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DYNAMICS OF CAPITAL FORMATION, CAPACITY CONSTRAINTS AND TRADE PATTERNS IN A MULTISECTORAL MODEL

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FOREWORD

This paper has been written as a background paper to the Tuscany case study which is a collaborative effort between the Regional and Urban Development Group at IIASA and the Regional Institute for Economic Planning of Tuscany (IRPET). The core of this joint study is the development of such applied models and methods which can be integrated into a decision supporting system for regional analysis, planning and decision-making.

The framework presented in this paper is designed to bring together the capacity formation process, which has a medium-term character, and short-term adjustment processes. The latter include, in this case, adjustments of interregional trade flows, international imports and so-called economic stabilization policies. From a more general point of view the suggested approach represents an attempt to formulate a dynamic multi-sectoral process of annual changes, with explicit recognition of the sequential change of capacity levels, investments and trade patterns.

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CONTENTS

1. INTRODUCTION: A Nested Dynamics Approach

This paper presents an attempt to formulate a dynamic input-output model which is given a biregional formulation as an application. The structure of the framework suggested is designed for numerical computations.

For closed input-output models it has been possible to provide solutions to systems in which production requirements are given by current use of commodities together with growth of production capacities [recent results are found in Johansen (1978), Åberg and Persson (1981)]. With regard to open multi-sectoral growth model the general approach has been to solve the model for a terminal date given a specified starting point; as such these medium- or long-term models are static in nature. A widely applied class of such models are the so-called MSG-models, founded on the work of Johansen (1960, 1974) [compare e.g. Bergman and Por (1980)]. In these models capital formation is introduced exogenously. Usually they contain no explicit analysis of the sequential time-dependency between production levels, capacity levels, and investments.

Attempts to capture the problem of consistency between production and capacity levels have resulted in "accelerator" type models, also constrained to a medium- or long-term perspective [see e.g. Lahiri (1976, 1977) and Persson (1980, 1981)]. The present paper may be viewed as a continuation of this latter approach, now with the objective of formulating a dynamic process of annual changes, with explicit recognition of the sequential change of capacity levels.

The dynamics are obtained by interlinking a medium- and a short-term perspective in the same model. The framework is a quantity model (without prices considered) which should be possible to extend to a price-quantity model, for example, of the kind described in Persson and Johansson (1982). The framework includes sector specific compositions of investment deliveries, gestation lags and biregional trade.

In section 2 the change process is outlined. The process is nested in the sense that first a medium-term problem is solved, then this solution is used to determine a short-term outcome of the model. Together these two types of solutions define the dynamic process.

In section 3 we analyze the existence of solutions and describe the properties of the algorithm for the nonlinear medium-term formulation of the model. We also explain and illustrate why the problem of capital formation cannot be solved with a short-term model version.

In section 4 the biregional formulation is introduced. Self-regulation is illustrated by modeling adjustments of interregional and international trade.

The appendix contains empirical illustrations of the approach, based on the biregional model version.

2. TIME ASPECTS OF CAPACITY CONSISTENCIES

2.1 Capital Formation and Capacity Adjustments

Investment is both a source of demand for current output and a determinant of the change of production capacity. This double role of capital formation brings about a two-sided consistency constraint in a multisectoral growth model in which new capacity in each sector is created with the help of deliveries of investment goods from different sectors in the economy. The consistency requirement means that (i) current output cannot exceed the existing capacity, and (ii) the increase of capacity can only be obtained with the help of the goods made available by different sectors, including imports.

We shall study the process of capacity change in a bi-regional model. However, in order to illuminate the fundamental aspects of the change process we shall not introduce any regional specifications in this section. The model may be described as follows in quantity terms:

$$x = Ax + h + v \tag{2.1}$$

where $A = \{a_{ij}\}$, $x = \{x_i\}$, $h = \{h_i\}$, $v = \{v_i\}$,

and where

x_i = production output from sector i

h_i = investment deliveries from sector i

v_i = remaining final demand for the output from sector i

The investment delivery h_i consists of a sum

$$h_i = \sum_j h_{ij} \tag{2.2}$$

where h_{ij} denotes the investment deliveries from sector i to sector j. Between two single years t and t+1 with the capacity $\bar{x}(t)$ and $\bar{x}(t+1)$, respectively, one may observe $\Delta\bar{x}(t+1) = \bar{x}(t+1) - \bar{x}(t)$ and $\Delta\bar{\bar{x}}_j(t+1) = \Delta\bar{x}_j(t+1) + \rho_j\bar{x}_j(t)$ for each j, where

$$\begin{aligned} \Delta\bar{x} &= \{\Delta\bar{x}_i\} \text{ denotes the net increase of capacity} \\ \Delta\bar{\bar{x}} &= \{\Delta\bar{\bar{x}}_i\} \text{ denotes the gross increase of capacity} \\ \rho &= \{\rho_i\} \text{ denotes the rate of capacity removal} \end{aligned} \tag{2.3}$$

Using (2.1) - (2.3) one may formulate the first type of constraints on the investment and capacity change processes

$$\begin{aligned} & h(t) \leqq x(t) - Ax(t) - v(t) \\ & x_j(t+1) \leqq \bar{x}_j(t+1) = \bar{x}_j(t)(1-\rho_j) + \Delta\bar{x}_j(t+1), \text{ all } j \end{aligned} \tag{2.4}$$

Our next objective is to relate the investment sequence $h(t), h(t+1), \ldots$ to the associated sequence $\Delta\bar{\bar{x}}(t+1), \Delta\bar{\bar{x}}(t+2), \ldots$ Having described this relation, we have also specified our second constraint on the capacity change process.

2.2 Medium-Term Projections and Short-Term Solutions

The desired future capacity in every sector is determined by expected demand. Let x^* be the projection of the desired future capacity and let x^o be the current capacity vector. In order to reach x^*, it is necessary to fill the gap $x^*-x^o-r = \Delta\bar{\bar{x}}$, where r represents capacity removed during the period. The gap $\Delta\bar{\bar{x}}$ is filled by means of investment deliveries, $h(\Delta\bar{\bar{x}})$ from the different sectors of the economy.

Suppose now that the expected future net output is v^*, and that we want to calculate the value of x^* which equals the summarized demand $Ax^* + h(\Delta\bar{\bar{x}}) + v^*$. A model $x = Ax + h(\Delta\bar{\bar{x}}) + v^*$ obviously satisfies our requirement of consistency between future capacity x^* and investment deliveries $h(\Delta\bar{\bar{x}})$ which make it possible to reach the level x^*. However, empirically observed ratios $h_{ij}/\Delta x_j$ are usually so high that the solution set of the model is empty for an expanding economy. By reformulating the model in a certain medium-term setting it has been possible, as described in section 3, to bypass this problem. However, this approach has also hampered the short-term analysis of the economic process as a whole [see Persson (1980), (1981)].

The strategy we are suggesting as a means to solve the problem indicated above interlinks a medium- and short-term perspective sequentially. In this way a dynamic one-year process may be obtained such that $\phi_t(\bar{x}(t)) = \bar{x}(t+1)$.

In the model outlined in the sequel, the gestation lag is one year. This is sufficient to illustrate how lags can be introduced. Naturally, when the model is used to analyze cycles it becomes essential to have an accurate specification of the lag structure.

Consider now a medium-term period consisting of T years. Let x^* be the desired capacity T years ahead, based on the expected development of demand. In year t the existing capacity $\bar{x}(t)$ is known. Suppose that we know how the investors behave in order to fill the medium-term gap $x^* - \bar{x}(t)$. Then it is possible to determine the gross capacity change in year $t+1, \Delta\bar{\bar{x}}_j(t+1)$, on the basis of information about the rate of removal and the pair $(x^*, \bar{x}(t))$. Therefore, we can write

$$\Delta\bar{\bar{x}}_j(t+1) = \psi_j(x^*, \bar{x}(t))x_j(t) \ , \tag{2.5}$$

where ψ_j may reflect that capacity grows linearly or geometrically. In the latter case ψ_j can be written as

$$\psi_j(x^*, \bar{x}(t)) = [x_j^*/\bar{x}_j(t)]^{1/T} + \rho_j - 1$$

if $x_j^* \geq \bar{x}_j(t)$ and where $\psi_j(x^*, \bar{x}(t)) = 0$ otherwise. This implies that the investments also will follow a geometric growth path.

Since we have assumed a one-period gestation lag, the investments, $h(t)$, will be determined by $\Delta\bar{\bar{x}}(t+1)$ so that $h(t) = h(\Delta\bar{\bar{x}}(t+1))$. We shall assume that $h_{ij}(t)$ is obtained as the product $k_{ij}\Delta\bar{\bar{x}}_j(t+1)$, where k_{ij} is a non-negative, fixed element of the investment coefficient matrix K. This yields

$$\begin{aligned} h(t) &= h(\Delta\bar{\bar{x}}(t+1)) \\ h(\Delta\bar{\bar{x}}(t+1)) &= K\Delta\bar{\bar{x}}(t+1) \geqq 0 \end{aligned} \tag{2.6}$$

At time t, $\bar{x}(t)$ is given and $h(t)$ is determined by (2.6). Therefore, the requirement in (2.4) will imply that in each single period t, the net output, $v(t)$, has to adjust so that $x(t) \leqq \bar{x}(t)$.

Given the assumptions made we may at each time t make a forecast, x^*, with regard to expected production T years ahead, i.e., for the terminal year of a moving medium-term period. With this approach we are able to solve for capacity, production, investment and net output in each year for a continuing sequence of years.

2.3 Capacity Constraints and Short-Term Adjustments

We have described the demand for fixed capital (capacity) as a medium-term concept. In the short-run the capacity is given and the one-year solutions must contain adjustments such that (i) final demand adapts in case of capacity shortage, and (ii) idle capacity obtains when the demand is too low.

The short-term system is given by (2.4) - (2.6) and can be formulated as $x(t) = Ax(t) + h(t) + v(t)$, where $x(t) \leqq \bar{x}(t)$ and $h(t) = h(\Delta\bar{\bar{x}}(t+1))$ constrains the solution. Since $v(t)$ is the only variable we can adjust in the short-run, we shall consider the following specification of $v(t)$:

$$\begin{aligned} v(t) &= g(t) + c(t) + e(t) - m(t) \\ g(t) &= \text{vector of public consumption} \\ c(t) &= \text{vector of private consumption} \qquad (2.7) \\ e(t) &= \text{vector of exports} \\ m(t) &= \text{vector of imports} \end{aligned}$$

The short-term solution is obtained by determining $v(t)$ which is levelled by means of variations in any of the components specified in (2.7). The government may influence $g(t)$ directly and $c(t)$ indirectly through alterations of the taxation/subsidy policy. If excess demand remains after such controls, the system may be self-regulated by a reduction of the different $e(t) - m(t)$, i.e., reduced exports and increased imports. In the interregional model specified in section 4, interregional trade patterns are also changed in a self-regulated manner.

3. MEDIUM TERM CAPACITY PROJECTIONS

3.1 Balanced and Unbalanced Capacity Projections

We have described in section 2 how the model operates and can be solved for any single year, given a projection x^* of the expected capacity level T years ahead. Having done this, our major concern now is how to generate the projections x^*.

We shall distinguish between two alternatives. The first approach determines x^* without any explicit consideration of the balance between the capacity, investments and net output T years ahead. Therefore, we may say that this method generates unbalanced projections. The second approach generates a projection, v^*, of net output T years ahead. Given this projection we determine x^* from the balance $x^* = Ax^* + h(\Delta\bar{\bar{x}}^*) + v^*$. In this case we may say that the capacity projection x^* is balanced with regard to $\Delta\bar{\bar{x}}^*$ and v^*.

We may think about an unbalanced projection in the following way. For each sector i we make a forecast, α_i, about the capacity change between year t and t+T, so that $x_i^* = \alpha_i \bar{x}_i(t)$. Then we may form the diagonal matrix $\alpha = \langle\alpha_i\rangle$ with the coefficients α_i as elements so that

$$x^* = \alpha\bar{x}(t)$$

The matrix α may reflect the past growth in the different sectors while at the same time having its level scaled in such a way that the economy grows at the same rate as the labor force (including adjustments for productivity changes). This kind of projection is all we need to generate annual solutions of the kind described in section 2.

Consider now the second approach to generating a medium-term projection. In this case we introduce a diagonal matrix α_v such that $v^* = \alpha_v v(t-1)$. Observe that due to the time-lag of the investment process, the value of $\bar{x}(t)$ is given one year earlier than that of $v(t)$.

Having established v^*, we observe that the constraints in (2.4) must apply also for year t+T. This means that $x^* \leqq \bar{x}(t+T)$. Moreover, if $x_j^* > \bar{x}_j(t)(1-\rho_j)^T$, there is a need for investment (capacity creation) in this sector. Therefore, we make use of (2.5) to formulate the balance problem in year t+T, based on the projection of $v^* = \alpha_v v(t-1)$. This yields

$$\begin{aligned} x^* &= Ax^* + h(\Delta\bar{\bar{x}}^*) + \alpha_v v(t-1) \\ \Delta\bar{\bar{x}}_j^* &= \Delta\bar{\bar{x}}_j(t+T+1) = \psi_j(x^*, \bar{x}(t))x_j^*(t+T) \end{aligned} \qquad (3.1)$$

which in all essence is the same balance as that formulated in (2.1).

3.2 Solution Method for a Balanced Projection

Consider the system in (3.1). It may be compressed to the following form:

$$x = F(x) + v \tag{3.2}$$

where $v = v^*$, and $F(x) = Ax^* + k(x^*)$ with $k(x^*) = h(\Delta\bar{\bar{x}}^*)$ and $\Delta\bar{\bar{x}}^*$ defined in (3.1). From the latter formula it should be obvious that, with $\bar{x}(t)$ given, $\Delta\bar{\bar{x}}^*$ is determined by x^* by means of the functions ψ_j.

Formula (3.2) defines a nonlinear system which can be solved by means of an algorithm based on the following recursive scheme:

$$x^{n+1} = F(x^n) + v \; ; \; x^o = v \tag{3.3}$$

where n denotes the n'th iteration step. Under quite general conditions applied to the form of ψ_j in (2.5) and h in (2.6) one may ascertain that F is a continuous and monotonous operator such that

$$x^1 \geqq x \geqq 0 \Rightarrow F(x^1) + v \geqq F(x) + v$$

To indicate that each solution is determined conditional to a given level of v in (3.1), we may denote the solution set of (3.2) - (3.3) by

$$S_v = \{x \geqq 0 \; : \; x \geqq F(x) + v\} \tag{3.4}$$

Given this set we may restate a proposition in Persson (1980), based on the assumption that F is continuous and monotonous.

Proposition 1. Given our assumptions about F we may state (i) if S_v is non-empty for a given v, there exists a solution $x = F(x) + v$,

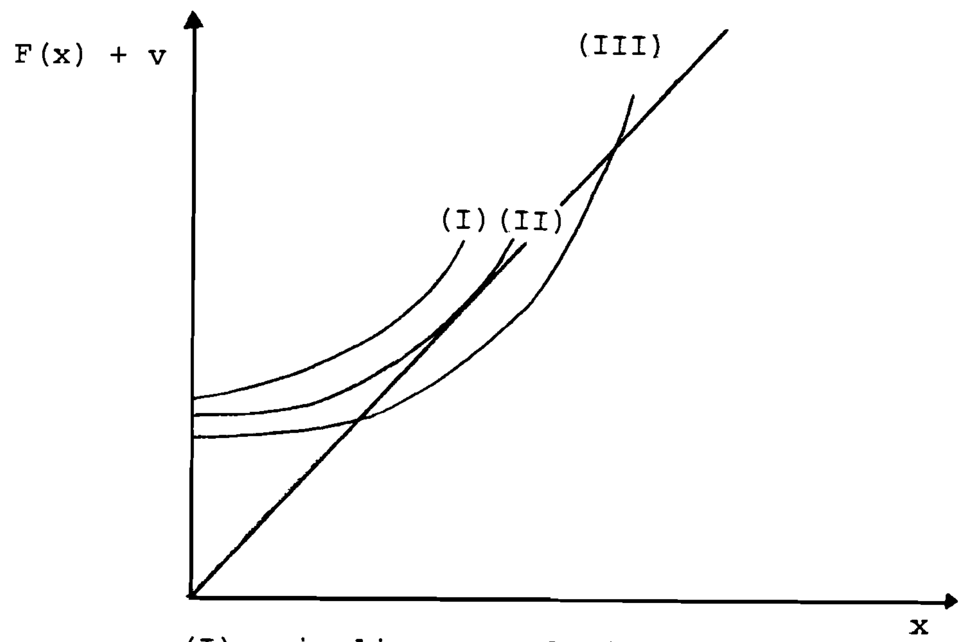
(ii) if S_v is bounded and contains at least two different vectors, there are at least two solutions to the system,

(iii) there is a solution to the model for a given v if, and only if, the sequence in (3.3) converges,

(iv) the limit of the sequence in (3.3) is a solution to the model and this solution is lowest in value in each component among solutions,

(v) for solutions x and x' lowest in value associated with v and v', $v' \geqq v \Rightarrow x' \geqq x$.

It can be shown that the character of S_v depends critically on the length of the time horizon, i.e., on T. As indicated in the subsequent section the crucial feature is the maximum eigenvalue of the Jacobian of F(x) in (3.2). This value should not exceed unity. In Figure 1 the nature of the problem is depicted. In case (I) there is no solution, in case (II) there is one solution and in case (III) there are two solutions.



(I) implies no solution,
(II) exactly one solution, and
(III) two solutions.

Figure 1. Solutions to the medium-term model.

As indicated in Figure 2 we have now arrived at a point where our approach may be summarized.

Remark 1: The same model is used in two different time perspectives which are interlinked in a consistent way each single year. Letting (i) and (ii) denote the short and medium term perspectives, respectively we have

(i) $x_i(t) = \Sigma a_{ij}x_j(t) + \Sigma k_{ij}\psi_j(x^*,\bar{x}(t))\bar{x}_j(t) + v_i(t)$,

(ii) $x_i^* = \Sigma a_{ij}x_j^* + \Sigma k_{ij}\psi_j(x^*,\bar{x}(t))x_j^* + v_i^*$,

where the k_{ij}'s are introduced in (2.6). In (i) the investment deliveries emerge as exogenously given, since $\bar{x}_j(t)$ is fixed. In case (ii) we solve for x^* contingent on a fixed v^*. Therefore, this second case constitutes a fixed-point problem.

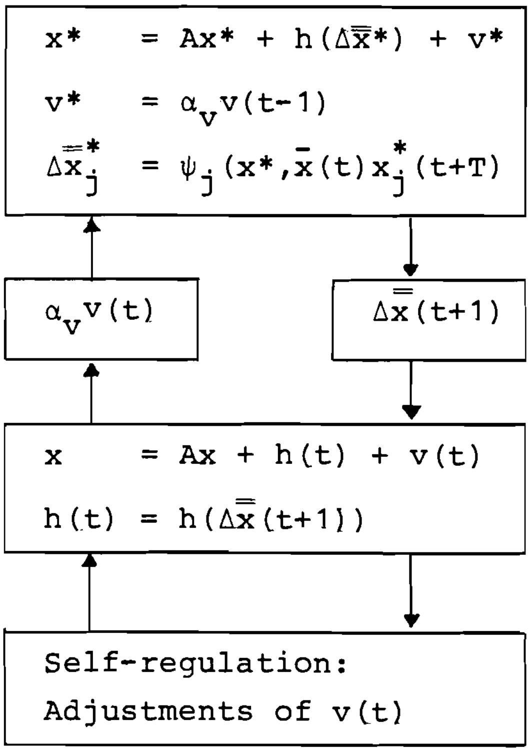


Figure 2. Illustration of how the short and medium term perspectives are interlinked within the same model.

3.3 Different Forms of Capacity Change

In (2.5) a capacity change function, $\psi_j(x^*,\bar{x}(t))$, was introduced. It may be written as

$$\psi_j(x^*,\bar{x}(t)) = \Delta\bar{\bar{x}}_j(t+1)/\bar{x}_j(t)$$

If the capacity is assumed to change with a geometric growth rate, then ψ_j has the following specification:

$$\psi_j(x^*,\bar{x}(t)) = [x^*_j/\bar{x}_j(t)]^{1/T} + \rho_j - 1 \tag{3.5}$$

where ρ_j is the rate of capacity removal in sector j.

Suppose that we want to approximate (3.5) with a linear expression. Then we have to specify ψ_j with respect to time which yields

$$\begin{aligned} \psi^t_j(x^*,\bar{x}(t)) &= [x^*_j - \bar{x}_j(t)]/[T\bar{x}_j(t)] + \rho_j \\ \psi^T_j(x^*,\bar{x}(t)) &= [x^*_j - \bar{x}_j(t)]/[T\bar{x}_j(t+T)] + \rho_j \end{aligned} \tag{3.6}$$

In order to illustrate the solution to (3.1) we may reformulate the expressions by introducing $k_{ij}(x^*) = k_{ij}\psi_j(x^*,\bar{x}(t))$ for (3.5) and $k_{ij}(x^*) = k_{ij}\psi^T_j(x^*,\bar{x}(t))$ for (3.6). We may then form the matrix $K(x^*) = \{k_{ij}(x^*)\}$. Then $K(x^*)x^* = K\Delta\bar{\bar{x}}^*$, where K is the investment matrix introduced in (2.6). From the properties of ψ_j or ψ^T_j we have that $k_{ij}(x^*) \geq 0$.

The linear version of capacity change provides an excellent opportunity to illustrate the role played by the time horizon.

Remark 2. Consider the system in (3.1) formulated as $x = Ax + K(x)x + v^*$, where $K(x)$ is determined by (3.6), and where v^* is given. Suppose that, for a certain value of T, the maximum eigenvalue of the Jacobian of $[Ax + K(x)x]$, $J(x)$, exceeds unity. Then this value can always be reduced below unity by increasing the time horizon, T, for which the projection of v^* is made.

The statement is self-evident from the character of a typical element, $J_{ij}(x)$, of $J(x)$ as can be seen below

$$J_{ij}(x) = a_{ij} + k_{ij}/T$$

Since $\Sigma_j a_{ij} < 1, \Sigma_j [a_{ij} + k_{ij}/T]$ will also approach a value below unity as T grows. The importance of this is obvious, since a solution to (3.1) or, equivalently, (3.2) exists only if the maximum eigenvalue of $J(x)$ does not exceed unity [see Persson (1980)].

4. DYNAMICS OF THE BIREGIONAL MODEL

4.1 Structure of the Biregional Model

The general form of the biregional model presented in this section has been described in detail by Martellato (1982a), (1982b). Our intention is to include the biregional model in the dynamic framework presented in the preceding sections.

The model has two regions denoted by $r,s \in \{1,2\}$. For each region r the following quantity relation is assumed to hold between demand and supply:[1)]

$$\begin{aligned} x^r + m^r = B^{rr}[A^r x^r + q^r + h^r + e^r] &+ \\ + B^{rs}[A^s x^s + q^s + h^s + e^s] & \end{aligned} \tag{4.1}$$

where $x^k = \{x_i^k\}, m^k = \{m_i^k\}, g^k = \{g_i^k\}, h^k = \{h_i^k\}, e^k = \{e_i^k\}$, $B^{rk} = \{b_i^{rk}\}, A^k = \{a_{ij}^k\}, k = r,s$, and where for region k

x_i^k = output from sector i,

m_i^k = import of sector i products

g_i^k = residual final demand for sector i products,

h_i^k = investment deliveries from sector i,

1) This assumption is specific for the TIM-model, in which the regions are Tuscany and the rest of Italy. See Martellato (1982a), (1982b).

e_i^k = export of sector i products

a_{ij}^k = input-output element

b_i^{rk} = trade coefficient referring to deliveries from region r to region k of sector i products.

The balance equations in (4.1) should be compared with those in (2.1). The assumption made implies that the delivery, x_i^{rs}, of sector i products from region r to region s is determined by the demand in region s as described below

$$x_i^{rs} = b_i^{rs}[\sum_j a_{ij}^s x_j^s + g_i^s + h_i^s + e_i^s] \tag{4.2}$$

Consider now the following two matrices:

$$B = \left[\begin{array}{c|c} B^{11} & B^{12} \\ \hline B^{21} & B^{22} \end{array}\right]$$
$$A = \left[\begin{array}{c|c} A^1 & \\ \hline & A^2 \end{array}\right] \tag{4.3}$$

and let $x = (x^1,x^2)$, $h = (h^1,h^2)$, $g = (g^1,g^2)$, $e = (e^1,e^2)$, $m = (m^1,m^2)$. Then we have

$$x = B[Ax + g + h + e] - m \tag{4.4}$$

Finally, let $D_i^r = \Sigma a_{ij}^r x_j^r + g_i^r + h_i^r + e_i^r$. From (4.1) we have that $\Sigma x_i^r + m_i^r = \Sigma(b_i^{rr} + b_i^{sr})D_i^r$ which yields $(b_i^{11} + b_i^{21})D_i^1 + (b_i^{22} + b_2^{12})D_i^2 = D_i^1 + D_i^2$, for any $D_i^1 > 0$ and $D_i^2 > 0$. Hence,

$$b_i^{rr} = 1 - b_i^{sr}\ , \text{ for } r = 1,2;\ r \neq s \tag{4.5}$$

4.2 Determining Capacity Change and Annual Investments

In this section we shall apply the framework introduced in section 3.3 to the model structure presented in the preceding section. Therefore, we define our investment matrix K as follows:

$$K = \left[\begin{array}{c|c} K^1 & \\ \hline & K^2 \end{array}\right]$$

$K^r = \{k^r_{ij}\}$ is a matrix of investment coefficients, referring to region r.

Consider the linear formulation in (3.6). Let I be the unit matrix and let $< \rho >$ be another diagonal matrix of regionally specified removal rates. Then we can define $D(x^*)$ as follows:[1]

$$\begin{aligned} D(x^*) &= Ax^* + g(T) + e(T) + \\ &+ (1/T)K[(I - T < \rho >)x^* - \bar{x}(0)] \end{aligned} \tag{4.6}$$

where x^* denotes the value of x associated with v^*, and where $\bar{x}(0)$ is the initially given capacity at time $t = 0$. From (4.6) we may formulate a biregional version of (3.1) in the following way

$$\begin{aligned} F(x) &= [B - MB][D(x) - g - e] \\ v &= B[g + e] - MBg \end{aligned} \tag{4.7}$$

where M is a diagonal matrix of import coefficients such that the import vector, m, equals $MB[Ax + g + h]$. In this way one obtains the system $x = F(x) + v$. From a solution to this system we derive, as shown in (3.6), the net capacity change, $\Delta\bar{x}^r_j(1)$ in year $t = 1$; from ρ_j and $\Delta\bar{x}^r_j(1)$ we determine the investments, $h^r_i(0)$, in year $t = 0$ and the capacity, $\bar{x}^r_j(1)$, in year $t = 1$ for all i and r, as shown below:

$$\begin{aligned} h^r_i(0) &= \sum_j k^r_{ij}(\Delta\bar{x}^r_j(1) + \rho^r_j\bar{x}^r_j(0)) \\ \bar{x}^r_j(1) &= \bar{x}^r_j(0) + \Delta\bar{x}^r_j(1) \end{aligned} \tag{4.8}$$

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[1] Here we utilize the linear formulation of the capacity change path, since the exercises presented in Appendix were done with this version of the model.

The derived solutions in (4.8) constitute inputs to the determination of the annual short-term outcome.

4.3 Regional Capacities and Trade in the Short-term

Consider the system in (4.1) and introduce the auxiliary variable y_i^r. Together with (4.8) we obtain the system

$$
\begin{aligned}
& m_i^r(t) + y_i^r(t) = b_i^{rr}(t) D_i^r(t) + b_i^{rs} D_i^s(t) \\
& y_i^r(t) \geq x_i^r(t) \qquad\qquad (4.9) \\
& x_i^r(t) \leq \bar{x}_i^r(t)
\end{aligned}
$$

From this we can see that $y_i^r(t) > \bar{x}_i^r(t)$ implies excess demand or shortage of regional capacity, and that $y_i^r(t) < \bar{x}_i^r(t)$ implies excess capacity in the region. We shall assume that the economy responds to such imbalances in two steps. First the interregional trade coefficients are adjusted. To denote this the interregional trade matrix may be expressed as $B = B(y,\bar{x}), y = (y^1, y^2)$.

If excess demand still remains after this adjustment, the gap is closed by additional imports. Therefore, the short-term import vector, $\hat{m}$, is determined as follows:[1)]

$$\hat{m} = MB(y,\bar{x})[Ax + g + h] + \max\{(y-\bar{x}), 0\} \qquad (4.10)$$

The major part of the import may be characterized as complementary, while the adjustment part may be called competitive or "gap determined". Combining (4.9) and (4.10) yields

$$\hat{m}_i^r + x_i^r = b_i^{rr} D_i^r + b_i^{rs} D_i^s \qquad (4.11)$$

which represents the short-term solution, given that b_i^{rr} and b_i^{rs} signify the adjusted trade coefficients.

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1) If $y_i^r(t) > \bar{x}_i^r(t)$ for a non-tradeable good, balance cannot be obtained through trade adjustments. Instead, final demand has to be reduced, e.g., by decreasing private and public consumption.

4.4 Short- and Medium-term Adjustment of Interregional Trade

The complete scheme for the interaction between the short- and medium-term perspective should include both (i) the capacity constraint impact on short-term interregional trade, and (ii) the impact on capital formation of adjustments in interregional trade coefficients. Such an interaction is illustrated in Figure 6.

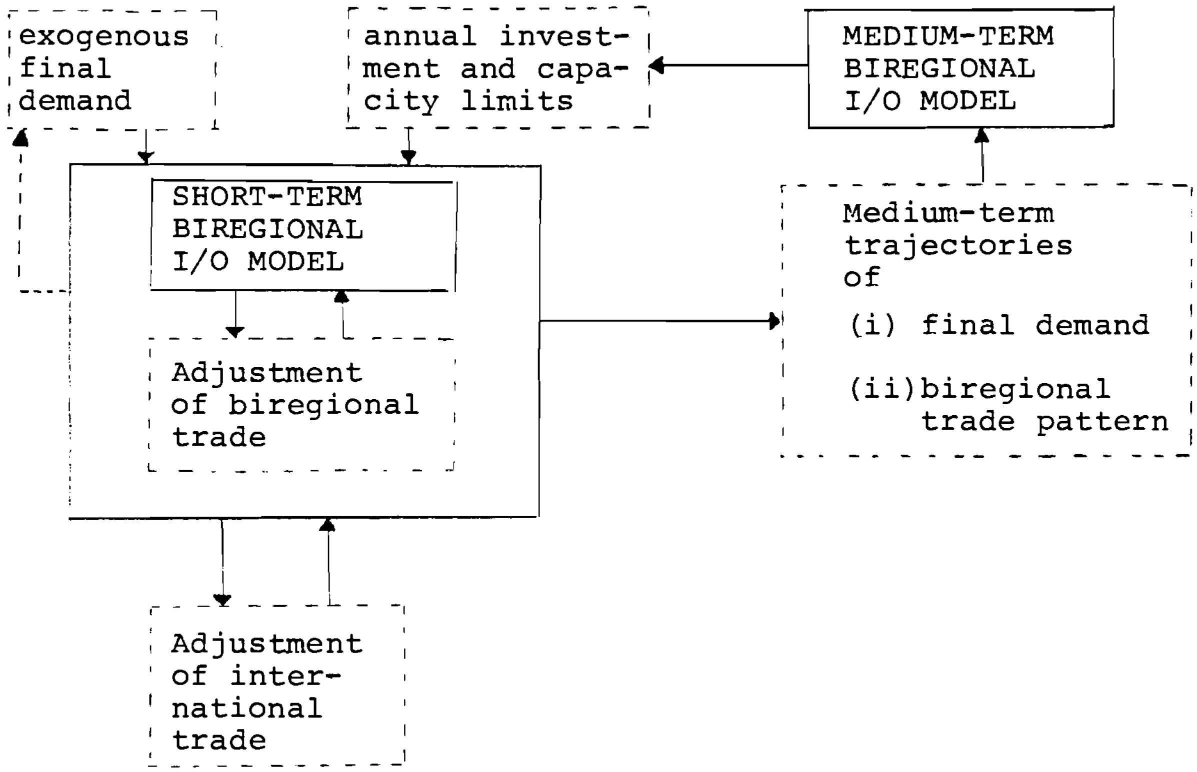

Figure 6. Interaction between short- and medium-term adjustments.

The specification of the system in (4.7) must contain matrices M and B which are assumed to be valid at time T. Let these matrices be M(T) and B(T). For the short-term analysis we need similar matrices M = M(0) and B = B(0), where the latter may represent the trade pattern which obtains if the short-term solution is characterized by interregional balance without any adjustments.

Combining (4.9) and (4.10) we may define

$$
\begin{aligned}
F^{(o)}(x) &= [I-M]B^{(o)}Ax \\
f^{(o)} &= [I-M]B^{(o)}[g + h] + B^{(o)}e \\
G^{(o)}(x) &= F^{(o)}(x) - \max\{(F^{o}(x) + f^{o} - \bar{x}), 0\}
\end{aligned}
\tag{4.12}
$$

where I denotes the identity matrix and $B^{(o)} = B(0)$ before any adjustments have occurred. From (4.12) we can express the short-term problem as

$$x = G^{(o)}(x) + f^{(o)} \tag{4.13}$$

where $G^{(o)}$ and $f^{(o)}$ are defined given $B^{(o)}$. The system in (4.13) can be solved by means of the same type of iteration as described in (3.2). Obviously G^{o} is continuous and monotonous. Having solved (4.13) given $B^{(o)}$ it remains to examine whether $B^{(o)}$ satisfies that criterion of interregional balance which we wish to apply. Suppose that this is not fulfilled and that we are adjusting $B^{(o)}$ to $B^{(1)}$. Then we have to solve the new system $x = G^{(1)}(x) + f^{(1)}$. If we signify $B^{(n)} = B(y^{(n-1)}, \bar{x})$, where $y^{(n-1)}$ equals the solution value of $F^{(n-1)}(x) + f^{(n-1)}$, we obtain the general formulation

$$
\begin{aligned}
x &= G^{(n)}(x) + f^{(n)}, \text{ given that} \\
B &= B(y^{(n-1)}, \bar{x})
\end{aligned}
\tag{4.13'}
$$

This formulation has the same solution properties as the initial step in (4.13).

As regards the interregional trade balance, one may contemplate several types of criteria for the setting outlined in (4.13'). We shall mention two alternatives which are characterized further in a section of appendix 1.

The first alternative relates to a suggestion by Martellato (1982b). In this case the interest is focused on the capacity tension in each region, γ_i^r, which is defined as follows

$$\begin{aligned}\gamma_i^r &= (y_i^r - \omega_i y_i^s)/y_i^s \\ \omega_i &= \bar{x}_i^r/\bar{x}_i^s\end{aligned} \tag{4.14}$$

The approach suggested by Martellato relies on an estimated functional relationship between each coefficient b_i^{rs} and the associated capacity tension γ_i^r so that $b_i^{rs} = b_i^{rs}(\gamma_i^r)$. Referring to the adjustment procedure indicated by (4.13′), the solution to the short-term problem is obtained for balanced interregional trade when

$$b_i^{rs(n+1)} = b_i^{rs}(\gamma_i^{r(n)}) \tag{4.15}$$

A weaker criterion for interregional trade balance may be stated with the help of the variable $\xi_i^r = (\bar{x}_i^r - x_i^r) \geq 0$ for $r = 1,2$. With this criterion we accept a solution as balanced if

$$\begin{aligned}&\text{(i)} \quad \xi_i^1 = \xi_i^2 = 0 \text{ , or otherwise} \\ &\text{(ii)} \quad \Sigma\xi_i^r > 0 \Rightarrow \Sigma y_i^r = \Sigma x_i^r\end{aligned} \tag{4.16}$$

If a solution to (4.12) does not satisfy condition (4.16) there must exist a surplus $\xi_i^r > 0$ which can be distributed to region $s \neq r$ so that the total foreign import of sector i products can be reduced.

With regard to the selection of target-year trade matrices B(T), B(T+1), etc., we shall just point out that such a matrix is used in (4.7) under explicit assumption about interregional balance. A fundamental problem is that such a matrix implies a specific investment process which is obvious from (4.6) and (4.7). Hence, selecting a matrix B(T) is indeed a choice of capacity location over the set of regions.

5. CONCLUDING REMARKS

It is important to observe that the approach we have suggested in this paper raises many new questions for multi-sectoral modeling. In particular, it suggests the possibility of analyzing short-term stability problems and cycles as well as economic stabilizing policies in a multisectoral framework. It does so by connecting medium-term expectations and consistency constraints with short-term consistency constraints and adjustment processes, including explicitly recognizable economic policies. In particular, one should note that short run capacity tensions in the model can be interpreted as implicitly referring to inflation-creating processes. Of course, this perspective calls for a multisectoral model framework which includes relative prices (including wages and interest rates), labor and capital markets. In fact, this defines a research program.

APPENDIX: Empirical Illustration

In this appendix we shall illustrate the model formulation $x = F(x) + v$ in (3.1) where $F(x)$ and v have a biregional specification as in (4.7). The first target year is 1980 for which v^* has been estimated. The target values v^* (1981), $v^*(1982)$ etc. have been obtained by applying different growth rates of v^* with $v^*(1980)$ as base so that $v^*(t) = v^*(t-1)(1+\lambda)$, $\lambda>0$. In this way investments have been calculated for 1975,..., 1979 and capacity change for the years 1976,...,1979. In table A : 1 the total investments are calculated for three different growth rates and compared with the realized sequence of aggregate capital formation.

In table A : 2 capacity change for each sector in Tuscany and the Rest of Italy are illustrated. The assumed annual growth in final demand is 6 percent. One should observe that with this specification the model gives higher investments and lower net capacity increase than exhibited by observed series. The explanation is simple: during 1975-1980 the rate of capacity removal was several percent lower than for the whole period 1970-1980. In the model exercise of table A : 2 the latter (and higher) pattern of removal was applied. The effect of this is further illustrated in table A : 3, in which observed capacity levels* are compared with levels generated by the model. Moreover, investment deliveries with regard to Tuscany 1977 are described as calculated in the model and as observed.

Due to lack of recorded time series, no real evaluation of the growth model has been possible. Therefore, the appendix merely gives an arbitrary example of a sequential one-year application of the medium-term model.

Table A:1 Total investment in Italy 1975-1979.

	Annual investments in the model when the growth of final demand (1980-1975) has been set to:			Observed annual investments
	4%	10%	6%	
1975	17946	30866	21868	25776
1976	20354	35951	25069	26090
1977	22551	41119	28044	26214
1978	24530	46462	30889	26127
1979	26361	52132	33651	27649

*The method for calculation of the "observed" capacity levels is described in Westin, Johansson and Grassini (1982).

Table A:2 Capacity change in Tuscany and the Rest of Italy 1976-79, according to growth model.

	Net change of capacity in percent			
	1976	1977	1978	1979
1 Agriculture	+ 7 (+ 0)	+ 7 (+ 1)	+ 6 (+ 2)	+ 6 (+ 3)
2 Coal and Oil	+ 1 (-18)	+ 3 (-17)	+ 3 (-16)	+ 4 (-15)
3 Other Energy	+ 1 (- 3)	+ 2 (- 2)	+ 3 (+ 0)	+ 4 (+ 1)
4 Minerals	- 3 (-12)	- 1 (- 9)	+ 1 (- 7)	+ 2 (- 5)
5 Non-metal	- 2 (- 6)	+ 0 (- 4)	+ 2 (- 1)	+ 3 (+ 0)
6 Chemicals	- 4 (- 5)	- 3 (- 3)	- 1 (- 1)	+ 0 (+ 0)
7 Metal Products	- 1 (- 7)	+ 1 (- 4)	+ 3 (- 2)	+ 4 (+ 0)
8 Machinery	+ 6 (- 8)	+ 7 (- 5)	+ 7 (- 3)	+ 7 (+ 0)
9 Other Machinery	+11 (-11)	+12 (- 8)	+11 (- 5)	+10 (- 3)
10 Electrical	+11 (- 8)	+10 (- 5)	+10 (- 3)	+ 9 (- 1)
11 Transport	+ 7 (- 4)	+ 7 (- 1)	+ 7 (+ 0)	+ 7 (+ 2)
12 Meat	+42 (+ 2)	+26 (+ 3)	+19 (+ 4)	+15 (+ 4)
13 Milk	+50 (+ 0)	+29 (+ 1)	+20 (+ 2)	+15 (+ 3)
14 Other Food	+ 0 (+ 1)	+ 1 (+ 2)	+ 2 (+ 3)	+ 3 (+ 3)
15 Beverages	+ 2 (- 1)	+ 3 (+ 0)	+ 3 (+ 1)	+ 4 (+ 2)
16 Tobacco	- 2 (+ 5)	- 1 (+ 5)	+ 1 (+ 6)	+ 2 (+ 6)
17 Textiles	+ 0 (+ 0)	+ 1 (+ 1)	+ 2 (+ 2)	+ 3 (+ 3)
18 Footwear	+ 5 (+ 3)	+ 5 (+ 3)	+ 5 (+ 4)	+ 5 (+ 4)
19 Wood Products	- 2 (- 6)	+ 0 (- 4)	+ 1 (- 2)	+ 2 (+ 0)
20 Paper Products	- 2 (- 6)	+ 0 (- 4)	+ 1 (- 2)	+ 2 (- 1)
21 Rubber Products	- 4 (- 4)	- 2 (- 2)	- 1 (+ 0)	+ 1 (+ 1)
22 Other Manufact.	- 3 (+ 2)	- 1 (+ 3)	+ 0 (+ 3)	+ 2 (+ 4)
23 Construction	+ 4 (- 5)	+ 5 (- 2)	+ 6 (+ 0)	+ 6 (+ 2)
24 Commerce	+ 4 (+ 1)	+ 5 (+ 2)	+ 5 (+ 3)	+ 5 (+ 4)
25 Hotels	+ 4 (+ 3)	+ 4 (+ 4)	+ 5 (+ 4)	+ 5 (+ 5)
26 Transport	+ 1 (+ 0)	+ 2 (+ 1)	+ 3 (+ 2)	+ 4 (+ 3)
27 Communication	+ 3 (+ 0)	+ 4 (+ 1)	+ 4 (+ 2)	+ 5 (+ 3)
28 Credit	-12 (-15)	-12 (-14)	-10 (-13)	- 8 (-11)
29 Housing	+ 6 (- 4)	+ 6 (- 3)	+ 6 (- 1)	+ 6 (+ 0)
30 Services	+ 3 (+12)	+ 4 (+10)	+ 4 (+ 9)	+ 5 (+ 8)
31 Non-market Services	+ 0 (+ 4)	+ 1 (+ 4)	+ 2 (+ 4)	+ 3 (+ 5)
32 Total Economy	+ 2 (- 5)	+ 3 (+ 1)	+ 3 (+ 0)	+ 4 (+ 2)

Remark: Results for "Rest of Italy" are given within brackets. All calculations are based on an "expected" annual growth rate of 6 percent in final demand 1980-1984.

Table A:3 Comparison between model result and observed outcome.

	Capacity in model divided by observed capacity. Percent 1978		Investment deliveries in Tuscany 1977	
	TUSCANY	REST OF ITALY	In Model	Observed
1	121	100	3	2
2	114	55		
3	106	85		
4	103	74		
5	94	90	11	12
6	95	78		
7	91	88	65	71
8	94	87	325	355
9	100	60	61	66
10	91	85	138	151
11	98	88	194	287
12	258	97		
13	202	91		
14	103	97		
15	120	102		
16	76	100		
17	84	98		
18	75	120	1	1
19	76	83	76	74
20	97	87		
21	77	93	11	11
22	81	91	1	1
23	110	96	1390	1310
24	105	98	84	97
25	106	107		
26	82	93	18	21
27	102	94		
28	61	56		
29	114	79		
30	102	121		
31	87	106		
Total	96	90	2378	2461

Remark: For these calculations final demand has been assumed to have an annual growth rate of 6 percent after 1980.

REFERENCES

Åberg, M., and H. Persson (1981) A Note on a Closed Input-Output Model with Finite Life-Times and Gestation Lags. *Journal of Economic Theory, vol. 24, No.3.*

Bergman, L., and A. Por (1980) A Quantitative General Equilibrium Model of the Swedish Economy. *IIASA WP-80-04*, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Johansen, L. (1960) *A Multisectoral Study of Economic Growth*, (Second enlarged edition 1974), North-Holland, Amsterdam.

Johansen, L. (1978) On the Theory of Dynamic Input-Output Models with Different Time Profiles of Capital Construction and Finite Life-Time of Capital Equipment. *Journal of Economic Theory, vol. 16.*

Lahiri, S. (1976) Input-Output Analysis with Scale-Dependent Coefficients. *Econometrica vol.44, No.5.*

Lahiri, S. (1977) Efficient Investment and Growth Consistency in the Input-Output Frame: An Analytical Contribution. *Econometrica vol.45, No.8.*

Martellato, D. (1982a) The Tuscany Interregional Input-Output Model (TIM): Mathematical Structure and Preliminary Results. *IIASA CP-82-30*, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Martellato, D. (1982b) Interregional Substitution Effects with Capacity Constraints. *IIASA Collaborative Paper* (forthcoming), International Institute for Applied Systems Analysis, Laxenburg, Austria.

Nikaido, H. (1968) *Convex Structures and Economic Theory.* Academic Press, N.Y.

Persson, H. (1980) On Extensions of a Medium-Term Input-Output Model. Working Paper, Department of Mathematics, Royal Institute of Technology, Stockholm.

Persson, H. (1981) A Way of Modeling Investment in a Multisectoral Model. Working Paper, Department of Economics, Gothenburg University.

Persson, H., and B. Johansson (1982) A Dynamic Multisector Model with Endogenous Formation of Capacities and Equilibrium Prices: An Application to the Swedish Economy, *IIASA PP-82-9*, International Institute for Applied Systems Analysis, Laxenburg, Austria.

Royden, H.L. (1963) *Real Analysis.* Macmillan, N.Y.

Westin, L., B. Johansson, and M. Grassini (1982) Estimation of Capital Matrices for Multisectoral Models: An Application to Italy and Tuscany. *IIASA WP-82-92*, International Institute for Applied Systems Analysis, Laxenburg, Austria.