GROWTH, CONFLICT AND CRISIS IN THE
URBAN SYSTEM: A NEO-MARXIAN APPROACH
TO MODELING INTER-URBAN ECONOMIC DY-
NAMICS

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FOREWORD

Many large urban agglomerations in the developed countries
are either experiencing population decline or are growing at
rates lower than those of middle-sized and small settlements.
This trend is in direct contrast to the one for large cities
in the less developed world, which are growing rapidly. Urban
contraction and decline is generating fiscal pressures and
fueling interregional conflicts in the developed nations; ex-
plosive city growth in the less developed world is creating
problems of urban absorption. These developments call for the
reformulation of urban policies based on an improved under-
standing of the dynamics that have produced the current patterns.

During the period 1979-1982, the former Human Settlements
and Services Area examined patterns of human settlement trans-
formation as part of the research efforts of two tasks: the
Urban Change Task and the Population, Resources, and Growth
Task. This paper was written as part of that research acti-
vity. Its publication was delayed, and it is therefore being
issued now a few months after the dissolution of the HSS Area.

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ECONOMIC DYNAMICS

1. INTRODUCTION

Most nations currently face severe problems as a result
of the uneven spatial impact of social and economic change.
In Western Europe and North America changes in the international
division of labor and the competitiveness of traditionally im-
portant economic sectors has created major problems. National
governments face the joint responsibility of paying the social
costs associated with private disinvestment from manufacturing
areas such as the American Manufacturing Belt, the Ruhr, North-
eastern France and central Belgium while at the same time social
infrastructure must be provided elsewhere (in the Southwest of
the United States, Southeastern England and Bavaria). Further-
more, these crises and conflicts, manifest because capital is
more spatially mobile than the labor force, occur at a time
when funds available for all kinds of social programs are under
threat.

In the Third World also such crises exist, although the
problems here are due to over-concentration rather than a re-
versal of previous trends. Some countries are benefitting from
capital that is mobile internationally, creating explosive

economic growth in places such as South Korea and Taiwan as the dual side of employment declines in the developed capitalist economies. Other countries have to struggle more to attract investment, but in every case the spatial destination of that investment within nations creates national problems. Typically the locations of employment growth, and population growth, are highly localized (often reflecting the extraction-oriented spatial arrangement of a country inherited from colonial days). A few cities attract the lion's proportion of investment, creating a spatial imbalance that leads to problems of over-urbanization and squatter-settlements in the core, together with stagnation and under-utilized resources in the periphery, as capital and labor respond to these differentials.

In Eastern Europe, a high priority has traditionally been given to deliberately planning the spatial arrangement of pro-duction and distribution of goods in order to avoid such problems. This is captured in the pursuit of explicitly spatial goals: diminishing differences in economic activity levels between regions; and equalizing living conditions between city and countryside. Despite this attention, however, it is evident that regionally coordinated production has not solved the problems of distributing the outputs. In addition, Eastern Europe may be facing some of the broad reversals in the spa-tial patterns of internal development being experienced in the advanced capitalist and social democratic societies (Korcelli, 1981), for some of the same reasons.

An axiom underlying this paper is that it is in the settle-ment system where these conflicts are manifest most. In the developed economies this is clear; most economic activity is non-agricultural and is (with the exception of extractive industries) almost by definition carried out in towns and cities. Thus it is in the cities of the Northeastern United States and the Ruhr area that social problems concentrate (Bluestone and Harrison, 1980; Glickmann, 1981; Wegener, 1982); indeed some parts of these regions experience growth even as the the metropoli decline. In addition, although there is much talk and evidence of non-metropolitan growth in these countries

this is clearly not a "return to the farm". This point is em-
phasized by Usbeck (1982) for the German Democratic Republic.
Rather it refers to a rearrangement of the non-agricultural
settlement system; a system defined by the spatial relations
between residences, non-farm jobs and services. Similarly in
the Third World the social problems revolve around undesir-
able urbanization trends in the core and the lack of non-agri-
cultural opportunities in the periphery.

For these reasons it seems essential to focus attention
on the settlement system of a country in evaluating the sub-
national impacts of new economic trends. It becomes necessary
to clarify the ways in which economic growth and decline are
localized in some parts of the urban system; and how change
is, or is not, then transmitted to other parts of the system.
In this context the urban system as a whole must be the focus
of attention, as it is the channel through which socio-economic
change reaches individuals.

The urban change group at IIASA has been concerned with
this issue, but has focused primarily on demographic change.
This is a vital component, since the improvement of individual
well-being requires knowledge of where people will be. How-
ever, an accurate forecasting even of population change requires
knowledge of how the economic constraints and opportunities,
which condition behavior, themselves evolve. Indeed, to go
further, economic change is in turn influenced by population
patterns as captured in the notion of demo-economics (Willekens
and Rogers, 1977). In order to quantitatively analyse the types
of location-specific, but nationally important, problems out-
lined at the start, it is necessary to develop a model of eco-
nomic production and dynamics at the inter-urban scale of re-
solution that matches the scale used by Korcelli (1981) in his
demographic research. This complementarity is necessary in
order to ultimately link together models of economic and demo-
graphic change. The purpose of this paper to construct a con-
ception of how certain aspects of economic change can be mo-
deled for a system of cities in a manner that is capable of

incorporating the types of crises and conflicts that are cur-
rently besetting many nations.

2. A MARXIAN PARADIGM FOR THE INTER-URBAN ECONOMY

The economic components that the model to be developed
will attempt to capture will be the way in which private en-
trepreneurs: produce commodities in the various cities; pur-
chase inputs and labor for production; market the outputs;
and reinvest their profits in urban economic growth. With
such a model it should be possible in principle to analyse
changes in levels of economic activity, changes in the demand
for labor, patterns of inter-urban trade, and also the prices
at which goods are sold in the various cities. Further, a
framework that explicitly models prices and the profits
available for reinvestment should make comparisons possible
of the relative advantages that different cities possess
for production, leading to statements about the future
prospects of various locations as growth nodes. This would
certainly have the potential of giving insight into the spa-
tial dynamics of investment that are causing problems in many
countries.

In restricting attention to these elements it is immedi-
ately clear that a major economic dimension, the public eco-
nomy, has been excluded. Thus, as it stands the model would
be at most applicable to capitalist economies, and even for
those the picture is partial at best. It would most probably
be misleading for example to attempt to include in this model
commodities, such as housing, that experience extensive state
intervention, and the significant group of publicly provided
goods and services must be completely excluded. Furthermore,
the assumption will be made that all private entrepreneurs are
fully integrated into the market economy, which excludes large
groups of economic actors in the Third World. Such simplifi-
cations reflect theoretical inadequacies as much as the

complexity of the real world. A descriptive model of the public space-economy, for example,does not yet exist despite extensive normative research (Lea, 1979; Leonardi, 1981).

Some other drastic simplifications will also be made initially, although I shall attempt to show later how these may be relaxed. Fixed capital will be ignored, despite its demonstrable importance in explaining how rapidly a city can lose investment funds once its production equipment is fully depreciated in value and out of date. Costs of *in situ* replacement can simply exceed costs of relocating production. Also the assumption wil be made that the market is made up of many individual enterpreneurs with no individual power to influence outcomes, although in reality there are powerful corporations in many sectors, whose economic power may transcend that of even national governments, and whose profits and investment decisions are not governed by the capital market (Hymer, 1979; Pred, 1976; Westaway, 1974).

Most importantly, I have deliberately chosen to adapt a non-neoclassical paradigm to modeling the inter-urban economy, in contradistinction to previous research. This approach draws on the classical political economy of Marx and Ricardo, as developed by the so-called past-Keynesians starting with Sraffa (1960). Since this choice may be controversial, I shall attempt to briefly outline the advantages and disadvantages of each as a model of the urban system in order to document my choice.

2.1. The Neoclassical Approach

In general there have been very few attempts to model the economic dimension of a system of cities, but those few that do exist tend to adopt a neoclassical approach. There are, however, many applications of neoclassical analysis to regional systems and similar conclusions apply to the urban case. A recent example is due to Smith (1975) and is typical in that he assumes that an inter-regional economy may be described by a series of dispersed locations each producing goods using two

homogeneous inputs: 'capital' and labor. Each city (region) is then described by an aggregate production function, in this case of the Cobb-Douglas type:

$$Y_i = K_i^{\alpha} \cdot (L_i \cdot e^{\rho t})^{1-\alpha} \tag{1}$$

Here Y_i is the money value of goods produced in city i, K_i is the stock of 'capital' available, L_i is the size of the labor force and the time trend $e^{\rho t}$ is introduced to capture the effects of technical change. ρ (less than zero) and α (between zero and one) are empirical constants.

Taking logarithms and differentiating with respect to time, it may be concluded that the rate of growth of the monetary value of output per worker in a city depends on the local availability of 'capital' and labor:

$$\dot{y}_i^* = \alpha \cdot \left[\dot{K}_i^* - \dot{L}_i^* \right] + (1-\alpha)\rho \tag{2}$$

where $\dot{K}^* = d \log K/dt$; $\dot{L}^* = d \log L/dt$ and $\dot{y}^* = d \log (Y/L)/dt$.

The second key element of the neoclassical paradigm is the assertion that the local rate of return on each factor is given by its marginal productivity:

$$r_i = \partial Y_i / \partial K_i = \alpha \cdot Y_i / K_i \tag{3}$$

$$w_i = \partial Y_i / \partial L_i = (1-\alpha) Y_i / L_i \tag{4}$$

where r_i is the local profit rate (or rate of return on capital) and w_i is the wage rate in city i. The rate of return is inversely related to the local relative physical abundance of each factor. If factors are mobile, they are postulated to flow to the city with the higher rate of return (or greater scarcity of the factor). This leads to an equalization of relative factor availabilities in all cities, and thus to the same rate of growth throughout the urban system; a stable

dynamic equilibrium.

If factors are not mobile, cities will specialize and trade commodities. In this case the Heckshes-Ohlin-Sammelson (HOS) theory of trade that derives from the neoclassical paradigm will ensure that cities specialize in those products which require more use of the locally abundant factor. Comparative advantage will once again ensure an equilibrium that is advantageous to each city.

It is evident, then, that the neoclassical approach allows a reduction of the complexity of urban system change to a few elementary variables. This is its great advantage. It is possible then to make basic predictions for urban systems, as has been done, by Henderson (1977, pp.81-82):

> For an economy facing fixed prices set in international markets for its two manufactured and traded goods, an increase in the endowment of L will lead to an increase (decrease) in the production of the L- (K-) intensive good, say x (z), in that economy with unchanged factor proportions in either industry. In a system of cities these results are effected by an increase in x-type cities and a decrease in z-type cities...

> In terms of growth theory... if we specify the basic growth model, assuming constant returns to scale at the national level and subsuming the spatial characteristics of cities, then the properties of simple growth models will follow directly.

The neoclassical picture is a harmonious one. Wages and profits are purely technical considerations being dependent only on the physical abundance of production factors. There are no conflicts of interest between social groups in the urban system, and the forces of competition ensure steady progress towards a stable pattern of equal growth rates in all cities, punctuated only by exogenous disturbances. Unfortunately this picture crucially depends on the assumption of a homogeneous capital good, and a relaxation of this

assumption demolishes the image.

2.2. A Classical Alternative

There are two reasons for seeking a constructive alternative to the neoclassical approach for urban production dynamics. The first has been outlined above and will be detailed below; the validity of neoclassical results depends crucially on accepting an unrealistic assumption. The empirical validity of any model that is this sensitive must always be questioned. The second reason is more empirical. Even a casual student of urbanism in the advanced capitalist societies will see little evidence of harmony or strong equilibrating forces. Social groups in cities are in continual conflict; some cities prosper while others are in a state of crisis. It is for precisely these reasons that urban-related research has become so popular. Of course this contradiction between theory and reality may be seen as simply being due to the existence of unions, corporations and government, elements that make the market imperfect, or due to external shocks. However, this seems to be insufficient, if only because despite the great variety of forms in which market economies have occurred in the last two hundred years there has yet to be an example where such conflicts and instabilities have not been an important part of the urban system. It will be shown below that once the assumption of a homogeneous capital good is relaxed then a macro-economic conceptualization results where social conflict is present even in a competitive capitalist society.

If the aggregate neoclassical approach can simplify the concept of capital only at the expense of giving predictions about urban production dynamics that are qualitatively different from a multi-commodity approach, then it seems worth investigating an alternative where the full variety of economic commodities is recognized. After all, the commodities required for consumption and production are very different; even for the latter there is a tremendous range of machinery, supplies and financial resources that all qualify as capital

goods. Each is produced to different degrees in various cities and thus has its own pattern of inter-urban trade.

One way of representing this complexity is to follow the tradition pioneered by Sraffa (1960) and since pursued by Garegnani (1970), Morishima (1977), and Pasinetti (1977). In this neo-Ricardian or neo-Marxian conceptualization, a model of interdependent production and reproduction of many commodities in the absence of substitutability of inputs is developed. A long debate (Harcourt, 1972) has established the following. First, it is quite possible that when capital (labor) becomes more expensive then it is profitable for an economy to choose a new tehcnology that is _more_ capital (labor) intensive than the technology previously in use. This pheno-menon, variously known as capital reversal (Robinson, 1953-54) or a positive real Wicksell effect (Burmeister, 1980), negates some keystones of neoclassical wisdom: that the degree of employment a factor must be positively related to its physical abundance, and that the price of that factor must be inversely related to its physical abundance. Instead, the profit rate and the wage rate can no longer be entirely determined by the production methods. Rather their relative magnitudes must depend on social conditions. Further, profits and wages are paid out of the same 'pie', the monetary surplus made in an economy. Hence there is an inevitable conflict between capi-tal owners and those providing labor (Sraffa, 1960). Society is no longer a harmonious collection of individuals whose in-terests, mediated by the invisible hand of competition, balance out to give rise to collective welfare. Rather it is made up of two broad groups with conflicting interests that struggle over the division of the economic surplus. A dynamic equili-brium, of a classical rather than neoclassical type (Walsh and Gram, 1980), can still exist but it is _un_stable, perhaps explaining why perfectly competitive markets have never per-sisted in the real world. These conclusions represent drama-tic contrasts to the view of economic development inherited from the neoclassical approach, especially since they arise from the apparently harmless step of relaxing the assumption

a one commodity world. The purpose of the balance of this paper will be to apply this neo-Marxian approach to urban system development and to determine whether in this application also the intuitive ideas inherited from the neoclassical approach must be overhauled.

3. THE BASIC MODEL

Consider a set of M commodities and an urban system composed of J cities. For the purpose of this paper it will be assumed that a 'city' is in fact an urban centered region incorporating a rural hinterland. In practice this introduces a series of problems due to the interlocking hierarchical nature of urban centered regions; a problem to be treated elsewhere (Sheppard 1982). The advantage, however, is that all commodity production is included. In the formulation developed here it will be assumed that each producer buys inputs from other producers in fixed proportions per unit of output. Thus the interdependences between producers and cities can be represented as an inter-urban input-output matrix of commodity flows. This assumption of a Leontief technology imposes rather strict conditions on producers, but it has been shown in the non-spatial case that the essential conclusions from the approach are not altered by relaxing this assumption (Roemer, 1981). Define the commodity flows, then, as a_{ij}^{mn}; the quantity of good m bought from city i that is used, per unit of good n produced in city j. These flow coefficients are assembled in a MJ by MJ inter-urban matrix, $\underset{\sim}{A}$, capturing the physical quantities demanded by each producer of every other producer in the urban system. The quantity of commodity n produced in city i at time t is given by x_{it}^{n}, and the price of that good at the factory gate is p_{i}^{n}.

An essential condition in order for production in the urban system to persist is that the system is capable of reproducing itself. One way of stating this is that the quantities necessary to produce one unit of each commodity should

sum to less than one for each commodity. For a given trading pattern this condition is:

$$\underset{\sim}{A} . \underline{i} \leq \underline{i} \tag{5}$$

where $\underline{i}$ is a vector of ones. A necessary and sufficient condition for growth is that the inequality in (5) hold, since a surplus of output over inputs can only occur in this case. We will designate this condition as representing a productive urban system. We shall also assume that there are no completely isolated sub-systems of production within the urban system and that matrix $\underset{\sim}{A}$ has no two eigenvalues that are identical. These seem reasonable assumptions for any empirical application. It then follows that $\underset{\sim}{A}$ is irreducible and primitive. Hence all eigenvalues of $\underset{\sim}{A}$ have modulus less than the largest row sum of $\underset{\sim}{A}$; i.e., they have modulus less than one in a productive system (Seneta, 1981).

With respect to any such matrix of inter-urban flows, three circuits can be defined: quantities produced, prices and labor values.

3.1. The Price Circuit

The rule of price determination applied is that prices be set in such a way that each producer in each city and sector makes the same rate of profit, r. In an advanced capitalist society with transactions dominated by a capital market, money will flow from low profit sectors and cities to high profit ones, with prices fluctuating accordingly as the relative supply and demand of the different products varies. Thus for producers of good j in city n:

$$p_j^n = (1+r) \left[\sum_i \sum_{m \in I} a_{ij}^{mn} p_i^m + \sum_i \sum_{m \in I} a_{ij}^{mn} \tau_{ij}^m p_i^t + w_j l_j^n \right] \tag{6}$$

where p_i^t is the price of a unit transportation hired in city i;

τ_{ij}^{m} is the number of units of transportation necessary to ship a unit of m from city i to city j, w_j is the money wage rate per hour in city j, and l_j^n is the number of hours of labor hired to produce a unit of m in city j. Thus the factory gate price is the sum of the cost of capital good inputs from the set {I} of capital goods, plus transport costs of those inputs, plus the wage, all incremented by a rate of profit. In this formulation it is assumed that all inputs including labor are bought at the beginning of the production period, although paying wages at the end has little effect on the outcome (Steedman, 1977).

The wage is given by the money value of goods consumed by workers, under the assumption that no wages are saved. For purposes of exposition we shall assume that the real wage, given by a consumption vector $\underline{b}$ of wage goods consumed per day by a worker and his/her family, is exogenous and the same in each city. Wage goods consumed in a city need not however be produced there. Rather there is an inter-urban flow to meet wage demands represented by b_{ij}^n; the amount of wage good n bought in city i that is consumed by a worker in city j per day. If the real wage consumption vector is:

$$\underline{b} = (b_1, \ldots, b_n) \tag{7}$$

then there are constraints on flows:

$$\sum_i b_{ij}^n = b_n \qquad \forall j \tag{8}$$

The hourly money wage in city j is then the sum of costs of producing and assembling the real wage:

$$w_j = T^{-1} \left[\sum_i \sum_{n \in II} b_{ij}^n \cdot p_i^n + \sum_i \sum_{n \in II} b_{ij}^n \cdot \tau_{ij}^n \, p_i^t \right] \tag{9}$$

where {II} is the set of wage goods contained in the consumption vector, and T is the length of the work day.

Define input-output coefficients in the following way:

$$a_{ij}^{mn} = \begin{cases} a_{ij}^{mn} & m \in \{I\} \text{ (capital goods)} \\[2ex] b_{ij}^{n} \cdot l_{j}^{n} \cdot T^{-1} & m \in \{II\} \text{ (wage goods)} \\[2ex] \sum_{m \in I} a_{ij}^{mn} \, {}_{ij}^{m} + \sum_{n \in II} b_{ij}^{m} \tau_{ij}^{m} & m = t \quad \text{(transport goods)} \end{cases}$$

then if (9) is substituted into (6), and these definitions are applied, equation (6) becomes:

$$\underline{p}' = (1 + r) \cdot \underline{p}' \cdot \underset{\sim}{A} \tag{10}$$

where $\underline{p}' = \left[p_{1}^{1} \ldots \ldots p_{J}^{M} \right]$ is the vector of prices.

Equation (10) is a characteristic equation for the non-negative primitive matrix $\underset{\sim}{A}$. From the Perron-Frobenius theorems we know that there exists a positive vector $\underline{p}'$ of relative prices, associated with the largest eigenvalue μ_1 of $\underset{\sim}{A}$. Further, $\mu_1 = (1 + r)^{-1} < 1$ because $\underset{\sim}{A}$ is productive. Thus it can be concluded that positive equilibrium prices and a profit rate can be solved for, that in a productive inter-urban economy the profit rate will be positive, and that only relative prices can be solved for since there is one unknown more than the number of equations in (10). Note that to solve (10) it was necessary to specify the level of the wage (in real terms) via $\underline{b}$. Thus wages and profits are not a technical matter; one of them is fixed by processes outside the production circuit. This is certainly in accord with the importance that is placed on wage negotiations as a determinant of profit rates in Western economies. Indeed in the United States there has been much discussion about the rate of geographical difference in money wages as an influence in the movement of industry to cities of the South and West

(Bulestone and Harrison, 1980; Watkins, 1980).

3.2. The Labor Value Circuit

In recent years there has been a spate of research developing explanations of urban development from a Marxist tradition (Harvey, 1973; 1983; Dear and Scott, 1981; Tabb and Sawers, 1978; Santos, 1979; Castells, 1979). A common theme is this literature is the use of labor values (Marx, 1967) as a key element in prognosticiations about urban change. Following Sraffa (1960), investigation by a number of authors has revealed that labor values may be calculated using the framework outlined here in a manner consistent with Marx's (1967) original definitions (Morishima, 1973; Abraham-Frois and Berrebis, 1979; Zalai, 1981; Lipietz, 1982). Exploiting, for example, the definition of the labor value of a good as being equal to the sum of direct labor involved plus the labor value of all non-labor direct inputs:

$$\lambda_j^n = 1_j^n + \sum_i \sum_{m \in I} a_{ij}^{mn} \cdot \lambda_i^m + \sum_i \sum_{m \in I} a_{ij}^{mn} \tau_{ij}^m \lambda_i^t \tag{11}$$

where λ_j^n is the labor value of one unit of good n as produced in city j. Defining the vectors: $\underline{\Lambda}' = \left[\lambda_1^1, \ldots, \lambda_J^M \right]$; $\underline{1}' = \left[1_1^1, \ldots, 1_J^M \right]$, then equation (11) becomes:

$$\underline{\Lambda}' = \underline{1}' + \underline{\Lambda}' A^* \tag{12}$$

$$\underline{\Lambda}' = \underline{1}' \cdot [I - \underline{A}^*]^{-1}. \tag{13}$$

where $\underline{A}^*$ is the inter-urban commodity trading matrix with all wage goods excluded. An element a_{ij}^{mn*} of $\underline{A}^*$ is defined as:

$$
a_{ij}^{mn*} = \begin{cases} a_{ij}^{mn} & m \in \{I\} \\ \\ 0 & m \in \{II\} \\ \\ \sum_{m \in I} a_{ij}^{mn} \cdot \tau_{ij}^{m} & m = t \end{cases}
$$

In the aspatial Western literature it has been shown, using the so-called Fundamental Marxian Theorem, that a positive profit is only possible if the labor value of an hourly wage is less than the labor value of one hour of work (unity). Indeed, defining the rate of exploitation of labor, e, as the rate of uncompensated to compensated labor (in labor value terms):

$$
e = \frac{1 - T^{-1} \sum_{n} b_{n} \cdot \lambda_{n}}{T^{-1} \sum_{n} b_{n} \lambda_{n}} \tag{14}
$$

where λ_{n} is the labor value of wage good n.

It can be shown that e > r; i.e., that exploitation of labor is necessary for positive profits in the economy.

These ideas have been examined for a spatial economy (Sheppard, 1981) and several interesting features have emerged. First, the labor value of the wage varies from city to city because of the fact that goods are traded between cities. The labor value of the wage in city i is:

$$
v_{i} = T^{-1} \cdot \left[\sum_{i} \sum\sum_{m \in II} \left(b_{ij}^{m} \cdot \lambda_{i}^{m} + b_{ij}^{m} \cdot \tau_{ij}^{m} \cdot \lambda_{i}^{t} \right) \right], \tag{15}
$$

which clearly depends on the inter-urban purchasing patterns of wage goods, and on the labor value of transportation needed to assemble those goods. Consequently the rate of exploitation varies, since exploitation is defined [see (14)] in the inter-urban economy as:

$$e_i = \frac{1 - v_i}{v_i} \qquad (16)$$

Secondly, it has been shown that in dynamic equilibrium, the rate of profit is always greater than or equal to the mean rate of exploitation in the urban system, where the mean rate of exploitation is:

$$\bar{e} = \frac{L - \bar{v}}{\bar{v}} \qquad (17)$$

Here L is the total hours worked by labor, equal to $\sum_i \sum_m x_i^m . l_i^m$ and $\bar{v}$ is the mean value of the wage weighted by the total number of workers in each city:

$$\bar{v} = \sum_i v_i \sum_m x_i^m \, l_i^m. \qquad (18)$$

Further, the rate of profit is always less than the highest rate of exploitation in the urban system.

Thus it is possible that in some cities the rate of exploitation may be zero or even negative. All that is necessary is that the mean rate be positive in order to guarantee positive profits. This is of considerable interest, since workers in some cities can benefit from economic growth due to positive profits without being exploited, because of high exploitation rates elsewhere. As a result it would seem to be in the interest of such workers to support current production relations. Alliances would be made then between social classes within one city and conflicts would occur within one social class at different locations as geographic interest groups are partially substituted for social alliances (Urry, 1981).

This form of analysis of conflicts in a developing urban system is premised on a contentious issue in social science theory; the degree to which actions respond to the implicit

system of labor values. It has been argued that in an economy
people respond to prices, and thus an analysis using labor
values as the driving force is not realistic. Indeed Marxists
have also come to question the validity and necessity of an
analysis founded only on labor values (Lippi, 1979; Hodgson,
1980-81). Without taking sides in this debate, the type of
approach advocated here does allow parallel analyses using
prices or labor values as the value system. Thus it is possi-
ble to compare the predictions based on each approach allowing
analysis to concentrate on those issues where conflicting con-
clusions are reached. In this limited sense, the possibility
exists of a comparison of Marxist with other less radical
approaches.

3.3. The Quantity Circuit

In terms of physical quantities I have argued that the
urban system must be productive in order to reproduce itself.
If this were not the case some fundamental changes would have
to occur in the very structure of the social system. Further,
in a capitalist society any surplus produced can be expected
to be reinvested in higher levels of production. Under eco-
nomic competition reinvestment is necessary even to retain
one's share of the market. Thus a model of the circulation
of commodities must be dynamic; some statement about how the
surplus is used is necessary. One starting point is with a
state of dynamic equilibrium. This is defined by two pro-
perties: each production sector in each city is growing at
the same rate, and products produced in one period are de-
manded for further production in the next time period. These
conditions can be formally written as:

$$\underline{x}_{t+1} = (1 + g)\underline{x}_t \qquad (19)$$

$$\underline{x}_t = \underset{\sim}{A} \cdot \underline{x}_{t+1} \qquad (20)$$

where $\underline{x}'_t$ is the vector $\left[x'_{1t}, \ldots, x^m_{Jt} \right]$ of production quantities, and g is the growth rate. The right hand side of (20) represents demands for inputs at time t+1; the left hand side of course represents outputs at time t. Substituting (19) into (20):

$$\underline{x}^* = (1 + g) . \underline{A} . \underline{x}^* \qquad (21)$$

Equation (21) is, like equation (10), a characteristic equation which has a unique solution for a positive rate of growth and a positive equilibrium vector of relative production quantities, $\underline{x}^*$. Indeed (10) and (21) are dual solutions, and we can conclude that the eigenvalue in each case is identical. Thus for dynamic equilibrium the rate of growth must equal the rate of profit for the urban system, and the quantities produced by each sector in each city must conform in relative size to the elements of the principal right hand eigenvector of $\underline{A}$. The equality of r and g depends on the assumption that workers save nothing whereas capitalists reinvest all their savings. If only a proportion, s_c, of capitalists earnings are saved:

$$g = s_c r \qquad (22)$$

If in addition a proportion s_w of the worker's wage is re-invested:

$$g = s_c r + s_w . W_k \qquad (23)$$

where W_k is the ratio of the money value of the real wage to the money value of capital goods (Sheppard, 1983a). Thus it is possible to conclude that in order for the urban system to remain in equilibrium the rate of growth will be less than or equal to the rate of profit, unless workers save a great deal and the wage bill is a high proportion of total production costs. This introduces a conflict, because high wages implies lower profits. Thus it is likely that the rate of profit

represents an upper bound on growth rates. Indeed under certain conditions workers' savings will not affect the equilibrium rate of growth at all (Pasinetti, 1962).

3.4. The Instability of Equilibrium

Before completing analysis of the equilibrium state it is important to establish whether the equilibrium is stable. It is easy to demonstrate that this is not the case. The difference equation (20) can be rewritten as:

$$\underline{x}_{t+1} = \underline{A}^{-1} \underline{x}_t \tag{24}$$

This is unstable since the roots of $\underline{A}^{-1}$ are the reciprocal of the roots of $\underline{A}$ and thus all the roots of this first order dynamic model lie outside the unit circle, which is sufficient to guarantee instability.

The instability of equilibrium is a conclusion of fundamental importance. Instability is also a feature of the multiple commodity neoclassical models (Burmeister, 1980) but is even more fundamental here. There do not even exist a set of prices which if imposed by central planners on the urban system would then guide it onto the path of smooth accumulation. Once the production vector $\underline{x}$, defining this knife-edge equilibrium, is deviated from in any way production patterns become highly unstable. Entrepreneurs become caught between two types of crises: a realization crisis whereby profits cannot be fully reinvested, and a disproportionality crisis whereby the goods produced do not match future demands for these goods. Away from dynamic equilibrium, the rate of growth necessary to avoid one crisis does not match the rate of growth necessary to avoid the other. As a result producers in the various cities continually face one as the other crisis. Interestingly the location of the producers facing each type of crisis depends on the pattern of inter-urban trade, $\underline{A}$ (Sheppard, 1983b). It

is quite possible that the trade pattern may take such a form that entire cities are faced by one or the other crisis, although this has not yet been investigated.

Hahn (1966) has referred to such instabilities as "the golden nail in the coffin of capitalism". This is because it suggests the conclusion that there exists no self-correction mechanism whereby the competitive market can direct the urban system onto a path of crisis free accumulation and economic growth. Furthermore, this model as it stands suggests that even a central pricing policy will not achieve this end. Rather, government intervention of a more fundamental nature is required to avoid socially unacceptable instabilities. Despite the extreme simplifications of this model, it does then give some insight into why government has been forced to play a central role in subsidizing economic production; and why in many countries the existence of an explicit urban policy is the norm. Traditionally many economic activities, varying from provision of social infrastructure and transportation to publicly funded education, housing, and social welfare services, have dominated those budgets of federal and local governments allocated to urban areas. Such costs essentially reduce production costs by subsidizing such costs either directly, or indirectly through provision of an educated and partially subsidized labor force. It was seen above that even if the urban system were in equilibrium the achievable rate of growth depends on the rate of profit. Thus it is perfectly natural that government intervention aimed at avoiding crisis and changing urban growth rates consists of policies directed at affecting production costs and thus profit rates (Broadbent, 1977; Scott, 1980).

When the urban system is out of equilibrium, the growth rate differences experienced by various producers will become translated into unequal rates in various cities. One necessary element of government intervention must then be an attempt to regulate urban growth rates directly, when indirect influences via the local profit rate fail to produce

the desired results. Thus, for example, in the United Kingdom
national policies have included subsidies to encourage location
of industry in the cities of economically stagnant areas, and
restrictions on office space in order to force service activi-
ties to locate in other cities than London (Rees and Miall, 1981;
Danson, 1982; Daniels, 1977). In other cases where explicit
national policies have not been developed, as in the United
States, local governments attempt to alleviate the problem
individually. Thus over the last five years there has been
a desperate, if ultimately self-defeating, attempt by some
U.S. cities to bid for industry in competition against other
cities (Glickmann, 1981). On the other hand, attempts to
deal with such problems on the local level have led to ur-
ban fiscal crises as cities such as New York and Cleveland
have been forced to borrow beyond their means (Alcaly and
Mermelstein, 1977). Although a study of such examples of
public response to urban system crisis is beyond the scope
of this paper, the framework here does suggest why such inter-
vention has been necessary.

4.   A SYSTEM OF SPATIAL MACRO-ECONOMIC ACCOUNTS

    Once a value has been assigned to production in the urban
system, it is possible to divide the value of production into
various components. This section will show how such macro-
economic components can be further composed to quantify how
much is flowing within and between the various cities. This
in turn allows assessment of the balance of trade for each
city; a concept that is surely as important in determining
urban prosperity as its international counterpart is for
national economic analyses.

4.1. Monetary Accounting

The monetary value of production (X) can be divided into that necessary for replacement of capital good inputs at current production levels (Y), plus the so-called "net income" which equals wages (W) plus profits (Π) (cf Harris 1977):

$$X = Y + W + \Pi \tag{25}$$

These concepts are defined as:

$$X = \underline{p}'\underline{x} = (1 + r)\underline{p}'.\underset{\sim}{A}.\underline{x} \tag{26}$$

$$Y = \underline{p}'.\underset{\sim}{A^*}.\underline{x} \tag{27}$$

$$W = \underline{p}'.\underset{\sim}{G}.\underline{x} \tag{28}$$

$$R = r.\underline{p}'.\underset{\sim}{A}.\underline{x} \tag{29}$$

using equation (10), and the definition of total income as being the sum of all quantities produced multiplied by their price. $\underset{\sim}{G}$ is the inter-urban matrix of flows of wage goods plus transport necessary to distribute them, with an element g_{ij}^{mn} defined as:

$$g_{ij}^{mn} = \begin{cases} 0 & m \text{ is capital good} \\ b_{ij}^{m}.1_{j}^{n}.T^{-1} & m \text{ is a wage good} \\ \underset{m \in \Pi}{\sum} b_{ij}^{m} \tau_{ij}^{m} & m \text{ is the transport sector} \end{cases}$$

Thus

$$\underset{\sim}{A} = \underset{\sim}{A^*} + \underset{\sim}{G} \tag{30}$$

and equation (25) is satisfied. Note that equations (26)-(29) can be applied for any observed vectors $\underline{p}$ and $\underline{x}$ and matrix $\underset{\sim}{A}$.

Those flows may be disaggregated for the different cities by employing the following definition. If $\underset{\sim}{M}$ is an inter-urban matrix of flows, define a component matrix $\underset{\sim}{M}_{ij}$ where all the elements of $\underset{\sim}{M}$ are equal to zero except for those representing a flow from city i to city j.

$$
\underset{\sim ij}{M} = \quad \text{city i} \quad
\begin{array}{c}
\text{city} \\
\text{j} \\
\left[
\begin{array}{c c c c}
0 & 0 & 0 & 0 \\
& 0 & 0 & 0 \\
& 0 & m_{ij}^{mn} & 0 \\
0 & 0 & 0 & 0
\end{array}
\right]
\end{array}
$$

It then follows that

$$
\underset{\sim}{M} = \sum_i \sum_j \underset{\sim ij}{M} \tag{31}
$$

The total money flowing directly from city i to city j is then equal to the money value of all goods from city j sold in city i:

$$
Y_{ij} = (1 + r) \cdot \underset{\sim}{p}' \cdot \underset{\sim ji}{A} \cdot \underline{x} \tag{32}
$$

The trade balance of city j is then:

$$
\overline{Y}_j = \sum_i (Y_{ij} - Y_{ji})
$$

$$
= (1 + r)\underset{\sim}{p}' \sum_i (A_{ji} - A_{ij}) \underline{x} \tag{33}
$$

This may be applied to any of the components of Y (Sheppard,

1982a). For example the flow of those payments from city i to city j that represent profit earned on investment is:

$$\Pi_{ij} = r \cdot \underline{p}' \cdot \underset{\sim}{A}_{ji} \underline{x} \tag{34}$$

4.2. Labor Value Accounting

The total labor value of production (L) may be subdivided in an analogous manner into constant capital (C), variable capital (V) and surplus value (S) (Marx, 1967; Morishima, 1973; Harris, 1977):

$$L = C + V + S \tag{35}$$

where

$$L = \underline{\Lambda}' \cdot \underline{x} = \underline{\Lambda}' \underset{\sim}{A}^* \underline{x} + \underline{L}' \underline{x} \tag{36}$$

$$C = \underline{\Lambda}' \underset{\sim}{A}^* \underline{x} \tag{37}$$

$$V = \underline{\Lambda}' \underset{\sim}{G} \cdot \underline{x} \tag{38}$$

$$S = \underline{\Lambda}' \underset{\sim}{G} (\underset{\sim}{E}) \underline{x} \tag{39}$$

For details, see appendix. $(\underset{\sim}{E})$ is a diagonal matrix containing the rate of exploitation of labor in city i, e_i, in all entries pertaining to sectors of city i.

Thus, for example, the labor value traded from city i to city j as a result of observed trading patterns, l_{ij}, is:

$$l_{ij} = \underline{\Lambda}' \underset{\sim}{A}^*_{ij} \cdot \underline{x} \tag{40}$$

4.3. Applications

Oddly there has been little analysis of urban growth from the point of view of trade balances. No doubt this is because capital and labor are more mobile within than between nations, and neoclassical wisdom implies that trade is only of interest when factors are immobile. This emphasis can be challenged. Factor mobility is far from perfect; and in particular labor is less mobile than capital. It is for this reason that rapid shifts in investment patterns in western urban systems have caused major social problems (Bluestone and Harrison, 1980; Glickmann, 1981).

The selectivity of migration in favor of younger and more well off population groups (Lansing and Mueller, 1967; Rogers and Castro, 1981; Glantz, 1975; Walker, 1978; Sheppard, 1980) implies that some individuals are likely to remain immobile despite incentive schemes (Beaumont, 1979). Relocation costs cannot be eliminated (Gober-Myers, 1978) and thus wages will not be equalized in all cities even in the presence of high labor mobility. Indeed, as argued above, the model used here implies that wages need not respond to the supply of labor in the same way in all cities since local social and political determinants of the division of surplus into wages and profits may vary within the system. Greenwood (1975) and others have found that wage differentials have in fact increased in response to migration, and Higgins (1972; 1981) among others has argued that such disparities may be beneficial for the nation. All of this means that labor will not be priced identically in all cities. Since the costs of heterogeneous capital inputs will vary from city to city depending on which other cities produce these goods and on how they are traded, it is inevitable that cities specialize by producing only certain goods. Indeed it is common knowledge that such systematic differences exist between cities (Alexandersson, 1956, Smailes, 1953).

As a consequence of this, it is not satisfactory to accept the supposition that specialization and trade is not important within an urban system, even if factors of production are mobile.

The prospects for a city will then depend on its position as a specialized partner in the national (and international) economy. Growth will depend on the investment behavior of entrepreneurs and of the public sector, but investment behavior is constrained by investment funds. Consequently it is vital to know the monetary terms of trade faced by various cities, and in particular in which cities profits accumulate. The accounting mechanisms described here permit precisely that sort of analysis. Further they may form the foundation for applying knowledge about how international trade affects national economic development to asking the question of how inter-urban trade affects the prospects faced by individual cities.

The flow of labor values has played an analogous role in Marxist analyses. Work by Emmanuel (1973) pioneered the notion of unequal exchange; whereby even if the monetary terms of trade balance there may be an unequal exchange of labor value. He and others predict that on an international scale this can lead to unequal development if a (Third World) country trades away more labor resources than it receives (Amin, 1976). Such arguments have also been extended to analysis of inter-regional and inter-urban growth rate differences (Lovering, 1978; Lipietz, 1980). The existence or absence of unequal exchange can be ascertained by comparing the ratio of price terms of trade to labor value terms of trade. For example unequal exchange exists between cities i and j if (Barnes, 1982):

$$\frac{Y_{ij}}{Y_{ji}} \neq \frac{l_{ij}}{l_{ji}} \tag{41}$$

While there are many fundamental problems with the notion of unequal exchange (see Bettelheim in Emmanuel, 1973; Roemer, 1981; Gibson, 1980; Barnes 1982) the ability to compare money flows to labor value flows once again allows a comparison of growth patterns as projected by the type of model developed here with predictions founded in the Marxist approach.

5. SOME ELABORATIONS: RESPONSE TO CRISIS

The urban system model as developed thus far is subject
to a severe limitation; the inter-urban trading matrix A has
been taken as exogenous. In fact, inter-urban interactions
will change as the system evolves (Sheppard, 1979; 1980) and
if this is not taken into account, any forecasts made would be
limited to projections based on current interdependencies.
Furthermore, the above analysis has shown that when A is
constant the accumulation process is unstable. If capita-
lists attempt to introduce changes in A, however, there is
always the possibility that through these changes some stable
urban growth paths may be introduced. Thus it is essential
to attempt to theorize about the structure and possible dyna-
mics of A.

The matrix A, however, is not just a spatial interaction
matrix, since it also includes information about production
technologies and about the wage paid to workers and other work-
ing conditions. Thus as producers face accumulation crises
and seek to alter A they may select changes in any of these
factors. From this perspective trade, technological change
and labor negotiations are different aspects of the same pro-
blem; choice of a production process in space and time. Al-
though each of these three aspects are interrelated, I shall
limit myself to describing the effects of each in isolation.

5.1. Dynamic Trading Patterns

Producers of a particular commodity n in city i face sup-
pliers of each necessary input in a number of cities. In a
perfectly informed and flexible inter-urban commodity market
all would buy form the one cheapest producer. However, realis-
tically this is not case since producers are imperfectly in-
formed, information is changing, and contracts with suppliers
are inflexible. It makes sense then to adopt a model of trad-
ing patterns that covers a broad range of possibilities ranging

from uninformed acts to the fully efficient case. One such model is as follows:

$$a_{ij}^{mn} = a_{j}^{mn} \cdot \left[\exp\left\{ -\beta q_{ij}^{m} \right\} / \sum_{k} \exp\left\{ -\beta q_{kj}^{m} \right\} \right] \tag{42}$$

where a_{j}^{mn} is the input of m, per unit of n produced, in city j; q_{ij}^{m} is the delivered price of a unit of m in city j when bought from a supplier in city i, and β is a constant measuring the efficiency of trade. Assume f.o.b. pricing:

$$q_{ij}^{m} = p_{i}^{m} + \tau_{ij}^{m} p_{i}^{t} \tag{43}$$

Equation (42) is a multinomial formulation stating that the expected amount of m bought from i is equal to the technical requirement for m multiplied by the expected probability that a supplier in i will provide m. The last term is inversely related to the delivered cost from i relative to other cities. β is an index of efficiency since it can be shown that in the limit as β tends to infinity the trading pattern converges to the fully efficient pattern. When β equals zero trading is entirely arbitrary (Evans, 1973; Williams, 1977). It has recently been shown that this formulation need not be based on the restrictive conditions of Weibull distributed utilities, as previously thought (Leonardi, 1982).

The following dynamic model of the relation between pricing patterns and trading can then be constructed (Sheppard, 1983a):

$$\underline{p}_{t}' = (1 + r_{t}) \cdot \underline{p}_{t}' \cdot \underline{A}_{t} \tag{44}$$

$$q_{kj}^{m}(t) = p_{k}^{m}(t) + \tau_{kj}^{m} p_{i}^{t}(t)$$

$$a_{ij}^{mn}(t+1) = a_{ij}^{mn} \left[\exp\left\{ -\beta q_{ij}^{m}(t) \right\} / \sum_{k} \left\{ \exp -\beta q_{kj}^{m}(t) \right\} \right]$$

These equations capture in a simple form the way that trading
patterns adjust to given prices, and in turn how new prices
are formed as a result of changed trading patterns. Equili-
brium is defined by a pricing and trading pattern where:

$$\underset{\sim}{A}_t = \underset{\sim}{A}_{t+1} \tag{45}$$

A number of numerical simulations have shown that invariably
a stable pattern of trading and pricing is achieved as a re-
sult of this dynamic adjustment process, with the rate of
profit increasing over time to some upper equilibrium value.
Furthermore equilibrium tends to be approached rapidly, parti-
cularly if the trading pattern is initially well connected
(Sheppard,1983a).

These experimental results have implications for real
world stability of a competitive urban system. As producers
respond to inter-urban price differentials, adjusting their
production method by substituting between alternative suppliers,
an equilibrium trading pattern, profit rate and price vector
are rapidly achieved. This equilibrium depends on the techno-
logy, the wage and the efficiency of trade. But if these are
given, the dynamic seems to be one of rapid adjustment. But
these equilibrium values, because they seem highly stable,
will now represent constant parameters determining accumula-
tion dynamics. However, it was shown that when $\underset{\sim}{A}$ is constant
over time, then the accumulation dynamics are unstable. Thus
it can be concluded that trading flexibility alone is not
sufficient to make the growth patterns more stable, except
possibly during the relatively short period of adjustment
toward trading equilibrium. Attempts to avoid accumulation
crisis by adjusting trading patterns within the urban system,
then, seem to be at best short run, and ultimately futile.

Another type of trading response to crisis, although in
a sense outside the terms of reference of this paper, is worth
mentioning. This is an opening of trade relations with the
outside world. A natural response to disproportionality crises

is to obtain materials in short supply from abroad and sell excess production there. This has of course been an important aspect of international economic development. The paradigm developed here is quite capable of extensions in this direction.

Indeed we already know that only those goods will be sold abroad for which the relative price in the international market is less than that within the urban system, with the converse holding for imports. This is the only situation in which individual entrepreneurs stand to profit from trade (Steadman, 1979). Because prices for a good vary between cities, this means that different cities may be able to export different goods, illuminating why a great diversity of goods is often traded by a country. If $r = g$, consumption will always increase, as a result of trade, but the total gains from trade for the nation may not be positive. This is because it may be rational for individuals to specialize in producing goods that do not increase collective welfare (Metcalfe and Steadman, 1974). Indeed when a spatial dimension is incorporated, the possibility that trading may not be beneficial is apparently very real (Barnes, 1982).

5.2. Altering the Labor Process

Private enterpeneurs faced with an accumulation crisis have a number of strategies available to them that directly affect the labor force. These include: processes of rationalization which may or may not include laying off labor; reduction of fringe benefits such as vacation time; and reductions in the money wage. Each of these may be interpreted in terms of its impact on the coefficients of the matrix $\tilde{A}$. Rationalization can be realized by reducing requirements for capital goods (through less wasteful production processes), or for labor inputs, l_i^n. To the extent that rationalization implies a more efficient utilization of current labor, this is equivalent to increasing the length of the workday, T. Reduction of time off is also equivalent to increasing T; whereas

reducing the money wage involves a reduction either in savings s_w, or in the basic real wage consumed, b. Note that all these changes represent a reduction in the size of some entries in A.

Whether such changes are introduced equally throughout the system, or just in one city and/or one economic sector, the effect is the same. There will be a rise in the general equilibrium rate of profit. This is because, by the Perron-Frobenius theorems for non-negative matrices, any reduction in even one element of A will lead to a reduction in the size of the largest eigenvalue $\mu[=1/(1+r)]$, which means in turn an increase in the rate of profit. Further, this increase implies that a greater proportion of the monetary surplus accrues to capital, with a smaller proportion consequently accruing to labor.

The existence of an inverse relation between the rate of profit and the size of the real wage is a basic feature of this approach (Sraffa, 1960; Morishima, 1973; Sheppard, 1981). It is because of this that the process of economic production and accumulation is inherently conflict-ridden. In neoclassical general equilibrium analysis it has been possible to avoid this social conflict by assuming with Walras that each individual has a sufficient initial endowment of means of production (Weintraub, 1979). This can be disputed, however, on two grounds. First, it is simply not accurate historically and thus can provide no basis for deducing that economic equilibrium in any real competitive economy is in the best interest of all (Sheppard, 1981). Second, Roemer (1982) has recently shown that if a market for labor exists in a competitive capitalist economy then it follows that the actors in the economy will be divided into five classes ranging from pure capitalists (who employ labor) to pure laborers (who work for others and are exploited).

Thus, notwithstanding the fact that workers have some indirect influence over private production via stockholding and pension funds (Rifkin and Barber, 1978), it is still the case in the Western economies that profits accumulate predominantly in one sector of society (to entrepeneurs and

corporations) while wages accrue elsewhere (to workers and unions). Thus two conflicting, or overlapping, interest groups exist in the urban system that are bound to struggle over their shares of economic prosperity.

It was argued above that from this paradigm it logically follows that considerable government intervention is a necessary feature in Western urban systems. An explicit recognition of the social conflicts generated by capitalist production informs us about the social struggles that in turn underly government intervention. It is then quite logical and consistent with the economic system that government policies for the cities will fluctuate as the different social classes take turns in exerting their influence (Glickmann, 1981; Offe, 1972). An approach which does not reveal the social conflicts is not as well able to explain these phenomena.

Within the urban system, however, there are some important modifications of this picture that must be introduced. As discussed earlier, the geographical dimension to economic production means that social conflicts can be substituted by geographical conflicts. An understanding of how such conflicts come about will require an analysis not only of how production evolves in the urban system, but also of how social conflicts develop in each city. An illustration of this can be given by means of a brief example. There has in recent years been a 'counterurbanization' trend in the United States whereby industry and population are abandonning the heavily urbanized areas of the Northeast and Midwest. One contributing factor to this has clearly been the high labor costs in these areas, itself a result of the local social changes due to a longer history of manufacturing that have led to high unionization levels (Hansen, 1982). As a result, industry has invested elsewhere and those parts of the population able to move have also voted with their feet (Bluestone and Harrison, 1980). The cities of the South and West benefitting from such shifts have a cheaper labor force. However that labor force, although being paid less, is experiencing a boom in employment

that is clearly for the time being is in its favor. It is then quite predictable that these trends are supported by the working population there, as is evidenced in persistent resistance to unionization and in the continued support for Federal Republican policies fostering this trend. Whether this is also in the long run in the interest of these people is another question; some industries are already moving on to cities of of the Third World where labor is even cheaper. However once again the framework developed here seems to have the potential for illuminating processes of economic change in the urban system and their social and political correlates.

5.3. Technical Change

The third way of coping with crisis is to introduce process or product innovation. I shall only discuss the former here. One way of treating this problem is to introduce an infinite set of innovations, allowing a continuous substitution of inputs depending on their relative prices. This process could be represented in a manner analogous to that described for the trading component above. More generally, the solution to such a continuous substitution process, in the absence of scale economies, can be described by replacing the linear Leontief technology with a general convex production set. Roemer (1981) has shown that a Marxian dynamic equilibrium always exists. However, the result is a stable production set of inter-industry production relations, implying a stable input-output matrix. This is then just as subject to accumulation crises.

A second approach that is more realistic is to assume that new technologies are qualitatively different from one another. This distinguishes technical change from the type of fine-tuning of production methods that can be achieved through altering trade patterns or introducing rationalization. It also allows consideration of major new techniques that can revolutionize production processes and thus introduce major crises into an inter-urban economy. Such an approach, for

example, would provide a means for analyzing the impact of cheap micro-processors and automated office equipment on the urban system. This type of discrete shift in technology has always been the approach favored by the post-Keynesians.

An an illustration, suppose that the technology used before the innovation has coefficients a_j^{mn}, whereas that used afterwards has coefficients d_j^{mn}. For a given level of efficiency of the trading system, inter-urban input-output coefficients that represent a trading equilibrium in each case, given by matrices $\underset{\sim}{A}$ and $\underset{\sim}{D}$, can be calculated. We wish to compare the efficiency of these two technologies.

In Figure 1, schematic plots are provided of the various values of r that are possible for different levels of the real wage (indexed by the workday T). Line DD' represents the combinations of r and T feasible for the new technology $\underset{\sim}{D}$, and AA' represents that for the old technology $\underset{\sim}{A}$. If the current rate of profit in the urban system is r_a, then the new rate of profit with the new technology, if real wages are held constant, is r_d. Notice that in this example, it is only for a certain range of profit rates that the new technology is cost saving and thus more profitable (Roemer, 1981). The possibility of the same technique (A) being best at high and low profit rates indicates the complexity ignored in neo-classical accounts that assume equality between factor pricing and marginal productivities (Harcourt, 1972). Of course in any real analysis comparing two technologies the costs of transition from one to the other would have to be taken into account.

What are the implications for this change on the patterns of production within the urban system? This will depend on the rules governing the locations of investment and disinvestment by individual capitalist, and in this area the macro-economic theory is not helpful. For example the issue of transition between techniques has only been considered as a planned transition between two states of dynamic equilibrium (Spaventa, 1973). But since the equilibria are unstable
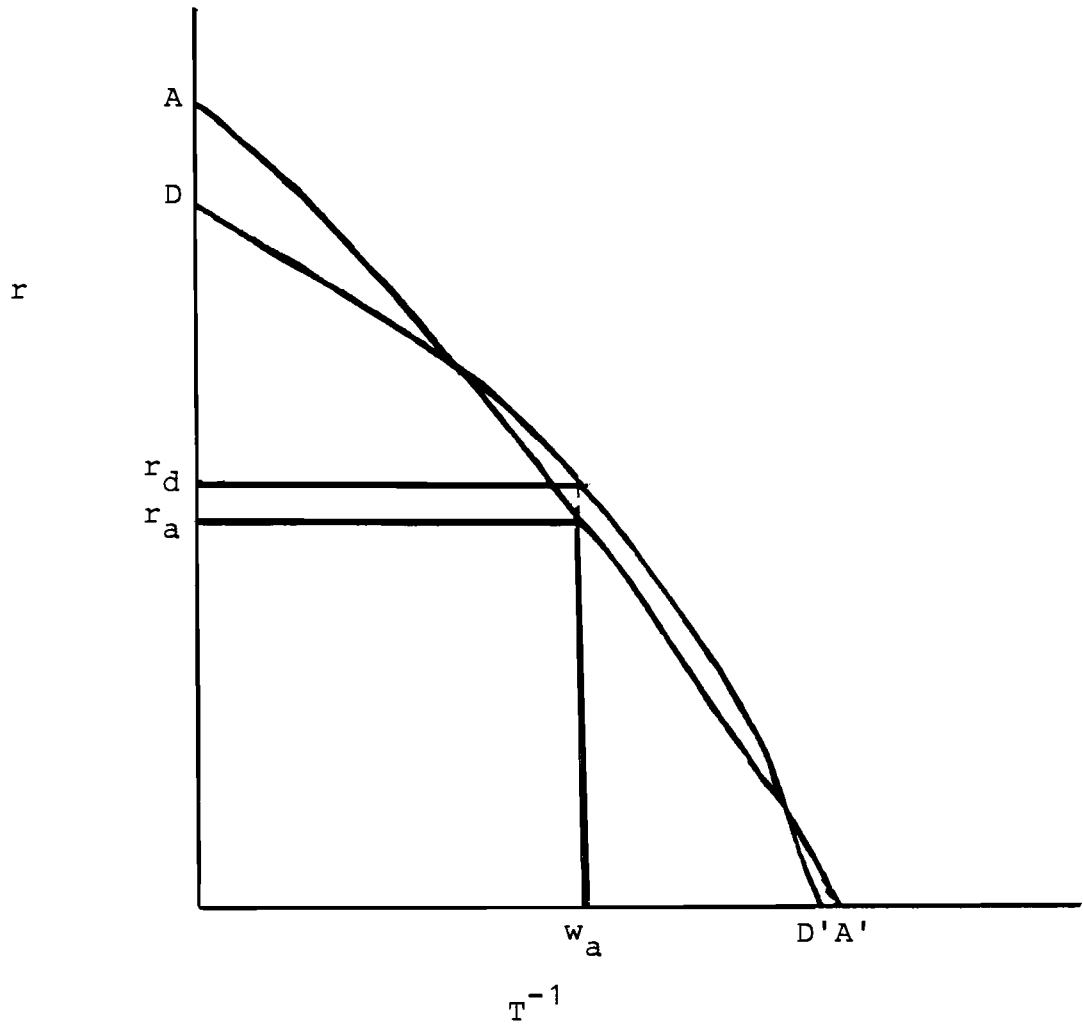
Figure 1. Choice between two technologies.

this does not seem very helpful.

6. SOME INVESTMENT RULES

In order to properly model city growth it is necessary to have some statement about how entrepeneurs choose to invest their money. This issue has not been researched in detail, but four illustrative examples will be presented.

6.1. Balanced Investment

One simple hypothesis, perhaps more normative than descriptive, is that investment occurs in such a way that supplies of each product match demands:

$$\underline{x}_t = \underset{\sim}{A} \cdot \underline{x}_{t+1}$$

or

$$\underset{\sim}{A}^{-1} \underline{x}_t = \underline{x}_{t+1}$$

This rule is, as pointed out earlier, unstable; sectors rapidly achieve negative production levels which is clearly unrealistic.

6.2. Retained Earnings Investment

A second rule is that producers in each sector in a city reinvest all their profits in their own industry. Alternatively, with the rate of return equal everywhere, investment in each industry is proportional to its size (Sheppard, 1983b):

$$\underline{x}_{t+1} = (1 + r)\underline{x}_t$$

Again this rule is unrealistic since only when the economy is operating on the unstable equilibrium ray is it achievable without an immediate crisis between supply and demand.

6.3. Weberian Investment

Suppose that producers in a sector m will invest in any location where m is produced, depending on the profitability of those locations. The extreme example of this occurs in Weberian location theory where all investment occurs at the most profitable location. The relative profitability of different locations will depend on their competitive position in the urban system. This in turn will depend on the prices that can be charged to customers compared to prices charged by competing cities.

By definition, the mean delivered market price, $\bar{p}_i^m$, charged for good m produced in city i is equal to the delivered price in each destination city weighted by the quantity sold there:

$$\bar{p}_i^m = \sum_j \sum_n q_{ij}^m \cdot a_{ij}^{mn} \cdot x_j^n \tag{46}$$

recalling that q_{ij}^m, equal to p_i^m plus $\tau_{ij}^m p_i^t$, is the delivered price at city j.   This price gives an immediate index of the overall price competitiveness of producers of m in city i relative to other cities. However, the overall competitive position of a producer in city i will also depend on the competitive position of his customers, weighted by their total demand for the product. This may be calculated as follows. Define a diagonal matrix $(\underset{\sim}{X}*)$ which contains as entries the production quantity vector $\underline{x}$ with its values divided through by the largest entry in $\underline{x}$. Then the matrix $\underset{\sim}{A}.(\underset{\sim}{X}*)$ is a matrix of direct demands of good m in city i for input to sector n in city j. The matrix $[\underset{\sim}{I} - \underset{\sim}{A}(\underset{\sim}{X}*)]^{-1}$ is then a full matrix of the relative size of all direct and indirect demands of each producer from each other producer.

Define v_j^n as the overall competitive position of a producer of m in city j. It is postulated here that v_j^n is given by:

$$v_j^n = \sum_{m,i} h_{ji}^m \cdot \bar{p}_i^{-m} \tag{47a}$$

where h_{ji}^{mn} is the total demand for good n from city j as input to producing m in city i. Employing equation (46) and recasting as matrices:

$$\underline{v} = [I - \underset{\sim}{A}(X^*)]^{-1} \cdot \underline{\bar{p}} \tag{47b}$$

where $\underline{v}$ and $\underline{\bar{p}}$ are vectors containing v_j^n and $\bar{p}_i^{-m}$. Thus:

$$\underline{v} = [I - \underset{\sim}{A}(X^*)]^{-1} \cdot \underset{\sim}{Q} \cdot \underline{x} \tag{47c}$$

where $\underset{\sim}{Q}$ is a MJ by MJ matrix with entries $\left\{ q_{ij}^m a_{ij}^m \right\}$.

Since highly competitive locations are those where the index v_j^n is low, the money invested by sector n in city j can be represented as:

$$I_j^n = \Pi_n \exp\left\{ -\alpha v_j^n \right\} / \sum_k \exp\left\{ -\alpha v_k^n \right\} \tag{48}$$

where α is an index of the spatial efficiency of investment (higher values representing greater concentration of investment in the most advantageous cities) and Π_n is the total profit in sector n:

$$\Pi_n = r \cdot \sum_j p_j^n x_j^n \tag{49}$$

In terms of interurban flows of investment:

$$I_{ij}^n = r \, p_i^n x_i^n \left[\exp\left\{ -\alpha v_j^n \right\} / \sum_k \left\{ \exp \ -\alpha v_k^n \right\} \right] \tag{50}$$

where I_{ij}^n is the amount of money earned by producers of good n in city i that is invested in the same sector in city j.

6.4. Marxian Investment

A final investment rule is generated on the hypothesis that all producers selling good n in city j must sell at the same price, q_j^n. For example, q_j^n could be the price of the marginal (most expensive) seller, or it could be some mean price. As a result, producers in different cities, forced to adjust their delivered prices to be no higher than q_j^n in a competitive market, will face different rates of profit due to relative location. This corresponds also to a Marxian location theory where local variations in the profit rate attract investment (Harvey, 1982). The total profits made by a producer of good m in city i, according to this rule, are:

$$\Pi_i^m = \sum_j \sum_n \left[r\, p_i^m + \left(q_j^m - q_{ij}^m \right) \right] a_{ij}^{mn} x_j^n \tag{51}$$

This is the sum of initial profits plus the gains (or losses) due to a deviation between the f.o.b. price at the uniform rate of profit and the actual price prevailing in the market of city j. The rate of profit for a producer of m in city i is then:

$$r_i^m = \Pi_i^m / p_i^m \sum_j \sum_n a_{ij}^{mn} x_j^n \tag{52}$$

Suggesting the investment rule:

$$I_j^n = \Pi \cdot \left[\exp\left\{ \alpha r_j^n \right\} \Big/ \sum_k \sum_m \exp\left\{ \alpha r_k^m \right\} \right] \tag{53}$$

where Π, the total profit made in the urban system, is equal to $r.\underline{x}'.\underline{p}$, and α is once again the efficiency of investment.

The application of any of these investment rules to examining a particular urban system could not be performed immediately as two other related problems remain unsolved. The first is a linking of investment dynamics with the

resulting mismatches of supply and demand. This will require
introduction of stocks to absorb some fluctuations (Goodwin,
1976). The second involves construction of a plausible me-
chanism showing how these mismatches may affect the short run
prices of products, introducing a feedback between market dis-
equilibrium and accumulation.

7. EXTENSIONS

7.1. Corporate Organization

One of the advantages of this paradigm is that it is
fairly straightforward to introduce some basic concepts of
institutional economics in order to model the behavior of
corporations within the urban system. In terms of data,
the most important concept is a matrix showing the degree
to which corporations in each city own subsidiaries in other
cities. Sheppard (1983a) introduced the well known suggestion
of Kalecki (1938) that producers in a sector make a rate of
profit proportional to the degree of concentration of owner-
ship in a sector. Applying this notion to a city:

$$r_i = r + \gamma c_i \qquad (54)$$

where c_i is the proportion of all production in the urban
system owned by firms headquartered in city i. For such a
system, prices may still be calculated if γ is known, by
forming the non-linear eigenvalue problem:

$$\underline{p}' = (1 + r) \cdot \underline{p}' [\underline{A} + \gamma (1+r)^{-1} (\underline{C}) \underline{A}] \qquad (55)$$

where $(\underline{C})$ is a diagonal matrix with entry c_i in all main dia-
gonal locations referring to city i. These results can be
used to calculate the total profits controlled by each city.

Pred (1976) among others (Hymer, 1979; Westaway, 1974) has suggested that those cities with more corporate head-quarters are better off because of the stability of these corporations. In fact a city will only benefit from the existence of corporate headquarters if those companies do invest heavily in their 'home city'. This need not be the case; what is in fact needed is a theory of investment be-havior by corporations. The key difference between corporate investment and that by individual entrepreneurs is that the former can take advantage of direct investment in their own subsidiaries, which is often more lucrative than the capital market. Using the elegant notions about direct investment due to Hymer (1976) it is possible to construct a theory in-corporating direct investment that postulates how profits are re-used for growth in an urban system (Sheppard, 1983a). Investment in city j, I_j is:

$$I_j = \pi^p \left(u_j / \sum_k u_k \right) + \pi^{d1} \left(r_j / \sum_k r_k \right) + \pi^{d2} \left(c_{ij} / \sum_k c_{ik} \right) \tag{56}$$

where u_j is the total value of production in city j; c_{ij} is the proportion of all production owned by companies in city i that is located in city j, π^p is portfolio investment funds available through the capital market, π^{d1} is direct investment funds oriented toward local profit rates, and π^{d2} is direct investment funds spent directly through subsidiaries. This hypothesis can in turn be embedded in a dynamic model of urban system change in a corporate economy (Sheppard, 1982a).

The approach to date is crude, but the problem is impor-tant. Corporations form an increasingly important component of production in urban systems, and it is well knwon that be-havior is distinctively different from that of autonomous capi-talists. One area where these differences are particularly striking is in the greater flexibility corporations have of shifting production between different locations. Among other effects this gives considerable bargaining power in obtaining favorable conditions in cities where they invest (Glickmann, 1981).

7.2. Joint Production

The important limitation of assuming no joint production of several products in one sector, as well as that of neglecting fixed capital, may be circumvented by exploiting the parallels between the conception developed here and the general theory of production due to Von Neumann (Von Neumann, 1945; Morishima, 1973; Morishima and Catephores, 1978). This approach can also adequately incorporate land as a fixed non-produced resource (cf Pasinetti. 1979). In applying this to the urban system we redefine the matrix $\underset{\sim}{A}$ to be possibly rectangular, with entry a_{ij}^{mn} representing the number of units of product m from city i used in a unit intensity of operation of _process_ n in city j. The number of processes, represented by columns, may clearly not equal the number of products given by the rows of $\underset{\sim}{A}$. Second, we introduce a block-diagonal matrix $\underset{\sim}{B}$, with the same dimensions as $\underset{\sim}{A}$, where b_i^{mn} represents the amount of commodity m produced, per unit intensity of operation of process n, in city i. If $\underset{\sim}{B}$ is a diagonal square matrix there is no joint production.

In economic equilibrium two conditions must be met. The value of all goods produced must not exceed the cost of inputs incremented by the rate of profit:

$$\underset{\sim}{p}'_{t+1} \cdot \underset{\sim}{B} \leq (1+r_t) \underset{\sim}{p}'_t \underset{\sim}{A} \qquad (57)$$

This implies that competition acts in the urban system ensuring that no producer makes more than the collective rate of profit. However in addition, processes making less than the collective rate of profit will be abandonned. Thus if (57) is post-multiplied by $\underset{\sim}{x}_t$, we would expect that $\underset{\sim}{x}_t$ equals zero in all cases where the strict inequality holds. Then if we only retain in (57) those processes which are actually operated:

$$\underset{\sim}{p}'_{t+1} \underset{\sim}{B} \underset{\sim}{x}_t = (1+r) \underset{\sim}{p}'_t \underset{\sim}{A} \underset{\sim}{x}_t \qquad (58)$$

Second, total demand cannot exceed supply:

$$\underset{\sim}{B} \cdot \underset{\sim}{x}_t \geq \underset{\sim}{A} \underset{\sim}{x}_{t+1} \tag{59}$$

For an urban system this condition is most restrictive. It states that the total demand for production of m in city i, due to the level of operation of all processes in all cities, cannot exceed the amount produced in the previous time period. Thus no substitution between producers of the same good in different cities is allowed.

If we postulate the dynamic equilibrium condition:

$$\underset{\sim}{x}_{t+1} = (1 + g) \underset{\sim}{x}_t,$$

and substitute this into (59);

$$\underset{\sim}{B} \cdot \underset{\sim}{x}_t \geq (1 + g_t) \underset{\sim}{A} \underset{\sim}{x}_t \tag{60}$$

If goods are in excess supply they will have a price of zero. Therefore premultiplying (60) by $\underset{\sim}{p}'_{t+1}$ and retaining only products with non-zero prices will eliminate all goods where the strict inequality in (60) holds. Thus:

$$\underset{\sim}{p}'_{t+1} \cdot \underset{\sim}{B} \cdot \underset{\sim}{x}_t = (1 + g_t) \underset{\sim}{p}'_{t+1} \underset{\sim}{A} \underset{\sim}{x}_t \tag{61}$$

where $\underset{\sim}{p}'_{t+1}$ are the actual prices at time t+1. If in addition to a dynamic equilibrium of quantities there is also a stationary price equilibrium, the $\underset{\sim}{p}'_{t+1}$ equals $\underset{\sim}{p}'_t$ and (58) becomes:

$$\underset{\sim}{p}'_t \cdot \underset{\sim}{B} \cdot \underset{\sim}{x}_t = (1 + r_t) \underset{\sim}{p}'_t \underset{\sim}{A} \underset{\sim}{x}_t \tag{62}$$

Comparing (61) and (62) it can be seen that in price and quantity equilibrium both equations must hold, and thus the rate of growth will equal the rate of profit. Equations (61) and (62) then describe the path of dynamic equilibrium for all

goods that are not in excess supply, and all processes which make the collective profit rate.

It can be shown that for an $\underset{\sim}{B}$ and $\underset{\sim}{A}$ representing a productive economy, positive prices, equilibrium quantities and profit rate will exist (Gale, 1960). Indeed this model, widely recognized for its abilities to handle fixed capital and also process innovation through the elimination of unnecessary sectors, is a natural generalization of the paradigm described in detail here. However there is much work to be done before it can be adapted for urban system analysis.

7.3. Inclusion of Demographics

An approach that seriously intends to replicate development trends in an urban system cannot ignore demographic developments. On the one hand, shortages in the labor market place bounds on the rate of growth in a city. On the other hand excess labor will push wages down, as well as creating unemployment with all the subsequent political measures and social expenditures that this necessitates. In order to determine how such imbalances come about, it is necessary to specify how demographic trends respond to economic factors. Fortunately this is an area where a great deal of research has accumulated (Greenwood, 1975; Easterlin et al. 1980; Kelley, 1980; Rogers & Williamson, 1982; Rogers and Williams, 1982), which can be drawn on in extending the economic framework into a demo-economic model of urban systems dynamics. For this purpose the owrk of Gordon and Ledent (1980, 1982; Ledent and Gordon, 1980), who have linked an interregional population model with a somewhat simpler and less theoretically sophisticated economic model, provides a useful focus.

8. CONCLUSION

The purpose of this paper has been to introduce a para-
digm for modeling the inter-urban economy that draws on the
post-Keynesian and Marxian paradigms developed over the last
twenty years by European and some American economists. More
detailed analysis can be found in Sheppard (1983a, 1983b,
1981). This provides a constructive alternative to the much
criticized neoclassical paradigm, an alternative that has
more theoretical foundation than a simple construction of
input-output models and more attention to detail than a
Keynesian income accounting framework. On the credit side
of this approach is its apparent flexibility. Analysis of
the basic framework gives insights into the existence of,
and conflicts between social groups in society. It allows
for a natural explanation of why government intervention of
a fundamental level has been a consistent feature in western
urban systems,and indicates the role of social groups in in-
fluencing this intervention. It treats equilibrating economic
dynamics as the exception rather than the rule in urban systems,
which is consistent with what apparently is occurring, and thus
indicates the source of economic as well as social crises and
of how they may spread through a country. In all these res-
pects there is a striking contrast to the smooth harmonious
picture of economic change to be found in neoclassical analy-
sis. Further direct elaboration of the basic framework allows
a ready incorporation of corporate organizations and fixed
capital; topics that are often difficult to handle even in
a purely aspatial economic model. Finally a number of people
have argued that the basic framework is so general, being
essentially a model of production interdependencies, that it
should be applicable in centrally planned and other non-
capitalist social systems (Rowthorn, 1974; Roemer, 1982).

On the debit side is the problem of complexity. A model
of this kind obviously has extraordinary data requirements,
making the possibility of empirical calibration of the full
framework seem very remote at this time. This of course is

not a unique problem: multi-regional population projections and computable neoclassical general equilibrium models face the same problems. The value of laying out a paradigm in its full complexity, however, is two-fold. First, given this full specification, it should be possible through some judicious simulation to determine in which ways the model can be aggregated without losing its essential properties. A reduced form may then be empirically estimated. Evidently one unacceptable aggregation, for example, is to reduce non-labor inputs to a homogenous capital good. Second, the paradigm offers a way of conceptualizing an urban system, the key relations within it, and the general causes behind crises that can occur in the real world. Indeed this type of aid to scientists is often as valuable as a detailed empirical test, if it helps indicate in a general manner how a particular complex system should be approached by those seeking to understanding it.

Finally, it is clear that many of the conceptual implications that have been drawn in this paper have involved significant leaps of faith from a restricted artificial framework to real world problems. Whether such leaps are justified can only be revealed by further research, both empirical and theoretical, which attempts to provide a more detailed analysis of the more speculative suggestions introduced here. Certainly any attempt at a full empirical calibration will require a large data collection effort, and thus should only be carried out given a reasonable confidence that the framework is at least plausible.

APPENDIX: Labor Value Accounts in the Urban System

A.1. Derivation of labor value accounting

Taking the equation for labor values, and post-multiplying by an arbitrary production vector $\underline{x}$:

$$\underline{\Lambda}' \underline{x} = \underline{\Lambda}' \cdot \underset{\sim}{A}^* \cdot \underline{x} + \underline{L}'\underline{x} \qquad (A.1)$$

But in each sector the value of direct labor is equal to the labor value of the wage, multiplied by one plus the rate of exploitation (Marx, 1967):

$$l_j^n = (1 + e_i) v_i \, l_j^n = v_j^n + s_j^n \qquad (A.2)$$

$$e_i = (1 - v_i)/v_i \qquad (A.3)$$

where v_j^n is the constant capital in sector n city j, and s_j^n is the surplus value. Define the diagonal matrix (V) with entries equal to v_i for each production sector of city i; a matrix of labor value wages. Then from (A.2):

$$\underline{L}' = \underline{L}'[\underset{\sim}{I} + (\underset{\sim}{E})](\underset{\sim}{V}) \qquad (A.4)$$

where $(E) = [I - (V)].(V)^{-1}$. And (A.1) becomes:

$$\underline{\Lambda}'\underline{x} = \underline{\Lambda}'A^*.\underline{x} + L'[\underline{I} + (\underline{E})](\underline{V})\underline{x} \qquad (A.5)$$

However $\underline{L}'(V) = \underline{V}'.(L)$, where $\underline{V}'$ is a one by MJ vector containing entries v_i, and (L) is a diagonal matrix with the values of $\underline{L}'$ arranged along the diagonal. Thus:

$$\underline{\Lambda}'\underline{x} = \underline{\Lambda}'A^*\underline{x} + \underline{V}'(\underline{L})x + \underline{V}'(\underline{L})(\underline{E})x \qquad (A.6)$$

Now from equation (15):

$$v_j 1_j^n = T^{-1}\left[\sum_i \sum_{m \in \amalg} b_{ij}^m \left(\lambda_i^m + \tau_{ij}^m \lambda_i^t\right)\right].1_j^n \qquad (A.7)$$

whence, using the definition of $\underset{\sim}{G}$ in equation (30):

$$\underline{V}'(L) = \underline{\Lambda}'G \qquad (A.8)$$

Therefore:

$$\underline{\Lambda}'\underline{x} = \underline{\Lambda}'\underline{A}^*\underline{x} + \underline{\Lambda}'\underline{G}\underline{x} + \underline{\Lambda}'\underline{G}(\underline{E})\underline{x} \qquad (A.9)$$

In equation (A.9) the first term on the right-hand side represents the labor value of capital inputs (constant capital), the second term represents the labor value of the wage (variable capital), and the third term represents surplus value (variable capital multiplied by the rate of exploitation).

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