Economic Modelling 165 (2026) 107836

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.journals.elsevier.com/economic-modelling

The social cost of carbon and optimal carbon pricing: Economic impacts in a dynamic decentralized market model

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ARTICLE INFO

Dataset link: DICE2016 Numerical Solution (Original data)

Keywords:
Optimal carbon pricing
Integrated assessment modeling
Dynamic optimization
Social cost of carbon

ABSTRACT

Carbon taxation is widely regarded as a key instrument for addressing climate change, with the Social Cost of Carbon (SCC) often proposed as the benchmark for optimal policy design. Despite the widespread use of the SCC as a policy benchmark, it remains an open question under which conditions alternative carbon pricing concepts, in particular the Social Marginal Abatement Cost (SMAC), implement the socially optimal allocation in dynamic decentralized economies. This paper develops a dynamic decentralized market framework based on a DICE-type integrated assessment model to study the implementation of optimal carbon pricing. We show that while both the SCC and the SMAC can induce efficient abatement by firms under certain conditions, only the SCC generally ensures socially optimal household behavior through its effects on wages and interest rates. In contrast, the SMAC is sufficient only under restrictive conditions, such as interior abatement solutions. Overall, our results clarify the conditions under which different carbon pricing concepts implement the first-best allocation and highlight the broader welfare relevance of the SCC in dynamic decentralized economies.

1. Introduction

Taxing carbon emissions is considered one of the most effective policy instruments for combating climate change. Imposing a price on carbon emissions allows for internalization of the side effects of economic activities on climate development. Integrated Assessment Models (IAMs) are the primary tools for conceptualizing economic-climate interactions. IAMs typically consist of both a climate module and an economic module, which interact with each other through feedback loops. Economic activities affect the climate module through carbon emissions that drive climate change. Conversely, developments in the climate module impact economic production via climate damages, including effects from temperature variation and natural disasters. The level of detail in each module can vary significantly and can be adjusted to suit the specific requirements of the research question.

The foundation for IAMs was laid by William D. Nordhaus through the development and introduction of the famous **D**ynamic **I**ntegrated **C**limate **E**conomy (DICE) model in the 1990s (Nordhaus, 1993). The DICE model was among the first to connect the dynamics of temperature and CO_2 emissions with a neoclassical economic growth model and remains in its various iterations a workhorse in the field of climate economics. The consideration of the model from a social point of view allows us to derive the first-best solution (i.e., the optimal path of the economy taking social welfare rather than isolated interests into account) and provides a key metric of climate economics, namely the **S**ocial **C**ost of **C**arbon (SCC). The SCC is defined as the aggregated (discounted) economic and social cost of one additional ton of CO_2 emissions (and a marginal – infinitesimally small – unit of CO_2 emissions in continuous terms) along the socially optimal path.

This modeling contrasts (i) differing approaches taken by other IAMs that derive the SCC on the basis of modified models (in terms of CO_2 emissions) and (ii) empirical SCC estimations (see Tol (2023)). These differences not withstanding, all approaches employ the SCC as a metric to measure the discounted and monetized stream of damage associated with a marginal ton of CO_2. From a static perspective, a standard Pigouvian argument would then suggest that the SCC can be directly employed to inform the value of a carbon tax or the equilibrium price that carbon markets should converge to. This explains the popularity of the SCC as a policy benchmark in the literature and policy. However, whether this Pigouvian argument holds true in a *dynamic* economy has been investigated rarely and incompletely.[1]

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[1] For details see the literature review in Section 2.

https://doi.org/10.1016/j.econmod.2026.107836
Received 12 November 2025; Received in revised form 2 September 2026; Accepted 2 September 2026
Available online 22 September 2026
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Considering the marginal abatement costs arising from the social-planner solution, one can alternatively argue that setting the carbon price equal to the **S**ocial **M**arginal **A**batement **C**osts (SMAC) provides the right incentive for firms to conduct the first-best level of emission abatement, as firms (in simple terms) end up equating their marginal costs to the marginal price. Hence, marginal abatement cost curves are the focal point of numerous empirical studies (Kesicki and Ekins, 2012; Wu et al., 2023).

Despite the widespread use of both concepts in the climate-economics literature, it remains an open question whether the SCC or the SMAC constitutes the appropriate carbon tax benchmark in a dynamic decentralized market economy, where firms choose abatement while households determine intertemporal consumption and savings. Without explicitly specifying the decision-making processes in the production sector, the household sector, and the tax-transfer system, this question is not straightforward and depends on the underlying model structure.

In order to address this issue, we formulate a decentralized market economy with a production and a household sector. The sectors are connected via the capital stock and labor, both employed by firms and provided by households. Firms pay taxes on the GHG emissions caused during the production process. The taxes are redistributed to households via a lump-sum payment. For this model, we can show that although both the SCC and the SMAC imply optimal abatement decisions by firms, only the SCC, through its effects on wages and the interest rate, leads to socially optimal consumption and savings decisions on the household side. In particular, once the economy operates at the abatement frontier defined by technological constraints, the SMAC as a carbon tax is no longer sufficient to replicate the first-best allocation.

A central contribution of this paper is therefore not to revisit the classical Pigouvian decentralization result itself, but to clarify the precise relationship between the SCC and the SMAC within a dynamic IAM framework. In doing so, the paper helps to bridge the gap between the normative interpretation of carbon pricing in social planner IAMs and its implementation in decentralized market economies. While both concepts are frequently treated as interchangeable policy benchmarks in the IAM literature, our analysis shows that their equivalence is not a structural property of decentralized climate-economy models. Instead, it depends critically on the nature of the optimal solution and the underlying economic environment. To study this issue formally, we develop an explicit dynamic decentralization framework for DICE-type models that allows the systematic comparison of planner allocations and decentralized equilibria under alternative carbon tax rules. Within this framework, we show that SCC and SMAC coincide only under specific conditions, whereas the SCC remains the welfare-relevant optimal carbon tax once additional constraints or richer economic frictions are taken into account.

In the present paper, we focus on one particularly transparent source of such divergences: technological limits to emission abatement as represented by the upper bound on the abatement rate in DICE. Such limits are not merely a modeling artifact. A broad literature points to bio-physical constraints (e.g. Staples et al., 2017) and technological limitations (e.g. Moriarty and Honnery, 2008; Rootz'en and Johnsson, 2013) that restrict the feasible speed and extent of decarbonization. While these constraints may evolve over time as new technologies emerge, they are likely to remain relevant features of real-world climate mitigation pathways and therefore have important implications for the design of optimal carbon pricing policies.

The remainder of the article is structured as follows. Section 2 provides an overview of the literature on optimal emission taxation, the social cost of carbon, and marginal abatement costs for CO_2 emissions. Section 3 briefly revisits the SCC and SMAC, including their definitions and derivations based on the DICE2016 model. Section 4 defines the decentralized market setup with a firm and a household sector and discusses the impact of the carbon price on the decision making of firms and households. Section 5 shows that the SCC is the appropriate carbon tax to reach the first-best solution in the decentralized model. Section 6 discusses specific results and properties of a future with net-zero emissions. Section 7 adds a thorough discussion of our results and on extensions. Section 8 concludes.

2. Literature review

The literature relevant to the present paper can be grouped into four closely related strands: environmental taxation, climate-economy modeling and integrated assessment models, the literature on the SCC, and studies on the SMAC. We briefly review each of these strands before positioning the contribution of the present paper.

After the concept of an environmental tax was first introduced by Pigou in the 1920s (Pigou, 1920) the literature on optimal environmental taxation expanded significantly in the 1990s. Bovenberg and van der Ploeg (1994), Boyd et al. (1995), Parry (1995), and Bovenberg and Goulder (1996) all consider static general equilibrium models for the evaluation of the effects of environmental taxation on general welfare with different specific policy questions in mind. Farzin (1996) and Rubio and Escriche (2001) consider dynamic (game-theoretic) models of optimal use of environmental resources and highlight weaknesses of the Pigouvian tax, which needs to be adjusted to reach the social optimum when accounting for stock externalities or market imperfections beyond the externality in focus. A related strand of the literature studies the interaction between environmental taxation and pre-existing distortions in the broader tax system. In particular, the double-dividend literature investigates whether carbon tax revenues recycled through reductions in distortionary labor or capital taxes may generate additional welfare gains beyond environmental improvements (Goulder, 1995). More recent contributions analyze these interactions in dynamic macro-climate settings with endogenous savings and fiscal distortions (Barrage, 2020; Goulder and Hafstead, 2018). These studies highlight that environmental taxation affects household decisions not only through environmental externalities, but also through income, labor-supply, and savings incentives.

Although the literature on environmental taxation expanded further, it still almost exclusively considered models with exhaustible resources. With the adoption of the Kyoto Protocol in 1997 the economic literature also expanded into the field of climate economics. Carmona et al. (2009) discuss the formation of the carbon price in an optimal stochastic control framework, and Lin and Li (2011), among others, began investigating the effects of a carbon tax on CO_2 emissions empirically. More recently, the climate-economics and climate-finance literature has increasingly focused on the transmission of climate-related risks and environmental policies through financial markets and investor behavior. For example, Dimitriadis et al. (2024) analyze the role of environmental characteristics in the pricing of crash risk across major stock sectors, while Dimitriadis et al. (2025) study international spillovers and investor preferences in climate-related financial markets. These contributions highlight the growing importance of integrating climate policy analysis with broader macroeconomic and financial-market dynamics.

Building on these foundations, a more recent strand of the literature studies optimal carbon pricing within integrated assessment models and dynamic general equilibrium frameworks. In 2014 (Golosov et al., 2014) published an influential paper on the optimal taxation of fossil fuels in general equilibrium. The authors introduce an economy of households, a final good sector, and multiple dirty and clean energy sectors. With emission damages conceptually similar to the DICE model the authors derive the SCC in a highly generalized form. After introducing the decentralized setting of the economy a proof that the SCC is the optimal carbon tax is discussed. However, the proof, as presented in the paper,

focuses on the decision between clean and dirty energy input factors and does not touch on the impacts of the carbon tax on household choices and, specifically, on whether the SCC implies first-best savings plans.

Rezai and van der Ploeg (2017) analyze a carbon tax with the intent to facilitate the energy transition and implicitly describe the SCC as the optimal carbon tax, still without providing or referring to an explicit proof. Similarly (Wiskich, 2024) discusses the socially optimal carbon tax in combination with research and innovation subsidies and refers to the SCC as the tax rate implementing the first-best solution, but without providing a formal definition of the SCC in his framework. In a different strain of climate economics (e.g. Matsumura et al., 2024) DSGE models are used to assess the effects of different carbon taxes in more extensive and involved models including multiple production sectors and consumption goods. However, due to their complexity, analytical insights in these models are hard to come by.

Adão et al. (2024) compare the effectiveness of a Pigouvian tax approach with subsidies on the adoption of renewable technologies in a framework based on the work of Golosov et al. (2014). van der Ploeg and Rezai (2021) present multiple extensions to the framework of Golosov et al. (2014) and introduce necessary adjustments to the optimal carbon tax within these scenarios. Fernández-Villaverde et al. (2024) provide an extensive general overview of IAMs and macroeconomic modeling in climate economics and briefly discuss the optimal carbon tax in the DICE model, but only to conclude that the optimal carbon tax is equal to the SCC if the socially optimal abatement rate is below 100%. The present paper fills the analytical gap with respect to the household side by rigorously proving that the SCC is the optimal carbon tax in a DICE-type IAM. In so doing, it further emphasizes the economic relevance of the SCC. Although the SCC has been studied extensively from both a theoretical and empirical perspective (Tol, 2023), its relevance has recently been questioned, e.g., in the works of Pindyck (2019) and Khabarov et al. (2022).

Lastly, we also discuss the relevance of SMAC which have been the focal point of many empirical studies. McKitrick (1999) provides a definition for the marginal abatement cost curve from the perspective of a firm and illustrates the implications of non-differentiability on tax policies. Kesicki and Ekins (2012) call for caution in the use of marginal abatement cost curves, as their estimates are entangled with a high degree of uncertainty. Vogt-Schilb and Hallegatte (2014) discuss the aspect of the optimal timing of investments in abatement, as early investments could lead to lock-in into technologies not suitable to meet long-term emission targets. In a comprehensive review (Huang et al., 2016) address different approaches to the estimation of marginal abatement cost curves and Wu et al. (2023) consider the marginal abatement cost of CO_2 for different abatement options.

Against this background, the present paper contributes to this literature by developing a dynamic decentralization framework for DICE-type IAMs that allows for a systematic comparison between planner allocations and decentralized equilibria under alternative carbon tax concepts. In particular, we formally characterize the conditions under which the frequently assumed equivalence between SCC and SMAC holds or breaks down. The Discussion and Extensions section further discusses richer IAM settings that may generate additional channels through which divergences between SCC and SMAC can arise.

3. SCC and SMAC

For our analysis, we draw on a continuous time version of the DICE2016 model (Nordhaus, 2017), formulated as an optimal control model, with three modules in order to obtain the first-best solution. Capital stock $K(t)$ (capital module), (atmospheric and lower ocean) temperature $T(t)$ (temperature module), and mass of greenhouse gasses $M(t)$ (atmospheric, upper ocean, lower ocean; GHG module) are considered the state variables. The savings rate $s(t)$ and abatement effort $\mu(t)$ serve as control variables. In line with the DICE2016 model, we assume abatement effort to be non-negative but possibly bounded from above, i.e., $\mu(t) \in [0, \overline{\mu}(t)]$. However, in contrast to the (numerical) representation of the DICE2016 model, we do not assume that the upper bound to abatement is given only by the extent to which carbon dioxide removal (CDR) is possible, implying that $\overline{\mu}(t) \geq 1$.[2] Indeed, we expressly allow for the possibility that $\overline{\mu}(t) < 1$ even by a large margin. This is reflective of hard technological or biophysical constraints on the scope for abatement (regardless of the cost).[3] The total economic output $Y(t)$ (produced using capital stock $K(t)$ and available labor $L(t)$ with total factor productivity $A(t)$) is reduced by the cost associated with the abatement effort $\Lambda(\mu, t)$ and damages depending on atmospheric temperature $\Omega(T^{AT}(t))$. The savings rate defines the share of the remaining net output $Q(t)$ that is saved for the next period. The remainder is used for consumption, which enters the (social welfare) utility function. The aggregate social welfare over time is consequently maximized by a (hypothetical) social planner. Economic production causes GHG emissions that induce changes in the different temperature layers via the interaction of the GHG and temperature modules. Table 1 summarizes the key variables and functions.

An extensive presentation of the DICE2016 model in continuous time, its functional forms, and the corresponding model parameters can be found in Appendices A and D. The results obtained from this continuous time implementation are consistent with the outcomes of the original discrete time version.

The Social Cost of Carbon is defined as the marginal willingness to pay total consumption $C(t) = L(t) \cdot c(t)$ for a marginal reduction of emissions (annual emissions of GHGs) $E(t)$. In formal terms, the SCC corresponds to the marginal rate of substitution $\psi(C(t), E(t))$ between total consumption and emissions with total welfare (i.e., the value-function of the full optimization problem (A.1)) remaining constant. Hence,

$$\mathrm{SCC}(t) := \psi(C(t), E(t)) = \frac{\mathrm{d}C(t)}{\mathrm{d}E(t)} = -\frac{\xi^E(t)}{\xi^C(t)}, \tag{1}$$

where $\xi^C(t)$ and $\xi^E(t)$ are the generalized shadow prices of $C(t)$ and $E(t)$, respectively.

Compared to the standard formulation of the DICE model in discrete time, in which consumption and emissions are treated as optimization variables, they are functions of the state and control variables in a continuous time model. Hence, the SCC is related to the actual controls and shadow prices of the state variables in this continuous time framework. Following tedious calculations, which have been relegated to Appendix C, we obtain the following expression for the SCC:

$$\mathrm{SCC}(t) = -\alpha \frac{\lambda^{M^{AT}}(t)}{\left(\lambda^K(t) - \left(\overline{\lambda^s}(t) - \underline{\lambda^s}(t)\right)/Q(t)\right)}, \tag{2}$$

[2] In the present paper, CDR is represented in the stylized DICE model as abatement rates exceeding 100%, implying negative net emissions. This abstraction neglects potentially distinct technological, sectoral, and cost structures of dedicated CDR technologies. A richer treatment would distinguish between conventional emission abatement and active carbon removal activities with potentially separate marginal cost schedules and policy instruments (see e.g. Edenhofer et al., 2023; Franks et al., 2023).

[3] For an extended discussion of emission efficiency, emission reduction and negative emissions in the DICE2016 model we refer to Appendix B.

Table 1
Variables and functional forms of the full continuous-time DICE model (A.1).

Control variables	Abatement rate	$\mu(t) \in [0, \overline{\mu}(t)]$
	Savings rate	$s(t) \in [0, 1]$
State variables (modules)	Economic/production capital	$K(t)$
	Temperature (atmospheric, ocean)	$T(t) = \left(T^{AT}(t), T^{OC}(t)\right)'$
	Greenhouse gases (atmospheric, upper ocean, lower ocean)	$M(t) = \left(M^{AT}(t), M^{UP}(t), M^{LO}(t)\right)'$
Functions	Utility function for total population	$U(c(t), L(t)) = L(t)u(c(t)) = L(t)\frac{c(t)^{\beta}-1}{1-\beta}$
	Total economic output (gross world product)	$Y(t) = A(t)K(t)^{\gamma}L(t)^{1-\gamma}$
	Damages from increasing atmospheric temperature (share of GDP)	$\Omega(T^{AT}) = \psi^1\left(T^{AT}\right)^{\psi^2}$
	Investments cost in abatement of GHG emissions (share of GDP)	$\Lambda(\mu, t) = \theta^1(t)\mu^{\theta^2}$
	Net economic output	$Q(t) = \left[1 - \Lambda(\mu(t), t) - \Omega(T^{AT}(t))\right]Y(t)$
	Total GHG emissions	$E(t) = (1-\mu(t))\,\sigma(t)Y(t) + E_{land}(t)$
	Final good consumption (per capita)	$c(t) = \frac{(1-s(t))Q(t)}{L(t)}$
Parameters	Atmospheric retention ratio	$\tilde{\alpha} = (\alpha, 0, 0)'$
	Upper bound for abatement rate	$\overline{\mu}(t)$
	Total factor productivity	$A(t)$
	Labor/world population	$L(t)$
	Carbon intensity of production	$\sigma(t)$

where $\lambda^{M^{AT}}(t)$ and $\lambda^{K}(t)$ depict the (dynamic) shadow prices for atmospheric GHGs and the capital stock respectively.[4] This is the classic *shadow price of GHG over shadow price of capital*-relation of the SCC including (often neglected in the literature) a correction term that is active if the savings rate lies on the boundary ($\overline{\lambda^s}(t)$ resp. $\underline{\lambda^s}(t)$) denoting continuous multipliers that are only active in case of $s(t) = 0$ (resp. $s(t) = 1$)). The parameter α denotes the atmospheric retention rate of GHG emissions. Equally, the SCC can be expressed as the present monetary value of the stream of future utility loss that is generated through temperature-related reductions in economic output, as triggered by the marginal unit of carbon (see Withagen, 2022).

From the optimality condition for the abatement intensity we can derive Eq. (3a), i.e., the optimal abatement decision.[5] According to this, the marginal abatement cost Λ_{μ}, i.e. the marginal loss in gross economic output, is equated to the carbon-intensity-weighted SCC, i.e., $\sigma \cdot \mathrm{SCC}$, adjusted for possible boundary settings. Assuming that the classic Inada-conditions apply to the utility function[6] it holds, that $\overline{\lambda^s} = 0$ (for all t), and (3a) can be further simplified to (3b) (for the sake of a clearer notation we suppress the time argument whenever necessary):

$$\Lambda_{\mu} = \sigma \cdot \left[\mathrm{SCC} \cdot \left(1 - \frac{\overline{\lambda^s}/Q}{\lambda^K + \underline{\lambda^s}/Q}\right) - \frac{1}{\sigma Y}\frac{(\overline{\lambda^{\mu}} - \underline{\lambda^{\mu}})}{\lambda^K + \underline{\lambda^s}/Q}\right], \tag{3a}$$

$$\Lambda_{\mu} = \sigma \cdot \left[\mathrm{SCC} - \frac{1}{\sigma Y}\frac{(\overline{\lambda^{\mu}} - \underline{\lambda^{\mu}})}{\lambda^K + \underline{\lambda^s}/Q}\right]. \tag{3b}$$

For inner solutions of the abatement rate $\overline{\lambda^{\mu}} = \underline{\lambda^{\mu}} = 0$ holds and (3b) reduces further to

$$\mathrm{SCC}(t) = \frac{\Lambda_{\mu}(t)}{\sigma(t)}. \tag{4}$$

As marginal abatement costs increase in μ, this equation demonstrates in a very simple way that a (ceteris paribus) higher willingness to forfeit consumption to decrease emissions implies a higher abatement rate. On the other hand, a lower carbon intensity of production, σ, implies lower abatement efforts, i.e., a smaller μ for the same SCC.

The SMAC captures the socially optimal marginal abatement costs or, in other words, the social costs of avoiding one additional marginal unit of CO_2 emissions. In the continuous time DICE2016 model, this corresponds to the marginal change in economic net-output for a marginal reduction of emissions, i.e.,

$$\mathrm{SMAC}(t) := \psi(Q(t), E(t)) = \frac{\mathrm{d}Q(t)}{\mathrm{d}E(t)} = -\frac{\xi^E(t)}{\xi^Q(t)}, \tag{5}$$

which is structured analogously to the SCC. Here, $\xi^Q(t)$ denotes the generalized shadow price of the net output $Q(t)$. By following a similar analysis as for the SCC we arrive at

$$\mathrm{SMAC}(t) = \frac{\Lambda_{\mu}(t)}{\sigma(t)}. \tag{6}$$

[4] The simplicity of the formula strongly depends on the functional specifications of the DICE2016 model. As emissions enter the carbon cycle only through the atmospheric layer, the shadow-price $\lambda^{M^{AT}}$ of the carbon accumulated in the atmospheric layer is the only one appearing in the formula. In contrast, the DICE2023 model contains an updated carbon module which includes the distribution of the carbon emissions into four different layers as well as a state variable capturing all historically accumulated emissions. In this case Eq. (2) changes to

$$\mathrm{SCC}(t) = \frac{-\lambda^M(t) \cdot (\xi^1, \xi^2, \xi^3, \xi^4, 1)^T}{\left(\lambda^K(t) - \left(\overline{\lambda^s}(t) - \underline{\lambda^s}(t)\right)/Q(t)\right)}$$

with $\lambda^M(t)$ being the five-dimensional co-state vector. All theoretical derivations can also be conducted for this more generalized framework, and the interpretation of the different terms remain the same. However, to keep the results more tractable we focus on the formulation of DICE2016 which is also frequently used in other models.

[5] For the optimality conditions derived from the Pontryagin Maximum Principle and the derivation of Eq. (3a) see Appendix D

[6] In this case it can never be optimal to set the savings rate $s = 1$ as this would coincide with consumption being equal to 0.

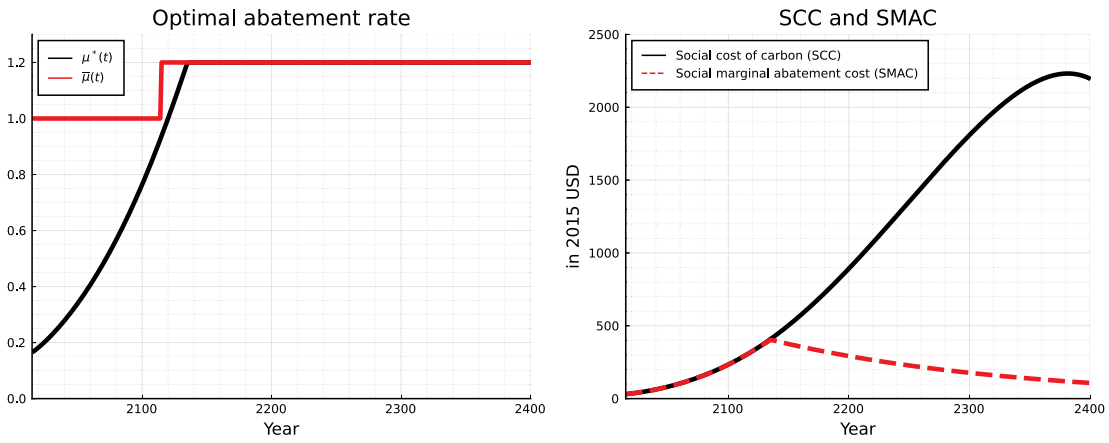

Fig. 1. Optimal abatement rate and its upper boundary (left panel), comparison of SCC and SMAC (in USD) (right panel) from the year 2015 to 2400.

Recalling Eq. (4), we see that

$$\mathrm{SCC}(t) = \mathrm{SMAC}(t) \tag{7}$$

holds when the optimal abatement strategy is not on the upper or lower bound. However, once the optimal abatement reaches its upper bound, a wedge arises between the two values, as described by Eq. (3b). The numerical illustration shown in the right panel in Fig. 1 for the standard DICE2016 calibration (for the parametrization and full solution see Appendices A and D) shows that the discrepancy between the two values can become quite substantial (as also presented in Khabarov et al. (2022)). Furthermore, the left panel of Fig. 1 distinguishes two periods with different optimal abatement decisions, reflected by the relation of the SCC and the SMAC:

2010–2135: In this period, the optimal abatement rate is positive but below the upper bound $\overline{\mu}(t)$. This implies $\underline{\lambda^{\mu}} = \overline{\lambda^{\mu}} = 0$, which means (according to (7)) that the SCC is equal to the SMAC. Moreover, the first order condition (3) reveals that the abatement efforts can be chosen to offset the future damages of additional emissions, i.e., to balance investment costs (in terms of lost production) and the value of future damages.

2135 - end: In year 2135 the optimal abatement level reaches the upper bound, i.e. $\mu(t) = \overline{\mu}(t)$ for $t \geq 2135$. Therefore, $\underline{\lambda^{\mu}} = 0$ and $\overline{\lambda^{\mu}} > 0$, which (by (3b) and (6)) implies a substantially higher SCC, i.e.,

$$\mathrm{SCC}(t) \gg \mathrm{SMAC}(t), \qquad t \geq 2135. \tag{8}$$

Intuition can be obtained by the meaning of the SCC (dynamic marginal rate of substitution) and considering again the first order condition (3b). While the SCC expresses the monetary value of future damages caused by the marginal unit of emissions, the SMAC expresses the marginal costs accruing at the optimal abatement level. As future damages exceed the possible abatement costs, the SCC on the right hand side of (3b) is adjusted downward by the marginal value of the boundary of the abatement rate. Hence, the level of abatement cannot keep up with future damages due to the technological upper bound $\overline{\mu}(t)$.

Combining Eqs. (3b) and (5) we can derive the term for the difference between SCC and SMAC as

$$\mathrm{SCC} - \mathrm{SMAC} = \frac{1}{\sigma Y} \frac{(\overline{\lambda^{\mu}} - \underline{\lambda^{\mu}})}{\lambda^K + \underline{\lambda^s}/Q}. \tag{9}$$

In case of the illustrations in Fig. 1 with the abatement rate reaching the upper bound and the savings rate being strictly positive (not shown in this figure), Eq. (9) can be simplified to

$$\mathrm{SCC} - \mathrm{SMAC} = \frac{1}{\sigma Y} \frac{\overline{\lambda^{\mu}}}{\lambda_K}. \tag{10}$$

Since $\overline{\lambda^{\mu}}$ captures the shadow-price of the upper bound $\overline{\mu}$, we find an intuitive interpretation for the difference, which can reach quite substantial monetary levels (as shown in Fig. 1). Similarly to the interpretation of the SCC $\overline{\lambda^{\mu}}/\lambda_K$ transforms the shadow-price of $\overline{\mu}$ into a willingness to pay measure in economic units. Since this ratio is also divided by total abatable emissions σY, the difference SCC-SMAC measures the willingness to pay for an extension of the technological or political barrier of abatement per unit of abatable emissions.

Connecting to the discussion of technological constraints, Eq. (10) can be rearranged to obtain an expression for the marginal value of relaxing the upper abatement constraint:

$$\overline{\lambda^{\mu}} = \sigma Y \, (\mathrm{SCC} - \mathrm{SMAC}) \, \lambda_K. \tag{11}$$

The shadow value of the constraint is therefore directly proportional to the difference between the SCC and the SMAC. Intuitively, this wedge measures the welfare gain that could be achieved from an incremental expansion of the technologically feasible abatement frontier.

4. Decentralized economy

In this section, we present an adaptation of the standard DICE2016 model and discuss the effects of carbon pricing on decentralized decision making within the economy. To avoid confusion in the following sections, we will denote the standard DICE2016 as the SPDICE model and the decentralized adaptation as the DCDICE model.

We formulate an economy with a production and a household sector. For both sectors we consider simple set-ups which are structurally consistent with the SPDICE formulation. In the production sector, we assume a market with perfect competition between homogeneous firms, spot-maximizing their profits (i.e., not considering dynamic aspects). For the representative household, we assume an infinitely living/dynastic household maximizing aggregated utility over the whole time horizon (surpassing the normal lifetime of an individual).

4.1. The firm's problem

Assume a representative firm that maximizes profits at every given point in time (ignoring dynamic effects of the climate module and dynamic-strategic effects with the household side). Thus, firms can decide on the amount of labor L to employ, the amount of capital K to use in production, and the intensity of abatement μ given the current set of prices and total factor productivity $A(t)$. The profits (to be maximized) at time t are given by

$$\Pi(t,\mu,K,L) = \left[1-\Omega(T^{AT}(t))-\Lambda(t,\mu)\right] AK^{\gamma}L^{1-\gamma} - R(t)\cdot K - w(t)\cdot L - p(t)\cdot E \qquad \longrightarrow \max_{L,K,\mu} \tag{12}$$

$$E = (1-\mu)\sigma(t)A(t)K^{\gamma}L^{1-\gamma} \tag{13}$$

with $R(t)$ being the per unit rent of production capital, $w(t)$ being the wage rate per unit of labor, and $p(t)$ being the carbon price/tax rate, i.e., the costs per unit of emissions. These three prices are exogenous for the firm, $R(t)$ and $w(t)$ are set at market equilibrium and $p(t)$ by a policy maker. Atmospheric temperature $T^{AT}(t)$ is also treated as an exogenous variable because individual firms believe to be too small to have an impact on aggregate emissions and climate developments.

As abatement is bounded between zero and an upper bound $\overline{\mu}(t)$ we need to define the Lagrangian as[7]

$$\mathcal{L}(t,\mu,K,L) = (1-\Omega(T^{AT})-\Lambda(t,\mu))AK^{\gamma}L^{1-\gamma} - R(t)\cdot K - w(t)\cdot L - p(t)\cdot(1-\mu)\sigma(t)A(t)K^{\gamma}L^{1-\gamma} + \overline{\xi^{\mu}}(\overline{\mu}(t)-\mu) + \underline{\xi^{\mu}}\mu, \tag{14}$$

from which the set of first order (and complementary slackness) conditions is obtained:

$$\frac{\partial\mathcal{L}}{\partial K} = \gamma\frac{Q}{K} - R - p\gamma\frac{E}{K} = 0 \tag{15a}$$

$$\frac{\partial\mathcal{L}}{\partial L} = (1-\gamma)\frac{Q}{L} - w - p(1-\gamma)\frac{E}{L} = 0 \tag{15b}$$

$$\frac{\partial\mathcal{L}}{\partial \mu} = -\Lambda_{\mu}\cdot Y + p\sigma Y + (\underline{\xi^{\mu}} - \overline{\xi^{\mu}}) = 0 \tag{15c}$$

$$0 = \overline{\xi^{\mu}}(\overline{\mu}-\mu) \tag{15d}$$

$$0 = \underline{\xi^{\mu}}\mu. \tag{15e}$$

Similarly to before, $\overline{\xi^{\mu}}$ and $\underline{\xi^{\mu}}$ denote Lagrange multipliers (for the abatement rate) within the firm's problem. A reformulation of (15a)–(15c) in factor prices yields

$$R = \gamma\frac{Q}{K}\cdot\left[1-\frac{p(1-\mu)\sigma}{(1-\Omega-\Lambda)}\right] \tag{16a}$$

$$w = (1-\gamma)\frac{Q}{L}\cdot\left[1-\frac{p(1-\mu)\sigma}{(1-\Omega-\Lambda)}\right] \tag{16b}$$

$$p = \frac{\Lambda_{\mu}}{\sigma} + \frac{(\overline{\xi^{\mu}} - \underline{\xi^{\mu}})}{\sigma Y}. \tag{16c}$$

Note that carbon pricing drives a wedge between the marginal product of capital and labor and their respective factor prices. Emissions are abated to the point that the marginal abatement cost equals the carbon price for an interior choice of the abatement rate. Marginal abatement costs exceed (fall short of) the carbon price in the case of a zero (maximum feasible) abatement corner solution, respectively. As firms are assumed to be in perfect competition, profits are zero in the market equilibrium. Hence,

$$\left(1-\Omega(T^{AT})-\Lambda(\mu)\right)AK^{\gamma}L^{1-\gamma} = R\cdot K + w\cdot L + p\cdot E \tag{17}$$

holds, which is a useful relationship for further calculations.

4.2. The household's problem

To be comparable to the SPDICE model we assume $L(0)$ infinitely living homogeneous households (maximizing upon the finite time horizon T) as common in all variants of the DICE model. In contrast to the firm, the household takes the intertemporal component of its assets into account. Furthermore, we assume that individuals are altruistic towards their offspring and therefore adjust their period-utility for the number of their

[7] We can omit the non-negativity multipliers for capital and labor as the Cobb–Douglas production function guarantees optimal decisions with strictly positive K and L.

offspring. This is reflected by multiplying period utility with the term $\exp\left\{\int_0^t n(s)\,\mathrm{d}s\right\}$ with $n(t) := \frac{\dot{L}(t)}{L(t)}$ being the exogenous population growth rate. Overall, the households maximize their discounted intertemporal utility subject to the asset dynamics, i.e.,

$$\max_{c(t)} \int_0^T \exp\{-\rho t\} \exp\left\{\int_0^t n(s)\,\mathrm{d}s\right\} u(c(t))\,\mathrm{d}t \tag{18a}$$

$$\dot{a} = R \cdot a + w + \frac{p \cdot E}{L} - \delta_K a - c - n \cdot a, \qquad a(0) = a_0 \tag{18b}$$

$$0 \leq c \leq R \cdot a + w + \frac{p \cdot E}{L}, \tag{18c}$$

Thereby, $a(t)$ denotes household per-capita assets, which develop according to the difference between net-income and consumption (both in per capita terms). The household gross income consist of income through interest on capital $R \cdot a$, labor income w, as well as the governmental redistribution of the emission tax revenue per-capita $\frac{pE}{L}$.[8] The net-return on capital also has to account for depreciation, as captured by the term $-\delta_K a$. Furthermore, the level of per-capita assets is also affected by the population growth rate $n(t)$ leading to the inclusion of the term $-n \cdot a$.

Per-capita consumption $c(t)$ is constrained by 0 from below and by the gross income from above, as assets held by the household cannot be used for consumption directly (see Eq. (18c)).[9] This assumption mirrors the assumption that $s \in [0, 1]$ in the social planner setting.

The household discounts utility at rate ρ. To be consistent with the SPDICE model, we assume, that the household supplies labor inelastically in proportion (normalized to one) of the population size and that it adopts the same utility function as the social planner. Note that this implies altruism in the sense that the utility is proportional to the population size. Hence, the household aggregates the utility of all offspring.

The standard Pontryagin Maximum Principle (Pontryagin, 1987) yields the first order condition

$$u_c = \xi^a + \overline{\xi^c} - \underline{\xi^c} \tag{19}$$

and the adjoint equation defining the dynamics of the shadow price of household assets

$$\dot{\xi^a} = (\rho - n)\,\xi^a - \left(R - \delta_K - n\right)\xi^a - R\overline{\xi^c} = \left(\rho - R + \delta_K\right)\xi^a - R\overline{\xi^c}, \tag{20}$$

together with the transversality condition $\xi^a(T) = 0$. Owing to the decentralized market structure (the firm decides on abatement, the household on savings) $\overline{\xi^c}$ and $\underline{\xi^c}$ appear in the first order condition for savings (19), which stands in contrast to the SPDICE model, where they already show up in the optimality condition for abatement (see Eq. (3)).

The dynamics of household consumption levels can be described through the consumption Euler-equation, which is obtained from the derivative with respect to time in Eq. (19). Assuming consumption being strictly positive (which is guaranteed if the utility function fulfills the standard Inada-conditions as also assumed in the SPDICE model) and below the upper bound of total income, we can derive

$$\frac{\dot{c}}{c} = \frac{u_c}{-u_{cc} \cdot c}\left(R - \delta_K - \rho\right). \tag{21}$$

Hence, analogously to classic Ramsey-type growth models, the growth-rate of consumption follows the gap between the market interest rate and the discount rate over time.

If consumption is on the upper bound, i.e., $c = R \cdot a + w + p \cdot E/L$ the growth rate of per-capita consumption follows the growth rate of total income per-capita. However, as described by Eq. (17), this term is equal to the per-capita net production in the economy and hence $c = Q/L$. To sum up, the consumption rate develops according to

$$\frac{\dot{c}}{c} = \begin{cases} \frac{u_c}{-u_{cc} \cdot c}\left(R - \delta_K - \rho\right) & \text{if } \overline{\xi^c} = 0 \\ \frac{\frac{d}{dt}(1 - \Omega(T^{AT}) - \Lambda(\mu, t))}{(1 - \Omega(T^{AT}) - \Lambda(\mu, t))} + \frac{\dot{A}}{A} - \gamma\left(\delta_K + n\right) & \text{if } \overline{\xi^c} > 0 \end{cases} \tag{22}$$

4.3. Market equilibria

Two goods are traded between production and the household sector: Labor and capital respectively assets. First, we do not consider unemployment in this framework, hence the wage rate w adjusts until the labor market fully clears and labor demand by the firms $L^d(t)$ (as defined in Eq. (16b)) meets the labor supply of the household sector, consisting of the whole population $L(t)$.

Second, the capital market is assumed to clear at all points in time. Contrary to labor supply, capital supply is endogenous and defined by the aggregated assets held by the households, i.e. $a(t)L(t)$. These aggregated capital supply has to match the capital demand as defined in Eq. (16a). Furthermore, we assume that all production capital at the start is distributed equally among the homogeneous households, i.e. $a_0 = K_0/L_0$. All market equilibrium conditions are summarized in Eq. (23):

$$a_0 = \frac{K_0}{L_0} \quad , \quad L^d(t) = L(t) \quad , \quad a(t)L(t) = K(t). \tag{23}$$

4.4. Impacts of the carbon price on market equilibria and economic outcomes

Before addressing the optimal carbon tax rate in Section 5, this section examines how the carbon price affects general equilibrium outcomes in the decentralized economy. In particular, we study how changes in the carbon price influence factor prices and, through them, the interaction between firms' demand for capital and labor and households' savings and labor supply decisions.

[8] In Section 7 we discuss and outline the implications of alternative emission tax revenue schemes.

[9] Hence, the transversality condition $a(T) \geq 0$ for households to have non-negative assets at the end of the time horizon is fulfilled automatically and it does not have to be included in the subsequent analysis.

To this end, we consider an unanticipated *blip* in the carbon price. Following Johannesson et al. (1997), a blip is defined as a temporary and arbitrarily short-lived change in a model parameter, which serves as the continuous-time analogue to a one-period perturbation in discrete-time models. This approach allows us to derive local comparative statics around the equilibrium.

In the production sector, the blip is equivalent to a marginal parameter change in a static optimization problem, as firms are assumed to be spot-profit maximizers. This permits us to characterize the marginal effects of the carbon price on abatement decisions, as well as on equilibrium wages and the interest rate. These results then serve as the basis for analyzing the induced adjustments in household consumption and savings behavior.

Proposition 1. *The marginal impact of a change in the carbon price p on the abatement rate μ chosen by the firms follows Eq. (24).*

$$\frac{d\mu}{dp} = \begin{cases} \frac{\sigma}{\Lambda_{\mu\mu}} & \text{if } \mu \in (0, \overline{\mu}) \\ 0 & \text{else} \end{cases} \tag{24}$$

For a convex abatement cost function Λ (as is the case in the DICE calibration) we find that a marginal increase in the price increases the abatement rate for an interior solution while a boundary solution is not affected by a price change.

Proof. Assuming an interior solution for the abatement rate μ, we can first calculate the total derivative of Eq. (16c) with respect to p with $\overline{\xi^\mu}$ and $\underline{\xi^\mu}$ being equal to zero and obtain the first case of (24) after simple rearrangements.

In case of the abatement rate being on the lower or upper bound for abatement, the price does not impact the rate and Eq. (16c) reacts to marginal changes in p through adjustments of the shadow-prices $\overline{\xi^\mu}$ or $\underline{\xi^\mu}$ respectively. Hence, we obtain $\frac{d\mu}{dp} = 0$. Finally, for a convex abatement cost function it holds that $\Lambda_{\mu\mu} > 0$, which directly implies the last proposition statement. ■

Building on the first proposition, we can trace how changes in the carbon price affect firms' production costs and thereby propagate through the system in general equilibrium, ultimately determining the equilibrium factor prices w and R.

Proposition 2. *The effect of an unexpected marginal change in the carbon price p on the factor price w and R follows Eqs. (25) and (26).*

$$\frac{dR}{dp} = -\gamma \frac{Y}{K}\sigma(1-\mu) = -\gamma \frac{E}{K} \tag{25}$$

$$\frac{dw}{dp} = -(1-\gamma)\frac{Y}{L}\sigma(1-\mu) = -(1-\gamma)\frac{E}{L} \tag{26}$$

Therefore, the marginal change in each factor price is proportional to the industrial emissions per production factor input times the production elasticity. Specifically, the signs of $\frac{dR}{dp}$ and $\frac{dw}{dp}$ are opposite to the sign of industrial emissions E, where we note that $E < 0$ is tantamount to a setting of CDR.

Proof. As an instantaneous change in the carbon price only affects future decisions by the households, the current supply of capital a is unaffected by the change in p, i.e. $\frac{da}{dp} = 0$. As the capital market is assumed to clear at all points in time, the current demand of capital K is unaffected. Similarly, the labor force L is exogenous and therefore, labor demand remains unaffected by a carbon price change under labor market clearance. In the following, we present the calculation for R but the analogous calculation holds for w. Rewriting Eq. (16a) to $R = \gamma \frac{Y}{K} \cdot \left[(1-\Omega-\Lambda) - p(1-\mu)\sigma\right]$ and taking the total differential we obtain

$$\frac{dR}{dp} = \gamma \frac{Y}{K}\left[-\Lambda_\mu \frac{d\mu}{dp} - (1-\mu)\sigma - p\sigma\left(-\frac{d\mu}{dp}\right)\right] = \gamma \frac{Y}{K}\left[-\sigma \frac{\Lambda_\mu}{\Lambda_{\mu\mu}} - (1-\mu)\sigma - \Lambda_\mu\left(-\frac{\sigma}{\Lambda_{\mu\mu}}\right)\right] = -\gamma \frac{Y}{K}(1-\mu)\sigma \tag{27}$$

In the calculation we have also used the results of Proposition 1. ■

According to Proposition 2 an increase in the carbon price affects equilibrium factor prices, and thereby propagates to household decisions through two distinct channels. It is shown that an increase in the carbon price leads to a reduction in factor prices in proportion to the factor shares (factor elasticities) and emissions per unit of factor input as long as final goods production generates CO_2 emissions. Conversely, in the presence of CDR an increase in the carbon price actually raises factor prices.[10]

The impact of a marginal unexpected increase in the carbon price on the household allocation is composed of two effects: (i) a change in the current income/budget that is available for consumption or savings; (ii) a change in the growth rate of consumption. We first consider the impact on current disposable income and show that a marginal increase in the carbon price at time t lowers the available budget B at that same point in time.

$$B := R \cdot a + w + \frac{p \cdot E}{L} \tag{28}$$

$$\frac{dB}{dp} = a \cdot \frac{dR}{dp} + \frac{dw}{dp} + \frac{E}{L} + \frac{p}{L} \cdot \frac{dE}{dp} = a \cdot \left(-\gamma \frac{E}{K}\right) + \left(-(1-\gamma)\frac{E}{L}\right) + \frac{E}{L} + \frac{p}{L} \cdot \left(-\frac{d\mu}{dp}\sigma Y\right) \tag{29}$$

$$= -\gamma E \frac{a}{K} + \gamma \frac{E}{L} - \frac{p}{L}\sigma Y \cdot \left(\frac{\sigma}{\Lambda_{\mu\mu}}\right) = -\gamma E \frac{a}{K} + \gamma \frac{E}{L} - \frac{p}{L} \cdot \left(\frac{\sigma^2 Y}{\Lambda_{\mu\mu}}\right) \tag{30}$$

Using the market clearing in the capital market, i.e., $a \cdot L = K$ it follows that

$$\frac{dB}{dp} = -\frac{p\sigma^2 Y}{L\Lambda_{\mu\mu}} < 0. \tag{31}$$

[10] This might seem quite unintuitive and we again refer to Appendix B for a more general and extended discussing on emission reduction and CDR in the DICE model.

Intuitively, the costly increase in abatement effort that is triggered by the carbon price increase lowers the GDP that is available for consumption or saving. Notably, this holds regardless of whether or not emissions are abated or removed from the atmosphere.

Regarding (ii), the Euler-equation (22) allows us to the assess the impact of changes on the growth rate of consumption. For the case of an interior solution, we see that the growth rate of consumption increases with the market interest rate R. In a scenario with positive emissions, an increase in the carbon price reduces the interest rate R and consequently lowers the growth rate of consumption, which is tantamount to an advancement of consumption.

These two individual effects are hard to assess in combination. Still, we are able to show that the impact on consumption and savings is identifiable.

Proposition 3. *The effect of an unexpected sufficiently small increase in the carbon price p over a sufficiently short period of time leads to a decrease in household consumption and a decrease in savings in case of positive emissions, i.e.*

$$\frac{dc}{dp} < 0 \quad , \quad \frac{d\dot{a}}{dp} < 0. \tag{32}$$

For the proof we refer to Appendix E.2.

This proposition shows that, for sufficiently small changes in the carbon price over a sufficiently short period of time, the induced change in the slope of the consumption path is not strong enough to overturn the direct effects on household behavior. In particular, this implies (i) no increase in instantaneous consumption in the case of positive emissions, and (ii) no decrease in instantaneous savings in the case of negative emissions.

We summarize all findings on the effect of a change in the carbon price on the behavior of economic actors and the market outcomes in Corollary 4.

Corollary 4. *The effects of an unanticipated marginal change of the carbon-price over a marginal period of time on the economy can be summarized as follows:*

- *Factor prices:*

$$\frac{dR}{dp} = -\gamma \frac{Y}{K}\sigma(1-\mu) = -\gamma\frac{E}{K} \quad , \quad \frac{dw}{dp} = -(1-\gamma)\frac{Y}{L}\sigma(1-\mu) = -(1-\gamma)\frac{E}{L} \tag{33}$$

- *Firm and household decisions:*

$$\frac{dc}{dp} < 0 \quad , \quad \frac{d\dot{a}}{dp} < 0. \quad , \quad \frac{d\mu}{dp} = \begin{cases} \frac{\sigma}{\Lambda_{\mu\mu}} & \text{if } \mu \in (0, \overline{\mu}) \\ 0 & \text{else} \end{cases} \tag{34}$$

- *Economic outcomes:*

$$\frac{dE}{dp} = -\frac{\sigma^2 Y}{\Lambda_{\mu\mu}} < 0 \quad , \quad \frac{dB}{dp} = -\frac{p\sigma^2 Y}{L\Lambda_{\mu\mu}} < 0 \tag{35}$$

5. The optimal carbon price

In this section, we discuss the optimal carbon pricing which allows the society to achieve the first-best outcome in the DCDICE model. Rewriting the first order optimality condition of firms in perfect competition (16c) and recalling the first order condition for abatement in the first-best solution (see Eq. (3b)) we find that they are structured similarly:

$$\Lambda_\mu = \sigma \cdot \left[p - \frac{(\overline{\xi^\mu} - \underline{\xi^\mu})}{\sigma Y} \right] \qquad \text{Market solution} \tag{36a}$$

$$\Lambda_\mu = \sigma \cdot \left[\text{SCC} - \frac{1}{\sigma Y} \frac{(\overline{\lambda^\mu} - \underline{\lambda^\mu})}{\lambda^K + \underline{\lambda^s}/Q} \right] \qquad \text{Social planner solution} \tag{36b}$$

This comparison provides the basis for the following proposition, which characterizes the optimal carbon price that decentralizes the first-best outcome.

Proposition 5. *Setting the carbon price equal to the social cost of carbon derived by the SPDICE model, i.e., $p(t) = SCC(t)$, provides the right incentive for firms to implement the socially optimal abatement rates. This carbon price, furthermore, creates a relationship between the shadow-prices of the social planner model and the firms in this perfectly competitive market:*

$$\frac{\overline{\xi^\mu} - \underline{\xi^\mu}}{\sigma Y^{DC}} = \frac{1}{\sigma Y^{SP}} \cdot \frac{\overline{\lambda^\mu} - \underline{\lambda^\mu}}{\lambda^K + \underline{\lambda^s}/Q}. \tag{37}$$

Proof. First, consider the following two cases:

1. For interior solutions in both cases under $p = \text{SCC}(t)$ the monotonous marginal abatement cost function directly implies identical solutions for μ. Interior solutions furthermore imply $\overline{\xi^\mu} = \underline{\xi^\mu} = 0$ resp. $\overline{\lambda^\mu} = \underline{\lambda^\mu} = 0$ and hence, Eq. (37) is fulfilled trivially.
2. If we find boundary solutions on either boundary, the solutions are by definition identical. This also implies that the terms Λ_μ, σ, and p resp. SCC are identical in Eqs. (36b) and (36a). Hence, Eq. (37) holds and Eqs. (36a) and (36b) can be fulfilled at the same time.

Now we will proof by contradiction, that an asymmetric solution with an interior solution for one setting and a boundary for the other is not possible.

Consider the interior solution $\mu^* \in (0, \overline{\mu})$ in the SPDICE solution and assume a solution on the upper bound $\hat{\mu} = \overline{\mu}$ in theDCDICE setting. This implies

$$\Lambda_\mu(\overline{\mu}) = \sigma \cdot \left[\text{SCC} - \frac{(\overline{\xi^\mu} - \underline{\xi^\mu})}{\sigma Y^{DC}} \right] > \Lambda_\mu(\mu^*). \tag{38}$$

As $\Lambda_\mu(\mu^*) = \sigma \cdot \text{SCC}$ holds as well as the complementary slackness condition $\underline{\xi^\mu}\overline{\mu} = 0$, we can derive

$$-\frac{\overline{\xi^\mu}}{Y^{DC}} > 0. \tag{39}$$

Hence, $\overline{\xi^\mu} < 0$, which is a contradiction to the Pontryagin maximum principle, which requires $\overline{\xi^\mu} \geq 0$. The proofs by contradiction for

- a solution on the lower boundary for $\hat{\mu}$ in the DCDICE setting,
- an interior solution for $\hat{\mu}$ and an upper or lower boundary solution for μ^* in the SPDICE setting,
- differing boundary solutions in the DCDICE and SPDICE setting

follow analogously. ■

This result has an important implication for the case in which the optimal abatement rate is located at the boundary of the feasible set. In this case, the shadow prices $\overline{\xi^\mu}$ and $\underline{\xi^\mu}$ are only non-zero if the abatement constraint binds, reflecting the wedge between the marginal abatement cost Λ_μ/σ and the carbon price p. Hence, setting the carbon price equal to the socially optimal marginal abatement cost from the Social-Planner-DICE model (i.e., $p = \text{SMAC}$), the firms still choose the socially optimal abatement rates.

This important conclusion is summed up in Proposition 6.

Proposition 6 (*Optimal Abatement Incentives*)**.** *In the decentralized market DCDICE solution (12) and (18), firms choose the socially optimal abatement rate if the carbon price $p(t)$ equals either the SMAC(t) or the SCC(t). In scenarios in which the SCC and SMAC do not coincide, any carbon price $p \in [SMAC(t), SCC(t)]$ or $p \in [SCC(t), SMAC(t)]$ also implies the socially optimal abatement rates being chosen by the firms.*

Proof. First, for interior solutions of the SPDICE solution the SMAC and SCC coincide and the statement of the proposition is trivially fulfilled. For socially optimal solutions on the boundary it holds per definition that SMAC $= \Lambda_\mu/\sigma$ and using Eq. (16c), the monotonicity of Λ_μ, and the non-negativity of the shadow-prices $\overline{\xi^\mu}$ and $\underline{\xi^\mu}$, we find that setting $p = \text{SMAC} + \epsilon$ implies that firms choose the socially optimal boundary solution.

Note that depending on which boundary solution (upper or lower) is socially optimal, either SMAC$(t) \leq$ SCC(t) or SCC$(t) \leq$ SMAC(t) holds. This follows directly from the non-negativity constraints for $\overline{\xi^\mu}$ and $\underline{\xi^\mu}$ and the complementary slackness conditions $\overline{\xi^\mu}(\overline{\mu} - \mu) = 0$ and $\underline{\xi^\mu}\mu = 0$.[11] ■

Having established that firms choose the socially optimal abatement level across a range of carbon prices satisfying SMAC$(t) \leq p \leq$ SCC(t), the next critical question to address is: at which carbon price do households also behave optimally — that is, choose the socially optimal savings rate? This question is rarely discussed in the literature and e.g. is also missing in the proof of the optimal carbon price in Golosov et al. (2014). To address this question, we first present a characterization of the optimal solution of the full decentralized model (12) and (18) including the firm and household sector under the carbon price being set equal to the SCC of the SPDICE solution (see Proposition 7). Subsequently, we will prove that this solution of the decentralized model aligns with the first-best solution.[12]

Proposition 7 (*Necessary Optimality Conditions*)**.** *The profile $(c, \mu, a, \xi^a, \overline{\xi^s}, \underline{\xi^s}, \overline{\xi^\mu}, \underline{\xi^\mu})$ of household consumption (c), firm abatement (μ), household assets a, and 5 co-state variables is an optimal solution of the decentralized model (12) and (18) with the carbon price being set equal to the SCC, if the following equations are fulfilled:*

$$u_c = \xi^a + (\overline{\xi^c} - \underline{\xi^c}) \tag{40a}$$

$$\frac{\Lambda_\mu}{\sigma} = SCC - \frac{(\overline{\xi^\mu} - \underline{\xi^\mu})}{\sigma Y} \tag{40b}$$

$$\dot{a} = R \cdot a + w + \frac{SCC \cdot E}{L} - (\delta_K + n)a - c \tag{40c}$$

$$\dot{\xi}^a = \left[\rho + \delta_K - \gamma \frac{Q}{K} \cdot \left(1 - \frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right)\right]\xi^a - \frac{\gamma}{K}\left(\left[1 - \frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right] Q\overline{\xi^c} - \frac{\left(\xi^a + \overline{\xi^c}\right)}{\lambda^K + \underline{\lambda^s}/Q}(1-\mu)(\overline{\lambda^\mu} - \underline{\lambda^\mu})\right) \quad , \quad \xi^a(T) = 0 \tag{40d}$$

where (40a) and (40b) are reformulations of the first order conditions of the households and the firms, resp., and (40c) and (40d) the dynamic equations for household assets and their shadow prices.

Proof. (40a) and (40b) have already been derived in (19) and (16c) for $p = \text{SCC}$. (40c) is the definition of the asset dynamics. (40d) is obtained by several manipulations presented in Appendix E.4. ■

[11] SMAC$(t) \leq p \leq$ SCC(t) holds in the standard DICE2016 calibration, but can be theoretically violated in different settings. In the DICE2016 calibration the marginal abatement cost are 0 for $\mu = 0$ and the SCC is always non-negative. For positive initial marginal abatement costs, which could e.g. result from the introduction of a lower bound for the abatement rate, and a very low SCC the optimal abatement rate could reach the lower bound and SMAC$(t) >$ SCC(t) could in fact occur.

[12] In general we can only proof that setting $p = \text{SCC}$ is sufficient for reaching the social optimum in the decentralized setting. However, proving that $p = \text{SCC}$ is a necessary condition is not possible in all cases. For a related discussion see Section 6.

To prove that p = SCC implements the first-best solution into the DCDICE model, i.e., overcomes the failure of the proposed market setup, we follow a two steps process. First, we show that translating the optimal trajectories of the first-best solution into the variables of the decentralized version provides a solution which fulfills the necessary optimality conditions presented in Proposition 7. Second, we argue that the first-best solution is not only a candidate for the optimal solution of the decentralized model, but is in fact an optimal solution. This is summarized in Proposition 8.

Proposition 8 (*SCC as the Optimal Carbon Price*)**.** *The control, state and co-state variable trajectories corresponding to the (first-best) social planner solution*

$$Z^{SP} := \left(s^{SP}, \mu^{SP}, K^{SP}, \lambda^{K,SP}, \overline{\lambda^s}^{SP}, \underline{\lambda^s}^{SP}, \overline{\lambda^\mu}^{SP}, \underline{\lambda^\mu}^{SP}\right) \tag{41}$$

translated into variables of the decentralized model, as by

$$c^* = \frac{(1-s^{SP})\cdot Q^{SP}}{L}, \qquad \mu^* = \mu^{SP}, \qquad a^* = \frac{K^{SP}}{L}, \qquad \xi^{a,*} = \lambda^{K,SP} \tag{42a}$$

$$\underline{\xi^c}^* = \overline{\lambda^s}^{SP}/Q, \qquad \overline{\xi^c}^* = \underline{\lambda^s}^{SP}/Q, \qquad \overline{\xi^\mu}^* = \frac{\overline{\lambda^\mu}^{SP}}{\lambda^{K,SP} + \underline{\lambda^s}^{SP}/Q^{SP}}, \qquad \underline{\xi^\mu}^* = \frac{\underline{\lambda^\mu}^{SP}}{\lambda^{K,SP} + \underline{\lambda^s}^{SP}/Q^{SP}}, \tag{42b}$$

satisfy the necessary optimality conditions of the DCDICE model (40a)–(40d) as described in Proposition 7 and describe the first-best solution as the optimal solution.

Proof. As shown in Appendix D the first order condition for savings and abatement in the social planner optimum can be formulated as

$$\frac{U_c}{L} = \lambda^K + \frac{(\underline{\lambda^s} - \overline{\lambda^s})}{Q}. \tag{43a}$$

$$\Lambda_\mu = \sigma \cdot \left[\text{SCC} - \frac{1}{\sigma Y}\frac{(\overline{\lambda^\mu} - \underline{\lambda^\mu})}{\lambda^K + \underline{\lambda^s}/Q}\right] \tag{43b}$$

Using the definitions of (42a) and (42b) it can be easily shown, that these equations are equivalent to the FOCs (40a) and (40b) of the DCDICE model.

Regarding the $\dot{a}$-dynamics, we can first show, that using the factor prices and the equilibrium values in the market for assets and labor leads to

$$R\cdot a + w + \frac{p\cdot E}{L} = \frac{Q}{L}. \tag{44}$$

This equation (for its derivation see (E.1) in Appendix E) also shows the no-profit property of the solution in perfect competition and holds if the capital market is in equilibrium at every point in time, i.e. $a = K/L$.

The $\dot{a}$-dynamics of the decentralized model can be rewritten using this equation as

$$\dot{a} = \left(R\cdot a + w + \frac{p\cdot E}{L}\right) - (\delta_K + n)a - c = \frac{Q}{L} - (\delta_K + n)a - (1-s)\cdot\frac{Q}{L} = s\frac{Q}{L} - (\delta_K + n)a \tag{45}$$

The solution for capital K^{SP} of the SPDICE model solves the following differential equation for capital per-capita

$$\frac{d}{dt}\left(\frac{K}{L}\right) = \frac{\dot{K}}{L} - \frac{K}{L}\cdot\frac{\dot{L}}{L} = \frac{s\cdot Q - \delta_K K}{L} - \frac{K}{L}\cdot n = s\frac{Q}{L} - (\delta_K + n)\frac{K}{L}. \tag{46}$$

As, furthermore, $a(0) = K^{SP}(0)/L(0)$ holds per definition in the DCDICE setting, we have shown, that the solution for capital per-capita from the SPDICE solution K^{SP}/L solves the differential equation of household assets in the DCDICE model.

Finally, following the derivations presented in Appendix E, we can show that the co-state dynamics for λ^K in the social-planner solution and ξ^a in the decentralized setting respectively can be rewritten as

$$\dot{\lambda}^K = \left(\rho + \delta_K - \gamma\frac{Q}{K}\left[1 - \frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right]\right)\lambda^K - \frac{\gamma}{K}\left(\underline{\lambda^s}\left[1 - \frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right] - (1-\mu)(\overline{\lambda^\mu} - \underline{\lambda^\mu})\right), \tag{47a}$$

$$\dot{\xi^a} = \left[\rho + \delta_K - \gamma\frac{Q}{K}\cdot\left(1 - \frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right)\right]\xi^a - \frac{\gamma}{K}\left[Q\overline{\xi^c}\cdot\left(1 - \frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right) - (1-\mu)\cdot(\overline{\lambda^\mu} - \underline{\lambda^\mu})\left(\frac{\xi^a + \overline{\xi^c}}{\lambda^K + \underline{\lambda^s}/Q}\right)\right]. \tag{47b}$$

Using the definitions in (42a) and (42b) we can directly see, that $\lambda^{K^{SP}}$ solves the ODE for ξ^a. As the social planner solution fulfills all four necessary optimality conditions of the decentralized model it is a candidate for the optimal solution.

Optimality can finally be proven by considering another solution $\hat{Z}$ (different from Z^{SP} from Eq. (41)) fulfilling the optimality conditions as described in Proposition 7 with an objective value $\hat{V}$ strictly greater than the SPDICE solution V^{SP}. The objective function of the solution planner aggregates the utilities of the $L(0)$ identical households and hence for the maximized objective value W^{SP} in the social-planner setting $W^{SP} = L(0)V^{SP}$ holds. Furthermore, the social planner can undertake all actions of the decentralized setting also in the centralized model. Therefor, if $L(0)\hat{V} > W^{SP}$ holds, the social planner would have taken the solution $\hat{Z}$ as the socially optimal one. This contradicts the fact, that $\hat{Z}$ is different from the social optimal solution and concludes the prove. ■

The proposition answers the question whether the SCC or the SMAC should serve as tax rate to implement the social optimum into a decentralized market setup. A key condition for this equivalence is that the equilibrium interest rate satisfies

$$R^* = \gamma\frac{Q}{K}\cdot\left[1 - \frac{\text{SCC}(1-\mu)\sigma}{(1-\Omega-\Lambda)}\right]. \tag{48}$$

Combined with the Keynes–Ramsey rule in Eq. (22), this relationship shows how climate damages enter the intertemporal consumption decision through the equilibrium interest rate. In particular, higher future climate damages, reflected by a higher SCC, reduce the equilibrium interest rate

and thereby lower the welfare-optimal growth rate of consumption. For the interest rate to match R^* it is necessary that the carbon price is set equal to the SCC for all points in time with the optimized abatement rate being below or above 1. Choosing the carbon price different to the SCC, what is specifically the case for p =SMAC, the co-state profile of the social-planner solution does not solve the co-state dynamics of the household assets and consequently does not fulfill the necessary optimality conditions. This implies, that the decentralized solution and social planer solution diverge and entails that the solution of the decentralized model is not socially optimal.

Proposition 8 extends to more general functional specifications of DCDICE, provided they preserve an analogous qualitative structure. The current specification, taken from the DICE2016 model, facilitates the analytical treatment and interpretation of the results. In particular, binding control or mixed constraints imply that the SMAC no longer fully internalizes marginal climate damages and therefore fails to implement the first-best allocation in the decentralized economy. By contrast, pure state constraints, such as temperature ceilings considered in some IAMs, do not directly enter the first-order conditions, implying that the equivalence between SCC and SMAC remains unaffected in such settings.

In a decentralized economy with competitive firms and households, the appropriate Pigouvian tax is therefore the SCC, as it reflects the discounted stream of future damages from additional greenhouse gas emissions rather than the contemporaneous marginal cost of abatement underlying the SMAC. Even when firms are constrained by technological limits on abatement (i.e., $\mu = \bar{\mu}$), a carbon price below the SCC would still fail to internalize future climate damages and would distort intertemporal consumption, savings, and factor prices. In this sense, the SCC plays a dual role: it ensures correct static abatement incentives and simultaneously aligns dynamic household decisions through wages and interest rates (see Eqs. (26) and (25)), thereby steering the economy toward the socially optimal growth path.

6. Optimal carbon pricing in a net-zero future

As discussed in previous sections, a wedge between the SCC and the SMAC arises once the socially optimal abatement rate reaches its technologically feasible upper bound. For the numerical illustration based on the DICE2016 parametrization, we assume that CDR becomes available in 2115 and that the optimal abatement rate reaches its upper bound shortly thereafter. In a counterfactual scenario without CDR but under a net-zero constraint, Fig. 1 shows that the upper bound $\overline{\mu}(t) = 1.0$ is reached around 2120. At this point, emissions from production are fully eliminated, so that industrial emissions vanish. As Eqs. (16a) and (16b) show, the decentralized allocation becomes independent of the carbon price once $\mu = 1$, provided the carbon price is sufficiently high to sustain full abatement. Hence, in the absence of CDR but in the presence of the technological capability to avoid all emissions, using the SMAC as the carbon-price p also leads to the socially optimal solution. In fact, any $p \geq$ SMAC after 2120 results in the socially optimal solution.

Still, the SCC provides a more robust approach for the carbon-tax, as the SMAC is only the optimal choice for $\overline{\mu} = 1$ exactly. For any other $\overline{\mu}$, emission are either not taxed enough for $\overline{\mu} < 1$, as would be the case for a limited technological abatement capability, or CDR is not subsidized enough for $\overline{\mu} > 1$. This in turn, results in distorted saving incentives and, thus, in a distorted growth trajectory. As summarized in Corollary 4 a lower carbon price would increase savings, increase emissions, and thereby accelerate the temperature developments and consequent damages through the instantaneous increase in emissions as well as higher future economic output. Conversely, if abatement technologies are represented by smooth convex cost functions and the socially optimal solution remains interior, the SCC and the SMAC coincide. Hence, within the DICE-based framework considered in this paper, the divergence between the SCC and the SMAC arises from binding technological abatement constraints rather than being a general property of decentralized IAMs.

The result that the SCC should be used as the carbon price need not hold for all sectoral and market structures. Consider, for instance, forms of imperfect competition among firms, vertical or horizontal differentiation into two or more production sectors (with sector-specific GHG footprints of products and processes), heterogeneous households with differing savings incentives, or institutional aspects, relating to e.g., the implementation of the tax-redistribution scheme (see Section 7 for a discussion).

Furthermore, the result is specific to the DICE2016 framework and should be interpreted in light of its simplifying assumptions. In particular, richer climate modules (as in DICE2023) introduce multiple greenhouse gases and thus multiple externalities, which may require a more differentiated set of social cost measures.

Similarly, the presence of carbon removal technologies may introduce additional instruments such as a social cost of carbon removal (SCCR), as discussed in Edenhofer et al. (2023) and Franks et al. (2023). These extensions lie beyond the scope of the present analysis, which isolates the interaction between abatement constraints, factor prices, and optimal carbon taxation in a single-externality environment.

7. Discussion and extensions

The current paper is based on the stylized DICE2016 framework, which relies on several characteristic simplifying assumptions.

First, DICE2016 considers only a single source of greenhouse gas emissions, namely CO_2. More recent IAMs, including the latest version of DICE (DICE2023), explicitly account for additional greenhouse gases and other climate forcing agents, leading to a broader concept of climate externalities. In such settings, abatement activities typically reduce not only CO_2 emissions but simultaneously other greenhouse gas emissions as a by-product. Consequently, the optimal emission tax can no longer be represented solely by the SCC, but should instead reflect a combination of the SCC and the social costs associated with other greenhouse gases (SCGHG). This observation already illustrates that identifying the SCC alone as the universally optimal tax rate is, in general, too restrictive. Second, the model assumes a representative infinitely lived household (or, equivalently, a representative agent framework over a very long horizon), thereby abstracting from heterogeneity across individuals with respect to age, income, health, or other socioeconomic characteristics. Third, the model is deterministic and therefore neglects uncertainty regarding climate damages, technological progress, or tipping points. Fourth, DICE adopts a globally aggregated perspective and does not capture regional heterogeneity in emissions, damages, or mitigation incentives. Finally, technological change is represented only in a highly stylized manner and does not explicitly model endogenous innovation or sectoral transitions.

Within this stylized setting, the present paper develops a framework that endogenizes emission taxes in a decentralized economy in order to implement the socially optimal allocation. The analysis focuses on the DICE2016 setup in which the only explicit friction is the upper bound on abatement rates. The paper demonstrates that the frequently stated equivalence between the SMAC and the SCC holds only for interior solutions, i.e., whenever the optimal abatement rate remains strictly below its upper bound. By contrast, once the economy operates at the abatement constraint, only the SCC constitutes the optimal emission tax rate. Consequently, the analysis shows that the common argument of using the SMAC as the optimal tax rate can, in general, be misleading. However, under alternative revenue-recycling schemes, setting the carbon tax equal to the (SCC) may fail to replicate the socially optimal allocation. To illustrate this, we briefly discuss three alternative mechanisms replacing the standard lump-sum transfer: (i) a wage subsidy, (ii) a consumption subsidy, and (iii) a capital interest subsidy.

(i) Due to the inelastic labor supply characteristic of the DICE2016 and models alike, a wage subsidy acts as a neutral redistribution mechanism. It mirrors the budget dynamics of a lump-sum transfer, leaving household optimization undistorted. Consequently, the SCC remains the optimal carbon tax. In more generalized frameworks with elastic endogenous labor supply, this result must not hold and the SCC could fail to implement the social optimum.

(ii) As demonstrated by the household Euler equation, a uniform consumption subsidy does not alter the intertemporal growth rate of consumption. Because aggregate subsidy payments match the redistributed carbon revenues, asset dynamics remain unaffected. Thus, the SCC continues to characterize the optimal carbon tax.

(iii) Unlike the previous schemes, a subsidy on capital returns introduces an intertemporal distortion by artificially altering savings incentives and modifying the consumption Euler equation. Consequently, this policy interacts with the macroeconomic equilibrium, implying that a simple Pigouvian tax equal to the SCC is no longer sufficient to achieve the socially optimal allocation.

This sketch highlights, that it is important to specify the context in which carbon taxation is implemented to assess whether the SCC is an applicable measure. Furthermore, real-world climate–economy interactions are substantially more complex, as reflected by the broad IAM literature that relaxes many of the restrictive assumptions underlying DICE2016. The framework proposed in this paper should therefore be interpreted as a testbed for evaluating optimal emission tax rules whether based on the SCC, the SMAC, a combination of both, or augmented by additional effects originating from richer model structures.

To illustrate this point further, we briefly sketch one possible extension. In contrast to DICE2016, suppose that the population is heterogeneous with respect to age, resulting in overlapping generations with asynchronous planning horizons. Such a framework is developed in Lambertini and Wrzaczek (2025), where the production side is modeled similarly to the present paper, while the household sector follows a continuous overlapping-generations (OLG) structure. The aggregate capital stock is built up from individual asset holdings, implying the classical capital dynamics both at the aggregate level and along each individual life cycle. Importantly, that framework does not explicitly model abatement activities and therefore does not feature an abatement constraint, but instead assumes convex emission damages. The paper studies the implementation of socially optimal climate policy through emission taxation. In particular, it shows that constant tax rates can implement the social optimum only in the steady state. Along the transitional dynamics, time- and age-dependent emission taxes are required. Moreover, the paper proves that time-dependent taxes can decentralize the socially optimal emission path; however, they simultaneously generate intergenerational redistribution effects and frictions, implying that climate policy implementation becomes inherently a second-best problem.

Enriching the DICE2016 setup of the current paper with an OLG-structured population further underscores our main argument. In the first-best allocation, consumption is perfectly smoothed across age groups at every point in time. This intertemporal and intergenerational efficiency condition directly affects the SMAC, since marginal abatement costs depend on the decentralized consumption and savings behavior of heterogeneous households. The SCC, by contrast, measures the marginal damage generated by an additional unit of emissions and therefore internalizes the distortion arising from inefficient decentralized capital accumulation and consumption allocation. This suggests that the SCC becomes the relevant welfare benchmark even in the absence of binding abatement constraints, as it additionally internalizes intertemporal and intergenerational inefficiencies in decentralized household behavior.

To further substantiate our arguments, we provide the following sketch of the model. We add abatement to the Lambertini and Wrzaczek (2025) setup and highlight that the capital stock in this extension is the sum of individual capital stocks (as constructed in the above model), which, however, corresponds to the OLG structured population. For convenience, we therefore define $A(t,a)$ as the individual stock of assets (owned by an a-year individual at t) and $K(t)$ as the capital of the economy (obtained via cross-cohort aggregation). The first-order condition for abatement in the social planner's problem then takes the familiar form,

$$SMAC(t) = \mu(t) = \frac{\xi^E(t)}{\xi^K(t)} = SCC(t). \tag{49}$$

However, the *aggregate* social cost of carbon, $SCC(t)$, can be decomposed into *individual* social costs of carbon associated with all current and future cohorts. These quantify the damages caused by a marginal increase in emissions over the remaining lifetime of each individual:

$$SCC(t) = \underbrace{\int_0^{\omega} N_0 S(t,a) ISCC(t,a) \mathrm{d}a}_{\text{cohorts already born at } t} + \underbrace{\int_t^{\infty} N_0 ISCC(s,0) \mathrm{d}s}_{\text{future born cohorts}}. \tag{50}$$

Here, the individual social cost of carbon, $ISCC(t,a)$, is derived in the standard way as the ratio of the shadow price of emissions to the shadow price of capital. In contrast to the representative-agent framework, however, the latter is given by the shadow value of the individual's asset holdings, $\xi^A(t,a)$.

Following the analogous arguments (with analogous derivations), as in Lambertini and Wrzaczek (2025) it can be shown that the SMAC implements the social optimum only once the economy has reached its steady state. Along the transitional dynamics, however, this result no longer holds due to the heterogeneous remaining lifetimes of individuals. Thus, an efficient tax/price scheme has to consist of the SCC or SMAC on the firm side, but on the $ISCC$ (assuming a balanced budget of the government) on the individual/household side. Equation (73) in Lambertini and Wrzaczek (2025) shows that the individual consumption taxes (that is linked to emissions via the production process) equal

$$\tau_I(t,a) = S(t,a) ISCC(t,a) - 1. \tag{51}$$

If government expenditures are zero, the corresponding emissions tax levied on firms is equal to the aggregate $SCC(t)$. Otherwise the emissions tax has to be adjusted to account for government expenditures.

This extension therefore substantially strengthens the central insight of the current paper. In the benchmark DICE2016 framework, the SCC emerges as the optimal tax rate once the economy hits the abatement constraint, whereas the SMAC coincides with the SCC only for interior solutions. In the richer OLG framework, however, the SCC remains the welfare-relevant optimal tax instrument even without binding abatement constraints, because it additionally corrects for intergenerational inefficiencies induced by decentralized household behavior. Hence, the analysis suggests that the SCC possesses a much broader normative relevance than implied by the standard equivalence argument between SCC and SMAC frequently stated in the IAM literature. More generally, the extension illustrates that the equivalence between SCC and SMAC is not a structural property of climate-economy models, but instead depends critically on the underlying economic environment and the nature of the prevailing distortions. Once additional frictions such as heterogeneous households, decentralized savings decisions, or intergenerational redistribution effects are taken into account, the SCC retains its role as the welfare-consistent corrective tax, whereas the SMAC reflects only technological mitigation costs. This distinction becomes particularly relevant in richer IAM frameworks where multiple distortions interact simultaneously.

8. Conclusions

In this paper, we formulate a decentralized market setup with both production and household sectors to study the broader economic impacts of carbon pricing. Our analysis shows that the choice of carbon price influences not only firms' abatement decisions but also household consumption and savings decisions through its effects on factor prices. Specifically, we derive the marginal effects of changes in the carbon price on key variables, including the wage rate, interest rate, abatement decisions, consumption levels, savings decisions, and the consumption growth rate.

Moreover, we demonstrate that both the Social Cost of Carbon (SCC) and the Social Marginal Abatement Costs (SMAC), or any carbon price set within this range, imply a socially optimal level of abatement in a perfectly competitive market as long as the optimal abatement decision remains interior. Once the economy operates at the abatement constraint, only the SCC constitutes the welfare-consistent optimal carbon price. Furthermore, we show that optimal household savings and the resulting capital accumulation that underpins socially optimal economic development are achieved only when the carbon price is set equal to the SCC.

Our results contribute to the ongoing discussion on the appropriate choice of a carbon price or carbon tax. Specifically, our analysis suggests that the SCC, rather than the SMAC, is crucial for addressing market failures arising from unaccounted economic damages caused by climate change in decentralized decision-making. While the SMAC reflects the marginal technological costs of emission reductions, the SCC additionally internalizes the broader economic distortions induced by climate change through their effects on decentralized consumption and savings decisions. This conclusion extends the *Pigouvian tax* argument from the static theoretical literature on environmental economics to a dynamic decentralized market framework with endogenous capital accumulation.

The analysis further demonstrates that the distinction between SCC and SMAC is not specific to the stylized DICE2016 setup considered in the core analysis, but persists in richer economic environments where their equivalence generally breaks down. In particular, the extension to an overlapping-generations economy shows that the SCC remains the welfare-relevant tax instrument even in the absence of a binding abatement constraint, as it corrects for intergenerational inefficiencies arising from decentralized household behavior. More generally, this highlights the broader normative relevance of the SCC beyond the frequently stated equivalence between SCC and SMAC in the IAM literature, especially once additional heterogeneity and market frictions are taken into account.

Future research will address remaining limitations of the present framework. Planned extensions include richer sectoral structures and alternative forms of imperfect competition (e.g., oligopoly and monopolistic competition), as well as refinements of the economic, temperature, and climate modules within integrated assessment models (IAM). Such extensions are likely to affect firms' responses to carbon taxation. Furthermore, we will consider the implementation of carbon taxes in economies with heterogeneous consumers. In the presence of distortionary labor or capital taxation, the optimal carbon tax generally deviates from the pure SCC benchmark due to additional tax-interaction and revenue-recycling effects, as emphasized in the double-dividend literature on environmental taxation. Developing such a second-best framework is therefore beyond the scope of the present paper but constitutes a particularly promising avenue for future research. More generally, existing real-world carbon pricing schemes (including taxes on specific fuels, energy products, or automobiles) differ substantially from the stylized setup considered in this paper and may generate additional efficiency and distributional effects.

Glossary

CDR Carbon Dioxide Removal.

DCDICE Decentralized adaptation of the DICE2016.

DICE Dynamic Integrated Climate Economy.

GDP Gross Domestic Product.

GHG Green-house Gas.

IAM Integrated Assessment Model.

SCC Social Cost of Carbon.

SMAC Socially optimal Marginal Abatement Cost.

SPDICE Standard DICE2016 model in the continuous time formulation presented in (A.1).

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work the author used ChatGPT4.0 in order to improve the readability and flow of the manuscript. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

We thank two anonymous reviewers for their helpful and insightful comments that improved the paper

Appendix A. Continuous time DICE2016 model

The standard DICE2016 model can be formulated in continuous time as presented in (A.1).

$$\max_{s(t),\mu(t)} = \int_{t_0}^{\mathcal{T}} e^{-\rho(t-t_0)} U\left(s(t), \mu(t), K(t), T^{AT}(t)\right)\, \mathrm{d}t \tag{A.1a}$$

$$\dot{K}(t) = s(t)Q(t) - \delta_K K(t) \qquad K(0) = K_0 \tag{A.1b}$$

$$\dot{T}(t) = \zeta T(t) + \tilde{\xi} F(M^{AT}(t), t) \qquad T(0) = T_0 \tag{A.1c}$$

$$\dot{M}(t) = \Phi M(t) + \tilde{\alpha} E(t) \qquad M(0) = M_0 \tag{A.1d}$$

$$s(t) \in [0,1], \quad \mu(t) \in [0, \overline{\mu}(t)], \qquad \forall t. \tag{A.1e}$$

Table A.2 summarizes all variables, functional forms, and external inputs of the model. Table A.3 contains all model parameters and initial values of the state variables.

Appendix B. Emission efficiency, emission reduction and negative emissions in the DICE model

The authors of the DICE model do not provide details on the distinct interpretations of the abatement rate $\mu(t)$ (control variable) and the carbon intensity of production $\sigma(t)$ (exogeneous parameter), although they enter the affect the translation of economic output to GHG emissions identically (see Table A.2). One might argue, that reductions in $\sigma(t)$ include the adoption of more renewable energy sources in the power supply of economic production and $\mu(t)$ refers to the end-of-pipe type abatement in the form of carbon capture and storage (CCS) or carbon capture and utilization (CCU). This interpretation is not fully compatible with emission reduction efforts being costly in terms of net-economic output while improvements in the carbon intensity not affecting the economic sector at all.

However, since our analysis is purely theoretical it is still viable as we can also attribute a switch towards renewable energy sources towards a higher abatement rate μ and set σ equal to a constant or parameterize it to account for changes in the energy-intensity of production, which could result from a change in the future composition of agricultural, industrial, and service industry.

In any case, we are not fully convinced of the modeling decision to incorporate CDR through a potential abatement rate above 1, i.e. $\overline{\mu}(t) > 1$. This approach implies that the level of CO_2 removal from the atmosphere depends on the carbon-intensity of production and the gross economic production itself. Hence, reductions in the carbon intensity of production decrease the efficiency of CDR simultaneously, which does not appear to be a plausible interaction. In future work, we plan to introduce an extended IAM with explicit consideration of CCS, CCU, and CDR. Nevertheless, in this paper we base our analysis on the original DICE2016 model to illustrate the research question for a well-established workhorse in the IAM literature.

Table A.2
Variables and functional forms of the full continuous-time DICE2016 model (A.1).

Control variables	Abatement rate	$\mu(t) \in [0, \overline{\mu}(t)]$
	Savings rate	$s(t) \in [0,1]$
State variables (modules)	Economic/production capital	$K(t)$
	Temperature (atmospheric, oceanic and biosphere)	$T(t) = \left(T^{AT}(t), T^{OC}(t)\right)'$
	GHG (atmospheric, upper ocean, lower ocean)	$M(t) = \left(M^{AT}(t), M^{UP}(t), M^{LO}(t)\right)'$
Functions	Utility function	$U(c,L) = L\frac{c^{\beta}-1}{1-\beta}$
	Total economic output (gross world product)	$Y = AK^{\gamma}L^{1-\gamma}$
	Net output	$Q = \left[1 - \Lambda(\mu,t) - \Omega(T^{AT})\right]Y$
	Damages from increasing atmospheric temperature	$\Omega(T^{AT}) = \psi_1 \left(T^{AT}\right)^{\psi_2}$
	Investments in abatement of GHG emissions	$\Lambda(\mu,t) = \theta_1(t)\mu^{\theta_2}$
	Per-capita consumption	$c = (1-s)\frac{Q}{L} = (1-s)\left[1 - \Lambda(\mu,t) - \Omega(T^{AT})\right]\frac{Y}{L}$
	Total GHG emissions	$E = (1-\mu)\,\sigma(t)Y + E_{land}$
	Increase of temperature by accumulated GHG	$F(M^{AT},t) = \eta \log_2\left(\frac{M^{AT}}{M}\right) + F^{EX}(t)$
Exogenous functions	Total factor productivity	$A(t) = A_{2015} \cdot \exp\left\{-g_{A0}/\delta_A \cdot (e^{-\delta_A \cdot (t-2015)} - 1)\right\}$
	Labor/world population	$L(t) = \left(\mathcal{L}_0 + \mathcal{L}_1 \cdot e^{-\mathcal{L}_2 \cdot (t-2015)}\right)^{1/\mathcal{L}_3}$
	Carbon intensity of production	$\sigma(t) = \sigma_{2015} \cdot \exp\left\{-g_{\sigma 0}/\delta_\sigma \cdot (e^{-\delta_\sigma \cdot (t-2015)} - 1)\right\}$
	Maximal abatement rate	$\overline{\mu}(t) = \begin{cases} 1.0 & \text{if } t < 2115 \\ 1.2 & \text{if } t \geq 2115 \end{cases}$
	Land use emissions	$E_{land}(t) = E_{land}(2015) \cdot e^{-\delta_E \cdot (t-2015)}$
	Non-industrial GHG	$F^{EX}(t) = \begin{cases} fex_0 + \frac{(t-2015)}{85} \cdot (fex_1 - fex_0) & \text{if } t < 2100 \\ fex_1 & \text{if } t \geq 2100 \end{cases}$
	Abatement cost scaling factor	$\theta_1(t) = p_0^{BS} \cdot e^{-\delta^{BS}(t-2015)}\sigma(t)/\theta_2$

Table A.3
Parameters and initial conditions of the full continuous-time DICE2016 model (A.1).

Initial conditions	Initial capital stock	K_{2015}	223	[trillion$]
	Initial temperature deviation from pre-industrial level	T_{2015}	(0.85,0.0068)	[°C]
	Initial GHG stocks	M_{2015}	(851,460.1740)	[GtC]
	Initial Population size	L_{2015}	7.403	[billion people]
	Initial carbon intensity	σ_{2015}	0.35032	[tCO2/1000$GDP/year]
	Initial TFP	A_{2015}	5.115	[–]
	Initial land-use emissions	$E_{land}(2015)$	2.6	[GtCO2/year]
	Initial price of backstop technology	p_0^{BS}	0.55	[1000$]
Parameters	Discount rate	ρ	0.015	[–]
	Time horizon	$\mathcal{T}$	2515	[year]
	Capital depreciation rate	δ_K	0.1	[–]
	Relative risk aversion	β	1.45	[–]
	GHG-forcings-temperature impact coefficient	$\tilde{\xi} = (\xi, 0)'$	0.0201	
	Temperature diffusion matrix	ζ	$\begin{pmatrix} -0.0256379 & 0.0017688 \\ 0.005 & -0.005 \end{pmatrix}$	
	Carbon diffusion process	Φ	$\begin{pmatrix} -0.024 & 0.0392 & 0 \\ 0.024 & -0.0406 & 0.000293023 \\ 0 & 0.0014 & -0.000293023 \end{pmatrix}$	
	Equilibrium concentration of carbon in atmosphere	$\tilde{M}$	588	[GtC]
	Carbon dioxid to carbon weight factor	$\tilde{\alpha} = (\alpha, 0, 0)'$	0.27	
	Capital elasticity	γ	0.3	
	Green-house-gas forcings scaling factor	η	3.6813	
	Climate damages scaling factor	ψ^1	0.00236	
	Climate damages exponent	ψ^2	2	
	Abatement cost exponent	θ^2	2.6	
	Rate of change of land use emissions	δ_E	0.023	
	Rate of change of price of back-stop technology	δ^{BS}	0.005	
	Total factor productivity	g_{A0}	0.0152	
		δ_A	0.005	
	Carbon intensity	$g_{\sigma 0}$	−0.0152	
		δ_σ	−0.001	
	Population dynamic	$\mathcal{L}_0$	1.38718	
		$\mathcal{L}_1$	−0.07950	
		$\mathcal{L}_2$	0.0268	
		$\mathcal{L}_3$	0.134	
	Non-industrial GHG forcings	fex_0	0.5	
		fex_0	1.0	

Appendix C. Derivation of the social cost of carbon (SCC)

Let $V(C, E)$ be the value-function of problem (A.1) depending on the profiles of total consumption and emissions. Suppose that the level set of V given by $\{V = V(C^*, E^*)\}$ can be locally parametrized as the graph of a function $C(E)$

$$\{(C(E), E) : E \in (E^* - \epsilon, E^* + \epsilon)\} \tag{C.1}$$

(which is true in a neighborhood of (C^*, E^*) if $\partial_C V(C^*, E^*) \neq 0$). The Social Cost of Carbon (SCC) is defined as the Marginal Rate of Substitution (MRS) of consumption for emissions:

$$\text{SCC} = \frac{dC}{dE}. \tag{C.2}$$

Since V is constant on the level set,

$$0 = \frac{dV}{dE}(C(E), E) = \partial_C V \frac{dC}{dE} + \partial_E V = \partial_C V \cdot \text{SCC} + \partial_E V$$

which yields

$$\text{SCC} = -\frac{\partial_E V}{\partial_C V} = -\frac{\Lambda_E}{\Lambda_C}. \tag{C.3}$$

Since E and C are not state variables and V does not depend on them explicitly, we identify which quantities are directly impacting V, compute the derivatives of such quantities with respect to E and C, and then apply the chain rule to $\partial_E V$ and $\partial_C V$.

E enters the dynamics of M^{AT}:

$$\dot{M}^{AT} = \phi_{11} M^{AT} + \phi_{21} M^{UP} + \alpha E \tag{C.4}$$

$$\begin{aligned} \frac{d}{dt}(M^{AT} + \Delta M^{AT}) &= \phi_{11}(M^{AT} + \Delta M^{AT}) + \phi_{21} M^{UP} + \alpha(E + \Delta E) \\ &= \phi_{11} M^{AT} + \phi_{21} M^{UP} + \alpha E + \phi_{11} \Delta M^{AT} + \alpha \Delta E \\ &= \dot{M}^{AT} + \phi_{11} \Delta M^{AT} + \alpha \Delta E \end{aligned}$$

yielding

$$\Delta \dot{M}^{AT} = \phi_{11} \Delta M^{AT} + \alpha \Delta E \tag{C.5}$$

$$\Delta M^{AT}(t) = \int_0^t e^{\phi_{11}(t-s)} \alpha \, \Delta E(s) \, ds \tag{C.6}$$

The derivative $\frac{dM^{AT}}{dE}(t)$ is to be intended in the sense of Fréchet, i.e. the variation of $M^{AT}(t)$ that is produced by $\Delta E = 1$ in the small interval preceding t and 0 otherwise ("blip"):

$$\frac{dM^{AT}}{dE}(t) = \lim_{\epsilon \to 0^+} \frac{1}{\epsilon} \int_{t-\epsilon}^{t} e^{\phi_{11}(t-s)} \alpha \, ds = \frac{\alpha}{\phi_{11}} \lim_{\epsilon \to 0^+} \frac{e^{\phi_{11}\epsilon} - 1}{\epsilon} = \alpha$$

So $\partial_E V = \partial_{M^{AT}} V \frac{dM^{AT}}{dE} = \lambda^{M^{AT}} \alpha$.

C **enters the utility function** $U = U(c, L(t)) = U\left(\frac{C}{L(t)}, L(t)\right)$:

$$\frac{dU}{dC}(t) = \partial_c U \frac{1}{L} \tag{C.7}$$

The derivative $\frac{dV}{dU}$ is again a Fréchet derivative:

$$\frac{dV}{dU}(t) = \lim_{\epsilon \to 0^+} \frac{1}{\epsilon} \int_{t-\epsilon}^{t} e^{-\rho(s-t)} \, ds = 1 \tag{C.8}$$

by continuity of the integrand function. So $\partial_C V = \frac{dV}{dU} \frac{dU}{dC} = \partial_c U \frac{1}{L}$.

Putting everything together,

$$SCC = -\frac{\partial_E V}{\partial_C V} = -\alpha \frac{\lambda^{M^{AT}} L}{\partial_c U} \tag{C.9}$$

Appendix D. Optimality conditions for the social-planner-DICE model

Using the Pontryagin Maximum-Principle for full optimal control model (A.1) implies the following first-order optimality conditions. Note that in the terms $(\rho - \Phi)$ and $(\rho - \zeta)$ the discount rate ρ has to be read as a 3-by-3 and 2-by-2 diagonal matrix respectively.

$$0 = U_s + \lambda^K Q - \overline{\lambda^s} + \underline{\lambda^s} \tag{D.1a}$$

$$\overline{\lambda^s} \geq 0 \quad , \quad \overline{\lambda^s} \cdot (1 - s) = 0 \quad , \quad \underline{\lambda^s} \geq 0 \quad , \quad \underline{\lambda^s} \cdot s = 0 \tag{D.1b}$$

$$0 = U_\mu - \left(\lambda^{M^{AT}} \alpha \sigma + \lambda^K s \Lambda_\mu\right) Y - \overline{\lambda^\mu} + \underline{\lambda^\mu} \tag{D.1c}$$

$$\overline{\lambda^\mu} \geq 0 \quad , \quad \overline{\lambda^\mu} \cdot (\overline{\mu} - \mu) = 0 \quad , \quad \underline{\lambda^\mu} \geq 0 \quad , \quad \underline{\lambda^\mu} \cdot \mu = 0 \tag{D.1d}$$

$$\dot{\lambda^K} = \left(\rho + \delta_K - s\gamma \frac{Q}{K}\right) \lambda^K - U_K - \lambda^{M^{AT}} \frac{Y}{K} \gamma \alpha (1 - \mu) \sigma \qquad \lambda^K(T) = 0 \tag{D.1e}$$

$$\dot{\lambda^M} = \lambda^M (\rho - \Phi) - (\lambda^{T^{AT}}, 0, 0) \xi F'(M^{AT}) \qquad \lambda^M(T) = (0, 0, 0) \tag{D.1f}$$

$$\dot{\lambda^T} = \lambda^T (\rho - \zeta) - (1, 0) \left(U_{T^{AT}} - \lambda^K s Y \Omega' T^{AT} - \chi\right) \qquad \lambda^T(T) = (0, 0) \tag{D.1g}$$

We can furthermore derive the following functional relationships:

$$U_s = U_c \cdot c_s = -U_c \frac{Q}{L} \tag{D.2a}$$

$$U_K = U_c \cdot c_K = U_c \frac{(1 - s) Q_K}{L} = \frac{U_c}{L} (1 - s) \gamma \frac{Q}{K} \tag{D.2b}$$

$$U_\mu = U_c \cdot c_\mu = U_c \frac{(1 - s) Q_\mu}{L} = -U_c \frac{(1 - s) \Lambda_\mu Y}{L} \tag{D.2c}$$

These equations allow us to simplify the FOC (D.1a) to

$$\frac{U_c}{L} = \lambda^K - \frac{\overline{\lambda^s} - \underline{\lambda^s}}{Q} \tag{D.3}$$

and (D.1c) to

$$-\lambda^{M^{AT}} \alpha \sigma Y - \left(\overline{\lambda^\mu} - \underline{\lambda^\mu}\right) = U_c \frac{(1 - s) \Lambda_\mu Y}{L} + \lambda^K s \Lambda_\mu Y \tag{D.4}$$

$$-\lambda^{M^{AT}} \alpha - \frac{\left(\overline{\lambda^\mu} - \underline{\lambda^\mu}\right)}{\sigma Y} = \frac{\Lambda_\mu}{\sigma} \left[\frac{U_c}{L} (1 - s) + \lambda^K s\right] \tag{D.5}$$

$$-\lambda^{M^{AT}} \alpha - \frac{\left(\overline{\lambda^\mu} - \underline{\lambda^\mu}\right)}{\sigma Y} = \frac{\Lambda_\mu}{\sigma} \left[\lambda^K - \frac{(\overline{\lambda^s} - \underline{\lambda^s})(1 - s)}{Q}\right] \tag{D.6}$$

$$-\lambda^{M^{AT}} \alpha - \frac{\left(\overline{\lambda^\mu} - \underline{\lambda^\mu}\right)}{\sigma Y} = \frac{\Lambda_\mu}{\sigma} \left[\lambda^K + \underline{\lambda^s}/Q\right] \tag{D.7}$$

$$\Lambda_\mu = \sigma \cdot \left[\frac{-\lambda^{M_{AT}} \alpha - \frac{(\overline{\lambda^\mu} - \underline{\lambda^\mu})}{\sigma Y}}{\lambda^K + \underline{\lambda^s}/Q}\right] \tag{D.8}$$

$$\Lambda_\mu = \sigma \cdot \left[\frac{SCC \cdot U_c/L - \frac{(\overline{\lambda^\mu} - \underline{\lambda^\mu})}{\sigma Y}}{\lambda^K + \underline{\lambda^s}/Q} \right] \tag{D.9}$$

$$\Lambda_\mu = \sigma \cdot \left[SCC \cdot \left(\frac{\lambda^K - (\overline{\lambda^s} - \underline{\lambda^s})/Q}{\lambda^K + \underline{\lambda^s}/Q} \right) - \frac{1}{\sigma Y} \frac{(\overline{\lambda^\mu} - \underline{\lambda^\mu})}{\lambda^K + \underline{\lambda^s}/Q} \right] \tag{D.10}$$

$$\Lambda_\mu = \sigma \cdot \left[SCC \cdot \left(\frac{\lambda^K + \underline{\lambda^s}/Q - \overline{\lambda^s}/Q}{\lambda^K + \underline{\lambda^s}/Q} \right) - \frac{1}{\sigma Y} \frac{(\overline{\lambda^\mu} - \underline{\lambda^\mu})}{\lambda^K + \underline{\lambda^s}/Q} \right] \tag{D.11}$$

$$\Lambda_\mu = \sigma \cdot \left[SCC \cdot \left(1 - \frac{\overline{\lambda^s}/Q}{\lambda^K + \underline{\lambda^s}/Q} \right) - \frac{1}{\sigma Y} \frac{(\overline{\lambda^\mu} - \underline{\lambda^\mu})}{\lambda^K + \underline{\lambda^s}/Q} \right] \tag{D.12}$$

Taking into account that the Inada-conditions hold for the utility function U, we know that it can never be optimal to have consumption level 0. This would correspond to a savings rate $s = 1$. Hence we know, that $\overline{\lambda^s} = 0$ holds for the whole time horizon and (D.1c) can be further simplified to

$$\Lambda_\mu = \sigma \cdot \left[SCC - \frac{1}{\sigma Y} \frac{(\overline{\lambda^\mu} - \underline{\lambda^\mu})}{\lambda^K + \underline{\lambda^s}/Q} \right] \tag{D.13}$$

Appendix E. Derivations for decentralized DICE model

E.1. Proof of no-profit condition

Using the market equilibrium values for the factor prices and the assumption that the capital and labor markets always clear we can proof that the total household budget corresponds to the net economic output per capita.

$$\begin{aligned} R \cdot a + w + \frac{p \cdot E}{L} &= \gamma \frac{Q}{K} \cdot \left[1 - \frac{p(1-\mu)\sigma}{(1-\Omega-\Lambda)} \right] \cdot a + (1-\gamma) \frac{Q}{L} \cdot \left[1 - \frac{p(1-\mu)\sigma}{(1-\Omega-\Lambda)} \right] + \frac{p \cdot E}{L} \\ &= \left[1 - \frac{p(1-\mu)\sigma}{(1-\Omega-\Lambda)} \right] \cdot \left(\gamma Q \frac{a}{K} + (1-\gamma) \frac{Q}{L} \right) + \frac{p \cdot E}{L} \\ &\underbrace{=}_{a = K/L} \left[1 - \frac{p(1-\mu)\sigma}{(1-\Omega-\Lambda)} \right] \cdot \frac{Q}{L} + \frac{p \cdot E}{L} \\ &= \frac{Q}{L} - \frac{p(1-\mu)\sigma}{L} \frac{Q}{(1-\Omega-\Lambda)} + \frac{p \cdot (1-\mu)\sigma Y}{L} = \frac{Q}{L} \end{aligned} \tag{E.1}$$

E.2. Proof for Proposition 3

Consider a marginal change of the carbon price during a time interval $[t, t+\Delta t]$. For the reaction of the household to be optimal, the following condition has to be satisfied. The marginal change of assets at the end of the time period measured in utility units has to be equal the marginal change in utility derived from consumption throughout the time period. This relationship can be formally stated as

$$\underbrace{\frac{da(t+\Delta t)}{dp} \cdot \xi^a(t+\Delta t) \exp\left\{ -\int_t^{t+\Delta t} (\rho - n(\tau))\, d\tau \right\}}_{=: \frac{d\widetilde{W}}{dp}} = \underbrace{\frac{d}{dp} \left\{ \int_t^{t+\Delta t} \exp\left\{ -\int_t^{\tau} \left(\rho - n(t')\right) dt' \right\} u(c(\tau))\, d\tau \right\}}_{=: \frac{d\widetilde{V}}{dp}} \tag{E.2}$$

Furthermore the dynamic equation for assets implies the following relationship between assets and consumption:

$$a(t+\Delta t) = \exp\left\{ \int_t^{t+\Delta t} R(\tau) - \delta - n(\tau) d\tau \right\} a(t) + \int_t^{t+\Delta t} \exp\left\{ \int_\tau^{t+\Delta t} R(t') - \delta - n(t') dt' \right\} \cdot \left[w(\tau) + \frac{p(\tau)E(\tau)}{L(\tau)} - c(\tau) \right] d\tau \tag{E.3}$$

Using the first-order optimality conditions and the Euler equations and after some tedious calculations we arrive at

$$\begin{aligned} \frac{dc(t)}{dp} = c(t) \frac{\gamma E}{K} \cdot &\left\{ \frac{1}{\beta} \frac{e^{(\mathcal{A}+\mathcal{B})\Delta t} \cdot (\mathcal{A}+\mathcal{B})\Delta t - e^{(\mathcal{A}+\mathcal{B})\Delta t} + 1 - \frac{1}{2} - \frac{(\mathcal{A}+\mathcal{B})\Delta t}{2} + e^{(\mathcal{A}+\mathcal{B})\Delta t}/2}{(\mathcal{A}+\mathcal{B}) \cdot (e^{(\mathcal{A}+\mathcal{B})\Delta t} - 1)} \right\} \\ - \gamma a(t) \frac{E}{K} \frac{(\mathcal{A}+\mathcal{B})}{2} \cdot \frac{\Delta t}{(e^{(\mathcal{A}+\mathcal{B})\Delta t} - 1)} + \gamma \frac{E}{L} \frac{(\mathcal{A}+\mathcal{B})}{2\mathcal{A}^2} \times &\left[\mathcal{A} \left(1 - \frac{p\sigma}{\gamma \Lambda_{\mu\mu}(1-\mu)} \right) \cdot \frac{e^{\mathcal{A}\Delta t} - 1}{e^{(\mathcal{A}+\mathcal{B})\Delta t} - 1} + \left(\frac{pE}{K} + \frac{w}{a} \right) \cdot \frac{(1 + \mathcal{A}\Delta t - e^{\mathcal{A}\Delta t})}{e^{(\mathcal{A}+\mathcal{B})\Delta t} - 1} \right] \end{aligned} \tag{E.4}$$

with

$$\mathcal{A} := \delta + n(t) - R(t) \tag{E.5}$$

$$\mathcal{B} := \frac{R(t) - \delta - \rho}{\beta} \tag{E.6}$$

Building the limit $\Delta t \to 0$ and using the rule of *L'Hospital* we arrive at

$$\frac{dc(t)}{dp} = -\frac{1}{2} \frac{p\sigma^2 Y}{\Lambda_{\mu\mu} L} < 0 \tag{E.7}$$

and thereby providing the proof for the proposition.

E.3. Reformulation of the co-state dynamics of capital in the social-planner model

Starting from the co-state dynamics presented in Eq. (D.1e).

$$\dot{\lambda^K} = \left(\rho + \delta_K - s\gamma\frac{Q}{K}\right)\lambda^K - U_K - \lambda^{M^{AT}}\frac{Y}{K}\gamma\alpha(1-\mu)\sigma \qquad \lambda^K(T) = 0$$

Using the FOC for the Social-Planner-DICE model and the fact that

$$U_K + \lambda^{M^{AT}}\alpha\sigma\frac{Y}{K}\gamma(1-\mu) = \lambda^K\cdot\left[(1-s)\gamma\frac{Q}{K}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right)\right] - \frac{(\overline{\lambda^s}-\underline{\lambda^s})}{Q}\left[(1-s)\gamma\frac{Q}{K}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right)\right]$$
$$-(1-\mu)(\overline{\lambda^\mu}-\underline{\lambda^\mu})\frac{\gamma}{K} - \lambda^K s\Lambda_\mu(1-\mu)\frac{\gamma Y}{K}. \tag{E.8}$$

we can rewrite the last two terms of Eq. (D.1e) as

Note that

$$(1-s)(\overline{\lambda^s}-\underline{\lambda^s}) = -(1-s)\underline{\lambda^s} = -\underline{\lambda^s} \tag{E.9}$$

what lets us simplify the previous term to

$$U_K + \lambda^{M^{AT}}\alpha\sigma\frac{Y}{K}\gamma(1-\mu) = \lambda^K\cdot\left[(1-s)\gamma\frac{Q}{K}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right) - s\gamma\frac{Q}{K}\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right] + \underline{\lambda^s}\frac{\gamma}{K}\cdot\left[1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right]$$
$$-(1-\mu)(\overline{\lambda^\mu}-\underline{\lambda^\mu})\frac{\gamma}{K}. \tag{E.10}$$

Inserting this equality into (D.1e) finally leads to

$$\dot{\lambda^K} = \left(\rho + \delta_K - \gamma\frac{Q}{K}\left[1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right]\right)\lambda^K - \frac{\gamma}{K}\left(\underline{\lambda^s}\left[1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right] - (1-\mu)(\overline{\lambda^\mu}-\underline{\lambda^\mu})\right). \tag{E.11}$$

E.4. Reformulation of the co-state dynamics of the household assets

We start from the co-state dynamics of household assets presented in Eq. (20).

$$\dot{\xi^a} = \rho\xi^a - \xi^a(R-\delta_K) - \frac{R}{L}\overline{\xi^c} = (\rho+\delta_K-R)\xi^a - \frac{\overline{\xi^c}}{L}\cdot R \tag{E.12}$$

For the interest rate for production capital, we can derive with $p = \mathrm{SCC}$

$$R = \gamma\frac{Q}{K}\cdot\left[1-\frac{\mathrm{SCC}(1-\mu)\sigma}{(1-\Omega-\Lambda)}\right] = \gamma\frac{Q}{K}\cdot\left[1-\frac{\left[\frac{\Lambda_\mu}{\sigma}+\frac{1}{\sigma Y}\frac{(\overline{\lambda^\mu}-\underline{\lambda^\mu})}{\lambda^K+\underline{\lambda^s}/Q}\right](1-\mu)\sigma}{(1-\Omega-\Lambda)}\right] = \gamma\frac{Q}{K}\cdot\left[1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)} - \frac{(1-\mu)\sigma}{\sigma Y(1-\Omega-\Lambda)}\cdot\frac{(\overline{\lambda^\mu}-\underline{\lambda^\mu})}{\lambda^K+\underline{\lambda^s}/Q}\right]$$
$$= \gamma\frac{Q}{K}\cdot\left[1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)} - \frac{(1-\mu)}{Q}\cdot\frac{(\overline{\lambda^\mu}-\underline{\lambda^\mu})}{\lambda^K+\underline{\lambda^s}/Q}\right] \tag{E.13}$$

Using this formula for the interest rate R we can derive the following formulation for the dynamics of $\dot{\xi^a}$.

$$\dot{\xi^a} = \left[\rho+\delta_K-\gamma\frac{Q}{K}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)} - \frac{(1-\mu)}{Q}\cdot\frac{(\overline{\lambda^\mu}-\underline{\lambda^\mu})}{\lambda^K+\underline{\lambda^s}/Q}\right)\right]\xi^a - \overline{\xi^c}\cdot\gamma\frac{Q}{K}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)} - \frac{(1-\mu)}{Q}\cdot\frac{(\overline{\lambda^\mu}-\underline{\lambda^\mu})}{\lambda^K+\underline{\lambda^s}/Q}\right) \tag{E.14a}$$

$$\dot{\xi^a} = \left[\rho+\delta_K-\gamma\frac{Q}{K}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right)\right]\xi^a + \gamma\frac{Q}{K}\cdot\left(\frac{(1-\mu)}{Q}\cdot\frac{(\overline{\lambda^\mu}-\underline{\lambda^\mu})}{\lambda^K+\underline{\lambda^s}/Q}\right)\xi^a$$
$$-\gamma\frac{Q}{K}\cdot\overline{\xi^c}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right) + \gamma\frac{Q}{K}\cdot\overline{\xi^c}\cdot\frac{(1-\mu)}{Q}\cdot\frac{(\overline{\lambda^\mu}-\underline{\lambda^\mu})}{\lambda^K+\underline{\lambda^s}/Q} \tag{E.14b}$$

$$\dot{\xi^a} = \left[\rho+\delta_K-\gamma\frac{Q}{K}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right)\right]\xi^a + \frac{\gamma}{K}Q\cdot\left(\frac{(1-\mu)}{Q}\cdot\frac{(\overline{\lambda^\mu}-\underline{\lambda^\mu})}{\lambda^K+\underline{\lambda^s}/Q}\right)\left(\xi^a+\overline{\xi^c}\right) - \gamma\frac{Q}{K}\cdot\overline{\xi^c}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right) \tag{E.14c}$$

$$\dot{\xi^a} = \left[\rho+\delta_K-\gamma\frac{Q}{K}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right)\right]\xi^a + \frac{\gamma}{K}\left[(1-\mu)\cdot(\overline{\lambda^\mu}-\underline{\lambda^\mu})\left(\frac{\xi^a+\overline{\xi^c}}{\lambda^K+\underline{\lambda^s}/Q}\right) - Q\overline{\xi^c}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right)\right] \tag{E.14d}$$

$$\dot{\xi^a} = \left[\rho+\delta_K-\gamma\frac{Q}{K}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right)\right]\xi^a - \frac{\gamma}{K}\left[Q\overline{\xi^c}\cdot\left(1-\frac{(1-\mu)\Lambda_\mu}{(1-\Omega-\Lambda)}\right) - (1-\mu)\cdot(\overline{\lambda^\mu}-\underline{\lambda^\mu})\left(\frac{\xi^a+\overline{\xi^c}}{\lambda^K+\underline{\lambda^s}/Q}\right)\right] \tag{E.14e}$$

Data availability

All necessary files for replication are available on the public Mendeley repository attached.

DICE2016 Numerical Solution (Original data) (Mendeley Data)

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