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OIL PRICES, INDUSTRIAL PRICES AND OUTPUTS: A GENERAL EQUILIBRIUM MACRO ANALYSIS

Graciela Chichilnisky*

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INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
2361 Laxenburg, Austria

FOREWORD

This is one of three papers derived from research on North–South trade performed in the System and Decision Sciences Area during the summer of 1982. The aim of this research was, first, to construct a model of North–South resource trade and then to use it as a framework for further work in gaming, negotiations, and interactive decision making.

In this paper, a two-region general equilibrium macro model is constructed to explore the impacts of oil prices on output, employment and the prices of goods in industrial economies. It is shown that the effects of an increase in the price of oil depend considerably on the initial prices. The author examines different regimes, looking at their policy implications and in particular the possibility of cooperative pricing policies.

ANDRZEJ WIERZBICKI
Chairman
System and Decision Sciences

ABSTRACT

A two-region general equilibrium macro model is constructed to explore the impacts of oil prices on output, employment and prices of goods in industrial economies.

The industrial region is a *competitive market economy* that produces two goods (consumer and industrial) with three inputs (capital, labor and oil). It trades industrial goods for oil with another region. The oil-exporting region is a *monopoly* which sets the price of oil. The general equilibrium of the model determines endogenously the price and level of output of industrial goods, the volume of exports and imports, and the utilization and prices of factors in the industrial economy.

The results show that an increase in oil prices can have a number of outcomes. Depending on the initial oil price, the real revenues of the oil exporter may either *increase* or *decrease*. The rate of profit and net value of output in the industrial economy may also either *decrease* or *increase*, depending on initial prices. This paper examines different regimes, looking at their policy implications, and in particular the possibility of the importer and exporter adopting cooperative pricing policies.

A computer program (in BASIC) describing the model together with a number of runs are given in the Appendix.

OIL PRICES, INDUSTRIAL PRICES AND OUTPUTS: A GENERAL EQUILIBRIUM MACRO ANALYSIS

Graciela Chichilnisky*

1. INTRODUCTION

The increased activity of OPEC in the early seventies produced some of the most significant changes in the international economy in the post-war period. These changes coincided with a renewed awareness of the problems produced by the exhaustibility of natural resources. An extensive literature on the economics of exhaustible resources developed, based on the study of intertemporal optimality and efficiency of depletion paths in one-sector growth models, and looking at the effects of market structure on price and depletion paths, e.g., Stiglitz (1974, 1976), Sweeney (1977) and Dasgupta and Heal (1979).

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*Professor of Economics, Columbia University.

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While it is true that the intertemporal issues relating to depletion rates are the ones that most readily spring to mind when considering extractable resources, there are in fact a number of issues related to the pricing of such resources that can be analyzed in a static general equilibrium context. Amongst these are the effects of resource prices on the relative prices of different goods and services, their effects on international terms of trade, and their effects on the macroeconomic equilibria of the consuming countries. The prices of extractable resources are of course not unique in having such market effects, but it is nevertheless the case that these effects include some of the most widely-debated impacts of resource pricing policies. There is probably at least as much concern about the effects of oil prices on the macroeconomic equilibria of the consuming countries, on international terms of trade, and on the international distribution of wealth,* as there is about their effects on depletion rates. This paper emphasizes the behavior of international resource markets and the limitations this imposes on the plans of both exporters and importers.

Recent work in international economic theory has dealt with some aspects of these problems, e.g., Corden (1971) studied the short-run impact of oil prices within a one-sector (IS-LM) analysis of the world economy in which the redistribution of world income in favor of OPEC is seen as raising overall propensities to save. However, it is not possible to study the specific effects of oil policies on the importing countries using this model. Findlay and Rodriguez (1977) and Buiter (1978) studied a Fleming–Mundell model incorporating imports of intermediate goods, where the nominal price of oil is an exogenous variable to which a particular small open economy has to adjust. In a more

*The Bariloche model (see Chichilnisky, 1978) explored the long-run effects of self-sufficient growth patterns on output and distribution in the different regions, but no conclusions were drawn about the functioning of international markets, or about the main effects on employment, output and prices within the industrial oil-importing economies.

recent piece, Findlay (1983) studies the relation between the volume of oil exported by a cartel and the levels of output and employment within an oil-importing country, using a model in which the nominal price of one aggregate good is determined endogenously.* Other recent pieces of work in this area are due to Dixit (1981), Harkness (1980), Sachs (1983) and Djajic (1981).

In contrast with these works, this paper is an attempt to construct a model that is able to explain the domestic general equilibrium responses of an oil-importing country, a *competitive market* economy, to the pricing policies of a monopolistic exporter. The model formalizes the notion that the demand faced by the exporter shifts as a consequence of his own actions, through the impact of oil prices on the general equilibrium of the importing region. OPEC is one example of a monopolistic organization which should be interested in the general equilibrium implications of its own actions. Similar effects have already been formalized by Pearce (1953, 1956), Hahn (1977) and more recently by Hart (1982), the latter two in a general equilibrium context.** It will be shown here that consideration by the monopoly of these general equilibrium effects leads to policy implications rather different from those obtained using partial equilibrium analysis or standard general equilibrium models.

*However, the assumptions of Findlay's model rule out the study of certain important relationships between oil supplies, output and prices when the cartel's actions affect both the demand and the supply of the importing region, an issue which is at the core of much of the present North–South debate. For instance, in Findlay's model, the nominal wage in the importing region is assumed to be fixed, and, furthermore, real wages and rates of interest have no feedback on employment, except through real balance effects. Furthermore, the importing region produces one aggregate good, with the result that an increase in oil supplies unequivocally increases output and employment, and reduces the price level in this region. In the long run, through the assumption of a positive effect on demand of an increase in employment, the oil cartel is seen not to affect the level of employment, but to decrease the returns to factors instead.

**These publications relate to our model in the sense that they study the general equilibrium responses of demand to monopolistic policies. Pearce (1953, 1956) studied a closed economy in which total demand responds in a general equilibrium fashion to the actions of a monopoly and explored the implications of this response. More recently, Hahn (1977) studied the 'conjectural equilibria' of a closed economy, in which a firm attempts to consider the general equilibrium repercussions of its own actions. It appears that these works are quite closely related to the approach we follow here, and, indeed to the approach that seems most appropriate in the case of OPEC. In a recent piece, Hart (1982) examines in a general equilibrium framework the case of a monopolistic producer that can affect its own market. His approach contrasts with that adopted here in that the effects of the firm's policies are on other firms, rather than on the demand the firm itself faces.

It is clear that the optimal pricing policy for OPEC will depend upon the elasticity of demand for oil in the oil-importing countries and that this elasticity must be a *total* one, taking into account the effect of oil prices on output and employment in these countries. But, as mentioned in Dasgupta and Heal (1979, Chapter 11), this elasticity is likely to change because the equilibrium state of the industrial economy varies with different oil prices. Hence the total demand function facing the exporter may be very complex and have an elasticity that varies significantly with the price, reflecting overall changes in output, employment and prices within the industrial economy. A better understanding of the behavior of this elasticity would be of great value in predicting the consequences of alternative pricing policies. The present model provides a first step towards such an understanding. Moreover, when the general equilibrium effects of the monopoly's policies are taken into account, it is readily perceived that oil prices may have a non-trivial effect on the prices of goods imported by the oil exporter and thus on the "real" elasticity of demand for oil, measured in terms of the power of the oil exporter to purchase industrial goods. These ideas are formalized in this paper: the effects of oil price changes are traced across the equilibria of the model through the functioning of all markets in the importing region (markets for consumer and industrial goods, and for the three factors of production).

We prove here that if oil prices are initially low, an increase in the price of oil leads to an increase in the real revenues of the oil exporter in terms of industrial goods imported. Thus the real elasticity of demand for oil is greater than -1 in this case, and the optimal policy of the oil exporter is to increase prices.

However, after an oil price level $\bar{p}_{\vartheta}$ has been reached, further increases in price produce the opposite result: the real revenues of the oil exporter decrease. This is because increases in p_ϑ above $\bar{p}_\vartheta$ may in fact reduce total exports of industrial goods, either because of a fall in the rate of profit and the output of industrial goods, or because of income effects within the industrial countries. In the latter case, increases in domestic demand exceed the increase in the supply of industrial goods as the price of oil increases. Therefore the general equilibrium effect of higher oil prices is to reduce the volume of industrial goods exported.

A similar non-linear response takes place in the revenues of the industrial country: at low oil prices an increase in the price of oil leads to higher returns on domestic capital, and to higher levels of overall output, while the opposite occurs when oil prices start from a high level. The specific conditions under which these different "regimes" prevail, and their effect on output and employment within the industrial economy, provide the subject of this paper. Computer simulations of the situations obtained by varying the (exogenous) price of oil are given in the Appendix.

A further issue which has been extensively discussed, and on which this paper may throw some light, is whether the relationship between capital and energy is one of complementarity or substitutability. Empirical evidence on this is ambiguous (see Berndt and Wood, 1979), suggesting that in some countries the former is true and in others the latter. In the following analysis, we shall show that in some circumstances an increase in the price of oil will lead to an increase in the general equilibrium return on capital, whereas in others a decrease is observed. An increase in the return on capital can be interpreted as implying that, at the aggregate level, capital and oil are behaving as substitutes (an increase in the price of a factor will raise the demand for, and prices of, substitute factors), while a decrease can be interpreted as showing complementarity between these factors. Hence the aggregate cross-equilibrium relationship between capital and energy prices may display either complementarity or substitutability, depending on the values of parameters and endogenous variables. This point is developed further in Chichilnisky and Heal (1982).

2. A GENERAL EQUILIBRIUM MACRO MODEL

The model used here is based on Chichilnisky (1981), but has been extended to include oil as an imported input which is traded for industrial goods. There are a number of features which have been introduced primarily to keep the comparative static analysis tractable, and which are not essential to the results. These are the assumption of fixed-coefficient production processes, and the assumption that all wages are consumed in good B. The first (Leontief) assumption yields simple price equations, while the assumption concerning consumption simplifies certain cross-equilibrium relationships. Both can be relaxed without changing the basic qualitative features of the results — for example, similar results can be obtained from Cobb–Douglas production functions and from more general savings behavior. However, the increase in complexity is considerable.

The industrial economy is represented by a competitive general equilibrium model with two produced goods and three factors of production. In addition to the production and savings assumptions mentioned above, it is assumed that factor supplies in the industrial economy are sensitive to real factor prices. The supply of labor is thus an increasing function of the real wage, and the supply of capital an increasing function of the real return on capital. The assumption about labor supply is routine, but that concerning capital supply perhaps merits some comment. What we have in mind is a situation in which the economy has a capital stock composed of machinery of differing vintages and thus differing efficiencies. The fraction of this that is actually used at equilibrium will therefore depend on the factor prices, and will increase with the price of capital.* Alternatively this assumption could represent a form of factor mobility into or out of the region. The dependence of factor supplies upon their real rewards is assumed to be linear. It should be noted that in spite of the linearity of the factor supply equations and production functions, the supply, demand and production aspects of the model interact in such a way that its equilibrium relationships and comparative static properties are highly non-linear: they are in some cases of the fourth order.

The industrial country produces a basic consumer good and an industrial good, denoted by B and I respectively. There are three inputs to production: labor (L), capital (K) and oil (ϑ). Oil is not produced domestically. In order to simplify the analysis, the production functions of this country are of the fixed-proportion type

$$B^S = \min \left(L^B / a_1 , \vartheta^B / b_1 , K^B / c_1\right) \quad , \tag{1}$$

where L^B , ϑ^B and K^B denote inputs of labor, oil and capital into the production of the consumer good, and a_1 , b_1 and c_1 are the inverses of factor/output coefficients. Similarly, the production function for the industrial good is

$$I^S = \min \left(L^I / a_2 , \vartheta^I / b_2 , K^I / c_2\right) \quad . \tag{2}$$

The associated or 'dual' price equations (assuming competitive behavior) are then

$$p_B = a_1 w + b_1 p_\vartheta + c_1 r p_I \tag{3}$$

$$p_I = a_2 w + b_2 p_\vartheta + c_2 r p_I \quad , \tag{4}$$

*As an example, one could think of oil-consuming power plants of differing vintages and thus costs. The higher-cost plants are commonly brought into and out of production as relative prices change.

where w denotes wages, p_{ϑ} denotes the price of oil, r the quasi-rent of capital, p_I the price of the industrial good, and p_B the price of the consumer good. The factor rp_I is a proxy for the user's cost of capital, which enters as a cost. Although the two production functions both specify factor use in fixed proportions, they are assumed below to have very different oil use/output coefficients. In addition, it is shown that the patterns of demand imply substitutability between B and I. These two properties of the model imply that there is a considerable amount of substitution in the use of factors in the economy as a whole. In certain cases studied below this also implies significant changes in the elasticity of demand for oil as an input. We assume that factor supplies are sensitive to prices. If the price of the consumer good is taken as the unit of measurement, then labor supply is responsive to real wages:*

$$L^S = \alpha(\frac{w}{p_B}) , \quad \alpha > 0 \quad , \tag{5}$$

and capital is a function of the rate of profit r, i.e.,

$$K^S = \beta r , \quad \beta > 0 \quad . \tag{6}$$

Next we formulate the demand behavior, postulating that at equilibrium the value of basic goods consumed equals wage income, i.e.,

$$p_B B^D = wL^S \quad .^{**} \tag{7}$$

The market equilibrium conditions are

$$L^D = L^B + L^I \tag{8a}$$

$$K^D = K^B + K^I \tag{8b}$$

*Although α is taken to be positive here, it may in general be either positive or negative depending on whether labor supplies respond positively or negatively to wage increases. The response may be negative when leisure has a high level of utility, i.e., there is a "backward bending" labor supply.
**Demand behavior outside equilibrium is not considered here.

$$\vartheta^D = \vartheta^B + \vartheta^I \tag{8c}$$

$$K^S = c_1 B^S + c_2 I^S \quad (\text{i.e., } K^S = K^D) \tag{8d}$$

$$L^S = a_1 B^S + a_2 I^S \quad (\text{i.e., } L^S = L^D) \tag{8e}$$

$$B^D = B^S \tag{8f}$$

$$\vartheta^D = \vartheta^S + X_\vartheta^D \tag{8g}$$

$$I^D + X_I^S = I^S \tag{8h}$$

$$p_I X_I^S = p_\vartheta X_\vartheta^D \quad , \tag{8i}$$

where X_I^S denotes exports of I, X_ϑ^D denotes imports of oil and the superscripts D and S indicate domestic demand and supply. The last equation is a balance of payments condition. We assume that the industrial country produces no oil, so that all of the oil used must be imported ($\vartheta^D = X_\vartheta^D$).

The national income identity (national demand equals national income) for this model

$$p_B B^D + p_I I^D = wL^S + rp_I K^S \tag{9}$$

is always satisfied at equilibrium, when all markets are cleared.*

To summarize, the model's exogenous variables are the coefficients $a_1, a_2, b_1, b_2, c_1, c_2$, and the parameters α and β representing the

*This is easily verified by substituting for p_B and p_I from (3) and (4) in the left-hand side of (9):

$$\begin{aligned} p_B B^D + p_I I^D &= (a_1 w + b_1 p_\vartheta + c_1 r p_I) B^D \\ &\quad + (a_2 w + b_2 p_\vartheta + c_2 r p_I)(I^S - X_I^S) \\ &= w(a_1 B^S + a_2 I^S) + rp_I(c_1 B^S + c_2 I^S) \\ &\quad + p_\vartheta(b_1 B^S + b_2 I^S) - X_I^S(a_2 w + c_2 r p_I + b_2 p_\vartheta) \\ &= wL^S + rp_I K^S + p_\vartheta X_\vartheta^D - p_I X_I^S \quad . \end{aligned}$$

Since $p_\vartheta X_\vartheta^D = p_I X_I^S$, we obtain (9).

responses of domestic factor supplies to prices.

The model can be formalized more concisely as a system of five behavioral equations, (1), (2), (5), (6), (7), and nine equilibrium conditions, (8a–8i). The endogenous variables are: supply of I, I^S; demand for I, I^D; exports of I, X_I^S; supply of B, B^S; demand for B, B^D; supply of labor, L^S; demand for labor, L^D; capital supplied, K^S; capital demanded, K^D; oil supplied, ϑ^S; oil demanded, ϑ^D; oil imported, X_ϑ^D; and the prices in all five markets, i.e., rate of profit, r; wages, w; price of B, p_B; price of I, p_I; and the price of oil p_ϑ.

We therefore have 14 equations in 16 unknowns: in the usual general equilibrium fashion, the system can be solved up to one parameter value, taking one good as a numeraire. We choose the price of oil to be the exogenous parameter. The prices that emerge for the other goods are therefore relative prices. Because the terms of trade between oil and industrial goods represent a significant endogenous variable in the model, and in order to facilitate possible empirical interpretation of the results, we choose B to be the numeraire, i.e., $p_B = 1$. The oil exporter is therefore assumed to adopt a pricing policy by which he sets the price of oil relative to that of B.

3. OIL PRICES, INDUSTRIAL PRICES AND EXPORT REVENUES

We now study the general equilibrium responses of the industrial economy to changes in the exogenously determined price of oil. This is very much a comparative statics exercise, and involves a certain amount of computation (see the Appendix for details). The basic point is that as the price of oil varies, the equilibria of the model will in general form a one-parameter family, i.e., they will describe a curve in the space of endogenous variables. Along this curve, the prices of goods, wage and interest rates, output levels, relative prices of imports and exports (i.e., the terms of trade) and the quantity of exports are all endogenously related. We now study the behavior of these variables across different equilibria. In order to do this, we make certain assumptions about the production of consumer and industrial goods that simplify the computations. These are:

1. $M = a_1 b_2 - a_2 b_1 > 0$, i.e., the consumer good is relatively labor-intensive and the industrial good relatively oil-intensive.

2. $c_1 = 0$, so that the consumer good requires no capital inputs.

3. b_1 is small, so that the consumer good requires little in the way of oil inputs.

Assumption 2 is not strictly necessary to obtain the results: all that is required is that B be significantly less capital-intensive than I, so that there can be substitution in the aggregate use of factors. One could think of B as a non-traded relatively labor-intensive commodity, such as services. Computer runs of this model with c_1 small, but not zero, give similar results (see the Appendix).

It is also shown in the Appendix that the cross-equilibrium relationship between the price of the industrial good relative to that of the consumer good (p_I) and the price of oil relative to that of the consumer good (p_ϑ) is:

$$\frac{\partial p_I}{\partial p_\vartheta} = \frac{M[a_1 + b_1 \gamma p_\vartheta (b_1 p_\vartheta - 1)] - (a_2 + p_\vartheta M)\gamma b_1 (2 b_1 p_\vartheta - 1)}{[a_1 + \gamma b_1 p_\vartheta (b_1 p_\vartheta - 1)]^2} \quad , \tag{10}$$

where

$$\gamma = \frac{\alpha c_2^2}{\beta a_2} \quad .$$

Equation (10) implies that when condition 3 is satisfied (b_1 is small), and $\alpha c_2^2 < 2\beta a_1 a_2$ (this is case A of Theorem 1 – see later), the price of the industrial good relative to that of the basic good (p_I) increases as the relative price of oil increases. Intuitively it is not surprising that a small b_1 has this result; the industrial good is more oil-intensive than the basic good, so that its price should increase with the price of oil. When b_1 is not small, so that neither good is clearly more oil-intensive, the relative prices of the two goods may move in a more complex way in response to changes in the price of oil p_ϑ.

We next study how the rate of profit r varies with the price of oil. The relevant expressions are:

$$r = \frac{\alpha c_2 b_1}{\beta a_1 a_2}(p_\vartheta - b_1 p_\vartheta^2) \tag{11}$$

$$\frac{\partial r}{\partial p_\vartheta} = \frac{\alpha c_2 b_1}{\beta a_1 a_2}(1-2b_1 p_\vartheta) \quad ,$$

so that r is a quadratic function of p_ϑ, r is zero when p_ϑ is zero or $1/b_1$ and r attains its maximum at $p_\vartheta = 1/2b_1$.

These relationships are displayed in Figure 1. Their interpretation is related to the point made in the introduction about the complementarity or substitutability relationships between capital and oil emerging from the model. An increase in the price of oil always has two effects on the demand for capital: a substitution effect and an income effect. The substitution effect causes substitution of capital for oil. This occurs in the present model, not because of a shift along isoquants (there are fixed factor proportions in each firm), but because the relative prices of oil- and capital-intensive goods change, causing a change in demand patterns, production patterns and thus relative levels of employment of factors. The income effect simply reflects the fact that an increase in the price of oil reduces income and thus demand in the industrial economy. This will tend to depress the return on capital. What Figure 1 shows is that the substitution effect of an oil price increase dominates at low oil prices, and the income effect at higher prices. The current conventional wisdom that increases in oil prices depress the return on capital in industrial economies can thus be interpreted in terms of this model as showing that current oil prices must be in the range $1/2b_1$ to $1/b_1$.

The dependence of r on p_ϑ is also related to the issue of whether oil and capital are complements or substitutes. This point is developed further in Chichilnisky and Heal (1982). Here we just note that (6) shows that the supply of capital increases with r, so that the amount of capital used at equilibrium must increase and then decrease with the price of oil. This means that, across equilibria, the cross-price elasticity of demand between oil and capital is first positive and then negative, implying a switch from substitutability to complementarity. Note that the elasticity referred to here is defined in Chichilnisky and Heal (1982) as a total cross-price elasticity.

It is shown in the Appendix that exports from the industrial to the oil-producing country satisfy

$$X_I^S = \beta\left(\frac{\alpha_1 r}{D} - r^2\right) \tag{12}$$

and

$$\frac{\partial X_I^S}{\partial p_\vartheta} = \beta\left(\frac{\alpha_1}{D} - 2r\right)\frac{\partial r}{\partial p_\vartheta} \quad .$$

Two cases can be distinguished here. The first is when r is bounded below $\alpha_1/2D$, in which case $\partial X_I^S/\partial p_\vartheta$ has the same sign as $\partial r/\partial p_\vartheta$. Alternatively, if r can exceed $\alpha_1/2D$, then $\partial X_I^S/\partial p_\vartheta$ will vary from negative to positive as p_ϑ increases, while $\partial r/\partial p_\vartheta$ varies from positive to negative. The first case is easy to interpret: as the price of oil and the interest rate rise, the supply of capital increases, and with it the supply of capital-intensive industrial goods. This facilitates an increase in the export of industrial goods. Conversely, when the interest rate falls, capital employed, production of industrial goods and the volume of exports all decrease. This is shown in Figure 1. The second case is more complex, and is portrayed in Figure 2. As p_ϑ increases, and r with it, exports rise as long as $r < a_1/2D$. At this point exports fall as r continues to rise, then rise as r falls, until r once more reaches $a_1/2D$, at which point both X_I^S and r move down as p_ϑ rises. What is happening in this case is that as p_ϑ rises, r increases and with it the supply of capital and the output of industrial goods, as before. However, in this case the parameters of the system are such that an increase in profits, all of which are spent on industrial goods (equations (9) and (7)), leads to an increase in the demand for these goods which exceeds the supply. The difference between supply and demand, which is exports, therefore falls. We can confirm this by noting that this occurs when

$$r_{max} > a_1/2D \quad ,$$

which can happen if and only if

$$c_2^2 > \frac{2\beta a_1 a_2}{\alpha} \quad .$$

c_2 is the inverse of the capital/output ratio in the industrial goods sector: the higher the value of c_2 and the lower the value of β (the responsiveness of capital to interest rates), the smaller the response of industrial output to the supply of capital and hence to r and p_ϑ. And it is obvious that the smaller the response of industrial output to capital supply, the more likely it is that output will fall short of the increase in demand, leading to a drop in exports.

We have now prepared the ground for the main results of this section.

Theorem 1. *If the initial price of oil p_ϑ is low, an increase in this price will increase the volume of industrial goods exported X_I^S. However, once the price* *of oil has reached a certain value $\bar{p}_\vartheta$, further increases in this price will lead to a decrease in the volume of industrial goods exported X_I^S. There are then two possibilities:*

A. $\alpha c_2^2 \leq 2\beta a_1 a_2$. *In this case, increases in the price of oil to above $\bar{p}_\vartheta = 1/2b_1$ decrease the rate of profit r in the industrial economy. This decreases the total capital available, and thus decreases the domestic supply of industrial goods and the volume available for export (see Figure 1).*

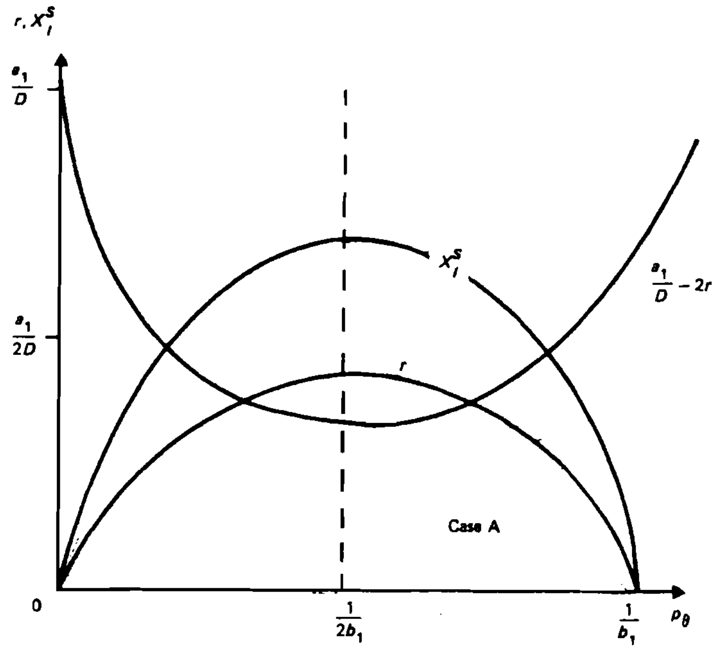

Figure 1. Case A: $\alpha c_2^2 < 2\beta a_1 a_2$, i.e., r is always bounded below $a_1/2D$. In this case the volume of industrial exports X_I^S initially increases with the price of oil. For $p_\vartheta > 1/2b_1$, however, X_I^S decreases with further increases in p_ϑ. This is due to the effect of oil prices on the rate of profit in the industrial economy.

B. $\alpha c_2^2 > 2\beta a_1 a_2$. *Now* $\bar{p}_\vartheta = (\frac{1}{2b_1}(1 - (\frac{\gamma - 2a_1}{\gamma})^{\frac{1}{2}})$. *Between $\bar{p}_\vartheta$ and $1/2b_1$, increases in the price of oil raise the rate of profit but lower the exports of industrial goods. For* $\frac{1}{2b_1} < p_\vartheta < \frac{1}{2b_1}(1 + (\frac{\gamma - 2a_1}{\gamma})^{\frac{1}{2}})$, *an increase in oil* *prices lowers the rate of profit and raises exports, and for higher oil prices both exports and the rate of profit fall as p_ϑ increase (see Figure 2).*

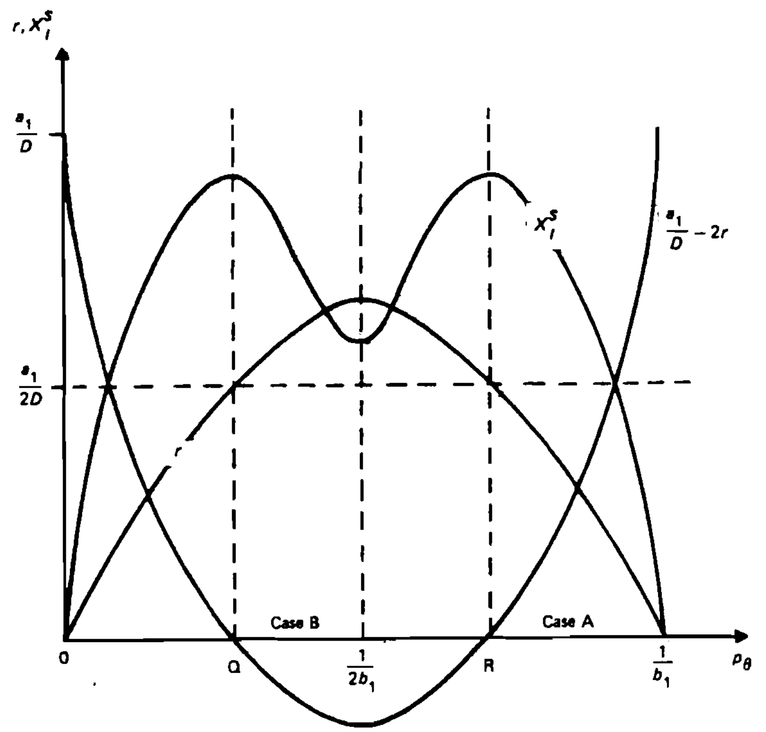

Figure 2. Case B: $\alpha c_2^2 > 2\beta a_1 a_2$, so that r exceeds $a_1/2D$ for $(1/2b_1)(1-[(\gamma-2a_1)/\gamma]^{1/2}) \le p_\vartheta \le (1/2b_1)(1+[(\gamma-2a_1)/\gamma]^{1/2})$. These limits are indicated as Q and R, respectively, in the figure. This result is due to cross-equilibria income effects: increases in the price of oil increase the rate of profit r, but this produces increases in demand that exceed increases in the supply of industrial goods within the industrial economy. This also happens in Case A, but since r decreases with p_ϑ in this latter case, the total effect is that the volume of exports increases with p_ϑ.

The proof of this theorem is given in the Appendix.

The final issue that we shall discuss in this section is the relationship between the price of oil and the elasticity of demand faced by the oil exporter. We are interested in the real elasticity of demand: the real revenue of the oil exporter is $p_\vartheta X_\vartheta^S / p_I$ (as only industrial goods are imported), and it is then natural to define the real elasticity as

$$\partial\eta = \frac{(X_\vartheta^S / p_\vartheta)}{\partial p_\vartheta} \cdot \frac{p_\vartheta p_I}{X_\vartheta^S} \quad .$$

It is shown in the Appendix that

$$\eta > -1 \quad \text{if and only if } \frac{\partial X_I^S}{\partial p_\vartheta} > 0$$

$$\eta < -1 \quad \text{if and only if } \frac{\partial X_I^S}{\partial p_\vartheta} < 0 \quad .$$

The behavior of the elasticity of demand with respect to the price of oil is then as shown in Figures 3 and 4 for cases A and B of Theorem 1.

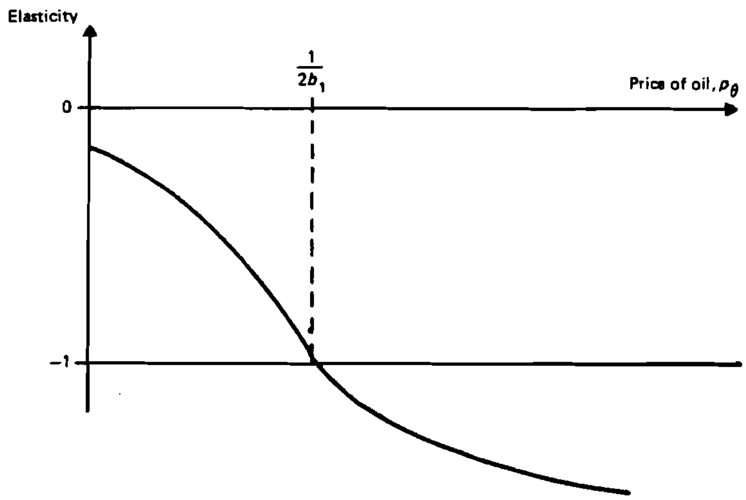

Figure 3. Elasticity of demand as a function of p_ϑ when $\alpha c_2^2 \leq 2\beta a_1 a_2$ (Case A).

These results confirm our expectation that the elasticity of demand facing the oil exporter will change as the equilibrium of the oil-consuming country changes.* A demand relationship as complex as that shown in Figure 4 has

*The nominal elasticity of the demand for oil ε can also be computed directly from its definition

$$\varepsilon = \frac{\partial\vartheta^D}{\partial p_\vartheta}\left(\frac{p_\vartheta}{\vartheta^D}\right).$$

From (12)

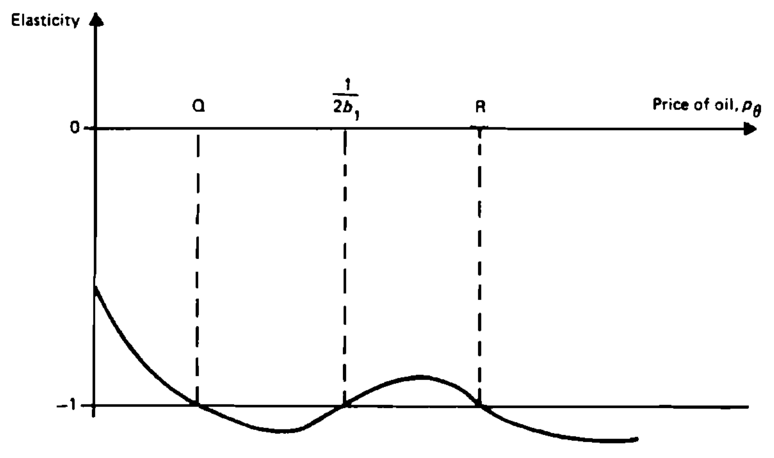

Figure 4. Elasticity of demand as a function of p_ϑ when $\alpha c_2^2 > 2\beta a_1 a_2$ (Case B). Here Q and R are given by $p_\vartheta = (1/2b_1)(1 \pm [(\gamma-2a_1)/\gamma]^{1/2})$.

significant implications for the optimal long-run monopoly pricing policy, which involves prices increasing at the same rate as the demand elasticity changes (Dasgupta and Heal, 1979, Chapter 12).

4. INDUSTRIAL OUTPUT AND EMPLOYMENT

We study now the effect of variations in the price of oil on the overall levels of output and employment in the industrial country, across equilibria.

We shall denote by Y the total value of (net) domestic output, i.e., the value of total output minus the value of the imported input:

$$Y = p_B B^S + p_I I^S - p_\vartheta X_\vartheta^D \quad .$$

$$\varepsilon = \frac{2b_1 p_\vartheta (a_1 b_2 + a_2 b_1)(r - b_1 p_\vartheta)}{(1 - b_1 p_\vartheta)(a_1 b_2 b_1 p_\vartheta + a_2 b_1 (1 - b_1 p_\vartheta))}$$

where

$$r = \frac{a_1 b_2 - 2a_2 b_1}{2(a_1 b_2 + a_2 b_1)} \quad ,$$

and is therefore also seen to vary with the price of oil p_ϑ.

Y is the overall domestic value-added in the industrial economy. Since at equilibrium $p_\vartheta X_\vartheta^D = p_I X_I^S$ and $B^S = B^D$, it follows that

$$Y = p_B B^D + p_I I^D \quad . \tag{13}$$

Our next aim is to compute changes in the equilibrium values of Y and of total overall employment L as the price of oil p_ϑ varies. We first study the effect on employment.

Note that by (3), when $c_1 = 0$

$$w = \frac{1-b_1 p_\vartheta}{a_1}$$

$$\frac{\partial w}{\partial p_\vartheta} = -\frac{b_1}{a_1} \tag{14}$$

Therefore, since $L = \alpha w$, *increases in p_ϑ always decrease overall employment* when $\alpha > 0$. When $c_1 \neq 0$, $\partial w / \partial p_\vartheta$ can be either positive or negative.

Next we study the effect of oil prices on consumption. From (5), (6), (7), (9) and (13)

$$Y = \alpha w^2 + p_I \beta r^2$$

so that

$$\frac{\partial Y}{\partial p_\vartheta} = 2\alpha w \frac{\partial w}{\partial p_\vartheta} + \beta\left(r^2 \frac{\partial p_I}{\partial p_\vartheta} + 2 p_I r \frac{\partial r}{\partial p_\vartheta}\right) \quad . \tag{15}$$

Note that because $p_B B^D = \alpha w^2$ and $p_B = 1$, and $c_1 = 0$

$$\frac{\partial B^D}{\partial p_\vartheta} = 2\alpha w \frac{\partial w}{\partial p_\vartheta} = (b_1 p_\vartheta - 1)\frac{2\alpha b_1}{a_1^2} \quad . \tag{16}$$

Therefore, when $\alpha > 0$ and $0 \leq p_\vartheta \leq 1/b_1$, *domestic consumption of B (B^D) decreases as the price of oil increases. This does not necessarily happen when $c_1 \neq 0$.*

Finally, we use the above analysis to study how output Y responds to changes in p_ϑ. Since $r = 0$ when $p_\vartheta = 0$, (14) and (15) imply that overall output is a decreasing function of p_ϑ for small values of p_ϑ. However, since $\partial r / \partial p_\vartheta \geq 0$ for $p_\vartheta \leq 1/2b_1$, and $b_1 \sim 0$, Y is an increasing function of the price of oil when p_ϑ is greater than some small value (denoted by p_ϑ^3). This is due to the fact that, as p_ϑ increases, the value of the demand for I, $p_I I^D = p_I \beta r^2$, also increases, since $\partial r / \partial p_\vartheta \geq 0$ and $\partial p_I / \partial p_\vartheta \geq 0$ when $p_\vartheta \leq \frac{1}{2b_1}\left[1 + \left\{\frac{\gamma - 2a_1}{\gamma}\right\}^{1/2}\right]$, see Figure 2. Since b_1 is rather small, the increase in $p_I I^D$ in (13) will exceed the decrease in the value of demand for B, B^D, so that Y increases with the price of oil.

Finally, note that when p_ϑ exceeds a certain value p_ϑ^4, $\partial Y / \partial p_\vartheta$ becomes negative again, because $r = 0$ when $p_\vartheta = 1/b_1$, $\partial r / \partial p_\vartheta < 0$ for $p_\vartheta > 1/2b_1$, and $\partial B^D / \partial p_\vartheta < 0$, and $r(\partial p_I / \partial p_\vartheta)$ is bounded above by $-(M\alpha c_2 b_1 / \beta a_1 a_2) p_\vartheta$, from (10) and (11). We therefore have the following situation (see Figure 5):

Theorem 2. *When the initial price of oil p_ϑ is close to zero, an increase in this price lowers the level of net overall domestic output in the industrial economy. When oil reaches a price p_ϑ^3, however, further price increases will increase overall output in the industrial economy. When p_ϑ reaches a value $p_\vartheta^4 > p_\vartheta^3$, increasing the price of oil still further will reverse the situation once again, overall output now decreasing with increasing p_ϑ. The overall level of employment in the economy decreases as the price of oil rises, as does the level of consumption of basic consumer goods.*

The fact that domestic employment and consumption of basic goods decrease monotonically with increases in the price of oil will occasion little surprise.

What is less obvious is that there is a range of oil prices for which value-added increases with increases in the price of oil p_ϑ. As indicated above, this occurs because, over a certain range of values, increases in p_ϑ lead to the substitution of capital for oil, and hence to increases in both profits and demand.

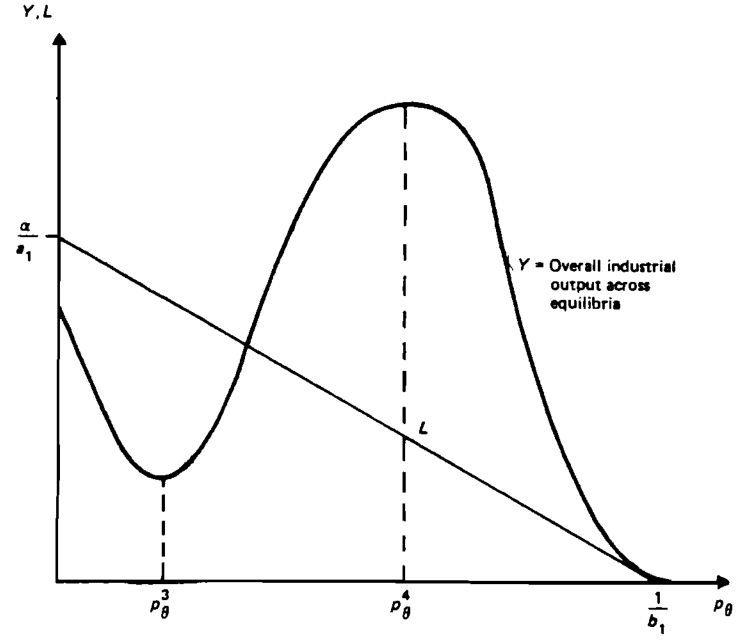

Figure 5. The behavior of overall output Y and employment L with increases in the price of oil, across equilibria.

5. CONCLUSIONS

In the previous sections we presented a simple general equilibrium model of an oil-consuming country, and used this to analyze the macroeconomic responses of such a country to changes in the price of oil. We studied the responses of outputs, prices, profits and consumption levels in the oil-consuming country, and showed how these depend on parameter values and the price of oil. A number of conclusions were relatively straightforward — for example, if $c_1 = 0$ employment and consumption of B decrease as the price of oil rises. Others, however, were less obvious, suggesting that the full impact of an oil price increase is very complex. It was shown that profitability first rises and then falls with an increase in the price of oil. The switch reflects the changing importance of income and substitution effects. Substitution of oil by capital bids up the return on capital, athough beyond a certain point this is outweighed by the demand-reducing effects of higher oil prices. Also not immediately apparent is the fact that in certain regions an increase in the price of oil may *increase* total value-added in the oil-using economy. This is because in some cases it leads to an increase in profitability, as noted above, and hence to increased demand.

It emerges from this analysis that there are certain situations in which an increase in oil prices may cause profitability and output in the industrial economy to rise, even though employment and consumption of basic goods will fall. In other situations, all four variables will decrease as the oil price rises. In the former case, one cannot say that the increase in oil prices is unambiguously harmful: indeed, taking GNP as a welfare index, it could be judged beneficial. Another issue that we examined was the effect of oil price changes on the export of industrial goods to the oil producing country. This can be rather complex: the plot of real exports aganst oil prices may have either one or two maxima, which implies that the cross-equilibrium demand function facing the oil exporter may be far removed from the simple functions often used in dynamic studies. We also characterized precisely how the elasticity varies with the price of oil.

One interesting implication of these results is that there may be situations where an increase in the price of oil will benefit both oil exporters (by raising their real revenues) and oil importers (by raising their profits and GNP). These two groups are therefore not always playing a zero-sum game.

APPENDIX

The Relationships Between P_ϑ and the Equilibrium Values of Endogenous Variables

First note that the production functions (1) and (2) yield equations for the demand for factors L, K, and ϑ at each level of output, assuming that the factors are used efficiently:

$$L^D = B^S a_1 + I^S a_2 \tag{A.1}$$

$$K^D = B^S c_1 + I^S c_2 \tag{A.2}$$

$$\vartheta^D = B^S b_1 + I^S b_2 \quad . \tag{A.3}$$

Equations (A.1) and (A.2) imply that when factors are used efficiently

$$B^S = (c_2 L^S - a_2 K^S)/D \tag{A.4}$$

$$I^S = (a_1 K^S - c_1 L^S)/D \quad , \tag{A.5}$$

where D is the determinant of the matrix

$$\begin{bmatrix} a_1 & a_2 \\ c_1 & c_2 \end{bmatrix}$$

The price equations (3) and (4) can be regarded as a system of two equations in two variables, w and r, when p_ϑ is a constant. From these equations we obtain

$$w = \frac{(p_B - b_1 p_\vartheta)c_2 - (p_I - b_2 p_\vartheta)c_1}{D} \quad \text{(A.6)}$$

$$r = \frac{a_1(p_I - b_2 p_\vartheta) - a_2(p_B - b_1 p_\vartheta)}{Dp_I} \quad . \quad \text{(A.7)}$$

Substituting L^S and K^S from (5) and (6), and w and r from (A.6) and (A.7) into (A.4), we obtain the equilibrium values of the supply of consumer goods B^S as a function only of the price of industrial goods p_I:

$$B^S = (c_2\alpha w - a_2\beta r)/D = \frac{\alpha c_2}{D^2}(c_2 + p_\vartheta N - c_1 p_I) + \frac{\beta a_2}{D^2}\left(\frac{p_\vartheta M}{p_I} + \frac{a_2}{p_I} - a_1\right) \quad , \quad \text{(A.8)}$$

where

$$M = a_1 b_2 - a_2 b_1$$

$$N = c_1 b_2 - c_2 b_1 \quad .$$

Similarly, substituting the expressions for K^S, L^S, w and r into (A.5) leads to

$$I^S = \frac{\beta a_1}{D^2}\left(a_1 - \frac{p_\vartheta M}{p_I} - \frac{a_2}{p_I}\right) + \frac{\alpha c_1}{D^2}(p_I c_1 - c_2 - p_\vartheta N) \quad . \quad \text{(A.9)}$$

Now, from the demand relation (7) and the accounting identity (9), we have at equilibrium

$$I^D = rK^S \quad . \quad \text{(A.10)}$$

Therefore, when $p_B = 1$, the equilibrium relation $B^S = B^D$ can be rewritten, using (5), (A.6) and (A.8), as follows:

$$\alpha c_2(c_2+p_\vartheta N-c_1p_I)+\beta a_2\left(\frac{p_\vartheta M}{p_I}+\frac{a_2}{p_I}-a_1\right) \tag{A.11}$$
$$=\alpha[(1-b_1p_\vartheta)c_2-(p_I-b_2p_\vartheta)c_1]^2$$

while from (6), (A.9) and (A.10), $I^S = I^D + X_I^S$ is

$$\beta a_1\left(a_1-\frac{p_\vartheta M}{p_I}-\frac{a_2}{p_I}\right)+\alpha c_1(p_Ic_1-c_2-p_\vartheta N) \tag{A.12}$$
$$=\frac{\beta}{p_I^2}[a_1(p_I-b_2p_\vartheta)-a_2(1-b_1p_\vartheta)]^2+X_I^S$$

From equation (9), equations (A.11) and (A.12) are not independent at equilibrium. The implicit function theorem implies that one can obtain, at least locally, a function $p_I = p_I(p_\vartheta)$ from (A.11). Therefore, since p_ϑ is given, the value of p_I at equilibrium can be obtained. This, with (A.8) and (A.9), yields the equilibrium supply of B and I, B^S and I^S. Equilibrium values of wages and profits w and r can be deduced from (A.6) and (A.7), and the equilibrium use of inputs K^S and L^S from (5) and (6). This allows I^D to be calculated from (A.10), so that the volume of industrial exports X_I^S can be found; the volume of oil imported can then be computed from (8i). The model is therefore 'closed', i.e., its equilibria are predetermined (and locally unique) for given p_ϑ. When p_ϑ is changed, the equilibrium values of all endogenous variables will change. In particular, the volume of industrial exports X_I^S and their price p_I will change, and our next goal is to compute how they change in relation to each other across equilibria. We now make some assumptions which simplify the computations, and which are discussed in more detail earlier in the text.

1. $M = a_1b_2 - a_2b_1 > 0$, i.e., B is more labor-intensive and I more oil-intensive.

2. $c_1 = 0$, i.e., B requires no capital inputs.

3. b_1 is small, i.e., B requires only a small input of oil.

From (A.11), using assumption 2, one can obtain an explicit expression for $p_I = p_I(p_\vartheta)$:

$$p_I = \frac{a_2 + p_\vartheta M}{\gamma b_1 p_\vartheta (b_1 p_\vartheta - 1) + a_1} \quad , \tag{A.13}$$

where

$$\gamma = \frac{\alpha}{\beta} \frac{c_2^2}{a_2}$$

and

$$M = a_1 b_2 - a_2 b_1$$

Consider now the possible range in which p_ϑ can vary. From the price equation (3), $w \geq 0$ implies $1/b_1 \geq p_\vartheta \geq 0$, since $c_1 = 0$. From (A.13) we have, when $p_\vartheta = 0$,

$$p_I = \frac{a_2}{a_1} \quad , \tag{A.14}$$

and when $p_\vartheta = \frac{1}{b_1}$,

$$p_I = \frac{b_2}{b_1} \quad . \tag{A.15}$$

Note that since $M > 0$, $a_2/a_1 < b_2/b_1$. We can now study the change in the price of the industrial good p_I as the price of oil increases:

$$\frac{\partial p_I}{\partial p_\vartheta} = \frac{M[a_1 + \gamma b_1 p_\vartheta (b_1 p_\vartheta - 1)] - (a_2 + p_\vartheta M)\gamma b_1 (2 p_\vartheta b_1 - 1)}{[a_1 + \gamma b_1 p_\vartheta (b_1 p_\vartheta - 1)]^2} \tag{A.16}$$

$$= \frac{f(p_\vartheta)}{a_1 + \gamma b_1 p_\vartheta (b_1 p_\vartheta - 1)^2}$$

The sign of (A.16) is therefore that of the quadratic function $f(p_\vartheta) = -(M\gamma b_1^2)p_\vartheta^2 - (2a_2\gamma b_1^2)p_\vartheta + a_1M + a_2\gamma b_1$, which is illustrated in Figure A1.

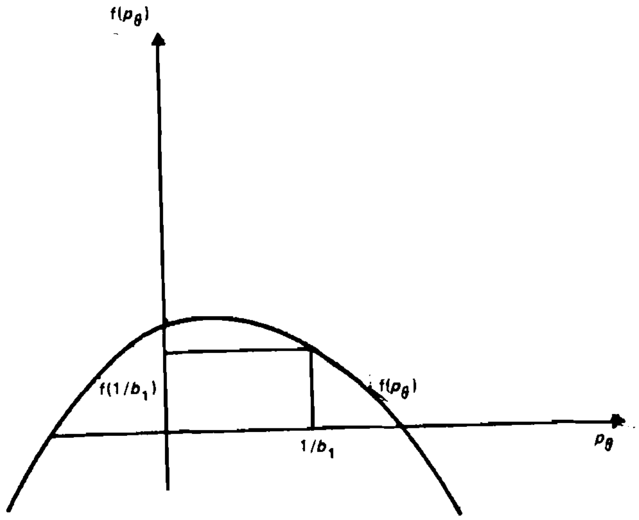

Figure A1. Plot of the quadratic function $f(p_\vartheta) = -(M\gamma b_1^2)p_\vartheta^2 - (2a_2\gamma b_1^2)p_\vartheta + a_1M + a_2\gamma b_1$.

It is easy to see that $f(p_\vartheta) = 0$ has only one positive root p_ϑ^*. In order that $\partial p_I / \partial p_\vartheta$ be positive for all p_ϑ between 0 and $1/b_1$, it is necessary and sufficient that $f(1/b_1) > 0$. But $f(1/b_1) = -M\gamma - \gamma a_2 b_1 + Ma_1 > 0$ and so

$$b_1 < \frac{b_2(\beta a_1 a_2 - \alpha c_2^2)}{\beta a_2^2} \qquad M > b_2\gamma \quad .$$

Therefore $\partial p_I / \partial p_\vartheta > 0$ if and only if

$$\frac{\beta a_1 a_2}{\alpha c_2^2} > 1 \text{ and } b_1 < \frac{b_2(\beta a_1 a_2 - \alpha c_2^2)}{\beta a_2^2} \quad ,$$

i.e., if we have case A discussed in the text $(\alpha c_2^2 \leq \beta a_1 a_2)$ and b_1 is small. Now, from (3)

$$w = \frac{1-b_1 p_\vartheta}{a_1} \tag{A.17}$$

and thus (4) implies

$$r = \frac{1}{c_2} - \frac{a_2 + p_\vartheta M}{a_1 c_2 p_I} \quad . \tag{A.18}$$

Substituting for p_I from (A.13) we obtain

$$r = \frac{\alpha c_2 b_1}{\beta a_1 a_2}(p_\vartheta - b_1 p_\vartheta^2) \quad . \tag{A.19}$$

Therefore $r = 0$ both when p_ϑ is zero and when it assumes its maximum value $1/b_1$. The change in the rate of profit as the price of oil varies is

$$\frac{\partial r}{\partial p_\vartheta} = \frac{\alpha c_2 b_1}{\beta a_1 a_2}(1 - 2p_\vartheta b_1) \quad . \tag{A.20}$$

Since

$$\frac{\partial r}{\partial p_\vartheta} = 0 \qquad p_\vartheta = \frac{1}{2b_1} \quad , \tag{A.21}$$

and r is quadratic in p_ϑ, it follows that the rate of profit is an increasing function of p_ϑ for $p_\vartheta < 1/2b_1$, and a decreasing function for $p_\vartheta > 1/2b_1$. Since r attains its maximum value when $p_\vartheta = 1/2b_1$, the maximum value of r is

$$r_{\max} = \frac{\alpha c_2}{4\beta a_1 a_2} \quad . \tag{A.22}$$

Note that the conclusion in fact requires only condition 2 above, i.e., that $c_1 = 0$.

We shall next analyze the response of the volume of industrial goods exported to changes in the price of oil. Since $X_I^S = I^S - I^D$, we deduce from (6), (A.5) and (A.10) that

$$X_I^S = \beta\left(\frac{a_1 r}{D} - r^2\right) \quad . \tag{A.23}$$

Therefore

$$\frac{\partial X_I^S}{\partial p_\vartheta} = \beta(\frac{a_1}{D} - 2r)\frac{\partial r}{\partial p_\vartheta} \quad . \tag{A.24}$$

Note that $r_{\max} > a_1/2D$ if and only if

$$\alpha c_2^2 > 2\beta a_1 a_2 \quad . \tag{A.25}$$

Proof of Theorem 1

Consider first the case where $\alpha c_2^2 < 2\beta a_1 a_2$. Then r is always bounded above by $a_1/2D$. In this case the sign of $\partial X_I^S/\partial p_\vartheta$ is the same as that of $\partial r/\partial p_\vartheta$, from (A.24). Furthermore, $\partial r/\partial p_\vartheta > 0$ for $p_\vartheta \leq 1/2b_1$ and $\partial r/\partial p_\vartheta < 0$ for $p_\vartheta > 1/2b_1$ from (A.19)–(A.22). Here as the rate of profit increases, the supply of industrial goods increases more than does the domestic demand for these goods, since $I^S = a_1\beta r/D$ and $I^D = \beta r^2$ (from (6), (A.5) and (A.10)), and $a_1/2D > r$, so that $\partial(I^S - I^D)/\partial r = \beta[(a_1/D) - 2r] > 0$.

Therefore, since $X_I^S = I^S - I^D$ and by (A.24) $\partial X_I^S/\partial p_\vartheta = \beta[(a_1/D) - 2r](\partial r/\partial p_\vartheta)$, the volume of industrial goods exported at equilibrium X_I^S will increase with increases in the price of oil for $p_\vartheta < 1/2b_1$. For $p_\vartheta > 1/2b_1$, however, this relation is reversed: increases in the price of oil will now decrease the volume of industrial goods exported across equilibria. In both cases considered here this change in the reaction of industrial exports X_I^S to increases in the price of oil p_ϑ is related to the change in the response of the rate of profit to increases in the price of oil p_ϑ, as illustrated in Figure 1.

Consider now the case where $\alpha c_2^2 > 2\beta a_1 a_2$. In this case $r > a_1/2D$ for values of p_ϑ near $1/2b_1$. When $r > a_1/2D$, $\partial I^S/\partial r < \partial I^D/\partial r$, so that the increase in the supply of industrial goods is exceeded by the increase in the domestic demand as the rate of profit rises, i.e., the cross-equilibrium income effect dominates the price (or substitution) effect. In this case, the reaction of the volume of industrial exports to changes in the price of oil, $\partial X_I^S / \partial p_\vartheta$, depends both on the sign of the change in the rate of profit r , $\partial r / \partial p_\vartheta$, and on the sign of $(a_1/D) - 2r$. In fact $\partial X_I^S / \partial p_\vartheta = 0$ $\quad$ $\partial r / \partial p_\vartheta = 0$, or $a_1/D = 2r$. This occurs when $p_\vartheta = (1/2b_1)(1 \pm [(\gamma - 2a_1)/\gamma]^{1/2})$. It follows that we have a succession of different situations as p_ϑ increases from 0 to $1/b_1$:

I. $\dfrac{\partial r}{\partial p_\vartheta} \geq 0$ and $r \leq \dfrac{a_1}{2D}$, i.e., $\dfrac{\partial X_I^S}{\partial p_\vartheta} \geq 0$

II. $\dfrac{\partial r}{\partial p_\vartheta} \geq 0$ and $r > \dfrac{a_1}{2D}$, i.e., $\dfrac{\partial X_I^S}{\partial p_\vartheta} \leq 0$

III. $\dfrac{\partial r}{\partial p_\vartheta} \leq 0$ and $r > \dfrac{a_1}{2D}$, i.e. $\dfrac{\partial X_I^S}{\partial p_\vartheta} \geq 0$

IV. $\dfrac{\partial r}{\partial p_\vartheta} \leq 0$ and $r \leq \dfrac{a_1}{2D}$, i.e. $\dfrac{\partial X_I^S}{\partial p_\vartheta} \leq 0$.

This is illustrated in Figure 2, and completes the proof of the theorem.

Behavior of the Demand Elasticity Facing the Oil Exporter

Firstly, note that the "real" revenue of the oil exporter is $p_\vartheta X_\vartheta^S / p_I$, which in an international market equilibrium equals X_I^D, where X_I^D is the amount of industrial goods imported and X_ϑ^S is the amount of oil exported.

Now by definition the real elasticity of demand for oil is given by

$$\eta = \frac{\partial (X_\vartheta^S / p_I)}{\partial p_\vartheta} \cdot \left[\frac{p_\vartheta p_I}{X_\vartheta^S} \right]$$

since the demand is basically the export demand.

If R denotes *real revenues*, then across equilibria we have for the oil exporter

$$R = \frac{p_\vartheta X_\vartheta^S}{p_I} \quad .$$

It follows that

$$\frac{\partial R}{\partial p_{\vartheta}} = \frac{\partial(X_{\vartheta}^{S}/p_I)}{\partial p_{\vartheta}} p_{\vartheta} + \frac{X_{\vartheta}^{S}}{p_I}$$
$$= \frac{X_{\vartheta}^{S}}{p_I}(\eta + 1) \quad .$$

Therefore $\partial R/\partial p_{\vartheta} > 0$ if and only if $\eta > -1$, or $|\eta| < 1$ for $\eta < 0$, and $\partial R/\partial p_{\vartheta} < 0$ if and only if $\eta < -1$, or $|\eta| > 1$, for $\eta < 0$. Since $R = p_{\vartheta} X_{\vartheta}^{S}/p_I = X_I^{D}$, it follows that $\eta > -1$ when $p_{\vartheta} < \bar{p}_{\vartheta}$ and $\eta < -1$ when $p_{\vartheta} > \bar{p}_{\vartheta}$.

Computer Simulations for the Industrial Country

Run 1. The following parameter and factor response values were used:

$\alpha = 1.00$	$b_1 = 0.10$
$\beta = 2.00$	$b_2 = 0.20$
$a_1 = 0.30$	$c_1 = 0.00$
$a_2 = 0.20$	$c_2 = 0.60$

Table A1. Results of the first run, with $c_1 = 0$. These results are illustrated in Figure A2.

Exogenously set price of oil (p_ϑ)	Values of endogenous variables					
	p_I	r	w	$p_I X_I^S$	Y	X_ϑ^D
0.0	0.666667	0.	3.33333	0.	11.1111	1.111
0.5	0.855199	0.2375	3.16667	0.580556	10.1243	1.161
1.0	1.09589	0.4500	3.00000	1.20000	9.44384	1.2
1.5	1.40351	0.6375	2.83333	1.84167	9.16857	1.227
2.0	1.79487	0.8000	2.66667	2.48889	9.40855	1.24
2.5	2.28571	0.9375	2.50000	3.12500	10.2679	1.25
3.0	2.88288	1.0500	2.33333	3.73333	11.8012	1.244
3.5	3.56955	1.1375	2.16667	4.29722	13.9318	1.227
4.0	4.28571	1.2000	2.00000	4.80000	16.3429	1.2
4.5	4.91909	1.2375	1.83333	5.22500	18.4274	1.161
5.0	5.33333	1.2500	1.66667	5.55556	19.4444	1.111
5.5	5.43689	1.2375	1.50000	5.77500	18.9022	1.05
6.0	5.23810	1.2000	1.33333	5.86667	16.8635	0.977
6.5	4.82940	1.1375	1.16667	5.81389	13.8587	0.894
7.0	4.32432	1.0500	1.00000	5.60000	10.5351	0.8
7.5	3.80952	0.9375	0.83333	5.20833	7.39087	0.694
8.0	3.33333	0.8000	0.666667	4.62222	4.71111	0.577
8.5	2.91498	0.6375	0.500000	3.82500	2.61933	0.45
9.0	2.55708	0.4500	0.333333	2.80000	1.14673	0.31
9.5	2.25462	0.2375	0.166667	1.53056	0.282127	0.16
10.0	2.00000	0.	0.	0.	0.	0.

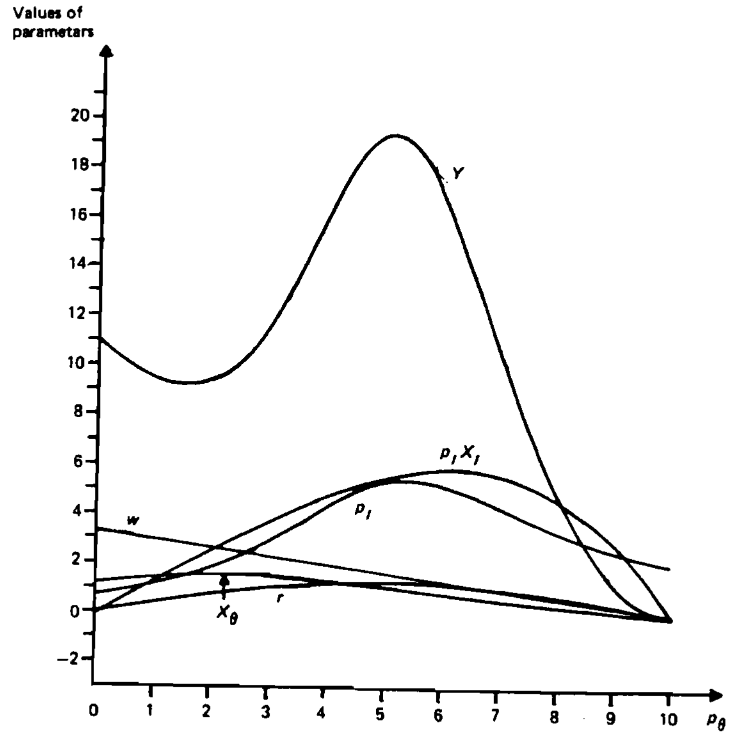

Figure A2. Behavior of the endogenous variables as the price of oil increases (run 1).

Run 2.

The following parameter and factor response values were used:

$\alpha = 1.00$	$b_1 = 0.10$
$\beta = 2.00$	$b_2 = 0.20$
$a_1 = 0.30$	$c_1 = 0.10\times10^{-2}$
$a_2 = 0.20$	$c_2 = 0.60$

Table A2. Results of the second run, with $c_1 \neq 0$. These results are illustrated in Figure A3.

Exogenously set price of oil (p_ϑ)	Values of endogenous variables					
	p_I	r	w	$p_I X_I^S$	Y	X_ϑ^D
0.0	0.66890	0.5574×10^{-2}	3.33332	0.337150×10^{-15}	11.1111	1.01
0.5	0.85861	0.243452	3.16597	0.580648	10.1251	1.16
1.0	1.10132	0.456505	2.99832	1.20033	9.44897	1.20
1.5	1.41239	0.644688	2.83030	1.84227	9.18463	1.228
2.0	1.80953	1.807918	2.66179	2.48953	9.44742	1.244
2.5	2.30945	0.946045	2.49272	3.12497	10.3476	1.249
3.0	2.91912	1.05883	2.32303	3.73121	11.9419	1.243
3.5	3.61864	1.14595	2.15284	4.29064	14.1388	1.225
4.0	4.33894	1.20707	1.98254	4.78579	16.5742	1.196
4.5	4.95548	1.24203	1.81282	5.20000	18.5754	1.155
5.0	5.33000	1.25113	1.64444	5.51801	19.3906	1.103
5.5	5.38881	1.23504	1.47782	5.72565	18.6234	1.041
6.0	5.16262	1.19450	1.31278	5.80860	16.4559	0.968
6.5	4.75047	1.12996	1.14877	5.75144	13.4507	0.884
7.0	4.25777	1.04152	0.98521	5.53761	10.2080	0.791
7.5	3.76058	0.929042	0.82168	5.14990	7.16683	0.686
8.0	3.30079	0.792336	0.65794	4.57096	4.57735	0.571
8.5	2.89531	0.631209	0.49390	3.78351	2.55107	0.445
9.0	2.54664	0.445510	0.32955	2.77048	1.11951	0.307
9.5	2.25047	0.235131	0.16490	1.51491	0.276035	0.159
10.0	2.00000	0.385923×10^{-1}	0.	0.257568×10^{-15}	0.595746×10^{-32}	0.000017

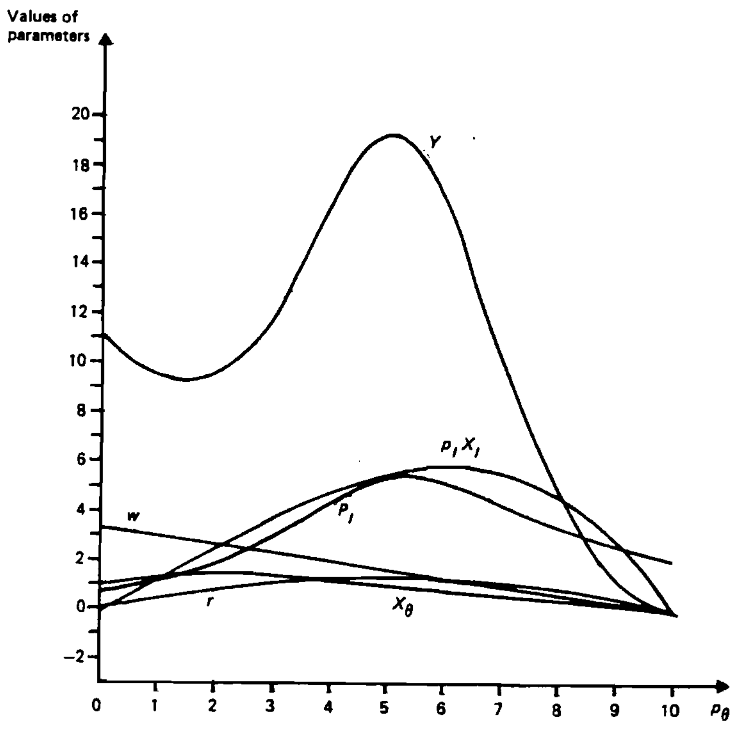

Figure A3. Behavior of the endogenous variables as the price of oil increases (run 2)

Listing of the computer program (in BASIC)

The computer program used to obtain the results given in the previous section is listed below. It was designed to run on a Sharp microcomputer. The parameters and variables discussed in the text are related to the variables used in the computer program as follows:

Text	Program	Text	Program	Text	Program	Text	Program
a_1	A1	M	M	w	W	I^D	ID
b_1	B1	D	D	L	L	Y	Y
c_1	C1	α	AL	K	K	X_I^S.	X
a_2	A2	β	BE	I^S	IS	p_I	IP
b_2	B2	p_ϑ	PO	B^S	BS	terms of trade	TT
c_2	C2	r	R	$X_\vartheta^{S,D}$	OL	solutions	S1,S2,S3

Program Listing

```
10 LF2
11 TEXT
12 CSIZE 1
20 INPUT "ENTER ALPHA AND BETA";AL,BE
30 INPUT "ENTER A1,B1,C1";A1,B1,C1
40 INPUT "ENTER A2,B2,C2";A2,B2,C2
45 INPUT "PRICE OF OIL";PO
46 LPRINT "(";AL;",";BE;")"
```

```
47 LPRINT "(";A1;",";B1;",";C1;")"

48 LPRINT "(";A2;",";B2;",";C2;")":LF2

60 H = 1 - B1*PO

70 M = A1*B2 - A2*B1

80 GA = AL*C2*C2/(BE*A2)

90 N = C1*B2 - B1*C2

100 D = A1*C2 - A2*C1

110 A5 = AL*C1*C1

120 B5 = AL*C1*(C2 - 2*C1*B2*PO - 2*C2*H)

130 C5 = AL*(C2*C2*H*H+C1*C1*B2*B2*PO*PO+2*C1*C2*B2*PO*H-C2*C2-C2*POXN)

          +BE*A1*A2

140 D5 =-A2*BE*(PO*M+A2)

150 IF A5 = 0 THEN GOTO 400

160 P5 = B5/A5:Q5 = C5/A5:R5 = D5/A5

170 A6 = (1/3)*(3*Q5-P5^2)

180 B6 = (1/27)*(2*P5^3-9*P5*Q5+27*R5)

190 J5 = (B6^2/4)+(A6^3/27)

200 IF J5 >= 0 THEN GOTO 250

210 Z5 = ACS ((-B6/2)/SQR(-A6^3/27))):A9 = SQR(-A6/3)

220 S1 = 2*A9*COS(Z5/3)-P5/3

230 S2 = 2*A9*COS(Z5/3+120)-P5/3

240 S3 =2*A9*COS(Z5/3+240)-P5/3

245 GOTO 420

250 A7 = -B6/2+SQR(J5):B7 = -B6/2-SQR(J5)

260 A8 = SGN(A7)*(ABS(A7))^(1/3)

270 BS = SGN(B7)*(ABS(B7))^(1/3)

280 IF J5 = 0 THEN GOTO 300
```

```
290 S1 = A8+B8-P5/3:S2 = -1:S3 = -1:GOTO 420
300 S1 = A8+B8-P5/3:S2 = -(A8+B8)/2-P5/3:S3 = -1:GOTO 420
400 S1 = -D5/C5:S2 = -1:S3 = -1
420 LET CO = 0
430 IF CO = 0 THEN LET IP = S1
440 IF CO = 1 THEN LET IP = S2
450 IF CO = 2 THEN LET IP = S3
460 IF CO>2 THEN GOTO 745
470 IF IP>O THEN GOTO 500
480 LET CO = CO+1
490 GOTO 430
500 R = (A1*(IP-B2*PO)-A2*H)/(D*IP)
510 W = (H*C2-(IP-B2*PO)*C1)/D
520 L = AL*W
530 K = BE*R
540 IS (A1*K-C1*L)/D
550 BS = (C2*L-A2*K)/D
560 OL = BS*B1+IS*B2
570 ID = R*K
580 X = IS-ID
590 IF ABS(OL)<=0^(-8) THEN LET TT = 0
600 IF ABS(OL)>10^(-8) THEN LET TT = X/OL
610 Y = BS+IP*ID
620 LPRINT "PO =";PO:LF1
630 LPRINT TAB2;"O =";OL:LF1
640 LPRINT TAB4;"R =";R:LF1
650 LPRINT TAB6;"W =";W:LF1
```

```
660 LPRINT TAB8;"IP =";IP:LF1

670 LPRINT TAB10;"NNI =";Y:LF1

720 LET CO = CO+1:GOTO 430

745 LET PO = PO+(.1/B1)

750 IF PO>(.9/B1) THEN GOTO 999

760 GOTO 60

999 END
```

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