subsystem

This subsystem includes activities in the secondary processing of agricultural production (cheese, butter, canned meat, juice, flour, etc.). If the production capacities in this subsystem are fixed, then the state equations for this processing subsystem are similar to those of the primary processing subsystem.

The state variables are defined as follows

$s_k^x(t)$ $(k = 1,\ldots,K_s^x)$ is the stock of the product of type m produced by secondary processing of the livestock subsystem (i.e. cheese, butter, canned meat, bacon);

$s_k^y(t)$ $(k = 1,\ldots,K_s^y)$ is the stock of the secondary product of type m from perennial crops (i.e. juice, canned fruit, frozen goods); and

$\tilde{s}_k^y(t)$ $(k = 1,\ldots,\tilde{K}_s^y)$ is the stock of the secondary product of the type m from annual crops (i.e. wheat flour, sugar).

Selling activities (control variables) are defined as follows

$$s_k^{x-}(t),\ s_k^{y-}(t),\ \tilde{s}_k^{y-}(t)$$

Thus the state equations can be written as

$$s_k^x(t + 1) = s_k^x(t) + \sum_m d_{mk}^x q_{mk}^x(t) - s_k^{x-}(t) \tag{50}$$

$$s_k^y(t + 1) = s_k^y(t) + \sum_m d_{mk}^y q_{mk}^y(t) - s_k^{y-}(t) \tag{51}$$

$$\tilde{s}_k^y(t + 1) = \tilde{s}_k^y(t) + \sum_m \tilde{d}_{mk}^y \tilde{q}_{mk}^y(t) - \tilde{s}_k^{y-}(t) \tag{52}$$

where

d_{mk}^x is the amount of products of type m required per unit of activity k for livestock;

d_{mk}^y for perennial crops; and

$\tilde{d}_{mk}^y$ for annual crops.

In matrix form

$$s^x(t + 1) = s^x(t) + D^x(t)q^x(t) - s^{x-}(t) \tag{53}$$

$$s^y(t + 1) = s^y(t) + D^y(t)q^y(t) - s^{y-}(t) \tag{54}$$

$$\tilde{s}^y(t + 1) = \tilde{s}^y(t) + \tilde{D}^y(t)\tilde{q}^y(t) - \tilde{s}^{y-}(t) \tag{55}$$

The annual and perennial crop subsystems are related to the output of agricultural production. Subsystems dealing with inputs of agriculture will now be considered. First, the subsystem for utilizing purchased goods (e.g., fertilizers, pesticides, fuel, electricity) will be described.

1.3.6 Utilization of Purchased Inputs

Let $z_m(t)$ be the stock of the purchased inputs of type m $(m = 1,\ldots,M)$ such as fertilizers, pesticides, and fuel.

Therefore we can write for all stored goods

$$z_m(t + 1) = z_m(t) + z_m^+(t) - \left[\sum_i \alpha_{mi} x_i(t) + \sum_j \beta_{mj} y_j(t) + \sum_j \tilde{\beta}_{mj} \tilde{y}_j(t)\right] - \left[\sum_k \gamma_{mk}^x q_{mk}^x(t) + \sum_k \gamma_{mk}^y q_{mk}^y(t) + \sum_k \tilde{\gamma}_{mk}^y \tilde{q}_{mk}^y(t)\right] \tag{56}$$

where

α_{mi}, β_{mj}, $\tilde{\beta}_{mj}$ represent the consumption of purchased inputs of type m per unit of livestock, perennial and annual crops;

γ^x_{mk}, γ^y_{mk}, $\tilde{\gamma}^y_{mk}$ are the consumption of purchased inputs of type m per unit of activity k for the processing of animals, for perennial, and for annual crop products.

In matrix form

$$z(t+1) = z(t) + z^+(t) - [\alpha x(t) + \beta y(t) + \tilde{\beta}\tilde{y}(t)] - [\Gamma^x q^x(t) + \Gamma^y q^y(t) + \tilde{\Gamma}^y \tilde{q}^y(t)] \quad (57)$$

For nonstorable goods and services (e.g., electricity) the state equation (56) is replaced by the static balance equation

$$z_m^+(t) - \left[\sum_i \alpha_{mi} x_i(t) + \sum_j \beta_{mj} y_j(t) + \sum_j \tilde{\beta}_{mj}\tilde{y}_j(t)\right] - \left[\sum_k \gamma^x_{mk} q^x_k + \sum \gamma^y_{mk} q^y_k + \sum \tilde{\gamma}_{mk} \tilde{q}^y_k\right] = 0 \quad (58)$$

1.3.7 Capacity Subsystem of Physical Resources

All production activities are dependent on the availability of production capacities. In the case above, they were supposed to be either fixed or changing over time, but were given exogenously.

When investments are depreciated, it is important to describe the development of the production capacity. In fact, this is one of the most crucial subsystems when considering the development of the agricultural system because land has to be extended, additional machinery purchased, new buildings erected, people hired and trained. All of these require resources and analysis of long-term benefits. The subsystem of physical resources can be formulated as simply as the previously described.

Let

$c_n(t)$ $(n = 1,\ldots,N)$ be the physical resource of type n (buildings, machinery, storage, etc.) available at the beginning of period t;

δ_n be the depreciation rate of the asset of type n;

$w_{nr}(t)$ $(r = 1,\ldots,R)$ be the intensity of activity of type r (purchasing of various types of tractors, construction of cow barns, etc.) in period t for increasing the physical resource of type n;

d_{nr} be the amount of increase in the physical resource of type n when using activity r at unit level for one time period; and

$c_n^-(t)$ be the physical resource of type n removed from the system during period t (e.g., disposal).

The state equations are then defined as

$$c_n(t + 1) = (1 - \delta_n)c_n(t) + \sum_{r=1}^{R} d_{nr}w_{nr}(t) - c_n^-(t) \qquad (59)$$

where

$c_n(t)$ are the state variables; and

$w_{nr}(t)$, $c_n^-(t)$ are control variables.

If we were to include time lags, our state equations would be modified as follows

$$c_n(t + 1) = (1 - \delta_n)c_n(t) + \sum_{r=1}^{R} d_{nr}w_r(t - \tau_r) - c_n^-(t) \quad (60)$$

where

τ_r is the time required for the full depreciation of activity r.

This system may have initial physical resources which are inconsistent with desired ones. Therefore, from a practical point of view, it is necessary to consider not only the construction of new capacities but also the reconstruction of existing assets. In this case, the state equations (59) should be rewritten as follows

$$c_n(t+1) = (1-\delta_n)c_n(t) + \sum_{r=1}^{R} d_{nr}w_{nr}(t) - \sum_{s=1}^{N} c_{ns}(t) + \sum_{s=1}^{N} \chi_{sn}c_{sn}(t) - c_n^-(t) \qquad (61)$$

where

$c_{ns}(t)$ $(n,s = 1,\ldots,N)$ is the decreasing physical resource of type n at step t where n began reconstruction to become the physical resource of type s (for example, the modernization of technology, or changing the type of activity); we call this process "conversion $n \rightarrow s$"; and

χ_{sn} is the conversion coefficient which shows the increase in physical resource n due to reconstruction of one unit of physical resource s.

Thus, the total increase in the physical resource n at step t due to conversion from all possible physical resources will be

$$\sum_{s} \chi_{sn}c_{sn}(t)$$

and the total decrease in the physical resource n at step t due to conversion to other physical resources will be

$$\sum_{s=1}^{N} c_{ns}(t)$$

Obviously, for each n and t

$$(1-\delta_n)c_n(t) - \sum_{s=1}^{N} c_{ns}(t) \geq 0 \qquad (62)$$

Usually the process of reconstruction takes more than one step. In this case, the above equations become

$$c_n(t+1) = (1-\delta_n)c_n(t) + \sum_{r=1}^{R} d_{nr} w_{nr}(t-\tau_r) - \sum_{s=1}^{N} c_{ns}(t-\tau_{ns}) + \sum_s \chi_{sn} c_{sn}(t-\tau_{sn}) - c_n^-(t) \tag{63}$$

where

τ_{ns} is the time (number of steps) needed for converting $n \rightarrow s$.

The values of the physical resources $c_n(t)$ are derived from the state equations (59). Generally, using different values of physical resources $c_n(t)$, $n = 1,\ldots,N$, (i.e. tractors of different types, separate buildings) the available capacities $c_g(t)$ can be combined for a specific g-th operation (for example, harvesting can be done with various types of combine harvesters). Then

$$c_g(t) = \sum_{n=1}^{N} \mu_{gn} c_n(t) \qquad (g = 1,\ldots,G) \tag{64}$$

where

μ_{gn} are coefficients showing the per unit physical resources (i.e. tractor power) required for the g-th operation. Frequently, either $\mu_{gn} = 1$ for $g = n$, or $\mu_{gn} = 0$. Then (64) becomes

$$c_n(t) = c_n(t)$$

The subsystem of physical resources is related to all the other agricultural subsystems. Therefore, the general constraints on the availability of physical resources for the system as a whole are

$$\sum_i \lambda^x_{gi} x_i(t) + \sum_j \lambda^y_{gj} y_j(t) + \sum_j \tilde{\lambda}^y_{gj} \tilde{y}_j(t) + \sum_m \sum_k \lambda^{q^x}_{gmk} q^x_{mk}(t) + \sum_m \sum_k \lambda^{q^y}_{gmk} q^y_{mk}(t) + \sum_m \sum_k \tilde{\lambda}^{q^y}_{gmk} \tilde{q}^y_{mk}(t) \le \sum_n \mu_{gn} c_n(t) \tag{65}$$

where

λ^{x}_{gi}, λ^{y}_{gj}, $\tilde{\lambda}^{y}_{gj}$

and $\lambda^{q^x}_{gmk}$, $\lambda^{q^y}_{gmk}$, $\tilde{\lambda}^{q^y}_{gmk}$ are coefficients expressing the requirements of physical resource g for livestock, annual crop production, and perennial crop production and for the primary processing of livestock, annual and perennial crop products.

The general equation (65) covers most cases of resource constraints. Therefore, for specific problems, most of the coefficients are zero.

In order to complete the system, certain control variables may need to be constrained (e.g., due to land availability, disease control, environemtal requirements). This is done by stating further inequalities.

1.3.8 Water Supply Subsystem

The water supply subsystem is one of the most important in agriculture. The planning of irrigation projects is treated separately by Dean et al. 1973; Glickmann and Allison 1973; Heady 1971. Here, we consider a simplified water supply model as a special case of a model of resource development.

Let

$q_s(t)$ $(s = 1,\dots,S)$ be the water supply of type s available at the beginning of time period t;

$\tilde{q}_s(t)$ be the increase of this capacity during time period t; and

δ_s be the depreciation rate of the asset of type s.

Then the state equation for developing the water supply subsystem will be

$$q_s(t + 1) = (1 - \delta_s)q_s(t) + \tilde{q}_s(t)$$

The demand constraints for water are similar to equation (65)

$$\sum_i \lambda_i^x x_i(t) + \sum_j \lambda_j^y y_j(t) + \sum_j \lambda_j^{\tilde{y}} \tilde{y}(t) + \sum_{m,k} \lambda_{mk}^{q^x} q_{mk}^x(t)$$
$$+ \sum_{m,k} \lambda_{mk}^{q^y} q_{mk}^y(t) + \sum_{m,k} \tilde{\lambda}_{mk}^{q} \tilde{q}_{mk}^y(t) \leq \sum_s \mu_s q_s(t) \qquad (67)$$

where λ's coefficients express the water requirements of livestock, annual and perennial crops.

1.3.9 Financial Subsystem

This subsystem summarizes the costs and benefits of all the above activities described largely in physical terms. There is a wide range of possible methods to calculate costs according to the type of economic and accounting system. Therefore, we only describe the general elements of the financial subsystem. The specific accounting procedure used by the farm will dictate the exact form of the equation and the constraints on the system.

Return in Period t

$$\sum_i p_i u_i^-(t) + \sum_j p_j v_j^-(t) + \sum_n p_n k_n^-(t) + \sum_m p_m^x z_m^{x-}(t) + \sum_m p_m^y z_m^{y-}(t)$$
$$+ \sum_m \tilde{p}_m^y \tilde{z}_m^y(t) + \sum_m p_m^{q^x} s_m^{x-}(t) + \sum_m p_m^{q^y} s_m^{y-} + \sum_m \tilde{p}_m^{\tilde{q}^{\tilde{y}-}} \tilde{s}_m^{\tilde{y}-} = f^r(t) \qquad (68)$$

where

$f^r(t)$	is the total amount of return in period t; and
p_i, p_j, etc.,	are the prices or the appropriate indicators (such as income and profit).

Expenditures

$$\sum_m p_m^x z_m^{x+}(t) + \sum_m p_m^y z_m^{y+}(t) + \sum \tilde{p}_m^{\tilde{y}} \tilde{z}_m^{\tilde{y}+}(t) + \sum p_m^+ z_m(t)$$
$$+ \sum_i \sum_g p_g^c \lambda_{gi}^x z_i(t) + \sum_j \sum_g p_g^c \lambda_{gj}^y y_j(t) + \sum_j \sum_g p_g^c \tilde{\lambda}_{gj}^y \tilde{y}_j(t)$$
$$+ \sum_m \sum_k \sum_g p_g^c \lambda_{gmk}^{q^y} q_{mk}^y(t) + \sum_m \sum_k \sum_g p_g^c \tilde{\lambda}_{gmk}^{q^y} \tilde{q}_{mk}^y(t) = f^e(t) \qquad (69)$$

where

$f^e(t)$ is the amount of expenditures in period t; and

p_g^c are the expenses of the g-th resource, including depreciation.

Money Balance

$$z_p(t+1) = z_p(t) + f^r(t) - f^e(t) \tag{70}$$

with

$$z_p(t) \geq 0$$

where

$z_p(t)$ is the income generated by the system.

Investments

$$\sum_i p_i u_i^+(t) + \sum_j p_j v_j^+(t) + \sum_n \sum_r p_{nr} w_{nr}(t) = f^i(t) \tag{71}$$

where

$f^i(t)$ is the amount invested in period t.

The investments may be restricted by

$$f^i(t) \leq z_p(t) \tag{72}$$

or

$$f^i(t) \leq z_p(t) + \bar{f}^i(t) + f^d(t) \tag{73}$$

where

$\bar{f}^i(t)$ is the exogenously given upper limit of investment funds from external sources.

Fixed Capital

$$z_c(t + 1) = z_c(t) + f^i(t) \tag{74}$$

where

$z_c(t)$ is the net value of fixed assets.

This section completes the description of separate agricultural subsystems. In the following section we shall discuss the linkage of separate subsystems to an integrated system.

1.4 Integrated Agricultural Models

1.4.1 Cattle Breeding - Crop Production Model

We will begin with the simplest case; for planning the expansion of a combined livestock-crop production farm. What follows is an application of the DLP model for a large dairy farm (Swart 1975, Propoi 1979a). The system is divided into two subsystems: the livestock subsystem (dairy cattle) and the crop production subsystem for animal feed (Figure 4). Production and storage capacities (i.e. arable land, machines, silos) are supposed to be fixed, and the only control activities are the annual crop mix and the sale of animals.

In general, the livestock subsystem can be described by the equations (14) and the crop production subsystem by the equations (24) with the constraints (25) to (27).

The objective of the system is, in general, to maximize

$$J(u,v,y,w) = \sum_{t=0}^{T-1} [\alpha(t)\ x(t) + \beta(t)\ u^-(t) + \beta'(t) w^-(t)] - \sum_{t=0}^{T-1} [\gamma(t)\ z(t) + \delta(t)\ u^+(t) + \delta'(t)\ w^+(t) + \rho(t)\ y(t)] \tag{75}$$

where

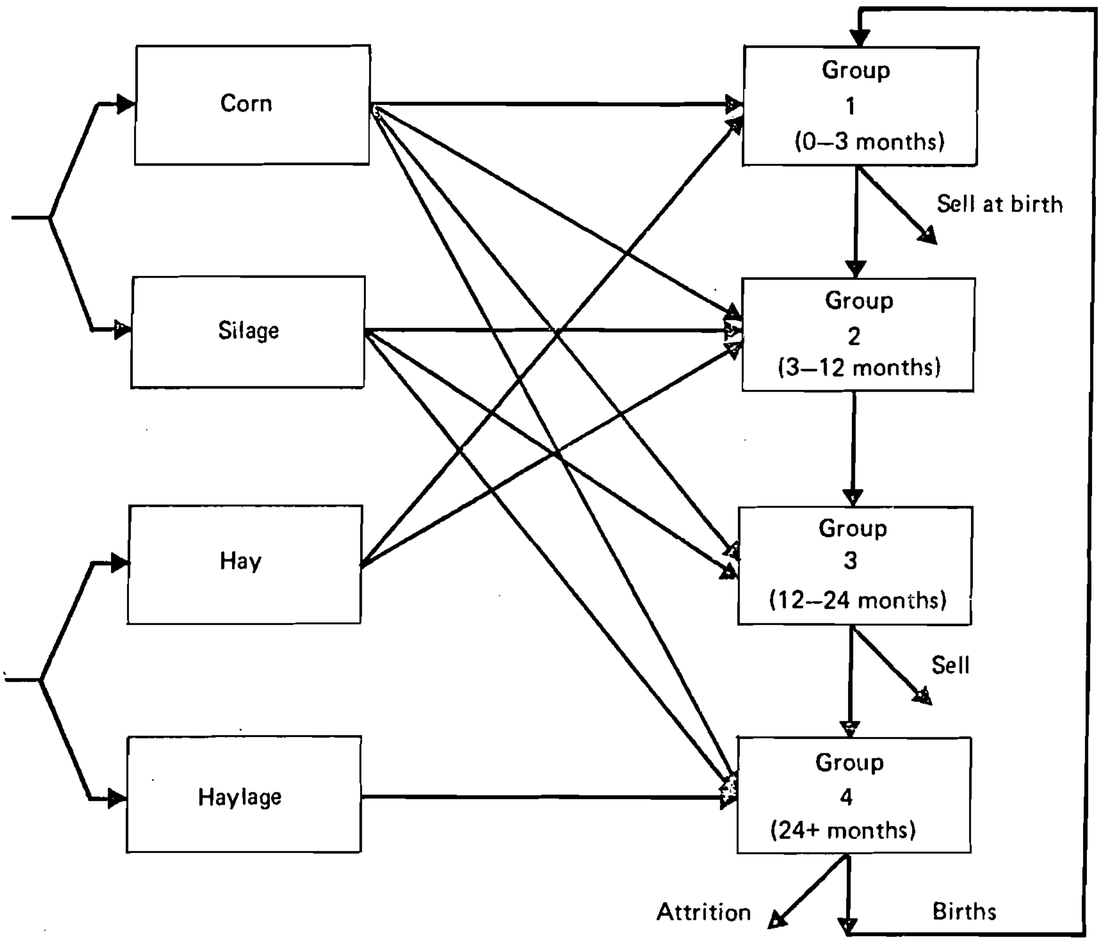


Figure 4. Cattle Breeding--Crop Production Model

$\alpha(t) = \{\alpha_i^a(t)\}$ is the per unit revenue from animals of type i and group a in time period t after deducting the cost of care and other expenses (except (feed-producing expenses);

$\beta(t) = \{\beta_i^a(t)\}$ is the return per animal of type i and group a, sold at time period t;

$\beta'(t) = \{\beta_k(t)\}$ is the return for crop k, sold at time t;

$\gamma(t) = \{\gamma_k(t)\}$ is the cost of storing a unit of crop k during period t;

$\delta(t) = \{\delta_i^a(t)\}$ is the expense per animal of type i and group a purchased at time t;

$\delta(t) = \{\delta_k(t)\}$ is the expense per unit of crop k purchased at time t; and

$\rho(t) = \{\rho_k(t)\}$ is the expense of growing one hectare of crop k at time t.

Finally, the problem can be formulated as follows.

Problem. The controls $u^+(t)$, $u^-(t)$, $y(t)$, $w^+(t)$, $w^-(t)$ are to be determined so that they satisfy the state equations (14) and (24) with the given constraints (25) to (27) and maximize the objective function (75).

Example 1. Planning model of a dairy farm

In the concrete case of a dairy farm the cattle are divided into four groups (Figure 4).

The number of milk-producing cows (Group 4) at year t is $x_1^4(t)$. During the year, each milk-producing cow has one calf, and approximately one half of all calves will be bulls, the other half heifers. Consequently

$$x_1^1(t) = 0.5x_1^4(t) - v_1^1(t)$$

$$x_2^1(t) = 0.5{}_1^4(t) - v_2^1(t)$$

where

$v_1^1(t)$, $v_2^1(t)$ are the number of heifers and bulls sold at birth.

Calves are not sold as long as they belong to Group 2 (see Figure 4). Furthermore, calves grow from Group 1 to Group 2 in the same year. Hence

$$x_1^2(t) = x_1^1(t)$$

$$x_2^2(t) = x_2^1(t)$$

The cattle in Group 2 will belong to Group 3 in the following year, and all the bulls at that age are sold. Hence

$$x_1^3(t + 1) = x_1^2(t) - v_1^3(t)$$

$$0 = x_2^2(t) - v_2^3(t)$$

The cattle in Group 4 have a culling rate of approximately 30% each year. At the same time Group 4 is enlarged by the heifers of Group 3 from the previous year. Hence

$$x_1^4(t + 1) = x_1^3(t) + 0.3x_1^4(t)$$

In the crop production subsystem, two types of crops are considered; corn and alfalfa, which make corn grain and silage, and hay and haylage. Thus there are four different equations.

1) For silage

$$z_1(t + 1) = z_z(t) + a_1 y_1(t) - [b_{11}^2 x_1{}^2(t) + b_{12}^2 x_2{}^2(t)$$
$$+ b_{11}^3 x_1{}^3(t) + b_{11}^4 x_1{}^4(t)]$$

where

b_{1i}^a $(i = 1,2;\ a = 2,3,4)$ are coefficients depicting the yearly per capita consumption of silage by the various cattle groups of the herd; and

a_1 is a coefficient of the yield of silage (in tons) per hectare.

The storage for silage is limited:

$$z_1(t) \leq \bar{z}_1(t)$$

2) For corn

$$a_2 y_2(t) - \sum_{a,i} b_{2i}^a x_i^a(t) = 0$$

where

a_2 and b_{2i}^a $(a = 1,2,3,4;\ i = 1,2)$ are coefficients similar to those for silage.

It is assumed that there is no corn storage at the farm.

3) For haylage

$$z_3(t + 1) = z_3(t) + a_3 y_3(t) - b_{31}^4 x_1{}^4(t)$$

(Haylage is consumed only by cattle in Group 4), with

$$z_3(t) \leq \bar{z}_4(t)$$

where

$w_4(t)$ is the amount of hay purchased in year t.

Evidently, all variables are nonnegative. Furthermore, the land available for cultivation is limited

$$y_1(t) + y_2(t) + y_3(t) + y_4(t) \leq Y$$

The problem is to maximize the total profit during the planning period T

$$J = \sum_{t=0}^{T-1} \Big[\alpha_1{}^4 x_1{}^4(t) + \{\beta_1{}^1 v_1{}^1(t) + \beta_1{}^3 v_1{}^3(t) + \beta_2{}^3 v_2{}^3(t)\}$$
$$- \{\alpha^1 x_1{}^1(t) + \alpha^2 x_2{}^1(t) + \alpha^2 x_1{}^2(t) + \alpha^2 x_2{}^2(t) + \alpha^3 x_1{}^3(t)\}$$
$$- \Big\{\rho_1 y_1(t) + \rho_2 y_2(t) + \rho_3 y_3(t) + \rho_4 y_4(t)\Big\}$$
$$- \delta_4 w_4(t) - \{\gamma_1 z_1(t) + \gamma_2 z_2(t) + \gamma_3 z_3(t) + \gamma_4 z_4(t)\}\Big]$$

where

$\alpha_1{}^4$ is the revenue from milk of one cow of Group 4;

α^1, α^2 and α^3 are the cost of care and other expenses for Groups 1,2 and 3; and

β_i^1, β_i^3, ρ_j, γ_j, δ_4

$(i = 1,2,; j = 1,2,3,4)$ are coefficients similar to those in equation (75).

1.4.2 Water Supply - Agricultural Production Model

In this section we will briefly describe a simple model of the interaction of the water supply and agricultural production.

The problem is how to distribute the common resources (money, labor, etc.). which are needed for the development of both the water supply and agricultural production, so that the maximum output of agricultural production is achieved (see Figure 5). The agricultural production submodel includes crop production and livestock breeding. The livestock breeding model is described by the equations (14).

For simplicity, we only look at irrigated crop production.

Let

$y(t)$ be the total area of arable irrigated land available at the beginning of time period t;

$v^+(t)$ be the increase in arable land; and

$v^-(t)$ be the decrease.

The state equations for the development of the irrigated land

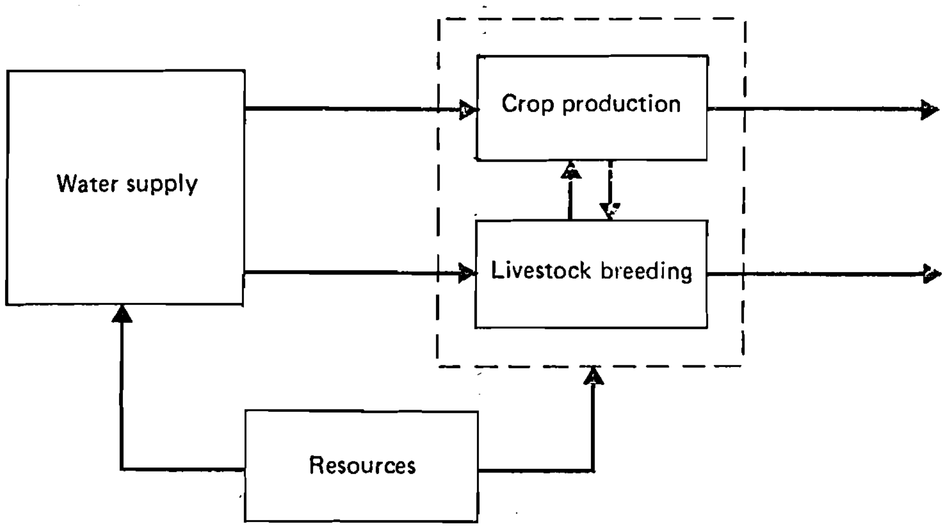


Figure 5. Water Supply--Agriculture Production Model

will be

$$y(t + 1) = y(t) + v^{+}(t) - v^{-}(t) \tag{76}$$

Let $y_j(t)$ be also the number of hectares used for crop j at time t. Clearly,

$$\sum_j y_j(t) \leq y(t) \tag{77}$$

and, in turn, $y(t) \leq L(t)$, where $L(t)$ is the total arable land available at time t.

Then the demand for water in the agricultural subsystem will be

$$\sum_j \lambda_j^y y_j(t) + \sum_i \lambda_i^x x_i(t) = d(t) \tag{78}$$

where

$x_i(t)$ is the number of animals of type i at time t calculated in equations (14); and

$y_j(t)$ is calculated for perennial crops in (39).

For more details of the water demand model see Section 5 of this volume.

On the other hand, the supply of water is

$$\bar{d}(t) = \sum_s \mu_s q_s(t) \tag{79}$$

where

$q_s(t)$ is the water production capacity of type s at time period t and is calculated in (66).

Clearly,

$$\bar{d}(t) \leq d(t) \tag{80}$$

The condition (80) links the water supply and crop production subsystems (figure 5). Moreover, these subsystems have common resource constraints for their operation and development. They can be written as

$$\sum_j r^y_{kj} y_j(t) + \sum_i r^x_{ki} x_i(t) + \sum_s r^q_{ks} q_s(t)$$

$$+ \sum_j r^{y+}_{kj} v^+_j(t) + \sum_i r^{x+}_{ki} u_i(t) + \sum_s r^{q+}_{ks} q^+_s(t)$$

$$\leq r_k(t) \tag{81}$$

The problem is to define a common development program for the water and agricultural subsystems (under common resource limitations), which maximizes the output of agricultural production in the whole system.

1.4.3 Agro-Industrial Complex

An agro-industrial complex may include the following three subsystems: agricultural production (including livestock, perennial and annual crop subsystems), primary utilization of agricultural products, and possibly, their secondary processing (Figure 6). The capacity subsystem and the financial subsystem should also be added. These subsystems were described previously. The way these subsystems are linked depends on the structure of the complex. An example of an agro-industrial complex is given in Section 4 of this volume (see also Il'uskonok, 1980).

Here only the objective functions are mentioned. In fact, this is equivalent to a multiobjective optimization problem and several numerical runs with different objective functions and different assumptions on the coefficients of the model are needed.

Some of the objective functions, which have been applied to investment anlysis, are listed:

-- maximization of the present value of future consumption

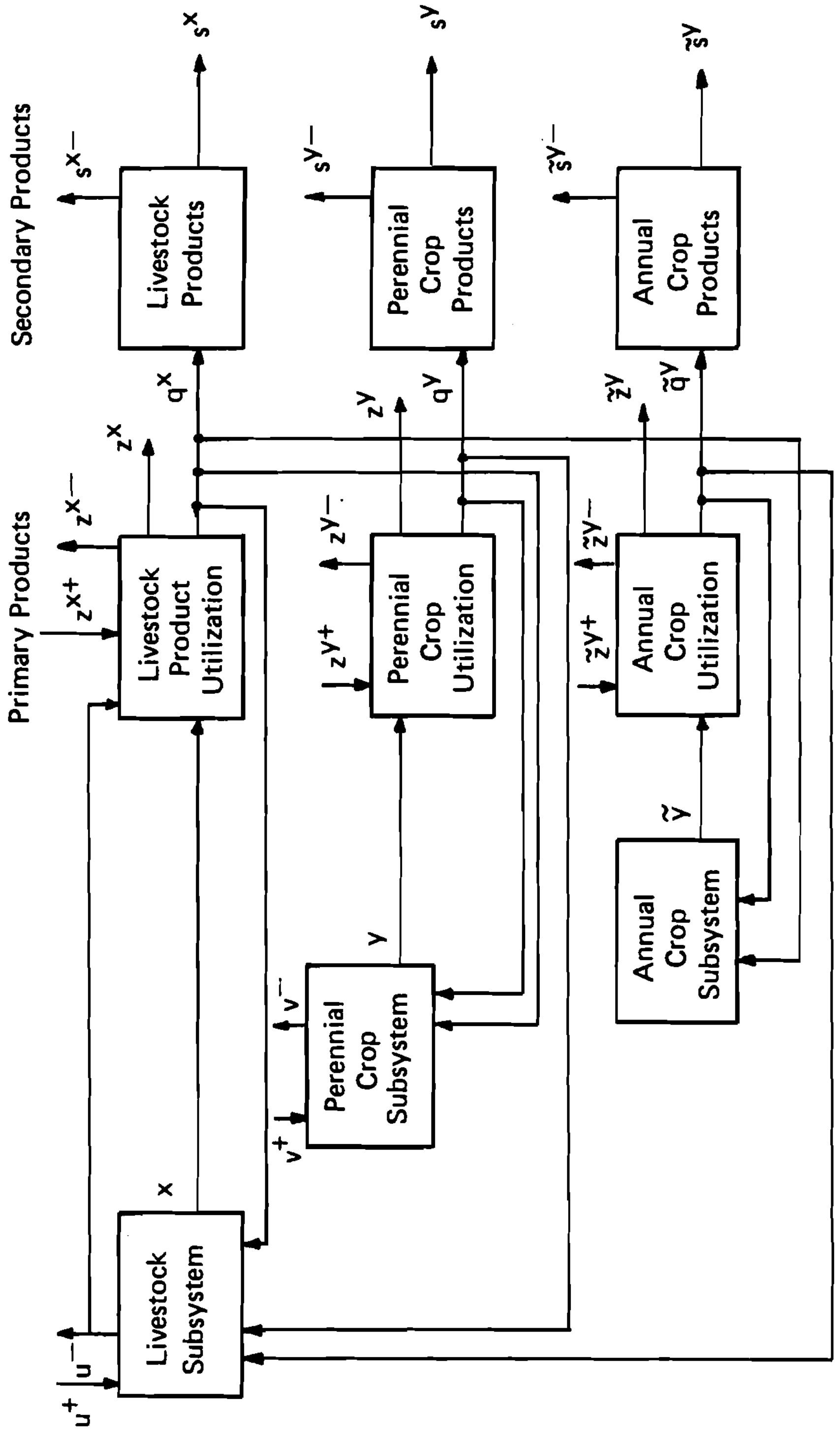
-- maximization of the present value of future return (profits), when profits are withdrawn at the end of each accounting period, and when profits are reinvested as they are realized

-- maximization of the discounted cash flow

-- maximization of the present value of future cash flows

-- maximization of terminal net worth.

For example, for the problem in this paper the following objective functions may be considered



PROGRAM OF PRODUCING SUBSYSTEMS

$$\text{Max} \sum_{t=1}^{T} \omega(t) z_p(t)$$

where

$\omega(t)$ is the discount coefficient

or

$$\text{Max } z_c(T) + z_p(T)$$

where

$z_c(T)$ is the fixed capital in the terminal year T.

1.5 Conclusion

The DLP model of an agro-industrial complex described in this paper has been used as the methodological framework for the agro-industrial development model of the Silistra region, in Bulgaria. With the cooperation of IIASA scientists, the Silistra model has been constructed by Bulgarian experts as part of a large scale integrated regional development program of the Silistra region (see Section 4 of this volume). Parallel to this modelling effort, the Silistra Water Demand Model has also been elaborated (see Section 5 of this volume).

Other applications of DLP models in agriculture are discussed by Boussard (1971); Carter et al. (1977); Chien and Bradford (1976); Cocks and Carter (1970); Csaki (1977); Csaki and Varga (1976); Dean et al. (1973); Olson (1971) (1972); Propoi (1979a); Propoi (1979b); Il'ushonok (1980). Based on these studies, the following can be concluded:

-- The experience gained from practical applications has proved that DLP models are adequate for planning agricultural development projects. Compared with other possible approaches, the main advantage of DLP models lies in the possibility of analyzing the dynamics of agricultural production. This is especially important when large investments are involved.

-- DLP models for agro-industrial development may be considered at various levels of aggregation. Our experience indicates that disaggregation beyond a certain, generally rather moderate level, does not significantly improve the quantity of useful information gained by using DLP models. On the other hand, large scale models are both computationally and due to lack of sufficient data difficult and expensive to handle.

-- The agricultural models discussed in this paper can all be reduced to a canonical DLP form, which can be considered as an LP problem with a special staircase structure of the constraint matrices. Therefore, conventional LP packages can, in principle, be used, and have been used, to solve these problems. However, special DLP methods, which take into account the dynamics of the problem, seem much more promising for future research. Computational aspects of DLP are discussed by Propoi (1979b).

-- When separate DLP agricultural submodels are to be linked in a whole system, different approaches are possible. One approach is to build an integrated DLP model, which describes all the activities in the whole system. The agro-industrial model delineated in Section 4 of this volume is one example. In other approaches the submodels are considered independently, and linkage is implemented iteratively. (Basically, this is carried out either by supply-demand conditions or by sharing a common resource, such as joint investment or labor. Methodological questions related to the linkage of optimization models are discussed by Propoi (1979a); Propoi (1979b); Kallio et al (1979).

-- A reliable data base is necessary for successfully testing and applying the model. The elaboration of model parameters and technical coefficients for technological options by traditional methods has slowed down the whole Silistra modeling work. The possibilities for computerized data preparation will have to be investigated for any further applications.

-- The DLP model assumes constant prices of inputs and outputs; in other words, linearity is assumed. If products and prices are a function of the scale of the production process, which well may be the case in large scale projects, then the model should be reformulated as a nonlinear programming model. In practice, the appropriate sensitivity analysis by using parametric programming techniques often gives good approximations to the nonlinear solutions while retaining the computational efficiency of linear programming.

-- Another objection to DLP is that it is a deterministic approach to a problem which has many stochastic elements. Here again, alternative techniques may be conceptually superior (e.g., quadratic programming, stochastic programming), but operational problems are more formidable because of massive data requirements. Furthermore, it could be argued that some of the annual stochastic variations may be relatively minor compared to the more critical sources of uncertainty in models with a long range planning horizon; for example, changes in the general level of prices, yields, and the variables due to technological change and general economic conditions.

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2. UNCERTAINTY AND DYNAMIC LINEAR PROGRAMMING MODELS IN AGRICULTURE: RECENT ISSUES IN THEORY AND PRACTICE*

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Summary

This paper is devoted to the following aspects of dynamic linear programming models (DLP) in agriculture:

(i) risk and uncertainty

(ii) data handling with matrix generators

(iii) the length of the planning horizon

Although at first glance these questions may seem completely independent, they are in fact closely interrelated.

2. UNCERTAINTY AND DYNAMIC LINEAR PROGRAMMING MODELS IN AGRICULTURE: RECENT ISSUES IN THEORY AND PRACTICE.

J. M. Boussard

2.1. Introduction

When modeling agricultural decisions by applying the techniques of dynamic linear programming (DLP) one major question arises--that of uncertainty. This plays an even greater role in dynamic rather than in static linear programming (LP) studies because plans are made not only for immediate application, but for the remote future. The actual value prices and technological coefficients will have in the future is not known, regardless of the quality of the data. However, the analysis should not be hindered by the mere observation that future data are unknown: although they are not known with certainty, some relevant information is nevertheless available. It is often possible at least to assign a range of variation to specific data. Frequently, the probabilities associated with various alternative assumptions can be specified, which give more information about a set of events although it is less than knowing exactly which alternative will actually occur. In this case one speaks of risk rather than uncertainty.

Often, analysts replace problems of uncertainty by a risk problem for which they have obtained a probability distribution from their data. This custom is widely established, so that its relevance will not be discussed further in this paper.

The important thing is that in whatever way future expectations are described, it is possible to take account of their variability in LP models. And, given the time dimension which by definition is included in dynamic linear programming, it is most desirable to make use of this possibility of dealing with variability in these kinds of models. The main purpose of this paper is to review this facet of DLP.

However, risk and uncertainty considerations are not only mere refinements to be introduced in DLP, they also greatly modify the solutions. More important, they often guarantee the existence of a bounded solution over an infinite planning horizon, although the corresponding "non-risky" model may be unbounded. More generally, they shorten the "useful" planning horizon--i.e., the planning horizon that is necessary in order to make decisions in the present. This aspect will be examined in this paper and some examples will be provided.

First, we shall turn our attention to some preliminary problems of introducing risk and uncertainty in an ordinary static LP model of farm decisions.

2.2. Uncertainty

2.2.1. *Uncertainty and programming models in static setting.*

The first recorded attempt to introduce uncertainty into a farm linear programming model is that of Freund (1956). He developed an LP model of North Carolina farmers, and found that the solution he obtained was unsatisfactory:

(i) The optimal crop patterns generated by these models were much "better" than the actual production plans applied by farmers.

(ii) Farmers, even those who were open-minded, were extremely reluctant to apply these optimal plans.

This could have been explained by an alleged "irrationality" of North Carolina farmers. However, such an explanation was really too simplistic. The differences between optimal and actual plans were obviously due to the fact that uncertainty considerations were excluded from the model. In order to introduce them, Freund made use of the "portfolio selection model" previously developed by Markowitz (1952, 1970) for financial analysis.

Instead of maximizing the expected income, $\overline{z}=E(z)$ he maximized a utility function F, defined by:

$$F=\overline{z} - \alpha\sigma_z^2 \tag{1}$$

where σ_z^2 is the variance of the random income z and α is a "risk aversion coefficient".

With x_i being the level of activity i, c_i the random income level per unit of activity i,

$$E(c_i) = \overline{c_i}$$

and

$$E(c_i \cdot c_j) = c_{ij}$$

then

$$\overline{z} = \sum_i \overline{c_i}\, x_i$$

Thus the function F could be written as

$$F = \sum_i \overline{c_i}\, x_i - \alpha \sum_{i,j} c_{ij}\, x_i\, x_j \tag{2}$$

instead of

$$\overline{z} = \sum_i \overline{c_i}\, x_i \tag{3}$$

Apart from this modification of the economic function, the rest of the model (i.e. the constraints) remained unchanged. Freund then observed that the solutions were reasonably close to those which the farmers who had supplied the data used in the experiment. The consequences of his findings were two-fold.

(i) It was possible to use linear programming not only for normative purposes, in order to compute better plans than those applied by farmers, but also for descriptive purposes, in order to depict in a model the actual behavior of the farmers under the assumption that they are rational decision makers. This is now the most popular application of LP models in agriculture. It enables the derivation of various types of supply curves, the measurement of cross elasticities with respect to prices either for outputs or for inputs, and the simulation of possible effects of various agricultural policies. Thus, it is a major tool for rural economists.

The "normative" approach is now almost obsolete because the difference between a LP plan, where the constraints have been realistically defined, and the plans that a farmer makes after careful consideration of his specific problems is so small that it does not justify the cost of solving the LP problem even if a model already exists. LP models are justified from a normative point of view only for very large and complex agro-businesses or socialist farm cooperatives.

(ii) It was necessary to consider uncertainty in order to account for the behavior of the farmers. In Freund's example, the difference between the plans "with uncertainty" and the plans "without uncertainty" was far from marginal, the average income of the plan "with uncertainty" being about 50% less than the average expected income "without uncertainty". Admittedly, one example of this kind cannot be generalized without precaution. The difference could have been explained by specific situation of North Carolina farmers. However, the same kind of comparison was repeated hundreds of times throughout the world for twenty years, not only with the Markowitz model adapted by Freund, but also with other types of decision models. Almost without exception, it was concluded that uncertainty reduces the farmers' average income by about half and that it reflects the existing discrepancy between the average income maximizing plans and the plans which are actually used by farmers.

Therefore, uncertainty has become a necessary component in any model of farmer behavior. The precise way in which uncertainty is reflected in a model may differ from the classical portfolio selection model. Other methods of dealing with uncertainty in agricultural management problems have been proposed. From a practical viewpoint they may be classified according to whether or not they make use of linear programming methods.

Attention was given first to non-linear methods, since they seemed more justified from a theoretical point of view. Some methods are related with Roy's "safety first" principle (Roy, 1952), which consists in minimizing the probability that current income falls under a given "catastrophic" income. Others start with the chance constrained approach of maximizing $\bar{z}$, the expected income, under the constraint (Telser, 1955)

$$\text{prob}\ (z < z_o) < \varepsilon$$

where $\bar{z} = E(z)$
z sub o is a "catastrophic" income; and
ε a small probability

A variant of this model proposed by Kataoka (1963) is "maximize z_o, subject to

$$\text{prob}\ (z < z_o) < \varepsilon$$

It became quickly clear that practical reasons (such as lack of computer programs and lack of data) prevented the application of non-linear methods on a large scale. Even the most popular of them, the Freund-Markowitz model, is practically never used because of the difficulties associated with the estimation of the unitary variance/convariance matrix, and with the handling of quadratic programming code (without mentioning the estimation of the "risk aversion coefficient").

If we turn our attention toward those models which can be implemented in a strict linear programming framework, we find a number of models with varying degrees of sophistication. The most simple, and most easy to use, is probably the "flexibility constraint" model, which consists of bounding the level of each activity by two equations of the form

$$(1 - \beta_i)\, x_{it-1} < x_{it} < (1 + \beta_i)\, x_{it-1} \quad (4)$$

where x_{it} is the level of the activity i at time t and $\beta_i \geq 0$ is a "flexibility coefficient". Thus the rate of change of the level of activity i between time t and time t + 1 is situated between $-\beta_i$ and $+\beta_i$. This set of flexibility constraints was initially proposed by Henderson (1959), and then extended by Day (1963) who demonstrated empirically its ability to reproduce the farmer's behavior. Aigner (1972) showed the equivalence of this set of constraints with Telser's (1955) chance constrained approach. Of course, the equivalence is not strict; the flexibility constraint provides only an approximation of Telser's problem.

The simplicity of this model has made it extremely popular. In fact, in the absence of a more sophisticated system of security constraints, or of discounting for risk in the objective function, having some kind of flexibility constraint in a programming model of farmers' behavior is an absolute necessity. Otherwise, the plans derived from the programming model differ too much from the plans applied by the farmers and the model cannot be of any practical use. However, this way of taking account of risk, uncertainty, and reluctance to change, is far from being perfectly satisfactory. It is no more than a sophisticated rule of thumb, the shortcomings of which become apparent as soon as one assesses the proper values to be attributed to the β_i coefficients.

A better system was proposed by McInnerney (1967, 1969) under the heading of "game against nature". The underlying idea is quite simple. Let c_{it} be the observed unitary income of activity i at time t. The farm income at time t, under the production plan $\{x_i\}$ would have been

$$v = \sum_i c_{it}\, x_i$$

McInnerney's "maximin" criterion consists in maximizing the minimum feasible v as defined by the above constraints for t = -1,...,-T. Then the optimal v^* is the maximum, over all the states of nature which actually occurred from time -T till the present, of the minimum income generated by the optimal production plan under these states of nature.

Thus, denoting A the matrix of the coefficients of the constraints of the original LP model, C the matrix of the c_{it}, and $\hat{1}$, a column vector of 1, the model is defined as:

$$\max v$$

subject to:

$$\begin{Bmatrix} A & 0 \\ C & -\hat{1} \end{Bmatrix} \begin{Bmatrix} x \\ v \end{Bmatrix} \leq \begin{Bmatrix} b \\ 0 \end{Bmatrix} \quad (5)$$

where b is the vector composed of the right hand sides of the technical constraints in the "without risk" model*

The magnitude of T depends upon the data available from past records. In practice, it seldom exceeds 10 to 15 years. Care should be taken when using this model in an inflationary situation, to reflate the past prices in order to take account of the depreciation of money.

* Apart from risk considerations, the set of technical constraints is represented by $Ax \leq b$.

The computational difficulties are small. The main shortcoming stems from the lack of any risk aversion parameters. Perhaps this is the reason for the system's relative lack of success.

Nevertheless, it has been modified by Low (1974) in the following way: instead of maximizing the minimum of v, a constraint is imposed on v which must be greater than a prescribed minimum income z_o. Thus the expected income is maximized under the constraint that the current income cannot be less than z_o. This provides the model with well-defined "risk aversion parameter", i.e., the value of the minimum income z_o. Then the problem is

$$\max\,(\bar{c},o)\begin{Bmatrix}x\\v\end{Bmatrix} \tag{6}$$

under the constraints

$$\begin{Bmatrix}A & 0\\C & -\hat{1}\\0 & -1\end{Bmatrix}\begin{Bmatrix}x\\v\end{Bmatrix} \leq \begin{Bmatrix}b\\0\\-z_o\end{Bmatrix}$$

where $\bar{c}$ stands for a row vector of the average unitary incomes.

Boussard and Petit (1967) presented, under the heading of "focus loss constrained programming" (FLCP) the following linear model.

Let c_i be the (random) income from the unitary level of activity i, $\bar{c}_i$ its expectation, and p_i the unitary "possible loss": p_i is such that

$$\text{prob}\,(\bar{c}_i - c_i < p_i) < \varepsilon$$

where ε is a "reasonably" small probability. Let us define L by

$$L = \sum \bar{c}_i\, x_i - z_o$$

where z_o is a "minimum income" and L is an "admissible loss" at the farm level. If P is the diagonal matrix of p_i the model is then described by

$$\max\,(\bar{c},x) \tag{7}$$

under the constraints

$$\begin{Bmatrix}A & 0\\P & -\hat{1}/k\\\bar{c} & -1\end{Bmatrix}\begin{Bmatrix}x\\L\end{Bmatrix} \leq \begin{Bmatrix}b\\0\\z_o\end{Bmatrix}$$

where k is a fixed scalar coefficient.

This model can be considered as a linear approximation of a quadratic constraint of the form

$$\gamma_o = \text{prob}\,(\sum_i c_i\, x_i < z_o) < \varepsilon\ (k)$$

assuming cov $(x_i, x_j) = 0$, and that γ_o depends only on the mean and variance of $\sum_i c_i\, x_i$ (Kennedy and Francisco, 1974).

Hazell (1971) proposed a model, which presents some similarities with the preceding ones. The main difference is that the focus loss constraints are replaced by the following set of equations: let L_s be the absolute deviation

$$L_s = |\sum_i c_{is}\, x_c - \sum_i \bar{c}_i\, x_i|$$

where c_{is} is the unitary income of activity i under the state of nature s.

Then one minimizes $\sum_s L_s$. Eventually, one maximizes $\sum_i \overline{c_i} x_i$ under the constraint

$$\sum_s L_s \leq S(\overline{z} - z_o)$$

where S is the number of states of nature, z_o is the minimum income, and $\overline{z} = \sum_i \overline{c_i} x_i$. With this last formulation, the model is similar to the FLCP described above, except that the average sum of deviations is taken into account, instead of the maximum deviation. In addition, the meaning of the matrix P is changed: instead of being a diagonal matrix of estimated possible unitary losses, P is now a full, not necessarily square, matrix of observed $\overline{c_i} - c_{is}$. Here again, as shown by Thompson and Hazell (1972) this model can be considered as a linear approximation of a chance constrained programming model*. However, the restriction: $\text{cov}(x_i, x_j) = 0$ is relaxed in this case. More generally, Duloy and Norton (1975) provided a general method for approximating quadratic constraint by a set of linear equations.

Thus, a great variety of decision models under uncertainty have been applied by agricultural economists. The study of farmers' behavior provides a perfect test for these kinds of models. Can we draw any general conclusions about the relative qualities of the various models?

Surprising as it may seem, they are more or less equivalent from the point of view of their ability to depict farmers' behavior, (Boussard, 1970). All of them finally reduce the mean expected income, increase the diversification of crops, and generate optimal plans which are more or less similar to those which are actually put into operation by farmers. More precisely, the results of these models depend upon the value of some risk aversion parameters (such as the "catastrophic income" in the chance constrained programming model, or the α coefficient in the Markowitz model when the objective function is max $(\overline{z} - \alpha \sigma_z^2)$, or the β_i's in the flexibility constraint model, etc.). It is always possible to assign these parameters suitable and probable values which guarantees that the results correspond with reality. Therefore, the problem lies not so much in the choice of a specific decision model under uncertainty among the great variety of such existing models but rather in the choice of values for the risk aversion parameters.

In this respect, the financial situation of the firm plays an important role: the farmer who knows that he will not get any additional credit will not have the same attitude towards risk as his colleague who has completed the repayment of his loans. Baker and Barghawa (1974) depicted fairly well the behavior of Indian peasants and, explained their reluctance to adopt high yield varieties of rice.

Similarly, when relating the z_o parameters of their model to the level of indebtedness, Boussard and Petit (1967) could successfully explain the farmers' refusal to adopt irrigation methods in southern France. The same kind of study has been done in many countries and under different circumstances. The crucial fact is that uncertainty prevents investments, and without investments it is not possible to grow the most profitable crops. The major consequence of uncertainty must be reflected in the investment activities. But these activities are not normally included in static models such as those we have reviewed in

* Actually, the approximation is better with MOTAD than with FLCP.

this section. They can only be introduced when the model is concerned with investment planning decisions. And since these investment planning decisions are normally studied with DLP, the introduction of risk and uncertainty in agricultural programming models leads very naturally to considerating dynamic multi-period linear programming models.

2.2.2. *Uncertainty in Dynamic Setting*

Most of the models just described can easily be transferred to the case of DLP. No problems arise with those models where uncertainty is introduced in the constraints. It is only necessary to replicate T times the relevant static constraint in order to obtain the set of constraints of a T-period DLP.

The problem is a little more complicated when uncertainty is introduced into the objective function, as with the Freund-Markowitz model. This is an additional reason for not making use of this particular set of constraints. Nevertheless, there are no reasons for not discounting the utility associated with each period, and then summing it up over the entire planning horizon. Thus, the Freund model would result in maximizing

$$U = \sum_t \frac{1}{(1+\rho)^t} \left\{ \sum_i \overline{c}_{it} \, x_{it} - \alpha \sum_i \sum_j C_{ijt} \, x_{it} \, x_{jt} \right\} \tag{8}$$

with ρ being the discount factor, and $\overline{c}_{it}$, x_{it}, and C_{ijt} being defined at each time period t. This implies that there are no intertemporal correlations of the utilities at different time periods (i.e., c_{it} is stochastically independent from c_{it-1}). No simple solutions exist for the case when c_{it} depends on c_{it-1}. The major feature of introducing uncertainty in a dynamic model is that it shortens the planning horizon. This question will be examined in the following section.

2.3. Objective Function and the Length of the Planning Horizon

With DLP models, the total number of activities is proportional to the number of time periods, in other words to the length of the planning horizon. The computational cost is roughly proportional to a power of the length of the planning horizon. Since computational costs are a very real problem in any LP model, this is a very strong incentive for exploring all the possibilities of reducing the length of the planned horizon.

Another problem is the choice of an objective function. In the classical one period farm programming problem, it is often assumed that the maximand should be the income of the farmer. Even in this static framework, the validity of this option may be questioned. We have just seen that taking uncertainty into account often leads to the use of complicated nonlinear objective functions. Within the multiperiod framework, the problem is more complex because the major source of capital stock increment is in saving. Therefore saving must be determined endogenously. There are two main approaches:

(i) To consider the consumption function as a constraint, and maximize any other utility function.

(ii) to compute the utility as an increasing function of consumption--for instance, to maximize the sum of the discounted value of consumption for each year of the planning horizon.

This second approach is consistent with the classical theory of investment. This is why it was adopted in early studies. However, authors quickly became aware that it raised serious problems because towards the end of the planning horizon, it was optimal to sell all existing assets in order to convert them into consumption. This undesirable effect of dynamic optimization may be avoided by considering that the market value of existing assets at the end of the planning horizon reflects their ability to sustain consumption beyond the planning horizon. Thus, their discounted value can be added to the discounted value of consumption within the planning horizon. The following function is maximized:

$$F = \sum_{t=1}^{T} \frac{c_t}{(1+\rho)^t} + \frac{1}{(1+\rho)^T} \sum_{i=1}^{n} p_{iT}\, x_{iT} \tag{9}$$

with c_t being the consumption at time t, T the end of the planning horizon, ρ the rate of actualization, p_{iT} the market price of asset i at time T, x_{iT} the level of the stock of asset i at time T.

On the other hand, the first approach is more flexible because economic functions are used which do not depend on consumption levels.

The central problem, however, is the length of the planning horizon. Modigliani (1951) pointed out the theoretical solution to this problem. Even if a decision maker has great confidence in his plans, he knows that he will have to revise them as he receives more information. Hence long term plans are not necessarily intended to be carried out, but rather to utilize all available information in order to make the best possible decision for the first period. Therefore, the planning horizon should be the shortest time needed to make a plan which will enable taking decisions about the first time period. More recently, Hammond and Mirrless (1973) and Hammond (1975) refined this idea with their concept of "agreeable plan".

For instance, buying a harvester is a poor investment if the farmer is not reasonably sure of growing cereals for at least a few years. But the decision to grow cereals during the next few years implies limiting other activities during that time span, such as cattle breeding. This will have consequences for the feasible growth rate of the livestock over a longer period, in general, than the life of the harvestor. This in turn implies the need for other decisions in the future. As can be seen from this example, these decisions are interdependent, and therefore one theoretically needs to make a plan for an infinite time span.

Common sense, however, shows that the planning horizon must be bounded. Furthermore, the greater the uncertainty, the shorter is the planning horizon. In this respect common sense corresponds to the mathematical theory of the "turnpike". The Turnpike Theory was first stated in the famous book of Dorfman, Samuelson and Solow (1958). They studied a special case of multiperiod linear programming models, which could be formulated as follows

Maximize cx_T under the constraint:

$$-A\, x_{t-1} + Bx_t \leq 0 (t=1,\dots,T) \tag{10}$$

where x_0 is given, c is a row vector of utility coefficients, x_t is the vector of activities at time period t, and A and B are coefficient matrices independent of time. The problem is maximizing the utility value of the assets at the end of the planning horizon with technologies remaining constant. If the planning horizon is long enough, there exist T_1 and T_2 $T_1 \leq T_2 < T$ such that for $T_1 < t \leq T_2$, x_t^* the optimal value of x_t, is given by $x_t^* = \rho^* x^*$

where ρ^* is a scalar and x^* is a vector depending only on A and B. Then $\rho^* = x_t^* / x^*_{t-1}$ is the maximum feasible rate of growth of any weighted sum of the activity levels, that is, of any $c\, x_T$ (this is why it is represented with a star). Moreover, a corresponding theorem holds for the dual values.

Whenever $T > T_1$ the optimal solution for the first t_i period (i.e. all the periods t such that $t \leq T_1$ is independent of T. Therefore, for this type of model, a bounded planning horizon exists in the Modigliani sense. At the same time, it proves the importance of the parameters of the objective function, since any set of p_i will give the same results for x_t^* when $t < T_1$, although different p_i may imply a different T_1.

This analysis has several practical consequences for the design of farm models. Most important is a simple and practical rule for determining the planning horizon: Start with a relatively short planning horizon T_1 and market prices p_i. Compute x_{11}, the optimal solution for period 1 with planning horizon T. Then add one period to the program, and compute the optimal solution x_{12}: (the optimal solution for period 1 with planning horizon T_2). if $x_{11} = x_{12}$ then T_1 is long enough. Otherwise, continue with T_2 in place of T_1 (Boussard, 1971).

This rule has two serious shortcomings:

(i) It is necessary to compute endogenously the amount of money reinvested in the system (the remaining part of the cash flows out, presumably as consumption expenses). This can be done with a linear consumption function, relating the level of consumption of each year to the level of the income of that year. In principle, the linear form of this function is not imposed by linear programming, since it is always possible to linearize an even more complicated consumption function. However, in order to guarantee the unicity of the solution, the marginal consumption propensity needs to increase with income, a somewhat odd requirement, which implies the necessity of using a linear consumption function.

(ii) Similarly, because the theorem is only valid for a linear objective function, all the decisions rules under uncertainty were not applicable.

Recent generalizations of the Turnpike Theorem (McKenzie, 1976) eliminate these objections. His results, similar to those which have already been referred to, guarantee the existence of a planning horizon even in the case of a non-linear problem with a concave objective function. In addition, it is possible to prove that the introduction of uncertainty, in whatever way it is included in the model, shortens the planning horizon (Boussard, 1977).

2.4. Data Handling: Matrix Generators

On a practical level, data handling and computational costs are major obstacles to the extended use of dynamic optimization techniques. Data handling is even a tedious task with ordinary static agricultural programming problems. In the case of multiperiod models. the difficulty is multiplied by the number of time periods. This is why the use of a suitable matrix generator is necessary in order to use this approach. Several relevant programs have been developed during the last few years.

One example is the program GEMAGRI (Boussard, 1979) developed at the INRA (Institut National de la Recherche Agronomique). Activities are either "annual" or "perennial". Annual activities "consume" or "produce" certain "commodities" at different "dates" within the year. For instance, assuming that the beginning of the year is October 1st, the activity "wheat growing" consumes one hectare of the commodity "land" from October 15 to July 14. It produces 5 tons of the commodity "wheat" between July 1 and July 14. When used for generating a multiperiod linear programming matrix, these data generate the subactivities "wheat" for each block of the multiperiod matrix. Similarly, perennial activities are split into "sections", each section lasting one or more years. For instance, the activity "growing apple trees" is divided into the following sections:

(i) Section 1. Planting year: it lasts one year, and requires funds and labor, and produces nothing.

(ii) Section 2. Young apple trees: it lasts three years, produces nothing, and consumes only land.

(iii) Section 3. Production years: it lasts 30 years, produces apples, and consumes land, labor, and funds each year.

(iv) Section 4. Removing year: it lasts one year, produces apples and wood, and consumes land, labor, and funds.

The production and consumption in each section are used to compute the relevant coefficients in each block of the multiperiod matrix. One such activity is generated in each block, thus allowing for the possibility of planting a new orchard at the beginning of each period.

The blocks do not necessarily represent one year. Some may represent any specified number of years. The coefficients of the perennial and annual activities are computed accordingly.

Each commodity involved in production or consumption may or may not correspond to one or several equalities (or inequalities) depending on the type of commodity. If the type is "annual", then one row corresponding to this commodity is generated in each block of the periodic LP matrix. The corresponding equation expresses the requirement that the sum of the productions of this commodity be greater than the sum of the consumptions. If the commodity is "time related", then periods within the year are defined together with the commodity. For instance, time period A begins the first of October and lasts until the end of March. Time period B begins the first of May and last until the end of August. One row is generated for each time period, in each block of the LP matrix. The dates at which consumption and production take place instruct the program to input the right coefficient to the right row, either for annual or perennial activities.

There are some other types of commodities. "Intermediate" commodities are given a price, and the values of each of these commodities are aggregated into one "liquidity" commodity. With this feature it is very easy to formulate the cash flow constraints. "Overall" commodities correspond to one row across all time periods, thus allowing for the possibility of defining intertemporal constraints, etc. In addition, a few classical submatrices are generated on request by specifying a particular option: financing constraints (with built-in borrowing activities defined by the rate of interest and the date of reimbursement), security constraints (of the type described in Section 2.11 under the heading "focus loss constrained programming"), cash flow equilibrium constraints, etc.

Apart from the fact that the task of collecting data and feeding them into the computer is greatly simplified, the main advantage of this type of program is that it provides a data check list that can be easily read by people unacquainted with programming. This especially enables a quick verification of the validity of the data by agricultural economists and other data suppliers.

At the same time, it provides an easy way of quickly changing the most controversial parameters of any DLP model, especially the number of time periods. This parameter is indicated on only one data record, which is extremely easy to change. Also, some other key parameters, such as the discount factor in the objective function, or the form of the objective function itself (present value of the consumption, net worth at the end of the horizon, etc.) are easily changed, so that solutions to the same problem with different assumptions can be quickly compared. The example which follows in Section 2.3 provides an illustration of the possibilities of this program. Without this computer program, the task of running this example would have been tremendous.

The matrix generator has proven its efficiency in speeding the data collection and debugging of large multiperiod agricultural LP models. For instance, a five period LP model with a planning horizon of twenty years and a matrix larger than 400 x 400 was built in less than one week. Preparing the data without the matrix generator would otherwise have required at least three months. Nevertheless, this particular program is only one example of many at other institutions. For instance, another matrix generator in use at the World Bank is a very powerful tool for handling any kind of algebraic relationships and casting them into the framework of a LP model. Without a matrix generator, building a DLP is out of the question for all practical purposes.

2.5. An Example

In this example, we shall consider four basic activities: cereals, open-field vegetables, dairying, and greenhouses. The basic technical constraints are: land (summer and winter), labor (summer and winter), humus balance (expressing that the number of tons of organic matter used by cereals and greenhouses does not exceed the number of tons of organic matter equivalent produced by fodder crops and cattle), and irrigation water. The technical coefficients are given in the Table 1. In addition, incomes and costs are given in Tables 2 and 3.

Table 1. Technical Coefficients

	Activities			
Constraints	Cereals (ha)	Vegetables (ha)	Dairying (ha)	Greenhouses (1000 m^2)
winter labor (hours)	21.0	200.0	290.0	750.0
summer labor (hours)	31.0	270.0	230.0	614.0
winter land (ha)	1.0	0.5	1.0	0.1
summer land (ha)	1.0	0.6	1.0	0.1
organic matter (tons)	0	10.0	-18.0	18.0
irrigation water (100 m^3)	16.0	35.0	55.0	7.0

(positive numbers represent consumption, negative numbers represent production)

2.5.1. *A static case without uncertainty*

Let us consider three types of farms. Each of them have two permanent workers producing together 1730 manpower hours in each season, summer and winter. The farm type A has one ha of land, and a large amount of funds; 45000 units of account. Farm B has 15 ha of land, and only 120 units of account. Farm C is larger; 60 ha of land, but it also has only 120 units of account.

Let us maximize the current income under the five constraints of land, labor, and organic matter availability (see Table 1), assuming that the purchase

Table 2. Financial data in units of account

Activities	Production of Cereals (ha)	Vegetables (ha)	Dairying (ha)	Greenhouses ($1000\ m^2$)
receipts	22.5	76.0	50.0	400.0
cash requirement	5.5	20.0	11.0	170.0
gross margin	17.0	56.0	39.0	230.0
possible loss	2.0	12.3	3.0	100.0

Table 3. Miscellaneous Costs

	u.a.
cost of one hour of temporary manpower in summer	0.05
cost of one unit of greenhouse ($1000\ m^2$)	700.0
cost of one ha of purchased land	300.0
cost of one ha of land sold	200.0
cost of hiring a permanent worker (per year)	120.0
cost of 100 m^3 of water	0.07
cost of one ton of organic matter	0.35

(u.a. stands for "unit of account")

of organic matter is feasible. The problem is then to solve:

$$\max Cx$$

under

$$Ax \leq b$$

with

$$A = \begin{pmatrix} 21.0 & 200.0 & 290.0 & 750.0 & 0 \\ 31.0 & 270.0 & 230.0 & 614.0 & 0 \\ 1.0 & 0.5 & 0.1 & 0.1 & 0 \\ 1.0 & 0.6 & 0.1 & 0.1 & 0 \\ 0.0 & 10.0 & -18.0 & 18.0 & -1 \end{pmatrix} \tag{11}$$

$$C = \left[17 - 16 \times 0.07,\ 56 - 35 \times 0.07,\ 39 - 55 \times 0.07,\ 230 - 7 \times 0.07 \right] \tag{12}$$

and

$$b_A = \begin{Bmatrix} 1730 \\ 1730 \\ 1 \\ 1 \end{Bmatrix} \quad b_B = \begin{Bmatrix} 1730 \\ 1730 \\ 15 \\ 15 \end{Bmatrix} \quad b_C = \begin{Bmatrix} 1730 \\ 1730 \\ 60 \\ 60 \end{Bmatrix} \qquad (13)$$

The solutions are given in Table 4

Table 4. Solutions of the simplest problem

Right hand side	Cereals (ha)	Vegetables (ha)	Dairying (ha)	Horticulture (1000 m^2)	Gross Margin u.a.
A	0.77	0.0	0.0	2.28	522
B	14.81	0.0	0.0	1.89	857
C	55.80	0.0	0.0	0.0	886

Is such a picture satisfactory? From a certain view point, yes: this is actually the kind of cropping system used by many farmers in these situations.

Farmers having a large area of land and a small amount of capital such as Farmer C, grow mainly cereals. Medium sized farmers, such as farmer B, grow a mix of various crops. Very small farmers concentrate their resources on greenhouse cropping. From another point of view, it is surprising to find the cereal grower also having a significant number of greenhouses. Similarly one would have expected farmer B to grow open-field vegetables rather than cereals. These results are explained by the fact that horticulture is extremely profitable so that in each case as many resources as possible are concentrated on horticulture. This is not always possible for two reasons. First building a greenhouse is costly (see Table 2). With farmer C's existing funds he would not even be able to build 1000m^2 of greenhouses. Second, the farmer risks bankruptcy if he invests all his assets in horticulture

2.5.2. *Static case with uncertainty*

An illustration of this problem is given by the optimal solution of the problem when a security submatrix is introduced in the model. This security submatrix is identical to equation (7) with k = 3. The same problems as before are solved with the new matrix A' instead of A:

$$A' = \begin{Bmatrix} 21 & 200 & 290 & 750 & 0 & 0 \\ 31 & 270 & 230 & 614 & 0 & 0 \\ 1.0 & 0.5 & 1.0 & 0.1 & 0 & 0 \\ 1.0 & 0.6 & 1.0 & 0.1 & 0 & 0 \\ 0 & 10 & -18 & 18 & -1 & 0 \\ 15.88 & 53.55 & 35.15 & 229.51 & -0.35 & -1 \\ 2 & 0 & 0 & 0 & 0 & -0.33 \\ 0 & 12.3 & 0 & 0 & 0 & -0.33 \\ 0 & 0 & 3 & 0 & 0 & -0.33 \\ 0 & 0 & 0 & 100 & 0 & -0.33 \end{Bmatrix} \qquad (14)$$

Assuming the minimum income being 120 units of accounts, the vectors on the right hand side become

$$b_A = \begin{bmatrix} 1730 \\ 1730 \\ 1 \\ 1 \\ 0 \\ 120 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad b_B = \begin{bmatrix} 1730 \\ 1730 \\ 15 \\ 15 \\ 0 \\ 120 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad b_C = \begin{bmatrix} 1730 \\ 1730 \\ 60 \\ 60 \\ 0 \\ 120 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

The results are given in Table 5.

Table 5. Solution of the static model with a security constraint.

	Cereals	Vegetables	Dairying	Greenhouses	Gross Margin
A	infeasible solution				
B	13.47	2.36	0.0	1.10	683.3
C	55.80	0.0	0.0	0.0	886.4

For farmer C, the security constraints are not binding, so that his solutions are identical to the previous ones. For farmer B, security has its cost: the gross margin is lowered by 174, now 683 instead of 857. The crop system is deeply modified, the area of greenhouses having been greatly reduced. For farmer A, the modifications are even more stringent, since no solution is feasible under these constraints. These solutions are much more consistent with reality, except of course for farmer A. In his case, the results are mainly due to the fact that, in this kind of static model, it is impossible to take account of the large amount of liquidity he owns. Since in the model, he cannot use it for acquiring any additional inputs, his money is useless. Therefore it is necessary to take account of the initial fund constraint, as well as to allow for the hiring of man-power, the buying of land, the construction of greenhouses, etc. An annual fund constraint would not take this properly into account: building a greenhouse which will last 10 years, is not equivalent to hiring a worker for only one year. It is therefore necessary to build a multiperiod program reflecting the way available funds should be allocated between short-term (annual) and long-term (several years) investments. How will the current annual matrix be set up in such a case?

2.5.3. *A dynamic case*

Let us denote by

Q_t the number of hectares of land owned

G_t the number of hectares of greenhouses built in year t

L_t the number of hectares of land bought in year t

M_t the number of tons of organic matter purchased in year t

W_t the number of permanent workers hired in year t

O_t the number of hours of occasional workers hired in year t (in summer only)

X_t the number of hectares for crop j (j=1,...,4) in year t (X_t is a vector with four elements X, X_{tj}

E_t the amount of money borrowed in year t (borrowing is assumed to last 7 years, and to be reimbursed onthe basis of a constant annuity, the magnitude of which is a)

B_t the amount of short term borrowing

I_t the number of m^3 of water bought in year t

N_t the amount of money drawn from the cash flow for long term investment

S_t the number of hectares of land sold in year t

R_t the amount of money remaining from the previous year

C_t the consumption level of year t

P the annual discount factor

Then the annual current submatrix is given in Table 6 (comparable to matrix A). The righthand sides are computed as indicated below:

Row 1:	1700	(1700 being the quantity of labor supplied by the family workers)
Row 2:	1700	
Row 3 and 4:	0	
Row 5 and 6:	0	
Row 7:	$G_0 + \sum_{t=-10}^{-1} G_t$	
Row 8:	$R_{t-1} - \sum_{t=-7}^{-1} aE_t$	
Row 9:	0	
Row 10:	120	(120 being the minimal level of consumption required for the family labor force).
Row 11:	Q_{t-1}	

In the matrix generator, the above summations are automatically performed. For instance, G_t has a coefficient -1 in the 7th row of each block of order t+1, t+2,...,t+10. Similarly, the activity Q_t has the coefficient -1 in the 11th row of block t+1. Notice that row 11 is necessary to be sure that no land bought in year t will be sold by activity S_t. It is only possible to sell land bought in years t-1, t-2, etc...

Table 6. Annual current submatrix of the dynamic problem

	Constraints										
	Winter labour	Summer labour	Winter land	Summer land		Irrigation water	Green-houses	Funds, short-term	Funds long-term	Remaining cash	Land owned
Activities	1	2	3	4	5	6	7	8	9	10	11
X_{1t}	21	31	1.0	1.6		16		5.5		-17	
X_{2t}	200	270	0.5	0.6	10	35		70.0		-56	
X_{3t}	290	230	1.0	1.0	-18	55		11.0		-39	
X_{4t}	750	614	0.1	0.1	18	7	1	170.0		-230	
E_t									-1		
B_t								-1		0.10	
I_t						-1		0.07		0.07	
M_t					-1			0.35		0.35	
W_t	-1200	-1400						120		120	
O_t		-1						0.05		0.05	
G_t							-1		700		
L_t									300		
S_t								-200		200	1
N_t								1	-1		
R_t										1	
C_t										1	
Q_t			-1	-1							1

The objective function is given by

$$F = F_1 + F_2F + F_3 + F_4$$

where F_1 is the sum of discounted consumption

$$F_1 = \sum_{t=1}^{T} \frac{C_t}{(1+\rho)^t}$$

F_2 is the residual discounted value of land bought (at the price 200 u.a./ha)

$$F_2 = 200 \frac{Q_t}{(1+\rho)^{T+1}}$$

F_3 is the residual discounted value of greenhouses, built costing 700 u.a./unit

$$F_3 = \sum_{t=T-9}^{T} \frac{G_t}{(1+\rho)^{T+1}} \; \frac{700(T-t)}{10}$$

F_4 is the residual discounted value of debt over 7 years of borrowing (for one u.a. borrowed)

$$F_4 = \sum_{t=t-7}^{T} \frac{E_t}{(1+\rho)^{T+1}} \; \frac{1(T-t)}{7}$$

We now have to discuss the length of the planning horizon and the discounted rate. The main results are listed in Table 7 showing the influence of the length of the planning horizon and of the rate of discount on the optimal solution for farmers B and C.

Table 7. Results of various lengths of the planning horizon, and of two discount rates without security constraints.

	Discount rate 10%			Discount rate 20%		
	planning horizon			planning horizon		
Crop system for year 1	2 years	7 years	12 years	2 years	7 years	12 years
Farmer B						
Cereals (ha)	6.85	0.0	0.0	0.0	0.0	0.0
Vegetables (ha)	40.92	63.8	63.8	40.92	63.80	63.80
Dairying (ha)	0.0	0.0	0.0	0.0	0.0	0.0
Greenhouses (1000 m^2)	0.0	10.07	10.07	6.8	10.07	10.07
Consumption (u.a.)	0.0	0.0	0.0	0.0	0.0	0.0
Farmer C						
Cereals (ha)	23.0	0.0	0.0	0.0	0.0	0.0
Vegetables (ha)	158.0	250.0	250.0	158.4	250.0	250.0
Dairying (ha)	0.0	0.0	0.0	0.0	0.0	0.0
Greenhouses (1000 m^2)	0.0	36.0	36.0	23.4	36.23	36.0
Consumption (u.a.)	0.0	0.0	0.0	0.0	0.0	5639.04

With a two year planning horizon, the solution for the first time period is not independent of the length of the horizon. In fact, 7 years are hardly sufficient to give a good account of the optimal crop system, and 12 years are necessary to approximate the optimal level of consumption for year one.

This conclusion is practically independent of the discount rate, although, in principle, a shorter planning horizon is obtained with a higher discount rate.

The model was then modified in order to introduce security constraints. The last five rows and the last column of matrix A' were added to the standard annual submatrix of Table 6 (exactly as the matrix A' was deducted from matrix A, by adding these last five rows and the last one column). The results are given in Table 8.

Table 8. Results of various lengths of the planning horizon, and of two discount rates under security constraints.

	Discount rate 10%			Discount rate 20%		
	planning horizon			planning horizon		
Crop system for year 1	2 years	7 years	12 years	2 years	7 years	12 years
Farmer B						
Cereals (ha)	6.87	0.0	0.0	0.0	0.0	0.0
Vegetables (ha)	11.27	19.0	19.0	19.0	19.0	18.19
Dairying (ha)	0.0	0.0	0.0	0.0	0.0	0.0
Greenhouses (1000 m^2)	0.0	0.0	0.0	0.0	0.0	0.56
Consumption (u.a.)	0.0	0.0	0.0	770.0	1351.0	1418.0
Farmer C						
Cereals (ha)	27.16	0.0	0.0	0.0	0.0	0.0
Vegetables (ha)	40.10	70.0	70.0	70.0	70.0	70.0
Dairying (ha)	0.0	0.0	0.0	0.0	0.0	0.0
Greenhouses (1000 m^2)	0.0	0.0	0.16	0.0	0.16	0.16
Consumption (u.a.)	0.0	0.0	0.0	3046.0	5639.0	5639.0

The introduction of security constraints allows for a shortening of the planning horizon to 7 years. The consumption level, with a planning horizon of 7 years and a discount rate of 20%, is practically the same as with a planning horizon of 12 years.

Tables 9 and 10 may help to understand the reason for this result. They show the growth pattern of three farms, with two values for the discount rate where security constraints are included. The convergence of the three farms toward the same cropping pattern is striking. The ratio horticulture-to-vegetables tends to be about 12.33% independent of the starting ratio and the discount rate. But the behavior of the system as a whole is quite different depending on the discount rate: with a rate of 20%, the three farms have identical growth patterns, the amount of greenhouses being 0.73 units (1 unit = 1000 m^2) and the land with vegetable 5.92 ha. With a discount rate of 10%, each farm grows homothetically with a growth rate of about 15% per year. Since the starting point varies, the magnitude of growth of each farm differs.

Table 9. Growth pattern of each type of farm with security constraints and a discount rate of 10% over a 12-year planning horizon.

	Greenhouses ($1000\ m^2$)			Vegetables (ha)			Consumption (u.a.)		
Year	Farm A	Farm B	Farm C	Farm A	Farm B	Farm C	Farm A	Farm B	Farm C
1	2.66	0.0	0.0	21.62	19.0	70.67	0	0	0
2	2.70	0.78	3.08	21.96	17.18	63.69	0	0	0
3	3.16	1.67	6.55	25.69	16.74	62.12	0	0	0
4	3.62	2.30	8.78	29.48	18.70	71.45	0	0	0
5	4.17	2.77	10.59	33.93	22.55	86.14	0	0	0
6	4.80	3.27	12.51	39.06	26.61	101.74	0	0	0
7	5.53	3.84	14.75	44.98	31.25	119.95	0	0	0
8	6.37	4.50	17.33	51.81	36.62	140.94	0	0	0
9	7.34	5.26	20.31	59.69	42.81	165.17	0	0	0
10	8.46	6.14	23.75	68.79	49.96	193.13	0	0	0
11	9.22	7.35	28.40	74.96	59.63	230.96	0	0	0
12	10.68	8.39	32.53	86.86	68.23	264.51	7376	29135	9098

All profits are reinvested, and the consumption over the minimal level imposed by row 10 is zero.

The reason for this result is not difficult to understand. The maximal growth rate of the matrix (i.e., the ρ^* coefficient referred to just after equation 10) being 15% with a discount rate of 10%, it is always advantageous to differ consumption to the last year of the planning horizon, since the marginal value of one dollar saved is more than one dollar. However, with the discount rate at 20%, the marginal value of one dollar saved is smaller than the value of this dollar at the end of the horizon. Thus, an equilibrium is reached when the crop pattern obtains its optimal structure, no saving longer being advantageous.

These results demonstrate the importance of the discount rate on the long term behavior of each farm, as well as point out the differences between the results of the multiperiod program with those of the static program. The relationships between the static and the dynamic models are illustrated by Tables 11 and 12. Table 11 shows the results of the dynamic models without security constraints over seven years. Table 12 shows the solution of the corresponding static model. (The differences between Table 11 and Table 4 are due to the fact that since input buying is inhibited in Table 11, cultivation in greenhouses can

Table 10. Growth pattern of each type of farm with security constraints, and a discount rate of 20% over a 12-year planning horizon.

	Greenhouses (1000 m²)			Vegetables (ha)			Consumption (u.a.)		
Year	Farm A	Farm B	Farm C	Farm A	Farm B	Farm C	Farm A	Farm B	Farm C
1	0.73	0.56	0.167	6.77	18.19	70.0	3601	1418	5639
2	0.73	0.56	0.167	6.31	11.55	44.0	276	665	4353
3	0.73	0.73	0.73	5.92	9.79	27.0	109	502	2769
4	0.73	0.73	0.73	5.92	8.58	17.0	197	427	1702
5	0.73	0.73	0.73	5.92	7.54	10.6	197	366	820
6	0.73	0.73	0.73	5.92	6.66	8.0	197	315	412
7	0.73	0.73	0.73	5.92	5.92	7.1	197	242	329
8	0.73	0.73	0.73	5.92	5.92	6.4	197	233	290
9	0.73	0.73	0.73	5.92	5.92	5.92	197	256	262
10	0.73	0.73	0.73	5.92	5.92	5.92	0	62	233
11	0.69	0.73	0.73	5.92	5.92	5.92	351	325	233
12	0.73	0.73	0.73	5.92	5.92	5.92	513	517	517

not break even. This results in important losses in gross margins for farms A and B. No differences occur for farm C, because the optimum plan does not contain greenhouse cultivation.)

From these Tables we can see that the long term solution of the dynamic problem is identical to the solution of the static problem. However, the solution for the first year of the dynamic problem differs from the solution of the static problem, because the latter is not at all feasible. An adaptation of the cash flow is necessary before a long term static equilibrium can be reached. The same kind of result would have been obtained if security constraints were included.

We conclude that suppressing the input buying activities greatly modifies the optimal solutions.

2.6. Concluding Remarks

The introduction of uncertainty constraints in a linear programming model of a typical farm is likely to bring about a drastic change of the optimal solution. However, in a dynamic multiperiod model, introducing uncertainty in the model not only changes the solution, but also shortens the planning horizon.

In addition, the results show that, although reaching the turnpike may take a long time, the planning horizon is not very long: usually 7 to 10 years are

Table 11. Growth patterns of each type of farm, in a model without security constraints and without input buying activitis –discount rate: 20% – planning horizon: 7 years.

	Year						
	1	2	3	4	5	6	7
Farm A							
Consumption (u.a.)	3712.0	0.0	0.0	0.0	0.0	0.0	0.0
Greenhouses (1000 m^2)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Dairying (ha)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Cereals (ha)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Vegetables (ha)	1.66	1.66	1.66	1.66	1.66	1.66	1.66
Farm B							
Consumption (u.a.)	43.26	254.43	254.43	254.43	254.43	254.43	452.0
Greenhouses (1000 m^2)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Dairying (ha)	2.36	0.0	0.0	0.0	0.0	0.0	0.0
Cereals (ha)	0.0	11.98	11.98	11.98	11.98	11.98	11.98
Vegetables (ha)	4.25	5.03	5.03	5.03	5.03	5.03	5.03
Farm C							
Consumption (u.a.)	0.0	267.0	696.2	686.2	686.2	686.2	1055.6
Greenhouses (1000 m^2)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Dairying (ha)	2.36	1.26	0.0	0.0	0.0	0.0	0.0
Cereals (ha)	0.0	26.70	55.80	55.80	55.80	55.80	55.80
Vegetables (ha)	4.25	2.27	0.0	0.0	0.0	0.0	0.0

Table 12. Solution of the static model without input buying activities

Farm	Cereals (ha)	Vegetables (ha)	Dairying (ha)	Greenhouses (1000 m^2)	Gross Margin (u.a.)
A	0.0	1.66	0.0	0.0	88.9
B	11.98	5.03	0.0	0.0	459.6
C	55.80	0.0	0.0	0.0	886.1

sufficient to ensure that the solution for the first year is a good approximation of the optimal first step toward the turnpike. Convergence towards the optimal balanced growth path has been shown to be very quick even with three very different starting points.

Finally, this example has demonstrated the importance of the discount rate in comparison with the maximum rate of accumulation of the technology matrix. In the long-term a high discount rate produces a uniform solution which is identical for each type of farm without growth. On the contrary a small rate gives three homothetic solutions, each of which grows at a uniform rate in the long run.

Care must be taken when interpreting these results. They could imply that in a given region, characterized by one technology matrix, all farms should in the long term be identical or homothetic. This would only be the case if technology were not to change over time. In this kind of matrix technology also depends on prices, which are implicit in the security constraints and which are contained in those constraints expressing that funds must be available in order to buy inputs (rows 8 to 10 in the model). This implies that technology, even in the absence of technical progress, is not invariable through time. On the contrary, changing expectations and changing demand and supply may deeply modify technology from one year to the next. In the example illustrated above, almost every farm which tries to sell or to buy land would have changed the land market, and the development plans listed in Tables 9 and 10 could no longer be completed. Then the actual solutions would probably not differ very much from the solutions listed in Tables 11 or 12. More generally, changes in the technology matrix which are induced by price changes may favor or impede the growth of a specific type of farm and continuously change the conditions of the "access to the turnpike".

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3. APPLICATION OF A DYNAMIC MODEL OF FARM GROWTH IN NORTH IOWA

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Preface

This article includes application of a linear dynamic growth model for a private farm in north central Iowa. Growth problems are rather intense for these farms since a new operating firm must be initiated each generation with the limited capital which can be accumulated by a young farmer and his wife. Hence, the farm firm represented by a young farm husband and his wife must accumulate capital from various sources to help their business firm to grow. Of course, growth of the firm also will give rise to savings and the accumulation of capital. Growth and capital accumulation become endogenous variables which interact with each other.

The goal of this farm firm typically is to grow at a desired or acceptable speed which will augment family and net worth or wealth. As the farm grows, its income and capital will increase. Maximum income typically comes at the time the farmer retires when he rents his land to another farmer, typically a son. While his income will decline after retirement, his assets will continue to grow as long as his income exceeds his expenditure and savings occur. Upon the death of the older farmer and his wife, the farm firm that they possessed also ceases to exist.

The growth of the farm firm is a dynamic process. Each generation of private farmers must generate anew the capital with which it functions. Upon termination of a previous family and farm firm, the farm assets are usually divided equally among the surviving children. The majority having chosen other occupations, will transfer their capital share of the inheritance to other sectors of the economy. The remaining son must start with a smaller stake and again accumulate capital and generate growth. A problem of farm operators in that case is that of how farm size expansion or growth should take place. Should it be through more land producing cash crops? Should the land be rented or owned? Or should it be through intensive livestock operations and the buildings and facilities to go with them? In either case, the question is what mix or combination of crops, livestock and financial activities should be used for this growth pattern.

3. APPLICATION OF A DYNAMIC MODEL OF FARM GROWTH IN NORTH IOWA

Earl O. Heady
Ronald C. Kay

3.1. Purpose

The purpose of this study is to develop and apply a growth model for private farms in the productive Clarion-Webster soil area of northern Iowa. Farm size expansion has been clouded in recent years by rapidly increasing prices of land, labor wage rates and capital interest rates. Emphasis is on the application and the results of the model, although a basic mathematical model was specified for purposes of implementing the model.

A dynamic linear programming model is used to analyze farm firm growth in north central Iowa. The model, covering a planning period of eight years, identifies strategies which contribute most to farm growth under different initial resource restrictions, resource prices and objective functions. The initial resource base includes 320 acres. Of the 320 acres, half of the land is owned but has a mortgage on it, and the other half is rented under a cash-share lease. In the latter case, the landlord receives a half share of the grain crops and pays half the cost of seed, fertilizers and pesticides. The young operator has an initial equity of $96,000 in land, buildings and machinery. Buildings include adequate grain storage and swine facilities consisting of an open front shed with a concrete finishing floor for 100-groups, portable swine buildings for the sow herd and for finishing 50 groups on pasture. Cattle facilities include a bunker silo, open front shed and a feedlot with fence-line bunks for 100 feeder cattle. Machinery, including that for cropping activities on the 320 acres and for all livestock processes is available and has a depreciated value of $32,000. While 2,800 hours of family labor is available, 500 are used for farm overhead and 2,300 hours are distributed seasonally over the year into four time periods: 680 hours for September - November, 616 for December - January, 680 for April - June, and 324 for July - August.

Given this initial endowment, the study attempts to determine the optimal path into the future through purchase or rental of land, addition of specialized crops or crop combinations or addition of livestock and livestock facilities. Comparison of growth paths are made for two levels of initial operating capital.

3.2. Nature of the Model

The growth model is specified within the framework of dynamic linear programming. It is dynamic in the sense that time explicitly enters the model with all technical coefficients, variables, and resource restrictions dated to identify the time period in which they occur. The objective function is optimized over the entire planning period and the optimum activity mix in an earlier period is interdependent with that of a later period and vice versa.

Computer time requirements were fairly large due to the large number of inter-year relationships in the basic model. These relationships cause changes in plans for both preceding and following years whenever there is a change in the activities entering the solution in any year, or a change in the scale of any activities. Adjusting the effects of a change in an activity with strong inter-year relationships, tracing out these effects and performing the necessary calculations, consume a large amount of computer time. Each iteration provides only a small amount of progress toward an optimal solution.

The coefficient matrix for the basic model is of the block form shown in Figure 1. Coefficients for each individual year are represented by the larger blocks labeled a_{ij}^t (t=1,...,8) and can be seen to overlap in equations and variables with the coefficient matrix for the preceding and following years. This overlapping results from the inter-year relationships of the model.

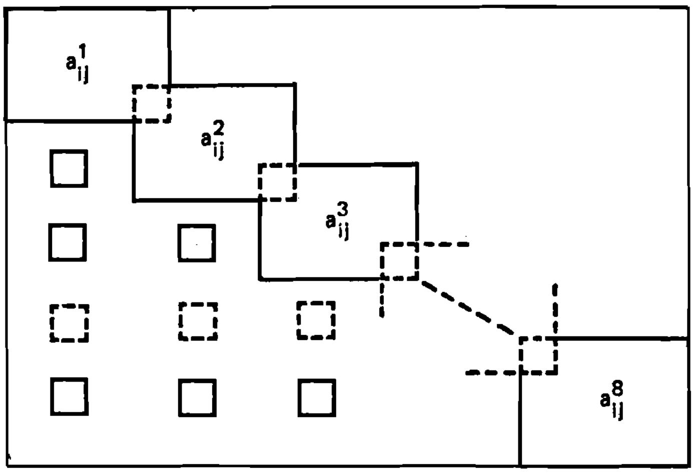

Figure 1. Block diagram of coefficient matrix.

Overlapping equations between adjoining years are needed by production activities in year t which produce intermediate products used by a production activity in year t + 1. This occurs, for example, in the corn and soybean production activities which produce a standing crop in the field during the year of planting. In turn, it is used as an input for the corn or soybean harvesting activity in the following year. The crop activities also produce crop stubble in each year which is used during the following year as a land restriction for the various crop activities, or by pigs farrowed in spring which are fattened in the fall. Land upon which corn is grown in one year will provide soil of a different productivity in the next year than will land upon which legume hay was grown. The overlapping equations transfer the inventory of all physical resources of one year to the next year.

Columns overlap between adjoining years to allow transfer of resources between years. Capital transfer between years is an example. The basic model also has overlapping columns for activities which add new physical productive assets to the resource restrictions. The smaller blocks in Figure 1 represent this type of column overlapping. Activities in them add to the resource restrictions and to fixed costs in all following years because of the depreciation, taxes and insurance payments which become annual obligations. Tax liability is also affected in all years following the purchase of a physical productive asset because the above items are also deductions from gross income when computing tax payments.

The programming tableau has 72 activities and 61 restrictions or accounting rows for each year in the model. Activities include 48 production, 16 resource acquiring, and 8 financial activities per year. Restrictions for each year consist of 33 physical production restrictions and 8 financial restrictions and financial accounting rows. The model's eight-year tableau contains 344 rows and 570 columns.

Each time period in the model runs from September 1 through August 31. This time period definition allows fall tillage practices, which are desirable (for the crop planted in the following spring), to lie within the same time period as the spring tillage and planting operations. Because of the relationships between the preceding and current crop, different types of fall tillage may or may not be necessary. The periods used cause the corn and soybean harvest to fall in one time period after planting, both crops are thus sold within the same harvest period.

3.2.1. *Crop activities*

Crop activities, defined on a two-year basis, and grown in year t, are related to the crop grown in year t-1 through effects on yield, fertilizer rates and carryover, amount and kind of chemicals used, and types of tillage practices needed. Fall plowing in one period is desirable for corn harvested in the next period because of its yield effect. It is generally possible to fall plow soybean, oat, and hay ground in year t in preparation for corn in t+1. The division of variable costs and crop yield between the tenant and landlord means that activities have to be specified separately for rented land and for owned land.

Crop harvesting activities are corn, soybeans, and corn silage which are planted in period t are harvested in t+1. The hay harvesting activity takes place in the same year in which the crop is grown, but the hay harvested is not considered to be available until the following time period. Different corn and soybean harvesting activities are included in owned land or rented land.

3.2.2. *Livestock activities*

A total of 21 livestock activities is included for each year in the basic model. Eleven are beef feeding and ten are various swine technologies. The three basic cattle feeding activities are good to choice steer calves, good to choice heifer calves, and good to choice yearling steers. Each of these three cattle feeding activities is considered at three different levels of technology.

The first beef feeding technology is the open feedlot type with a bunker silo for silage storage, a concrete feeding slab and fence-line bunks with feeding done by a tractor-pulled auger wagon. The second technology is similar, except silage storage is in upright silos and a fully automated feeding system is used. The third technology is capital-intensive consisting of a totally enclosed, building with environmental control and an automatic feeding system.

One swine technology is a single litter per year system wherein farrowing and raising take place entirely on pasture. The remaining swine farrowing and raising activities assume farrowing to take place in a central farrowing house but allow for different (a) combinations of litters per year, (b) timing of the farrowings, and (c) raising and fattening the hogs either on pasture, in an open front feeding shed, or in a totally enclosed finishing building. A feeder pig feeding activity is included as a possibility for use of the open front shed and totally enclosed building.

3.2.3. *Expansion activities*

The farmer can acquire additional physical resources necessary for growth through the model's expansion activities. A perfectly elastic supply curve is assumed for resources acquired. Additional land can be obtained by renting under a crop share lease or by purchasing with a mortgage or contract. Land purchased under a thirty-year equal payment mortgage requires a one-third down payment with an interest rate of eight percent a year. That, acquired with a land purchase contract requires a 15 percent down payment under a twenty-year equal principal payment contract with interest at 7 1/2 percent. The purchase or rental of additional land requires the purchase of additional machinery with a machinery capital investment of $130 per acre rented or purchased.

Additional labor can be hired on a yearly or seasonal basis. Seasonal labor can be hired separately in spring and fall periods. Full-time labor must be hired on a proportional basis in each of the four labor periods. Other expansion activities allow purchasing physical facilities and machinery if needed, to expand beef feeding and swine activities under any technology.

3.2.4. *Financial activities*

Six financial activities are included for each year. A capital borrowing activity allows, subject to a borrowing restriction, capital acquisition. The borrowing restriction is one dollar for each two dollar equity in land, buildings, machinery and swine. Borrowing of 100 percent is allowed in financing feeder cattle, a dollar borrowing restriction for each four dollar land purchase is used for land being purchased with a land purchase contract. A loan repayment activity forces operating loans to be repaid each year but allows immediate refinancing if sufficient borrowing capacity is available. A capital transfer activity transfers unused capital in one time period to the following time period.

An elementary family consumption function, C = a+bY, is included in the model with a minimum consumption of $7,566 (equal to a) required each year and a marginal consumption of 25 percent (b) of any additional net return above $7,566. A fixed cost activity pays fixed costs which accumulate in a fixed cost accounting row. An equality placed on this accounting row forces the payment of fixed costs each year. As additional fixed costs are incurred through the purchase of land, machinery or livestock facilities, the appropriate depreciation, taxes and insurance are added to the fixed costs which must be paid each year.

All monetary values in this manuscript are in terms of 1977 dollars. An alternative would have been to project alternative rates of inflation and maximize the ending period capital value.

Another drain on the capital flow of the business is Income and social security taxes withdrawn through a tax payment activity represent another drain on the capital flow of the business is An accounting row maintains a record of tax liability for each year. Appropriate adjustments are made in this row for taxable income as well as tax deductible operating expenses and depreciation. An equality in the tax liability accounting row forces payment of a combined income and social security tax each year after personal exemptions and a standard deduction.

3.2.5. *Relative prices*

Relative prices used in this study for inputs used and commodities produced are at a "real level" equal to the average for the previous ten years (but adjusted to a 1977 dollar value). Hog prices were computed on a monthly basis because of the wide variation in selling dates for the various swine enterprises. Prices for feeder cattle and corn purchased were computed on the same basis as commodity prices.

From the basic model described above, the variations explained earlier were made to provide alternative models for analyzing the effects of different resource prices and objective functions. The basic model is solved when larger (Model I) and smaller (Model II) amounts of capital are available. Also solutions were made when interest rates and labor wages were increased by 50 percent. The latter two solutions are not summarized in detail.

3.3. Model I results (higher capital)

The objective function used for Model I is the maximization of discounted net returns at a rate of 8 percent over the eight-year span. The objective function value, the total discounted net return for all years, is $407,819. Depreciation, taxes and insurance on physical assets are endogenously subtracted. Family consumption and tax payments are subtracted from the discounted income to indicate capital available for growth.

3.3.1. *Crop activities*

The model has complete freedom to develop a cropping sequence over the eight years. The optimum solution to Model I approximately selects a corn-soybean rotation on both owned and rented land (see Table 1). On the owned land, corn after soybeans and soybeans after corn are the two principal crop activities. In the first year, 10 acres of corn are raised after oats, and 14 acres of corn are raised after one year of hay because the amount of oat stubble and hay land specified at the beginning of the model is higher than hay requirements. Neither of these two crop activities is used again in any subsequent year. From year two through year eight, all corn is raised following soybeans or two years of hay (with the exception of year eight when there is some second-year corn). Oats on owned land are raised only as needed to establish new pasture. Owned land is used for pasture only as necessary for certain swine activities. The cropping pattern on rented land is principally corn and soybean alternatively. About 10 percent of rented land is in oats each year followed by corn or oats in the following year.

Low labor requirements explain the small amount of oats on rented land. Oats have a low spring labor requirement relative to corn and soybeans and no labor requirement for the fall time period. Oat harvest and straw baling have labor requirements in July-August when the corn and soybean crops have a very low labor requirement. Oats do not compete with corn and soybeans for fall labor and provide relatively little competition for spring labor.

Table 2 shows a marked shift to corn production in the eighth or last year. This change from a nearly equal emphasis on corn and soybean production in earlier years is caused in part by the need to set up the desired crop stubble in year seven and thus allow increased emphasis on corn production in year eight without going to a large amount of second-year corn. A type of end bias or "tail-end" problem is expected in a model of the type used, since it has a finite

Table 1. Summary of crop activities in Model I (acres).

	Owned Land								
Year	Corn after corn	Corn after soy-beans	Corn after oats	Corn after hay 1	Corn after hay 2	Soy-beans after corn	Oats after soy-beans	1st year hay	2nd year hay
1	0	46	10	14	0	76	0	62	2
2	0	70	0	0	2	69	6	0	6
3	0	68	0	0	6	71	1	6	0
4	0	63	0	0	0	74	8	1	6
5	0	72	0	0	6	63	2	8	1
6	0	54	0	0	1	78	10	2	8
7	0	77	0	0	8	55	2	10	2
8	60	55	0	0	2	24	0	2	10

	Rented Land						
Year	Corn after corn	Corn after soy-beans	Corn after oats	Soy-beans after corn	Soy-beans after oats	Oats after corn	Oats after soy-beans
1	14	171	44	148	0	49	0
2	0	121	52	241	0	0	37
3	0	217	0	194	41	0	41
4	0	226	49	253	0	0	49
5	0	210	0	282	53	0	57
6	0	314	69	266	0	0	66
7	0	199	45	408	26	0	87
8	0	514	108	343	0	0	0

planning horizon.

3.3.2. *Livestock activities*

Livestock activities entering the final solution for Model I are, after the first year, swine farrowing and raising only (Table 3). During the first year, livestock activities are restricted by the existing facilities. The structure of the model dictates that investment in additional livestock facilities in year one makes these facilities available only in year two.

Table 2. Summary of crops grown in Model I (acres).

	Owned Land				Rented Land				
Year	Corn	Soybeans	Oats	Forage	Corn	Soybeans	Oats	Total corn	Total soybeans
1	69	76	0	7	229	148	49	298	224
2	71	69	6	6	173	241	37	244	310
3	74	71	1	6	217	235	41	290	306
4	64	72	8	7	264	253	49	328	327
5	78	63	2	9	210	334	57	288	398
6	55	78	10	10	383	265	66	438	344
7	84	55	2	11	244	234	87	328	488
8	117	24	0	11	622	343	0	738	377

Table 3. Summary of livestock activities in Model I.

	Swine (litters)					Beef (head)
Year	Activity 1	Activity 3	Activity 4	Activity 6	Total litters	swine 1
1	0	40	10	10	60	45
2	70	40	0	0	110	0
3	88	40	0	0	228	0
4	107	40	0	0	147	0
5	135	40	0	0	175	0
6	150	40	0	0	190	0
7	187	40	0	0	227	0
8	187	40	0	0	227	0

In year one, 40 litters are raised in swine activity three, a four-litter system. Farrowing takes place in a central farrowing house and the hogs are raised in an open front shed with concrete feeding floor. These 40 litters utilize the farm's total capacity of the raising and fattening facilities. Another 10 litters are raised under swine activity four, a two-litter system with farrowing in the winter and summer utilizing portable buildings on pasture for both farrowing and raising. As a complement to this activity, another 10 litters are raised utilizing swine activity six, a two-litter system with spring and fall farrowing and both farrowing and raising taking place in portable buildings on pasture. Sixty litters are farrowed in year one for a total production of 450 market hogs.

The relatively high labor requirements of pasture farrowing and raising of hogs causes these activities to leave the solution after year one. During year one, the central farrowing house capacity is expanded and purchase of confinement hog raising facilities is begun. The annual production of 40 litters under swine activity three is continued and an additional 70 litters are farrowed utilizing swine activity one. These litters are farrowed in the central farrowing house and raised in the total confinement facility. After year two, the growth in the swine activities takes place entirely through expansion in the number of litters farrowed under swine activity one while maintaining forty litters per year from swine activity number three.

Shadow prices for swine indicate that activity two causes the least reduction in net return if forced into the optimum solution. This is the same swine raising activity as number one except it is a six-litter rather than a four-litter system. Because of the timing of the farrowing under the six-litter system (swine activity two) it has a higher per litter labor requirement during the fall and spring, when crop activities compete heavily for available labor. Shadow prices for swine activity two range from $11.28 to $28.71 per year. Feeder pig activities have higher shadow prices than the farrowing activities, indicating that buying and feeding feeder pigs does not compare favorably with farrowing and feeding, provided farrowing facilities are available or can be constructed.

The only cattle feeding activity entering the solution is beef activity one in year one. 45 steer calves are fed using an open front shed for shelter, a concrete slab with fence-line feed bunks and a tractor-pulled auger wagon for feeding. A feedlot of this type with a hundred head capacity initially exists on the farm but is used at less than 50 percent of capacity in the first year. It remains idle and goes into disinvestment in all following years. When compared with the swine activities, cattle feeding activities have a relatively large capital requirement per dollar of net return generated. With capital being the ultimate limiting growth factor in this model and the objective function being the growth of discounted net returns, cattle feeding does not compete favorably.

3.3.3. *Labor activities*

As shown in Table 4, 396 hours of full-time equivalent labor is hired in year one. Labor use increases each year thereafter up to 3,630 hours of full-time equivalent hired in year eight. One full-time man is assumed to supply 2,800 hours of labor per year.

With the exception of fall in year one, the maximum allowable part-time labor in each of the fall and spring time periods is hired in each year. The crop activities have a high demand for labor in the fall and spring and the labor hiring restriction in these time periods is quickly reached because of the labor needed for crop activities. Once the limit for hiring part-time labor is reached, additional labor in the fall and spring time periods must be obtained through hiring of full-time labor.

Some labor remains unused during the winter period in each of the eight years while the total labor available during the fall and spring periods is completely used. Labor available during the summer period of July and August is completely utilized in every year except the eighth.

Shadow prices for labor are shown in the last part of Table 4. Spring labor is most restrictive while winter labor is not restrictive.

Table 4. Summary of labor activities in Model I.

Year	Labor hired (hrs) Full time (fall)	Part time (spring)	Part time (fall)	Shadow prices for labor ($hr.) Period 1 (winter)	Period 2 (spring)	Period 3 (summer)	Period 4
1	396	250	250	$4.71	0	$48.86	$4.12
2	581	250	250	$24.59	0	$26.67	$1.25
3	815	250	250	$21.60	0	$21.70	$4.29
4	1,190	250	250	$19.92	0	$19.14	$1.63
5	1,728	250	250	$16.75	0	$15.97	$3.87
6	2,158	250	250	$14.78	0	$15.27	$0.55
7	2,905	250	250	$10.74	0	$9.48	$3.36
8	3,630	250	250	$10.66	0	$11.85	$2.11

3.3.4. *Expansion activities*

Expansion activities allow an increase in the physical resources and include land renting, land purchase, machinery purchase, and the construction of physical facilities to increase swine and beef activities.

Table 5 indicates that land renting and purchase of machinery to farm additional acres are the only two crop expansion activities which occur. A total of 857 acres of land is rented during the eight years. Beginning with the 160 acres of rented land and the 160 acres owned by the operator, a total of 1,175 acres of land is being farmed at the end of the eight years. Since one unit of machinery is purchased for each additional acre of land, 857 machinery units are purchased during the eight years. This is an additional machinery investment of $105,073.

The shadow prices on land purchase activities show that a substantial reduction in the objective function value would result from the purchase of more land than indicated. This is particularly true in the early years of the model when the relatively large amounts of capital needed to purchase land and to meet the yearly mortgage or contract payments in subsequent years, would make this capital unavailable for operating purposes or other expansion activities.

Swine farrowing capacity increases in each of the first six years of the model. A total of 42 farrowing stalls are added to the fifteen existing at the beginning, making a total capacity of 57 sows at one time. This increase of 42 stalls required an investment of $39,501 over the six year period. Swine raising and fattening facilities are expanded entirely by building construction in the total confinement system used by swine activity one. The equivalent of a total confinement building with a capacity of 468 market hogs is constructed during the first six years of the model. This represents an additional investment of $24,549 in swine facilities.

Table 5. Summary of expansion activities in Model I.

Year	Land rental (acres)	Machinery purchase (acre units)	Swine farrowing capacity (stalls)	Swine technology four (litters)
1	289	289	12	174
2	26	26	5	46
3	45	45	5	47
4	78	78	7	71
5	36	36	4	38
6	119	119	9	92
7	53	53	0	0
8	211	211	0	0
Total	857	857	42	468

3.3.5. *Financial activities*

Model I places no explicit limit on the growth rate except that dictated by (a) the amount of capital available at the beginning of each year of the model and (b) the resulting rate at which capital can be generated within the model itself for investment purposes. For these reasons, the financial activities that are concerned with capital use, capital borrowing and the necessary withdrawals of capital from the income stream for other than investment purposes are of particular significance. Table 6 includes a summary of the financial activities for Model I.

The capital borrowed in any year is the total amount borrowed for all purposes other than the purchase of land. Capital borrowing increases from $53,355 in year one to $123,072 in year eight, an increase of 131 percent. Capital is borrowed in the maximum amount allowable by the model in each of the eight years.

There is no unused borrowing capacity any year and the shadow prices on the borrowing restriction indicate that additional borrowed capital could be used profitably in each year. The shadow prices on a dollar of borrowing capacity range from 38.2 cents in year two down to 16.6 cents in year eight.

The second column of Table 6 indicates the amount of capital withdrawn from the income stream each year to meet fixed costs such as mortgage payments on the land being purchased, taxes and insurance on land, buildings and machinery, and a depreciation allowance on machinery and buildings. Interest on the capital borrowed shown in column one is not included in fixed costs. Fixed costs increase as additional physical assets are purchased to add to the production capacity of the farm firm. The amount of capital which must be withdrawn from net income each year in order to meet fixed costs increases from $20,257 in year one to $35,840 in year eight. Funds for consumption by the family is another drain on the capital flow.

The model allcws for any capital not needed in investment or production to be invested in a savings account. This activity does not enter into the solution for any year.

Table 6. Summary of financial activities in Model I.

Year	Capital borrowed	Family consumption
1	$55,355	$8,812
2	$61,272	$15,449
3	$69,151	$15,533
4	$71,234	$16,478
5	$89,398	$17,910
6	$101,132	$17,909
7	$114,640	$20,335
8	$128,072	$14,290

3.3.6. *General analysis*

The Model I solution shows that farm firm growth can be accomplished rather rapidly from internally generated and borrowed capital, given the assumptions of this model and the resources available at the outset. Undiscounted net income for each year is presented in column one of Table 7. Net income in year one is the lowest of any year, since gross income in year one is limited (a) to the amount received from the sale of crops assumed to be standing in the field at the beginning year of the model and (b) the income from livestock production activities which are restricted by the buildings in place at the beginning of year one. Net income reaches a maximum of $90,759 in year seven and dips in year eight due to the terminal nature of the model. (With a ninth year added, income would progress to $105,942 in year eight).

Another indicator of volume increase is annual cash flow. Cash flow, the total number of dollars flowing into the business from all sources, includes gross income and capital borrowing. A common indicator of farm size or farm growth is total acres farmed or the increase in acres farmed. The total acres farmed in the model increase from 320 acres to 1,175 in year eight. Hog production increases from 450 market hogs in year one to 1,703 in each of the last two years.

Because of the emphasis on land rented, an indicator of growth in the physical resources used for production activities is the value of resources controlled (Table 7). While the farm operator controls $735,360 of physical resources (land, buildings and machinery used in the production processes) at the beginning of the model, this increases to $2,814,700 in the eighth year.

Table 8 shows estimated marginal rate of return (shadow prices) on capital in each year. The marginal rate of return is much higher than interest rates in all years. Hence, if there were not capital restraints on borrowing, the firm could profitably use more capital. Investment capital has a lower rate of return than operating capital in all years except year one.

3.4. Model II Results (Less Capital)

Model I assumed a relatively large amount of initial capital. In addition to the $9,450 in cash, a crop in the field had a net value of $34,332 and there was an uncommitted $22,680 in borrowing capacity. To determine effects of a more

Table 7. Summary of growth indicators in Model I.

Year	Net income[a]	Total acres farmed	Market hogs sold	Value of resources owned[b]	Value of resources controlled[c]
0	[d]	320	[d]	$383,360	$735,360
1	$32,532	609	450	$450,249	$1,438,147
2	$61,364	635	822	$462,718	$1,509,285
3	$63,110	679	959	$477,799	$1,619,697
4	$68,939	757	1,100	$501,987	$1,814,975
5	$76,219	793	1,313	$514,180	$1,905,892
6	$77,332	912	1,427	$528,210	$2,208,289
7	$90,759	965	1,703	$554,557	$2,335,060
8	$72,970	1,175	1,703	$581,406	$2,814,700

[a] Undiscounted net income before any fixed costs have been deducted.

[b] Amount of mortgage on land being purchased has not been deducted.

[c] Includes both owned and rented resources.

[d] Not measured in model.

Table 8. Estimated marginal rate of return on capital, Model I (undiscounted values).

Year	Total capital (%)	Operating capital (%)	Investment capital (%)
1	17.9	22.6	26.7
2	27.4	31.9	25.3
3	26.2	30.0	24.0
4	24.6	27.6	22.8
5	23.3	25.6	21.7
6	22.1	23.8	20.5
7	20.4	21.4	19.3
8	19.3	19.3	a

restrictive initial capital allowance, Model II was developed with restrictions and coefficients the same as Model I with the exception of initial capital. Model II has $20,000 less capital than Model I, with no cash available, a $15,000, debt and a zero borrowing restriction. This change causes the value of the objective function to decline from $407,819 for Model I to $335,862 for Model II over the eight year period.

3.4.1. *Crop and livestock activities*

The number of acres farmed is reduced under Model II as initial capital becomes more restricting. Model I has 49 acres of oats in year one but Model II has none. With the exception of the first two years, however, there is very little change in the proportion of corn, soybeans and oats raised.

From years four through eight of Model II, the maximum allowable part-time labor is hired during the fall and spring time periods. The cropping pattern then becomes similar to that of Model I even though fewer acres are cropped.

Swine farrowing and raising activities in each year are similar for both models. Both have forty litters in a four-litter system utilizing a central farrowing house and an open front shed with cement feeding floor for finishing hogs. Both models have 10 litters raised in portable buildings for farrowing and finishing in a two-litter, winter-summer farrowing system. However, Model I utilizes the portable buildings on pasture to the maximum allowable by farrowing restrictions for the fall- spring, two-litter system, while Model II, with less capital, raises only one litter under this system. From years two through eight, both models fully utilize, by farrowing and raising forty litters, the open front finishing shed. Neither utilizes the pasture systems after the first year. The expansion in swine farrowing and raising which takes place in Model I is limited to a four-litter system utilizing a central farrowing house and a total confinement finishing building. Model II utilizes a slightly different growth path for swine activities, switching from a four-litter system to a six-litter system in years four to five, but returns to the four-litter system in year six. The number of litters farrowed and raised in the eigth year declines from 227 in Model I to 142 in Model II. It reaches a point in years four and five where operating capital is available to expand the swine production but there is insufficient investment capital for expanding farrowing facilities. Under these circumstances, there is a temporary shift to the six-litter system to allow more complete utilization of farrowing capacity. The shadow prices on swine farrowing and raising facilities in Model II are higher than in Model I until year six, when they become approximately equal thereafter. The only beef feeding activity in either model is steer calves fed in an open feedlot. This activity is used in year one for 45 of steers in Model I and for 12 in Model II.

3.4.2. *Labor activities*

With capital availability restricted in Model II, several changes take place in labor hiring and utilization. Model I hires full-time labor in year one and increases its amount each year, reaching a maximum of 3,630 hours in the eighth year. Model II does not hire any full-time labor until year five. By the fourth year, Model II specifies hiring the maximum allowable amount of part-time labor in both fall and spring and, with the exception of year five, continues to do so in all the following years. Not only does Model II use less full-time and part-time labor than Model I, it also has some unused operator labor in the early years.

3.4.3. *Expansion activities*

Table 9 compares expansion activities for solutions to Model I and Model II. They are the same but generally on a smaller scale in Model II, with some difference in their timing and relative emphasis. Model I has 289 added rented acres in year one with the necessary machinery purchased to farm them. With less capital available at the outset, Model II has only 61 additional rented acres in the first year. In the second year, Model I has 26 acres of added land rented with investment going into hogs, while Model II has 37 acres as it internally generates additional capital. Physical facilities for farrowing and raising swine show the expansion differences, indicated in Table 9.

Table 9. Comparison of expansion activities for Model I (upper figure in each cell) with Model II (lower figure in each cell).

Year	Additional rented (acres)	Machinery purchase (acre units)	Swine farrowing capacity (stalls)	Swine technology 4 (head capacity)
1	289	289	12	174
61		61	0	50
2	26		26	5
37	37	0	0	
3	45	45	5	47
92	92	2	52	
4	78		78	7
59		59	3	41
5	36	36	4	38
66	66	7	25	
6	119	119	9	
67	67	8	83	
7	53	53	0	0
	0	0	1	4
8	211	211	0	0
126		126	0	0
Total	857		857	42
	508	508	21	255

Reduced capital availability in year one causes corresponding reductions in levels of the various financial activities entering the solution of Model II. Model II is not able to acquire assets and expand borrowing to the levels of Model I. After year one, however, the difference in borrowing declines continuously. The major effect of reduced capital available in year one under Model II is to reduce the scale of production activities throughout the eight years. Table 10 provides a comparison of selected growth indicators for Model I and Model II. Model I uses 1,175 acres in year eight while Model II uses only 827. Market hogs sold in the eighth year are 1,703 in Model I as compared to 1,067, 37.3 percent fewer, in Model II. These figures again indicate some relative advantage in expanding production, over time through land renting relative to expansion of hog production, as capital is the more limiting resource. All financial and scale characteristics are lower in year eight. With another 10 years added to the time span of the models, however, Model II would have approximately the same ending values.

Table 10. Comparison of some growth indicators for Model I and Model II.

Growth indicators	Model I	Model II
Objective function (discounted net return for eight years)	$407,819	$335,862
Acres farmed in year eight	1,175	827
Market hogs sold in year eight	1,703	1,067
Value of assets owned in year eight	581,406	450,141
Value of assets controlled in year eight	2,814,700	193,861

3.5. General Discussion

The basic dynamic programming model when applied, supposes that prices and production functions are known with certainty in each year of the planning span. Generally, this certainty is not true, and the challenge in planning over-time is that of selecting decision processes which are consistent with both the degree of uncertainty involved and the decision-makers degree of risk aversion. If these facets were incorporated into a decision process involving time, the activities selected and the magnitude of farrowings and investment could differ considerably from those indicated in this paper. Similarly, if we knew the utility function of the farmer, an objective function of utility maximization (comparing the disutility of risk against the utility of money income) could be preferable to one of income maximization over time.

It is possible, however, that a farmer in a safe capital position, in lack of other knowledge, might make investment plans extending into the future based on some concept of average parameter values in each year. The model used here would best conform with his situation. However the farmer with a weak capital position, who might be bankrupt in case of large farrowings and a crop failure in a single year, might want to consider the distribution of outcomes if possible in each year and vary his production and investment mixes accordingly.

The objective function used in the basic model maximizes the present value of the stream of income over 8 years. Constant commodity and resource prices are used. In recent years land and other prices have moved up rather sharply under inflation. Under these conditions a model incorporating inflation and maximizing value of assets at the end of the period might be more appropriate. This formulation would emphasize buying rather than renting land.

4. MODEL FOR THE DEVELOPMENT OF A LARGE-SCALE AGRO-INDUSTRIAL COMPLEX

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Preface

At the end of 1976, a research group from the Institute for Social Management (Sofia, Bulgaria) was given the task of developing a mathematical model in order to define the optimal production structure of the "DRUSTAR" agro-industrial complex, in the Silistra Region of Bulgaria. This large and complex task could not be solved by traditional planning methods. Therefore, a mathematical modeling approach was applied.

A production program of the "DRUSTAR" agro-industrial complex was developed before 1976 by the firm "Globe Servicing International" in Chicago, based on a preliminary fixed structure of animal husbandry. Our group was faced with another task:to search for an optimal structure of animal husbandry and to elaborate ways of implementing this optimal structure given the available land resources.

At the beginning of 1977 we were visited by experts of IIASA, Dr. C. Csaki and Dr. A. Propoi, who were acquainted with the static model. At that time it was decided that a dynamic linear programming model should be built where the overall development of the complex could be traced over a long time period.

In March 1977 the research group presented their proposal for the model to IIASA. During the ensuing discussions scientists at IIASA made recommendations for refining the model and for further research.

After studying a new version of the dynamic linear programming model, and conferring with IIASA scientists in May 1977 in Sofia, a final model outline was developed. This version of the model was presented to IIASA in the fall of 1977 and was accepted as a basis for further implementation.

When applying the model to the problem of developing the agro-industrial complex, 60 alternatives were discussed with experts, technologists and leaders at different management levels (policy makers). On the basis of these discussions the model was continuously improved.

It was thus possible to use the results of the model runs as a basis for making decisions concerning the production structure of the "DRUSTAR" agro-industrial complex.

In this paper the results of this modeling exercise are discussed. First the mathematical model is briefly outlined, and then the major conclusions made on the basis of the model runs are presented. The working group in charge of the modeling project is grateful for the comments of experts at IIASA and the valuable contribution of scientists and experts of various Bulgarian institutions.

4. MODEL FOR THE DEVELOPMENT OF A LARGE-SCALE AGRO-INDUSTRIAL COMPLEX

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4.1. Agro-Industrial Complexes in Bulgaria

Since 1970 more than 93% of the arable land in the People's Republic of Bulgaria has become part of an agro-industrial complex (AIC). Nearly 160 AICs have been established with an average of 24,000 ha of arable land. The average land size of one complex varies from 3000 to 48000 ha depending on the available natural resources, climatic conditions, and the production structure of the particular region.

Large-scale agro-industrial complexes were needed for the following reasons:

(i) to accelerate the scientific and technical revolution by organizing agricultural production on an industrial basis.

(ii) to strengthen the links between agricultural firms in order to integrate the agro-food industry

(iii) to solve the social and economic problems of rural areas with greater speed.

In a specific region, an agro-industrial complex also includes all enterprises of the fodder and food industries as well as related enterprises, such as those dealing with the storing and distribution of chemicals (agro-chemical centers), the maintenance and repair of machinery and trucks, transportation and construction.

The basic organizational structure of an AIC is the land unit. The enterprises or land units of an AIC are not legally independent but rather, have their own management and accounts. The general investment policy, the allocation of different production funds and overheads, the marketing and income distribution are carried out by the management of the AIC. On the basis of targets given for the production of basic products, determined by central planners of the Ministry of Agriculture and Food Industry, the management of a complex develops the annual plans for farming.

The subject of this study is a large agro-industrial complex specialized in grain production and animal husbandry. The aim of this study is to optimize the production structure and to find ways of achieving this optimal structure. In solving such a complex task, different evaluation criteria are used for determining the best planning alternatives, while taking all the existing conditions into consideration.

There are basic steps to be taken in order to study alternative plans for an AIC:

- To define indicators for the evaluation of planning alternatives
- To take into consideration all conditions connected with agricultural production; analysis of climatic conditions, the degree of initial development of the complex, present and expected input availability of labor and other resources, trends in related scientific technological fields, trends in demand and prices, social and political conditions, etc.
- To choose the relevant mathematical methods and models as well as computers and programs.

Static models are not sophisticated enough to determine the optimal production structure and the means to achieve it. Preliminary studies have proven that dynamic linear programming is a more adequate method for solving this type of problem.

An important methodological question is to determine the time measurements (that is, the number of years at each time period) and the length of the planning horizon. In order to achieve a high degree of specialization in a given complex it is necessary to study the possibilities and effects of changes in the structure and in the technology of production over time. Some features of maintenance, large capital investments (e.g. assembly factories) and other factors do not allow for quick changes in the production structure.

Thus the modeling period (planning horizon) should cover 15 to 20 years. For agricultural production a period of one year is a suitable duration of one time increment. After 5 to 7 years, uncertainty about the information used in the model is greater. The duration of individual time periods in this case can be extended to 2, 3, or even 5 years. On the other hand, the capacity of the computers limits the model size, and it is time consuming to find a solution over a greater number of time periods. Considering the different economical, technical, and data requirements, the most suitable period for the investigation is about 15 years, and the time increment for the first five years is one year and for the remaining period 2 years.

When applying a mathematical model to the problem of developing production in agro-industrial complexes, this study included the following steps:

- to formulate the task and coordinate it with the regional and national authorities
- to determine the scope of the project; which activities have to be studied, which are to be included in the model with fixed dimensions, which are to be excluded; to include activities related to food processing industries (fodder processing, soybean, hemp, milk, meat, sugar, etc.)
- to coordinate with the authorities and specify criteria for the evaluation of the various alternatives, and to formulate the objective function
- to determine the planning horizon and time increments
- to clarify the conditions and resources (biological, physiological, agrarian, climatic, economic, etc.) with experts and decision makers
- to gather information on the technical and economic characteristics of different technologies in a form required by the model
- to construct the linear dynamic model from actual data, try and formulate the problem, and to analyze the validity of the model

- to do the run model
- to discuss the alternatives with regional and with state decision makers on various levels
- to make further runs of the model on the basis of these discussions
- to describe and analyze the results in detail which are then presented to the authorities responsible for the complex (Ministry of Agriculture and Food Industry, State Planning Committee, etc.)

4.2. Structure of a Linear Dynamic Model for the Development of the "DRUSTAR" Agro-Industrial Complex.

The model contains six relatively independent blocks which are interrelated. The general assumptions in the formulation of the model are as follows:

- Investment in irrigation is not modeled: the irrigation facilities available are given exogenously
- Local food demands must be satisfied by the complex (self sufficiency within the AIC)
- All processing industries in the district are owned by the complex. There is an economic mechanism which stimulates the separate subsystems of the complex towards maximization of the net income
- Modern technologies are used in the various activities of the complex (cropping, fodder industry, livestock-breeding, etc.)
- There is an unlimited market for the products of the complex

The model presented in section 1 of this volume was used to depict the following activities. In order to simplify this presentation the interested reader is referred to the relevant equations in section 1.

4.2.1. *Land utilization block*

The land utilization block of the model describes the land use patterns and the utilization of commodities produced by field crops. In the model all crops are considered which can be grown in the complex. Table 1 lists the corresponding commodities.

Table 1. Crops included in the model

1. Wheat
2. Barley
3. Sunflower
4. Soybean
5. Corn
6. Fodder Maize
7. Hemp
8. Beans
9. Tobacco
10. Alfalfa for hay
11. Alfalfa for flour
12. Alfalfa for consumption
13. Alfalfa for Silage
14. Alfalfa for seed

Crop production variables are given according to irrigated or non-irrigated production, assuming the use of certain technologies. Several possible technological options are considered. In the model a variable is given for silage maize as a second crop after wheat or barley.

The first group of constraints are related to the available land resources.

(i) the total available arable land

(ii) the available irrigated land

These are given as separate constraints assuming that the total arable land is fully used.

The second group of constraints describes land utilization patterns and crop rotation.

(i) crop rotation on irrigated or non-irrigated land

(ii) the size of the silage maize as a second crop

Special constraints describe the dynamics of the land used for alfalfa. The second group of equations describes the product utilization of crop production assuming that corn and wheat are sold by the complex, and sunflower and soybean can either be bought or sold. Separate balances are given for feeds.

4.2.2. *Modeling the development of livestock*

In the livestock part of the model variables are given according to various types, strains and utilization groups of possible activities of livestock production.

Livestock variables are grouped according to type (i.e. pig, sheep, dairy cows, etc.) and age, both of which determine the kind and level of product as well as the technologies needed for feeding and raising the animals. Six types of livestock production are included in the model: dairy cows, beef cattle, pigs, milk sheep, sheep for meat, and poultry. There are three age groups: young livestock for slaughter, young livestock for breeding, and mature livestock.

The purchase and sale of livestock is done at the beginning of every time period. The initial values of all the variables are determined for the time period to, which is the year 1977 in our application.

The livestock block of the model is formulated according to the six types of livestock production. In the model, by using the standard solutions of already published multi-stage linear programming models, the dynamics of these livestock production fields are described as:

(i) the development of the basic herd

(ii) the utilization of young livestock

(iii) the constraints on livestock breeding

Special constraints express the size of the initial stock and the limits on selling and buying various types of animals.

4.2.3. *Livestock feed block*

Each type of livestock consumes different quantities and kinds of fodder which have been produced by various crops in the complex . These quantities are measured in kilograms or in feed units. The required quantity is determined by the relevant technologies.

The diet of an animal can be defined in either of two ways; as a set of feeds and their quantities (in kilos) or as quantities of feed concentrates, hay, and green forage characteristic of a given diet. The second is the more convenient as it allows for greater choice of a production program for crops.

The following variables are included in the model:

(i) Feed balances expressing the production and consumption of different types of feed (concentrates, silage, hay, and green forage) expressed in feed units: there is a minimum and maximum number of feed units which can be consumed as a given type of feed

(ii) Additives, such as salt and calcium and other components such as oats, which are not produced in the complex

(iii) The number of feed units contained in a specific feed, obtained from a given crop and cultivated by using a specific technology per hectare of irrigated or non-irrigated land

(iv) The number of feed units in a given feed, obtained for a crop in a specific time period which can be stored for consumption during the subsequent time periods

(v) A coefficient of the amount of the stored crop which can not be used as fodder due to losses in nutritional value and physical or chemical changes.

4.2.4. *Livestock production block*

The following types of livestock for slaughter were included:

calves

cows

pigs

sows

lambs

weaned lambs

sheep

broilers

hens

Balance equations are given according to the major products of the livestock sector. The quantity of meat obtained from one of these types of livestock during a given time period is included. A technical coefficient shows how many kilos of meat of a given type of livestock are necessary to produce one kilo of processed meat.

Balance equations for milk production and milk products include the following products:

milk

yogurt

cheese

yellow cheese

butter

4.2.5. *Production resources block*

In this section of the model, the physical resources and their utilization are described as generally formulated in linear dynamic models. The resources needed to cultivate the land, grow a crop, transport and store the products are included in the model as well as resources used in the fodder industry, in building livestock shelters, and in the processing industry. All of these resources represent constraints on the development of the complex. The purchase of new machinery is also included in the model.

4.2.6. *Financial block*

In this block the financial flows related to the physical resources are described according to the following major groups of equations:

(i) capital investment for maintaining or enlarging production capacities

(ii) cost of purchasing livestock and machinery

(iii) revenues from crops and livestock breeding (wool, eggs, hides, etc.)

(iv) profit, taking the material and labor costs into account.

4.3. Objective Function

The existing situation in an agro-industrial complex determines the content of an objective function.

The most suitable economic long-term indicator of development seemed to be the net income. Maximizing net income leads to an increase in production to an effective production structure where production costs are kept to a minimum. Other indicators could be used, however. In complexes and enterprises where production is more narrowly specialized, basic changes in the production structure are not likely. Therefore, net production can be used as an indicator.

The net income or (net production) over the whole modeling period is the sum of the net income of each period (1 or 2 years). For all loans (and savings investments) the complex pays (or receives) an interest and workers want to get higher wages. Therefore not only in the total net income (or net production) must be considered, but also the value of that indicator during each period. Therefore, when the objective function expresses a net income, net production or another indicator which is maximized, the value of the indicator for each period must be discounted towards the end of the first period. If the objective function is an indicator which has to be minimized (for example, minimization of the reduced production costs), the discounting should be made at the last period with an interest rate equal to the rate paid to the bank. This might speed up the investment procedures.

Using net income, the objective function is formulated as follows:

$$\sum_{t=1}^{T_D} \frac{D(t)}{1+\varepsilon^{t-1}} \rightarrow \text{maximum}$$

where

ε is the interest rate paid by the bank (discount coefficient)

T_D is equal to the number of time periods in the planning horizon

$D(t)$ is the net income

4.4. Model Computation

4.4.1. *The data base*

Many sources of data information were considered while building the data base for the model. Some of them were known before the research was begun (research institutes, planning and developing firms) and others were then included after work was underway. The following information sources were used:

(i) research on possible objective function for development problems

(ii) crop growing and stock-raising technologies, their technical, technological, and economic properties

(iii) technologies for the fodder, milk processing, and meat processing industries

(iv) preliminary designs for servicing activities, (agro-chemical servicing, maintenance centers, measures against hailstorms and erosion, purification installations, etc.)

(v) program for the development of the irrigation system

(vi) program for the development of wine growing, horticulture, and fruit growing

(vii) plan for the development of the tobacco, beans, and hemp cultivation

(viii) limits on the development of various products as defined by experts

(ix) actual results of activities of the complex in 1977 and the plan for 1978.

(x) restrictions on production as defined by experts, taking into consideration climatic, technical, marketing, agronomic, economic, social, political and other factors

(xi) market and prices forecasts

4.4.2. *Sensitivity and scenario analysis*

In the process of the computation many changes were made in the model described in section 4.2 due to missing data, the modification of technologies, and the new requirements made by the experts. Some peculiarities of the computer program also imposed further modifications on the model.

First, all the restrictions connected with livestock feeding were modified. Certain levels of feed transformation efficiency were considered in the model with a given rate of usage for maize, barley, wheat, soybean, sunflower and maize silage. Further calculations were done by changing these levels.

Most significant modifications were made with respect to the forage in livestock diets. The quantity of rough forage given in processed form for different livestock must have a certain nutritional value (measured in food units). It is not important which crop makes the fodder nourishing. Thus only one restriction is made in food units. Apart from reducing the size of the model this leads to a complex evaluation of forage use.

The limited number of the technologies considered (especially in livestock breeding) does not allow for comparisons.

Besides the above mentioned modifications, the following changes were also made.

(i) For estimation of the maximum labor intensive month, September was chosen as the peak period of agricultural activities. For this month a balance equation was formulated which could easily be converted into a constraint.

(ii) The development of the processing capacities for forage and concentrates was included in the model. The increase in meat and milk processing did not justify inclusion in the model because only one milk and two or three meat processing plants were necessary (the model is not integer). Furthermore, reliable data could not be ensured--the cost of processing plants greatly depends on their capacity and on their location. Therefore, the necessary capacities were determined by balance equations, and the choice of the type, size and the time of building the individual plants were effected later on.

(iii) Equations were included in the block of financial indicators to calculate the need for hard currency for investments and operational expenses.

(iv) Initially, the model was designed for a period of 15 years. However, this greatly enlarged the size of the model and involved considerable computational difficulties. Thus, it was decided that the first five periods would be of one year each and the other five periods of two years each. The matrix was thus reduced by 30%.

4.4.3. *Parameters of the model*

The implemented model included the following parameters:

(i) from 790 to 801 constraints with 4 objective functions, 500 to 513 equations, 250 and 260 upper bounds (<) and 25 to 30 lower bounds (>);

(ii) 1041 non-negative variables

(iii) two possible sets of right-hand sides

(iv) two vectors of upper and lower bounds with 36 and 113 elements, respectively

(v) vector with the description of the rows

(vi) vector with the names of the variables

The matrix contained a total of 9400 to 9500 coefficients and had a density of 1.13.

4.5. Results and their Implementation

4.5.1. *Main scenarios*

More than a hundred model runs concerning alternative paths of development of the complex were computed differing not only in the constraints and limitations, but also in the coefficients. Every modification of the model was made on the basis of expert judgement by managers or technologists.

The large number of results can be divided for the most part chronologically into several groups:

(i) model validation

(ii) basic scenarios

(iii) scenarios exploring the limits to development

Although the first two groups of solutions cannot be considered final from a methodological point of view, they are of paramount importance in understanding the development processes of this agro-industrial complex.

The first group of model runs are connected to a great extent with the process of model validation. The large number of variables in the model led to a considerable increase in the time required for a computer run. For this reason, the problem of decreasing its size and aggregating some of the variables was seriously discussed. Subsequent modifications were made due to this aggregation process and to the lack of data.

As soon as the analysis and initial modifications were completed, solutions were computed, some of which were discussed in detail with a wide range of experts and technologists. A consensus was reached between the results of the computer runs and the common sense, intuition and professional knowledge of the experts. The second group of model runs were connected with a detailed clarification of the complex's development. In the first version of the model there were no major constraints with regard to the scale of development of the individual activities in the complex: this was because the most efficient sub-industries were to be identified.

The first model runs pointed out that the most effective branch of the livestock breeding is pig breeding. Furthermore, the development of pig breeding must be implemented on a large scale, and the overall development of the complex must be subordinated to this activity. The main discussions were concerning the scale of pig breeding, the necessary conditions (natural, production, sanitary, ecological, etc.) and the eventual possibilities of restricting this breeding. As a result of the discussions, it was concluded that pig breeding must be restricted and an upper limit of the number of fattening pigs and sows was determined. Without ignoring the most important activity of development, livestock breeding, all the remaining constraints were taken into consideration and the model was used to obtain much more realistic solutions.

As expected, the analysis proved that one of the most important constraints on development was the amount of arable land. Unfortunately, this constraint could not be related. However, particular importance was placed on the way in which the available land was to be used.

A complex estimation of the individual technologies used in livestock breeding disclosed a very serious problem. In some cases the feed technologies were not really suitable for the real conditions of the complex, which the model proved, because too high a share of grain led to low utilization of a huge mass of forage. First, an attempt was made to solve this problem by introducing an additional constraint on the use of forage. However, an analysis of the ensuing model runs showed that this was not the right way to solve the problem. The formal requirements for the use of not less than 50%, 70%, or 80% of the available forage led to changes in the logical structure of the solutions and therefore to unrealistic solutions. New technologies were proposed for livestock breeding which allowed the fuller use of forage. Practically all the technologies for cattle breeding and sheep breeding were changed, which led to new model runs and therefore new results.

Another important problem was the sensitivity of the solutions to the possibility of buying young animals (calves or lambs) for fattening. After detailed discussions with experts it was agreed that the "DRUSTAR" complex was not suitable for developing farms for fattening lambs purchased from other districts. It was concluded that lambs must be purchased at international prices, irrespective of the place they were bought.*

An alternative was to purchase lambs for fattening on the home market. As soon as an agreement was reached on the number of lambs to be purchased, particular attention was paid to sensitivity analysis. The sensitivity of the solutions was investigated for changing assumptions on the availability of capital, changing purchase prices in the crop growing and livestock breeding fields, changing the production structure to increase the production of grain in order to provide a livelihood for the population. We shall pay special attention to the first problem.

As a result of the first investigations, only a relatively low amount of the available capital funds was scheduled annually for building and assembly work within the complex. This proved to be a major constraint in the development of the complex. Over the 15 year modeling period its full potential could not be developed. For this reason a decision was made to increase the availability of

* All purchases and sales connected with the international market were calculated in the model according to international prices. Feed was given at net cost in the model because it was assumed that the needed feed would be produced within the complex. Feed additives were also calculated according to international prices.

capital for constructing physical capacities and to look at scenarios of development which would permit the complex to invest more capital in construction. Various scenarios of development were considered which would allow the complex to use 4 times and 8 times more capital for construction (see Figure 1). The scenarios, which allowed the complex to use up to 8 times more capital for construction and assembly work, practically does not introduce any important alterations in the strategies of construction within the complex. In other words beyond a certain limit the availability of capital no longer plays an important role but other constraints may limit the investment strategy.

Compared to the first scenario, 4 times more construction and assembly work was required. For this reason all future scenarios were mainly included with these assumptions.

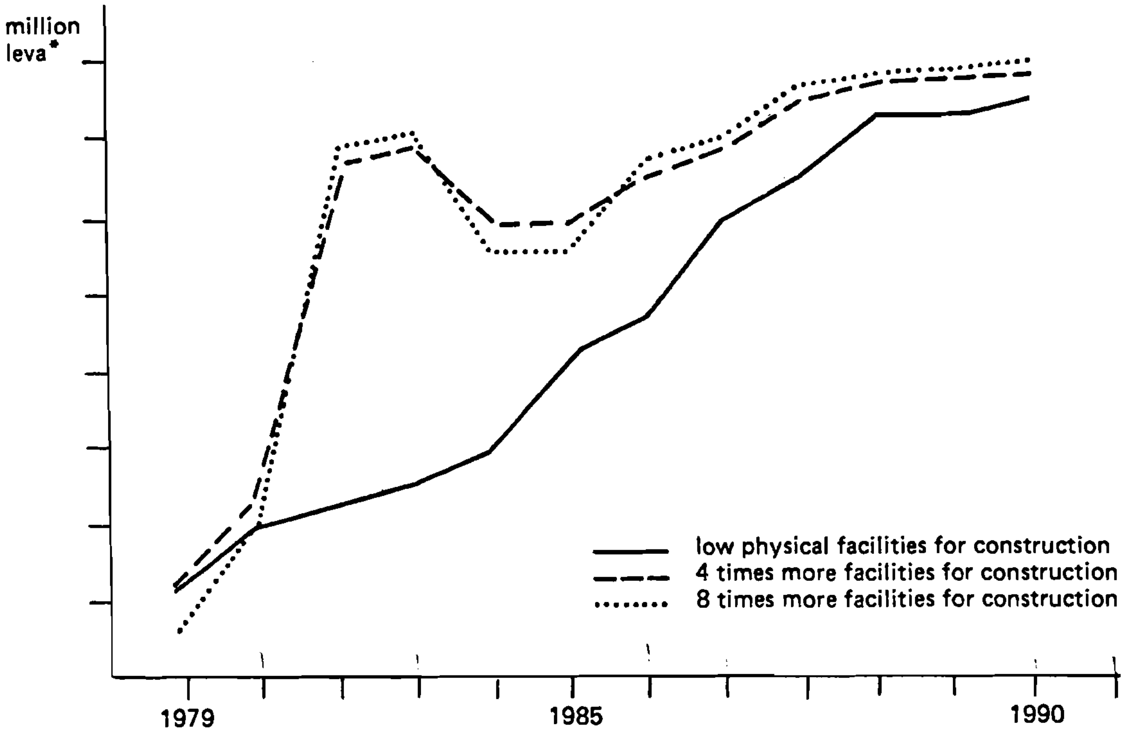

Figure 1. Net Production by assuming various construction scenarios. (The Bulgarian exchange rate was approximately US Dollars 1.10 per leva in 1981).

There were major differences in the basic scenarios. These differed mainly in the structure and rate of development of animal husbandry. The main basic scenario was focused on breeding pigs.

One scenario studied the development of milk production. Sheep-breeding also seemed to be an effective activity. Therefore, along with a considerable number of milk sheep, there was also a relatively large development of local sheep breeding.

The second scenario differed from the others in the possibility of purchasing calves for fattening. Thus it was directed toward meat production. In this scenario the total production and the total expenditure level for inputs were the highest.

In the third scenario the development of sheep breeding was limited (the total number of milk and meat sheep is limited from the above scenarios). Contrary to the other two scenarios poultry raising was also developed.

In the last three scenarios peak periods of capital investment were the years of 1982 and 1983. This was connected with the accelerated development of pig breeding.

To avoid unbalanced investments, a fourth scenario was proposed with more moderate capital investments. Here, the peak periods were not as clearly defined. This led to a slower development of pig raising which reached its maximum 2 to 3 years later than in the other scenarios. The total number of sheep was only a little larger than in the third scenario, but it was also constrained. There was a relatively intensive development of milk production and cattle breeding and a good development in raising poultry.

4.6. Development of Crop Production

The development of land use for individual crops was connected with satisfying the basic requirements of the development of the complex. It was assumed that all livestock were supplied with feed produced in the complex, that local demand had to be satisfied for some crop products such as flour, fruit, and vegetables, that there were requirements of crop rotation on irrigated and non-irrigated land, etc. The structure of crop production by year is shown in **Figures 2 and 3.**

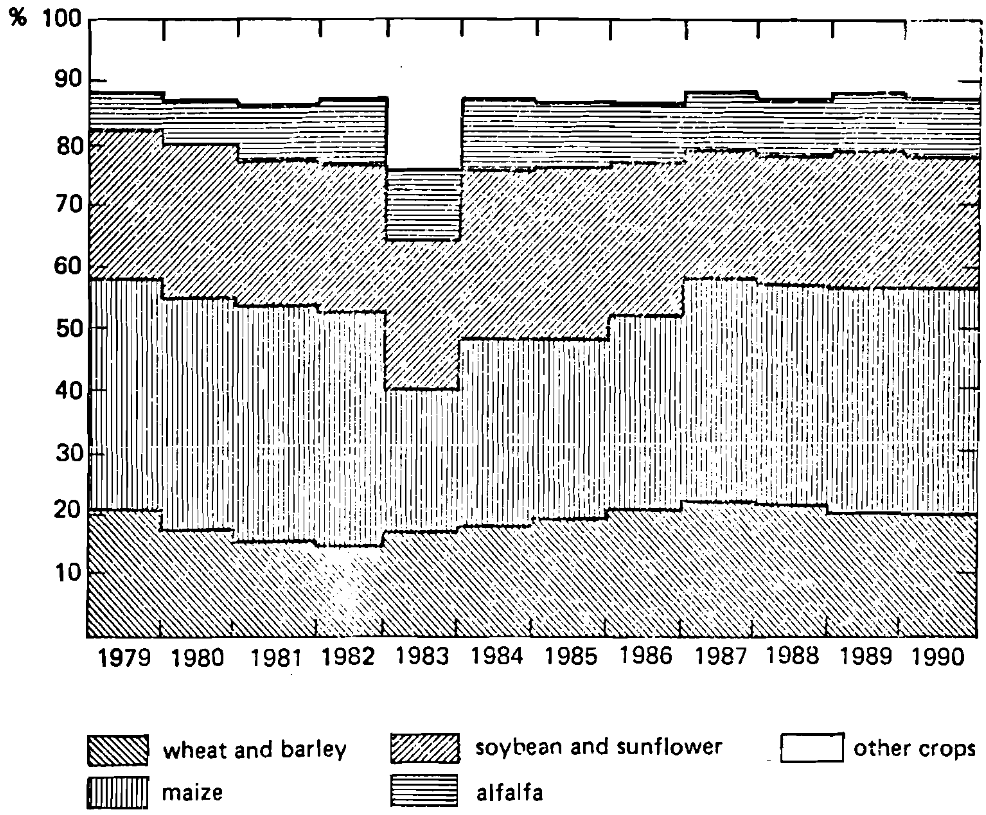

Figure 2. Crop structure.

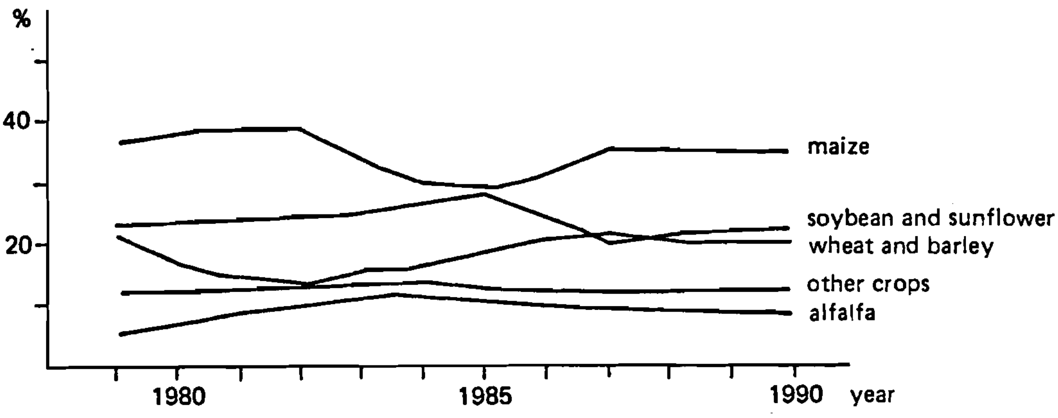

Figure 3. Share of major crops in total acreage.

During the first years of development, mainly from 1979 to 1982 when animal husbandry was still poorly developed, part of the land was used for grains which could then be sold.

The land used for wheat decreased in the period up to 1982. After that year their volume showed a slow increase. Although in the individual scenarios the development of land used for wheat differs, the decrease compared to 1978 was as follows: in 1980, the area used for wheat decreased nearly 2,3 times, in 1985 nearly 4.1 times, in 1990 nearly 4 times. The decrease in land for growing wheat was caused by the mono-diet of the livestock, by the necessity of using wheat as feed, by the structural change of livestock breeding (the increase in the number of pigs), and by the higher yields.

In the proposal for the development of the complex, made by the American firm "Globe Services", the land used for wheat growing decreased approximately 2.8 times due to the comparatively large share of wheat in some of the diets and and the lower yields used in the projection.

The tendency toward developing land used for barley was the reverse. By 1985 land increased 1.5 times as compared to the land cultivated in 1978 and 3.4 times by 1990.

There was a certain increase in land used for maize. The scenarios considered the following types of maize: maize for grain, silage maize collected as a full-grown plant in wax ripeness, seed producing maize. Silage maize was cultivated on about 1/5 of the total area used for maize. There was a certain increase in the area of grain maize up to 1984, due to the sale of grain maize during this period and to the fact that most wheat was planted on non-irrigated areas, whereas during the rest of the planning period wheat was sown exclusively on irrigated land previously used for livestock breeding.

At the beginning of the development period there was a tendency to increase the land used for sunflower cultivation. Although this differed in each scenario land increased on the average about 2.1 by 1980 times as compared to 1978. In all the scenarios the land used for sunflower then decreased, and in 1990 the cultivated area diminished to the size of 1978. The increase in land up to 1984 was again at the expense of sunflower cultivation because of livestock breeding and a more rational use of the non-irrigated areas.

From 1979 to 1982 the land used for soybean decreased in all the scenarios due to the increase in land for sunflower cultivation and to the fact that the soybean is grown almost exclusively on irrigated land. Furthermore, the low growth in cattle breeding during this period did not require great quantities of soybean. The land used for soybean gradually increased with the increase in livestock fed on soybean, and by 1990 the quantity increased 27 times since 1980. From 1985 to 1986 the soybean grown on irrigated land was replaced by grain maize which had better technological and economical indicators.

The area of lucerne decreased during the first years. This was connected with the lower percentage of the lucerne in livestock diets. During the 15 year planning period the land projected for beans and tobacco cropping corresponded to the plans made by the managers.

About one tenth of the total arable land was used for second crops. The possibility of growing repco as a first crop was also considered. The size of these areas is only determined by the need to satisfy the feed requirements of different livestock. The decrease in cropland was due to modern technologies with higher yields. The possibilities for increasing the area used for the second crops were investigated by:

(i) replacing silage maize as the main crop with silage maize as a second crop

(ii) increasing the usage of rough silage.

If these possibilities are used efficiently, the total land used for second crops could increase by about 2.5 times more than in previous scenarios.

Changes in land size used for vineyards, orchards, and other vegetables were made according to the plans developed by experts. As already noted, they are not subject to optimization within the framework of this model.

The development of individual crops and available land was connected to a great extent with an increase in irrigated land. According to the actual decisions made about the development of the complex, by 1990 irrigated land increased considerably. With the exception of the area necessary for the development of fruit and vegetable production and that already used for lucerne, the remaining irrigated land was allocated basically between two crops: soyabean and maize. During the last years of the development period, about 90% of the land used for soybean was irrigated. At the beginning of the development of the complex, grain maize was planted for the most part on non-irrigated land. More than half the maize cropping shifted to irrigated land by 1990. Part of the barley crop was cultivated on irrigated land due to the requirements of crop rotation. The results related to the irrigated areas are given in Figures 4 and 5.

Typical for the proposed production program was a considerable growth of production due to an increase of the level of intensification. In spite of an insignificant increase in land used in grain crops, the total grain production increased steadily and by 1985 was 1.2 times greater than in 1978, and 1.6 times greater than by 1990. The production of fodder corn (maize, barley and soybean) increased due to the potential development of cattle breeding. Thus, one problem caused by breeding cattle was solved: sufficient quantities of fodder became available.

With regard to other crops there was a relatively higher increase in the production of sunflower. By 1980 the production was 2.1 times greater than in 1978 and 1.6 times greater by 1990. The bean production increased steadily although the land size remained constant, due to the steady increase in yields of this crop.

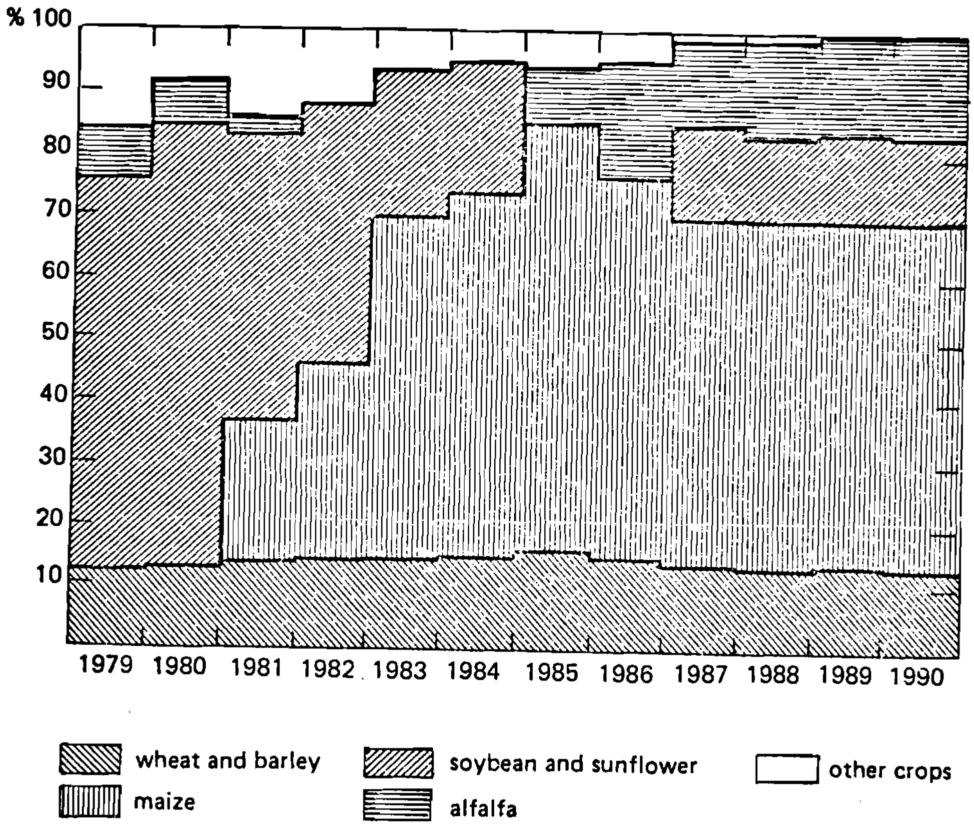

Figure 4. Use of irrigated land.

The production of hay and rough forage was determined by the dietary norms set for animal husbandry. Thus the production of lucerne straw was 2.8 times greater by 1990 than in 1978, of lucerne straw is 2.8 times greater in 1990 than in 1978, of repco silage 3.5 times, the maize silage collected in wax ripeness nearly 2 times, etc. The production of tobacco and hemp increased well due to intensification of production.

In the production program a considerable increase in the production of fresh and processed fruit and vegetables was planned in order to help satisfy local demand and to export products. The production of vegetables was 1.67 times greater by 1990 than in 1978; of grapes, 1.3 times; of fruit over 3 times.

4.6.1. *Development of livestock breeding*

Milk and meat production represented two paths of development in livestock breeding up to 1990. For breeding livestock for milk both the present technologies and a new technology for raising cows producing 5000 liters of milk annually were included in the model. Initial technologies were imposed because some farms were not for reconstruction and could not accommodate high yielding cows. Buildings were in good condition so renovation or rebuilding was not necessary. The development of livestock breeding was predetermined by the number of initial farms. Compared to the initial year, by 1990 the number of cows raised according to the presently available technologies decreased by more than 3 times. For the development of milk production, the cows yielding

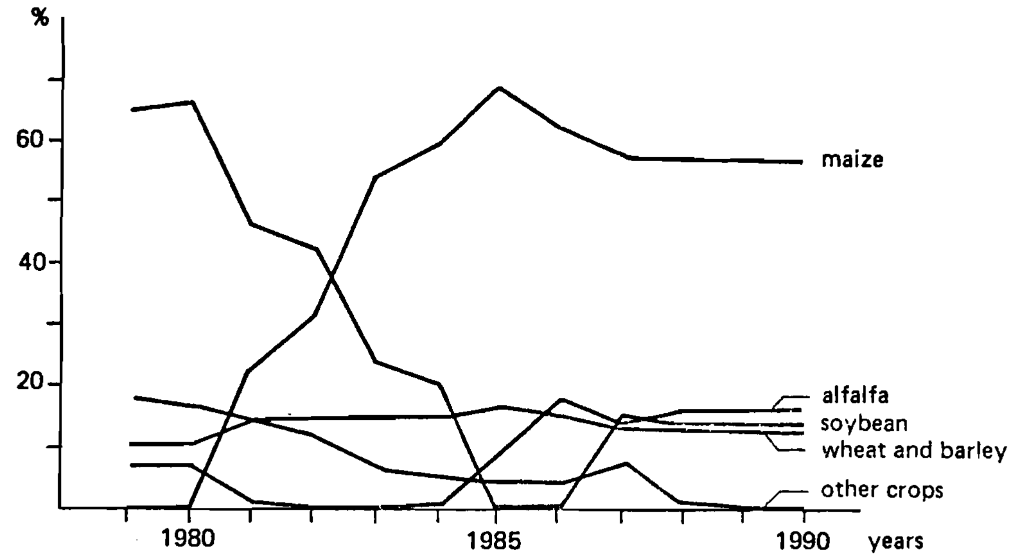

Figure 5. Relative proportion of irrigated area used for crop production.

5000 litres of milk were considered as the basic herd. Including a technology for raising cows yielding 4000 litres of milk proved to be relatively ineffective. This seemed justified because it would be possible to build modern dairy farms, and to increase the herd by purchasing cows abroad and by increasing the number of calves for breeding. With these assumptions, the number of cows with high yields was nearly three times greater by 1985 than in 1980, and nearly 5 times greater by 1990. Their raising was assumed to be on the same cattle farms, with the same organization and technology of raising as the cows yielding 5000 litres of milk. According to these technologies it was assumed that about 65% of the cows, after delivering their first calf, would be transferred to the basic herd of cows yielding 5000 litres. Thus the total number of milk cows in the complex was 1.5 times greater by 1985 than in 1980 and 2 times greater in 1990. Accordingly, the average yield of milk of the cow herd was 1.2 greater by 1985 than in 1980 and 1.3 times greater by 1990.

Due to the projected development of dairy cows, it was planned that the whole breeding stock required for the expansion of the cow herd would be raised in newly-built constructions after 1979.

Development of cattle-breeding for meat was proposed which had not been done in the complex and would begin in 1979. In various scenarios, the number of cattle for meat was greater in 1985 than in 1980 and 2 times greater by 1990. The increase in herd during the whole period was done by purchasing breeding stock and by breeding calves. For a better illustration, the dynamics of the development of cattle breeding over the development period is shown in Figure 6. Most of the cattle were dairy cows, the rest were used for meat production. The tendency to increase the number of fattened calves was evident in the scenarios. It was proposed that all the fattening calves would be raised on new sites beginning with 1982 according to technological requirements.

The fattening of calves was highly efficient. If it were possible to purchase more calves then this activity could be considerably extended which would increase the total production of the complex.

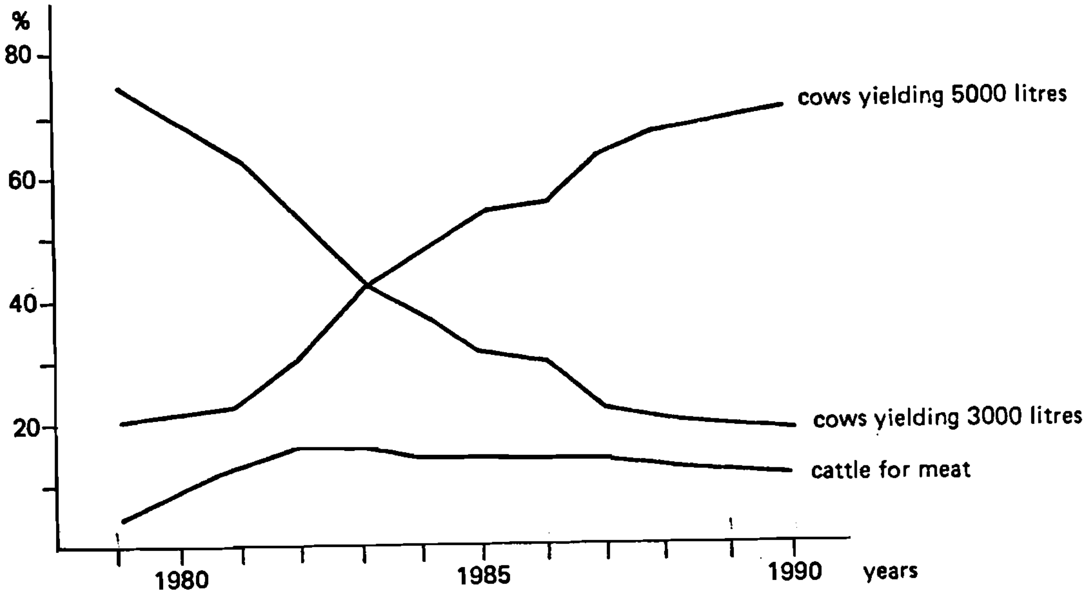

Figure 6. Development of cattle breeding.

The analysis made by specialists of sheep breeding showed that the sheep herd, raised for milk and meat according to the available technologies was not productive and had notably high labor costs and a low productivity. Therefore, it was proposed that current technology be gradually replaced by two new specialized and highly productive technologies--sheep, raised for wool and meat, and sheep, raised for wool and milk.

According to the analysis of the scenarios the development of sheep breeding for meat was as follows: by 1980 the meat producing sheep would represent 60% of the total sheep herd by 1985--110% and by 1990--126%.

In order to meet local needs for sheep's milk and to sell sheep's cheese it was proposed that a new group of milk and wool sheep with high milk yields and a high birthrate be developed, and organized by purchasing breeding stock up to 1985. In the last stage of development the total number of sheep producing milk and wool would represent about 17-20% of the total sheep herd.

The total number of sheep in 1980 would be 1.2 times greater than in 1978; 1.48 times greater by 1985 and 1.55 times greater by 1990. There could be a small decrease in the total number of sheep in 1982 and 1983 due to the need to replace old farms with new farms (see Figure 7).

Along with the development of sheep breeding, the number of lambs for fattening increased. By 1990 the number of fattened lambs was 1.6 times greater than in 1978.

Analyses of the various scenarios showed that sheep breeding (particularly for milk production) is a highly efficient activity (using the prices of 1978) and if, in future exports of sheep breeding were to be guaranteed, then the volume of production could be considerably enlarged.

Pig breeding showed the most intensive development of all the activities of livestock breeding in the scenarios. Its development was only restricted by the need to build pig farms. Compared to the initial year of 1978, by 1985 the pig-

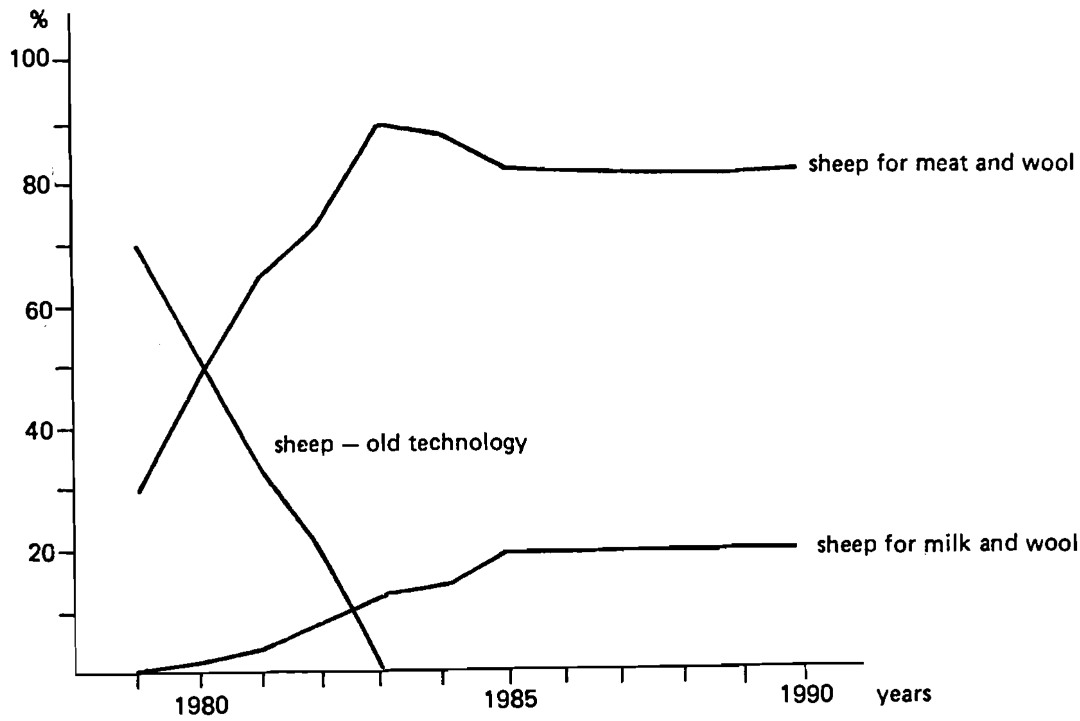

Figure 7. Sheep breeding structure.

breeding increased 3.8 times and 5 times by 1990. In figure 8 the development of livestock herds is shown.

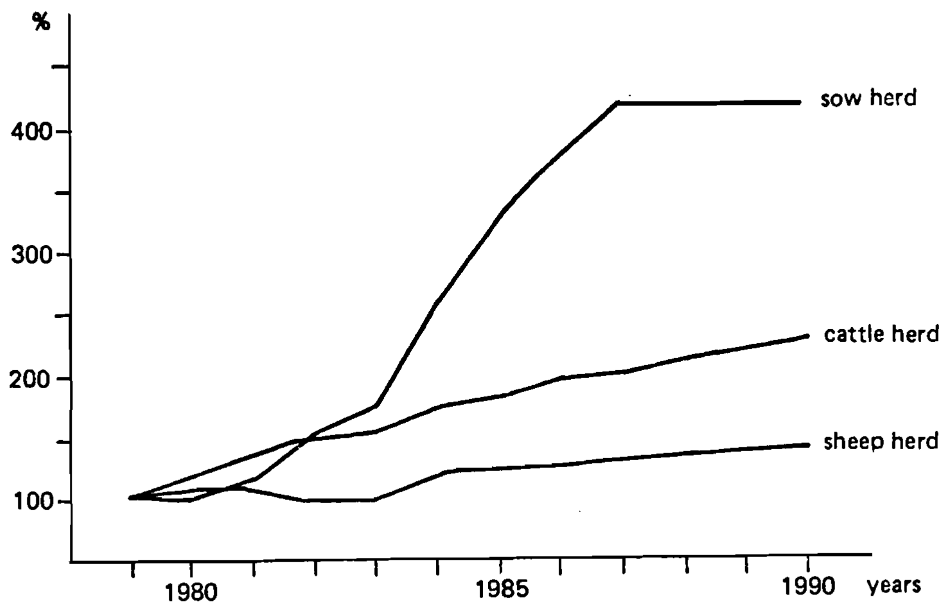

Figure 8. Dynamics of livestock breeding.

It is proposed that raising poultry could only be developed by reconstructing and modernizing the available sites of chicken farms. At a minimum of additional capital investment and an insignificant increase in labor, the efficiency of raising poultry would be improved.

4.6.2. *Production volumes of animal husbandry*

As a result of the increased production of fodder there would be a sharp increase in some basic animal products (see Table 2). In 1980 the production of meat was projected to be 1.4 times greater than 1978, 3.7 times greater by 1985, 4.7 times greater by 1990.

These high statistics were due to an increase in pork (more than 5 times the initial production volume). The total quantity of milk (reduced to 3.6% of its butter content) was 2.3 times larger by 1985 than in 1978 and by 3.2 times by 1990. The number of eggs could be 3 times greater by 1990 than in 1978. The wool production would increase 1.8 times according to the fourth scenario and 2.5 times according to the first scenario.

Table 2. Pattern of production volumes of animal husbandry (% in the total output).

Year	Cattle breeding (%)	Sheep breeding (%)	Sow breeding (%)	Egg production (%)
1979	20.4	14.3	63.7	1.6
1980	23.5	14.7	60.2	1.6
1981	25.0	14.5	59.2	1.3
1982	23.5	13.1	62.2	1.2
1983	21.0	11.6	66.3	1.1
1984	15.4	10.4	73.2	1.0
1985	14.3	8.7	76.2	0.8
1986	13.8	8.5	77.0	0.7
1987	13.3	7.6	78.4	0.7
1988	13.6	7.8	77.8	0.8
1989	14.2	7.9	77.0	0.9
1990	14.8	8.3	76.0	0.9

4.7. Scenarios Exploring the Limits to Development.

All the economic indicators in the four scenarios showed a tendency to increase steadily, the sharpest increase being from 1982 to 1985. Due to the intensive projected development of the complex, the total production would grow considerably. Compared to 1980, the total production of the complex in 1985 increased by 1.8 times and in 1990 by 2.1 times. The highest production was in the scenario where it was possible to purchase calves for fattening. The increase of production after 1982 was exclusively due to these calves. Production in the fourth scenario increased most slowly due to the relatively slow and constant rate of capital investment which also led to a certain slowing down of the development of livestock breeding, although at the end of the period, i.e., towards 1990, the development of livestock breeding was almost the same in all scenarios.

Figure 9 shows the volume and rate of growth, of the net production and the net income of the complex

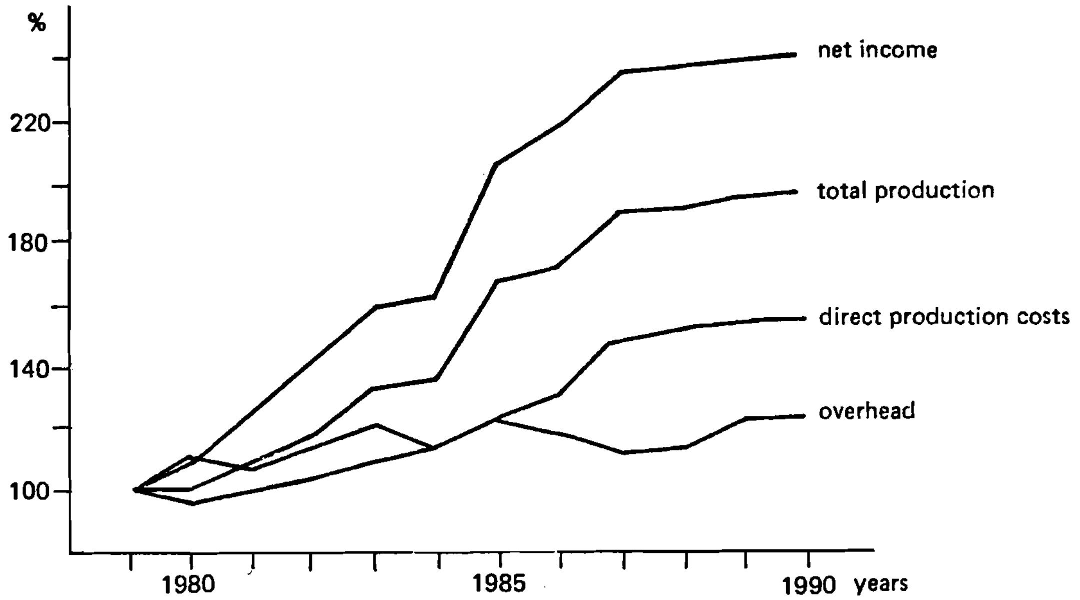

Figure 9. Dynamics of the basic economic indicators.

The maximum amount of capital in the first three scenarios was invested around 1982 to 1983. The highest capital investments was in the second scenario. After 1986 capital investments in all four scenarios dropped off sharply.

Due to the gradual increase in implementation of modern production technologies, the relative share of labor costs dropped. The production costs in each scenario were all approximately equal. By 1985 the production costs were about 1.5 times greater than in 1980 and about 2 times greater by 1990.

The labor used in crop production showed a clear tendency to decrease in number, due mainly to a decrease in available manual and unskilled labor. The average number of labor working on irrigated fields decreased from 1979 by about 13% by 1990. This was due not only to new technologies, to the relative increase of the areas of grain, to the mechanization of labor used for tobacco, beans, etc. After 1983 this average number of laborers remained practically stable. Similarly for orchards and vineyards the number of laborers in 1990 was about 30% less than in 1978. Figure 10 shows the number of labor which is projected to be employed over the 15 year development in various agricultural sectors.

Modern cattle breeding technologies have relatively low manual labor costs. With the gradual disappearance of old technologies and the introduction of new ones, a relative decrease in labor by about 14% was observed, in spite of the strong development of cattle breeding.

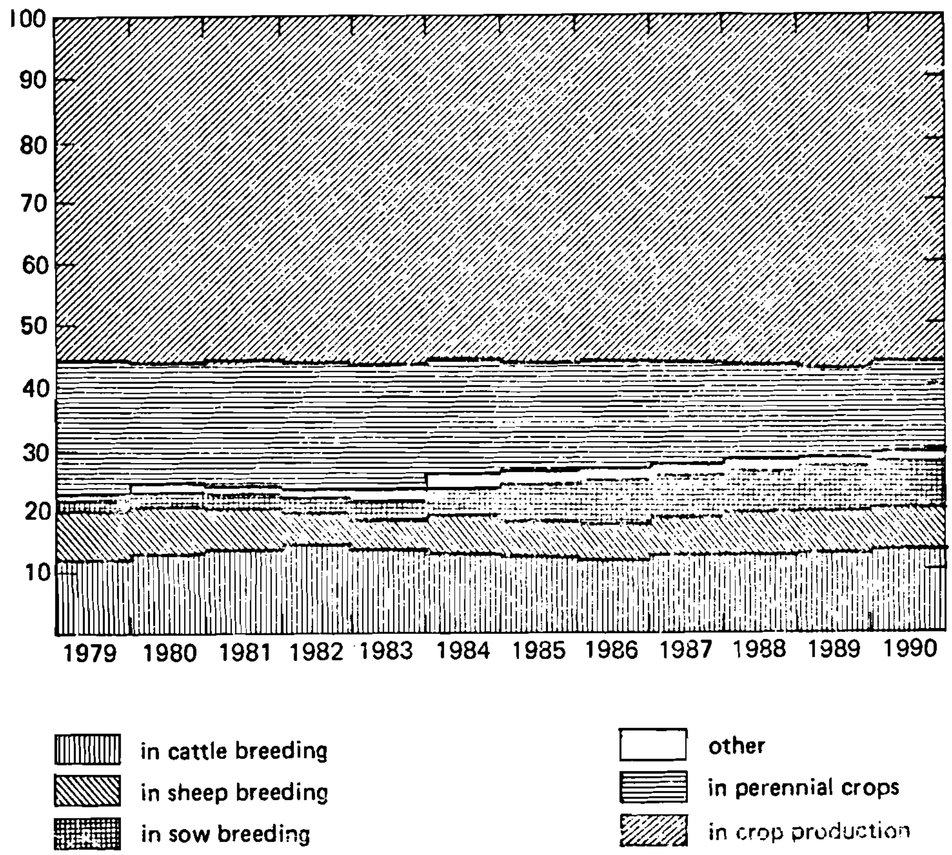

Figure 10. Percent of labor employed in different activities.

4.8. Conclusions

When applying the model, detailed information about many other activities needed to help develop the complex were gained. In the course of looking for alternative paths of development for the complex, possibilities were disclosed on the basis of model runs which would not otherwise have been noticed. A synthesis between the formal logic of the model and the experience of experts and managers makes the search for new paths of development of an agricultural complex more fruitful.

This modeling enterprise is unique in Bulgaria since up to now this type of problem--looking at the development problems of a large scale industrial complex with over 150,000 ha. of arable land--has not been solved by a computer.

The major accomplishment of this research was building a model of crop production and livestock breeding for agro-industrial complexes, in order to study their development over a certain time period. With this mathematical model fast optimal alternatives for management decisions can be obtained when new problems arise, or when more scenarios are analyzed. Furthermore, the optimal alternatives for the development of activities other than crop production and livestock breeding can be identified, provided that the required information is available.

This model makes a step toward introducing computer based systems for agricultural planning of large complexes. By using this research experience in the programming of yields from crop production and livestock breeding, setting up a model and data bank could be started which would automatically compile information on technologies of crop production, the basis of the soil and climatic conditions in each complex.

It must be stressed that it is necessary to update the data and make model runs on the basis of the latest data.

This dynamic linear programming model is an application of the model presented in section 1 of this volume. The optimum structure of an agro-industrial complex can be investigated using the model, provided that the basic directions for development are determined by the central planners. Problem solving concerning the specialization of a complex is outside the realm of the model. This can be done only with a national model of the food and agriculture system.

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5. DYNAMIC LINEAR PROGRAMMING MODEL FOR DERIVING AGRICULTURAL WATER DEMANDS

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5. DYNAMIC LINEAR PROGRAMMING MODEL FOR DERIVING AGRICULTURAL WATER DEMANDS

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5.1. Introduction

Water supply and demand are crucial factors in agricultural production. However emphasis has always been on studying the supply of water. The reasons for this are: the demand for water has been treated as a fixed requirement of development; the price of water is often negligible compared with the price of other resources; water has always been considered as an inexhaustible resource and therefore the opinion evolved that "if it is needed, it must be supplied". This attitude still prevails in some regions of the world.

In many countries agriculture is one of the most water consuming sectors of the economy. Developing the water supply system requires major capital investments as well as other resources (labor, materials) in order to meet this demand. The policy question is whether these resources should be used to augment the water supply,or by keeping the production targets at a desired level, use these resources to reduce the demand for water and thereby save water which can then be utilized more efficiently in other sectors of the economy.

This paper aims at developing a methodology for deriving agricultural water demands and their marginal benefits in order to assist in making decisions about the following:

(a) determining the scale of development of an agricultural water related activity over time (i.e. irrigated versus non-irrigated land, estimating irrigation and livestock water demand and the distribution over time and space

(b) evaluating various irrigation techniques

(c) determining the impact of water demand on the production process, including the availability of other inputs.

(d) estimating the time-dependent water demand function.

5.2. The Agricultural Production Process—General Characteristics of Water Use.

Agriculture is a complex system composed of several closely related subsystems (Figure 1). Water, together with other resources (machinery, labor, capital investments, fertilizers, pesticides, energy, etc.), enter into the crop and livestock production subsystems whose products are subsequently processed and then distributed (e.g. sold to the market).

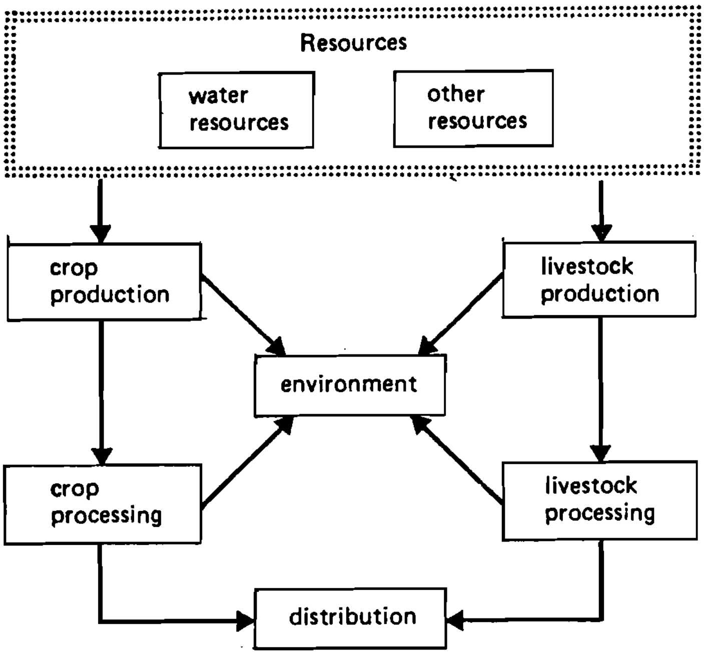

Figure 1. Elements of the agricultural system.

Water is used in all subsystems. In the crop production subsystem water is used to increase the productivity of crops by irrigation, while for the production of livestock water is used for drinking water for the animals as well as for cleaning. Both crop and livestock processing subsystems depend highly on the availability of water (processing of fruits, vegetables, and meat for the canning industry). The last subsystem--environment--may require substantial amounts of water to enhance its capacity to assimilate, to sustain wildlife, to leach salts from the soil, etc. In this paper the environment is not explicitly considered.

For our purposes, agricultural production can be formalized in the following way. The vector V(t) of products in each time period t is a function of water Q(t), other resources R(t) and the production alternatives X(t).

$$V(t) = f\,[Q(t), R(t), X(t)] \tag{1}$$

A given value of V(t) is determined by various combinations of resources and production alternatives. In other words, one resource, say water, can be substituted for another resource to obtain the same value of V(t). One of the aims of this paper is to reveal the relationship between the amount of water Q(t) and all other resources R(t), the production alternatives X(t) and the products V(t)

$$Q(t) = \rho\,[V(t), R(t), X(t)] \tag{2}$$

in such a way that certain goals are achieved for example the maximum production or the maximum net benefit. The relationship (2) can also serve as a forecasting tool to predict water demands Q(t) at certain future points in time.

5.3. A Dynamic Linear Programming Model of Water Demand

The aim of this section is to identify the relationship (2). To do this we will discuss the modeling of the subsystems shown in Figure 1 and link them in a dynamic water demand oriented model. Each subsystem is described in terms of state equations followed by constraints on the state and control variables. (see section 1 and 2 of this volume). In order to simplify the mathematical notation the equations will be written in vectorial form. All vectors will have an additional dimension if the total agricultural land is divided into regions.

5.3.1. *Resources subsystem*

Input resources can be divided into two categories: storable (water, machinery, including irrigation equipment, fuel, fertilizer and pesticides, labor) and nonstorable resources (e.g. electricity). Although these categories are rather fuzzy, e.g. if no reservoirs are available, water is also a nonstorable resource, they need different approaches in modeling.

First let us consider water as a storable resource. Let:

$W_q(t)$be the amount of water stored for irrigation (q=1), for animal drinking water (q=2), and for the processing subsystems (q=3) (state variable).

$Q_q^+(t)$be the amount of water available at time period t, (control variable)

$Q_q^-(t)$be the amount of water drawn for irrigation or for drinking at time t (control variable)

The state equations of water available for irrigation, for drinking, and for processing are

$$W_q(t+1) = W_q(t) + Q_q^+(t) - Q_q^-(t), \qquad (q = 1, 2, 3) \tag{3}$$

For all the other resources the same type of equation can be written

$$R_p(t+1) = (1 - \delta_{p)}\, R_p(t) + R_p^+(t) - R_p^-(t) \qquad (p = 1, \ldots, P) \tag{4}$$

This shows that the amount $R_p(t+1)$ of any storable resource p at time (t+1) equals the amount of resource p at the beginning of time period t (with possible depreciation) plus the amount of resource $R_p^+(t)$ made available to the the system minus the resource used by the system $R_p^-(t)$ at time t.

Nonstorable resources $R_n(t)$, n=1, . . . ,N, are used by the system during each time period t. Hence the general equation can be written as follows:

$$R_n(t) = \sum_j k_{nj}(t)\, X_j(t), \qquad n = 1,\ldots,N \tag{5}$$

where

$X_j(t)$ is the unit of production alternative j consuming the resources R_n at time t;

$k_{nj}(t)$ denotes the per unit consumption of resource R_n by the production alternative j at time t.

Note that (5) also holds for the utilization of storable resources $R_p^-(t)$.

Obviously, all state and control variables must be non-negative. Other constraints on the control variables are discussed in the following sections for specific subsystems using these resources.

5.3.2. *Crop production subsystem*

5.3.2.1. *Irrigated and nonirrigated land with* annual and perennial crops

Two different approaches are needed for building a dynamic model of annual and perennial crops. If the time period t is one year, only the control vector $\tilde{y}(t) = [\tilde{y}_{1r}(t), \ldots, \tilde{y}_{mr}(t), \ldots, \tilde{y}_{Mr}(t)]$ (r = 1, 2) is introduced for annual crops. The components of this vector are the amount of irrigated (r = 1) or nonirrigated (r = 2) land under annual crops m (m = 1,...,M) at time t. Perennial crops (lucerne, fruit trees, etc.) need more than one year to reach maturity. (see section 1 of this volume)

Let

$y(t) = [y_{1r}(t), \ldots, y_{jr}(t), \ldots, y_{Jr}(t)]$ be the irrigated or nonirrigated land (ha.) used for the perennial crop j at time t, y(t) is a state vector

$v^{+}(t) = [v_{1r}^{+}(t), \ldots, v_{jr}^{+}(t), \ldots, v_{Jr}^{+}(t)]$ be the irrigated or nonirrigated land (ha.) used for new plantings of the perennial crop j at time t, $v^{+}(t)$ is a control vector

$v^{-}(t) = [v_{1r}^{-}(t), \ldots, v_{jr}^{-}(t), \ldots, v_{Jr}^{-}(t)]$ be the irrigated or nonirrigated land (ha.) under the perennial crop j removed at time t, $v^{-}(t)$ is a control vector

$B = [b_{jk}^{r}]$ be a matrix indicating the rate of tree planting of type j on irrigated or non-irrigated land at time t.

Then the state and control vectors can be linked to form in the following state equation:

$$y(t+1) = By(t) + v^{+}(t) - v^{-}(t) \qquad (6)$$

5.3.2.2. *State equations and constraints of irrigated and nonirrigated* land.

If IL(t) is the amount of irrigated land available at time t, $IL^{+}(t)$ is the land introduced to irrigation at time t, and $IL^{-}(t)$ is the land removed from irrigation at time t, then the irrigated land IL (t+1) at time (t+1) is as follows:

$$IL(t+1) = IL(t) + IL^{+}(t) - IL^{-}(t) \qquad (7)$$

For the irrigated land IL(t) the following holds

$$IL(t) = \sum_{j=1}^{J} y_{j1}(t) + \sum_{l=1}^{M} \tilde{y}_{l1}(t) \qquad (8)$$

In addition $IL(t) \leq \overline{IL}(t)$, where $\overline{IL}(t)$ is the upper bound of irrigated land in the system at time t.

Similarly the following holds for nonirrigated lands

$$NL(t+1) = NL(t) + NL^{+}(t) - NL^{-}(t) \qquad (9)$$

$$NL(t) = \sum_{j=1}^{J} y_{j2}(t) + \sum_{l=1}^{M} \tilde{y}_{l2}(t) \qquad (10)$$

5.3.2.3. *State equations and constraints of arable land*

Let L(t) be the total arable land available at time t. $L^{+}(t)$ is land entering the arable land inventory at time t, and $L^{-}(t)$ land leaving the arable land inventory then the amount of arable land L (t+1) at time t+1 is

$$L(t+1) = L(t) + L^{+}(t) - L^{-}(t) \qquad (11)$$

with L(t) = IL(t) + NL(t) or, taking into account (8) and (9)

$$\sum_{r=1}^{2}\left[\sum_{j=1}^{J} y_{jr}(t) + \sum_{l=1}^{M} \tilde{y}_{lr}(t)\right]=L(t) \tag{12}$$

The arable land L(t) is further constrained by the total amount of arable land $\overline{AL}(t)$ available in the system at time t.

$$L(t) \le \overline{AL}(t) \tag{13}$$

The water $Q_l^-(t)$ drawn for irrigation is computed by the following equation

$$Q_l^-(t) = \sum_{j=1}^{J} a_{j(t)}\, y_{jl}(t) + \sum_{l=1}^{M} \tilde{a}_l(t)\, \tilde{y}_{l1}(t) \tag{14}$$

where $a_j(t)$ and $\tilde{a}_l(t)$ are the unit water requirements (m^3/ ha) of perennial and annual crops (see (3)).

To keep the soil productive, additional constraints for the rotation of crops are introduced. In dynamic models these constraints can be formulated in different ways. In this model the simplest way has been chosen. For both irrigated and nonirrigated areas the ratio between close grown crops (wheat, barley, etc.) and row crops (maize, soybean, etc.) is constrained by

$$A_{1r}(t) \le \left[\sum_{l=\text{close grown crops}} \tilde{y}_{lr}(t) \;/ \sum_{l=\text{row crops}} \tilde{y}_{lr}(t)\right] \le A_{2r}(t)$$

where $A_{1r}(t) \ge 0$ and $A_{2r}(t) \ge 0$ are determined by a subjective judgement on the continuous cropping potential of the cultivated area.

5.3.3. *Livestock production subsystem*

The livestock production subsystem uses some of the products of the crop production subsystem for animals of type i (e.g. cows, calves, sheep, etc.). Let

$x_i(t)$ be the number of animals of type i at time t (state variable),

$u_i^+(t)$be the number of animals of type i purchased or bred at time t (control variable),

$u_i^-(t)$be the number of animals of type i sold at time t (control variable)

$G=[g_{ij}]$be the matrix whose elements indicate what proportion of animals of type j will progress to type i in the next period (e.g. calves become cows).

The state equation for the livestock subsystem is (see 3) as follows:

$$x(t+1) = G\,x(t) + u^+(t) - u^-(t) \tag{15}$$

To assure a certain level of productivity the animals have to be fed by various feedstuffs, such as green forage, hay, silage, concentrated forage, and roughage. Furthermore, each animal diet is specified within certain upper and lower limits as described by the following inequalities

$$D^{min}\, x(t) \le F(t) \le D^{max}\, x(t) \tag{16}$$

$$\sum_{b} f_{bi}(t) => C_i(t) \tag{17}$$

$$F(t) = KV(t) \tag{18}$$

where

D^{min} and D^{max} are matrices whose coefficients indicate the minimum and maximum amount of feed unit content of the feedstuff b (b = 1, ..., B) fed to animal i at time t;

$F(t) = [f_b(t)]$ is a vector of the required amount of feed units of feedstuff b fed to all animals at time t; $f_b(t) = \sum_i f_{bi}(t)$;

$C(t) = c_i(t)$ is the total amount of feed units required by animal i;

$V(t) = [V1, \ldots, V_q, \ldots, V_Q]$ is a vector of the products to be used in the livestock production subsystem;

$K = [k_{bq}]$ is the feed unit content of product q used as feedstuff b.

The water resources $Q_2^-(t)$ for the drinking water requirements of livestock can be determined by the following equation

$$DR(t)\, x\,(t) = Q_2^-(t) \qquad (19)$$

where

$DR(t) = DR_i(t)$ is a vector of the unit water requirement of animal i at time t.

The amount $R_p^-(t)$ of all other resources used in the livestock production subsystem are computed by (4).

5.3.4. *Crop and livestock processing and marketing subsystems*

These three subsystems are closely related to each other. As in the case of input resources, crop and livestock products are either storable or nonstorable. State equations can be written for all storable products, while for nonstorable ones there are only balance equations similar to (4).

Let the state variables be defined as follows

$s^y(t) = [s_m^y(t)]$, $(m=1,\ldots,V_j)$ is the stock of product m produced by perennial crops at time t:

$H^y(t) = [h_{mj}^y(t)]$, $(j=1,\ldots,J)$ is a matrix whose coefficients indicate the amount of product m produced from one unit of perennial crop j

$Y^y(t)$ is a vector of perennial crop products

$s^{\bar{y}}(t) = [s_q^{\bar{y}}(t)]$, $(q=1,\ldots,V_1)$ is the stock of product q from annual crops at time t

$H^{\bar{y}}(t) = [h_{qj}^{\bar{y}}(t)]$, $(1\text{-}2,\ldots,M)$ is a matrix whose coefficients show the amount of product q produced from one unit of the annual crop j.

$Y^{\bar{y}}(t)$ is a vector of annual crop products

$s^x(t) = [s_n^x(t)]$, $(n=1,\ldots,N_x)$ is the stock of the product n of the livestock production subsystem

$H^x(t) = [h_{di}^x(t)]$, $(d=1,\ldots,I_d)$ is a matrix whose coefficients indicate the amount of product d produced from animal i. $U^x(t)$ is a vector of animal products.

The products $s^y(t)$ and $s^x(t)$ may also be sold to the market. The selling activities, which are considered as control variables, are denoted by $s^{y-}(t)$, $s^{\bar{y}-}(t)$, and $s^{x-}(t)$, respectively.

The state equations for these three subsystems can be written as follows:

for the perennial crop processing subsystem

$$s^y(t+1) = s^y(t) + H^y(t)\, Y^y(t) - s^{y-}(t) \qquad (20)$$

for the annual crop processing subsystem

$$s^{y}(t+1) = s^{y}(t) + H^{y}(t)\, Y^{y}(t) - s^{y-}(t) \qquad (21)$$

for the livestock processing subsystem

$$s^{x}(t+1) = s^{x}(t) + H^{x}(t)\, U^{x}(t) - s^{x-}(t) \qquad (22)$$

For processing a certain amount of resources are required. If the vector $H(t) = [H^{y}(t)\, Y^{Y}(t), H^{y}(t)\, Y^{y}(t), H^{x}(t)\, V^{x}(t)]$ is introduced, then the water used in these processing subsystems is

$$Q_3^-(t) = H(t)\, PR(t) \qquad (23)$$

where

RP (t) is a vector of unit water requirements of processing activities in the perennial and annual crop and livestock processing subsystems.

The other required resources are computed in the same way.

5.3.5. *Objective function*

In the particular case of studying agricultural water demand, it is obvious that an increase in demand requires the development of water resources system. Developing the water system is the primary objective. As this takes time, the goal is usually to maximize some socially oriented indicators; for example, the difference between the present value of future gross revenue flow and future cost flow (e.g. the present value of net benefit), or some other combination of these two indicators.

Each state or control variable contributes either to the benefit or to the cost of the system. The net contribution (e.g. benefit minus cost) per unit of activity for all variables Z(t) is denoted by the vector b(t). Then the objective function B for the agricultural system is

$$B = \max \sum_{t=1}^{T} \delta(t)\, b(t)\, Z(t) \qquad (24)$$

where

$\delta(t)$ is a discount coefficient at time t.

5.3.6. *Application of the model*

The dynamic linear programming model described above can be used to answer a broad spectrum of questions concerning water demand analysis, the development of irrigated land and the impact of scarce resources on agricultural production. A conventional approach of supply-demand analysis is given in the Appendix.

The described methodology was used for determining the irrigation and livestock water demands up to year 2000 of an agricultural area including about 40,000 ha. arable land located in the Silistra region, Bulgaria. The area is under intensive development to generate crop and livestock products to meet the local demands, as well as for export to other regions of the country.

There are 10 crops grown in this area: wheat, barley, maize grain, maize silage, second crop of maize silage grown after harvesting wheat and barley, soybeans, sunflower, orchards, vegetables, lucerne. These crops are grown on irrigated or nonirrigated land some of them using up to two technologies: one of the technologies is assumed to be prevailing in the area up to the year 1985 and the other is supposed to be introduced after 1985. Crop products are used for feed, for local demand, or for high quality exports. Four types of animals--dairy and beef cattle, sheep, hens and pigs--are raised and their products are again used to meet local demand and for export.

The dynamic linear programming model for the area in the Silistra region has 320 variables and 267 constraints. The 6 period model was run on a EC1020 computer.

Although the model is capable of describing and determining a variety of outputs, we shall only concentrate on some water related activities: irrigation and livestock water demands over time, irrigated land over time, net benefits versus water demand, and finally, the water demand function of the area.

The time horizon 1978-2000 was divided into 6 time periods:
1978-1980, 1980-1982, 1982-1985, 1985-1990, 1990-1995 and 1995-2000. The water demand projected over the six time periods is shown in Figure 2 for both normal and dry weather.

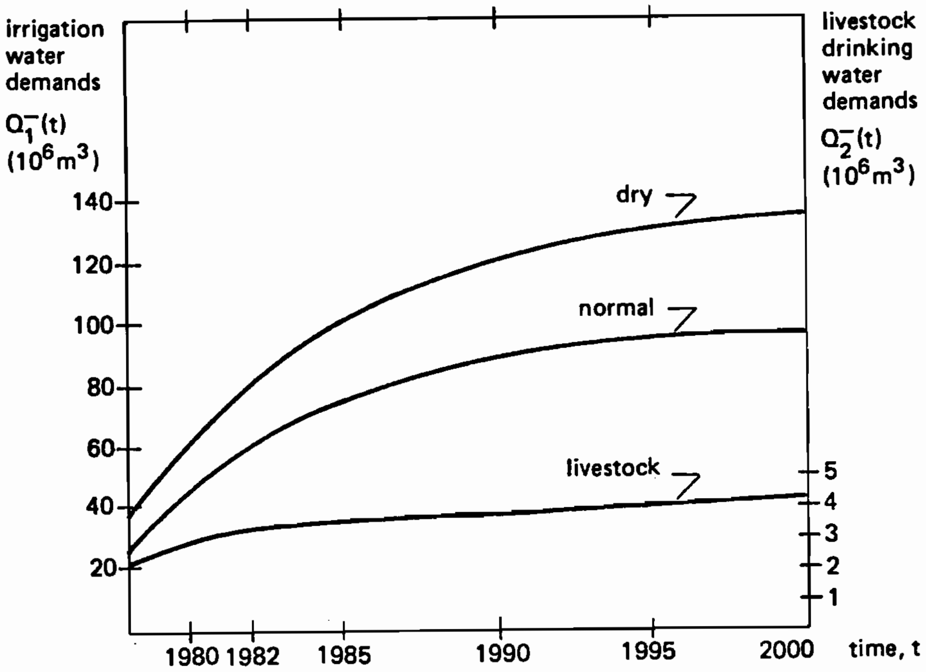

Figure 2. Projected water demand.

The normal weather conditions are defined by averaging 25 years of monthly rainfall data. A dry year is one in which the rainfall is exceeded on average in three years out of four. This is considered to be a representative measure for developing irrigation systems in Bulgaria. A general tendency shown in Figure 2 is the decreasing rate of water demand over time. This is a consequence of diminishing marginal returns on water (Figure 3) due to the irrigation of less productive land. Before all the arable land is irrigated, a point is reached where the marginal costs of additional irrigation equals the marginal net benefit. This point corresponds to about 135 x 10^6 m^3 of irrigation water and 31,000 ha. irrigated land. The corresponding water input coefficients are 4350 m^3/ha (435 mm) under dry conditions. Since an irrigation efficiency of 50% is assumed, this coefficient corresponds to a consumption of 217 mm of irrigation water.

The next important result is deriving a water demand function which relates the marginal value of water to the water demand, and the time t (Figure 4). It should be noted that to obtain this three dimensional relationship, a great

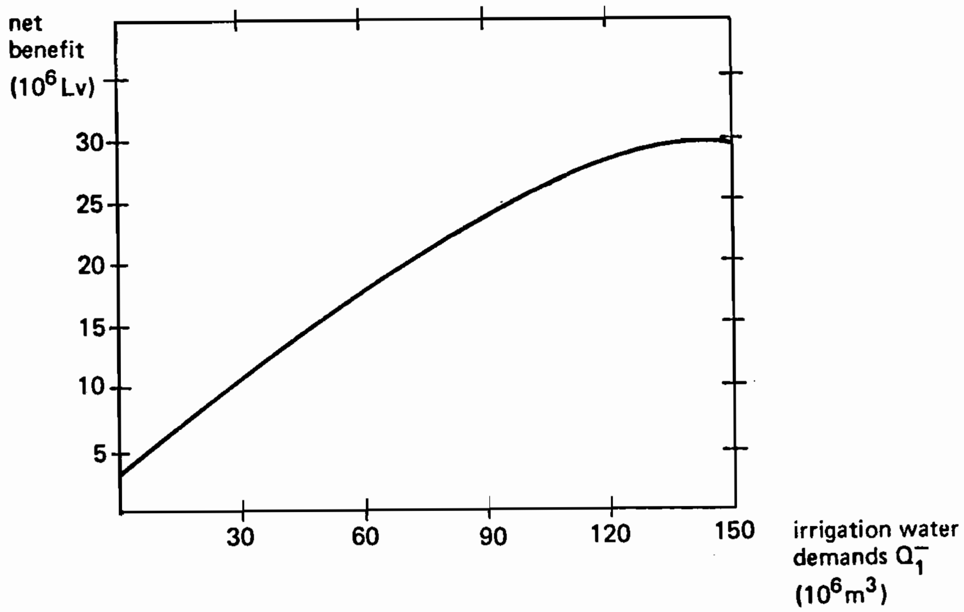

Figure 3. Returns for irrigation.

number of computations have to be carried out. For each computation, water demand is fixed at certain time periods, the model is run and a marginal value of water is obtained. If the model were static, for a fixed value of water demand we would get the corresponding marginal value of water. Thus after running the model r times, a static demand function with r points would be obtained. In the dynamic case however, the number of possible combinations of the values of water demand and with number of time periods k would be r^k. To reduce this, the following two hypotheses are assumed:

(a) the amount of water $Q_I^-(t_j)$ fixed at time period t_j, $j=1,\dots,T$, does not have an impact on the production process of the agricultural system and on the water resources at time t_{j-i}, $i=1,\dots,T$; $i < j$. In other words if the water supply is limited at certain time periods, this constraint influences the future but not the past of the agricultural process.

(b) if, according to Figure 2, the water demand at time t is $Q_I^-(t)$, then the model is only run for a finite number c of values $Q_{Ia}^-(t)$; $a=1,\dots,c$, satisfying the condition $0 \leq Q_{Ia}^-(t) \leq Q_I^-(t)$.

The demand function for irrigation water $Q_I^-(t)$ obtained under these conditions is shown in Figure 4. The optimal amount of water demand in the year 2000 corresponds to the equilibrium point A, which has the following coordinates: $Q_I^-(t) = 135 \times 10^6 m^3$ marginal value $m_Q = 0.017$, which is the assumed price of irrigation water. If the water price were zero, the equilibrium point at year 2000 would be the point A' which corresponds to a water demand of $Q_I^-(2000) = 150 \times 10^6 m^3$.

5.4. Conclusions

Deriving agricultural water demand by using dynamic linear programming models has certain advantages as well as some limitations. The most important advantage is that in forecasting future water demands, the entire dynamic production process is modeled. Therefore the possibility of substituting one

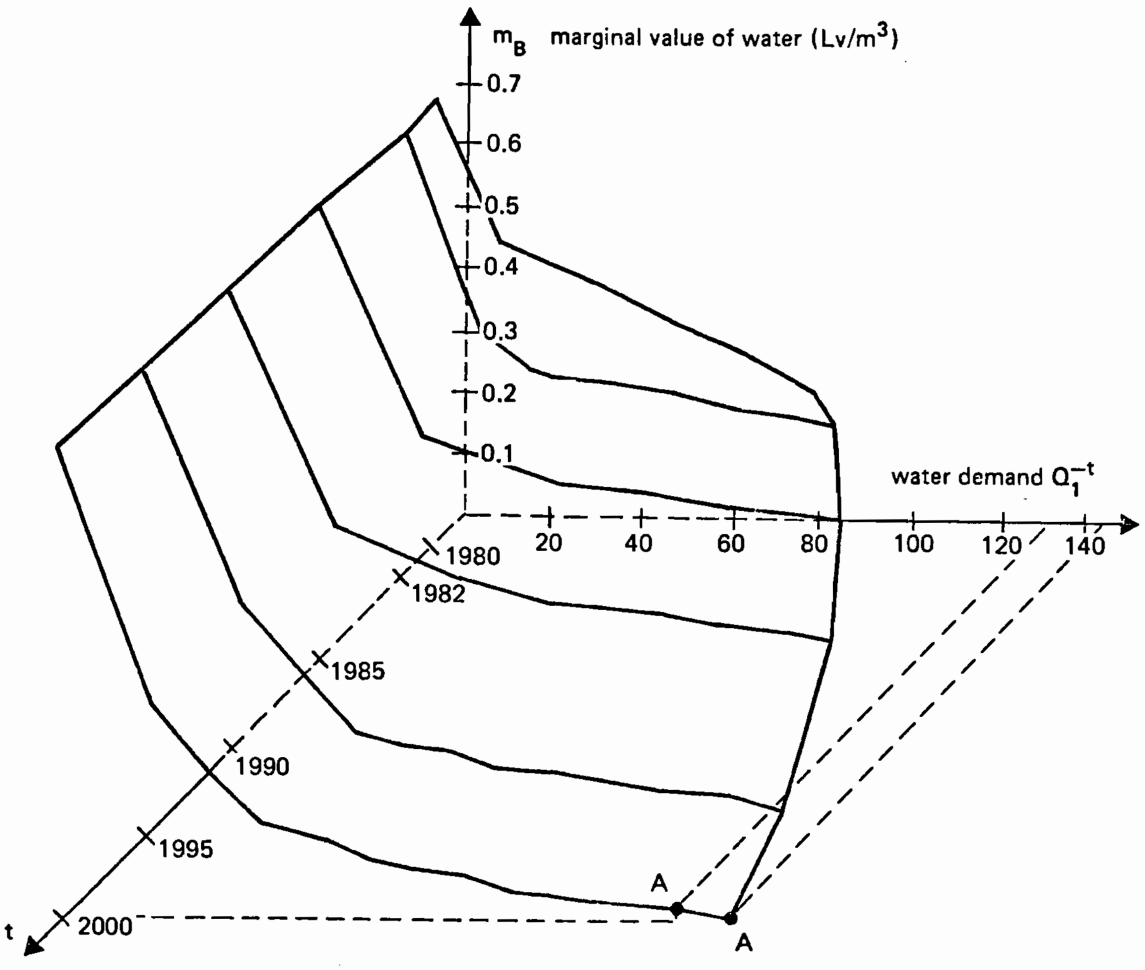

Figure 4. Water demand function.

resource can be taken into account for another as well as the reduction in the number of exogenous variables in comparison with other methods used in forecasting of water demands.

However, one must be careful in interpreting the results of this modeling exercise. The model assumes that all coefficients (prices, per unit consumption of resources, alternative technologies) are known for the model's time horizon, say the next 15-20 years. If this assumption is not justified, the model results are at most approximate. One way of overcoming this limitation is to couple the dynamic optimization model with a simulation model, which would generate a set of coefficients and alternative technologies under some general assumptions for development policies.

APPENDIX

Supply Demand Analysis.

The trade-off between increasing supply and reducing agricultural water demand is usually made by considering the following conceptual scheme (see Arrow, Hahn (1971) for details).

The benefit B of an agricultural firm is, in general, a function of the quantity $Q_D(t)$ of water used by the system at time period t, t=1,...,T and of production alternatives and other resources X(t), i.e.,

$$B(t) = B\left[Q_D(t), X(t)\right].$$

Under the assumption that the enterprise is benefit maximizing the quantity of water $Q_D(t)$ can be derived by solving the following optimization problem

$$\max \{B[Q_D(t), X(t)]\} \text{ for } \{Q_D(t), X(t)\} \varepsilon R_D, \qquad (A.1)$$

where R_D is a set of admissible values for $Q_D(t)$ and X(t).

Let $\left\{Q_D^o(t), X^o(t)\right\}$ be an optimal solution of (A.1).

Then the first derivative

$$\frac{\partial B}{\partial Q_D^o(t)} = m_B\left[Q_D^o(t)\right]$$

is the marginal value of the water demand Q_D^o at time t. This value indicates how much the objective function B would be reduced, if one unit of water at time t was removed from the system.

To meet the demand $Q_D(t)$, the agricultural system has to make use of water supply facilities which either already exist or need to be developed. The cost C of developing water supply facilities, such as reservoirs, pumping stations, etc., is a function of the amount of water $Q_S(t)$ and the existing technologies Y(t) needed to produce this amount:

$$C(t) = C[Q_S(t), Y(t)]$$

It is reasonable to assume that the supply facilities should be developed at minimum cost. Hence, the amount of water to be supplied to the agricultural system can be derived by solving the following optimization problem:

$$\min \{C[Q_S(t), Y(t)]\} \text{ for } \{Q_S(t), Y(t)\} \varepsilon R_S \qquad (A.2)$$

where R_S is a set of all the admissible values of $Q_S(t)$ and Y(t).

If $[Q_S^o(t), Y^o(t)]$ is an optimal solution of (A.2) then the first derivative

$$\frac{\partial C(t)}{\partial Q_S^o(t)} = m_S\left[Q_S^o(t)\right]$$

is the marginal value of the water supply Q_S^o at time t.

To ensure strict equilibrium in the demand-supply system, the following conditions must hold:

$$Q_D^o(t) = Q_S^o(t), \qquad t=1,\ldots,T \qquad (A.3)$$

$$m_B\left[Q_D^o(t)\right] = m_S\left[Q_S^o(t)\right], \qquad t=1,\ldots,T \qquad (A.4)$$

The equations (A.3) and (A.4) state the strict condition that at each time period t=1,...,T there must be an equilibrium between demand and supply. Sometimes this requirement is relaxed, when it is stipulated that there must be an equilibrium at the end of the development time horizon, or when the supply-demand system reaches a steady-state condition.

Under the assumption that $m_B(.)$ and $m_C(.)$ are monotonic with respect to their arguments, the solutions of (A.3) and (A.4) is the function

$$E\left[Q_D^o(t),\ Q_S^o(t),\ m_B(.),\ m_C(.)\right]$$

which is graphically represented in Figure (A.1)

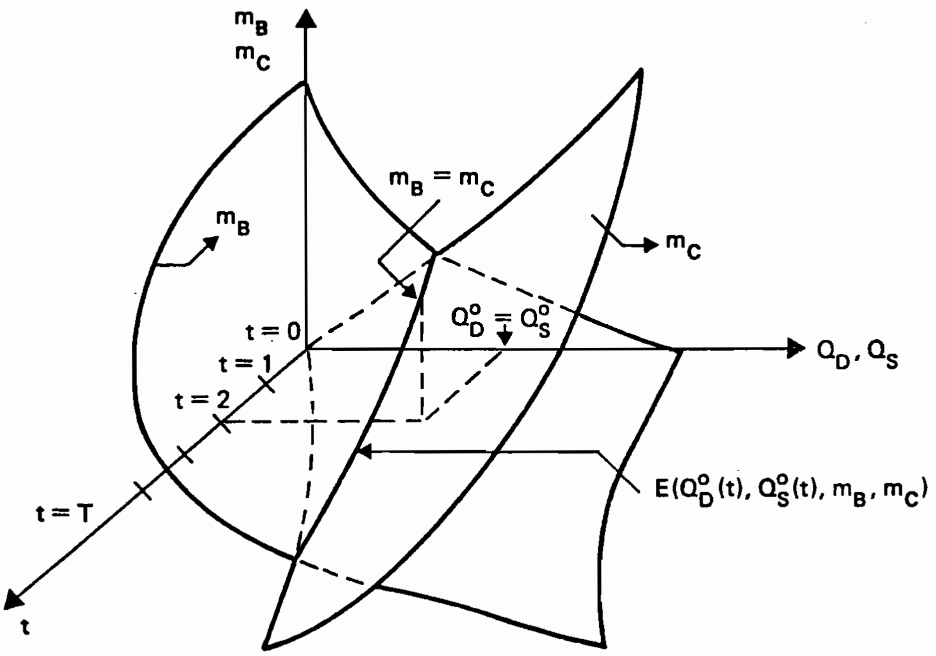

Figure A.1. Relations in the water demand model.

Note that the equations (A.3) and (A.4) reflect a pure market mechanism of water resource development. This mechanism is not an accurate description of water resources in either market or centrally planned economies. It just determines the socially optimal level of water resource development. In many cases, however, this level may be any solution of (A.3) and (A.4) if the equation (A.3) is satisfied but the equation (A.4) is an inequality, i.e.

$$Q_D^o(t) = Q_S^o(t), \qquad t=1,\ldots,T \tag{A.5}$$

$$m_B\left[Q_D^o(t)\right] - m_C\left[Q_S^o(t)\right] \geq B\ (\leq B), \qquad t=1,\ldots,T \tag{A.6}$$

where B is a positive number.

The inequality (A.6) implies that the supply-demand system is in quasi-equilibrium, i.e., there is pressure from the demand/supply side to increase the supply/demand, but decision makers prevent this for various other reasons. For example, in developing the water supply to meet the demand for drinking water, the marginal value of demand may be less than the marginal value of supply, and the system is still considered to be "in equilibrium" because this service, and the cost of providing it, are socially justified.

It is obvious, that whatever set of equations (A.3) and (A.4) or (A.5) and (A.6) is used in establishing an equilibrium in the water resource system, decision makers ought to receive information about marginal benefits, marginal costs and water demand in the agricultural system in order to make decisions about the development of this system.

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