Working Paper

RENEWABLE RESOURCE ECONOMICS – OPTIMAL RULES OF THUMB

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WP-84-84

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FOREWORD

The objective of the Forest Sector Project at IIASA is to study long-term development alternatives for the forest sector on a global basis. The emphasis in the Project is on issues of major relevance to industrial and governmental policy makers in different regions of the world who are responsible for forestry policy, forest industrial strategy, and related trade policies.

The key elements of structural change in the forest industry are related to a variety of issues concerning demand, supply, and international trade of wood products. Such issues include the development of the global economy and population, new wood products and substitution for wood products, future supply of roundwood and alternative fiber sources, technology development for forestry and industry, pollution regulations, cost competitiveness, tariffs and non-tariff trade barriers, etc. The aim of the Project is to analyze the consequences of future expectations and assumptions concerning such substantive issues.

In this article the supply of roundwood is discussed within the framework of renewable resource economics. Quantitative guidelines for forestry are derived and tested against possible disturbances to plantation managenent conditions. It is shown that certain rules of thumb for renewable resource management are robust with respect to a broad set of incidental disturbances, e.g., weather conditions, market fluctuations, etc.

Markku Kallio
Project Leader
Forest Sector Project

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Å E. Andersson (IIASA) and P. Lesse (CSIRO)

1. INTRODUCTION

In this paper we formulate a simple model for optimum management of large forests. When seeking the optimum management policy, we aim to reconciliate the ever-present conflict between the forest as a valuable capital resource to be exploited, and as a valuable biotope to be protected and preserved.

We are aware of previous work in this field, which already led to valuable results. Kilkki and Vaisanen (1969) developed an optimum thinning policy for Scotch pine stands in Finland. Their work has been further refined by Clark (1976). In both cases, the problem has been treated as that of linear optimum control leading to a "bang-bang" cutting policy.

There is also a large group of models using operations research approaches. These models can often be viewed as resource management simulators. A recent example is the integrated set of models developed in New Zealand by Garcia (1981), Levack and Jennings (1981), and Lee (1981, 1982). Large scale (20,000 variables, 10,000 constraints) mathematical programming model for forestry management were developed by Whyte and Baird (1982, 1983) and Dykstra (1984). (See also Kallio, Andersson, and Seppälä 1984 and the forthcoming book on renewable resource and forest economics by Lofgren and Johansson 1985).

In this paper, we concentrate on deriving qualitative rules of thumb plantation for management. This approach takes into account both economic and some simple ecological aspects of the problem. For an easy interpretation, we derive the qualitative rules using a *very* simple model at first. Later, we investigate the robustness of these rules, i.e., we test their validity by gradually relaxing some of the assumptions underlying the simple model.

2. A SIMPLE MODEL FOR OPTIMUM MANAGEMENT OF FORESTS

The aim of this section is to formulate a simple model capable of generating as clear-cut and hard-boiled conclusions as possible. We are primarily interested in management of large areas, and hence do not address the problem of the optimum period. The model is based on the following simplifying assumptions:

(a) Harvested volume h is proportional to the standing volume x, i.e. $h(t) = u(t)x(t)$ where $u(t) \in [0; u_{max}]$ is a control variable called *thinning effort*.

(b) The net revenue derived from the amount harvested p with prices assumed to be known is independent of the scale of harvesting and time. Accordingly, the net revenue per unit harvested, $\bar{p}$, is a constant.

(c) Growth of wood is given by the differential equation

$$\dot{x} = ax - bx^2, \tag{1}$$

which is treated as the only constraint relevant to our problem.

(d) Parts or the whole of the plantation can be sold at any time in a market. This assumption implies that there is no terminal date at which the optimization must end.

(e) The plantation owner maximizes the profit discounted over time by δ, which is the maximal real rate of interest in any segment of the capital market. Thus, the plantation owner seeks to

$$\underset{\{u\}}{\text{maximize}}\ \Pi = \int_0^\infty \bar{p}\ u\ x\ e^{-\delta t}\ dt \tag{2}$$

subject to constraint (1).

Following Clark (1976), we formally derive the Euler Lagrange equation corresponding to (1), (2). By inserting for $ux \doteq ax - bx^2 - \dot{x}$ into (2), we obtain

$$\max \int_0^\infty \bar{p}(ax - bx^2 - \dot{x}) \exp(-\delta t) dt.$$

The corresponding Euler Lagrange equation leads to the expression for the optimal path x^*

$$a - 2bx^* - \delta = 0 \tag{3}$$

and hence

$$x^* = \frac{a - \delta}{2b}. \tag{3'}$$

The equation (3) defines a singular solution to the linear optimum control problem (1), (2). The control rule is to use the maximum effort of harvesting u_{max}, whenever $x > x^*$, and stop thinning, if $x < x^*$. Along the optimum path, the optimum policy should satisfy

$$ux^* = ax^* - bx^{*2},$$

and hence if $x^* \neq 0$

$$u_{opt} = \frac{a + \delta}{2}. \tag{4}$$

Note that the optimal steady-state harvest, equal to $x\,u_{opt} = (a^2 - \delta^2)/4b$, is always *less than the maximum sustainable yield*, which equals to $a^2/4b$. The relative difference compared to maximum yield is $(\delta/a)^2$. For example, if the parameter a corresponds to 10% annual growth and the interest rate δ is 5%, then the difference is 25%.

The optimum thinning policy thus leads to a stationary (equilibrium) state. This equilibrium state is stable, provided that $a > \delta$, as can be shown by introducing a new variable $X = x - x^*$, and by transforming (1) into

$$\dot{X} = -\frac{(a - \delta)}{2} X - bX^2.$$

The asymptotic stability of the solution $X = 0$ can be proven, e.g., by using the positive definite Lyapunov function

$$V(X) = X^2 [\frac{a - \delta}{2} + bX]^2.$$

The equation (3'), (4) thus define a stable equilibrium point, which may be called a *bio-economic equilibrium*. We have just proven the following Proposition:

> PROPOSITION 1: *The maximization of net profit (2), subject to the growth equation (1), leads to a stable bioeconomic equilibrium given by equations (3'), (4), if the maximum effort of thinning exceeds the arithmetic average of the linear growth rate and the discount rate (4).*

The original problem can be easily transformed into an equivalent Hamiltonian maximization problem:

$$\max_{\{u,x\}} \int_0^\infty H dt = \int_0^\infty \bar{p}\; u\; x\; e^{-\delta t} - \delta(ax - bx^2 - ux) dt. \tag{5}$$

In equilibrium we expect

$$\frac{\partial H}{\partial u} = \bar{p} x e^{-\delta t} + \lambda x = 0 \tag{6}$$

As $x \neq 0$, in any sustainable plantation, this implies that $-\lambda = \bar{p} e^{-\delta t}$. As $\delta, \bar{p}$ are parameters, it follows that

$$\frac{d\lambda}{dt} / \lambda = \frac{\dot{\lambda}}{\lambda} = \delta$$

The problem of $\lambda \to 0$ can easily be avoided by inserting a requirement that the standing volume should be kept always above a certain positive value. Our second proposition can thus be formulated.

> PROPOSITION 2: *Under the condition specified in model (5), the rate of interest is equal to the relative rate of change of the plantation's value (measured in terms of the shadow price δ).*

The interpretation of this proposition is obvious. If the plantation value would change slower than the rate of interest requires, there is an inducement for the owner to invest in another object, carrying the increase of value δ. If the rate of value increase is larger than the rate of interest, resources are withdrawn from the alternative uses, until the condition is fulfilled. Such an adaptation could occur both in the non-forestry and the forestry markets.

This adaptation is closely related to the harvesting effort as can be observed by the following condition

$$\frac{-\partial H}{\partial x} = \dot{\lambda} = -p\ u\ e^{-\delta t} + \lambda a - \lambda 2bx - \lambda u, \tag{7}$$

or by substituting expression (6) into (7):

$$\dot{\lambda} = \lambda u + \lambda a - 2\lambda bx - \lambda u \tag{8}$$

or

$$\frac{\dot{\lambda}}{\lambda} = a - 2bx = \delta \tag{9}$$

Above it has been shown that $\dot{\lambda}/\lambda$ must equal the interest rate δ at an optimal trajectory. The only way of achieving this is by adjusting the size of the standing volume to this requirement. Such a procedure can be illustrated in a simple diagram (see Figure 1). The Figure shows how the steady state standing volume x^* increases from x_1^* to x_2^*, when the rate of interest changes from δ_1 to δ_2.

> *COROLLARY 1*: There is a danger of extinction of the whole standing volume if $a \leq \delta$; i.e., if the real rate of interest is larger than the coefficient of linear growth in the biological equation (1).

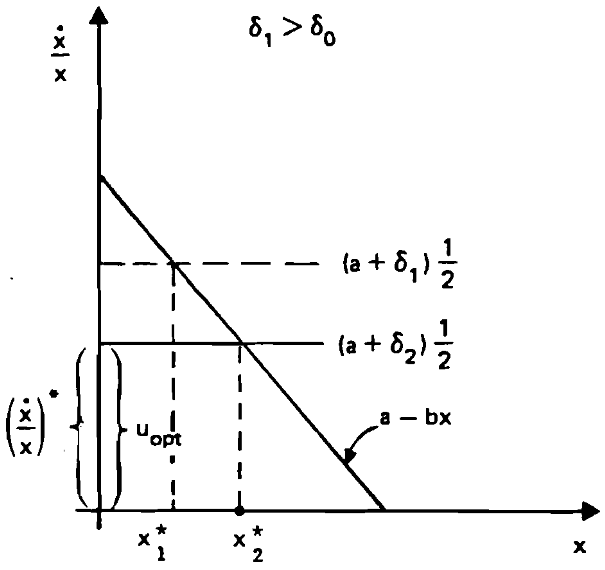


Figure 1. The relative role of growth $\frac{\dot{x}}{x}$ as function of standing volume x.

This can easily happen in a society where the industrial growth rate is high while the ecological conditions prevent a high biological growth rate. Parts of Japan and similar countries with a high rate of growth (and a correspondingly high real interest rate in their industrial sectors) fall into this category. These countries could lose certain biological species, presumably due to violating the condition specified by *Corollary 1*.

Corollary 1 thus illustrates a general rule that decision making in forestry management has to be based on both ecological and economic facts.

3. CONSEQUENCES OF USING THE FOREST AS A POLLUTION SINK

Forests act as powerful pollution sinks, especially in industrialized countries with moderate to warm and humid climatic conditions. Thus we would, for most situations, need to modify our analysis to include considerations of the forests being of value both as flow generators and as *stocks*.

The simple model is then reformulated in the following way:

$$\underset{\{x,u\}}{\text{maximize}} \int_0^\infty (\bar{p}\cdot u\cdot x+\omega\cdot x)e^{-\delta t}dt$$

$$\text{subject to: } \dot{x} = ax - bx^2 - ux \tag{10}$$

ω in this case denotes the net ecological value of the forest stock, i.e., the value of the forest as a pollution sink, which, for obvious reasons depends on the standing volume.

The corresponding Hamiltonian formulation is

$$\max_{\{u,x\}} \int_0^\infty Hdt = \int_0^\infty (\bar{p}ux+\omega x)E^{-\delta t} - \lambda(ax-bx^2-ux). \tag{11}$$

Following the same procedure as in the last section we obtain

$$pe^{-\delta t} = -\lambda \quad \text{and} \quad \dot{\lambda}/\lambda = \delta \tag{12}$$

The condition

$$\frac{\partial H}{\partial x} = -\dot{\lambda} = [pu+\omega]e^{-\delta t} - \lambda a+2\lambda bx+\lambda u$$

leads to

$$-\dot{\lambda} = +\omega e^{-\delta t}-\lambda a+2\lambda bx; \text{ i.e.,:}$$

$$-\dot{\lambda} = \omega(-\frac{\lambda}{p}) - \lambda(a-2bx); \text{ or}$$

$$\frac{\dot{\lambda}}{\lambda} = \frac{\omega}{\bar{p}}+a - 2bx = \delta \tag{13}$$

and this implies that the steady state (optimal) standing forest volume is determined by the expression

$$x^* = \frac{1}{2b}[a-\delta+\frac{\omega}{\bar{p}}]. \tag{14}$$

The determination of the rate of harvesting, corresponding to steady state, is a slight extension of the earlier results (cf. (4))

$$\begin{aligned} u_{opt} &= a - bx^* \\ &= \frac{1}{2}(a + \delta - \frac{\omega}{p}). \end{aligned} \tag{15}$$

The interpretation of equation (14) can be formulated as a *Corollary 2.*

> *COROLLARY 2* : If the standing volume of forest is valued *per se* (i.e., as a pollution-reducing ecological asset), the steady state standing volume increases proportionally to the fraction value of the forest *per se*, over net value of forest as a material harvested resource.

The procedure for determining w/p will not be addressed in this paper. However, determination of the relative valuation of the standing volume is one of the most complicated issues within the framework of public goods theory.

4. THE "SAFE" OPTIMUM MANAGEMENT OF FORESTS

The simple rules for optimum management of forests derived in the preceding sections were obtained on the basis of a mathematical model. This model is certainly not an exact representation of reality. The question arises, how sensitive are the management rules to *changes* of the model. For example, should the conclusions reached on the basis of the simple model remain valid for a whole class of plausible models, one would feel much more confident, when implementing the model in practice. Alternatively, the parameters of the simple model can be expected to depend on unpredictable exogenous factors (e.g., weather, insect attacks, etc.) and on slowly evolving variables (like age and other biological distribution factors). It is important to know how the parameter changes influence the management policies. To improve our understanding of these problems, the simple model of the preceding sections is modified and generalized and the conditions sufficient to preserve the validity of the management rules already derived are also investigated in Section 4.

4.1 The Modified Growth Equations

We commence by replacing the equation (1) by a more general equation, thus allowing for the influence of incidental disturbances. These disturbances may be due to biological, meteorological and other factors. Let the dynamics be given by a more general equation

$$\dot{x} = xF(x,u,\nu), \tag{16}$$

where $F(x)$ is a function, which is Lipschitz continuous in x, and let $u_{max} \geq u \geq 0$ denote the harvesting effort related to the rate of thinning by the equation $h = ux$. The variable ν summarizes the influence of the disruptive factors. We shall assume that equation (16) is integrable for $u \in \Omega_u$, $\nu \in \Omega_\nu$, which are the appropriate control and disruption sets.

To ensure that the rate of thinning is proportional to the biomass x, the function $F(x,u,\nu)$ is assumed to have the form

$$F(x,u,\nu) = -uF_1(\nu) + F_2(x,\nu).$$

The term $F_1(\nu) \geq 0$ can be interpreted as disturbance of the thinning rate due to incidental factors. The function $xF_2(x,\nu)$ then represents a general form of growth dynamics influenced by fluctuations of biological, meteorological and other nature. The reader can verify that (1) is a special case of (16), if $F_1 = 1, F_2 = a - bx$. The equation (16) has at least one equilibrium point $x = 0$. Let some other equilibrium point be denoted x_0, i.e., x_0: $F(x_0, u, \nu) = 0$ for some values of u_0, ν_0 from Ω_u, Ω_ν. In the vicinity of such an equilibrium point, the r.h.s. of equation (16) can be linearized:

$$xF(x, u_0, \nu_0) = x_0 F(x_0, u_0, \nu_0) + x_0 \frac{\partial F}{\partial x}\Big|_{x=x_0}(x - x_0) + \cdots$$

The equilibrium will be stable, if

$$\frac{\partial F}{\partial x}\Big|_{x=x_0} < 0;$$

or, more specifically, if

$$\frac{\partial F_2}{\partial x}\Big|_{x=x_0} < 0,$$

for some u_0, ν_0. The condition can be written

$$\frac{\partial}{\partial x}\left(\frac{\dot{x}}{x}\right)\Big|_{\substack{x=x_0 \\ u=u_0 \\ \nu=\nu_0}} < 0.$$

A dynamics satisfying $\frac{\partial}{\partial x}\left(\frac{\dot{x}}{x}\right) < 0$ has been called *compensatory* (cf. Clark, 1976).

The equilibrium point x_0, which depends on $u \in \Omega_u, \nu \in \Omega_\nu$ is thus stable, if the dynamics is *locally compensatory*. The equilibrium point $x_0(u, \nu)$ is stable with respect to the harvesting efforts from the set Δ_u, and with respect to the incidental factors from Δ_ν, if the dynamics is locally compensatory for all $u \in \Delta_u \subset \Omega_u$, and $\nu \in \Delta_\nu \subset \Omega_\nu$. Obviously, a model with dynamics (16), satisfying $\frac{\partial F_2}{\partial x}(x, \nu) < 0$ for $x > 0, \nu \in \Delta_\nu$, is compensatory for all harvesting efforts, and for all values of incidental factors ν from the set Δ_ν.

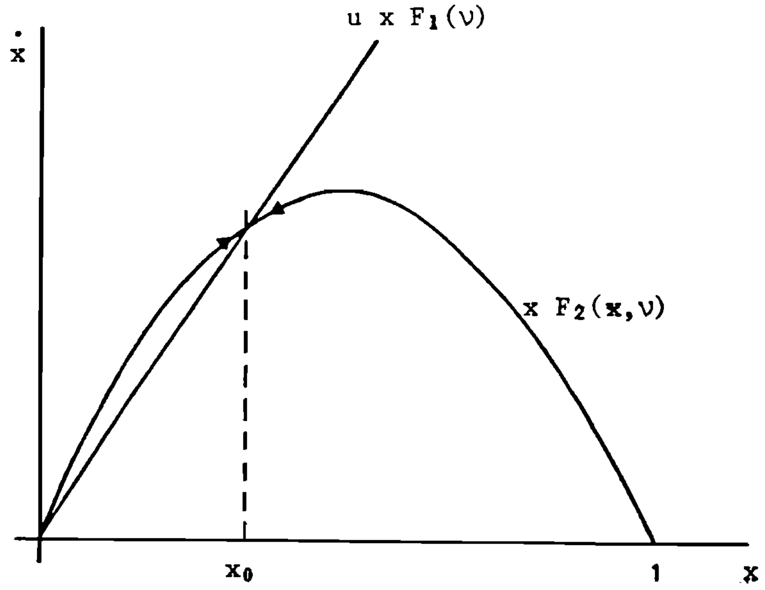
The model which is not compensatory, is said to be *depensatory*. The distinction between compensatory and depensatory dynamics, and the possibility of a transition from one to another, is a matter of crucial importance (c.f. Holling 1973). Indeed, as long as there is a chance that an environmental or economic factor could distort the dynamics to make it become depensatory, the optimal management policies could lead to environmental damage, and even to an extinction of the biotope through destabilization of the system (see Figure 2). Conversely, a general class of models (16), which is locally compensatory with respect to sufficiently 'large' sets Δ_u, Δ_ν, is safe to use. In particular, models with

$$\frac{\partial F_2}{\partial x}(x,\nu) < 0; \nu \in \Delta_u$$

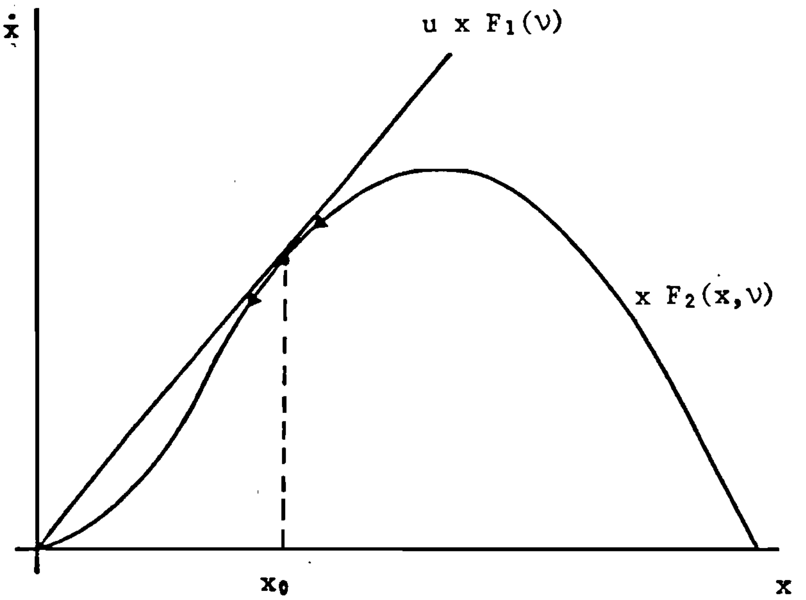
are safe for all harvesting efforts, and for all incidental disturbances from Δ_ν. In the following, we shall limit our attention to models of this type only.

4.2 Optimum Management in the Presence of Incidental Factors

In this paragraph, we shall combine the general dynamics of 4.1 with an optimality criterion including a more general 'cost' function. This function will be assumed either positive, in which case it will represent the harvesting cost, or negative, in which case it will be interpreted as a benefit derived from the forest in a non commercial way (e.g., acting as a pollution sink, cf. Section 3). We shall now proceed to derive the formulae for the steady state standing volume under optimal management, and for the relationship between the shadow price and the rate of



a) Compensatory dynamics; equilibrium at $x = x_0$ is stable.



b) Depensatory case; the dynamics having been distorted the equilibrium first becomes unstable since the system heads for extinction.

Figure 2.

interest.

The valuation equation for the model is now assumed to be of the form

$$I = \int_0^T (pux - C(u,\nu,x))e^{-\delta t}\,dt \tag{17}$$

where $C(u,\nu,x)$ is general cost function which depends both on the thinning effort, and on the incidental factors ν.

The cost function is assumed to satisfy the requirement

$$C = uxC_1(\nu) + C_2(x,\nu)$$

The cost function is separable into two parts: the first part is a linear function of the rate of thinning, influenced by disruptive factors. It may be called *production cost*. The second part is not directly under the control of the forester, and may be interpreted as additional costs or benefits, not immediately connected with the thinning operation, but attributable to the interaction between, for example, weather or biological calamities on the one hand, and the biomass on the other. This part can be called *environmental cost (benefit)*.

Denoting $P(\nu) = p - C_1(\nu)$ and calling it *producers price*, we can write (17)

$$I = \int_0^T [P(\nu)ux - C_2(x,\nu)] \exp(-\delta t)dt. \tag{17'}$$

This expression is to be compared with (2) and (10).

In contrast to the present model, the earlier model thus assumed a constant net revenue per unit harvested ($\bar{p}$). This revenue was considered independent of the disturbing factors, while the environmental cost/benefit was neglected. Following the standard procedure, we form the Hamiltonian

$$H = [P(\nu)ux - C_2(x,\nu)] \exp(-\delta t)$$
$$+\lambda[xuF_1(\nu) + xF_2(x,\nu)]$$

From the condition $\frac{\partial H}{\partial u} = 0$, we obtain for $x \neq 0$

$$P(\nu) \exp(-\delta t) = -\lambda F_1(\nu) \tag{18}$$

Under special circumstances, the equation (13) allows, to interpret the discount rate as the relative change of the shadow price λ. To reproduce the equation (9)

$$\frac{\dot{\lambda}}{\lambda} = -\delta$$

it is necessary to assume either $\frac{dv}{dt} \approx 0$, i.e. slowly acting disruptive forces or, alternatively,

$$\frac{dP}{dv} = \frac{P(\nu)}{F_1(\nu)} \frac{dF_1(\nu)}{d\nu}$$

which translates into $\frac{dlnP}{dlnF_1} = 1$, i.e. the producers price should have unit elasticity with respect to the disruptive effect on the thinning rate due to incidental factors.

Summing up: The discounting rate δ can be interpreted as the relative change of the shadow price (cf. 9) either if the disturbances of the environment are extremely slow, or if disturbances affect the producers price and harvesting rate in a similar manner.

Returning to the optimum management problem, we can obtain a condition connecting interest rate, consumers price, and value of stock, by using the equation $-\frac{\partial H}{\partial x} = \dot{\lambda}$ which yields after some algebra

$$\frac{F_1}{P}\frac{\partial C_2}{\partial x} + F_2 + x\frac{\partial F_2}{\partial x} = \delta \tag{19}$$

In a special case when $F_1 = 1$, $F_2 = a - bx$, $\frac{\partial C}{\partial x^2} = 0$, (19) becomes

$$a - 2bx = \delta \tag{20}$$

i.e., the expression (9) obtained in Section 2. Similarly, the equation (14) of Section 3 is obtained as a special case, when $F_1 = -1$, $F_2 = a - bx$, $C_1(\nu) = 0$, $C_2 = -\omega x_1$. We have thus arrived at a generalization of Proposition 1, and of Corollary 2.

PROPOSITION 3: *In a system with the general compensatory dynamics (16), the optimum management leads to a steady state determined by the discounting rate according to equation (19). Other things being equal, an increased discounting rate brings about a reduction of the standing volume if*

$$\frac{\partial \delta}{\partial x} = \frac{2\delta F_2}{\partial x^2} + x\frac{\partial^2 F_2}{\partial x^2} - \frac{F_1}{P}\frac{\partial^2 C_2}{\partial x^2} < 0$$

for all $\nu \in \Delta\nu$.

Proposition 4 thus extends and confirms the qualitative conclusions reached on the basis of the simple model, which remain valid irrespective of the influence of incidental factors in a wide range of situations; in particular, it is sufficient, if the function $F_2(x,\nu)$ is positive, decreasing and nonconvex in the standing volume, and if the environmental cost (benefit) function $c_2(x,\nu)$ is nonconcave in the same variable.

5. OPTIMUM THINNING POLICIES MADE SIMPLE (CONCLUSIONS)

On the basis of our considerations, we can draw the following conclusions:

(a) The growth of wood depends on the quality and maturity structures of the trees, and often in a secondary way on a number of incidental factors including weather, pollution and interaction (competition, symbiosis) with other organisms.

(b) The thinning rate is in a first approximation proportional to the standing volume of wood; the thinning rate per unit standing volume is determined by the forester according to the prevailing economic conditions, and it is also influenced by the incidental factors.

(c) An optimal thinning policy aims at maximizing discounted net profit over a period of time. Such policy leads in the long run to a steady state representing a bioeconomic equilibrium, i.e., a state at which the biological growth is balanced by the thinning operations.

(d) The bioeconomic equilibrium is determined, *in the first approximation*, by the equation (3') (cf. Proposition 1). The equation (3') can be given a very simple meaning, if one takes into account that $b = \frac{a}{K}$, where K is a carrying capacity of the habitat. The steady state standing volume is then $x^* = \frac{K}{2}(1-\frac{\delta}{a})$ i.e., it is equal to the standing volume corresponding to the maximum rate of growth (according to the simple growth model this happens when the standing volume reaches 50% of carrying capacity), reduced by the fraction $\frac{\delta}{\alpha}$, i.e., by the ratio of the rate of interest to the initial (constant) rate of growth of wood. This simple formula becomes more complicated, when the forest is considered not only as a source of wood, but also as a public good indispensable as atmospheric filter, reservoir of water, etc.

In this case, the steady state standing volume is increased, and the amount of increase depends on the ratio of ecological and commercial values of forest (cf. equation (14), Corollary 2). In general, the steady state depends not only on the rates of growth, interest, and on price, but also on the form of the cost/benefit function, and on the incidental factors. The general equation, from which the steady state standing volume can be obtained, is given in paragraph 4.2 (equation (19)).

(e) Having determined the optimal steady state, the optimal thinning policy is easy to formulate: Whenever the actual standing volume exceeds the steady state standing volume, use maximum thinning effort. As soon as the steady state standing volume is reached, stop thinning.

(f) It is important to know whether the unknown (incidental) factors can spoil the effect of the theoretically optimal thinning policy, for example by reducing irreversibly the standing volume below the steady state value. In an extreme case, this reduction can lead to a catastrophic destruction of forest. However, Proposition 3 specifies the conditions, under which the bioeconomic equilibrium is stable. The stability guarantees that the influence of incidental factors (e.g. draft, pest calamity, increased pollution, etc.), cannot lead to an irreversible damage so long as the influence is kept within predetermined limits. We hasten to add that this conclusion is not valid absolutely, but only in the context of what appears to be a large class of models. In other words, should reality be so complicated that its modeling required relaxation of the assumptions specified in Proposition 3, then the optimal thinning policy need not be safe for implementation. In the final analysis, the practitioner's experience must decide whether the theory and its underlying mathematical assumptions are of sufficient generality to accommodate the behavior of the particular ecological system at hand. If depensation phenomena seem to be of great potential importance there is probably a strong case for viability and permanence analysis using differential inclusion theory as proposed by Aubin and Sigmund (1984).

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