Working Paper

THE EFFECTS OF INTERNATIONAL TRANSPORTATION COSTS ON PRODUCTION LOCATION IN DEVELOPING COUNTRIES: A THEORETICAL APPROACH

Cynthia Griffin

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International Institute for Applied Systems Analysis
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FOREWORD

The objective of the Forest Sector Project at IIASA is to study long-term development alternatives for the forest sector on a global basis. The emphasis in the Project is on issues of major relevance to industrial and governmental policy makers in different regions of the world who are responsible for forestry policy, forest industrial strategy, and related trade policies.

The research program of the Project includes an aggregated analysis of long-term development of international trade in wood products, and thereby analysis of the development of wood resources, forest industrial production and demand in different world regions.

This paper is an outgrowth from the author's participation in the Young Scientists Summer Program at IIASA. The work was carried out in conjunction with the UNDIO/IIASA project on transprtation costs of forest products and their influence on the location of various processing activities. In particular, this project aims to provide the transportation cost data for the Global Trade Model, which is under development in the Forest Sector Project.

Markku Kallio
Project Leader
Forest Sector Project

ABSTRACT

Many developing countries are interested in determining the optimal level of forest sector production and an overall production program. This paper offers a theoretical approach to the aspects involved in determining such a production optimum which includes the development of a global model of forest resources, wood and fiber processing and international trade in forest products as a partial equilibrium economic model cast in the mathematical programming framework with linear constraints and a nonlinear objective function. The proposed model has been under investigation by the IIASA Forest Sector Project over the last several years. Therefore, rather than focusing on the continued development of the core model, the emphasis is now placed on proposing basic theoretical questions surrounding the forest sector to determine how transport costs, production costs, tariff and non-tariff barriers to trade, natural resource endowments and marketing costs will affect the location of various processing activities. This initial development will identify the need for a more structured empirical analysis in resolving these questions.

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INTRODUCTION

Many developing countries in the forest sector are investigating the possibility of substituting their raw export activity with processed products such as sawnwood and furniture with an interest in improving their overall economic competitive advantage in the world forest products market. Quite simply, policy makers for these developing countries are seeking to develop a production plan to motivate minimum total costs of commodity flows from the forest to the consumer.

The development of such a production plan is very complex in nature as it not only is concerned with the forest sector but with several other market sectors including the transportation sector. Therefore, within the last decade there has been a notable increased interest in analysis concerning the introduction of secondary and final forest production activities in developing countries. In any such analysis, one of the most important issues that is relevant to both policy makers and producers alike is the establishment of a production program with the development of the proper relationship between that program and the transportation sector. Therefore, the concern has become more than with just defining the proper mix of different levels of production activities considering the development of capacities and capabilities of a particular country's forest sector but also there is a concern with the cost of providing the proper transportation infrastructure which links the developing country with the world forest product market.

A number of studies have been done concerning the economies of the forest sector products on an international level. Although such studies as those done by Sedjo (1983), Wisdom (1982), and Dykstra and Kallio (1984) include innovative and extended discussions of the connection between the forest sector and the transportation sector, most of the existing studies do not analyze in detail the connection and trade-offs between the development of an optimal production mix to produce exports and the development of both inland and international transportation facilities.

Yeats (1981) presents a more detailed analysis of the relationship between ocean transportation costs in particular and the level of production activity. More specifically Yeats is interested in identifying what "factors work against domestic processing" in less developed countries which includes consideration of the influences of ocean transportation costs. *In general* Yeats concluded that:

- processing lowers bulk and stowage factors
- processing increases product value
- processing can increase the fragility of a product making it difficult to handle
- processing can lead to increased freight rates.

From his research, Yeats found that transportation costs actually tend to increase through a particular processing chain. Table 1a shows such a tendency with wood products exported from several less developed countries to the United States. Similar conclusions may be drawn from Table 1b. As the level of production technology increases, in general, both transport costs and the most favored nation tariff level also increases. Therefore, the structure of freight rates facing the less developed countries could be seen as a deterrent to the growth of processing industries.

Lipsey and Weiss (1974) look at yet another aspect of the complex relationship between transportation costs and the characteristics of a particular commodity. They conclude that route, vessel, and commodity characteristics must be taken into account to determine freight rates and could in fact lessen some of the "export deterrence" posed by transport costs. "The characteristic of trade route—the balance of liner and non-liner" trade is of particular importance. Other characteristics of the vessel would include size and type such as conventional bulk carriers as opposed to the new general purpose container vessels which ease the movement from inland to ocean modes of transport.

It is the purpose of this paper to use the existing model structure proposed by Dykstra and Kallio (1984) at IIASA to develop a connection between the location of secondary and final production activities in developing countries and the development of the necessary transportation infrastructure. The paper places emphasis on the theoretical development of such a model. However, the paper also maintains the overall goal of developing an empirically reliable planning tool for both policy makers and the forest sector producers.

Table 1a. Analysis of the Ad Valorem Indices of international transport costs in Indian exports to U.S. (%) (adapted from Yeats 1981).

SITC			Exporters	Primary	Intermediate
242-251-641	Wood in the rough paper pulp, paper and board	India	13.1	24.8	40.0
		Philippines	22.1	n.a.	37.2
242-243-631	Wood in rough, wood shaped, plywood	India	13.1	16.1	34.9
		Malaysia	n.a.	32.2	23.3
		Philippines	22.1	n.a.	37.9

Table 1b. South African Exports to U.S.

	Estimated Ad Valorem Rate (%)		
Processng chain	Transport	M.F.N. tariffs	Total
Paper			
Paper pulp (251)	14.4	0.0	14.4
Paper and board (641)	69.7	3.0	72.7
Paper articles (642)	34.8	7.2	42.0
Wood			
Rough wood (242)	29.6	0.0	29.6
Shaped wood (243)	23.9	0.0	23.9
Wood manufactures (632)	48.6	5.7	54.3

GLOBAL TRADE MODEL FOR FOREST PRODUCTS

During the last two years, the Global Trade Model (GTM) has been developed by the Forest Sector Project core team at the International Institute for Applied Systems Analysis (IIASA). The initial GTM permits the subdivision of the world into as many as 18 regions and 13 forest product categories. Using data from 57 timber producing regions, the GTM can be used to investigate potential structural changes in the forest sector over a 50-year time horizon. The basic GTM model structure is derived from a model of pulp and paper sector of North America to Buongiorno (1981) and Buongiorno and Gilless (1983a and b). It also can be viewed as an extension of the model developed by Adams and Haynes (1983) to study the structure of forest products market in North America. The GTM includes extensions that account for the interdependences between forest products which are developed from a common raw material base. The model contains implicit supply functions for every product within each producing region. These supply functions are assumed to be sensitive to prices of all products under consideration.

The structure of the GTM comprises five components:

- Demand for end products
- Supply of timber
- Supply of recycled paper
- Production
- World trade

It is assumed that the major forces influencing international exchange and production of forest products can be formulated as a model of a competitive market and profit maximizing producers which are subject to market imperfections such as quotas, tariffs, bilateral trade agreements and inertia of trade flow. The impetus for such a focus is the transportation costs. The world market price equilibrium is then derived while taking into account the current market imperfections.

Therefore, the GTM is set in a partial equilibrium economy cast in the mathematical framework of Samuelson (1952) with linear constraints and a nonlinear objective function. The problem of solving the market equilibrium for the regional profit is found by maximizing the sum of consumer surplus and producer surplus subject to materials balance constraints and is formulated as follows:

$$\underset{c_{ik}, y_{im}, e_{ijk}}{\text{maximize}} \left[\sum_{ik} \int_0^{c_{ik}} P_{ik}(c_{ik}) \mathrm{d}c_{ik} - \sum_{im} \int_0^{y_{im}} Q_{im}(y_{im}) \mathrm{d}y_{km} - \sum_{ijk} z_{ijk} e_{ijk} \right] \quad (1)$$

subject to

$$c_{ik} - \sum_{m} A_{ikm} y_{im} + \sum_{j} (e_{ijk}) - e_{ijk} = 0 \quad \text{for all } i \quad (2)$$

$$0 \le y_{im} \le K_{im} \quad \text{for all } i, m \quad (3)$$

$$L_{ijk} \le e_{ijk} \le U_{ijk} \quad \text{for all } i, j, k \quad (4)$$

where indices i and j refer to regions, k to products, m to production activities, and

c_{ik} = consumption of product k in region i

$P_{ik}(c_{ik})$ = price of product k in region i associated with a particular level of consumption c_{ik}

y_{im} = level of annual production by process m in region i associated with a particular level of production activity m

$Q_{im}(y_{im})$ = marginal cost of production by process m in region i associated with a particular level of production y_{im}

z_{ijk} = unit cost of transporting commodity k from region i to region j

e_{ijk} = quantity of commodity k exported from region i to region j

A_{ikm} = net output of product k per unit of production for process m in region i

K_{im} = production capacity associated with process m in region i

L_{ijk}, U_{ijk} = lower and upper bounds on trade flows of product k between region i and j.

The objective function is the maximization of consumer and producer surplus solving for the equilibrium consumption, production and level of export flow. Equation (2) represents a type of materials balance constraint where consumption is set equal to production less net exports. Equation (3) and (4) represent resource and trade capacity constraints respectively.

Formulated in this manner the objective function measures the 'net social welfare' of a region. More precisely, the solution can indicate the quantity of timber to be harvested in each region, the quantity of harvested raw materials to be traded between regions or converted into final products within each region as well as the quantity of the final products to be traded among the regions considered. The dual solution to this problem gives an indication of marginal prices of both raw materials and final products.

However, for the purpose of this paper the focus is now drawn towards the last term of the objective function which represents the transportation costs. The GTM assumes that there is a competitive world market where the price differential between regions results in trade flows. Under this assumption, an equilibrium solution can be derived assuming there is one world market price and the price differentiation between nations is based solely on transportation costs which can include tariffs, subsidies, and inland, ocean as well as port transportation costs. To introduce this current treatment of transportation costs in the GTM it is useful to consider the following optimality conditions that explain the trading sector in terms of transportation costs z_{ijk} per unit of commodity k, price in region i, π_{ik}, and the shadow price, δ_{ijk}, of trade:

$$-z_{ijk} - \pi_{ik} + \pi_{jk} - \delta_{ijk} \leq 0 \quad \text{for all } i,j,k \tag{5}$$

$$(-z_{ijk} - \pi_{ik} + \pi_{jk} - \delta_{ijk})(e_{ijk} - L_{ijk}) = 0 \quad \text{for all } i,j,k \tag{6}$$

$$\delta_{ijk} \geq 0 \quad \text{for all } i,j,k \tag{7}$$

$$\delta_{ijk}(U_{ijk} - e_{ijk}) = 0 \quad \text{for all } i,j,k \tag{8}$$

these conditions may be interpreted as follows:

(a) if $e^{*}_{ijk} > L_{ijk}$, then $\delta_{ijk} = \pi_{jk}\pi_{ik} - z_{ijk}$

(b) if $U_{ijk} > e^{*}_{ijk} > L_{ijk}$, then $\pi_{jk} - \pi_{ik} = z_{ijk}$

(c) if $e^{*}_{jik} = L_{ijk}$, then $\delta_{ijk} = 0$ and $\pi_{jk} - \pi_{ik} \leq z_{ijk}$.

Further economic implications can be stated from the above conditions when we interpret the Lagrangian multiplier, δ_{ijk}, associated with the condition (a) it is apparent that if a commodity flow exceeds its lower bound then the marginal profit of trade will be the import price at region j less the export price at region i and transport costs. If trade is to occur, as condition (b) states, the price difference between region i and j must be at least equal to the transportation cost. And, if flow is on its lower bound, there is no incentive to trade since the price difference is less than transportation cost as stated in condition (c).

This presentation of transportation cost is very similar to the transportation problem of linear programming as will be seen in the following sections. However, it is difficult for a planner or production analyst to ascertain much of the interaction between the transportation sector and the forest product sector in this simple term. Especially in developing countries, one of the most important questions would be the proper allocation of capital and scarce resources between production development and transportation infrastructure improvements. A change in production technology may bring about new needs in transportation modes, time and capacity on all 'links' of the transportation network used by the producer and in some cases this would include port facilities as well. It is the purpose of the remainder of this paper to focus on developing a more complete description of the interaction between production levels and the efficient use of transportation infrastructure. The following sections develop a theoretical approach to integrating freight network concepts with the forest product trade analysis in the GTM to conceptualize an initial formulation of an integrated forest commodity flow—transportation network model.

THE TRANSPORTATION PROBLEM

To begin the transportation sector development it is beneficial to theoretically displace the transportation term from the GTM objective function and initially observe transportation costs exogenous to the model. For the purposes of this paper, the exogenously determined transportation costs will then be reintroduced into the original GTM objective function.

Extracting the transportation term of the objective function, we can readily see that we have the simplest case of a commodity flow problem: the transportation problem of linear programming. In such a problem, we seek to minimize transportation costs subject to known requirements for total demand. This implies that the producer of a good seeks those commodity flows that will minimize total transportation cost. Thus the formulation is consistent with the assumption that the producer is an overall profit maximizer. Therefore in its simplest form, the transportation problem may be written in standard form as follows:

$$\text{minimize} \sum_{i=1}^{R} \sum_{j=1}^{R} z_{ijk} e_{ijk} \tag{9}$$

subject to

$$-\sum_j e_{ijk} \geq -X_{ik} \quad \text{for all } i,k \tag{10}$$

$$\sum_i e_{ijk} \geq y_{ik} \quad \text{for all } j,k \tag{11}$$

$$e_{ijk} \geq 0 \quad \text{for all } i,j,k \tag{12}$$

where

X_{ik} = the total output of commodity k from region i

y_{ik} = the total import demand for commodity k from region i

z_{ijk} = the transportation cost for one unit of commodity k from region i to region j

e_{ijk} = the flow of commodity k from region i to region j

and the Lagrangian becomes:

$$L = \sum_i \sum_j z_{ijk} e_{ijk} + \sum_i \alpha_i (-X_{ik} + \sum_j e_{ijk}) + \sum_j \beta_j (y_{jk} - \sum_i e_{ij})$$

$$+ \sum_{\substack{i \\ j}} \gamma_{ijk} (0 - e_{ijk}) \tag{13}$$

the first derivative with respect to e_{ijk} is found

$$\frac{\partial L}{\partial e_{ijk}} = z_{ijk} + \alpha_{ik} - \beta_{jk} - \gamma_{ijk} = 0 \tag{14}$$

therefore

$$z_{ijk} = \beta_{jk} - \alpha_{ik} + \gamma_{ijk} \tag{15}$$

Here α_{ik} and β_{jk} are the dual variables of the transportation problem. They are interpreted here as shadow prices or location rents. From the above we then have the following Kuhn-Tucker condition:

(a) If $\gamma_{ijk} e_{ijk} = 0$; then $\gamma_{ijk} = 0$ and/or $e_{ijk} = 0$

Therefore

(b) If $e_{ijk} > 0$; then $\gamma_{ijk} = 0$ and $\beta_{ik} - \alpha_{ik} = z_{ijk}$

(c) If $e_{ijk} = 0$; then $\gamma_{ijk} \geq 0$ and $\beta_{jk} - \alpha_{ik} + \gamma_{ijk} = z_{ijk}$ or $\beta_{jk} - \alpha_{ik} \leq z_{ijk}$.

Condition (b) implies that for flows e_{ijk} to be positive, the differential in shadow prices must be equal to the transportation costs. Therefore, the rents are determined by the costs. Condition (c) states that if flows do not occur, the rent or 'price' difference is less than the transportation cost and trade could not occur. These are similar to the conditions derived with the GTM.

Briefly, from looking at the dual of the above formulation and applying the fundamental theorem of linear programming we know the objective function of the primal and dual are equal so that:

$$\sum_i \sum_j z_{ijk} e_{ijk} = \sum_j \beta_{jk} y_{jk} - \sum_i \alpha_{ik} X_{ik} \quad (16)$$

for

$$e_{ijk} > 0 \ , \ z_{ijk} = \beta_{jk} - \alpha_{ik} \quad (17)$$

Therefore, the rents are set so that the profit made by traders buying at α_{ik}, paying transportation cost z_{ijk} and selling at β_{jk} are exactly zero. Thus the transportation problem can be used as a model of a competitive spatial system apart from the original GTM objective function. Since we also require

$$\sum_{j=1}^{R} X_{jk} = \sum_{i=1}^{R} y_{ik} \quad (18)$$

then the number of independent constraints is $(2R - 1)$ where R is the number of regions involved. Therefore, it is impossible to solve for all the α_{ik} and β_{jk} even if we know which $e_{ijk} > 0$. So we must set one α_{ik} equal to zero and then find β_{jk} for $e_{ijk} > 0$, and continue to solve for all optimal values.

SPATIAL DISPERSION

Although for initial estimations this simple formulation is useful, it is widely accepted that the cost-minimizing solution given by the transportation problem does not really reflect the real world commodity flow pattern. This is due mainly to the fact that such a cost minimizing solution does not allow for shipments of a single product between regions in both directions since one of these two shipments might not be necessary and would be considered economically inefficient. Therefore, many more than $(2R - 1)$ flows can occur. This 'crosshauling' of commodities occurs because of two main reasons:

- Aggregation of commodities have often classified distinctly different groups of goods in the same commodity class which could appear as crosshauling
- Actual crosshauling of a good due to imperfect information, established trade patterns and product differentiation due to marketing and advertising.

Therefore, it is necessary to find a 'suboptimal' or differently constrained solution to this problem which recognizes these factors to develop a more realistic description of commodity flows. One approach introduced by Erlander (1977) is to observe the present level of crosshauling and construct our model to have a similar level by adding a constraint that is a measure of the level of dispersion or crosshauling expression in an entropy function:

$$S_k = -\sum_i \sum_j e_{ijk} \ln (e_{ijk}) \tag{19}$$

where

S_k = a scalar measure of the level of dispersion in an observed commodity flow matrix.

If unconstrained by other factors the range of S_k is

$$0 \leq S_k \leq -\ln (R) \tag{20}$$

where R represents the number of alternatives choices. Then, defining (0) ln (0) to be zero, S_k is equal to zero if all shipments are made on one alternative. S_k reaches its maximum value if the proportion of shippers choosing each alternative is equal. Therefore the magnitude of S_k is determined partly by the number of alternatives.

Adding equation (19) to the original transportation problem the new Lagrangian is

$$\begin{aligned} L = & \sum_i \sum_j \sum_k z_{ijk} e_{ijk} + \sum_k \frac{1}{\mu_k}(S_k + \sum_i \sum_j e_{ijk} \ln (e_{ijk})) \\ & + \sum_i \alpha_{ik} (-X_{ik} + \sum_j e_{ijk}) + \sum_j \beta_{jk} (I_{jk} - \sum_i e_{ijk}) \\ & + \sum_i \sum_j \gamma_{ijk} (0 - e_{ijk}) \end{aligned} \tag{21}$$

We now combine a modified equation (2) with the transportation model to describe the relationship that can be developed between the transportation sector and the forest product sector. To do so it is necessary to make these preliminary assumptions:

- Total exports of commodity k, E_{ik} and total imports, I_{ik}, are known for each region
- As with the original GTM, transportation costs alone determine interregional commodity flows
- Final demand in a given region is known.

The objective function of such a combined model must reflect the total interregional transportation costs associated with export/import activity. Therefore the transportation problem now takes on the following form:

$$\text{minimize} \sum_i \sum_j \sum_k z_{ijk} e_{ijk} + \sum_{i \in \psi} d_{ik} (E_{ik} + I_{ik}) \tag{22}$$

subject to

$$\sum_{i \in \psi} E_{ik} \geq E_i \quad \text{for all } k \tag{23}$$

$$\sum_{i \in \psi} I_{ik} \leq I_k \quad \text{for all } k \tag{24}$$

$$c_i - A_i y_i + \sum_i (E_{ik} - I_{ik}) = 0 \tag{25}$$

$$-\sum_i \sum_j e_{ijk} \ln (e_{ijk}) \geq S_k \quad \text{for all } k \tag{26}$$

$$e_{ijk} \geq 0 \quad \text{for all } i,j,k \tag{27}$$

$$E_{ik} \geq 0 \quad \text{for all } i,j,k \tag{28}$$

$$I_{ik} \geq 0 \quad \text{for all } i,j,k \tag{29}$$

where

E_i^k and I_i^k are the estimated amounts of exports and imports respectively of commodity k through ports in region i

ψ denotes the subset of regions that have adequate port facilities

d_{ik} is the cost of ex/importing one unit of k through a port in region i, with shipping costs and plus any other costs associated with port activity.

Equation (23) and (24) are introduced as target levels for exports and imports. The remaider of the constraints are already familiar. The new Lagrangian is formed as follows:

$$L = \sum_i \sum_j \sum_k z_{ijk} e_{ijk} + \sum_{i \in \psi} d_{ik} (E_{ik} + I_{ik}) + \sum_k \rho_{ik} (-I_k + \sum_{i \in \psi} I_{ik})$$

$$+ \sum_i \sum_k \gamma_{ik} (c_i - A_i y_i + (E_{ik} - I_{ik})) + \sum_k \frac{1}{\mu_k} (S_k + \sum_i \sum_j e_{ijk} \ln (e_{ijk}))$$

$$+ \sum_i \sum_j \sum_k \lambda_{ijk} (0 - e_{ijk}) + \sum_i \sum_j \sum_k \lambda_{ik_1} (0 - E_{ik}) + \sum_i \sum_k \lambda_{ik_2} (-I_{ik})$$

$$+ \sum_k \sigma_k (E_k - \sum_{i \in \psi} E_{ik} \tag{30}$$

Differentiating with respect to e_{ijk}, E_{ik} and I_{ik} the following optimality conditions are obtained:

$$\frac{\partial L}{\partial e_{ijk}} = z_{ijk} + \frac{1}{\mu_k} \ln (e_{ijk}) - \lambda_{ijk} = 0 \tag{31}$$

$$\frac{\partial L}{\partial E_{ik}} = d_{ik} + \gamma_{ik} - \lambda_{ik_1} - \sigma^k = 0 \tag{32}$$

$$\frac{\partial L}{\partial I_{ik}} = d_{ik} + \rho_{ik} - \gamma_{ik} - \lambda_{ik_2} = 0 \tag{33}$$

And

$$\left.\begin{aligned} \lambda_{ijk} e_{ijk} = 0 \ , \ \lambda_{ik_1} E_{ik} = 0 \ , \ \lambda_{ik_2} I_{ik} = 0 \\ \sum_k \sigma_k (E_k - \sum_{i \in \psi} E_{ik}) = 0 \\ \sum_k \rho (-I_k + \sum_{i \in \psi} E_{ik}) = 0 \\ \sum_k \frac{1}{\mu_k} (S_k + \sum_i \sum_j e_{ijk} \ln (e_{ijk}) = 0 \end{aligned}\right\} \tag{34}$$

Plus the original constraints. All these conditions are interpreted in the following manner:

- if $e_{ijk} > 0$, then $\lambda_{ijk} = 0$ and ln $e_{ijk} = \mu_k(\gamma_{ik} - z_{ijk}$; assuming z_{ijk} is finite, then $e_{ijk} > -\infty$
- if $E_{ik} > 0$, then $\lambda_{ik_1} = 0$ and $\gamma_{ik} = \sigma_k - d_{ik}$
- if $E_{ik} = 0$, then $\lambda_{ik} \geq 0$ and $\gamma_{ik} \geq \sigma_k - d_{ik}$

solving for z_{ijk} we now have:

$$z_{ijk} = -\frac{1}{\mu_k} \ln (e_{ijk}) - \alpha_i + \beta_j + \gamma_{ij} \tag{35}$$

MODAL CHOICE

Next we consider a possible choice between modes and routes. It is important to introduce the concept of the mode and route choice to more realistically develop a model of interregional transportation. For simplicity we now assume there is one specific commodity k such that we drop the commodity notation temporarily and introduce the following notation:

z_{ijm} is the cost from region i to region j by mode m

e_{ijm} is the flow from region i to region j by mode m

p_{ijm} is the proportion of flow from region i to region j on mode m and is equal to $e_{ijm} / E.$, $E. = \sum_i \sum_j e_{ijm}$

q_{ijm} is an *a priori* probability of a shipment from region i to region j on mode m

Then we can redefine S_k as

$$S_{km} = -\sum_i \sum_j p_{ijm} \ln \left[\frac{p_{ijm}}{q_{ijm}} \right] \tag{36}$$

and the original problem becomes

$$\text{minimize} \sum_i \sum_j \sum_m z_{ijm} p_{ijm} \tag{37}$$

subject to

$$\sum_j \sum_m p_{ijm} = \frac{E_i}{E_.} \quad \text{for all } i \tag{38}$$

$$-\sum_i \sum_j \sum_m p_{ijm} \ln \left[\frac{p_{ijm}}{q_{ijm}} \right] \geq S_{km} \tag{39}$$

$$p_{ijm} \geq 0 \tag{40}$$

where

$$z_{ijm} = \min_m (z_{ijm}) \tag{41}$$

From this structure of the model, the following properties are observed from the original optimality conditions and by solving for e_{ijm} we find:

$$e_{ijm} = \frac{E_i I_j \exp(-\mu z_{ijm})}{\sum_j \sum_m I_j \exp(-\mu z_{ijm})} \tag{42}$$

and the following conditions are determined:

- Shipments are proportional to E_i, I_j and the exponential of the transportation costs
- Destination and mode chosen are determined by an exponential function
- The denominator of equation (42) summarizes the accessibility from region i to all regions j via all modes m.

Further analysis develops a general form for the z_{ij}'s as:

$$\tilde{z}_{ij} = -\frac{1}{\mu} \ln \sum_m q_{ijm} \exp(-\mu z_{ijm}) \tag{43}$$

where $\tilde{z}_{ij}$ is seen as a weighted function of the modal costs.

NETWORK CHARACTERISTICS AND ROUTE CHOICE

So far we have considered the entropy constraint in terms of mode choice where the resulting model is a combined destination and route choice model with z_{ijm} assumed known. Now we move to a more realistic level of modeling where z_{ijm} is not known but instead depends upon the transportation 'link' costs which in turn depend on the amount flow transversing a particular link. Reducing the number of modes to one for simplicity in presentation, we can assume that transport cost can be defined directly in terms of operating costs and are reflected in rates charged for a particular service. Several approaches have been developed for estimating the level of freight rates. For now we simply assume that each link of the transportation network has a unit user cost v_a, such that it is an increasing function of annual flow, f_a:

$$v_a = v_a(f_a) \tag{44}$$

where the flow associated with a particular link a is defined as follows:

$$f_a = \sum_a \sum_i \sum_j \sum_R e_{ijkR} \delta_{ijkR}^a \tag{45}$$

and

e_{ijkR} = the amount of commodity k shipped from region i to region j on route R

d_{ijkR} = $\begin{cases} 1 & \text{if link } a \text{ is included in route } R \text{ from region } i \text{ to region } j \text{ for commodity } k \\ 0 & \text{otherwise} \end{cases}$

Since we have assumed that the shipper is a cost minimizer in general, if link costs are fixed the shipper will choose the route or combination of links that traverse the transportation network with minimal accumulated link cost. This behavior could congestion on the lowest cost link and therefore the basis for the assumption of costs being an increasing function of the flow on a link. One link can serve many different routes so its is easily seen that flows between each origin–destination pair are very interdependent between links. If, even under congestion we continue to assume that the wareowner seeks the minimum cost path, then a network equilibrium problem results. When the travel costs over all route pairs are equal and when no unused route has a lower transport cost we have Wardrop's (1952) user equilibrium. The solution to this problem results in each shipper minimizing his total transport costs.

The problem of network equilibrium for a single mode was introduced by Beckman et al. (1956) and made computationaly efficient by LeBlanc et al. (1975). The mathematical formulation of the model minimizes the total area under the link cost function to obtain an equilibrium solution. Therefore, our transportation problem objective function now becomes:

$$\text{minimize} \sum_a \int_0^{f_a} v_a(t)\mathrm{d}t + \sum_i \sum_j d_{ik}(E_{ik} + I_{ik}) \tag{46}$$

subject to

Equations (23)–(29), and (34).

The solution to this problem does not minimize total transport cost unless link costs are assumed constant. However, if link costs are not constant, a solution can be found by replacing equation (44) with

$$\sum_a f_a v_a(f_a) = \sum_a \int_0^{f_a} m_a(x)\mathrm{d}x \tag{47}$$

where

$$m_a(x) = \frac{d(v_a(f_a))f_a}{\partial f_a} = \text{marginal cost of link } a$$

and at the optimal solution for all used routes, R, connecting region i to region j:

$$z_{ijk} = \sum_a v_a(f_a)\delta_{ijR}^{ak} \tag{48}$$

Ultimately, equation (47) would replace the transportation term in equation (1). Several programming techniques such as bi-level mathematical programming reviewed by Kolstad and Lasdon (1983) and the nonlinear complementarity approach used by Fisk and Boyce (1983) have been applied to the integration of commodity flows and the transportation network activity. However, the consideration of these developments is beyond the scope of this paper. For now we seek to maintain the development of the transportation costs in a separate program and reintroduce these costs into the original objective function thereby maintaining the original formulation and optimality conditions of the original GTM while leaving the possibility of integrating these two problems as a future research opportunity.

However, the problem remains to identify cost functions for the link which in actuality is the cost function for a particular modal choice. The original user cost estimations were from Hassan and Wilson (1982) and Sedjo (1983). In general most research done in this area suggests that transportation costs are determined by the following network mode characteristics:

- Distance of voyage
- Size of shipment
- Loading and unloading terms
- Registry of ships (in the case of ocean transport)
- Volume of trade on a particular route
- Value of commodity
- Total route characteristics

Current research in this area provides many viable alternatives for determining a cost function for each link.

For the purpose of this study, we are concerned with determining and strengthening the explanation of the relationship between commodity flows and transportation costs. To establish the approach presented here satisfies that research we turn to a recent study done by Kim, Boyce and Hewings (1983) done with Korean data. In this study, 18 commodities grouped into four categories, of which lumber can be found in the fourth category, were studied. To develop a relationship between the transportation and commodity flows, the dispersion of flow constraint, equation (19) is normalized and an expression for flow is determined:

$$e_{ijk} = A_{ik} E_{ik} B_{jk} I_{jk} \exp(-\beta_k z_{ijk}) \tag{49}$$

Then, the transportation deterrence factor, β_k, and the observed level of dispersion is determined. The results can be seen in Table 2. From this table one can see that

if $\beta_k = 0$ then e_{ijk} is proportional to production and unrelated to transportation costs

if $\beta_k \to \infty$ then e_{ijk} approaches the solution of the simple transportation problem of linear programming.

As shown by Table 1, it is obvious that shippers of commodity group four, including paper products, are the most sensitive to transportation cost — which is in keeping with our original assumptions and our overall objective of depicting the relationship between the transportation and forest product sector.

Table 2. Results of Korean prototype commodity flow model (adapted from Kim, Boyce and Hewings, 1984).

Commodity group	S_k	β_k
1	5.30075	1.8330
2	4.79244	1.94442
3	5.57109	1.05553
4	4.72683	2.02776

Group 1: coal, limestone, petroleum, metallic minerals

Group 2: cement, non metallic materials

Group 3: agricultural products, livestock and silkworm, forest products fish

Group 4: food and tabacco, textile, lumber-wood-furniture, pulp and paper, chemicals, machinery, miscellaneous manufactured products.

CONCLUSION

Therefore, to summarize, by analyzing the transportation sector separately we have started with the simple transportation problem of linear programming and expanded it mathematically to include the concept of spatial interaction through the dispersion of flow constraint which allows us to reach a 'suboptimal' solution while considering the problems of aggregation and crosshauling. Reformulation of this constraint was shown to be useful in reflecting modal choice and established trade agreements. Then returning to the assumption of one mode we introduced network characteristics and route choice into the expanded transportation problem objective function.

Briefly, it was shown that the materials balance constraint of equation (2) in the GTM constraints could be introduced as a constraint on the transportation problem to clarify the connection between the transportation sector and the forest product sector. However, since it is the overall purpose to reintroduce the new transportation costs into the GTM, this constraint could be dropped in the subprogram.

The advantages of this approach are mostly found in the increased flexibility in defining which link, port, mode or combination of the latter is the most cost efficient to the producer/shipper of a product. In considering the overall objective, the forest sector producer in a developing country must find the proper balance between investment in new production activities and the possibility of investing in the transportation infrastructure. To such decision makers, it is extremely important to seek out the most efficient means of transport among all model and routes.

Although in this initial analysis, the GTM objective function still uses the fixed transportation cost term, we recognize that the assumption of constant transportation costs between regions is of little use to those who must decide what priority investments in both production technology and transportation improvements should be made. Therefore, the proposed 'submodel' provides some extended description of the issues involved in the transportation sector that could effect commodity flows.

The development and functional form of the model facilitates future model developments which would integrate the forest sector and the transportation sector more completely. Theoretically, such advances are already being developed (Griffin, Boyce and Kim 1983) for the integration of world trade flows and the transportation network in which the hierarchical interactions between the producer–trader–shipper and in some cases a group of governmental or institutional regulations are considered. Therefore, it appears that the framework of the model proposed here and its further development will provide a useful planning tool for developments, planners, producers, governments, and industries interested in the forest sector either directly or indirectly.

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