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# COMPUTER ASSISTED PROCEDURES IN WATER PROJECT EVALUATION

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## Abstract

The paper describes a methodology which can be used in solving water economy capital investment problems. These problems concern water economy expansion to meet an increasing demand for municipal, industrial, and other uses within given resource limitations.

A water economy is considered as an individiual branch of a region. Its connections and interactions with other branches of that region (industry, agriculture, etc.) are taken into account.

We presumed that the interaction of the regional management (Center) and the branch manager takes place within the framework of a centralized hierarchical system. The Center distributes the resources (money, manpower, etc.) among the managers of separate branches. The information a manager receives may be represented by the vector function of resources C(t) during the planning period $[t_o, T]$. The output information is considered as a scalar function I(t), which characterizes the loss to the branch if it is not sufficiently supplied with hydrotechnical structures.

The process of decision-making consists of three phases. The first phase is formulating all feasible alternatives for the development of a given regional water management system. The second phase is calculating all versions of the development program for the branch. The third phase is estimating the loss which the branch may incur with a given program of branch development. The latter two phases are provided with appropriate mathematical models and numerical algorithms.

This system of models differs from all known capital investment models in water economy in that it incorporates the process of building hydrotechnical structures over a planning horizon.

## I. Introduction

Mr. A., coordinator of a Center agency, has to appropriate a certain sum of money to Mr. B., his assistant, for expanding a water economy complex. Mr. A. is not interested in details and therefore asks only one thing: what would be the loss (J) from floods, lack of water, etc., if he were to allot a sum (C) from the budget for development of a water economy project.

Mr. B., unlike Mr. A., is interested not only in the total sum foreseen for development in his area of responsibility (water economy), but also in how much he is to receive, and when, for realization of water economy activities ($C(t)$, $t \in [t_o,T]$). Moreover, since he is a developer and the agency is responsible for any material loss, he also wishes to know what losses will be incurred in each year of the planning period ($J(t)$, $t \in [t_o,T]$).

## II. Dialogue

A.: Dear B., we have worked together with you for many years, and you are always moderate in your views and demands; I really appreciate these qualities of yours. But this time your demand for the next five-year period is exorbitantly high.

B.: My dear A! In the past, I, as well as the other representatives of the different branches of our region, acted as our intuition and experience prompted, in accordance with the information on the situation you provided us. But our experts have developed a system of procedures which allows us to consider the possible variants of our activity and estimate the consequences of any adopted solution in relation to the capital invested in the development of our branch. These studies led us to request the sum you consider so high, which proves to be necessary for the best development of our branch. All other solutions would lead to a less desirable result, and some of them could have irreversible negative consequences.

A.: You assert, then, that your demands are substantiated, and, that if I give you a considerably smaller sum than you ask for, sooner or later serious problems can be expected?

B.: Yes, that is right.

A.: Can you explain the essence of your system to me, without any details or mathematics?

B.: Our efforts are distributed in a certain way over time. You alloted us the sum (C) and I must give you my views on investments we'll need next year and in the more distant future.

A.: Are you considering the fact that I need to know more exactly the sum necessary for the next year or the next five-year period? For the more distant future, the data can be approximate.

B.: That makes the matter simpler but doesn't change it in principle. The sequence of the investments needed corresponds to the sequence of time intervals:

$t_i \rightarrow C_i$, $i = 0,1,\ldots,K$, $t_k = T$.

By the way, I am aware of the value of the investment $C_o$ you have allotted to our branch this year. But the values $C_1, C_2, \ldots, C_k$ should be substantiated by me and considered by you.

A.: We have planning values of $C_1, \ldots, C_k$, which you should use as guidelines.

B.: Only as guidelines, not more, since these planning values are based on information we supplied to you sometime earlier. The data are usually checked yearly, but new data for substantiating, for example, the value $C_{k+1}, \ldots$ are needed.

A.: That is, you want to develop a methodology of continuous estimation of $C_i$ $(i = 1,\ldots,k)$ on the basis of currently available information.

B.: We already have a system which can be used for such data estimation. As initial information we use not only the knowledge of the status quo but also all the conceivable water economy activities and units with their locations and main characteristics.

A.: Yes, I've seen your list. It's probably a bit too long and has some unnecessary items.

B.: We've undertaken investigations to find out what is necessary. In our list, each acticity or water economy unit having the number "i" corresponds to an appropriate investment $c_i$ necessary for its realization. Sometimes the experts can point to the exact jobs, their sequence, alternatives, of the program which exclude each other, etc. Such information represents different variants of development of a water economy system, each unit of which can accomplish various tasks simultaneously.

A.: In other words, the activities have to some extent been regulated, and the problem is to compile a schedule indicating the capital to be. But how do you define the effectiveness of the investments?

B.: We have a model, a prototype of our water economy system, which describes its development over time. It comprises all the elements now existing within the system: different parts of rivers, reservoirs, hydroelectric power stations, sluices, canals, etc. Moreover, it includes the construction and development of new elements (water economy structures). From different combinations of these elements a complicated network activity is formed, which corresponds to real and expected conditions and the usage of water resources in our area. Floods in the network correspond to the real flood capacity supplying our system during any given period and being distributed among water users (which we consider to be given, for simplicity). Each model element is characterized by a loss function, defined by the losses from floods or insufficient supply of water, which is determined by the initial data. The calculation of the model is done by a computer. In calculating the model, we have solved the problem of river flow regulation by means of reservoirs and water distribution among water users, which gives the minimum cumulative loss function for all existing elements of the system during the planning period.

A.: Thus you can establish a relationship between any development alternative of the water economy complex and the minimum value of the loss function. Consequently, if I assign you $\tilde{C}$ you can point out what damage it will cause, and by comparing different values of $\tilde{C}$ with the anticipated damage I can make sounder decisions. I assume that you tested the different values of $\tilde{C}$ and selected the one which to the greatest extent diminishes the function of your damages. However, if I give you not $\tilde{C}$, but a little less, the consequences won't be so drastic, or at least the losses could be recovered within a few years.

B.: (Dialing a number on the phone) Tell me your assessment of $\tilde{C}$. We'll know what damage can be expected and what $\tilde{C}_0$, $\tilde{C}_1$, $\tilde{C}_2$, etc. will be.

A.: All right, here's C'. (B. feeds C' into computer.) I would like to say that your system is not infallible. In two or three years your engineers will discover that a few more reservoirs, canals, etc., can be built. Thus, your set of alternatives is not complete, and consequently the system is not complete. Besides, it may turn out that the new alternatives for this complex development are much better than the old ones.

B.: They can easily be taken into account and incorporated into the system.

But the assessments will change; perhaps not rapidly or significantly, but they will change. Therefore, I can in principle make no decisions on investments significantly deviating from your evaluations, at least until our water economy system has been sufficiently investigated and developed.

B.: In due course, the possibilities of development will be more and more limited and costs will rise.

A.: That's true, but in due course. And it is also very important for me to gain time. (The telephone rings. A. answers.) Computer Services reports that damages will be significant; nevertheless, I can't give you more than C'.

B.: Well, I guess the decision has been made. I have no choice.

A.: I can't give you more money, but I have received a lot of useful information from you; I want other branch managers to do the same kind of work and will instruct them accordingly.

## III. Description of Models

The problem of water economy design and scaled evaluation of large administrative or economic regions is extremely complex and cannot be stated and solved as a single mathematical problem. The selection of an optimal or rational development project is a set of problems connected by means of data processing and informal decision-making procedures. But there is always a hope that by considering a sufficiently large number of development alternatives, one can insure against erroneous solutions. The use of simulation systems allows one to carry out this work rapidly, and thus to make decisions less expensive at their preliminary stages.

In this paper we consider man-machine decision-making procedures which concern the interaction between a regional water economy directive body and the top management (Center) of a region as a whole during development planning. We assume the water economy to be an independent branch in the region, connected with other branches in the region through information and resource flows.

The particularity of problems of rational water resource use is a combination of interest in increasing industrial output on the one hand and purely regional interests on the other hand (e.g. environmental considerations).

Let us assume that interaction of the Center and water economy management takes place within the framework of centralized management. The simplest relation between these bodies is shown in Figure 1. The proper information will allow the Center to attain its goals in the most rational way while taking into account the goals of its branches. Thus the Center will be able to distribute its deficit resources rationally. We also assume that the Center makes its decisions on the basis of existing uncertainty of information on the goals and possibilities of its branches.

The basic principles of resolution of problems of resource allocation among branches by the Center have been studied in [1], [2], and we will not dwell on them here. We are concerned with the problem of information for the regional directive body provided by the water economy management, and the problem of decision-making by the latter under some constraints.

We consider long-range planning for the water economy branch to be consistent with three stages (Figure 2). The first is formulation of all conceivable development alternatives; the second, calculation of all versions of the branch development programs (i.e. for building, expansion and reconstruction of water economy structures). The third stage is estimation of the loss (cumulative damage over all branches in the region due to insufficient development of water economy structures in the region) which may occur .ith the given program when the branch is in optimal short-term operation.

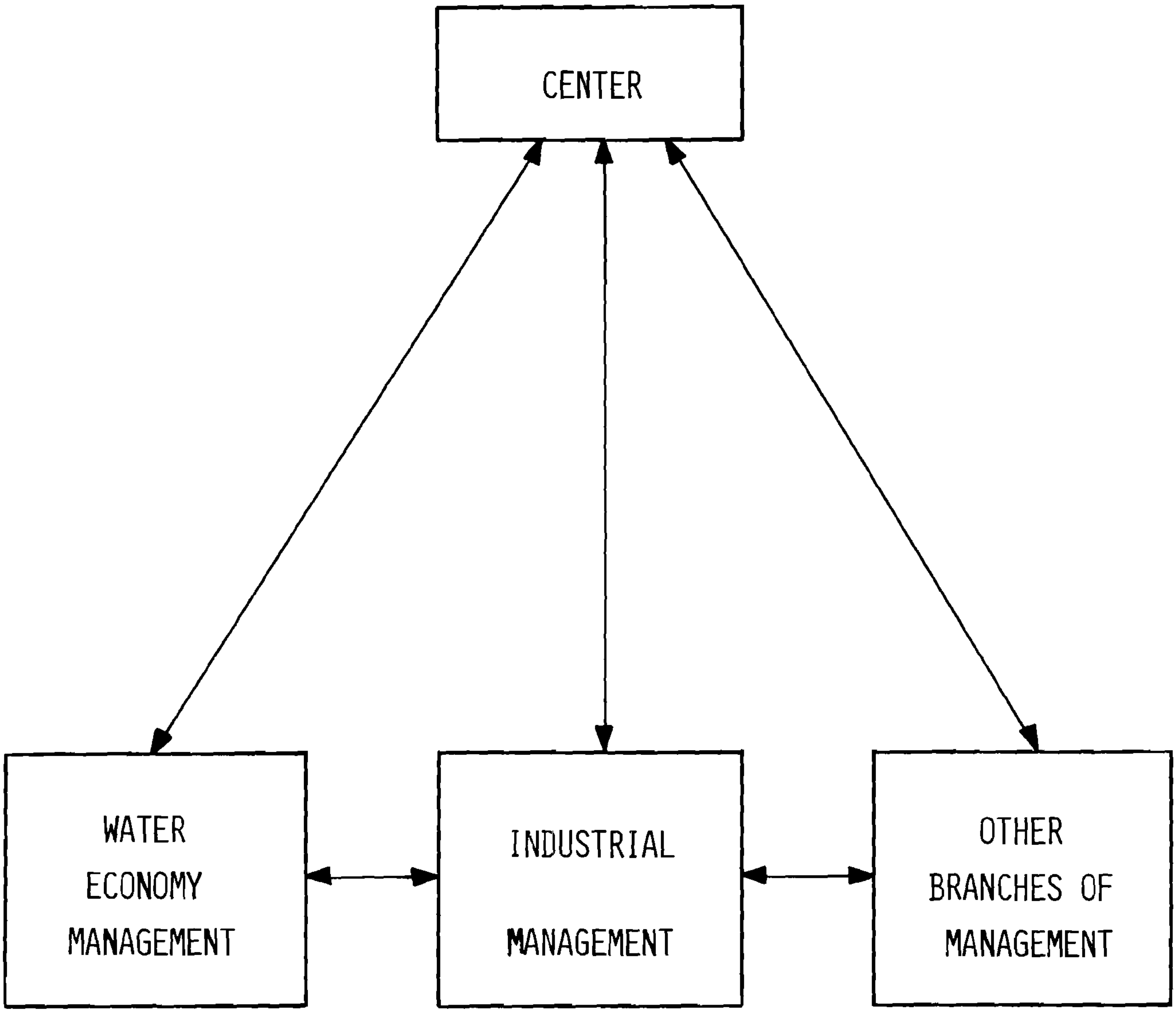



FIGURE 1. INFORMATION EXCHANGE DIAGRAM

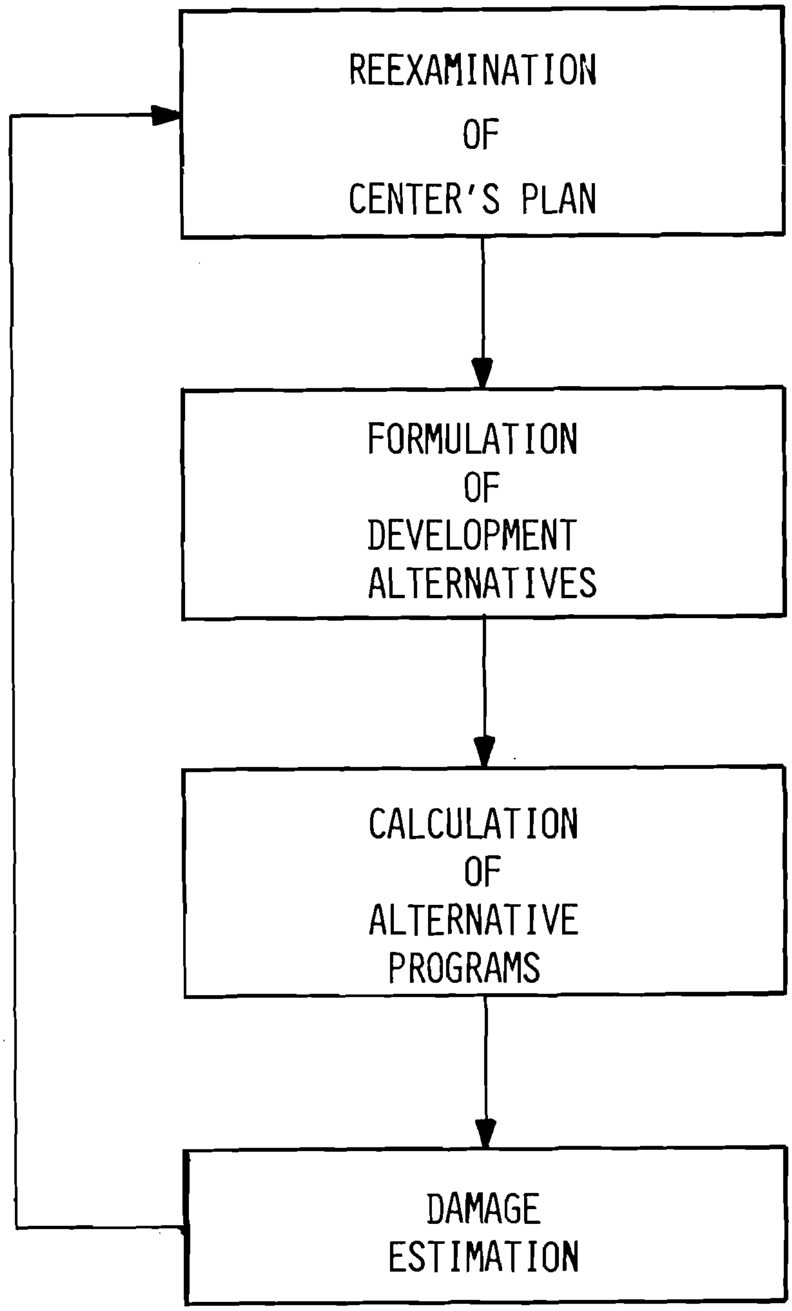



FIGURE 2. THREE BASIC STAGES OF PLANNING

It is easily seen that these planning stages are general enough to be applied to any other branch development planning.

## A. Principles of Interaction Among Center and Branch Management

Preliminary planning procedures take place in the existing administrative (management) system as an iterative process of information exchange among Center and branch managements, in which they adjust balance and correct plans and actions. This iterative process is the essence of planning.

In a general case, the information water economy management (WEM) receives may be represented by the vector function C(t) during the interval $[t_o,T]$, where $t_o$ is the starting time of the planning period and T is its terminal. An output of WEM is considered here as the scalar function J(t) which characterizes the damage suffered by the WEM branch in question. Knowing this function one can easily estimate the cummulative damage within the planning period,

$$\bar{J}(t_o,T) = \int_{t_o}^{T} J(t)dt \quad ,$$

or at a certain moment t in this period,

$$\bar{J}(t_o,t) = \int_{t_o}^{T} J(t)dt \quad .$$

In principle, it is not difficult to provide the Center with more detailed information, such as budget expenses including, for example, personnel training and improvement in specialists' skills.

It is clear from general considerations that the decision-making process of the Center for resource allocation for the development of the branches within its jurisdiction should be an iterative procedure. We may assume that the first iteration takes place when the Center informs all branches about preliminary values (for instance, outputs) which it intends to attain at the end of the planning period. Branches in turn inform the Center what additional resources they need to achieve the goals stated. Usually, at first iteration, information exchange can be expressed in terms of preliminary preparation of the development project. For the water economy branch such information includes data on additional water requirements through improvement and expansion of existing water economy structures and building of new structures.

At this stage WEM should consider all development alternatives for water requirements in other branches of the region, present a program of jobs (actions) which satisfy, to the greatest extent possible, the Center's requirements, and evaluate the resource and time needed to attain its goals. WEM also estimates possible damages during the planning period if the program is accepted (Figure 3). At the second iteration the Center informs all branches about resources it can provide (usually less than branch requirements), makes the desired planning values more precise and elaborates the first preliminary project of regional development. Now branches reexamine their plans from the point of view of possibilities of expansion and creation of new structures subject to given (by Center) resource limits.

Amended plans (or programs) are then given to the Center. At this iteration WEM obtains more precise information from all other branches about water requirements and limitations due to insufficient water economy development. On the basis of these data and the Center's priorities for different branches (water consumers), WEM determines its own priorities for water structure development, and for its own actions during a given planning period. WEM gives this information and corresponding damage estimations to the Center (Figure 4.).

It should be noted that the impossibility of attaining desired (ideal) planning values due to resource limits is one of the reasons for proposal formulation for further planning periods.

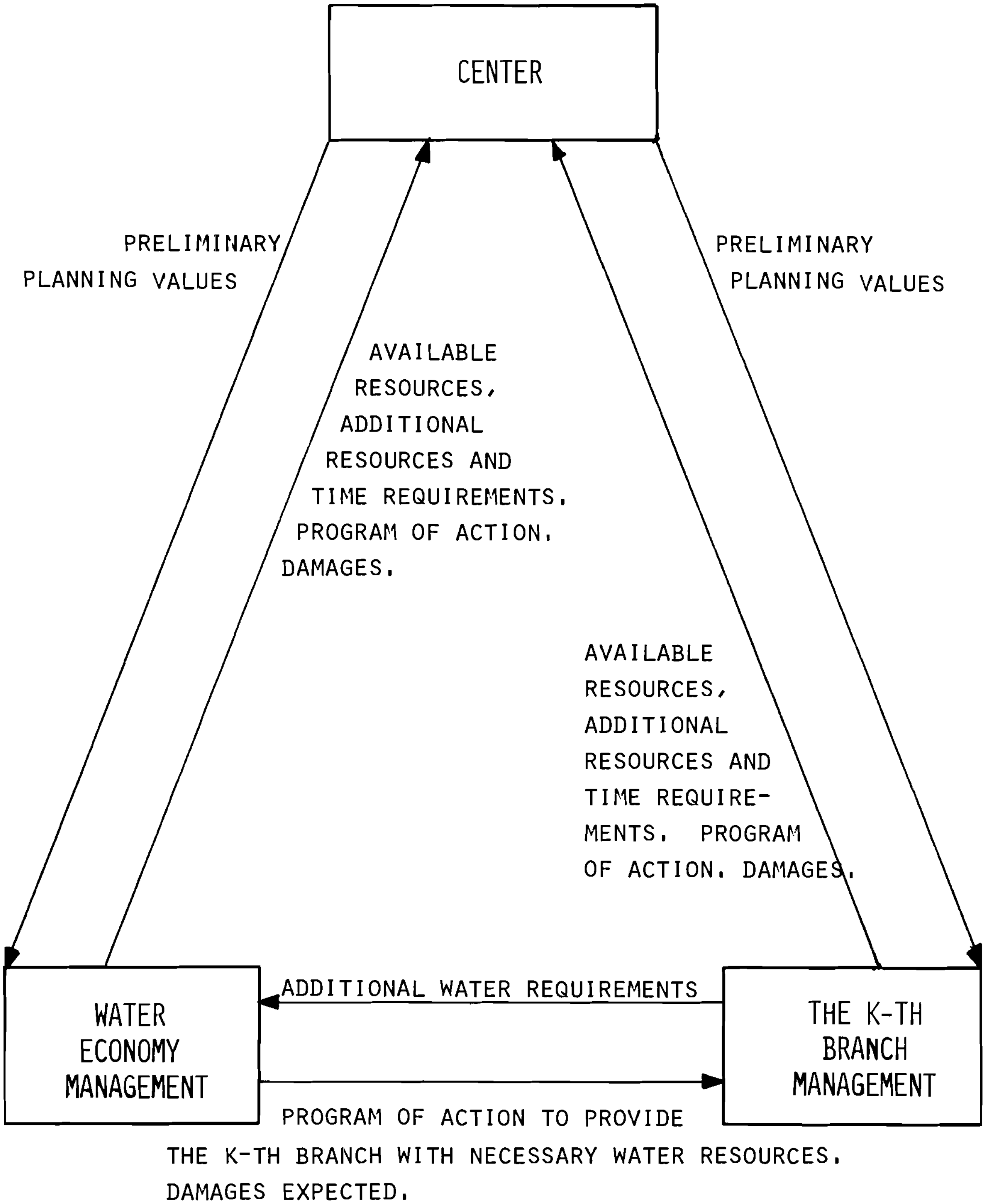



FIGURE 3. FIRST PLANNING ITERATION

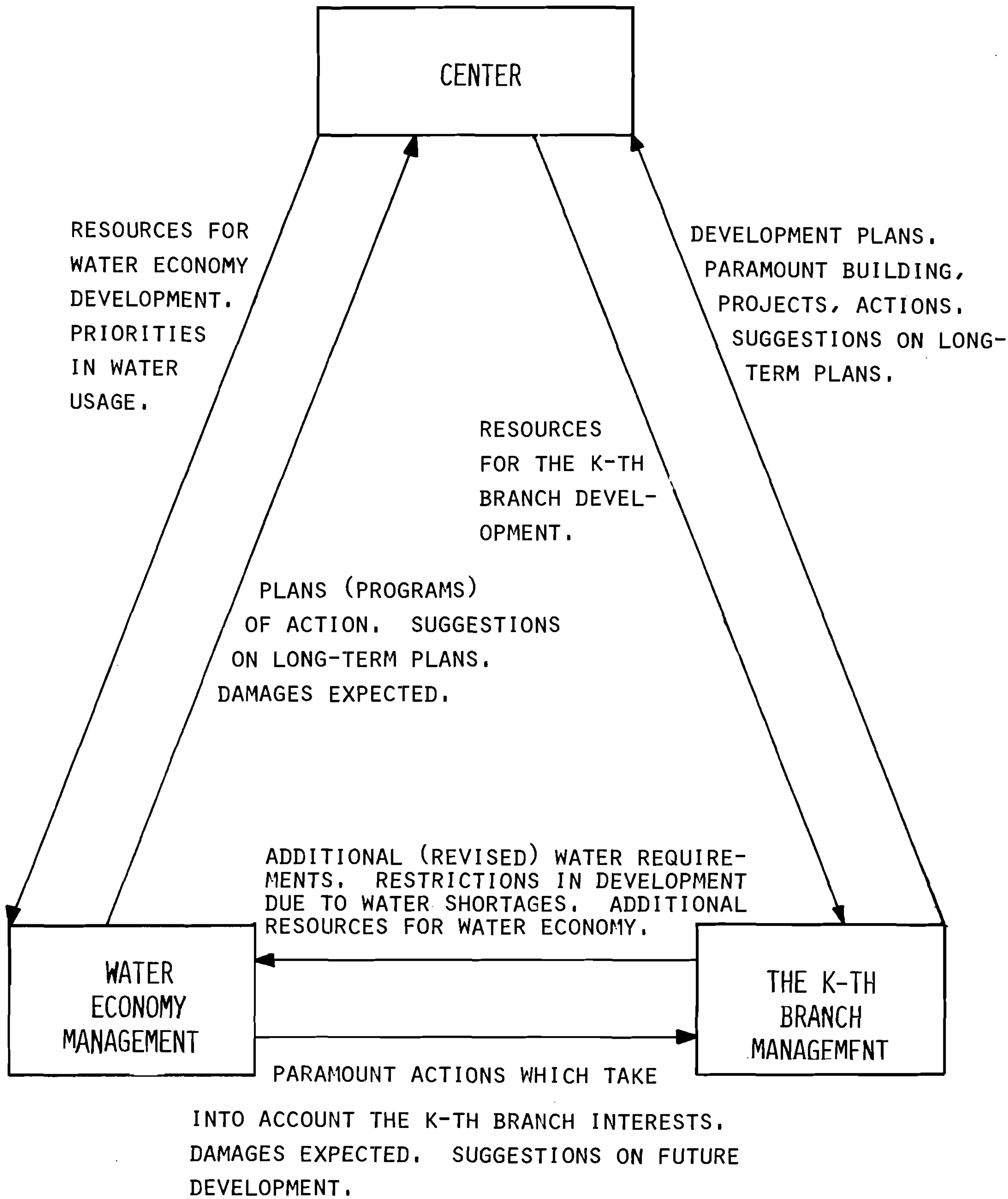



FIGURE 4. 2ND ITERATION OF PLANNING

At the third planning iteration, the Center may approve development plans for the branches, if these plans are sufficiently balanced and if they assure the Center's planning values and satisfy its social and political goals. Otherwise the Center revises preliminary plans with respect to its allocation among branches. In other words, the Center supplies all branches with information on re-examined resource distribution and development plans.

If the revised plan is accepted, the branches begin the plan execution, i.e. expansion of existing structures and building of planned structures, etc. In particular, they may change their operational policies for existing structures. In this case the branches give information to the Center about plan performance, amount and quality of production, profits, and damage which occurs due to insufficient branch development.

If the plan is not accepted by the Center, the second planning iteration should be repeated until all planning values are completely in balance with the resources required. Note that during this process suggestions on long-term planning become more and more precise. During projecting and designing of the construction, expansion and exploitation of plants and other structures, this information is transformed by the Center into a perspective plan for future planning periods. One of the most important points in the process is damage evaluation and comparison of resources spent with outputs.

Note that repetition of the second phase will lead at least to plan acceptance, and the planning procedure will be completed.

The planning procedure described has to be repeated for each planning period. Figure 5 illustrates Center and branch interaction at the third planning iteration.

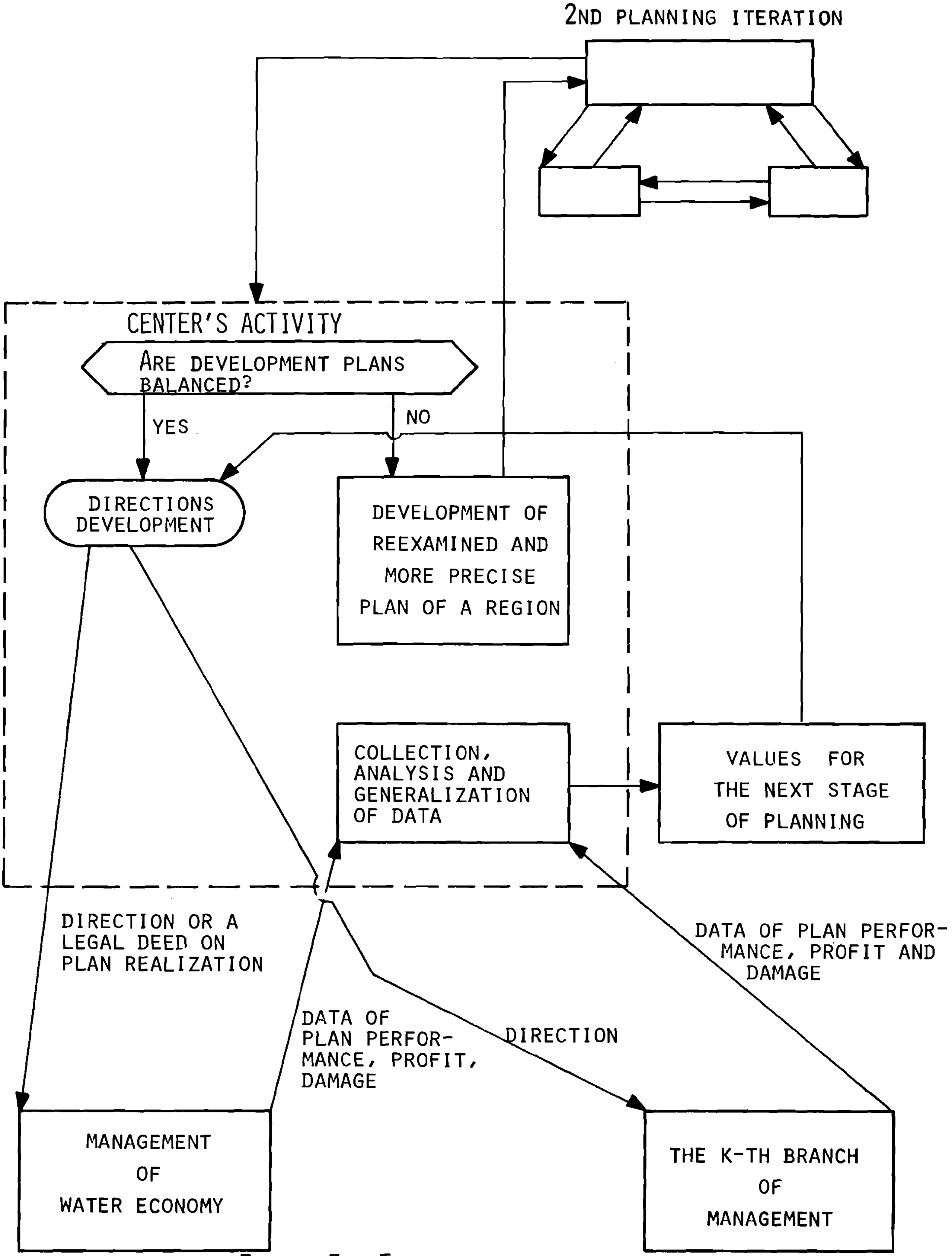



FIGURE 5. THIRD PLANNING ITERATION

## B. Formalized Description of Models

### 1. First phase of planning

As mentioned above, the first phase of planning is formulating all feasible (or conceivable) alternatives of water economy development. It is not a simple problem. We should note that in designing water structures and planning water management activities, particularly in the USSR, there is a certain sequence of preparing decisions. One of the obligatory components of this sequence is making a scheme of the complex use and protection of water resources of a river basin or a larger physical-geographical (economic) area. Such a scheme always contains the description of all feasible (at the moment of its conception) water management activities, the location and basic parameters of water management structures (reservoirs, navigable and drainage canals, artificial banks, etc.). Moreover, there are often more or less well founded assumptions about the possible sequence and timing of water management activities and of putting some structures into operation.

Actually, not all the scheme data are necessary in our approach. The composition and amount of minimally necessary data depend on the detail in which the concrete water economy processes are considered (on the established model level of aggregation of the real system). Thus, the first phase is an informal procedure realized by a group of water economy specialists. As a result, sets of actions will be formulated for each alternative, including the construction of reservoirs, canals, conduits, artificial banks, etc., which are to be taken within the planning period. Some of these actions can be interrelated by priority. Thus in a general case, to each alternative of a given regional water economy development corresponds a network of activities which will lead to achieving the goals of a given branch. Standard requirements for time and resources are given for each network activity which is indispensable for realization of the total.

## 2. Second phase of planning

The second phase of planning is calculating program alternatives. In this phase we have alternative versions of the development program of water economy structures, as well as information on receiving resources for the Center. It is necessary to distribute the available resources over time so that the program can be implemented in the best way from the point of view of the WEM. For this purpose we will use a dynamic model of program performance [3].

The dynamics of construction of water economy structures is described by a system of equations:

$$z(t + 1) = z(t) + u(t) \quad . \tag{1}$$

Here $z(t)$ is the vector of the job amount accomplished by time $t$ (phase variable in the model). For instance, if the construction of a hydropower unit consists of four stages, the equality 1/4 of the corresponding component of vector $z$ will correspond to the termination of the first construction phase. $u(t)$ is the intensity of job performance, in other words, the portion of the job carried out during period $[t,t + 1]$. $u(t)$ is the control in the model. If within the period $[t,t + 1]$ none of the water economy units is constructed, then $u(t) = 0$. We agreed on the condition that the process step is equal to one. In principle, it may be assumed equal to any positive quantity and is determined only by the specific time of the processes at the chosen aggregation level. It may be a day, a week, a month, a year, and so on.

The initial state is the volume of construction jobs accomplished by the beginning of the period of planning,

$$z(t_0) = z_0 \quad . \tag{2}$$

The job is regarded as completed by time $t$ if

$$z_j(t) = 1 \quad . \tag{3}$$

Technological constraints on the sequence of activities can be represented by a network (US type of representation, see the example in Figure 6.) or analytically as follows [4]:

$$u(t) \in v(z(t),t) \quad , \tag{4}$$

where $v(z(t),t)$ is a set of feasible controls at moment $t$ and under the obtained state $z(t)$. For example, if the activity network can be represented by Figure 6, then

$$v(z(t),t) \begin{cases} u_1(t) \geq 0, \qquad u_2(t) \geq 0 \\ u(t) \begin{cases} = 0 \text{ if } z_1(t) < 1 \text{ or } z_2(t) < 1 & ; \\ \geq 0 \text{ if } z_1(t) = 1 \text{ and } z_2(t) = 1 & ; \end{cases} \\ u(t) \begin{cases} = 0 \text{ if } z_1(t) < 1 \text{ or } z_2(t) < 1 & ; \\ \geq 0 \text{ if } z_1(t) = 1 \text{ and } z_2(t) = 1 & . \end{cases} \end{cases}$$

As the resources allocated by the Center for water economy activity or the intensity of their inflow are limited, these limitations must be present in our model. They are written in the following way:

$$\sum_{j=1}^{N} Q_j(z(t),u(t)) \leqq C(t) \quad , \tag{5}$$

where $Q_j$ is the vector of standard resource consumption by activity $j$ with the given state of the program $z(t)$ and the intensity of construction $u(t)$. $C(t)$ is the vector of the resources allocated for realizing the program.

Parallel with program formulation, an objective function must be formulated to evaluate the quality of program performance. The objective function reflects the preference

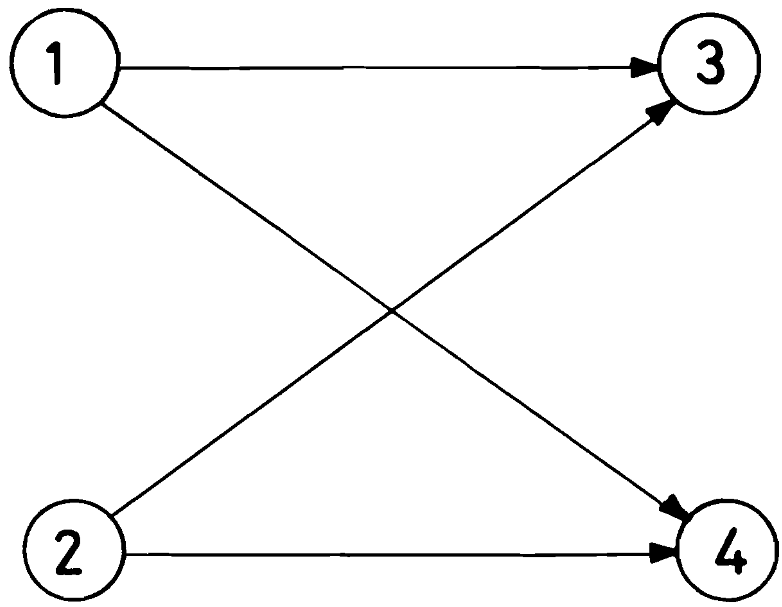



FIGURE 6. SIMPLE ACTIVITY NETWORK.

of regional management, perhaps including political, social, and other aspects. Generally speaking it is not confined to estimating damages caused by the insufficient development of water structures. One of the simplest objective functions of this kind is the time of program completion. Minimization of the time of program completion is justified, on the one hand by the branch's desire to achieve economic and social goals as soon as possible, and, on the other hand, by the need to reduce the damage the region may incur because of insufficient development. And indeed, the cummulative damage suffered by the region because of the absence of such structures is a nondecreasing funct·on of time. It applies equally to the probability of disasters (flood, droughts, etc.).

Thus, choosing the shortest plan of activity performance, the management insures itself to a certain extent against disasters and minimizes total damage. As we noted, time may not be the only feasible objective function when choosing a plan for realization of the given development alternative. One can use more sophisticated criteria and their superpositions ([5] - [7]).

The chcice of objective function is determined by the peculiarity of the given region and is one of its specifications. In the end, as was mentioned above, it is determined by the preferences of regional management.

As a result of the second phase, optimal schedules of water economy activities will be obtained for each alternative, i.e., vectors will be determined:

$$\xi^{(i)}(t) = (\{{}^{(i)}t_0{}^j\} \quad , \quad \{{}^{(i)}t_f{}^j\} \quad , \quad \{{}^{(i)}\tilde{u}^j(t)\} \quad , \quad \{{}^{(i)}\tilde{C}^k(t)\}) \quad ,$$

where

$^{(i)}t_o{}^j$ ($^{(i)}t_f{}^j$) = start (finish) time of activity j of program variant i.
$i = 1,\dots,L$ , $j = 1,\dots,N_i$ ;

L = total number of alternative programs;

$N_i$ = total number of activities in program i;

$^{(i)}\tilde{u}^j(t)$ = optimal intensity of activity j in program i at moment t;

$^{(i)}\tilde{C}^k(t)$ = resources of the k-th kind consumed by program i at moment t.

It is clear that

$$^{(i)}\tilde{C}(t) = \sum_{j=1}^{N} Q_j(^{(i)}z(t), ^{(t)}u(t), t) \quad ,$$

and

$$^{(i)}\tilde{C}^k(t) \leq C^k(t) \quad .$$

The case when $^{(i)}\tilde{C}^k(t) < C^k(t)$ means that at moment t a resource of the k-th kind is not being completely utilized and can, without prejudice to this branch program, be used in other branches of the region.

3. The third phase of planning

The third phase is estimation of damage under the given dynamics of branch development $\xi^{(1)}(t)$ for each variant $(1 = 1,2,\dots,L)$.

To estimate the damage we will use one of the network or linear models of runoff control, for example [8] or [9]. The simplest model can be written in the following way (below we will omit 1 corresponding to the number of the alternative).

The river system is represented as a network (without cycles). The nodes of the network are separate cross-sections of the river, and its tributaries on which water intake units (such as urban areas, irrigation canals, reservoirs) are situated. The arcs connecting the nodes are marked by arrows which show the direction of water flow (see Figure 7).

The equation of the system dynamics is as follows:

$$w_i(t + 1) = w_i(t) + \sum_{\ell \varepsilon \Gamma_i^-} F_{\ell i}(t) - \sum_{k \varepsilon \Gamma_i^+} F_{ik}(t) + F_{oi}(t) - F_{io}(t) \quad ; \tag{6}$$

This is a water balance equation at node (i), where

$w_i(t)$ = total volume of water in element i (phase variables in the model);

$F_{ij}(t)$ = volume of water running from element i (the network node) to element j during time $[t,t + 1]$ (controls in the model);

$F_{io}(t)$ = water withdrawal in element i within the interval $[t,t + 1]$ (transmission of water to another branch; controls in the model);

$F_{oi}(t)$ = water inflow into element i from surface and underground inflow, and precipitation (random factors);

$\Gamma_i^-$ = set of element (cross-sections, reservoirs, canals) which lie upstream;

$\Gamma_i^+$ = set of element into which water comes from element i.

The initial conditions are determined by the initial state of the basin at the initial moment of the planning period:

$$w_i(t_0) = w_{i,0} \quad . \tag{7}$$

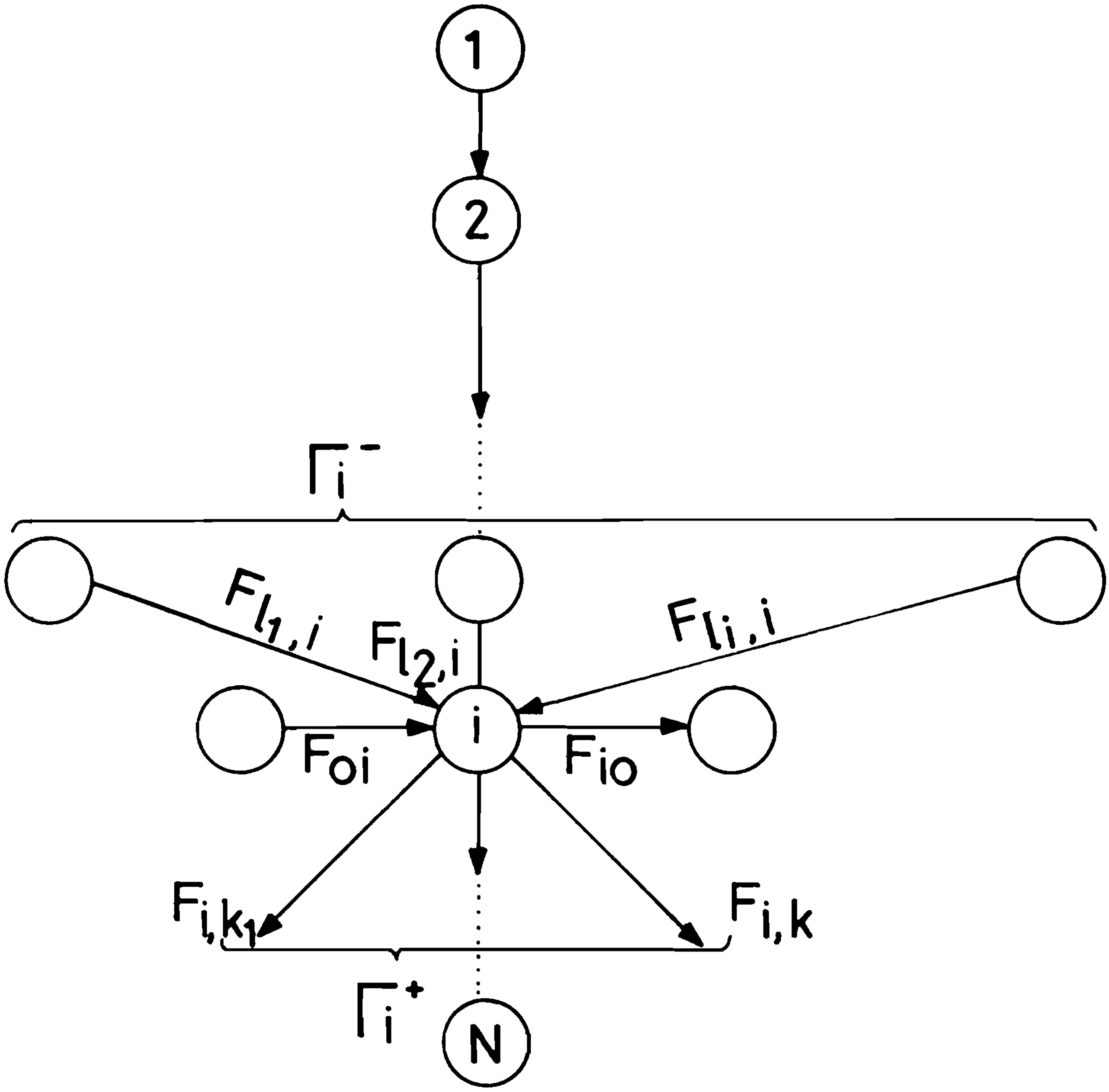



FIGURE 7. RIVER BASIN NETWORK DIAGRAM.

The values of $w_i(t)$ are limited by the maximum $(\bar{w}_i(t))$ and minimum $(\underline{w}_i(t))$ feasible capacity of water reservoir:

$$\underline{w}_i(t) \leq w_i(t) \leq \bar{w}_i(t) \quad . \tag{8}$$

If element i corresponds to a reach of a river or canal, then

$$w_i(t) = 0 \quad ,$$

as there is no accumulation of water in such reaches.

Besides, as canals (river reaches) have ultimate capacity there are necessarily corresponding constraints in the model:

$$0 \leq F_{ij}(t) \leq \bar{F}_{ij}(t) \quad , \tag{9}$$

where

$\bar{F}_{ij}(t)$ = minimum feasible capacity of canal or river reach ij at the t-th time step.

Dependence of quantities $\bar{w}_i$, $\underline{w}_i$ and $\bar{F}_{ij}$ on time simulates the development of water economy units over time during construction and expansion. These values directly depend on realization of the water economy development program. For example, if at moment t the construction of reservoir i has not yet begun, then $\bar{w}_i(t) = 0$. As construction is going on (the activities of program m, l, k are being performed), some units of the given reservoir are put into operation. These units are oriented towards the utilization of useful volumes; accordingly $\bar{w}_i^1$, $\bar{w}_i^2$, $\bar{w}_i^3$. This process can be described by the system of the following equations:

$$w_i(t) = \bar{w}_i^1\Theta(z^m(t) - 1) + \bar{w}_i^2\Theta(z^\ell(t) - 1) + \bar{w}_i^3\Theta(z^k(t) - 1) \quad ; \tag{10}$$

$$\bar{w}_i^1 + \bar{w}_i^2 + \bar{w}_i^3 = \bar{w}_i^* \quad ,$$

where $\bar{w}_i^*$ is the projected maximum capacity of reservoir i.

Relations similar to (10) could be written for $\underline{w}_i(t)$ and $F_{ij}(t)$.

Some processes related to floods can be easily taken into account in the model. Overflow occurs in canal i at moment t in case water inflow into reach i exceeds maximum feasible water outflow, i.e., the following condition holds:

$$\sum_{\ell \varepsilon \Gamma_i^-} F_{\ell i}(t) + F_{oi}(t) > \sum_{k \varepsilon \Gamma_i^+} \bar{F}_{ik}(t) + \bar{F}_{io}(t) \quad . \tag{11}$$

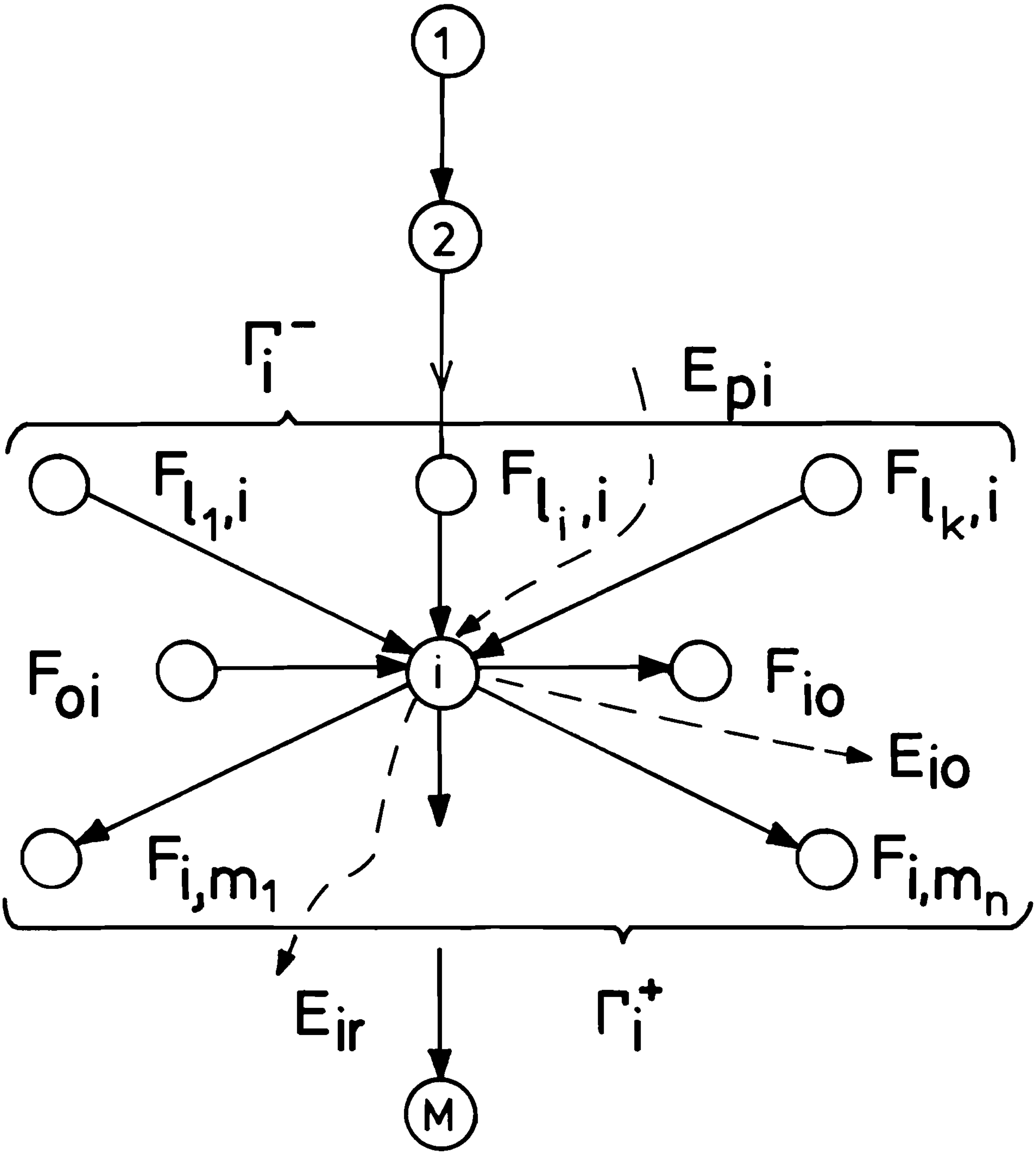
If an element corresponding to a reservoir is being considered, the condition

$$w_i(t) = \bar{w}_i(t) \tag{12}$$

should be added to (10). Superfluous water spreads about the environs of the given element in accordance with the relief. Assuming that the local relief does not change over time, we can introduce fictitious canals and reservoirs along which water spreads in case of flood (Figure 8). These could be connected with the river system elements lying both up- and down-stream.

Introducing the notations $E_{i\xi}(t)$ and $E_{\eta i}(t)$ for the amount of water running from element i to element $\xi$ and from $\eta$ to i at moment t as a result of overflow in this (or some other) reach, we obtain

$$\begin{aligned} &\sum_{\ell \varepsilon \Gamma_i^-} F_{\ell i}(t) + F_{oi}(t) - \sum_{k \varepsilon \Gamma_i^+} \bar{F}_{ik}(t) - \bar{F}_{io}(t) \\ &\qquad + \sum_{\eta \varepsilon \Gamma_i^-} \sum E_{\eta i}(t) \\ &\qquad = \sum_{\xi \varepsilon \Gamma_i^+} E_{i\xi}(t) + E_{io}(t) \quad . \end{aligned} \tag{13}$$




RIVER BASIN NETWORK DIAGRAM.

FIGURE 8

The above-mentioned fictitious canals are included in sets $\Gamma_i^-$, $\Gamma_i^+$ (also see Figure 8). The last item in the right part of (13) is irreparable losses of water caused by flood; in other words, it is the water which, having overflowed the lowland, partially stays there and evaporates. Note that in general irreparable losses are non-linear functions of $\sum_{\eta\varepsilon\Gamma_i^-} E_{\eta i}(t)$, $\sum_{\xi\varepsilon\Gamma_i^+} E_{i\xi}(t)$. $E_{io}(t)$ is determined by the amount of the water and the relief in the vicinity of the given river reach. The model described permits us to solve the problem of minimizing the damage with the given dynamics of water economy system development under conditions of flood and while other industries' demand for water is not sufficiently satisfied.

As there are random (uncontrolled) factors in the model (functions $F_{i0}(t)$, $t \varepsilon [0,T]$), problems of two kinds can be stated.

The first problem is searching for an optimum guaranteed strategy of short-range planning and operation management of a water economy system. In this case the objective function is as follows:

$$J_1(F) = \underset{\{F_{0i}(t)\}}{\mathrm{Max}}\ J(F) \qquad (14)$$
$$t = t_0, t_1, \ldots, T \ ;$$
$$i = 1, 2, \ldots, N \ ,$$

where

$$J(F) = \sum_{t=t_o}^{T} \left\{ \sum_{i=1}^{N} P_i^t \left( F_{\ell_1, i}(t), \ldots, F_{\ell_i, i}(t), \right. \right.$$

$$\left. F_{ik_1}(t), \ldots, F_{i,k_i}(t), F_{io}^*(t) - F_{io}(t) \right) \qquad (15)$$

$$\left. + \sum_{i=1}^{N} S_i^t \left( E_{\eta_1, i}(t), E_{i\eta_1}(t), E_{i\xi_1}, \ldots, E_{i\xi_i}, E_{io} \right) \right\} .$$

$P_i^t$ is a function which represents a penalty for irrational regulation of water (including hydro-energetics, navigation etc.) in reach i, and for not completely meeting consumer requirements. $F_{i0}^*(t)$ is a standard demand for water in reach i at moment t. $S_i^t$ is the damage caused by flood in the environs of reach i. Functions $P_i^t$ and $S_i^t$ can be defined in different ways; this is an independent problem not considered in the present paper.

Thus, the first problem is formulated in the following way:

$$J_1(F) \rightarrow \text{Min} \quad ,$$

under the constraints (6)-(9), (11)-(13).

The second problem is obtaining short-range managerial operating strategies for the water economy system which minimize the average damage (as far as realization sets $\{F_{0i}(t)\}$ are concerned). Let $J_2(F)$ be the mathematical expectation of damage with a given function of probability distribution for random quantities $F_{oi}(t)$, i = 1,...,N. Then the second problem is formulated as follows:

$$J_2(F) \rightarrow \text{Min} \quad ,$$

under the constraints (6)-(9), (11)-(13). In this case the distribution function can be constructed on the basis of statistical methods applied in hydrology (evaluation of distribution parameters [10], and simulation of stream-gauging rows [12]).

Despite the fact that the solution of the stochastic programming problem stated is very complicated and requires further development of probability theory methods and mathematical programming, one can use simple (heuristic) procedures for approximate evaluation of the damage. The following is such a procedure.

The distribution function given, $F_{oi}(t)$ $(i = 1,\ldots,N$ $t \in |t_o,T|)$ are generated. When the values of these quantities are obtained, a non-linear dynamic programming problem is solved:

$$J(F) \to \text{Min} \quad ,$$

subject to (6)-(9), (11)-(13).

$J(F)$ is determined by formula (15) and all $F_{oi}$ are fixed given functions. For $F_{oi}$ generated in this way, we find optimum control $F_{ij}(t)$, $i, j = 1,\ldots,N$, $t \in [t_o,T]$ and the corresponding value of the functional. Then the values of random quantities $F_{oi}$ are generated again and the process is repeated. Having performed many iterations one can take the least and the greatest values of the objective function (15) over the realization set as the lower and upper approximate damage estimations. Despite the obvious crudeness of this method the estimations obtained contain the information which characterizes possible damages and may prove to be sufficient for top management to make the decision.

As a result of the third phase one obtains the following specification for each alternative: estimation of damage (or benefit) with a given policy of investments $C(t)$ assigned by the Center. Besides, having made the calculations for each variant, one obtains information on the most critical (from the point of view of resources) time periods and on superfluous resources.

Thus, on the one hand, we are able to suggest to the Center various alternatives of the given water basin (branch) development, and on the other hand, we have constructive proposals for the improvement of the current situation if none of the suggested alternatives seems satisfactory to the Center.

## Conclusion

The problem of decision-making on rational investments in water economy is one of the most important in the economy of any modern country, especially in connection with the necessity of maintaining and improving the environment. On the other hand, the model suggested is general enough to be applied in a number of other sectors of the economy, particularly in those using limited natural resources.

The present paper is integrally related to the methodology of simulation system development, a technique suitable for designing a wide class of concrete systems of regional water resources use as well as procedures for choosing the rational project variant. The latter work is being carried on by the Water Resources Project of IIASA with the support of different national organizations. The aim is to investigate problems common to many applications and to elaborate a universal methodology of constructing and applying simulation systems. The major results of the research are given in [1]. Our work is not only functionally related to the models described in [1], but it also represents further development and application of a programmed approach to planning and managing complex water economy systems.

The procedure suggested can be used for decision-making now, as all the relevant models are developed and their software is at our disposal. However, we understand that many of the models could be considerably improved. It might, therefore be expedient to consider further research problems arising from the present work.

The first and probably the most difficult problem is "penetration" of formalized procedures into the first phase of preparing development alternatives for water economy system development. It applies primarily to the elaboration of forecast methods on the basis of aggregated models of system development.

The improvements relevant to the second phase concern multi-criterion optimization in problems of resource allocation over time and space with due regard for uncertainties of preliminary information.

The third phase of the procedure suggested requires perhaps the greatest effort in different areas: the development of models which describe loss calculation inherent in water economy system operation in urban areas and agriculture; development of a wider class of models describing the regional economy as a whole; and, finally, creation of efficient techniques for solving stochastic optimization problems.

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