WORKING PAPER

ESTIMATION OF FOREST PRODUCTS DEMAND AS AN INTERMEDIARY FUNCTION

Åke E. Andersson

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IIASA
International Institute
for Applied Systems Analysis

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2361 Laxenburg, Austria

FOREWORD

The objective of the Forest Sector Project at IIASA is to study long-term development alternatives for the forest sector on a global basis. The emphasis in the Project is on issues of major relevance to industrial and governmental policy makers in different regions of the world who are responsible for forestry policy, forest industrial strategy, and related trade policies.

The key elements of structural change in the forest industry are related to a variety of issues concerning demand, supply, and international trade of wood products. Such issues include the development of the global economy and population, new wood products and substitution for wood products, future supply of roundwood and alternative fiber sources, technology development for forestry and industry, pollution regulations, cost competitiveness, tariffs and non-tariff trade barriers, etc. The aim of the Project is to analyze the consequences of future expectations and assumptions concerning such substantive issues.

In this article the problem of demand forecasting is discussed from a quantitative point of view. It is shown that an intermediate demand approach is preferable to the common final demand procedures of forest product demand studies.

Markku Kallio
Project Leader
Forest Sector Project

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INTRODUCTION

Forest sector products are intermediary commodities. Less than 15 percent of total output is delivered to households for final consumption. A study of seven OECD countries shows a marked decline of the household share in paper purchases in the period 1965–1975. A projection for the US indicates that the household purchases of paper will drop below 10 percent of total output in the 1980s. Any development of improved methods for demand forecasting must consequently address the issue of intermediate demand and its determinants.

METHODS FOR ANALYZING INTERMEDIATE DEMAND

The classical approach to intermediate demand analysis is input–output modeling. An input–output model is based on an accounting framework of the following type:

Deliveries from	Deliveries to 1	2	3	4	5	6	7	Total output 8
1. Forestry	X_{11}	X_{12}	X_{13}	X_{14}	X_{15}	C_1	G_1	X_1
2. Wood processing	X_{21}	X_{22}	X_{23}	X_{24}	X_{25}	C_2	G_2	X_2
3. Paper processing	X_{31}	X_{32}	X_{33}	X_{34}	X_{35}	C_3	G_3	X_3
4. Other industries	X_{41}	X_{42}	X_{43}	X_{44}	X_{45}	C_4	G_4	X_4
5. Services	X_{51}	X_{52}	X_{53}	X_{54}	X_{55}	C_5	G_5	X_5
6. Households	X_{61}	X_{62}	X_{63}	X_{64}	X_{65}	..	X_{67}	X_6
7. Government	..	..	..	..	..	C_G	..	X_7
8. Total input	X_1	X_2	X_3	X_4	X_5	X_6	X_7	

Consistency requires that the value of total input $\sum_j X_{ij} = X_j = \sum_j X_{ij}$.
This means, for instance, that the total value of all sales from paper processing should equal the value of all purchases of raw materials, intermediary commodities ($X_{23} + X_{33} + X_{43} + X_{53}$), plus the cost of household deliveries of labor, capital, and land services. Governmental costs are often treated as delivery charges from industry even if there is no measurable delivery of government services (the lack of consistency in this respect is represented by double dots in the table).

In the classical input—output studies by Leontief (1951) and many others it has often been assumed that the interdependencies between sectors can be roughly represented by a fixed coefficient assumption. Thus the degree of interdependency is represented by the coefficients $a_{ij} = \frac{X_{ij}}{X_j}$, e.g., the interdependency between forestry (sector 1) and wood processing (sector 2) would be represented by two coefficients:

$$a_{12} = \left[\frac{\text{cost of delivery of forestry products to wood processing}}{\text{value of output of wood processing}}\right]$$

and

$$a_{21} = \left[\frac{\text{cost of delivery of wood processing products to forestry}}{\text{value of output of forestry products}}\right]$$

C_i and G_i would normally be treated as exogenous parameters and the forecasting problem is thus a question of solving the following set of linear equations (in which all relative prices are assumed constant for the forecasting period).

$$X_i = \sum_j a_{ij} X_j + C_i + G_i \;\; ; \;\; (i = 1, \ldots, 5)$$

or in matrix form

$$x = Ax + c + g \quad \text{where}$$

$$A = [a_i] \qquad c = \begin{bmatrix} C_1 \\ \cdot \\ \cdot \\ \cdot \\ C_n \end{bmatrix} \quad \text{and} \quad g = \begin{bmatrix} G_1 \\ \cdot \\ \cdot \\ \cdot \\ G_n \end{bmatrix}$$

The solution can be computed as

$$x = (I - A)^{-1}(c + g)$$

where $(c + g)$ is forecasted independently. The expression $(I - A)^{-1} = I + A + \cdots + A^{r-1}$ which indicates that the interdependencies cascade though the whole economic system. E.g., a dollar worth of services implies less than a dollar worth of paper, which implies a need for wood raw material, which implies a need for chemicals, which implies a need for ... and so on ad infinitum.

In any productive economy the sum of all direct and indirect needs is bounded, producible and compatible with a general equilibrium of the economy.

The use of forest sector products in different sectors of the economy is highly skewed. Unlike labor or energy these are inputs that seem to be reducible from most types of production. The connections to other sectors of the economy are highly structured as can be seen from Table 1. The table shows that for the mechanical wood processing two sectors stand out as important users – *Construction* and *Furniture*.

Different types of paper used by a limited group of industries are of special importance:

- Printing (printing and writing, newsprint)
- Food (packaging)
- Offices (printing and writing)
- Domestic trade (packaging)
- Chemicals (printing and writing)
- Hotels and Restaurants (household and sanitary)

A demand study should focus on the choice of technique of production in these user sectors.

CHANGING INPUT DEMAND STRUCTURE

It has been suggested that the constant price coefficients a_{ij} should be made functions of time in order to reflect the technological development of production processes. This implies no further problem in solv-

TABLE 1. The structure of intermediate deliveries in the Canadian forest sector 1971 (and 1978) (% of total output). Source: Statistics Canada, Input–Output Tables in Constant Prices.

Delivery to / Delivery from	Fabricated wood products	Furniture	Paper products	Constr.	Food	Printing	Dom. trade	Services	Accommod. food service	Chemicals	Offices	Rest
Forestry	51.7 (58.5)	0.2 (..)	37.7 (30.2)	0.5 (0.4)	.. (..)	.. (..)	.. (..)	.. (..)	.. (..)	.. (..)	.. (..)	9.9 (10.9)
Lumber & timber	26.1 (36.1)	5.2 (5.3)	.. (..)	58.3 (50.6)	.. (..)	.. (..)	.. (0.5)	.. (..)	.. (..)	.. (..)	.. (..)	10.4 (7.5)
Veneer & plywood	23.6 (24.9)	5.7 (4.7)	.. (..)	63.8 (65.5)	.. (..)	.. (..)	.. (..)	.. (..)	.. (..)	.. (..)	.. (..)	6.9 (4.9)
Other wood fabricated materials	10.5 (8.2)	2.7 (1.2)	21.5 (28.1)	54.6 (55.0)	1.3 (0.3)	.. ..	1.9 (1.7)	.. (..)	.. (..)	.. (..)	.. (..)	7.5 (5.5)
Pulp	..	..	87.7 (90.1)	..	..	..	..	..	..	5.2 (9.7)	..	7.1
Newsprint & other paper stock	0.5 (0.5)	0.5 (1.4)	45.8 (41.8)	8.0 (8.1)	2.0 (1.3)	32.4 (38.3)	2.7 (2.2)	.. (..)	.. (..)	0.4 (0.2)	1.1 (1.2)	6.6 (5.0)
Paper products	0.3 (0.4)	1.1 (1.1)	5.1 (7.2)	2.7 (3.9)	24.9 (23.0)	0.7 (1.4)	14.2 (14.8)	0.4 (0.3)	4.4 (4.9)	5.7 (4.6)	23.3 (19.7)	17.2 (18.7)

ing the input–output system of equations.

An example of the development of relative forest sector inputs is given in Table 2. In this table the value coefficient development is given relative to the value coefficient for other inputs into the construction sector – a major user of forest sector products.

The table indicates an average decline of 2 percent per year in the relative importance of wood deliveries to the construction sector. This development is, however, subject to one major qualification: what the table shows is the development of the ratio a_{wb} / a_{nb}, i.e., the value input–output coefficient for wood into buildings relative to the non-wood coefficient.

It can be shown that these figures can be decomposed into a sum of percentage relative price change and percentage relative quantity

TABLE 2. Forest sector construction input divided by other building material construction input. (Sources: UN–ECE Standardized Input–Output Tables of ECE Countries, 1965 and 1975. United Nations, 1977 and 1982, New York).

Country	1965	1975	Average annual change (%)
Australia	~0.30	~0.25	~–1.5
Austria	0.130	0.086	–3.4
Canada	0.233	0.213	–0.9
Czechoslovakia	0.134	0.121	–1.0
Denmark	0.234	0.266	+1.4
West Germany	0.259	0.187	–3.3
Italy	0.138	0.102	–3.0
Japan	0.319	0.277	–1.4
Netherlands	0.268	0.232	–1.4
Norway	0.492	0.477	–0.3
Portugal	0.211	0.143	–3.9
Spain	0.210	0.111	–7.1
Sweden	0.459	0.423	–0.7
United Kingdom	0.226	0.198	–1.9

change. Taking this into account the quantitative decline is still there and amounts to approximately one half to one percent per year.

The demand for inputs is rigidly fixed to the scale of operations in most input—output models. In reality choice of techniques is adaptable to prices through substitution. The problem of input substitution is therefore an important subject also in intermediary commodity demand studies.

THE INFLUENCE OF PRICES ON CHOICE OF TECHNIQUES AND INTERMEDIARY FOREST PRODUCT DEMAND

I have argued above that the demand for forest sector products should be based on ***intermediary demand*** theory.

One approach to this problem is to assume that the physical inputs per unit of physical output is determined by profit maximization at the level of the plant.

We can further assume that each plant is producing an output flow q_j per unit of time, being constrained by a production function $q_j = q_j(q_{1j}, \ldots, q_{ij}, \ldots, q_{nj})$. For simplicity we assume this function to be of the CES form, i.e.,

$$q_j = A\left[\alpha_{1j} q_{1j}^{-\rho_j} + \cdots + \alpha_{nj} q_{nj}^{-\rho_j}\right]^{-1/\rho_j}$$

where

q_{ij} = physical flow of input i into output j,

A, α_{ij}, ρ_j = parameters representing technology.

A short run maximization of profits, given input prices, p, corresponds to solving the problem

$$\max_{\{q_{ij}\}} \Pi_j = \bar{p}_j A \left[\sum_i \alpha_{ij} q_{ij}^{-\rho_j} \right]^{-1/\rho_j} - \sum_i \bar{p}_i q_{ij}$$

where $\bar{p}_i$ are the observed prices (or expected price, reflecting the observed prices).

The conditions of a profit maximum are

$$\frac{\partial \pi_j}{\partial q_{ij}} = 0 \; ; \; (i = 1, \ldots, n)$$

This implies that the input quantities should be adjusted so that:

$$\ln \left[\frac{q_{ij}}{q_{kj}} \right] = \delta_j \ln \left[\frac{\bar{p}_k}{\bar{p}_j} \right] + C_{kji} \; ;$$

$$\text{where } c_{kij} = \frac{1}{1 + \rho_j} \ln \frac{\alpha_k}{\alpha_i} \; ; \text{ and}$$

$$\text{where } \delta_j = \frac{1}{1 + \rho_j} \, .$$

As a special case of this technology representation we have the Cobb-Douglas production function. Such a C-D function is a good approximation if an estimated δ is not statistically significantly different from 1. In that case values of input–output coefficients that remain constant over time is also a necessary consequence (Klein 1974).

There are consequently three types of tests that are possible (provided we can make the necessary assumptions of a statistical nature for the estimations). We can test for a) $\delta_j \neq 0$; b) $\delta_j \neq 1$; and c) $\hat{\delta}_j$ for one pair of input being different from $\hat{\hat{\delta}}_j$ for another pair of inputs.

$\delta_j \neq 0$ indicates that price variation could be of importance for the choice of technique within the technology available (and portrayed by the production function chosen). $\delta_j \neq 1$ indicates that we have to rule out the Cobb-Douglas representation. $\hat{\delta}_j \neq \hat{\hat{\delta}}_j$ indicates that the functional form chosen is not a valid representation of the true technology.

These tests have been performed on a limited set of user sectors defined above, repair residential and non-residential construction, and a limited set of inputs.

Some results of the Canadian economy are presented in Table 3. It is clear from the table that we cannot accept only one of the three hypotheses to be tested: $\delta_j \neq 0$. We have, however, no ground whatsoever to accept the Cobb-Douglas representation and little reason to accept the CES-form. The analysis of intermediate demand should be built on another foundation.

AN ALTERNATIVE APPROACH TO INTERMEDIATE DEMAND ANALYSIS

In order to accommodate the results of the testing of the CES production function we can follow a track of complicating the production function to accommodate different elasticities of substitution and complementarities between inputs. Instead, another route has been followed. It has been argued by Diewert (1971) and McFadden & Fuss (1978) that a dual approach to input demand could be used. Their argument essentially is that cost functions are the dual of production functions, although it is often very hard to reconstruct the production function underlying a given (optimized) cost function. In some cases the duality is obvious. A unit cost function of Cobb-Douglas form in input prices has a Cobb-Douglas function as a dual production function (Nerlove 1965). A cost function, allowing for complements and differences in substitution

TABLE 3. Results for constant elasticity of substitution regressions for construction sectors of Canada based on data for period 1961-1974.

$$\ln\left[\frac{\text{Forestry prod.}}{\text{Fabr. wood prod.}}\right] = \underset{(4)}{-9.2}\ \ln\frac{P_F}{P_W} - \underset{(3)}{0.1t} \qquad R^2 = 0.6$$

Repair constr.

$$\ln\left[\frac{\text{Lumber \& Timber}}{\text{Fabr. wood prod.}}\right] = \underset{(3)}{-0.73}\ \ln\frac{P_{LT}}{P_{FW}} - \underset{(2.0)}{0.08t} \qquad R^2 = 0.97$$

Non res. constr.

$$\ln\left[\frac{\text{Lumber \& Timber}}{\text{Fabr. metal prod.}}\right] = \underset{(2)}{-0.6}\ \ln\frac{P_{LT}}{P_{FM}} - \underset{(3.0)}{0.03t} \qquad R^2 = 0.85$$

Non res. constr.

$$\ln\left[\frac{\text{Lumber \& Timber}}{\text{Cement prod.}}\right] = \underset{(3)}{-0.86}\ \ln\frac{P_{LT}}{P_{CP}} - \underset{(5)}{0.04t} \qquad R^2 = 0.91$$

Non res. constr.

$$\ln\left[\frac{\text{Lumber \& Timber}}{\text{Struct. Met. prod.}}\right] = \underset{(3.7)}{-1.95}\ \ln\frac{P_{LT}}{P_{SM}} + \underset{(1.3)}{0.03t} \qquad R^2 = 0.67$$

Res. constr.

$$\ln\left[\frac{\text{Panels}}{\text{Iron \& steel prod.}}\right] = \underset{(4.1)}{-1.84}\ \ln\frac{P_{Pan}}{P_{I\&S}} + \underset{(2.3)}{0.03t} \qquad R^2 = 0.71$$

Res. constr.

$$\ln\left[\frac{\text{Fabr. wood prod.}}{\text{Cement prod.}}\right] = \underset{(2.3)}{-1.4}\ \ln\frac{P_{FW}}{P_{CP}} + \underset{(3.4)}{0.05t} \qquad R^2 = 0.61$$

Res. constr.

elasticities between different inputs has been proposed by Diewert. This function is also proposed to be used in our study of forest product input demand.

The idea is quite straightforward and goes back to Hotelling (see Fuss and McFadden 1978) and Shephard (1953). It can be illustrated with a simple diagram (Figure 1). Suppose two inputs are free to be varied and that the goal of a plant manager is to minimize costs subject to an exogenously specified quantitative demand level and given input prices. It is fairly obvious that a slight variation of the factor price ratio at a given demand level will trigger a predictable response in terms of demand for the two inputs.

Shephard's theorem can be stated in the following simplified fashion (Diewert 1971). Assume that:

(1) A cost function, C, in input prices, p, and demand level, y.

(2) $C(y,p)$ is positive real valued, defined and finite for all $p \gg 0$ and $y > 0$.

(3) $C(y;p)$ is nondecreasing, left continuous and tends to infinity for $p \gg 0$.

(4) $C(y;p)$ is nondecreasing in p.

(5) $C(y;p)$ is linearly homogeneous in p for $y > 0$.

(6) $C(y;p)$ is concave in p.

(7) $C(y;p)$ is everywhere differentiable in p.

Then

$$\frac{\partial C}{\partial p_i} = x_i$$

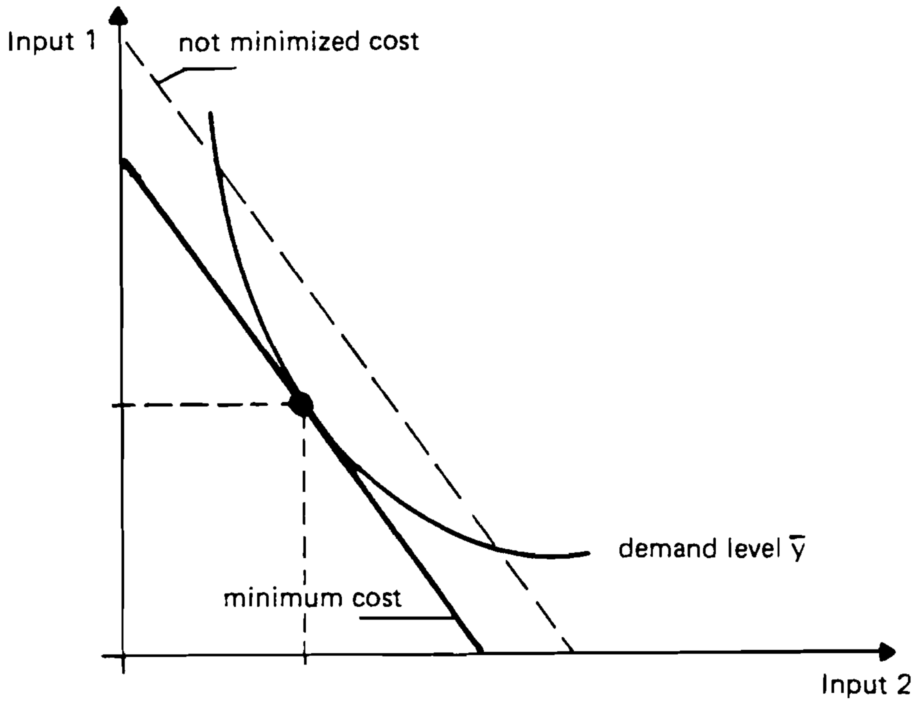

FIGURE 1. Cost minimization and factor demand.

A proof is given in Diewert (1971).

This theorem, based on economically reasonable assumptions, states in short that all input demands are uniquely determined if the price vector is given.

One econometrically amenable form suggested by Diewert and others is, the Generalized Leontief Cost Function

$$C(y,p_1,\ldots,p_n) = h(y)\sum_i\sum_j b_{ij} p_i^{0.5} p_j^{0.5}$$

for $p_i \geq 0$; $y > 0$ with $b_{ij} = b_{ji}$. $h(y)$ could be any continuously and monotonically increasing function, for instance, $h(y) = \alpha_0 y^{\alpha_1}$ with α_0, $\alpha_1 > 0$.

With α_0, $\alpha_1 = 1$ we have

$$\frac{\partial C}{\partial p_i} = x_i = y\sum_j b_{ij}[p_j / p_i]^{0.5} \; ; \; (i = 1, \ldots, n)$$

In order to generate the input–output coefficients needed for a full interdependency analysis of the economy we only have to divide x_i by y, i.e.,

$$a_{iy} = x_i / y = \sum_j b_{ij}[p_j / p_i]^{0.5} \; ; \; (i = 1, \ldots, n)$$

This permits the immediate formulation of a dual general equilibrium formulation. The whole set of input–output coefficients, depending on relative prices, can be arranged as a matrix of functions $A(p) = \{a_{ij}(p)\}$. This expression can be used to formulate the general equilibrium equation (in the dual space). In the sequel we disregard capital and other fixed inputs.

$$\underset{\text{revenue}}{p_i x_i} = \underset{\text{input cost}}{\sum p_j a_{ji}(p) x_i} + \underset{\text{labor cost}}{w_i a_{li} x_i} \; ; \; (1 = 1, \ldots, n) \tag{3}$$

where

w_i =household income per unit of labor input

x_i = total output (and input) of commodity i

x_i can be reduced from the system of equations

$$\lambda p = pA(p) + wa_l$$

If the use of labor would be determined in a substitution process, then relative price of labor would also be determined endogenously and the system could be reformulated:

$$\lambda p_1 = p_1 A_1(p_1)$$

where p_1 includes the relative wage rates.

This is a non-linear fixed point eigenvalue problem which has an equilibrium solution (Nikaido 1968). Once solved for the equilibrium p_1^* and A_1^* the quantitative equilibrium can be easily calculated using inversion of the equilibrium matrix, A_1^*.

ESTIMATION WHEN DATA ARE SCARCE

Statistical data good enough for the approach suggested above only exist for Norway and Canada. In these two countries time series of input–output tables are actually available. In most other countries only time series of ***unrelated*** input and output series are available (see Table 4). A series of construction output and furniture output does, for instance, exist in most countries. Similarly sawnwood and panel (apparent) consumption data does exist for most countries and so does the associated price indexes. It is thus fairly straightforward to form a basis for dual approach intermediate demand estimation.

It is possible to estimate macro-input–output functions

$$\left[\frac{x_i}{y_C + \gamma y_F}\right] = \sum_{j=1}^{N} b_{ij}(p_j/p_i)^{0.5} \;\; ; \;\; b_{ij} = b_{ij}$$

The size of the weight γ can be determined in a full input–output table containing data on the relative importance of the furniture sector as a user of sawnwood.

TABLE 4. Hypothetical data for estimation of macro intermediate demand function.

Year	Apparent consumption of sawnwood	Construction output	Furniture output	Price of sawnwood	Price of cement	Price of metals	Price of labor
0	$x_i(0)$	$y_C(0)$	$y_F(0)$	$P_i(0)$	$P_s(0)$	$P_m(0)$	$P_w(0)$
1	$x_i(1)$	$y_C(1)$	$y_F(1)$	$P_i(1)$	$P_s(1)$	$P_m(1)$	$P_w(1)$
2	$x_i(2)$	$y_C(2)$	$y_F(2)$	$P_i(2)$	$P_s(2)$	$P_m(2)$	$P_w(2)$
.	.	.	.	.	.	.	.
t	$x_i(t)$	$y_C(t)$	$y_F(t)$	$P_i(t)$	$P_s(t)$	$P_m(t)$	$P_w(t)$
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
N	$x_i(N)$	$y_C(N)$	$y_F(N)$	$P_i(N)$	$P_s(N)$	$P_m(N)$	$P_w(N)$

In order to uphold the symmetry constraint information on the use of building metals, cement and labor has to be organized in a form similar to Table 4. Analogous data tables can be assembled for most of the commodities of the GTM and most developed countries.

Ordinary Least Square is not a reasonable approach in such a constrained estimation procedure. In the estimations currently done on Canadian data, the "Seemingly Unrelated Regression" method has been chosen as a starting point for the econometric work. This work is to be reported in papers in Andersson et al. (1984).

CONCLUSIONS

Forest sector products are intermediate commodities. This implies that input–output models or their generalizations into programming models should be used as forecasting tools. The recent energy crisis has shown that in periods of substantially changing relative prices, substitution of inputs must be taken into account (see Bjerkhalt et al. 1983). It is argued above that combining interdependency analysis with substituion models of the Generalized Leontief Cost Function category is a reasonable methodology. Indeed, it is a superior method at least if compared to classical inpout—output modeling or substitution/interdependency analysis.

REFERENCES

Andersson, Å.E., R. Brännlund, and G. Kornai. 1984. The Demand for Forest Sector Products. WP-84-87. Laxenburg, Austria: International Institute for Applied Systems Analysis.

Diewert, W.E. 1971. An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function. *Journal of Political Economy*, 79(3).

Frenger, P. 1978. Factor Substitution in the Interindustry Model and the use of Inconsistent Aggregation. Chapter V.2 in Fuss and MacFadden (1978).

Fuss, M. and D. McFadden (eds). 1978. *Production Economics: A Dual Approach to Theory and Applications*. Amsterdam: North-Holland.

Klein, L. 1974. *A Textbook of Econometrics*. Englewood Cliffs, N.J.: Prentice Hall.

Leontief, W. 1951. *The Structure of the American Economy 1919–1929*, 2nd ed. New York: Oxford Univ. Press.

Nerlove, M. 1965. *Estimation and Identification of Cobb-Douglas Production Functions*. Amsterdam: North-Holland.

Nikaido H. 1968. *Convex Structures and Economic Theory*. New York: Academic Press.

Shephard, R.W. 1953. *Cost and Production Functions*. Princeton: Princeton University Press.