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## **ANNA'S LIFE EXPECTANCY**

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Every year the Office of the Actuary of the U.S. Department of Health and Human Services releases an estimate of the life expectancy for a newborn child: the most recent figure for a female was 78 years, and for a male, 71 years [1]. But this number assumes that over the course of the child's life there will be no further progress against mortality, an improbable supposition indeed. The actuaries who calculate official life expectancy estimates assume a baby born in a given year will experience the mortality rates for every age of that year throughout her life.

On March 20 of 1984 Anna Bodil Vaupel was born. The Office of the Actuary would give her a life expectancy, or what demographers denote by  $e_0^0$ , of not much over 78. A.B., however, should live much longer than that. Throughout this century great progress has been made against mortality in medicine, public health, sanitation, nutrition, safety, and other areas, and it seems implausible to suppose such progress will come to a halt now. There are several ways of thinking about the continuation of this

progress, and here we shall look at three: continued progress at the same rates, accelerated progress, and cause-specific, or disease-by-disease, progress. We will base our estimates on rough calculations that are hopeful in that we ignore catastrophes such as nuclear war.\* Although the estimates are approximate, the message is clear: Anna's life expectancy and the life expectancies of other newborn girls and boys is probably many years longer than the official estimates.

### **STEADY PROGRESS**

If progress against mortality at each age continues at the same rates as in the 1970s, how long can Anna B. expect to live? It turns out that if we do assume such steady progress against mortality, her expected life span is 90 years, fully 12 years more than official estimates give her. If conditions improve at the same rates indefinitely and Anna has a daughter at age 25, her daughter can expect to live to be 94. If the daughter has a daughter under the same conditions, Anna's granddaughter has an expected lifespan of 97.

### **ACCELERATED PROGRESS**

There is no need to restrict the calculations to assumptions of a constant rate of progress, however. A look at progress against mortality in the previous decades of the twentieth century reveals that in the 1970s the rates of such progress were greater than in previous decades at most ages. This acceleration may continue, and it is natural to wonder what its continuation would mean for persons being born now.

In fact, when extrapolations are made based on the acceleration of mortality progress from the 1960s to the 1970s, Anna's life expectancy increases to 93 years. When

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\*The methods used in this paper are discussed in the Appendix.

we account for acceleration in figuring male life expectancy, the result is 97 years, which would finally give males a higher  $e_0^o$  than females. Clearly this means that progress against male mortality accelerated greatly in the 1970s; little progress was made at most ages in the 1960s, and at many ages mortality rates in fact increased.

#### OTHER RATES OF PROGRESS

A recent report of the Office of the Actuary does make some conservative projections of future progress against mortality, although even this progress is not taken into account when figuring official estimates of life expectancy [2,3]. Since the actuarial figures do not go past the year 2050, we will assume mortality rates at all ages remain constant after 2050. Under these conditions A.B.'s life expectancy is 83, some five years higher than the official estimate.

Mortality rates improved at an average of almost 2 percent per year in the 1970s. If the 2 percent improvement rate is applied uniformly to all baby A.B.'s ages, her expected lifespan is 102 years. The great difference between this number and the 90 which results from extrapolating the current rates of progress, which **average** 2 percent, is attributable to the fact that more people die at older ages than at younger ages, whereas the rate of progress against mortality tended to less than 2 percent at older ages and greater than 2 percent at younger ages. Even if progress against mortality is only 1 percent per year at all ages, Anna can expect to become an octogenarian with an  $e_0^o$  of 87 years.

Table 1. Various estimates of the life expectancy of a newborn.

Circumstances	Baby A.B.'s $e_0^o$	Baby Boy's $e_0^o$
Official prediction	78	71
Continued progress against mortality as in 1970s	90	81
Progress continues to accelerate as it did between 1960 and 1980	93	97
Progress slows down as actuary projects	83	75
Progress is 1 per cent every year at all ages	87	79
Progress is 2 percent every year at all ages	102	94

### DISEASE-BY-DISEASE PROGRESS

But where specifically will this progress against mortality be made? Again, it seems reasonable to gain insight from extrapolating recent rates of progress against the several causes of death to see what A.B. will probably die from. Anna's  $e_0^o$  turns out to be 86 years after this extrapolation.

As seen in Table 2, A.B. would be more likely to die from heart disease than any other cause if current mortality rates persisted. If the rates of **improvement** against these causes continue, as the right columns of the table show, cancer will overtake all other causes of death and become the leader.

Some of these numbers may be surprising, but they are explained by the ages at which people die from these diseases today and the ages at which improvement is being made. For example, although many more people die from heart disease than from cancer today, and some progress is being made against cancer, most of that progress is made at younger ages, where the chances of A.B. dying are relatively low; in the case of heart disease, progress is being made at older ages, when the chances of her dying are high. In fact, while in recent years total average annual progress

Table 2. What will a newborn eventually die from?

Cause	Probability of dying from various causes			
	Assuming no progress		Continued progress	
	Female	Male	Female	Male
Heart disease	42%	40%	11%	9%
Cancer	18	20	44	67
Vascular disease	19	13	1	1
Violence	4	8	1	3
Respiratory causes	5	7	4	11
Congenital causes	1	1	1	1
Digestive diseases	2	2	1	<1
Diabetes mellitus	2	1	3	<1
Liver cirrhosis	1	2	1	2
Other causes	6	6	33	5
Total	100%	100%	100%	100%

against heart disease for females was close to 3 percent, it was slightly negative for cancer--that is, cancer mortality actually increased. As is apparent from the even more extreme numbers for male mortality, cancer trends have been still more dismal for males, averaging nearly a 1 percent annual increase.

The probability of Anna dying from violent causes, which include accidents, homicides, and suicides, is only around 1 percent and may be unrealistically low, although the point is debatable. The fact is that great progress has been made against death by violence at most ages, especially older ages. The accident death rate among the elderly is a good measure of how healthy they are, and so this progress may be a sign of improvements in health among the elderly. Extrapolating this progress over A.B.'s life yields the low value of 1 percent. But if **all** disease were eliminated, everyone would have to eventually die from violence, which would raise A.B.'s probability of dying from violence to 100 percent.

Models are sometimes more useful for counter-prediction than for prediction. If we view a 1 percent probability of dying from violence as highly unlikely, then we must believe that the rate of improvement cannot continue as it did in the 1970s.

#### **IF CANCER IS ELIMINATED**

Since cancer becomes such a major cause of death, it is natural to wonder how the eradication of cancer would affect life expectancy. The conventional answer has been that curing cancer would only increase expected lifespan by little more than two years [4]. This slight prediction of a cancer cure's impact is due to the fact that at **current** rates only about a fifth of males and females can expect to die from cancer. Like the conventional life expectancy estimates, however, this projection changes if we assume continued rates of progress against mortality.

If we take the cause-specific death rates projected above, most of which improve each year, and subtract the cancer mortality rates from each year of Anna's life, A.B.'s  $e_0^o$  increases to 96 years. This substantial increase of a full decade stems from the high chances, 44 percent, of Anna dying from cancer if current progress against the various causes of death continues. Among males, where cancer becomes even more dominant, accounting for two-thirds of all deaths, its eradication raises life expectancy from 74 to 88.

Eradicating today's number one killer, heart disease, only increases A.B.'s life expectancy from 86 to 87. A continuation of present trends in progress against mortality would mean that heart disease can be expected to become much less important, and thus its total elimination would do little to increase expected life span. Getting rid of **all** cardiovascular disease, in fact, only raises Anna's  $e_0^o$  by two years.



## **THE PROSPECTS FOR INCREASING LIFE EXPECTANCY**

The projections made here for baby A.B.'s lifespan may seem overly optimistic, but they may eventually turn out to be conservative if the predictions of certain biomedical scientists on future progress against mortality rates, and even the imminence of a breakthrough against aging itself, turn out to be correct. According to biologist Robert A. Weinberg of M.I.T., for example, the recent advances in genetic engineering will make great strides possible against such ailments as atherosclerosis, cancer, and diabetes both in our knowledge of what causes them and in our ability to treat them [4]. And U.C.L.A. gerontologist Roy Walford writes that his field is showing the early signs of a scientific revolution, with several equally credible competing hypotheses about the aging process being offered [5].

One set of theories about aging assumes that the process is caused by cellular damage. A number of medical scientists have done research into the oxygen free radicals which form during normal metabolic processes and which play a newly discovered role in cellular injury and perhaps in aging itself (e.g., [6, 7, 8]). Other scientists are experimenting with increasing the rate of cellular repair, which they say will increase life span substantially.

An alternate set of theories is based on the paradigm that aging is programmed into cells. One holds that the expression and repression of certain genes at certain ages is responsible for aging, and manipulation of this expression and repression could extend lifespan. Another says that the body releases a destructive hormone at a predetermined time; this hormone may soon be identified. A third theory holds that it is the breakdown of the immune system which holds the key to extending life expectancy [5].

If a revolution in the biological sciences does take place, by one of these theories or by a synthesis of them, the demographic impact will be drastic, with entirely new challenges presenting themselves to demographers and to society in general. Imagine the social, economic, and political consequences if Anna's life expectancy was 150 or even 200 years.

## APPENDIX

Use of the approximate formula for life expectancy,

$$e_0^o = \sum_{a=0}^w q(a)p(a)(a + .5) \quad , \quad (1)$$

where  $q(a)$  is the mortality rate at age  $a$ ,  $p(a)$  is the probability of being alive at age  $a$ , and  $w$  is the age beyond which no one lives, shows what needs to be calculated to find baby A.B.'s  $e_0^o$ . In all of our calculations reported here, we took  $w$  to be 200 years. In no case would the use of a higher  $w$  (of, say, 250 years) make any difference in the results; in nearly all cases, use of a lower  $w$  (of even 120) would not change the results by more than a fraction of a year.

### *Linear Progress Against Mortality:*

Using mortality rates for age 0 through age 119 in 1970 and comparing them with their 1980 counterparts [2, 3] allows extrapolation of the 1970s rates of progress over

Anna's life. First each  $q(a)$  for Anna must be modified according to the rates of progress observed in the 1970s. If  $R(a)$  equals the progress multiplier, or the number by which we multiply an old mortality rate to get a new mortality rate, and  $q_y(a)$  equals the mortality rate at age  $a$  in year  $y$ , then

$$R(a) = q_{80}(a) / q_{70}(a) .$$

The resulting number represents progress over an entire decade; it will be more useful to find progress over a single year. Since the rate of progress over a decade is given simply by the annual rates within that decade multiplier together, the average annual progress multiplier is found as follows:

$$R(a) = [q_{80}(a) / q_{70}(a)]^{0.1} . \tag{2}$$

To figure each  $q(a)$  for Anna B., simply use each  $R(a)$  raised to the appropriate power:

$$q(a) = q_{80}(a) [R(a)]^{(a+4)} . \tag{3}$$

The exponent must be  $(a+4)$  because Anna was born in 1984 rather than 1980.

Next the task is to figure the probability Anna will be alive at age  $a$ . The probability of her living to age  $a$  equals the probability that she lived to the previous age times the mortality rate for the previous age:

$$p(a) = p(a-1)q(a-1) \tag{4}$$

where  $p(0) = 1$ .

Now equation (1) yields Anna's life expectancy.

*Accelerated Progress:*

Just as before, dividing the rates of progress against mortality for a later decade by those of an earlier decade and finding the tenth root of the result gives the average annual rate of progress for each age. This average is probably not exactly true for any one year, but we can approximately represent it as applying to the midpoint of each decade:

$$R_{65}(a) = [q_{70}(a) / q_{60}(a)]^{0.1} \quad , \quad (5)$$

$$R_{75}(a) = [q_{80}(a) / q_{70}(a)]^{0.1} \quad . \quad (6)$$

For most ages,  $R_{75}(a)$  is less than  $R_{65}(a)$ ; that is,  $q_{80}(a)$  is less than  $q_{70}(a)$  by a greater percentage than  $q_{70}(a)$  is less than  $q_{60}(a)$ . To find out by how much progress accelerated at each age, divide the second decade's progress multiplier by the first decade's. As 1975 minus 1965 equals 10 years, raise the acceleration rate to the one-tenth power:

$$A(a) = [R_{75}(a) / R_{65}(a)]^{0.1} \quad . \quad (7)$$

At several ages, progress against mortality actually decelerated in the 1970s. We ignored this deceleration by changing the acceleration rate from the negative number signifying deceleration to 0, signifying no acceleration.

The next task is to find the mortality rates,  $q$ , for each of Anna's ages. Baby A.B.'s  $q$ 's must be based on the 1980  $q$ 's as modified by  $R_{75}(a)$  and  $A(a)$ . The mortality rate at age  $a$  for Anna is equal to the mortality rate for that age in 1980 times the rate of progress for that age between 1980 and 1981, times the rate of progress between 1981 and 1982, and so forth, until the year in question is reached:

$$a(a) = q_{80}(a) \times R_{80}(a) \times R_{81}(a) \times R_{82}(a) \times \dots \times R_{80+a+3}(a) \quad .$$

The answer to what  $R_{81}(a)$  and  $R_{82}(a)$ , etc., are, lies in  $A(a)$ , the acceleration multiplier. The rate of progress between 1980 and 1981 equals the rate of progress in 1975, our most recent and accurate figure, times the acceleration multiplier raised to the fifth power (to account for the passage of five years). Likewise,  $R_{81}(a)$  is  $R_{75}(a) \times A(a)^6$ .

Thus:

$$q(a) = q_{80}(a) \times R_{75}(a) \times A(a)^5 \times R_{75}(a) \times A(a)^6 \times R_{75}(a) \times A(a)^7 \times \dots \times R_{75}(a) \times A(a)^{(a+8)} .$$

It follows that

$$q(a) = q_{80}(a) \times [R_{75}(a)^{(a+4)}] \times [A(a)^x] ,$$

where  $x$  is the sum of the values of all exponents of  $A(a)$ . To find  $x$ , we use the rule Gauss discovered as a schoolboy for summing consecutive numbers. Note that

$$5 + 6 + 7 + 8 + 9 + 10 = (4 \times 6) + (1 + 2 + 3 + 4 + 5 + 6) ,$$

which equals  $[(4 \times 6) + (6 \times 7)] / 2$ . More generally, the value of  $x$  is given by

$$4 \times (4 + a) + [(a + 4) \times (a + 5) / 2] . \tag{8}$$

The same equations for  $p(a)$  and  $e_0^o$  are used as before.

*Using the Actuary's Estimates:*

The data on future mortality in the actuary's report is by decade rather than by year. Since mortality rates improve every year rather than every ten years, we must take the actuarial figures for each decade and interpolate yearly improvement. The projected rate of progress against mortality must be found, using the equations above.

As an example, assume we are given  $q(30)$  for the year 2010 and  $q(30)$  for 2020, but not for any years in between. The yearly progress multiplier is then

$$R = [(q_{20}(30)/q_{10}(30))^{0.1}] .$$

Suppose we want  $q(30)$  in the year 2014, the year Anna is 30. Now,  $q(30)$  in 2014 equals  $q(30)$  in 2010 times the yearly progress multiplier raised to the fourth power, to account for the passage of four years. Thus Anna's mortality rate is:

$$q_{14}(30) = q_{10}(30) \times R^{0.4} . \quad (9)$$

*Cause-Specific Mortality Rates:*

We used cause-specific period data released by the Office of the Actuary on mortality rates in 1977 and average annual rates of improvement between 1968 and 1978. Although these data are period data, or slices of time, we can use them to figure out Anna's cause-specific mortality rates. All we need do is improve all the mortality rates for each disease through all of A.B.'s life, using the same annual percentage improvements as were experienced in 1968-1978, then apply these rates to Anna's lifetime. The data we use only includes mortality rates up to the category 85+; for higher ages up to 119, we used total death rates subdivided, based on the proportions in the category 85+, into the ten causes; and after age 119, we extrapolated the cause-specific mortality rates based on the average annual rate of increase in mortality for ages 100 to 119.

We encountered a problem with the cancer rates, which becomes greater than 1 before age 200 because of the continued negative progress against them. Therefore we worked with  $\mu$ , the force of mortality, after age 119, because it is acceptable for  $\mu$  to become greater than 1. The formula we used to convert  $q$ 's to  $\mu$ 's is:

$$\mu(a) = -\ln(1 - q(a)) . \quad (10)$$

Before we figure life expectancy, we convert the  $\mu$ 's back to  $q$ 's, using the formula:

$$q(a) = 1 - e^{-\mu(a)} \quad . \quad (11)$$

*Eliminating Cancer:*

Clearly, the first step is to subtract the cancer death rates from the total death rates. But here again is a complication: by eliminating cancer, we must account for the increased exposure to the other diseases A.B. experiences each year since she did not die from cancer. In other words, the death rate after cancer eradication equals the total death rate minus the cancer death rate plus the additional death rate caused by slightly increased exposure to other causes of death.

Since people die on the average on July 1, we must allow for an additional half year of exposure to non-cancerous causes of death. Therefore, letting  $q_c$  denote cancer mortality rates and  $q'$  denote the total mortality rates after cancer is eradicated,

$$\begin{aligned} q'(a) &= q(a) - q_c(a) + q_c(a) \times [(q(a) - q_c(a)) / 2] \quad (12) \\ &= [1 + (q_c(a) / 2)] \times [q(a) - q_c(a)] \quad . \end{aligned}$$



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