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A RISK-ADVERSE APPROACH FOR RESERVOIR MANAGEMENT WITH APPLICATION TO LAKE COMO

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PREFACE

Analysis of problems concerned with the rational use of natural resources almost invariably deals with uncertainties with regard to the future behavior of the system in question and with multiple objectives reflecting conflicting goals of the users of the resource. Although effective mathematical tools have been made available during the last decades for solving such problems, there have only been few applications, even in the field of water resources, which is certainly the most developed one. The major reason for this is probably due to the fact that such mathematical tools are often quite abstract and sophisticated and are therefore of little help for the practitioners.

For these reasons, one of the issues addressed during the summer study "Real-time Management of Hydrosystems" organized by the Resources and Environment Area of IIASA in 1981, was the possibility of developing simple and heuristic methods for reservoir management that could directly take into account the experience and the preferences of the manager. The research was mainly conducted with reference to the case of Lake Como, for which substantial data were available. This paper describes a new approach towards operational management of a multipurpose reservoir, which explicitly takes into account the risk-adverse attitudes of the reservoir operator. An interesting comparison is made between operation rule developed this way and the other one developed earlier based on some stochastic optimal control concepts.

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ABSTRACT

A deterministic approach which avoids extreme failures in the management of a multipurpose reservoir is presented and discussed in the paper. The main feature of the method is to suggest a whole range of possible decisions which guarantee the efficient performance of the system. This allows the manager to choose the release which better fits with the additional informations or forecasts he might have, as well as to accommodate for secondary objectives which were not considered in the formulation of the problem. The results of the application of this approach to the management of Lake Como (Northern Italy) favourably compare with those obtained by a more traditional stochastic optimal control formulation and with the historical data.

CONTENTS

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1. INTRODUCTION

Actual operation of multipurpose reservoirs seems to prove that in most cases there is no great interest in optimizing the expected value of the objectives, as usually proposed in the literature (e.g. /1/,/4/, /5/). On the contrary, reservoir operation is very often aimed to avoid extreme and unacceptable failures of the objectives when the system is under stress. This is why reference is often made to very specific situations like "the most dry (wet) year of the century", "the highest recorded flood" and so on. In fact, it seems that managers, when selecting an operating rule, prefer to evaluate its performance by making reference to a particularly severe episode (or sequence of episodes), they have directly or indirectly experienced in the past. Consistently, in order to be safe, managers like to adopt that operating rule which best performs during that particular reference episode even if this entails a reduction of the average performance of the system.

Lake Como in Northern Italy is no exception. When the results of a detailed optimization study /3/, based on stochastic optimal control, have been presented to the manager, he recognized that he was not completely satisfied with the three objectives selected in that study (mean yearly agricultural deficit, average number of days of flood per year, and mean yearly hydroelectric production). Being risk-adverse, the manager showed a definite preference toward the possibility of avoiding failures of the system during severe and extreme hydrological episodes, like those he has experienced in the past.

This paper, which extends and applies some of the results illustrated by Orlovski et al./6/ for storage control problems, represents a first attempt to define operating rules which better account for this facet of the management attitude. The paper is organized in the following way. Next section describes the main physical, economic,and institutional features of Lake Como system. Sect. 3 introduces a deterministic (min-max) formulation of the risk-adverse management problem, while Sect. 4 briefly describes the application to Lake Como and compares the results with those obtained using the stochastic approach. The main characteristics of the min-max approach and some possible extensions are dealt with in the last section.

2. THE ACTUAL OPERATION OF LAKE COMO

Lake Como is a natural lake which drains a basin of 4508 km^2 in the central part of the Alps. It is operated as a multipurpose reservoir since the end of Wold War II and serves a number of downstream agricultural and hydroelectric users. The lake works as a seasonal reservoir with an annual cycle. It is filled during the snow-melt season(late spring-early summer) and emptied during the dry season (July-September) when water is needed for the irrigation of downstream areas. Then , it is filled again with autumn rains and slowly emptied during winter and spring for hydropower production. One of the main regulation problems is to prevent floods at the lake sites, particularly in the town of Como, which is the most densely populated area of the lake coast. At present the data necessary to develop a reliable cost-benefit analysis for the determination of the best operating rule are not available. Thus, the most natural approach is to model the problem as a multiobjective decision making process in which all benefits and damages are expressed in simple but representative units. The physical indicator selected to represent the satisfaction of the agricultural users is the total annual water deficit D expressed in millions cubic meters. A deficit situation occurs whenever the release from the lake during day τ

falls below the crop water demand w_τ, which is periodic during the year and obviously peaks in summer (see Fig. 1). The damages incurred by the municipality of Como (interruption of public services) can be indirectly quantified by the number F of days of flood per year, i.e. by the number of days in which the level of the lake exceeds that of the shore. Finally, as far as downstream hydropower production is concerned, a previous analysis (performed by Guariso et al./3/)has shown that it is rather insensitive to variations of the operating rule: for this reason this aspect of the problem will not be considered in the following.

When operating the regulation dam, the manager is constrained by a license act issued by the Ministry of Public Works. This act, agreed upon by all parties, states that the daily release r_τ can be freely selected whenever the lake level x_τ at the beginning of day τ is between two limits $\underline{x}$ and $\overline{x}$, which correspond respectively to -0.50m and 1.20m, as measured at the Fortilizio hydrometer. For this reason the interval $(\underline{x},\overline{x})$ will be called control range in the following. When the level of the lake reaches the lower limit $\underline{x}$ of the control range, the release r_τ must be equal to or smaller than the inflow a_τ so that the level does not decrease further (this constraint was imposed by the Ministry of Public Works to guarantee navigation and prevent sanitary problems). When, on the contrary, the level of the lake raises above $\overline{x}$, the manager must progressively open all the gates of the dam, in order to discharge as much water as possible, thus preventing too large floods on the lake shores.

A detailed statistical analysis(carried out by Garofalo et al./2/)has shown that the operation performed by the manager during the period 1946-78, can be satisfactorily approximated by an operating rule of the type

$$r_\tau^i = r(\tau, x_\tau^i, a_\tau^i)$$

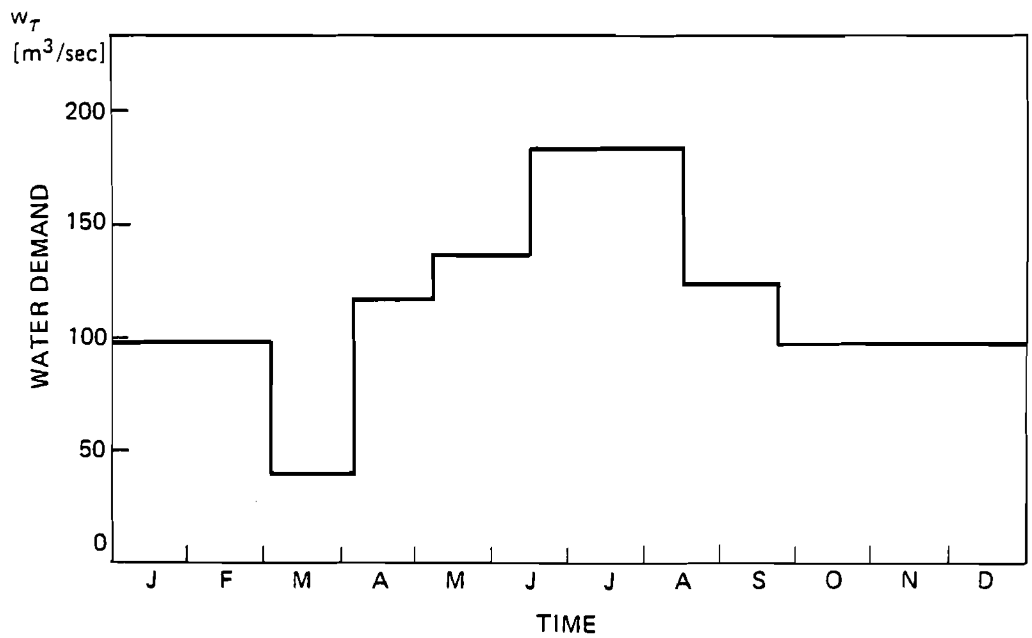

Figure 1. Seasonal variations of the agricultural water demand w_τ.

where x_τ^i represents the storage (level) of the lake at the beginning of day τ in year i, and r_τ^i and a_τ^i are the release and the inflow in the same day. The function r can be represented, for any particular day, as in Fig. 2 (for simplicity from now on the index i is omitted in the figures). The actual operating rule is thus increasing and convex with respect to x_τ^i in the control range, and is such that

$$0 \leq r\ (\tau, x_\tau^i, a_\tau^i) \leq S(x_\tau^i)$$

where $S(x_\tau^i)$ is the so-called stage-discharge function of the lake. This function gives, for any value of the level, the maximum amount of water which can be released in one day by keeping all the gates of the dam permanently open.

3. A MIN-MAX APPROACH

3.1 Problem formulation

The risk-adverse management problem will be formulated in this section as an optimal control problem, the solution of which guarantees a certain performance in terms of the objectives. The optimal operating rules $r(\cdot)$ will be selected by making explicit reference to their performance in particularly troublesome and specific situations formally defined by a set I of n one-year long daily inflow sequences, i.e.

$$I = \{\{a_t^i\}\ ;\ t=0,\ldots,364 \quad ; \quad i=1,\ldots,n\}$$

This reference set may contain recorded or synthetic sequences of inflows that the manager considers as particularly critical. For instance, in the case the reservoir is already in operation, one might consider as sequences of the reference set those corresponding to the most wet and dry

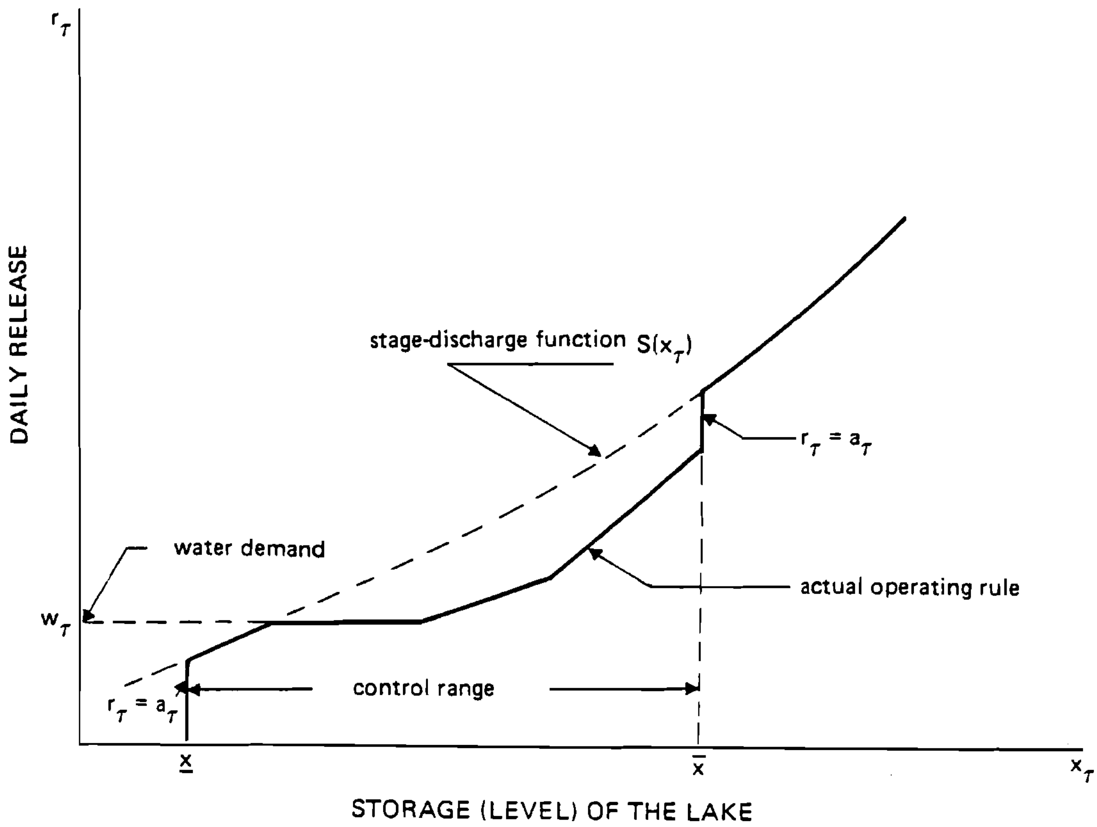

Figure 2. Actual operating rule of Lake Como.

years experienced by the manager. In doing so, the proposed operating rules may also be compared with the performance the manager was able to achieve in practice. Let us now indicate with D^i and F^i the water deficit in agriculture and the number of days of flood obtained by applying an operating rule $r(\cdot)$ during year $i(i=1,\dots,n)$ of the reference set. The value of D^i is the sum over year i of the daily water deficits d^i_τ given by

$$d^i_\tau = \begin{cases} 0 & \text{if } r^i_\tau \geq w_\tau \\ w_\tau - r^i_\tau & \text{if } r^i_\tau < w_\tau \end{cases}$$

Consistently, F^i is the sum over year i of the flood indicators f^i_τ given by

$$f^i_\tau = \begin{cases} 0 & \text{if } x^i_\tau \leq x_c \\ 1 & \text{if } x^i_\tau > x_c \end{cases}$$

where x_c is the level at which there are the first detectable flood damages (in our case the level of the main square of Como). Thus the problem can be formulated as a two-objective optimal control problem, in which the highest water deficit ($\max_i D^i$) and the highest number of days of flood ($\max_i F^i$) are minimized, that is

$$\min_{\{X_0, r(\cdot)\}} \left| \max_{1\leq i\leq n} D^i \qquad \max_{1\leq i\leq n} F^i \right| \tag{1}$$

where X_0 is a set of initial storages with non-empty intersection with the control range. The constraints of the problem are:

the continuity equation of the lake

$$x^i_{t+1} = x^i_t + a^i_t - r^i_t \qquad t = 0,\ldots, 364 \quad i = 1, \ldots,n \qquad (2)$$

the physical constraint

$$0 \leq r^i_t \leq S(x^i_t) \qquad t=0,\ldots,364 \qquad i = 1, \ldots,n \qquad (3)$$

the legal constraints

$$x^i_t \geq \underline{x} \qquad t=0,\ldots,364 \qquad i=1,\ldots,n \qquad (4a)$$

$$r^i_t = S(x^i_t) \quad \text{if } x^i_t > \overline{x} \qquad t=0,\ldots,364 \qquad i=1,\ldots,n \qquad (4b)$$

and the terminal constraint

$$x^i_{365} \in X_0 \qquad i = 1,\ldots,n \qquad (5)$$

This last constraint is imposed in order to avoid that a good performance in one year is followed by a very poor performance in the next year.

A feasible solution of problem (1-5) is a set X_0 $(X_0 \cap [\underline{x},\bar{x}] \neq \emptyset)$ of initial storages and an operating rule r(.) satisfying constraints (2-5). In general, the operating rule r(.) will be a function of the information currently available, i.e.

$$r^i_\tau = r(\tau,\ x^i_\tau,\ a^i_\tau,\ D^i_\tau,\ F^i_\tau) \qquad (6)$$

where $D^i_\tau = \sum_0^{\tau-1}{}_t\ d^i_t$ is the current water deficit in agriculture, and $F^i_\tau = \sum_0^{\tau}{}_t\ f^i_t$ is the current number of days of flood. A feasible solution $(X_0, r(.))$ is said to be efficient (or non dominated) if all other feasible solutions have at least one objective with a worse value.

In order to solve problem (1-5) we will first analyze two simpler problems. The first one (see Sect. 3.2) is called satisfaction of demand and consists of determining sets $X_0^{D^*}$ of initial storages, and operating rules of the form

$$r^i_\tau = r(\tau,\ x^i_\tau,\ a^i_\tau,\ D^i_\tau\ ,\ D^*) \qquad (7)$$

which are such that constraints (2-5) are satisfied and all yearly water deficits D^i are bounded by a given value D^*, i.e.

$$D^i \leq D^* \qquad i = 1, \ldots, n \qquad (8)$$

The second problem (see Sect. 3.3) is called flood protection and consists of finding sets $X_0^{F^*}$ of initial storages, and operating rules of the form

$$r^i_\tau = r(\tau,\ x^i_\tau\ ,\ a^i_\tau\ ,\ F^i_\tau,\ F^*) \qquad (9)$$

which can guarantee that the number of days of flood at the end of all reference years will be at most equal to a given value F^*, i.e.

$$F^i \leq F^* \qquad i = 1, \ldots, n \qquad (10)$$

The solutions of the two above problems will automatically point out (see Sect. 3.4) operating rules of the more general form

$$r_\tau^i = r(\tau,\ x_\tau^i,\ a_\tau^i,\ D_\tau^i,\ F_\tau^i,\ D^\star,\ F^\star) \qquad (11)$$

which can guarantee the satisfaction of both constraints (8) and (10) at the same time. Among this set of operating rules, a very simple procedure (see Sect. 3.5) will determine those which can guarantee the minimum value of $F^\star$(say F^0) for any given value of $D^\star$ (say D^0). These operating rules obviously solve the multiobjective problem (1-5) and are therefore efficient in the sense specified above. In general, these efficient operating rules are not unique. Thus, given the current value of information $(\tau,\ x_\tau^i,\ a_\tau^i,\ D_\tau^i,\ F_\tau^i)$ the solution algorithm suggests a whole range of possible releases r_τ^i. This means that, in normal conditions, the manager has still a certain freedom in making the final decision. He might, for instance, take into account secondary objectives which were neglected in the formal description of the problem. However, we will see that when hydrological conditions become critical, i.e. when the reservoir is almost empty or full, this freedom might disappear.

3.2 Satisfaction of demand

Let us now consider the problem of demand satisfaction, namely the problem of determining a set of initial storages $X_0^{D^\star}$ $(X_0^{D^\star} \cap [\underline{x},\bar{x}] \neq \emptyset)$ and a set of operating rules of the form (7) which can gaurantee the satisfaction of constraint (8). Obviously, solutions to this problem exist at least for sufficiently high values of $D^\star$. One of these solutions is the so-called minimum release policy which corresponds to discharge during day τ an amount of water which is as close as possible to the agricultural water demand w_τ. Taking into account all physical and legal constrains, it is easy to check that such a policy is given by (see Fig.3)

$$r_{min}(\tau, x_\tau^i, a_\tau^i) = \begin{cases} \min\{a_\tau^i, S(\underline{x})\} & \text{if } x_\tau^i = \underline{x} \\ S(x_\tau^i) & \text{if } \underline{x} < x_\tau^i < S^{-1}(w_\tau) \\ w_\tau & \text{if } S^{-1}(w_\tau) \leq x_\tau^i < \bar{x} \\ \min\{\max(a_\tau^i, w_\tau), S(\bar{x})\} & \text{if } x_\tau^i = \bar{x} \\ S(x_\tau^i) & \text{if } x_\tau^i > \bar{x} \end{cases} \qquad (12)$$

where $S^{-1}(\cdot)$ denotes the inverse of the stage-discharge function. Notice that the minimum release policy does not fully exploit the information currently available since it does not depend upon D_τ^i and $D^\star$.

The set $X_0^{D^\star}$ corresponding to the operating rule (12) can easily be determined by recognizing that the yearly water deficit in agriculture obtained by applying a given operating rule is a non increasing function of the initial storage x_0. Thus, the set $X_0^{D^\star}$ will have the form

$$X_0^{D^\star} = \{x_0 : x_0^D(D^\star) \leq x_0\} \qquad (13)$$

where the lower limit $x_0^D(D^\star)$ is the solution of the following mathematical programming problem.

Problem 0

$$x_0^D(D^\star) = \min x_0 \qquad (14)$$

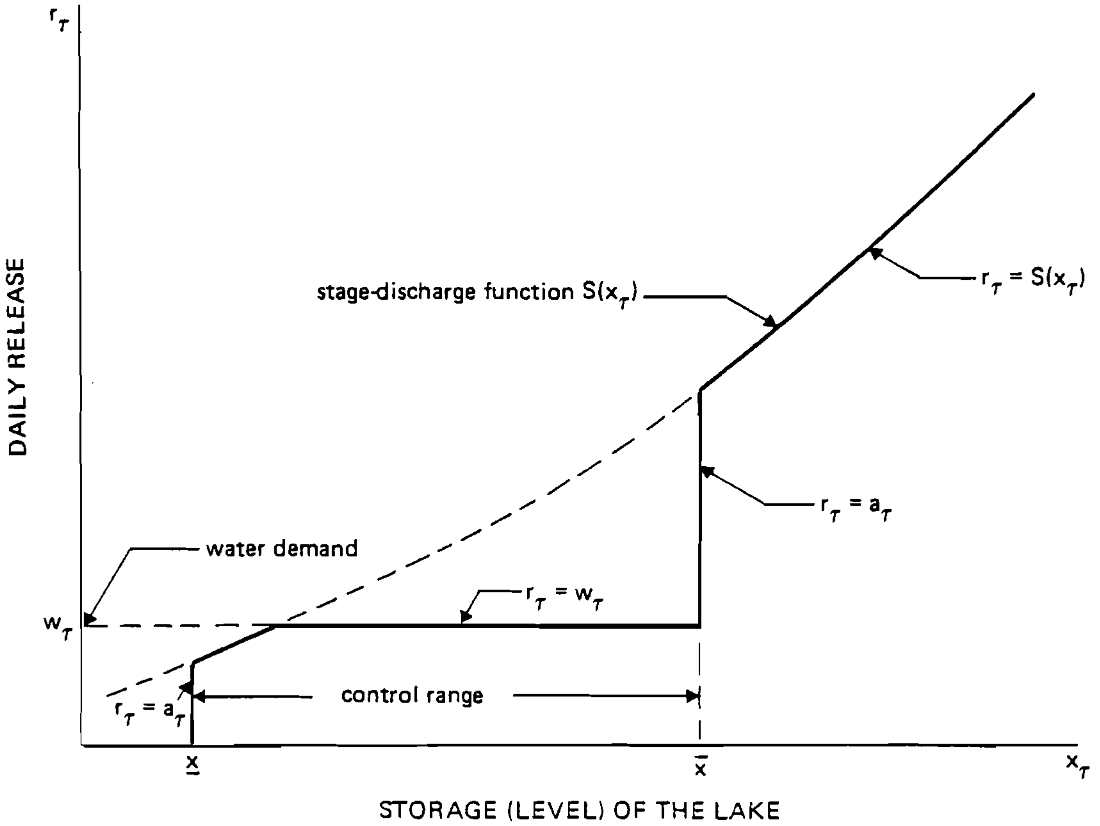

Figure 3. The minimum release policy $r_{min}(\tau, x_\tau, a_\tau)$.

subject to

$$x_0^i = x_0 \leq \bar{x} \qquad i = 1,\ldots,n \tag{15}$$

$$x_{t+1}^i = x_t^i + a_t^i - r_{min}(t, x_t^i, a_t^i) \quad t = 0,\ldots,364 \quad i = 1,\ldots,n \tag{16}$$

$$\sum_{0}^{364} {}_t \, d_t^i \leq D^{\star} \qquad i = 1,\ldots,n \tag{17}$$

$$x_0 \leq x_{365}^i \qquad i = 1,\ldots,n \tag{18}$$

The solution of Problem 0 can simply be found by recursively simulating the behaviour of the lake with $r_t^i = r_{min}(\cdot)$ for different values of the initial level x_0. If, at the end of a simulation, constraints (17) and/or (18) are not satisfied, x_0 must be increased before performing the next simulation. If, on the contrary, Eqs. (17) and (18) are satisfied with the strict inequality sign, x_0 must be decreased. Thus, a very simple one-dimensional searching procedure (e.g. bisection) can be used to determine $x_0^D(D^{\star})$.

Operating rules $r(\tau, x_\tau^i, a_\tau^i, D_\tau^i, D^{\star})$ satisfying constraint (8) can be found by noticing that the release during day τ must guarantee that the water deficit for the rest of the year will not exceed $(D^{\star} - D_\tau^i)$ and the terminal storage x_{365}^i will fall within the set $X_0^{D^{\star}}$. But this, in turn, can be accomplished provided that the level of the lake at the beginning of day τ does not drop below a minimum value, denoted by $x_\tau^D(D^{\star}, D_\tau^i)$, which can be computed by solving the following mathematical programming problem.

<u>Problem τ</u> (τ= 1, ..., 364)

$$x^D_\tau (D^\star, D^i_\tau) = \min x_\tau \tag{19}$$

subject to

$$x^i_\tau = x_\tau \qquad i = 1,\ldots,n \tag{20}$$

$$x^i_{t+1}=x^i_t+a^i_t-r_{min}(t,x^i_t,a^i_t) \quad t=\tau,\ldots,364 \quad i=1,\ldots,n \tag{21}$$

$$\sum_{\tau}^{364}{}_t\, d^i_t \leq D^\star - D^i_\tau \qquad i = 1,\ldots,n \tag{22}$$

$$x^D_0(D^\star) \leq x^i_{365} \qquad i = 1,\ldots,n \tag{23}$$

This problem can be solved by the same one-dimensional searching scheme used for Problem 0. Note, however, that the solution of Problem τ requires the knowledge of the value $x^D_0(D^\star)$ (see Eq. (23)). Thus, Problem 0 must be solved first, while all other Problems τ (τ= 1,...,364) are independent one from each other.

Finally, one can notice that a volume of water greater than the current demand w_τ can be released without any consequence on the management performance, provided the lake is sufficiently full and/or the inflow is sufficiently high. In fact, if

$$x^i_\tau + a^i_\tau - x^D_{\tau+1} (D^\star, D^i_\tau) \geq w_\tau \tag{24}$$

any release r^i_τ between w_τ and $x^i_\tau + a^i_\tau - x^D_{\tau+1} (D^\star, D^i_\tau)$ will leave the current value of the water deficit unchanged (in fact if $r^i_\tau \geq w_\tau$, then $D^i_{\tau+1} = D^i_\tau$). Moreover, if the release r^i_τ is lower than or equal to $x^i_\tau + a^i_\tau - x^D_{\tau+1}(D^\star,D^i_\tau) = x^i_\tau + a^i_\tau - x^D_{\tau+1}(D^\star,D^i_{\tau+1})$, it will generate a storage $x^i_{\tau+1} = x^i_\tau + a^i_\tau - r^i_\tau \geq x^D_{\tau+1}(D^\star, D^i_{\tau+1})$, which is indeed (by definition) the minimum value of $x^i_{\tau+1}$, which can guarantee the satisfaction of the objectives (see Eqs.(19-23)). On the contrary, if the lake is so empty that a release $r^i_\tau \geq w_\tau$ is infeasible, then an amount $r_{min}(\tau, x^i_\tau, a^i_\tau)$ of water is discharged and the current value of the deficit is updated. All this can be summarized (see also Fig. 4) by saying that any release r^i_τ such that

$$r_{min}(\tau, x^i_\tau, a^i_\tau) \leq r^i_\tau \leq \min \{S(x^i_\tau), \max [x^i_\tau + a^i_\tau - x^D_{\tau+1}(D^\star, D^i_\tau), r_{min}(\tau, x^i_\tau, a^i_\tau)]\} \quad (25)$$

will satisfy constraints (5) and (8). In other words, given the current information $(\tau, x^i_\tau, a^i_\tau, D^i_\tau)$ and the required performance $D^\star$, all operating rules which satisfy Eq. (25) will guarantee that $x^i_{365} \in X^{D^\star}_0$ and $D^i \leq D^\star$ for all years i. Fig. 4 shows that for sufficiently high values of the storage x^i_τ this implies the existence of a whole interval of feasible releases r^i_τ. On the contrary, if the lake is too empty Eq. (25) suggests a unique value for the release, namely $r_{min}(\tau, x^i_\tau, a^i_\tau)$.

3.3 Flood protection

The problem of flood protection consists of finding a set $X^{F^\star}_0$ of initial storages and a set of operating rules of the form (9) satisfying constraint (10) for a given value of $F^\star$. In order to solve this problem we follow the same approach outlined in Sect. 3.2. Therefore, we first introduce the maximum release policy

$$r_{max}(x^i_\tau, a^i_\tau) = \begin{cases} \min [a^i_\tau, S(\underline{x})] & \text{if } x^i_\tau = \underline{x} \\ S(x^i_\tau) & \text{if } x^i_\tau > \underline{x} \end{cases} \quad (26)$$

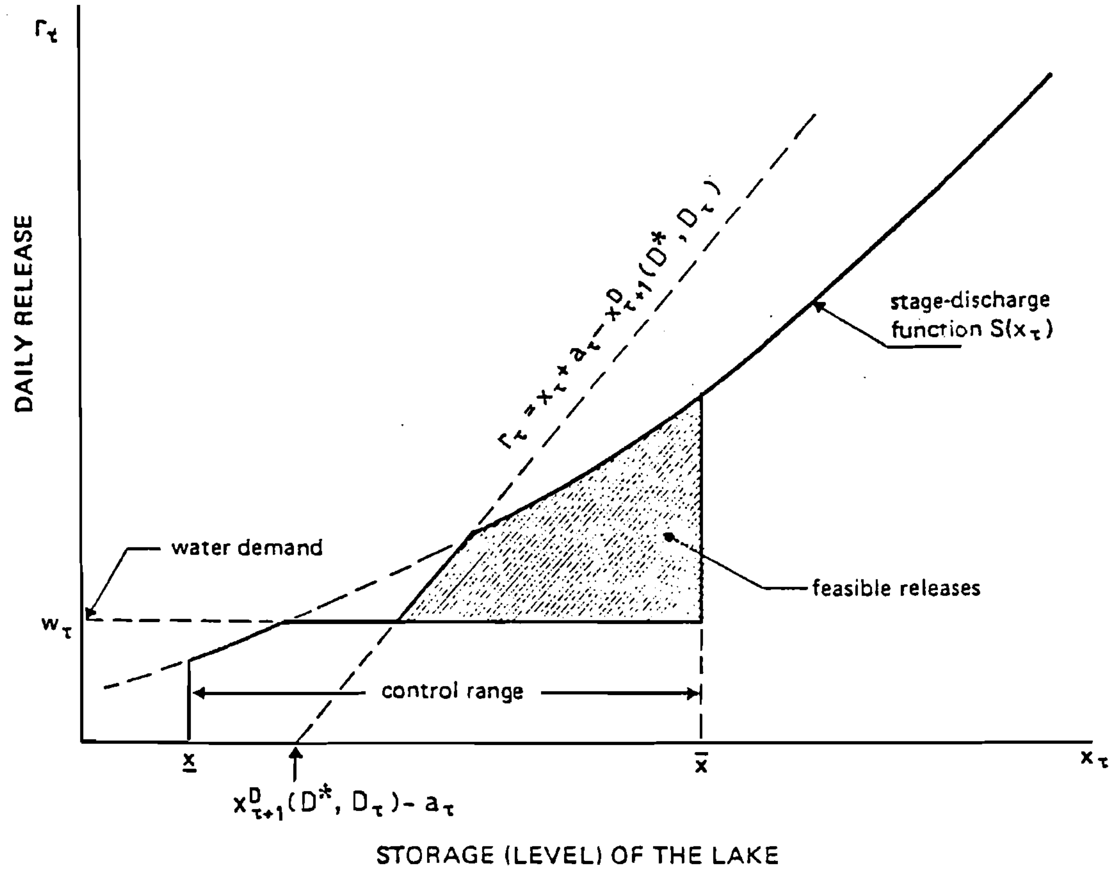

Figure 4. The set of releases r_τ which guarantee an agricultural deficit smaller than or equal to D^*.

which obviously minimizes the floods. Then, we notice that the number of days of flood F^i is a non decreasing function of the initial storage x_0, so that the set $X_0^{F^\star}$ is of the form

$$X_0^{F^\star} = \{ x_0 : x_0 \leq x_0^F (F^\star)\} \qquad (27)$$

where the upper limit $x_0^F(F^\star)$ can be obtained by solving a mathematical programming problem similar to the above Problem 0. Then, the value $x_0^F(F^\star)$ is used to formulate Problem τ ($\tau = 1,\ldots,364$) which specifies the maximum storage at time τ, called $x_\tau^F(F^\star, F_\tau^i)$, for which there exist operating rules of the form (9) which can guarantee no more than $(F^\star - F_\tau^i)$ days of flood during the rest of the year and the satisfaction of the terminal constraint $x_{365}^i \in X_0^{F^\star}$. These storages $x_\tau^F(F^\star, F_\tau^i)$, $\tau = 1,\ldots,364$ allow to define a lower limit to the daily release from the reservoir. More precisely, one must notice that the release r_τ^i can be smaller than $r_{max}(x_\tau^i, a_\tau^i)$ provided the lake is sufficiently empty and/or the inflow is sufficiently low. In fact, if

$$x_\tau^i + a_\tau^i - x_{\tau+1}^F(F^\star, F_\tau^i) \leq S(x_\tau^i) \qquad (28)$$

and

$$x_{\tau+1}^F(F^\star, F_\tau^i) < x_c \qquad (29)$$

(recall that x_c is the threshold level defining the flood), then any release r_τ^i between $x_\tau^i + a_\tau^i - x_{\tau+1}^F(F^\star, F_\tau^i)$ and $S(x_\tau^i)$ will not give rise to a flood since $x_{\tau+1}^i \leq x_c$ (i.e. $F_{\tau+1}^i = F_\tau^i$). Moreover, the same release will generate a storage $x_{\tau+1}^i$ smaller than or equal to $x_{\tau+1}^F(F^\star, F_{\tau+1}^i)$, which is indeed the maximum value of $x_{\tau+1}^i$, that can guarantee the satisfaction of constraint (10). Conversely, whenever inequality (28) is not satisfied, the release is set to the maximum feasible value $S(x_\tau^i)$ and the value of $F_{\tau+1}^i$ is suitably updated. These observations can be summarized (see also Fig. 5) by saying that any release r_τ^i such that

$$\min\{r_{max}(x_\tau^i,\ a_\tau^i),\ \max\ [x_\tau^i+a_\tau^i-x_{\tau+1}^F(F^\star,\ F_\tau^i),\ 0]\}\leq r_\tau^i\leq\ r_{max}(x_\tau^i,\ a_\tau^i) \quad (30)$$

will satisfy constraints (5) and (10).

Fig. 5 shows that for high values of the inflow a_τ^i the straight line $r_\tau^i = x_\tau^i + a_\tau^i - x_{\tau+1}^F(F^\star, F_\tau^i)$ may intercept the stage-discharge function $S(x_\tau^i)$ at a point $\tilde{x}_\tau$ with $\tilde{x}_\tau < \bar{x}$. In such a case the manager would open the gates of the dam even if he is not strictly obliged to do so by the license act. This has been actually done by the manager of Lake Como during the past few years.

3.4 Satisfaction of demand and flood protection

Let us now consider the case in which the manager wants to guarantee specified values $(D^\star,\ F^\star)$ of both the objectives (for example, $D^\star$ and $F^\star$ could be a percentage of the worst recorded values). If solutions to such problem exist, they will be constituted by the interceptions of the sets of initial storages and operating rules which solve the problems of demand satisfaction and flood protection. Thus, the set of the initial storages is specified by (see Eqs. (13) and (27)):

$$x_0^D\ (D^\star)\ \leq x_0\leq\ x_0^F\ (F^\star) \quad (31)$$

while the release r_τ^i is constrained by (see Eqs. (25) and (30))

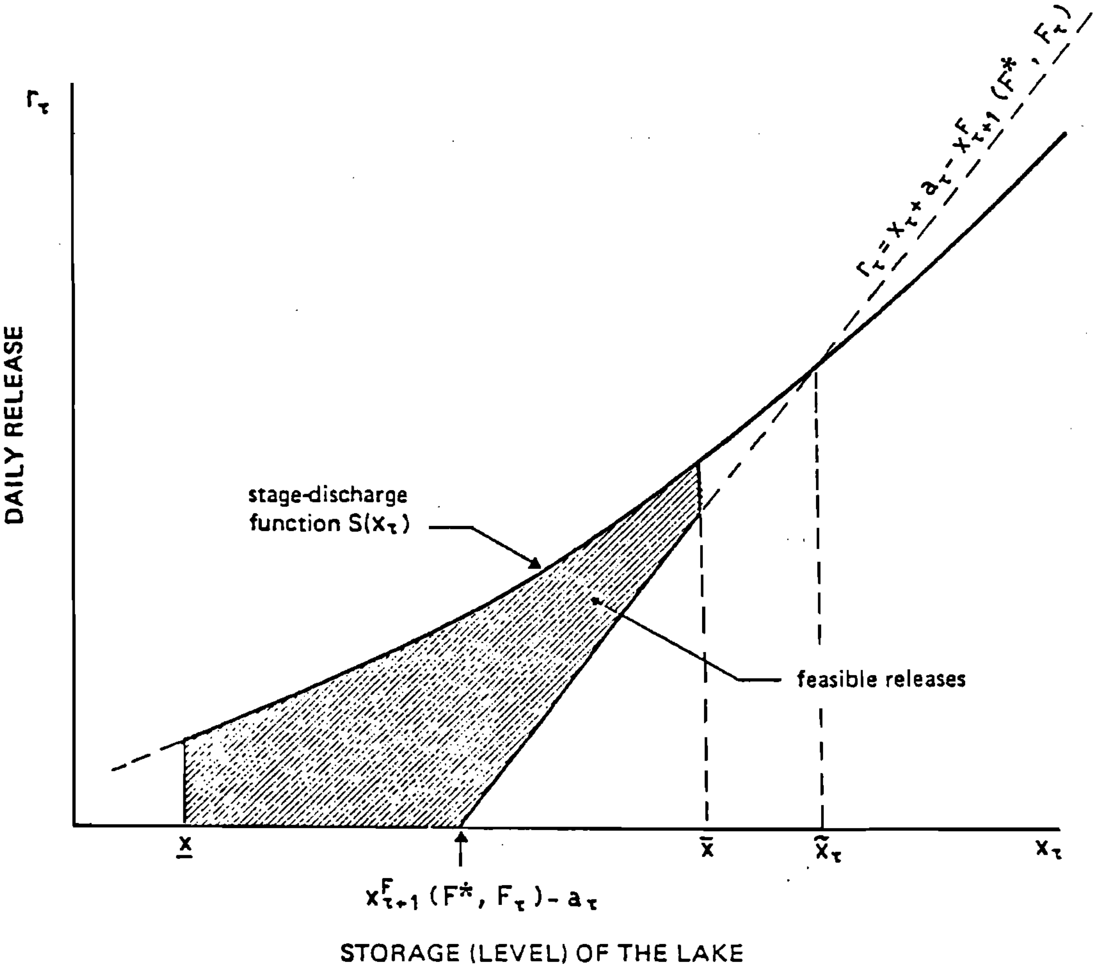

Figure 5. The set of releases r_τ which guarantee a yearly number of days of flood smaller than or equal to F^*.

$$\min\{r_{max}(x_\tau^i,\ a_\tau^i), \max\ [x_\tau^i+a_\tau^i-x_{\tau+1}^F(F^\star,\ F_\tau^i),0]\} \leq r_\tau^i$$
$$\leq\ \min\{\ S(x_\tau^i), \max[\ x_\tau^i+a_\tau^i-x_{\tau+1}^D(D^\star,\ D_\tau^i), r_{min}(\tau, x_\tau^i\ ,\ a_\tau^i)]\} \qquad (32)$$

This means that very often there is the possibility of choosing the release in a relatively wide range (shaded area in Fig. 6), but this freedom vanishes whenever the reservoir is too empty or too full, namely when the achievement of one of the two targets ($D^\star$ or $F^\star$) becomes critical. It is interesting to note that the operating rule shown in Fig.2 (which best interprets the data of the period 1946-1978) falls in each day of the year within the shaded region shown in Fig. 6 or remarkably close to it.

3.5 Efficient solutions

We can now point out a simple procedure for determining the efficient solutions (D^o, F^o) of the two objective Problem (1-5).

For this, let us suppose that a value D^o of water deficit in agriculture is fixed. Thus, the set of initial conditions and the set of operating rules which can guarantee the satisfaction of the terminal constraint $x_{365}^i \in X_0^{D^o}$ and of the target $D^i \leq D^o$ are given by Eqs. (13) and (25) with $D^\star = D^o$. The lowest storage within the set $X_0^{D^o}$ and the highest release satisfying Eq. (25) obviously minimize the number of days of flood. Therefore, one can simulate the behaviour of the lake with initial storage $x_0^D(D^o)$ and operating rule

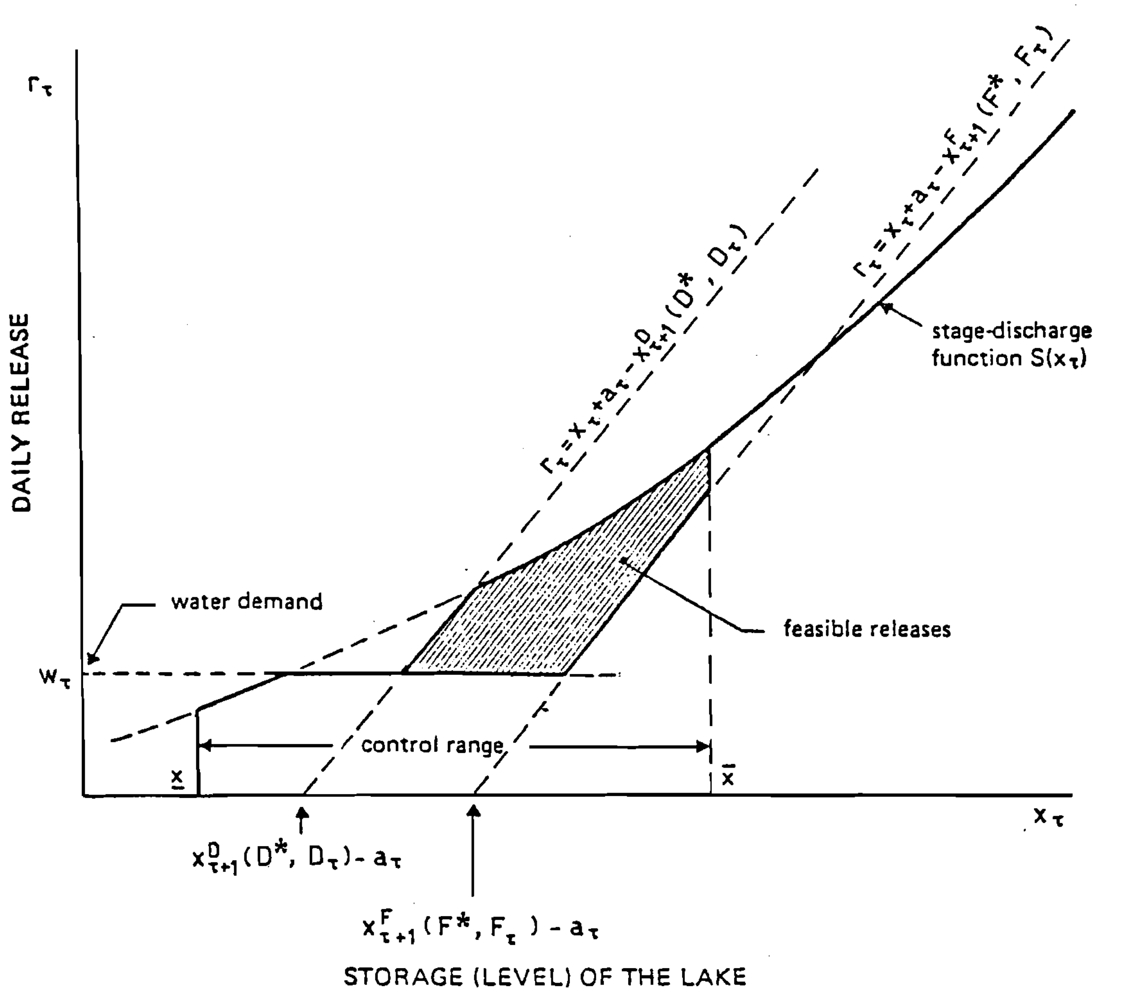

Figure 6. Feasible releases proposed by the min-max approach for the two objectives management problem.

$$r^i_\tau = \min \{ S(x^i_\tau),\ \max[x^i_\tau + a^i_\tau - x^D_{\tau+1}(D^o,\ D^i_\tau),\ r_{min}(\tau, x^i_\tau, a^i_\tau)]\}$$

for all the inflow sequences of the reference set. Thus, a certain number of days of flood F^i is obtained for each sequence i and obviously

$$F^o = \max_i \ F^i .$$

The efficient pairs (D^o, F^o) could also be found by fixing the value F^o and searching for D^o, which simply implies to simulate the behaviour of the lake starting from $x^F_o(F^o)$ with the operating rule given by the lower bound of Eq. (30) with $F^\star = F^o$.

Once an efficient pair (D^o, F^o) of the objectives has been found, the efficient operating rules and the set of initial storages are simply determined by substituting D^o and F^o for $D^\star$ and $F^\star$ in Eqs. (31) and (32).

4. APPLICATION TO LAKE COMO

In the case of lake Como, the min-max approach outlined in the previous section has been reduced to the following sequence of operations performed off-line. The seven most critical yearly inflow sequences of the period 1946-1981 were chosen to define the reference set I. Problem 0 was solved for different values of $D^\star$ by simulating the system behaviour for different values of the initial storage x_o selected by a one-dimensional search. It turned out that no solution existed for $D^\star$ smaller than 600 million cubic metres and that $x^D_o(600) = \underline{x}$ (obviously $x^D_o(D^\star) = \underline{x}$ for all $D^\star \geq 600$), which means that constraint (23) in Problem τ is a priori satisfied. In other words, in the case of lake Como, $x^D_\tau(D^\star, D^i_\tau)$

is only a function of $(D^{\star} - D_{\tau}^{i})$. This peculiar characteristic allows to determine $x_{\tau}^{D}(D^{\star}, D_{\tau}^{i})$ in the following very simple way. For each initial level x_{τ} in the control range, simulate over the rest of the year the behaviour of the lake with the operating rule $r_{min}(\tau, x_{\tau}^{i}, a_{\tau}^{i})$ for each inflow sequence i of the reference set, and store, as shown in the table of Fig. 7, the maximum deficit obtained in this way. This value corresponds to the smallest deficit $(D^{\star}-D_{\tau}^{i})$ which can be guaranteed from that day on. By inverting the table of Fig. 7, one can compute the function $x_{\tau}^{D}(D^{\star}, D_{\tau}^{i}) = x_{\tau}^{D}(D^{\star}-D_{\tau}^{i})$, which in this case is shaped as in Fig. 8.

In a very similar way one can determine the function $x_{\tau}^{F}(F^{\star}, F_{\tau}^{i})$ needed to solve the flood protection problem (see Sect. 3.3). In fact Problem 0 showed that it is not possible to guarantee less than 18 days of flood per year and that $x_{0}^{F}(18) = \bar{x}$, which implies $x_{0}^{F}(F^{\star}) \geqslant \bar{x}$ and $[\underline{x}, \bar{x}] \subset X_{0}^{F^{\star}}$ for any $F^{\star} \geqslant 18$. Since no flood ever happened in the history after November 15 neither one flood lasted more than ten days, the storage at the end of any year always falls within the control range and thus the terminal constraint is a priori satisfied also for the flood protection problem and $x_{0}^{F}(F^{\star}, F_{\tau}^{i}) = x_{0}^{F}(F^{\star}-F_{\tau}^{i})$. One can thus simulate the behaviour of the system with initial level x_{τ} and maximum release policy $r_{max}(x_{\tau}^{i}, a_{\tau}^{i})$ for each inflow sequence of the reference set, and determine the maximum number of days of flood over the rest of the year as shown in the table of Fig.9.

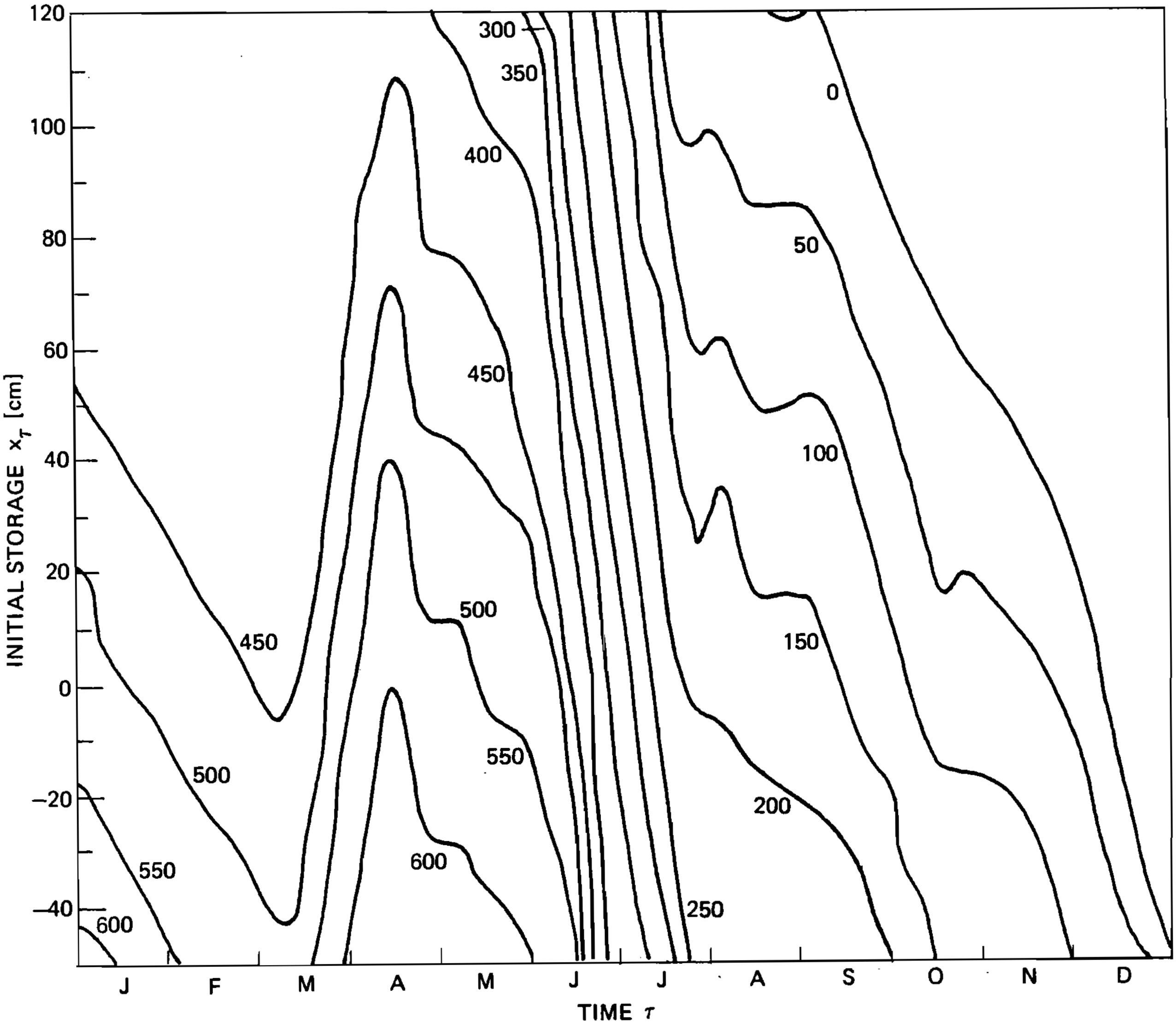

Figure 7. The maximum agricultural deficit obtained by simulating the system behaviour in the period from τ to the end of the year.

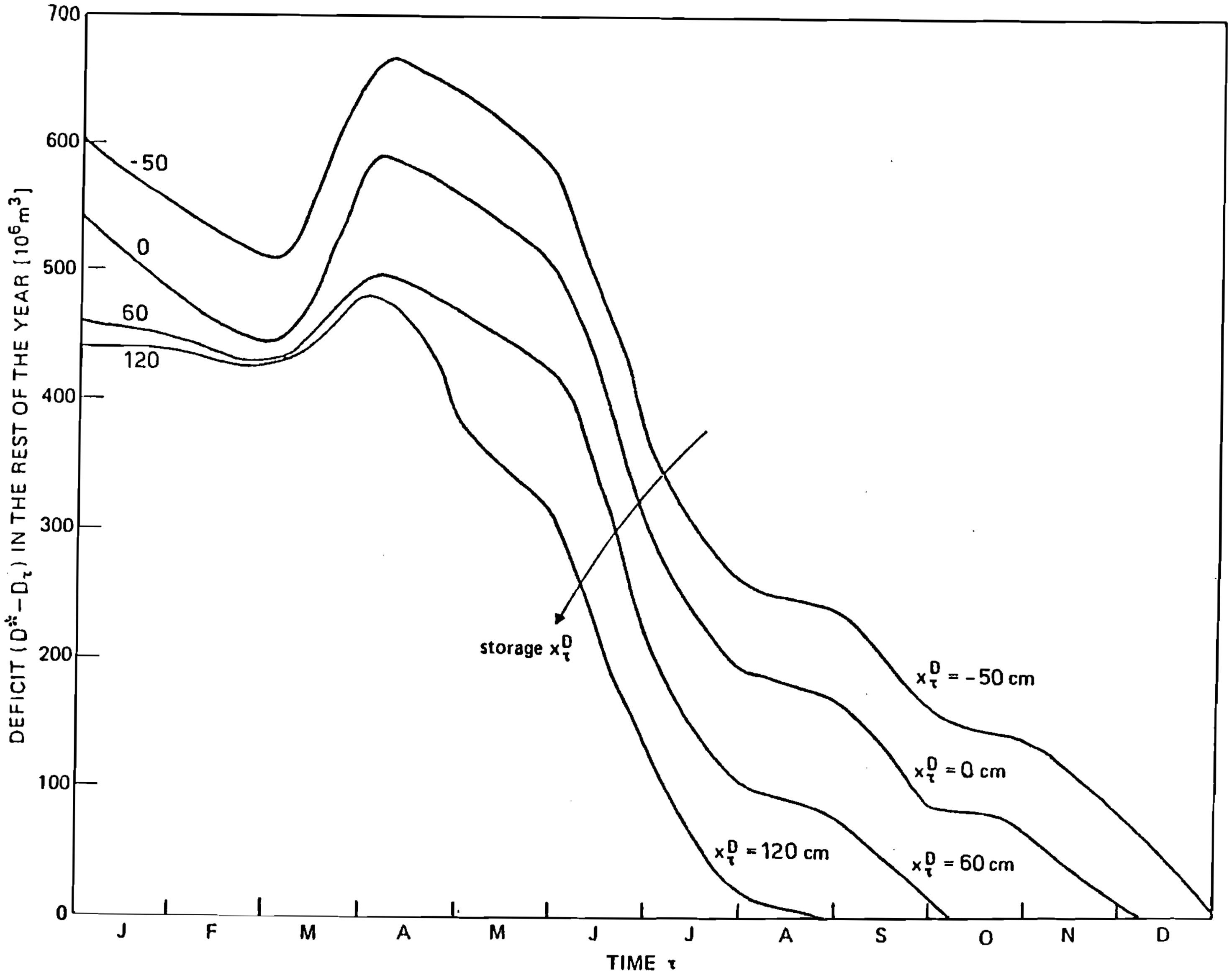

Figure 8. Curves of constant minimum initial storage (level) x_τ^D needed to guarantee a prescribed deficit (D^*-D_τ) from day τ to the end of the year.

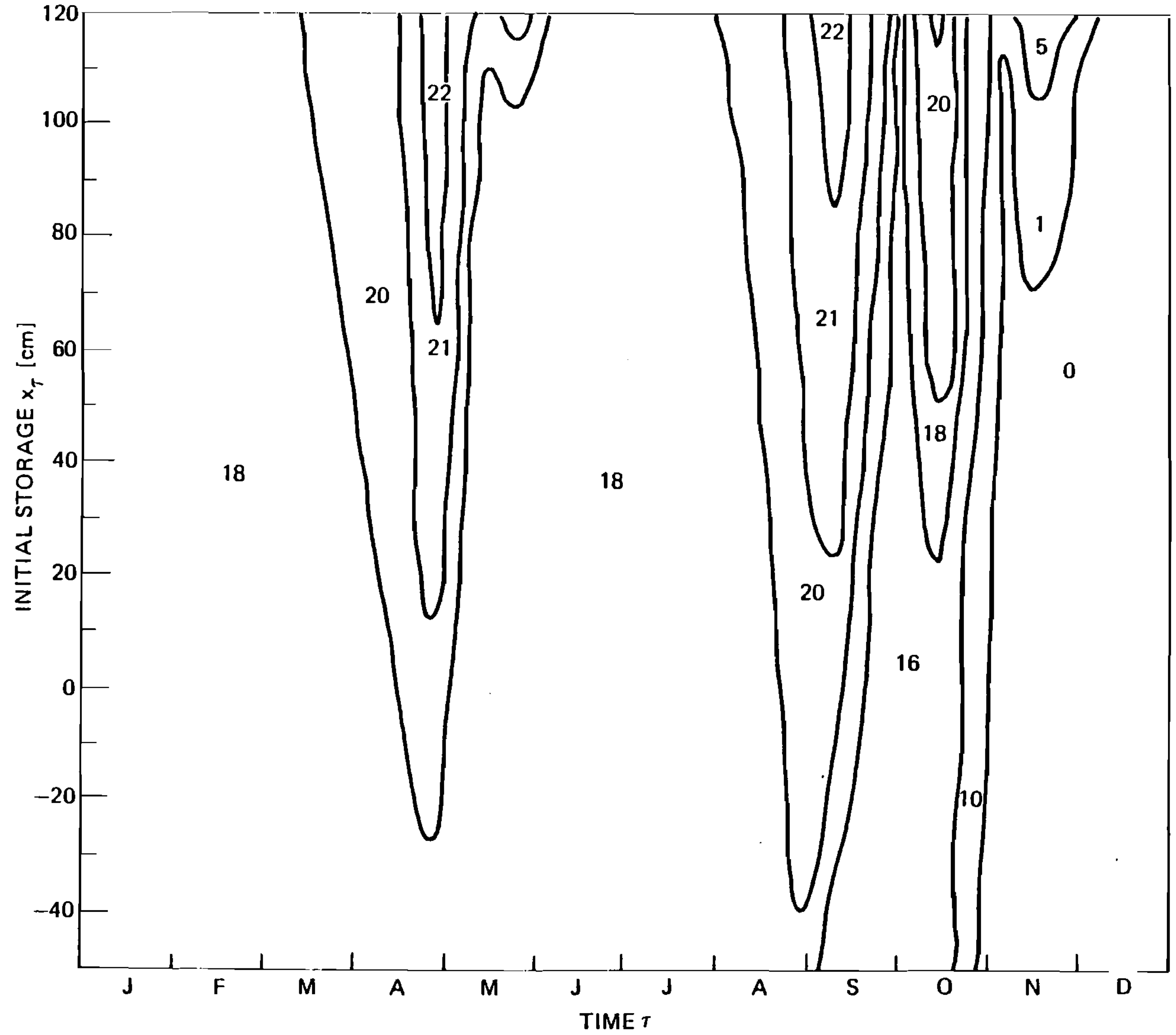

Figure 9. The maximum number of days of flood obtained by simulating the system behaviour in the period from τ to the end of the year.

This table represents a step-wise function since the number of days of flood is an integer. It appears from Fig. 9 that there are large areas of indifference in the space (τ ,x_τ). For example, it is not possible to guarantee less than 18 days of flood per year, but this value can be obtained for all initial storages x_τ in January and February. Lower storages should be used during the snow-melt season to achieve the same performance, but again the maximum number of days of flood is rather insensitive to the lake storage in June and July. These seasonal variations are in perfect agreement with the historical data. By inverting the table of Fig. 9, one obtains the function $x^F_\tau(F^\star, F^i_\tau) = x^F_\tau(F^\star - F^i_\tau)$ which is shown in Fig. 10.

Figs. 8 and 10 contain all the information necessary to find out the efficient solutions (D^o, F^o) of the risk—adverse management problem (1-5), provided the procedure outlined in Sect. 3.5 is used. The set of these efficient solutions is shown in Fig. 11 in the space of the objectives. In the same figure one can find the performance of the historical management (point H corresponding to an agricultural deficit of 750 million cubic metres and to 45 days of flood) and the "utopia" point U, which represents the independent and hence infeasible optimum of the two objectives (600 million cubic metres of deficit and 18 days of flood). Finally, point P represents the performance of the operating rule obtained by means of a classical stochastic approach and discussed in Guariso et al. /3/ (see below). Among all efficient solutions, the closest to the segment HU (see point X) has been suggested to the manager, This solution is clearly superior to the historical management. Indeed, improvements of 17% and 52% are possible for the maximum yearly agricultural deficit and the maximum yearly number of days of flood, respectively. On the contrary, the difference

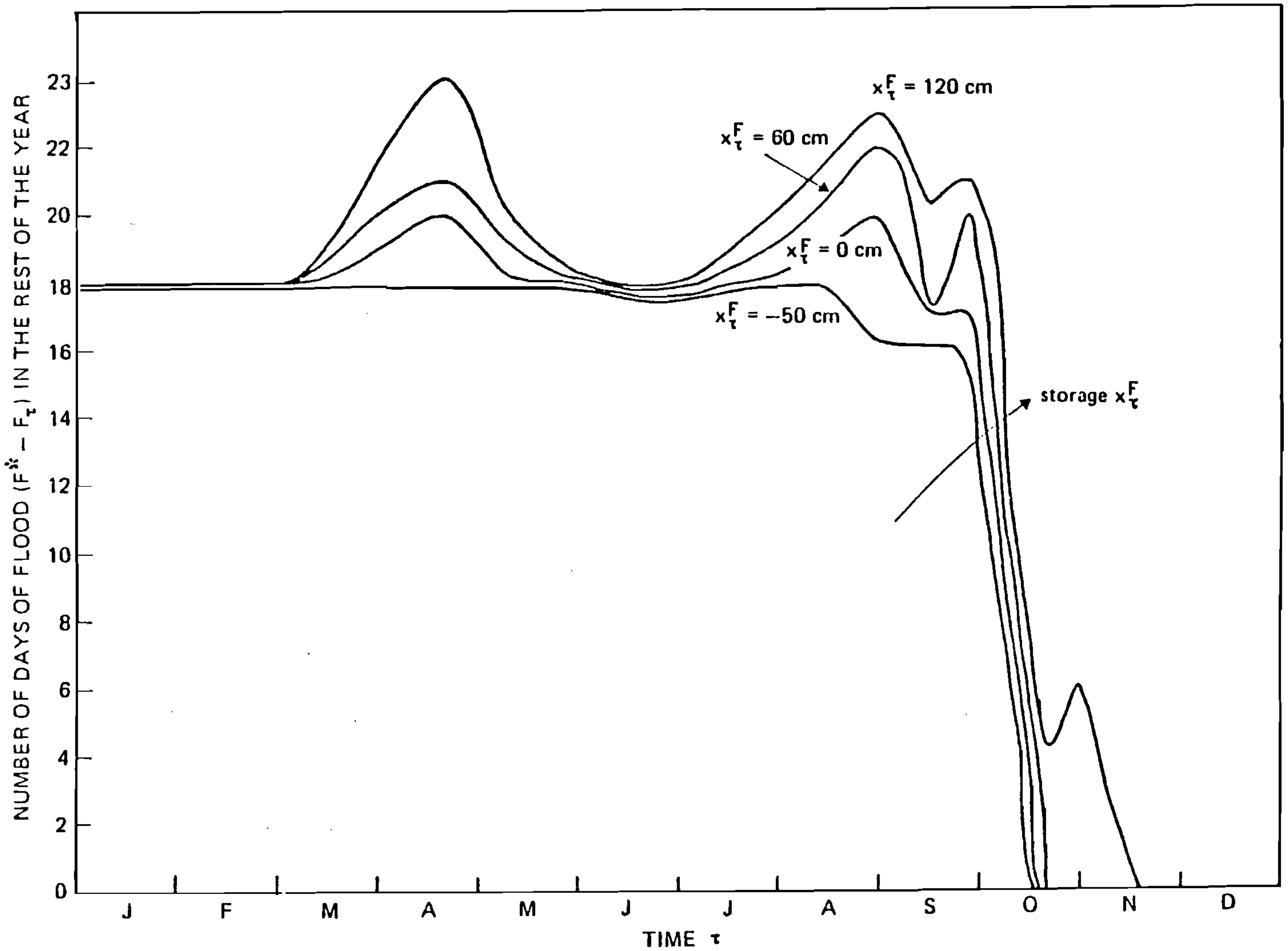

Figure 10. Curves of constant maximum initial storage (level) x^F_τ needed to guarantee a prescribed number of days of flood (F^*-F_τ) from day τ to the end of the year.

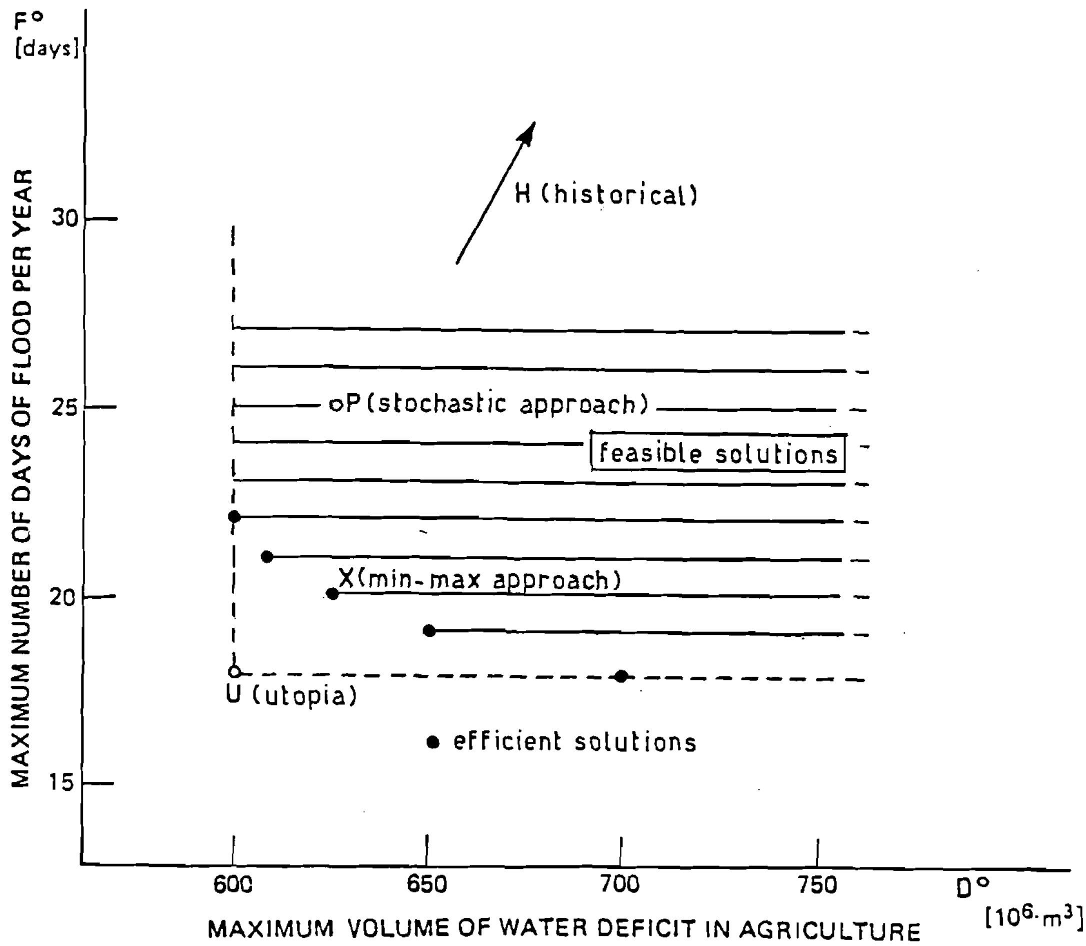

Figure 11. Feasible solutions (horizontal lines) and efficient solutions of the min-max approach: point X is the proposed solution and point P is the solution of the stochastic approach.

between point X and point P is only moderate. The stochastic approach would have in fact caused an agricultural deficit of 623 million cubic metres in the worst case, namely only 1% more than what achieved by the suggested min-max efficient operating rule. The difference between the deterministic and stochastic approach looks somehow greater when floods are considered (see Fig. 11).

As already mentioned, the stochastic approach followed by Guariso et al. /3/ models the decision making process as a multiobjective optimal control problem, but this time the objectives are the mean values ($E[\cdot]$) of the yearly water deficit in agriculture D and of the number F of days of flood per year. More precisely, the problem is given the following formulation

$$\min_{\{P\}} \quad | E[D] \quad E[F] |$$

subject to the continuity equation

$$x_{t+1} = x_t + a_t - r(t, x_t, a_t, p)$$

where the function r is a family of operating rules periodic over the year, p is a vector of unknown parameters to be determined through optimization, and $\{a_t\}$ is a one-year ciclostationary stochastic process. Clearly, the operating rules of the class $r(t,x_t,a_t,p)$ satisfy all legal and physical constraints of the problem.

The efficient solutions can be found by simulating for different values of p the behaviour of the system for a sufficiently long real or synthetic sequence of inflows. The parameter p is varied by means of a suitable criterion till the minimum value of $E[F]$ is achieved for any fixed value D^o of $E[D]$. Thus, the set of efficient solutions can be obtained by parametrically varying D^o. The results obtained by using the recorded sequence of inflows in the period 1946-1981 are summarized in Fig. 12, which shows all the efficient solutions and the absolute (and independent) minimum values of the objectives (utopia point U'). In this figure points H and X represent the performance of the historical management and of the suggested min-max operating rule in the same period. Finally, point P corresponds to the efficient operating rule suggested by Guariso et al. in /3/. It can be noticed that point P represents a substantial improvement with respect to the historical management. On the average, the agricultural deficit is reduced by 55% while the number of days of flood is 50% lower. On the contrary, the operating rule suggested by the min-max approach produces, in the average, much smaller improvements. In fact point X falls about half way between point P and point H.

Some interesting conclusions can be drawn from these comparisons. First, despite the apparent risk-adverse attitude of the manager, the historical data show that the past management does not seem to be closer to the solution suggested by the min-max analysis than to that of the stochastic approach. Second the performance of the risk-adverse approach is rather poor in terms of mean values of the objectives, while the stochastic approach seems to guarantee a satisfactory performance even when the system is under stress. For example, during the four most wet years of the reference set we would have 20, 20, 20 and 19 days of flood with the min-max operating rule, and 25,21, 19, and 10 days of flood with the stochastic approach. However, it is worthwile to notice that the min-

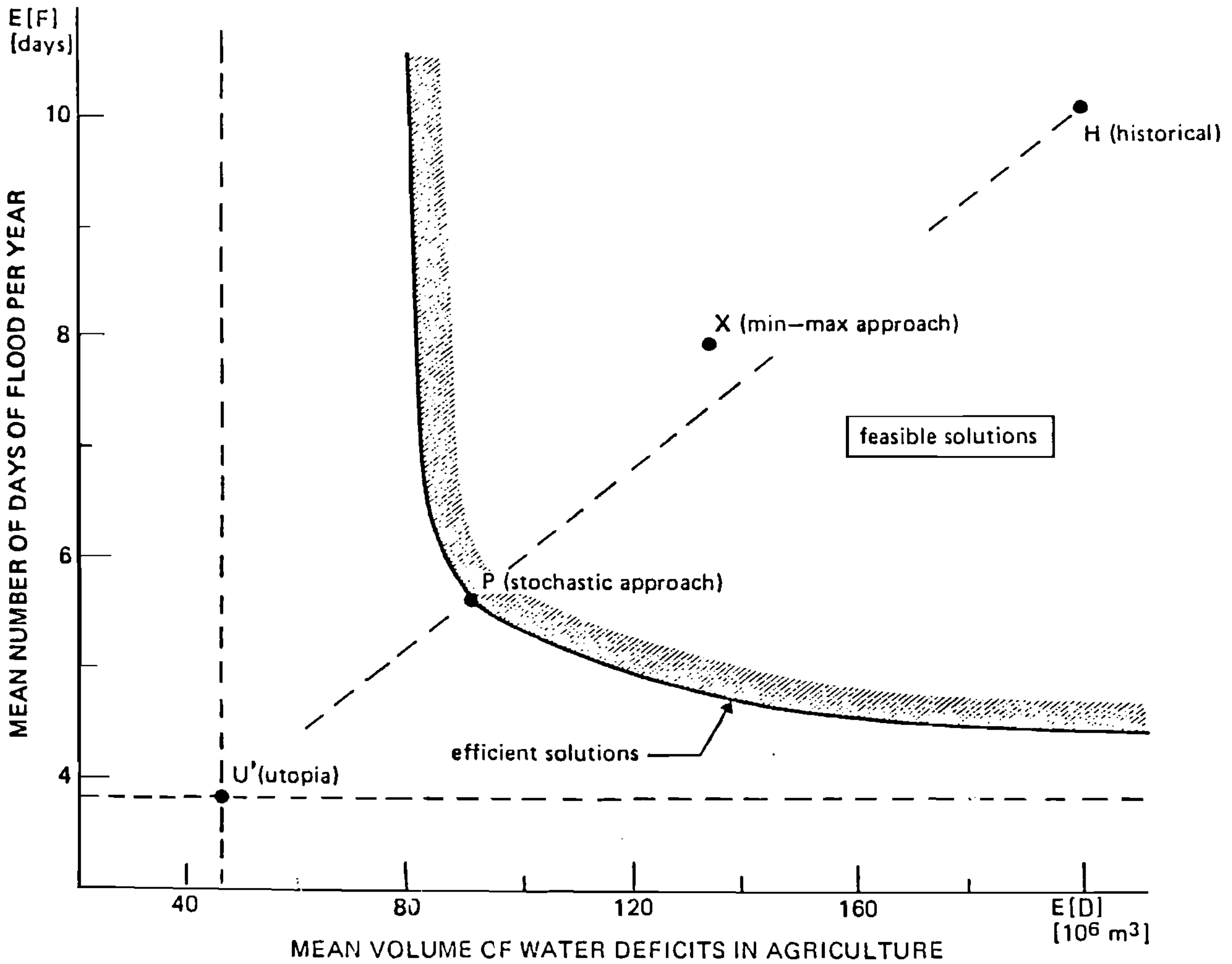

Figure 12. Feasible solutions (dashed area) and efficient solutions of the stochastic approach: point P is the proposed solution and point X is the solution of the min-max approach.

max approach is certainly more flexible than the other one, since it often allows to select the release within a specified range. This gives the manager the possibility to accomodate for secondary objectives,a fact that would require a complete reworking of the problem if the stochastic approach is followed.

5. CONCLUDING REMARKS

A deterministic (min-max)approach for the daily operation of Lake Como has been presented in this paper and compared with the more classical stochastic approach and with the historical data. The proposed operation, which is defined with the perfect knowledge of one day ahead inflow, performs much better than the historical one and about the same as the operating rule obtained through the stochastic approach.

The main characteristic of the deterministic methoa is that the daily release it not completely specified by the algorithm. Only a lower and an upper bound (see Fig. 6) are suggested to the manager who has still the freedom to select the final value of the release depending upon his judgement on the current and future situation of the system. Of course when conditions become critical (i.e. when the lake is almost empty or almost full) this slack disappears and the algorithm suggests a single value for the release. This value is the only one that would guarantee the prescribed performance if the future inflows would be one of the yearly sequences of a specified reference set.

Obviously, in the real application of the method future inflows can only be forecasted, so that this "guarantee" has no precise meaning. Nevertheless, the real performance of the system will strongly depend upon the representativeness of the reference set. This does not necessarily mean, however, that in order to improve the solution one should increase the number of inflow sequences in the reference set. This would in fact increase the time required to compute all the necessary tables. A more interesting suggestion to better the performance of the system is to use an "adaptive" reference set, which simply contains the sequences which are considered more significant for the current year. For example, if during a particular year, the snow-melt is over by May 30, there is no interest in considering after that date all those sequences in the reference set which have inflow peaks in June. This would imply, however, the use of an on-line computer to determine the feasible releases in real-time .

The final conclusion of the paper is that both the stochastic approach and the min-max approach seem to answer, in some way, to precise requirements of the manager. For this reason, it is probably useful to supply the manager with both optimal solutions. Indeed, this is what has been done in the case of Lake Como, where the optimal operating rules have been programmed on a microcomputer, which is used since then by the manager as an important support for the final decision.

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