WORKING PAPER

DERIVED DEMAND AND SUBSTITUTION FOR FOREST PRODUCTS BASED ON COBB-DOUGLAS AND CES PRODUCTION FUNCTIONS

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December 1984
WP-84-104

IIASA
International Institute
for Applied Systems Analysis

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FOREWORD

The objective of the Forest Sector Project at IIASA is to study long-term development alternatives for the forest sector on a global basis. The emphasis in the Project is on issues of major relevance to industrial and governmental policy makers in different regions of the world who are responsible for forestry policy, forest industrial strategy, and related trade policies.

The key elements of structural change in the forest industry are related to a variety of issues concerning demand, supply, and international trade of wood products. Such issues include the development of the global economy and population, new wood products and substitution for wood products, future supply of roundwood and alternative fiber sources, technology development for forestry and industry, pollution regulations, cost competitiveness, tariffs and non-tariff trade barriers, etc. The aim of the Project is to analyze the consequences of future expectations and assumptions concerning such substantive issues.

The research program of the Project includes an aggregated analysis of long-term development of international trade in wood products, and thereby analysis of the development of wood resources, forest industrial production and demand in different world regions. This article analyzes the robustness of derived demand with respect to assumptions concerning underlying production functions. As a case, the wood consumption in the Canadian construction sector is chosen.

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Project Leader
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ABSTRACT

Standard production theory with Cobb-Douglas and CES production functions is applied to derive demand functions for forest products. Time-series data for 1961–1978 from Canadian construction sector is employed for estimation, and sensitivity of the demand forecast is tested with respect to the choice of the production function.

CONTENTS

DERIVED DEMAND AND SUBSTITUTION FOR FOREST PRODUCTS BASED ON COBB-DOUGLAS AND CES PRODUCTION FUNCTIONS

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1. INTRODUCTION

The purpose of this note is to study demand (and thereby also substitution) of various production inputs in a given production sector. We are particularly interested in such derived demand and substitution for forest products, and therefore, the construction sector which is a major consumer of mechanical wood products was selected for this study.

Using standard production theory, we first derive the demand functions when the production sector is described by Cobb-Douglas or Constant Elasticity of Substitution (CES) production functions. These functions will be estimated for the construction sector in Canada, for each of the years between 1961 and 1978. Finally, estimates will be provided for the demand function parameters in 1990. Testing will be carried out concerning the sensitivity of demand with respect to the assumptions on the production function.

2. ASSUMPTIONS

Consider a production sector (e.g., the construction sector) whose production (quantity) y is characterized by a production function

$$y = y(q_1, q_2, \cdots, q_n) \ , \tag{1}$$

with q_i being the input (quantity) of production factor i, for $i=1,2,\cdots,n$. Let p_i be the price of factor i and $y = y^0$ a given level of output. Assume that the input mix (q_i) results from cost minimizing:

$$\underset{q_i}{\text{minimize}} \sum_i p_i q_i \tag{2}$$

$$\text{s.t.} \quad y^0 = y(q_i) \ . \tag{3}$$

The optimality conditions for the Lagrangian function L are:

$$\frac{\partial L}{\partial \mu} = 0 \text{ and } \frac{\partial L}{\partial q_i} = 0 \quad \text{for all } i \ , \tag{4}$$

where μ is the dual multiplier for constraint (3).

3. THE COBB-DOUGLAS CASE

Let the production function be specified as

$$y(q_i) = a \prod_i q_i^{\beta_i} \tag{5}$$

where a and β_i are parameters. Condition (4) yields

$$p_i - \mu \frac{\beta_i}{q_i} a \prod_i q_i^{\beta_i} = 0 \ , \tag{6}$$

multiplying by q_i, summing over i, and observing (3) we obtain from (6)

$$\sum p_i q_i = \mu y_0 \sum \beta_i \tag{7}$$

By strong duality, $\sum p_i q_i = \mu y_0$, and therefore

$$\sum \beta_i = 1 \ . \tag{8}$$

Equation (6) yields also

$$\frac{p_i q_i}{p_j q_j} = \frac{\beta_i}{\beta_j} \quad \text{for all } i,j. \tag{9}$$

Relations (8)–(9) shall be used below for estimation of parameters β_i. Thereafter (5) may be employed for estimation of parameter a.

Solving q_i from (9), for all $i \neq j$, and substituting in (3), we obtain

$$y^0 = a\prod_i \left[\frac{p_j/\beta_j}{p_i/\beta_i} q_j\right]^{\beta_i} \tag{10}$$

Observing (8) we obtain from (10)

$$q_j = (y^0/a)\prod_i \left[\frac{p_i/\beta_i}{p_j/\beta_j}\right]^{\beta_i} \ . \tag{11}$$

This determines the derived demand for all j. Equivalently, if q_j^0 is the base year consumption, η the production volume index and λ_i the price index for input i, for all i, then

$$q_j = \eta q_j^0 \prod_i \left[\frac{\lambda_i}{\lambda_j}\right]^{\beta_i} = \eta q_j^0 \lambda_j^{1-\beta_j} \prod_{i \neq j} \lambda_i^{\beta_i} \ .$$

4. THE CES CASE

Assume now that

$$y(q_i) = b\left[\sum_i \gamma_i q_i^{\alpha}\right]^{1/\alpha} \ , \tag{12}$$

where $b > 0$, $\gamma_i \geq 0$, $\sum \gamma_i = 1$ and $\alpha \leq 1$. Condition (4) yields

$$p_i - \mu \gamma_i q_i^{\alpha-1} b\left[\sum_j \gamma_j q_j^{\alpha}\right]^{(1-\alpha)} = 0 \ . \tag{13}$$

This implies

$$\frac{p_i}{p_j}\left(\frac{q_i}{q_j}\right)^{1-\alpha} = \frac{\gamma_i}{\gamma_j} \quad \text{for all } i,j. \tag{14}$$

Solving q_i for all $i \neq j$ and substituting into (3) yields

$$y^0 = b\left[\sum_i \gamma_i q_j^\alpha \left(\frac{p_i/\gamma_i}{p_j/\gamma_j}\right)^{\alpha/(\alpha-1)}\right]^{1/\alpha} \tag{15}$$

or

$$q_j = (y^0/b)\left[\sum_i \gamma_i \left(\frac{p_i/\gamma_i}{p_j/\gamma_j}\right)^{\alpha/(\alpha-1)}\right]^{-1/\alpha} \tag{16}$$

5. APPLICATION TO THE CONSTRUCTION SECTOR IN CANADA

Time-series data for 1961–78 from the Canadian input–output statistics was employed to study the derived demand in the construction sector.

For the Cobb-Douglas case, the output elasticity parameters β_i and the scale factor a were estimated for each 18 years and tabulated in Table 1, which also indicates the input categories employed. Also estimates for the year 1990 are given. The development of parameters β_i over time is illustrated in Figure 1.

The CES-case was studied in two cases: first with the value for $1/(1-\alpha)$, the elasticity of substitution equal to .5, and the second with the elasticity of substitution equal to 0.1. Tables 2 and 3 present the values of the distribution parameters γ_i and of the scale parameter b over 1961–78 as well as two sets of estimates for 1990 for both cases. The first set is a conservative estimate assuming the distribution parameters to stay at the level of year 1978. The second set assumes a radical trend of the 1970s to continue until 1990. Forecasts are also presented in graphical form in Figure 2.

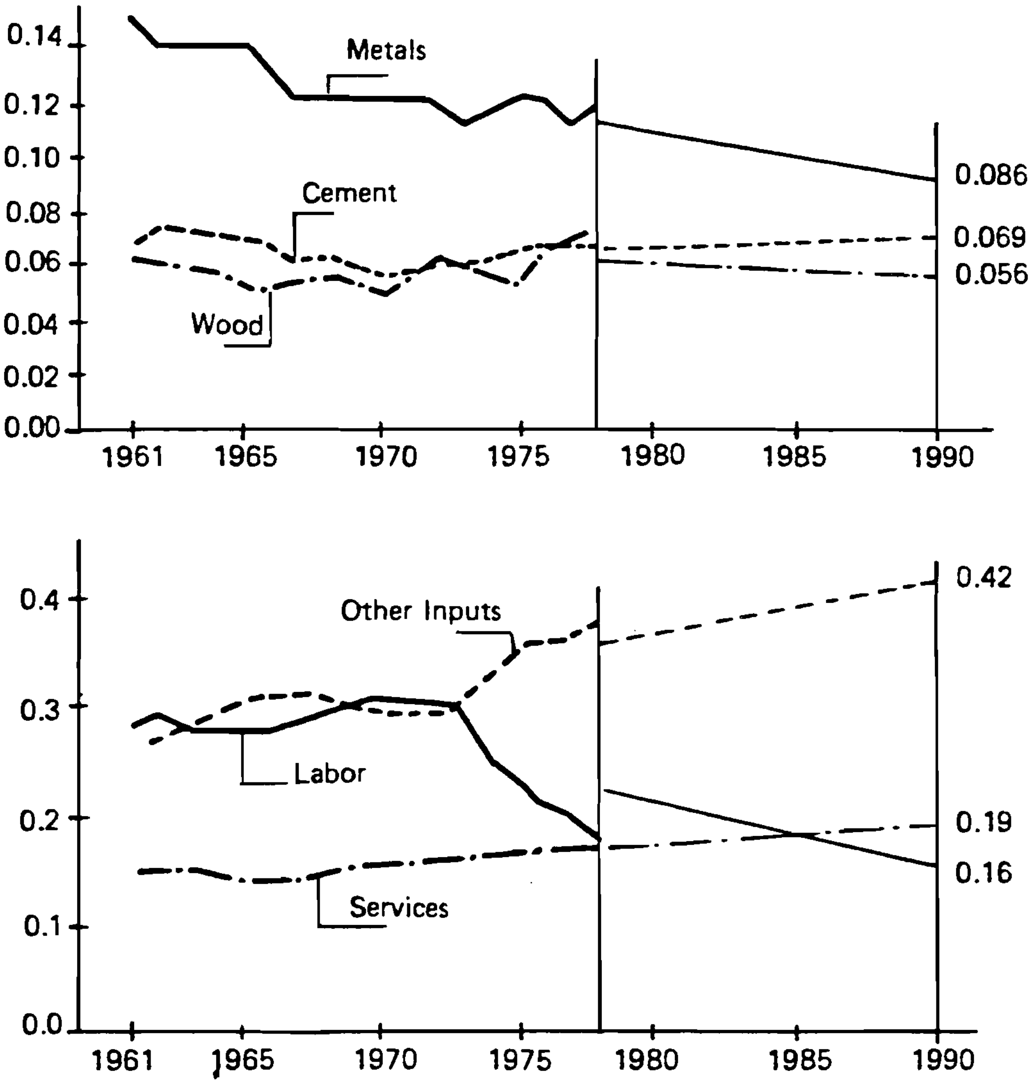

Figure 1. Elasticity parameters β_i for a Cobb-Douglas production function for the Canadian construction sector in 1961–78 and forecasts for 1990.

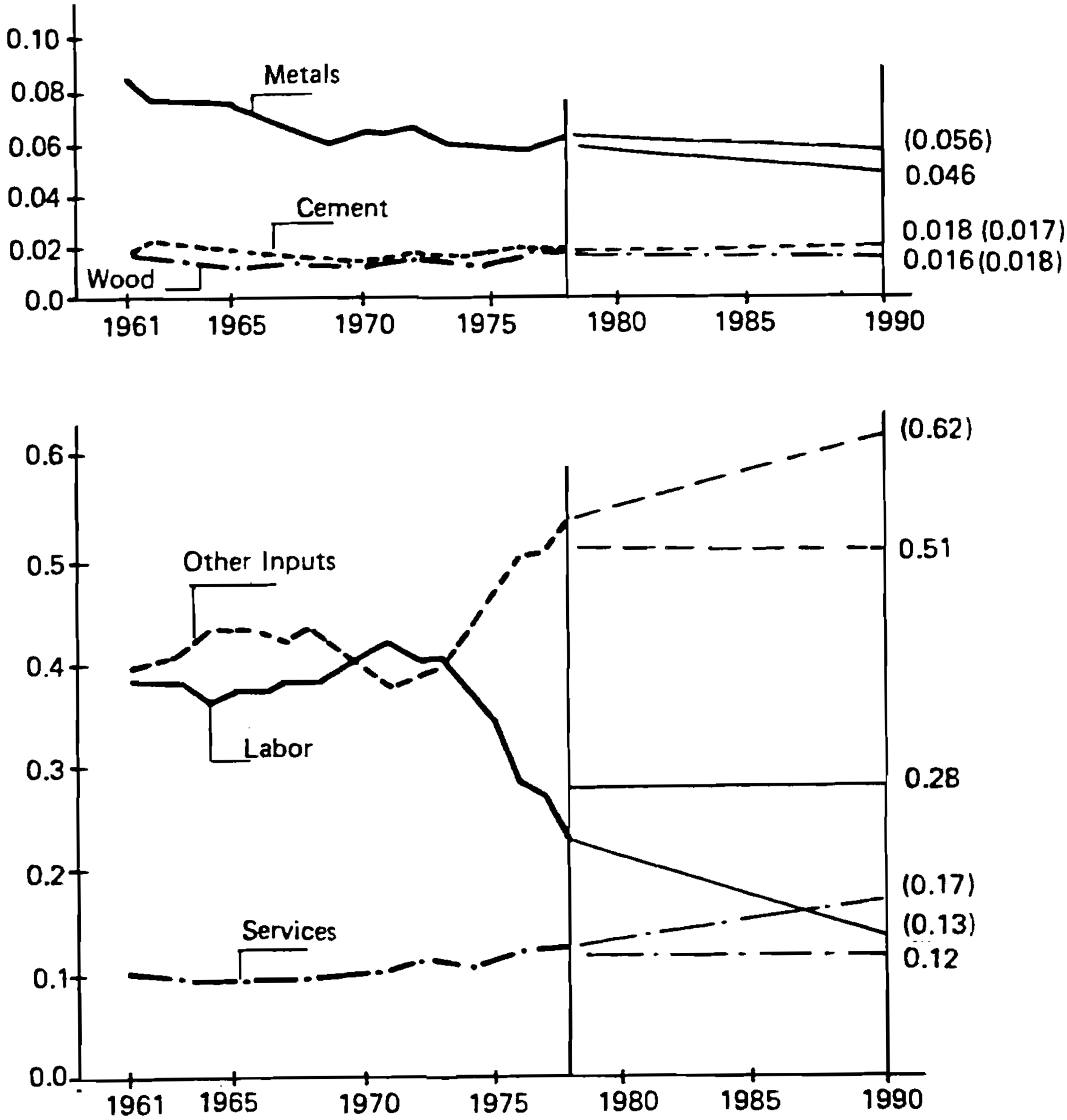

Figure 2. Distribution parameters γ_i for a CES production function (elasticity of substitution = 0.5) for the Canadian construction sector in 1961–78 and two sets of forecasts for 1980.

Applying formulas (11) and (16), the resulting demand forecasts for wood products for unit of output in the construction sector are shown in Table 4. Price indices and their forecasts for 1990 are given in Table 5. The results are also compared to the situation where prices would not change at all, or that relative prices were constant. The corresponding wood inputs in 1990 for the two price scenarios are .052 and .053, when the Diewert function approach is used (Andersson et al. 1984).

6. CONCLUSION

The results of Table 5 show that the choice of production function as such does not radically affect the derived demand levels: the difference between CD and CES results was only 0–4%, depending on the case. The results of the CES function proved also to be rather insensitive with respect to the value of elasticity parameter α, as well as to different forecast values of parameter γ_i. The effects of changes in relative prices appears to be more important for demand derived from the CD function than that from the CES function.

TABLE 1. Scale parameter α and elasticity parameters β_i for the Cobb-Douglas production function for the Canadian construction sector.

Year	α	β_i					
		Wood products	Metals	Cement	Labor	Services	Other
1961	5.2	.062	.15	.067	.28	.15	.26
1962	5.2	.060	.14	.074	.29	.15	.27
1963	5.2	.060	.14	.072	.28	.15	.28
1964	5.1	.058	.14	.071	.28	.14	.29
1965	5.0	.052	.14	.070	.28	.14	.30
1966	5.0	.051	.13	.068	.28	.14	.31
1967	4.9	.054	.12	.062	.29	.14	.31
1968	5.0	.057	.12	.063	.29	.15	.30
1969	4.9	.056	.12	.060	.30	.15	.30
1970	4.8	.049	.12	.055	.31	.15	.29
1971	4.9	.056	.12	.058	.31	.15	.29
1972	5.0	.061	.12	.060	.30	.16	.29
1973	5.0	.060	.11	.060	.29	.16	.30
1974	4.6	.057	.11	.061	.24	.16	.35
1975	5.0	.053	.12	.066	.23	.16	.34
1976	5.1	.067	.12	.066	.21	.17	.36
1977	5.1	.069	.11	.067	.20	.17	.36
1978	5.1	.071	.12	.066	.18	.17	.38
1990	5.1	.069	.086	.056	.16	.19	.42

TABLE 2. Scale parameter *b* and distribution parameters γ_i for the CES production function, elasticity of substitution equaling to 0.5, for the Canadian construction sector.

Year	b	γ_i					
		Wood products	Metals	Cement	Labor	Services	Other
1961	4.5	.017	.084	.017	.38	.098	.39
1962	4.4	.015	.076	.021	.38	.091	.40
1963	4.4	.015	.076	.020	.38	.091	.41
1964	4.4	.014	.076	.020	.36	.090	.43
1965	4.3	.011	.074	.019	.37	.088	.43
1966	4.3	.012	.068	.018	.37	.089	.43
1967	4.2	.012	.064	.015	.38	.090	.42
1968	4.2	.012	.061	.015	.38	.091	.43
1969	4.2	.012	.060	.014	.39	.095	.41
1970	4.2	.010	.065	.012	.41	.10	.39
1971	4.3	.013	.062	.014	.42	.10	.37
1972	4.4	.015	.064	.016	.40	.11	.38
1973	4.3	.012	.059	.016	.40	.11	.39
1974	4.0	.010	.056	.015	.30	.10	.50
1975	4.3	.011	.057	.017	.32	.11	.46
1976	4.4	.017	.057	.018	.28	.12	.50
1977	4.4	.017	.058	.018	.27	.12	.50
1978	4.4	.017	.061	.018	.22	.13	.54
1990 (conserv.)	4.4	.016	.056	.018	.28	.12	.51
1990 (radical)	4.4	.018	.046	.017	.13	.17	.62

TABLE 3. Scale parameter b and distribution parameters γ_i for the CES production function, elasticity of substitution equaling to 0.1, for the Canadian construction sector.

Year	b	γ_i: Wood products $x\,10^{-7}$	Metals $x\,10^{-4}$	Cement $x\,10^{-7}$	Labor	Services $x\,10^{-2}$	Other
1961	3.4	.415	.50	.30	.29	.018	.71
1962	3.4	.208	.30	.80	.27	.012	.73
1963	3.4	.198	.35	.63	.26	.013	.74
1964	3.3	.127	.35	.57	.20	.013	.80
1965	3.3	.050	.37	.59	.30	.013	.70
1966	3.3	.047	.28	.50	.33	.016	.67
1967	3.3	.085	.25	.21	.38	.019	.62
1968	3.3	.075	.21	.24	.31	.018	.69
1969	3.3	.062	.24	.17	.40	.025	.60
1970	3.4	.059	.42	.11	.52	.046	.48
1971	3.5	.206	.42	.31	.61	.072	.39
1972	3.5	.244	.54	.54	.58	.097	.41
1973	3.4	.045	.28	.46	.64	.067	.36
1974	3.4	.034	.22	.36	.38	.054	.62
1975	3.4	.038	.15	.46	.48	.072	.52
1976	3.4	.301	.17	.58	.28	.083	.72
1977	3.5	.379	.22	.72	.31	.102	.69
1978	3.4	.242	.21	.53	.11	.105	.89
1990 (conserv.)	3.4	.240	.19	.57	.30	.090	.70
1990 (radical)	3.4	.175	.093	.45	.042	.164	.94

TABLE 4. Price indices for inputs of construction sector.

Year	Wood products	Metals	Cement	Labor	Services	Other
1961	0.68	0.85	0.77	0.64	0.75	0.54
1962	0.70	0.85	0.76	0.66	0.75	0.55
1963	0.73	0.84	0.78	0.67	0.76	0.58
1964	0.75	0.85	0.80	0.68	0.77	0.61
1965	0.77	0.88	0.82	0.70	0.79	0.67
1966	0.80	0.90	0.85	0.72	0.81	0.74
1967	0.84	0.91	0.89	0.78	0.85	0.78
1968	0.92	0.91	0.91	0.82	0.88	0.77
1969	0.99	0.94	0.95	0.86	0.91	0.83
1970	0.91	0.98	0.98	0.94	0.96	0.89
1971	1.00	1.00	1.00	1.00	1.00	0.96
1972	1.16	1.04	1.03	1.02	1.04	1.00
1973	1.41	1.13	1.10	1.01	1.13	1.15
1974	1.52	1.40[1)]	1.28	0.97	1.29	1.42
1975	1.56	1.57	1.50	1.02	1.45	1.55
1976	1.76	1.66	1.62	1.06	1.59	1.74
1977	1.93	1.75	1.74	1.10	1.71	1.92
1978	2.18	1.88	1.87	1.11	1.85	2.03
1990	2.82	2.33	2.34	1.51	2.33	2.80

1) the original data material included an obvious error (original value for metals in 1974 was 0.40) and an estimated value was used. Table 1 to 3 were corrected accordingly.

TABLE 5. Demand for wood products in 1990 per unit of construction output.

Case	Prices from Table 4	Constant price	Difference (%)
Cobb-Douglas	.056	.066	−18
CES (α = −1, conservative)	.054	.060	−11
CES (α = −1, radical)	.057	.062	−9
CES (α = −9, conservative)	.056	.058	−4
CES (α = −9, radical)	.055	.056	−2
Diewert	.052	.053	−2

REFERENCES

Andersson, Å.E., R. Brännlund, and G. Kornai. 1984. The Demand for Forest Sector Products. WP-84-87. Laxenburg, Austria: International Institute for Applied Systems Analysis.