NOT FOR QUOTATION
WITHOUT PERMISSION
OF THE AUTHOR

MULTICRITERIA ANALYSIS FOR DEVELOPMENT PLANNING

P. Nijkamp* and H. Voogd**

January 1983
CP-83-5

*Dept. of Economics, Free University, Amsterdam, The Netherlands
**University of Technology, Delft, The Netherlands

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

CONTENTS

1.	INTRODUCTION	1
2.	SOME PRINCIPLES OF MULTICRITERIA ANALYSIS	2
3.	CONTINUOUS MULTICRITERIA METHODS	6
4.	DISCRETE MULTICRITERIA METHODS	10
5.	CONCLUDING REMARKS	16
References		17

MULTICRITERIA ANALYSIS FOR DEVELOPMENT PLANNING

P. Nijkamp
H. Voogd

1. INTRODUCTION

A particularly vexing methodological difficulty in the design and evaluation of planning proposals concerns the way in which choices are made. Clearly, the selection of the "best" action from several alternatives, each of which will have different outcomes or consequences, is very subjective. This problem grows in importance if the actions under consideration will ultimately determine the welfare and wellbeing of a region, as is often the case in development planning. Many planning activities in developing countries are concerned with the construction of new infrastructure, rather than the improvement of an existing regime more typical of developed countries. Hence, methods and techniques which facilitate the treatment of choice or classification problems can be of tremendous importance in development planning.

One important subset of such tools is that composed of approaches which use a multidimensional set of sometimes conflicting criteria or objectives to structure and solve a choice or classification problem. These are usually called *multicriteria methods*. The purpose of this paper is to assess various multicriteria approaches in terms of their potential use in development planning.

The structure of the paper is as follows. In Section 2 the nature of multicriteria analysis is discussed in some detail. A distinction is made between *continuous multicriteria methods* and *discrete multicriteria methods*. The first of these classes of methods is the subject of Section 3; the second is treated in Section 4. The final section summarises the conclusions of the study and points to some issues for future research.

2. SOME PRINCIPLES OF MULTICRITERIA ANALYSIS

A planner involved in the design or evaluation of plans or projects has to begin by considering several important questions. For example, what constitutes an exhaustive set of alternatives? Which variables are relevant in characterizing each project or plan? A fundamental question underlying the selection of variables is whether the variables regarded as important by the planner are in fact those perceived to be important by the public. This raises another question, viz., who should participate in the planning process, at what stage, and in what way. Answers to questions of this kind depend on the ability of the planning agency to gauge public opinion. This is a very complex procedural problem, which cannot be considered as a simple exercise in measurement or technical evaluation.

To obtain a better idea of the complexity involved, it is useful to think in more specific terms. Assume a finite and exhaustive set of *alternative plans*

$$P = \{p_1, p_2, \ldots, p_J\} \tag{2.1}$$

from which one plan must be chosen. Each plan can be evaluated by means of a finite set of *variables*

$$A = \{a_1, a_2, \ldots, a_N\} \tag{2.2}$$

which can be included in a finite set of *criterion functions*

$$F = \{f_1, f_2, \ldots, f_I\} \tag{2.3}$$

where the f's may be linear or nonlinear functions of the variables. The relationship between these factors can be illustrated as follows. Let $e_{i1}, e_{i2}, \ldots, e_{iJ}$ denote the values taken by the criterion function i for plans $1,2,\ldots,J$. Then the set of values attained by all criterion functions for all plans can be arranged in a matrix as follows:

criteria	alternative plans					
	p_1	p_2	$\cdots$	p_j	$\cdot\cdot$	p_J
f_1	e_{11}	e_{12}	$\cdots$	e_{1j}	$\cdot\cdot$	e_{1J}
f_2	e_{21}	e_{22}	$\cdots$	e_{2j}	$\cdot\cdot$	e_{2J}
.						
.						
.						
f_i	e_{i1}	e_{i2}	$\cdots$	e_{ij}	$\cdot\cdot$	e_{iJ}
.						
.						
f_I	e_{I1}	e_{I2}	$\cdots$	e_{Ij}	$\cdot\cdot$	e_{IJ}

This array is also known as an evaluation matrix, project-effect matrix, or effectiveness matrix. For instance, if the aim were to plan a transportation system the criteria may be road capacity, transportation costs, maintenance costs, accessibility of service centers, visual and aesthetic appeal, levels of pollution, etc.

There are many methods that could be used to reduce the amount of information included in the above matrix, most of which use information concerning the relative importance of the various e_{ij} scores (i.e., *priorities* or *weights*). This may result in a classification of the alternatives under consideration which may be used in the policy-making process. Such methods are called *discrete multicriteria methods*, where the work "discrete" implies that a finite number of explicitly formulated alternatives is considered. Some recent surveys and discussions of discrete multicriteria methods can be found inter alia in van Delft and Nijkamp (1977), Nijkamp (1979a, 1980), Rietveld (1980), Kmietowicz and Pearman (1981) and Voogd (1982).

Discrete multicriteria methods are especially suitable for problems in which the alternatives are precisely known. However, there are also many cases in which only the *dimensions* of the alternatives are known (e.g., a plan must include 'some' transportation infrastructure, 'some' housing, 'some' employment, etc.), but the exact value of each dimension is not fixed. This implies that a continuous number of alternatives must be taken into consideration; instead of explicit alternatives, there is additional information on the feasible area in which an optimal solution (i.e., 'best' plan) may be situated. This may be illustrated with the following brief example. Consider two criteria e and m, which should have values as large as possible, and the associated *feasibility spectrum* (shown in Figure 1).

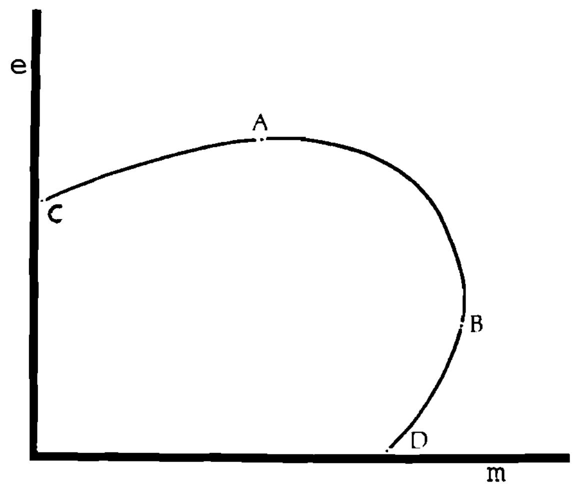

Figure 1. A feasibility spectrum for e and m.

This curve is usually called the *efficiency frontier* (or Pareto frontier, attainment-possibility frontier, set of nondominated points, set of noninferior points). It can be regarded as the locus of all points for which e cannot be increased without a decrease in m, and vice versa. Evidently, good solutions are represented by points on the efficient locus AB: any other point in the feasible set is dominated by a point on the arc AB. The choice among points on AB depends on the relative priorities of e and m and may be influenced, among other things, by institutional factors. A compromise solution (or compromise plan) of this type can ex post be defined as that efficient point (Pareto solution) which leads implicitly to the highest weighted total utility. The general form of such a model is:

$$\max_{\underline{a} \in K} \underline{f}(\underline{a}) \qquad (2.4)$$

where $\underline{f}$ is an I x 1 vector of criterion functions (also called objective functions), $\underline{a}$ is a J x 1 vector of decision variables, and K is a feasible area which defines the solution space. A multidimensional optimization (or programming) model of this type might, for example, be required to maximize production *and* employment *and* energy savings *and* system accessibility, subject to the constraints of limited resources and available technology.

Another central concept in this kind of multicriteria analysis is that of the *ideal point* (or reference point or utopia point). The ideal point $\underline{f}^o$ is defined as an I x 1 vector whose components are the maximum values of the individual criterion functions. This means that the elements f_i^o of $\underline{f}^o$ are defined as:

$$f_i^o = \max_{\underline{a} \in K} f_i(\underline{a}) \qquad (2.5)$$

The closer a point is to the ideal point, the better the alternative it represents. Obviously, the feasibility frontier also plays a central role in this approach, since it reflects the degree of conflict or complementarity between different possible outcomes. There are in fact infinitely many options, the number of which can be reduced by using a concept such as the ideal point. Because of the continuity of the alternatives under consideration, methods of this type will be called *continuous multicriteria methods*. This subject is treated inter alia in Keeney and Raiffa (1976), Bromley and Sfeir-Yonnis

(1977), Zeleny (1976b), Thiriez and Zionts (1976), Starr and Zeleny (1977), Bell et al. (1977), Nijkamp (1979a), Wierzbicki (1979a), Rietveld (1980), and Spronk (1981).

It is unfortunately the case that the information and data systems available for many regions in developing countries are generally rather poor; in particular, there is usually little quantitative (metric) data available for development planning. Consequently, it is also useful to classify multicriteria methods according to the accuracy of the data they require (see Table 1).

Table 1. A typology of multicriteria methods.

Type of method	Type of data	
	Hard	Soft
Discrete	I	II
Continuous	III	IV

Thus, multicriteria methods may be divided into *hard data methods* and *soft data methods*. Soft data methods are based on, for example, qualitative, fuzzy or ordinal data, while hard data methods are based on cardinal or metric data. Given the previous classification into discrete and continuous methods, it is clear that at least four main categories of methods may be distinguished (Table 1). These will be discussed in more detail in the next sections, where the soft data methods (i.e., categories II and IV) will be emphasized due to their importance for development planning.

3. CONTINUOUS MULTICRITERIA METHODS

There are many different continuous multicriteria methods currently in use (see Nijkamp, 1979a; Nijkamp and Spronk, 1979). The class of hard continuous methods includes utility function approaches (Farquhar, 1977; Fishburn, 1970), penalty models (Theil, 1968), goal programming (Lee, 1972; Charnes and Cooper, 1977; Spronk, 1981), min-max approaches (Nijkamp and Rietveld, 1976; Rietveld, 1980), reference point approaches (Zeleny, 1974, 1976a,b; Nijkamp, 1979b; Wierzbicki, 1979b; Lewandowski and Grauer, 1982), and hierarchical models (Nijkamp, 1977; Rietveld, 1980).

Utility methods are based on the assumption that the whole vector of relevant criteria or objectives can be translated by means of a weighting procedure into one utility function. This implies that (2.4) can be respecified as:

$$\max_{\underline{a} \in K} \phi = f(\underline{a}) \tag{3.1}$$

where ϕ is the master control of a scalar-valued optimization function. This approach has only limited value, since it presupposes a priori known quantitative trade-off rates.

Penalty models assume the existence of a set of desired achievement levels, reflected by an ideal vector $\underline{f}^{o}$. Any discrepancy between an actual value $\underline{f}$ and an ideal value $\underline{f}^{o}$ incurs a penalty calculated by means of a penalty function which could, for instance, be quadratic:

$$\min \sum_i w_i \, (f_i - f^{o})^2 \tag{3.2}$$

The coefficient w_i $(i=1,2,\ldots,I)$ represents the weight attached to deviating from the ideal value of criterion i. Evidently, the main difficulty in applying this kind of model is lack of information about appropriate penalty functions.

Goal programming methods are widely used to treat many different types of problems. They are essentially a subclass of penalty models for which the penalty function is defined as:

$$\sum_i w_i \, (f_i^{+} + f_i^{-}) \tag{3.3}$$

where f_i^{+} and f_i^{-} are the respective over- and underachievement of f_i with respect to the a priori specified achievement level

f_i^o for each criterion i. The plan that minimizes the penalty is considered to be the most attractive option. This approach is especially appropriate when used interactively so that the users can learn about the problem and modify their aspirations (achievement levels) accordingly.

Min-max approaches are based on the use of a matrix representing the pay-offs between conflicting objectives. The first step is the separate optimization of each criterion or objective function f_i:

$$\max_{\underline{a} \in K} f_i(\underline{a}) \text{ for all } i \tag{3.4}$$

The optimal value of each function from (3.4) is then denoted by $f_i^o(\underline{a}^i)$, where the vector of variables associated with this individual optimum is denoted by $\underline{a}^i$. A pay-off table representing the conflicts between the individual objectives is then constructed, each column corresponding to a given function and each row to the strategy a^i which represents an optimal solution with regard to the ith function. In each row i, we record the value achieved by each objective function when the strategy $\underline{a}^i$ is adopted. Thus, $f_1(\underline{a}^i)$ is the value of the first criterion or objective function that is obtained when strategy a^i is adopted, $f_2(\underline{a}^i)$ is the value of the second criterion under strategy a^i, and so forth. This pay-off table can then be used in several ways. For example, an equilibrium or min-max solution may be identified--this would be the solution that is nearest to the set of ideal solutions given on the main diagonal of the pay-off table (the values $f_i^o(a^i), i=1,2,\ldots,I$).

This approach is obviously especially appropriate when it is necessary to take into account different views of a problem in some explicit way. Each view is represented by a criterion (objective) function and the information given in the pay-off table may then be used to help the decision maker(s) to arrive at a compromise solution.

Reference point approaches are based on the concept of an ideal point already mentioned in the preceding section. They usually employ some kind of distance metric to assess the deviation between ideal solutions $f_i^o(\underline{a}^i)$ on the one hand and the set of efficient solutions $f_i(\underline{a}^*)$ on the other. The compromise solution is defined as the option in the set of efficient solutions for which the distance to the ideal solution is a minimum. It should be noted that there are also reference point approaches which are formulated in a goal programming framework, where the reference point represents a set of aspiration levels. This approach is also particularly appropriate when there is direct interaction between the user and the computer so that reference points can be modified during the course of the analysis.

Hierarchical models are based on the assumption that all criteria or objectives can be ranked in importance. Optimization is then carried out in a stepwise fashion, so that higher-ranking functions are optimized before those of lower rank. A tolerance parameter (or relaxation factor) can be specified for each function (except the most important), indicating the maximum deviation from the optimum $f_i(\underline{a}_i^o)$ considered acceptable by the users.

The hard continuous multicriteria methods described above have received considerable attention in the literature. The same is not true for soft continuous approaches, however, which undoubtedly represent a much less developed area of multicriteria analysis. Three different approaches may currently be distinguished: the fuzzy set approach (Chang, 1968; Bellman and Zadeh, 1970; Capocelli and De Luca, 1973), the stochastic approach (Donckels, 1975), and the soft econometric approach (Nijkamp and Rietveld, 1982).

Fuzzy set methods focus on criteria or constraints that are not sharply defined, so that the boundaries of the decision area are not marked out in an unambiguous manner. By using fuzzy set theory, it is possible to derive measures for the extent to which an element (e.g., aspect) belongs to a certain category. This information can be quantified by means of so-called membership functions, and can then be utilized in some hard optimization method. The definition of membership functions is crucial to the use of the fuzzy set approach.

The use of *stochastic approaches* in continuous multicriteria analysis has been quite uncommon up to now. Nevertheless, if qualitative or ordinal decision variables can be approximated by cardinal (metric) variables with a certain probability distribution, it is possible to construct a stochastic optimization model. The most probable compromise solution may then be identified using an appropriate hard multicriteria method. One of the main drawbacks of this approach is usually the lack of information about appropriate probability functions.

The *soft econometric approach* is perhaps the most promising basis for soft continuous multicriteria methods. This approach involves the transformation of qualitative or ordinal data input into metric units, which may then be analyzed further by means of an appropriate hard multicriteria method. Although no applications of this approach in the field of optimization are known, the general principles are certainly applicable in multicriteria analysis. This is still a relatively new technique, however, and further research is necessary.

It may be concluded from the above descriptions of continuous multicriteria methods that the development of soft methods is lagging far behind that of hard methods. For this reason the use of soft methods in development planning is currently very limited, despite promising progress in recent years. We will return to this point in the final section of this paper.

4. DISCRETE MULTICRITERIA METHODS

Discrete multicriteria methods are based on the existence of a fixed number of explicitly defined alternatives. The first step in all these methods is to construct an evaluation matrix, as explained in Section 2, since the purpose of this kind of multicriteria analysis is to make some kind of evaluation of the various alternatives available. However, such an evaluation is only possible if there is a *weighting scheme* which expresses the relative importance of the various scores.

In the past, cost-benefit analysis has been the method most commonly used to evaluate discrete alternatives. However, many projects or plans are concerned with outcomes or consequences which cannot be discussed in terms of prices, and this makes the cost-benefit approach inappropriate for complex decision making (see Nijkamp, 1977, for an extensive criticism). Related methods such as the planning-balance sheet method, cost-effectiveness analysis, and the shadow project approach are significant improvements upon traditional cost-benefit analysis for complex planning purposes, but provide no solution to the problem of judging incommensurate and intangible outcomes.

Instead of using (artificial) prices to assess the relative merits of these intangible outcomes, discrete multicriteria methods assign political priorities to certain criteria. These weights reflect the relative importance attached to the outcomes associated with each criterion. However, political weighting schemes are often difficult to infer from questionnaires or other procedures designed to reveal preferences (see Voogd, 1982). When such weights cannot be assessed a priori, the analyst may proceed in one of two ways: he may either (a) use general alternative scenarios as the basis for deriving alternative sets of weights; these scenarios may reflect alternative policy directions (views) or future policy choices (see Nijkamp and Voogd, 1979), or (b) use an interactive learning procedure during which relative priorities are specified in a stepwise manner (see van Delft and Nijkamp, 1977).

There are many discrete multicriteria methods, both for hard and for soft data. The following hard data methods will be considered here: the expected value method (Schimpeler and Grecco, 1968; Schlager, 1968; Kahne, 1975), the discrepancy analysis technique (Nijkamp, 1979a), the goals-achievement method (Hill, 1973), and the concordance approach (see Guigou, 1974; Roy, 1972; van Delft and Nijkamp, 1977).

The *expected value method* assigns weights to the criteria and treats these weights as "quasi-probabilities" which must add up to 1. Thus the expected value of the outcomes of each alternative plan can be calculated by multiplying the value obtained for each criterion by its appropriate weight and then summing the weighted values for all criteria. Essentially, the expected value method calculates the weighted average of all (standardized) criteria scores. This method implies a rather rigid approach since it assumes perfect linear substitution of the values for the various criteria, which is seldom true in practical applications.

Discrepancy analysis attempts to rank the plans according to their discrepancy from an optimum plan. This (hypothetical) optimum plan achieves a set of predefined goals. Statistical correlation coefficients are then used to identify the plan most similar to the reference plan. This method should be used with care, because the various discrepancies in the outcomes of a plan cannot be made sufficiently explicit.

The *goals-achievement method* links each criterion with a quantitative achievement level or target value. Evaluation essentially involves taking the achievement score for each criterion, and aggregating these to give a total achievement score for each alternative plan. The values are aggregated using a weighted summation procedure similar to that described above for the expected value method. The goals-achievement method is widely used in planning practice due to its simple and straightforward structure.

The *concordance approach* is also widely used. This method is based on a pairwise comparison of alternatives, thus using only the metric interval characteristics of the various outcome evaluations. The basic idea is to measure the degree to which the outcomes and their associated weights confirm or contradict the dominant pairwise relationships among alternatives. The differences in weights and the differences in evaluation scores are usually analyzed separately. This approach uses the available information reasonably well and can be considered as a useful type of discrete multicriteria model.

In recent years, much attention has been paid to the development of *qualitative* or *soft* evaluation techniques, with considerable practical success. As a result, many operational soft discrete multicriteria methods are now available.

The following approaches will be discussed here: the eigenvalue approach (Saaty, 1977; Lootsma, 1980), the extreme expected value method (Kmietowicz and Pearman, 1981; Rietveld, 1982), the permutation method (Paelinck, 1976), the frequency approach (van Delft and Nijkamp, 1977; Voogd, 1981a), the geometric scaling approach (Nijkamp and Voogd, 1979, 1981), and the mixed data approach (Voogd, 1981b, 1982).

The *eigenvalue approach* involves the pairwise comparison of alternatives (Saaty, 1977). This comparison is carried out using a nine-point scale, where the value 1 means that the two factors being compared are of equal importance while the value 9 implies that one is much more important than the other. A table is constructed for each criterion, in which the alternative plans are compared in a pairwise fashion with respect to that criterion. The criteria themselves are then compared in a similar way, resulting in a separate criteria evaluation table. The next step is to aggregate the information in each table using an eigenvalue procedure. This involves the calculation of quantitative evaluation scores and weights, which are then used in a weighted summation procedure to determine an aggregated appraisal score for each alternative plan. This approach therefore has the same drawbacks as the expected value method discussed earlier. In addition--and this is probably the most fundamental limitation of the approach--it is impossible for the user to relate the values of the criterion weights to the values obtained for the plan outcomes. In other words, the weighting is independent of the characteristics of the various plans.

The *extreme expected value method* can be regarded as an extension of the expected value method discussed above. It is still assumed that the scores achieved by each plan with respect to each criterion have quantitative properties, but in addition it is postulated that the probabilities (weights) are only known in a qualitative sense, i.e., only their ordinal properties are given. In essence, the aim of this approach is to determine the alternative with the maximum or minimum expected value. This is done by solving the following linear programming problem:

$$\max \text{ or } \min EV_j = \sum_i p_i e_{ij}$$
$$\text{subject to } p_1 \geq p_2 \geq p_3 \geq \cdots \geq p_I \geq 0 \qquad (4.1)$$
$$\sum_i p_i = 1$$

where EV_j denotes the expected value of alternative j and p_i is the probability associated with the evaluation e_{ij} of alternative j with respect to criterion i. Some elementary operations lead to maximum and minimum expected values, which may be used in a final assessment of the alternatives. However, Rietveld (1982) has shown that this assessment should not be made solely on the basis of the extreme values, but should also take into account certain values of EV_j generated for intermediate values of p_i.

The *permutation method* is based on a comparison of all possible final rankings of alternative plans in order to find the best 'final' ranking. For each hypothetical final ranking a score is calculated which measures how well this ranking corresponds to the (ordinal) values registered by each plan for each criterion. Instead of the original set of alternative plans we now have a new set of alternative configurations of rankings. Then, using a weighted summation procedure involving the extreme values of the ordinal weights, an appraisal score for each permutation is calculated. Given the extreme weight set used, the best final ranking of alternatives can thus be determined. The use of this approach is limited to problems involving only a few alternatives because of the number of permutations, although a more heuristic extension to deal with many alternatives is possible.

The *frequency approach* is also based on the pairwise comparison of alternatives. The basic idea of this approach is to transform the available ordinal information into information on a 'lower' (i.e., binary) scale, which is then treated as a frequency statistic. This approach also has the disadvantage that it may become rather cumbersome if a large number of alternatives and criteria are involved.

The *geometric scaling* approach is based on the principles of nonmetric multidimensional scaling. The basic idea of this approach is to transform a large amount of ordinal data into a small amount of metric (cardinal) data, such that the new cardinal configuration is as close as possible (has maximum goodness-of-fit) to the ordinal data. One limitation of this elegant approach is that it requires a fairly complicated computational algorithm. In addition, evaluation problems treated by this method should have a sufficient number of degrees of freedom to allow geometric scaling. This implies that unless sufficient ordinal information is available, no metric data can be extracted.

Except for the extreme expected value approach, which assumes cardinal evaluation scores, all of the soft multicriteria methods mentioned above deal with qualitative weights and qualitative assessments of the alternatives with respect to individual criteria. Consequently, most of these methods have already been used in planning practice, despite the fact that they have only just been developed. However, one much-voiced and persistent criticism of these techniques is that only the ordinal characteristics of the available quantitative information are utilized. Therefore the most recent research in this area has concentrated on the development of methods capable of dealing with *mixed data* i.e., evaluation matrices containing both quantitative scores and qualitative rankings. Nijkamp and Voogd (1981) have developed a mixed-data procedure based on the geometric scaling approach which obviously suffers from the same limitations as the simpler version mentioned above. Another set of methods has been developed by Voogd (1981b, 1982); these involve the construction of two measures: one dealing only with ordinal information and the other with cardinal information. By making various assumptions, the information from these measures can be aggregated into one appraisal score for each alternative. Thus, different mixed-data methods have been constructed using different sets of assumptions.

In conclusion, it can be said that a whole series of soft discrete multicriteria methods is now available, each method having its own particular advantages and disadvantages and making its own individual assumptions. These evaluation methods are especially useful for development planning problems because they require only a modest amount of information of modest quality.

5. CONCLUDING REMARKS

Multicriteria methods have become an integral part of modern planning methods and techniques. For global and macro decision problems and policy scenario analyses, hard continuous multiobjective methods have reached a stage of sufficient maturity that they can be and actually *are* applied in a wide variety of policy analyses. They may also be used to scan problems and to identify the main alternative lines of action. Hard discrete multicriteria methods have become very useful in micro decision problems and project evaluation problems. The soft variants of discrete multicriteria methods have also been successfully applied in many plan and project evaluation problems, although much work remains to be done on soft continuous models. The latter class of methods could be very useful, especially for planning and decision problems with limited or qualitative information, so that further research in this area is certainly justified.

One problem still remains to be discussed, viz., the problems of uncertainty regarding the application of various methods. Not all methods give the same results, so that a sensitivity analysis may be necessary (see especially Voogd, 1982). Clearly such a sensitivity analysis should only be carried out on a set of models preselected on the basis of methodological, theoretical, and empirical criteria.

The above survey has given a brief indication of the variety of multicriteria methods available for use in planning and policy problems with conflicting objectives. Some of these techniques may play an important role in development planning in lagging areas or countries. When reliable data is difficult to obtain, the soft variants may be especially helpful.

REFERENCES

Bell, D.E., R.L. Keeney, and H. Raiffa (1977), *Conflicting Objectives in Decisions*, Wiley, New York.

Bellman, R.E. and L.A. Zadeh (1970), Decision-Making in a Fuzzy Environment, *Management Science* 17(4), B41-B164.

Bromley, D.H. and A. Sfeir-Younis (1977), *Decision-Making in Developing Countries; Multiobjective Formulation and Evaluation Methods*, Praeger, New York.

Capocelli, R.M. and A. De Luca (1973), Fuzzy Sets and Decision Theory, *Information and Control* 23, 446-473.

Chang, C.L. (1968), Fuzzy Topological Spaces, *J. Mathematical Analysis and Applications* 24, 182-190.

Charnes, A. and W.W. Cooper (1977), Goal Programming and Multiple Objective Optimization, *European Journal of Operations Research* 1, 39-54.

Delft, A. van and P. Nijkamp (1977), *Multicriteria Analysis and Regional Decision-Making*, Martinus Nijhoff, The Hague/Boston.

Donckels, R. (1975), *Regional Multiobjective Planning*, Regional Science Research **Paper No. 8, University** of Louvain.

Farquhar, P.H. (1977), A Survey of Multiattribute Utility Theory and Applications, *Multiple Criteria Decision Making*, (M.K. Starr and M. Zeleny, eds.), North-Holland Publishing Co., Amsterdam, pp. 59-90.

Fishburn, P. (1970), *Decision and Value Theory*, Wiley, New York.

Guigou, J.L. (1974), *Analyse des Données et Choix à Critères Multiples*, Dunod, Paris.

Hill, M. (1973), *Planning for Multiple Objectives*, Monograph Series No. 5, Regional Science Research Institute, Philadelphia.

Kahne, S. (1975), A Contribution to Decision Making in Environmental Design, *Proceedings of the IEEE 1975*, 518-528.

Keeney, R.L. and H. Raiffa (1976), *Decision Analysis with Multiple Conflicting Objectives*, Wiley, New York.

Kmietowicz, Z.W. and A.D. Pearman (1981), *Decision Theory and Incomplete Knowledge*, Gower, Aldershot.

Lee, S.M. (1972), *Goal Programming for Decision Analysis*, Auerbach, Philadelphia.

Lewandowski, A. and M. Grauer (1982), The Reference Point Optimization Approach; Methods of Efficient Implementation, WP-82-26, IIASA, Laxenburg.

Lootsma, T.A. (1980), Saaty's Priority Theory and the Nomination of a Senior Professor in Operations Research, *European Journal of Operational Research* 4, 380-388.

Nijkamp, P. (1977), *Theory and Application of Environmental Economics*, North-Holland Publishing Co., Amsterdam.

Nijkamp, P. (1979a), *Multidimensional Spatial Data and Decision Analysis*, Wiley, New York.

Nijkamp, P. (1979b), A Theory of Displaced Ideals, *Environment and Planning A* 11, 1165-1178.

Nijkamp, P. (1980), *Environmental Policy Analysis*, Wiley, Chichester.

Nijkamp, P. and P. Rietveld (1976), Multi-Objective Programming Models, New Ways in Regional Decision-Making, *Regional Science and Urban Economics* 6, 253-274.

Nijkamp, P. and P. Rietveld (1982), Soft Econometrics as a Tool in Regional Discrepancy Analysis, *Papers of the Regional Science Association* 49 (forthcoming).

Nijkamp, P. and J. Spronk (1979), Analysis of Production and Location Decisions by Means of Multicriteria Analysis, *Engineering and Process Economics* 4, 285-302.

Nijkamp, P. and J.H. Voogd (1979), The Use of Psychometric Techniques in Evaluation Procedures, *Papers of the Regional Science Association* 42, 119-138.

Nijkamp, P. and H. Voogd (1981), New Multicriteria Methods for Physical Planning by Means of Multidimensional Scaling Techniques. *Water and Related Land Resource Systems* (Y. Haimes and J. Kindler, eds), Pergamon, Oxford, pp. 19-30.

Paelinck, J. (1977), Quantitative Multicriteria Analysis - Application to Airport Location, *Environment and Planning A* 9(8), 883-895.

Rietveld, P.(1980)*Multiple Objective Decision Methods and Regional Planning*, North-Holland Publishing Co., Amsterdam.

Rietveld, P. (1982), Using Ordinal Information in Decision-Making under Uncertainty, Research Memorandum 1982-12, Department of Economics, Free University, Amsterdam.

Roy, B. (1972), Decision avec Critères Multiples, *Metra* 11, 121-151.

Saaty, T.L. (1977), A Scaling Method for Priorities in Hierarchical Structures, *Journal of Mathematical Psychology* 15, 234-281.

Schimpeler, C.C. and W.L. Grecco (1968), The Expected Value Method, an Approach based on Community Structures and Values, *Highway Research Record* 238, 123-152.

Schlager, K. (1968), The Rank-Based Expected Value Method of Plan Evaluation, *Highway Research Record* 238, 153-158.

Spronk, J. (1981), Interactive Multiple Goal Planning for Capital Budgeting and Financial Planning, Kluwer Nijhoff, Boston/The Hague.

Starr, M.K. and M. Zeleny (eds.) (1977), *Multiple Criteria Decision Making*, North-Holland Publishing Co., Amsterdam.

Theil, H. (1968), *Optimal Decision Rules for Government and Industry*, North-Holland Publishing Co., Amsterdam.

Thiriez, H. and S. Zionts (eds.) (1976), *Multiple Criteria Decision Making*, Springer, Berlin.

Voogd, H. (1981a), Qualitative Multicriteria Evaluation Methods for Development Planning, *The Canadian Journal of Regional Science* 4, 73-87.

Voogd, H. (1981b), Multicriteria Analysis with Mixed Qualitative-Quantitative Data, Department of Urban and Regional Planning, Planologisch Memorandum, Delft University of Technology.

Voogd, H. (1982), *Multicriteria Evaluation in Urban and Regional Planning*, Pion, London (forthcoming).

Wierzbicki, A.P. (1979a), A Methodological Guide to Multiobjective Optimization, Working Paper WP-79-122, IIASA, Laxenburg.

Wierzbicki, A.P. (1979b), The Use of Reference Objectives in Multiobjective Optimization; Theoretical Implications and Practical Experience, WP-79-66, IIASA, Laxenburg.

Zeleny, M. (1974), *Linear Multiobjective Progamming*, Springer, Berlin.

Zeleny, M. (ed.) (1976a), *Multiple Criteria Decision Making*, Springer, Berlin.

Zeleny, M. (1976b), The Theory of Displaced Ideal, in M. Zeleny (ed.), *Multiple Criteria Decision Making*, Springer, Berlin, pp. 153-206.