

A UTILITY FUNCTION FOR TIME STREAMS  
HAVING INTER PERIOD DEPENDENCIES

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Abstract

Frequently a decision maker's preferences for consequences in a given period will depend on the particular outcome in the previous and/or following period. This paper gives a simple functional form which enables such preferences to be explicitly included in a utility function for time streams.

In evaluating alternative strategies which have impacts over time, many factors serve to complicate matters, and principal amongst them are the necessity of making tradeoffs between consequences in different periods and the uncertainty of the precise outcomes as to their magnitude and timing. It is precisely these issues that Von Neumann-Morgenstern utility functions [9] are able to resolve. Such a utility function serves as a preference (or value) function in that it provides an ordering over certain outcomes and its expected value provides a preference function over uncertain outcomes.

The difficulties associated with this approach arise when the number of time periods is large, for the dimension of the utility function is equal to the number of time periods; and whilst the assessment of a one-dimensional (or one-attribute) utility function is relatively easy and that of a two-dimensional utility function still practical, it soon becomes impossible without major simplifying assumptions.

The problem is to find reasonable assumptions which reduce the assessment of the utility function to a manageable level without losing the flexibility to reflect the decision maker's true preferences accurately, and without losing the property of the utility function as an evaluator of uncertain outcomes.

Both Meyer [ 8 ] and Fishburn [ 3 ] have given assumptions which do fulfil these requirements. Let  $X = X_1 \times X_2 \times X_3 \times \dots \times X_T$  be a set of time streams where  $X_i$  is the set of possible outcomes in period  $i$  whose elements are scalars or vectors.  $X_i$  will also be used to denote the attribute as well as its set of values.

Meyer assumes utility independence (see next section) to exist between the sets  $X_1 \times \dots \times X_j$  and  $X_{j+1} \times \dots \times X_T$  for each  $j = 1, \dots, T-1$  and shows that this implies that

$$u(x_1, \dots, x_T) = \sum_{i=1}^T c_i u_i(x_i) \quad c_i > 0$$

or (1)

$$\prod_{i=1}^T (a_i + b_i u_i(x_i)) \quad b_i > 0 ,$$

where  $u_i$  is a utility function assessed over only the set  $X_i$ .

Fishburn assumes Markovian dependence which, briefly stated, says that the decision maker is indifferent between two uncertain time streams if and only if the probability distribution over set pairs  $X_i \times X_{i+1}$  is the same for all  $i$ . This yields the form

$$u(x_1, \dots, x_T) = \sum_{i=1}^{T-1} \lambda_i(x_i, x_{i+1}) , \quad (2)$$

where

$$\begin{aligned} \lambda_i(x_i, x_{i+1}) = & u(x_1^0, x_2^0, \dots, x_{i-1}^0, x_i, x_{i+1}, x_{i+2}^0, \dots, x_T^0) \\ & - u(x_1^0, \dots, x_i^0, x_{i+1}, x_{i+2}^0, \dots, x_T^0) . \end{aligned}$$

In a recent study by the Ecology Project of a forest pest problem, the impacts on society were summarized by three attributes, profit from lumber (P), employment within the industry (E) and the recreational potential of the forest (R), each of these being time streams of attributes. The decision maker

felt that the recreation streams and the profit/employment streams were mutually utility independent, which yielded a form

$$u(p,e,r) = u_R(r) + u_S(p,e) + ku_R(r) u_S(p,e) ; \quad (3)$$

and that the assumptions leading to the product from (1) were appropriate for the utility function  $u_R$  of recreation, but that preferences for  $(p_i, e_i)$  pairs depended heavily on the employment figures in neighbouring time periods. Analysis showed that for a given level of  $e_{i-1}$  he was risk averse for values of  $e_i$  which were greater than  $e_{i-1}$  but risk prone otherwise. The Markovian dependence assumptions leading to form (2) were not felt to be sufficiently intuitively meaningful to be used.

The results that follow give a functional form for a general utility function for time streams which is less restrictive than, but can be specialized to, the forms of Meyer and Fishburn.

A final section returns to the forest pest problem and gives a functional form for  $u_S(p, e)$ .

#### Conditional Utility Independence

Consider a situation involving two attributes  $X_1$  and  $X_2$ , each of which may be vector valued. Then  $X_1$  is said to be utility independent of  $X_2$  if, in cases where only consequences for  $X_1$  are uncertain and the value of  $X_2$  is known with certainty, the decision maker's attitude towards risk taking is independent of the particular value at which  $X_2$  is fixed.

More precisely,

$$u(x_1, x_2) = f(x_2) + g(x_2) u(x_1, x_2^0) \quad (4)$$

for some fixed  $x_2^0 \in X_2$  where the only restrictions on  $f$  and  $g$  are that  $g(x_2) > 0$ . This form comes from the fact that two utility functions are strategically equivalent under uncertainty if and only if they are positive linear transformations of each other. If, in addition,  $X_2$  is utility independent of  $X_1$ , then

(see Keeney [5])

$$u(x_1, x_2) = u(x_1, x_2^0) + u(x_1^0, x_2) + k u(x_1, x_2^0) u(x_1^0, x_2) \quad (5)$$

for some constant  $k > 0$ , where  $u(x_1^0, x_2^0) = 0$ .

For a comprehensive discussion of utility functions and related independence assumptions see Keeney and Raiffa [6].

A generalization of utility independence is conditional utility independence (see Chapter 6 of [6]). For a situation having three attributes  $X_1, X_2, X_3$  (each possibly vector valued),  $X_1$  is conditionally utility independent of  $X_2$ , given  $X_3$ , if, whenever  $X_3$  is fixed,  $X_1$  is utility independent of  $X_2$ . In this case

$$u(x_1, x_2, x_3) = f(x_2, x_3) + g(x_2, x_3) u(x_1, x_2^0, x_3) \quad (6)$$

for some fixed value  $x_2^0$ .

If in addition  $X_2$  is conditionally utility independent of  $X_1$ , given  $X_3$ , then we will say that  $X_1$  and  $X_2$  are mutually conditionally utility independent and write  $X_1$  mcui  $X_2$ . The conditional "given  $X_3$ " will be omitted since it will always be assumed here that the conditioning is on all the attributes not explicitly mentioned.

Lemma 1: If  $X_1$  mcui  $X_2$ , then

$$u(x_1, x_2, x_3) = [1 - k(x_3) u(x_1^0, x_2^0, x_3)] [u(x_1, x_2^0, x_3) + u(x_1^0, x_2, x_3) - u(x_1^0, x_2^0, x_3)] + k(x_3) u(x_1, x_2^0, x_3) u(x_1^0, x_2, x_3),$$

where  $x_1^0, x_2^0$  and  $x_3^0$  are arbitrary values of  $X_1, X_2, X_3$ ;  $k(x_3)$  is an arbitrary function, and  $X_3$  represents all attributes other than  $X_1$  and  $X_2$ .

Proof: With  $X_3$  fixed at an arbitrary level  $\bar{x}_3$  and defining  $\bar{u}(x_1, x_2, x_3) = u(x_1, x_2, x_3) - u(x_1^0, x_2^0, x_3)$ , it can be seen from (5)

that

$$\bar{u}(x_1, x_2, \bar{x}_3) = \bar{u}(x_1, x_2^0, \bar{x}_3) + \bar{u}(x_1^0, x_2, \bar{x}_3) + k\bar{u}(x_1, x_2^0, \bar{x}_3) \bar{u}(x_1^0, x_2, \bar{x}_3) \quad (7)$$

where  $k$  now may depend upon the value  $\bar{x}_3$ . Since (7) will hold for all choices  $\bar{x}_3$ , resubstitution in (7) for  $u$ , explicitly recognizing  $k \equiv k(x_3)$  and rearranging terms, gives the result. ||

It will be most convenient to abuse normal notation slightly in order to make a number of subsequent formulas look neater and more meaningful. For each attribute  $X_i$  a value  $x_i^0$  is selected and the utility function scaled so that  $u(x^0) = 0$ . Then whenever an attribute is at its fixed level it will be omitted from explicit mention in the vector array of attribute values. Hence  $u(x_1, x_2^0, x_3^0)$  will be written  $u(x_1, x_3)$ ,  $u(x_1^0, x_2, x_3^0)$  as  $u(x_2)$  and so on. Thus when  $X_1$  mcui  $X_2$  we may write

$$u(x_1, x_2, x_3) = [1 - k(x_3) u(x_3)] [u(x_1, x_3) + u(x_2, x_3) - u(x_3)] \\ + k(x_3) u(x_1, x_3) u(x_2, x_3) .$$

The following lemma will enable us to use induction to prove the main result.

Lemma 2: If  $X_1$  mcui  $\{X_2, X_3\}$  then  $X_1$  mcui  $X_2$  and  $X_1$  mcui  $X_3$ .

Proof: Let  $X_4$  be a vector attribute combining all attributes not including  $X_1, X_2, X_3$ , if any.

If  $X_1$  mcui  $\{X_2, X_3\}$  then by Lemma 1

$$u(x_1, x_2, x_3, x_4) = [1 - k(x_4) u(x_4)] [u(x_1, x_4) + u(x_2, x_3, x_4) - u(x_4)] \\ + k(x_4) u(x_1, x_4) u(x_2, x_3, x_4) . \quad (8)$$

for some function  $k(x_4)$ . If  $X_3$  and  $X_4$  are fixed at any values  $\bar{x}_3, \bar{x}_4$ , say, and denoting  $u(\bar{x}_4)$  by  $c_1$ ,  $k(\bar{x}_4)$  by  $c_2$ ,  $u(x_1, \bar{x}_4)$  by  $u_1(x_1)$ ,  $u(x_2, \bar{x}_3, \bar{x}_4)$  by  $u_2(x_2)$ , then from (8) we have

$$u(x_1, x_2, x_3, x_4) = -c_1(1-c_1c_2) + (1-c_1c_2)(u_1(x_1) + u_2(x_2)) + c_2u_1(x_1) u_2(x_2) ,$$

which shows that for any fixed values of  $X_3$  and  $X_4$ ,  $X_1$  and  $X_2$  are mutually utility independent (compare (5)), and thus  $X_1$  m.c.u.i.  $X_2$ . Similarly  $X_1$  m.c.u.i.  $X_3$ . ||

The converse of the lemma is not always true although it is easily shown that  $X_1$  c.u.i.  $X_2$ ,  $X_1$  c.u.i.  $X_3$  does imply  $X_1$  c.u.i.  $\{X_2, X_3\}$ .

Theorem: Assuming that

- (i)  $\{X_1, X_2, \dots, X_{i-1}\}$  is mutually conditionally utility independent with  $\{X_{i+1}, \dots, X_T\}$  for all  $i = 2, \dots, T-1$ ,
- (ii) For each value  $x_i$  of  $X_i$  there exist values  $x_{i-1}^*$  of  $X_{i-1}$  and  $x_{i+1}^*$  of  $X_{i+1}$  such that

$$u(x_{i-1}^*, x_i) \neq u(x_{i-1}^0, x_i)$$

$$u(x_i, x_{i+1}^*) \neq u(x_i, x_{i+1}^0) ,$$

then for  $T \geq 4$  either

$$a) \quad u(x_1, x_2, \dots, x_T) = \sum_{i=1}^{T-1} u(x_i, x_{i+1}) - \sum_{i=2}^{T-1} u(x_i) ,$$

or

$$b) \quad u(x_1, x_2, \dots, x_T) = \left[ \prod_{i=2}^{T-1} (\lambda + u(x_i)) \right]^{-1} \left[ \prod_{i=1}^{T-1} (\lambda + u(x_i, x_{i+1})) \right]^{-\lambda} ,$$

where  $\lambda$  is a constant which may be taken as  $\pm 1$ .

Proof: First, fix  $X_i$  at level  $x_i^0$  for all  $i = 5, 6, \dots, T$ ; then by assumption (i) and Lemma 2 we have that  $\{X_1, X_2\}$  is mutually conditionally utility independent with  $X_4$ , and  $X_1$  is mutually conditionally utility independent with  $\{X_3, X_4\}$ . Regarding  $X_1, X_2$  as one vector attribute we may use Lemma 1 to give that



$$u(x_1, x_2, x_3, x_4) = [1-s(x_3) u(x_3)] [u(x_1, x_2, x_3) + u(x_3, x_4) - u(x_3)] \\ + s(x_3) u(x_1, x_2, x_3) u(x_3, x_4) ; \quad (9)$$

and regarding  $x_3, x_4$  as a single attribute Lemma 1 gives

$$u(x_1, x_2, x_3, x_4) = [1-k(x_2) u(x_2)] [u(x_1, x_2) + u(x_2, x_3, x_4) - u(x_2)] \\ + k(x_2) u(x_1, x_2) u(x_2, x_3, x_4) \quad (10)$$

for some functions  $s(x_3)$  and  $k(x_2)$ .

Substitution of  $x_1 = x_1^0$  in (9) gives

$$u(x_2, x_3, x_4) = [1-s(x_3) u(x_3)] [u(x_2, x_3) + u(x_3, x_4) - u(x_3)] \\ + s(x_3) u(x_2, x_3) u(x_3, x_4) , \quad (11)$$

and  $x_4 = x_4^0$  in (10) gives

$$u(x_1, x_2, x_3) = [1-k(x_2) u(x_2)] [u(x_1, x_2) + u(x_2, x_3) - u(x_2)] \\ + k(x_2) u(x_1, x_2) u(x_2, x_3) . \quad (12)$$

Now substitute (11) into (10) and (12) into (9), then subtraction of (10) from (9) gives that

$$A(x_2, x_3) [-u(x_2) u(x_3) + u(x_3) u(x_1, x_2) + u(x_2) u(x_3, x_4) - u(x_1, x_2) u(x_3, x_4)] \equiv 0$$

where

$$A(x_2, x_3) = s(x_3) - k(x_2) - s(x_3) k(x_2) [u(x_2) - u(x_3)] .$$

Suppose that there exist values of  $x_2, x_3$ , say  $x_2^*, x_3^*$ , such that

$$A(x_2^*, x_3^*) \neq 0 ,$$

then it must be that

$$\begin{aligned}
 & - u(x_2^*) u(x_3^*) + u(x_3^*) u(x_1, x_2^*) + u(x_2^*) u(x_3^*, x_4) \\
 & - u(x_1, x_2^*) u(x_3^*, x_4) \equiv 0 \quad \text{for all } x_1, x_4 . \quad (13)
 \end{aligned}$$

By assumption we may choose a value of  $x_4, x_4^*$  such that  $u(x_3^*, x_4^*) \neq u(x_3^*, x_4^0)$ ; hence from (13)

$$[u(x_1, x_2^*) - u(x_2^*)][u(x_3^*, x_4^*) - u(x_3^*, x_4^0)] = 0 ,$$

which implies that

$$u(x_1, x_2^*) = u(x_1^0, x_2^*)$$

for all  $x_1$  -- a contradiction to assumption (ii).

Hence  $A(x_2, x_3) \equiv 0$  .

Thus

$$s(x_3) = k(x_2) / [1 - k(x_2) u(x_2) + k(x_2) u(x_3)] ,$$

so that if  $k(x_2^*) = 0$  for some  $x_2^*$ , then  $s(x_3) \equiv 0$  (similarly  $s(x_3^*) = 0$  implies  $k(x_2) \equiv 0$ ); otherwise

$$s(x_3) = 1 / [k(x_2)^{-1} - u(x_2) + u(x_3)] ,$$

implying that

$$k(x_2)^{-1} - u(x_2) = \text{constant} = \lambda, \text{ say,}$$

$$\text{or} \quad k(x_2) = (\lambda + u(x_2))^{-1}, \quad (14)$$

$$\text{and} \quad s(x_3) = (\lambda + u(x_3))^{-1} . \quad (15)$$

Substituting (12), (14) and (15) into (9) gives

$$u(x_1, x_2, x_3, x_4) = [\lambda + u(x_1, x_2)][\lambda + u(x_2, x_3)][\lambda + u(x_3, x_4)] / [\lambda + u(x_2)][\lambda + u(x_3)] - \lambda \quad (16)$$

If  $k(x_2) \equiv s(x_3) \equiv 0$  then

$$u(x_1, x_2, x_3, x_4) = u(x_1, x_2) + u(x_2, x_3) + u(x_3, x_4) - u(x_2) - u(x_3) \quad (17)$$

Equations (16) and (17) show that the Theorem is true when  $X_i$  is fixed at  $x_i^0$  for  $i \geq 5$ . Now we proceed by induction, assume the theorem is true whenever  $X_i$  is fixed at  $x_i^0$  for  $i \geq n+1$  and consider the case when  $X_i$  is fixed at  $x_i^0$  for  $i \geq n+2$ .

First of all, regarding  $X_n' = X_n \times X_{n+1}$  as a single attribute, by induction we know that either

$$u(x_1, \dots, x_{n+1}) = \sum_{i=1}^{n-2} u(x_i, x_{i+1}) + u(x_{n-1}, x_n, x_{n+1}) - \sum_{i=2}^{n-1} u(x_i) \quad (18)$$

or

$$u(x_1, \dots, x_{n+1}) = \frac{\prod_{i=1}^{n-2} (\lambda + u(x_i, x_{i+1})) (\lambda + u(x_{n-1}, x_n, x_{n+1}))}{\prod_{i=2}^{n-1} (\lambda + u(x_i))} - \lambda \quad (19)$$

Similarly, regarding  $X_1 \times X_2$  as a single attribute we know that either

$$u(x_1, \dots, x_{n+1}) = u(x_1, x_2, x_3) + \sum_{i=3}^n u(x_i, x_{i+1}) - \sum_{i=3}^n u(x_i) \quad (20)$$

or

$$u(x_1, \dots, x_{n+1}) = \frac{(\theta + u(x_1, x_2, x_3)) \prod_{i=3}^n (\theta + u(x_i, x_{i+1}))}{\prod_{i=3}^n (\theta + u(x_i))} - \theta \quad (21)$$

If (18) and (20) are true simultaneously then simple comparison

gives

$$u(x_1, \dots, x_{n+1}) = \sum_{i=1}^n u(x_i, x_{i+1}) - \sum_{i=2}^n u(x_i)$$

as required.

If (19) and (20) are true simultaneously then fixing  $x_i$  at  $x_i^*$  except for  $i = 1, n+1$  and equating gives

$$u(x_1) + u(x_{n+1}) = -\lambda + \frac{(\lambda + u(x_1))(\lambda + u(x_{n+1}))}{\lambda} ,$$

implying that

$$\frac{1}{\lambda} u(x_1) u(x_{n+1}) = 0 ,$$

which if true would contradict assumption (ii). The same applies if (18) and (21) are true simultaneously.

If the remaining combination of (19) and (20) are true together then again fixing  $x_i$  at  $x_i^0$  for all  $i$  except  $1, n+1$  and equating we have that

$$-\lambda + \frac{(\lambda + u(x_1))(\lambda + u(x_{n+1}))}{\lambda} = -\theta + \frac{(\theta + u(x_1))(\theta + u(x_{n+1}))}{\theta} ,$$

implying that

$$\theta u(x_1) u(x_{n+1}) = \lambda u(x_1) u(x_{n+1}) .$$

This can be achieved without contradicting assumption (ii) only if  $\theta = \lambda$ .

As the method of induction may be continued up to and including  $n+1 = T$  the theorem is proved. ||

Note that because utility functions are unique only up to positive linear transformations it is possible to scale  $u(x_1, \dots, x_T)$  so that  $\lambda = \pm 1$ . The transformation  $u = \lambda u'$  if  $\lambda > 0$  or  $u = -\lambda u'$  if  $\lambda < 0$  will serve to cancel the  $\lambda$ 's.

The functional form of the utility function still requires the assessment of T-1 two-attribute utility functions

$$u_i(x_i, x_{i+1}) = u(x_1^0, x_2^0, \dots, x_{i-1}^0, x_i, x_{i+1}, x_{i+2}^0, \dots, x_T^0) \quad ,$$

with the restriction that

$$u_i(x_i^0, x_{i+1}) = u_{i+1}(x_{i+1}, x_{i+2}^0) \quad i = 1, \dots, T-2 \quad . \quad (22)$$

This can be simplified still further if the decision maker's preferences are stationary (Koopmans [7]), for then for each i

$$u_i(x, y) = \alpha_i u^*(x, y)$$

for some constant  $\alpha_i$ , where normally  $0 \leq \alpha_i \leq 1$ , and some function  $u^*$ . But because of (22)

$$\alpha_i u^*(x_i^0, x_{i+1}) = \alpha_{i+1} u^*(x_{i+1}, x_{i+2}^0) \quad ,$$

so if  $x_1^0 = x_2^0 = \dots = x_T^0$  then

$$\frac{\alpha_{i+1}}{\alpha_i} = \alpha, \text{ a constant.}$$

Corollary: The conditions of the theorem, plus an assumption of stationarity and  $x_i^0 = x^0$  for all i, implies that

$$u(x_1, \dots, x_T) = u^*(x_1, x_2) + \sum_{i=2}^{T-1} \alpha^{i-1} (u^*(x_1, x_{i+1}) - u^*(x_i, x^0))$$

or

$$u(x_1, \dots, x_T) = \left[ \prod_{i=2}^{T-1} (\lambda + \alpha^{i-1} u^*(x_i, x^0)) \right]^{-1} \left[ \prod_{i=1}^{T-1} (\lambda + \alpha^{i-1} u^*(x_i, x_{i+1})) \right]^{-\lambda}$$

where  $\alpha$  is constant and  $u^*(x^0, y) = \alpha u^*(y, x^0)$ .

The Application

Returning to the forest pest study which led to the fore-going theory, the decision maker felt that as long as he knew the levels of profit and employment in neighbouring years, his attitude to risk in any given year was independent of the profit/employment levels in other years. Hence regarding  $X_i = P_i \times E_i$  as a single attribute the conditions of the theorem seemed to be appropriate. This meant that the four-dimensional functions

$$u_i(p_i, e_i, P_{i+1}, e_{i+1})$$

were required for all  $i$ . But also, he felt that given everything else was fixed,  $(P_i, E_i)$  pairs were mutually utility independent with  $P_{i-1}$  and  $P_{i+1}$ .

Thus with  $X_1, \dots, X_{i-2}, X_{i+1}, \dots, X_T$  fixed his assumptions were

$$(P_i, E_i) \quad \text{mcui} \quad P_{i-1}$$

and  $(P_{i-1}, E_{i-1}) \quad \text{mcui} \quad P_i$  .

But these are exactly the assumptions used in the first part of the proof of the theorem (associate  $P_{i-1}$  with  $X_1$ ,  $E_{i-1}$  with  $X_2$ ,  $E_i$  with  $X_3$  and  $P_i$  with  $X_4$ ); hence

$$u(p_{i-1}, e_{i-1}, p_i, e_i) = u(p_{i-1}, e_{i-1}) + u(p_i, e_i) + u(e_{i-1}, e_i) - u(e_{i-1}) - u(e_i)$$

or

$$u(p_{i-1}, e_{i-1}, p_i, e_i) = \frac{(\lambda_i + u(p_{i-1}, e_{i-1})) (\lambda_i + u(p_i, e_i)) (\lambda_i + u(e_{i-1}, e_i))}{(\lambda_i + u(e_{i-1})) (\lambda_i + u(e_i))} - \lambda_i$$

If we assume stationarity of preferences in addition, it may be shown that  $\lambda_i = \lambda$  for all  $i$ , and the assessment of the utility function  $u_s(p, e)$  of (3) requires only two-attribute utility functions  $u_A(p, e)$  and  $u_B(e_1, e_2)$  together with a number of

constants, giving a final functional form of

$$\frac{\prod_{i=1}^T (\lambda + \alpha^i u_A(p_i, e_i)) \prod_{i=1}^{T-1} (\lambda + \alpha^i u_B(e_i, e_{i+1}))}{\prod_{i=1}^T (\lambda + \alpha^i u_B(e_i, e^0)) \prod_{i=2}^{T-1} (\lambda + \alpha^i u_B(e_i, e^0))}$$

where  $u_A(p^0, e) = u_B(e, e^0)$ ,  $u_B(e^0, e) = \alpha u_B(e, e^0)$  and  $\lambda$  and  $\alpha$  are constants.

A description of the forest pest problem and the preliminary analysis of the decision maker's preferences are reported in Holling et al. [ 4 ] and Bell [ 2 ] respectively. A future paper will discuss the assessment of the utility function in more detail.

#### Summary

It has been shown that it is not necessary to require utility independence amongst all attributes in a time stream in order to have a manageable assessment task, and that explicit account can be taken of preferences which depend on the values assumed by attributes in other periods.

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