WORKING PAPER

THE COMPUTER-AIDED DESIGN OF A SERVO SYSTEM AS A MULTIPLE-CRITERIA DECISION PROBLEM

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PREFACE

In this paper, Alexander Udink ten Cate treats the task of selecting the controller gains of a servo system as a multiple-criteria decision problem. In contrast to the usual optimization-based approaches to computer-aided design, inequality constraints are included in the problem as unconstrained objectives. This considerably simplifies the optimization process when the constraints are imposed on the trajectories of a dynamic system. The objectives are evaluated by simulation of the behavior of the dynamic system over time using the DIDASS/N software package. The author finds this approach promising for relatively small problems (up to ten objectives), especially because in the field of control engineering there are many simulation packages that could be linked to some version of DIDASS/N.

This research was carried out as part of the work on interactive decision analysis in the System and Decision Sciences Program.

A.B. Kurzhanski
Chairman
System and Decision
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ABSTRACT

The task of selecting the controller gains of a servo system is formulated as a multiple-criteria decision problem. The criteria are based on the unit step response of the system. The approach described here differs from the usual approach in that design constraints on the trajectories are included as additional criteria. Simulation runs are used to evaluate the criteria.

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1. INTRODUCTION

The computer-aided design of a control system can be regarded as an optimization problem in which several design and performance criteria have to be satisfied. This would normally take the form of a single-criterion optimization problem with the design and performance criteria taken as equality or inequality constraints (Mayne et al., 1982; Gustafson and Desoer, 1983). Such constrained optimization formulations appear very complex (Mayne et al., 1982), which might explain why only a few references to this approach can be found in the control engineering literature.

Another approach to computer-aided control-system design is based on developments in multiple-criteria decision making (see Zeleny, 1982, for an introduction), using the techniques of multi-objective optimization. This approach is intuitively more appealing, since in engineering the designer typically compares several conflicting criteria before arriving at some "optimal" or "best" solution. The use of multiobjective programming to solve static problems in chemical engineering has been reported by Grauer et al. (1983). Multiobjective methods capable of dealing with dynamical problems have been applied by Tabak et al. (1979) to the design of an aircaft control system and by Franke and Ester (1983) to a servo control system. These applications involve inequality constraints on the process dynamics, which complicates the solution.

In this paper it is proposed to regard the inequality constraints on the process trajectories as unconstrained objectives. This is possible because in design these constraints are not usually very strict. In the example presented here--a servo system--the objectives are evaluated by means of a simulation in the time domain. The optimization method chosen is the reference point approach (Wierzbicki, 1981), in which aspiration levels (or reference points) are set by the designer. An efficient solution is identified by minimizing the "distance" (measured according to some norm) between the reference point and the points in the Pareto set. The designer can then change his aspiration levels and perform another optimization in an interactive procedure very familiar in computer-aided design. Applications of this approach to large static problems (concerning a bridge and a camera lens), including the use of simulation to evaluate some objective functions, are described by Nakayama and Sawaragi (1984). The methods and related software (Grauer and Kaden, 1984) used in the present paper are the same as those used in Grauer et al. (1983); the main difference is that dynamical systems with design constraints on the trajectories are considered here, leading to a family of objective functions which are evaluated over time by means of simulation runs.

2. MULTIOBJECTIVE OPTIMIZATION

The main aim in multiobjective optimization is to find Pareto-optimal solutions. Loosely speaking, a solution is Pareto-optimal if it is not possible to improve the value of any of the objectives without causing the value of at least one of the others to deteriorate. This concept of optimality was developed because in multiobjective situations a straightforward comparison between objectives is not generally possible. The definition of Pareto-optimality usually leads to a set of possible solutions (the Pareto set), from which the user has to select one.

The example presented in this paper is based on the reference point approach to multiple-criteria analysis. The principle behind this method (Wierzbicki, 1981; Grauer and Kaden, 1983) is to rank the vectors of decision alternatives $\underline{q} \in \mathbb{R}^p$, $p \geq 2$, relative to a reference point $\underline{\bar{q}}$ which reflects the aspirations or preferences of the user. This ranking is based on the partial ordering $\underline{q}^1 \leq \underline{q}^2$ iff $q_i^1 \leq q_i^2$ $\forall i$; $\underline{q}^1, \underline{q}^2 \in \mathbb{R}^p$. The objectives $\underline{q}$ are functions of the decision variables $\underline{x}$.

A solution is now sought by minimizing an achievement scalarizing function $f(q-\bar{q})$. This identifies the point $\underline{\hat{q}}$ in the Pareto set closest to the reference point $\bar{q}$. Then, in an interactive procedure, a sequence of Pareto points $\{\underline{\hat{q}}\}$ is generated from a sequence of reference points $\{\underline{\bar{q}}\}$. A solution is found by letting the $\{\underline{\bar{q}}\}$ converge to $\{\underline{\hat{q}}\}$. Note that the decision variables $\underline{x}$ in $q(\underline{x})$ are subject to inequality constraints. An overview of the reference point approach is given in Grauer (1983).

The reference point approach forms the basis of the DIDASS/N software package (Grauer and Kaden, 1984). This package also calculates the utopia point $\underline{q}^*$, where $\underline{q}^*$ is the vector of solutions obtained by optimizing each of the objectives separately. In the nonlinear case treated in this paper, the following achievement scalarizing function is minimized:

$$s(w) = -\frac{1}{\rho} \ln \left\{ \frac{1}{p} \sum_{i+1}^{p} (w_i)^{\rho} \right\} \tag{1a}$$

$$w_i = \gamma_i [(\tilde{q}_i - q_i)/(\tilde{q}_i - \bar{q}_i)] \quad , \tag{1b}$$

where $\underline{\tilde{q}}$ is an upper limit to the sequence of reference points, $\rho \geq p$ is an arbitrary coefficient, and the γ_i, $i = 1,\ldots,p$, are weighting factors. The resulting single-criterion programming problem is then solved using the MINOS software package (Murtagh and Saunders, 1980).

3. DESIGN OF A SERVO SYSTEM

Consider the servo system shown in Figure 1 and described by the differential equation

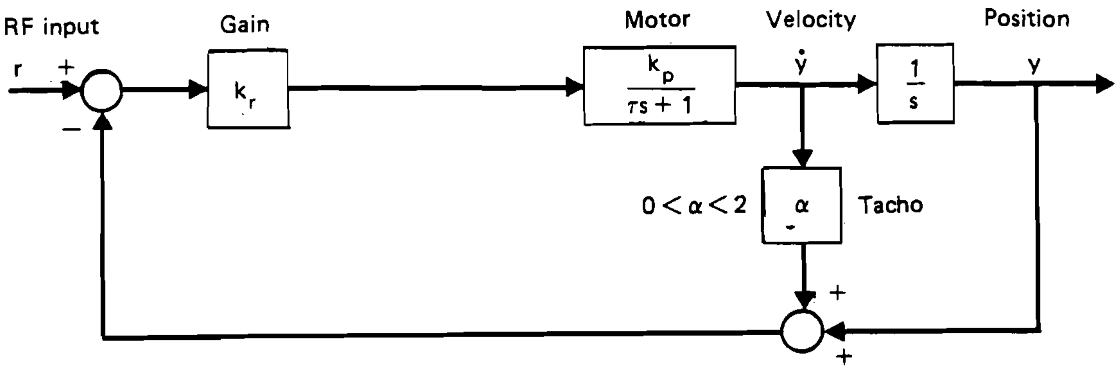

FIGURE 1. A servo system.

$$\frac{\tau}{k_p k_r} \frac{d^2 y}{dt^2} + \left(\frac{1}{k_p k_r} + \alpha \right) \frac{dy}{dt} + y = r(t) \quad , \qquad (2)$$

where τ = 10 seconds and k_p = 0.1. The problem is to select the values of k_r and α according to certain criteria. These criteria are based on the unit step response (Figure 2), and are the minimization of the rise time, the overshoot and the ITAE criterion

$$\text{ITAE} = \int_T^0 | \; e(t) \; dt \quad , \qquad (3)$$

where $e(t) \hat{=} r(t) - y(t)$. An upper limit is also imposed on the motor input. This leads to four objective functions (which should be minimized):

$$\text{obj1 (rise time)} = \min_{0 \le t \le T} \{t | y(t) \ge r(t)\} \qquad (4a)$$

$$\text{obj2 (overshoot)} = \frac{\max\{y(t) | 0 \le t \le T\}}{r(t)} \qquad (4b)$$

$$\text{obj3 (ITAE)} = \text{see eqn. (3)} \qquad (4c)$$

$$\text{obj4 } (L_{max}) = \max\{k_r e(t) | 0 \le t \le T\} \quad . \qquad (4d)$$

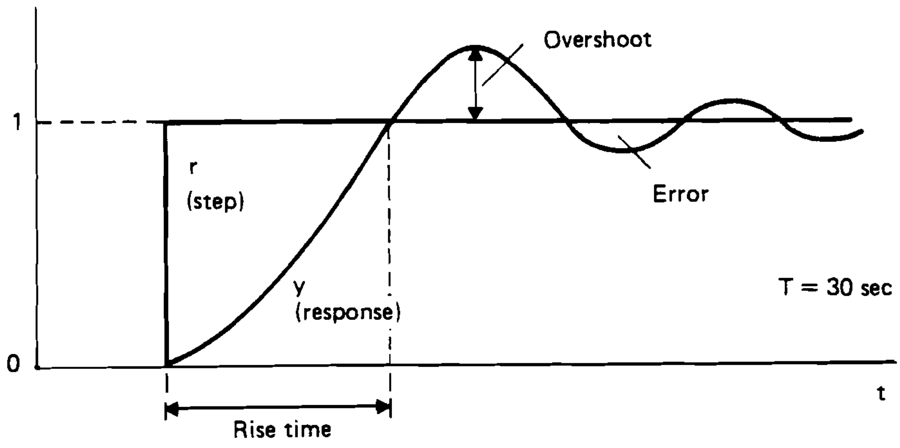

FIGURE 2. Unit step response.

The decision variables k_r and α have the following bounds:

$$1 \leq k_r \leq 50 \tag{5a}$$

$$0 \leq \alpha \leq 2 \quad . \tag{5b}$$

The objectives (4a-d) were evaluated in a simulation run of system (2), taking T = 30 sec. A simple Euler method with a time difference of 0.125 sec was used for integration. The problem was solved using the DIDASS/N software package (Grauer and Kaden, 1984), with the gradients calculated numerically taking a difference interval of 10^{-3}.

The final results obtained in a computer-aided design experiment are given in Table 1. The corresponding values of the decision variables are $k_r = 4.087$, $\alpha = 1.423$.

Table 1. Calculation of efficient points.

Objective	Scale	Reference point	Efficient point	Utopia point	Nadir point
obj1	1.0	10.0	10.125	4.125	25.625
obj2	1.0	1.2	1.305	1.026	1.344
obj3	1.0	50.0	72.3	2.63	106.2
obj4	1.0	4.0	4.3	1.0	50.0

The nadir values given in Table 1 are the worst (in this case the highest) values of each objective obtained on minimizing individual objectives separately. Thus the utopia point and the nadir point provide upper and lower guidelines for specifying the reference point. The table shows that the efficient point represents a reasonable compromise solution. The requirements of the third objective (eqn. 4c) are not completely satisfied, but in an engineering design the result would be quite acceptable.

In the example presented above, the objective values were obtained by simulation. In this particular case an analytical solution of the unit step response of the second-order system could also be obtained, which reduces the amount of computation considerably. However, for more complicated servo systems (especially those which include nonlinearities in the form of saturations or hysteresis) simulation is essential.

When using the DIDASS/N program to solve this particular problem, it was noted that the chosen optimization method (as implemented in MINOS/N) frequently fails. Therefore, other optimization methods should be employed in this type of multiple-objective decision making.

4. CONCLUSIONS

This paper treats the selection of the controller gains of a servo system as a multiple-criteria decision problem. Unlike the usual optimization-based approach to computer-aided design, inequality constraints are included in the problem as design criteria or objectives. This approach considerably simplifies the optimization process when the constraints are imposed upon trajectories of a dynamic system. The objectives are evaluated by simulation of the behavior of the dynamic system over time.

It is shown that this approach can yield satisfactory results when four objectives and two decision variables are involved. The results are obtained interactively using the DIDASS/N software package (Grauer and Kaden, 1984) based on the reference point method. It is felt that this approach is promising for relatively small problems (up to ten objectives), especially because in the field of control engineering there are many fast, block-oriented, FORTRAN-callable simulation packages which might be linked to some version of DIDASS/N. Also, the interactive procedure is very appealing to a designer in the control field. However, it was noted that the optimization algorithms employed does not always yield reliable results.

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