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(M,R)-SYSTEMS AS A FRAMEWORK FOR MODELING STRUCTURAL CHANGE IN A GLOBAL INDUSTRY

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ABSTRACT

The theory of metabolism-repair (M,R)-systems is developed as a means for mathematically characterizing an industrial firm, and a network of such (M,R)-systems is proposed as a suitable vehicle for describing an entire industry composed of several interacting firms. It is shown that virtually all of the important features of an industrial process including production, marketing, innovation, growth, decline, emergence of new firms and so on can be accommodated within the (M,R)-framework.

Theoretical issues associated with time-lags, dynamics, adaptation and selection are explored from the vantage point of (M,R)-systems, as are practical questions involving the application of the theoretical ideas to the world automotive industry. The paper concludes with a discussion of how (M,R)-system can be used as a means for comparison of entire industries using the mathematical machinery of category theory.

(M,R)-SYSTEMS AS A FRAMEWORK FOR MODELING STRUCTURAL CHANGE IN A GLOBAL INDUSTRY

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1. The Problem of Structural Change

With the arrival of rapid global communication facilities via satellites and the widespread availability of cheap worldwide transportation by air and sea, the phenomena of the transnational corporations (TNC) has emerged, carrying with it a total and complete re-shaping of the structure and operation of major industrial activities. Prior to the advent of the TNC, large industries were considered nationally; now, for example, the automotive industry is scattered throughout the world with firms engaging in design in one place, production in another and marketing and sales everywhere. The situation is further compounded and confused by a myriad of interlocking joint ventures, co-production agreements, partial mergers and so forth. What all of this amounts to is a discontinuous shift from one way of doing business to another and from one industrial paradigm, emphasizing national centralization and domestic markets, to a global structure transcending national boundaries and, to a great extent, local governmental control. The problem of industrial structural change is basically how to account for this transition, how to understand its implications for the future evolution and development of a given industry and how to gain some understanding of the way such changes can be directed, or managed, to avoid unnecessary chaos, disorder and economic upheavals during the transition periods from one structure to another.

To study the problem of structural change, a suitable conceptual framework is needed within which the various firms comprising the industry can play out their roles in interaction with their environment. Whatever framework is used, it must account for the way in which firms execute their design, production and marketing functions, as well as incorporate mechanisms whereby the firms can expand, merge, or even cease to exist. The conceptual scheme must also allow for the mutual interactions of the firms of the industry, both with each other, and with their external environment. The outside environment includes the suppliers of the raw materials and resources needed for the firm's activities, the consumers of the firm's product and the various environmental influences exerted by government regulators acting by the setting of the economic climate (through taxes, interest rates, exchange restrictions, etc.), and the business climate (through tariffs, quotas, import restrictions and the like).

The foregoing requirements for a conceptual framework for the study of industrial structural change have a strongly biological overtone, suggesting that the view of a global industry as a living multicelled organism may serve as a foundational metaphor for the framework we seek. The balance of this paper is devoted to the exploration of this idea. More specifically, the notion of an (M,R)-system (metabolism-repair system) is examined as a candidate for the type of theoretical construct needed to capture the main features of industrial structural change. Originally, (M,R)-systems were introduced into biology by Rosen [1-2] as a means to study cellular development of organisms by breaking away from the traditional bio-chemical types of analyses, and employing a purely relational analysis emphasizing the functional rather than structural organization of the system. This approach leads to the study of *classes* of abstract biologies and a means for their comparison rather than to detailed material analysis of a single organism. This is exactly the type of scheme needed to investigate industrial structural change, although as we go along it will become clear that certain biological aspects of the (M,R)-systems will require modifications in the industrial context. Nonetheless, the (M,R)-framework that follows, does, in our opinion, provide a suitable mathematical skeleton upon which to build an operational theory of industrial structural change.

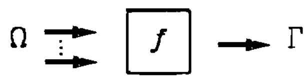
2. Production and Sales as an Input/Output Process

Our underlying basic hypothesis is that an industry such as the world auto industry, is composed of a collection of interacting firms receiving inputs from an external environment, processing these inputs into the products of the firm, which are in turn discharged back into the environment for which the firms receive additional resources (money, usually) to continue their activity. For a variety of reasons that will become clear later, it is most natural and convenient to regard the firms' outputs as the money they receive from their products rather than the products, themselves. In short, the real mission of a firm is to make money, not products, and the products are thought of as only a vehicle to facilitate this higher-level goal.

For the moment, let us concentrate upon the description of a single firm F as an (M,R)-system. We will return later to the case of several firms (an industry). Let Ω denote the set of environmental inputs received by F. In general, the elements of Ω are both *physical* inputs such as raw materials, labor, machinery and so forth, and the external *operating* inputs such as the economic, political and technological constraints of the general environment. The firm accepts an input $\omega \varepsilon \Omega$ and processes it via some internal production and marketing procedures to produce a marketable product which is then sold, thereby generating an output $\gamma \varepsilon \Gamma$, measured in monetary units. Here, due to the assumption that the observed output is money, we could take $\Gamma = R$, the real numbers. To maintain uniformity of scale between inputs and outputs, we can introduce prices for all environmental inputs, thereby converting all inputs into monetary equivalents. We shall omit this mathematically trivial, but possibly economically important step in the remainder of the paper. Thus abstractly the behavior of F is represented by a *metabolic* map

$$f: \Omega \longrightarrow \Gamma$$

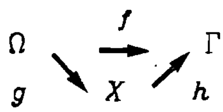
(Note: in the economics literature, it is common to call f a *production function*. To preserve our biological metaphor, we shall depart from this convention in this paper). Schematically, we can represent the production and sales component of F as



The foregoing picture is very familiar in the mathematical system theory literature, where it is termed an *external* or *input/output* description of the behavior of F [3-4]. If we want to focus attention on the manner in which the inputs from Ω are translated into revenue by means of specific products of the firm, then we must look at the *internal* behavior of F. Abstractly, what this means is that we must "factor" the metabolic map f through a state space X, using two maps g and h such that

$$g: \Omega \longrightarrow X, \quad h: X \longrightarrow \Gamma$$

In other words, we must find a space X and maps g and h such that the diagram



commutes.

In our industrial setting, the space X and the maps g and h have a very interesting interpretation: X represents the actual products that the firm produces (cars, TVs, lamps, drugs or whatever), while the map g specifies how inputs are transformed into products (a *production* map). The map h represents the manner in which the firm translates products into revenue, (i.e., h is a *marketing/sales* map).

For a variety of practical as well as mathematical reasons, it is customary to impose the additional requirements that the map g be *onto*, while the map h is *one-to-one*. Such a factorization of f is called *canonical*, and is essentially unique. These conditions have a very direct interpretation in the business setting: the production map being onto means that any level of production can be achieved if F is supplied with suitable inputs, i.e. there are no intrinsic limitations on the firm's ability to produce products given adequate raw materials and other resources. The requirement that the marketing/sales map be one-to-one just means that different levels of production generate different amounts of revenue. Or, put another way, two distinct levels of production cannot generate the same revenue for F. Procedures for generating such canonical factorizations of a metabolic map f are well-known in the system theory literature and will not be discussed further here (cf. [3-5]).

3. Innovations, Repair and Replication

The standard system-theoretic framework presented above for describing the metabolic behavior of a firm as an input/output map f (or, by its equivalent factorization through the production and marketing/sales maps g and h) would be perfectly adequate for the characterization of F if the firm were operating in a totally stable environment with no competition. However, the intrusion of real-world considerations into the firm's activities results in the need for the firm to continually engage in changes of its product, introduction of new production techniques and development of alternate marketing strategies if it is to remain a viable enterprise. In biological terms, the firm must repair damages *and* adapt or else enter a senescent phase ultimately resulting in its extinction. In biological organisms, the adaptation and repair is carried out by genetic programs which re-process the system output to renew the metabolic behavior of the organism. In the context of an auto firm, such a restoration of the metabolic activity can only result from the repair of different production and/or marketing procedures, i.e. renewal of the maps g and h. This can come about only through technological improvements, better managerial procedures and/or incorporation of new knowledge, i.e. through innovation. Our basic question here is: how can the modeling framework introduced earlier be extended in a natural fashion to account for the firm's need to "innovate or die?"

The key to answering this question is to note that the only way the firm can renew its metabolic activity is to utilize some part of its revenues to regenerate either its production processes or its marketing approach or both. Thus, the firm's "repair" mechanism must ultimately be a map that transforms the firm's output (revenues) into the desired metabolic structure. If we let $H(\Omega, \Gamma)$ denote the set of all possible metabolic processes, and if Φ_f denotes the repair map, we have

$$\Phi_f : \Gamma \longrightarrow H(\Omega, \Gamma).$$

Note that we explicitly indicate the dependence of the repair map upon the metabolism f since the objective of the repair procedure is to reproduce f which is an activity of the firm and, as such, is affected by the metabolic activity. We now have the following abstract diagram for a firm F as a *metabolism-repair (M,R)-system:*

$$\Omega \xrightarrow{f} \Gamma \xrightarrow{\Phi_f} H(\Omega, \Gamma).$$

As already noted, f represents the firm's procedures for operating upon the environment to produce or "metabolize" revenues, while Φ_f corresponds to the firm's "genetic" capacity to repair disturbances in metabolism arising from environmental fluctuations.

To complete the metaphor of the firm as a biological organism, we must address the issue of how to repair the repairers. The repair mechanisms were introduced to account for the fact that during the course of time, the firm's metabolic machinery will erode and decay, thereby requiring some sort of rejuvenation if the firm is to avoid extinction. Precisely the same argument applies to the repair mechanism, but it is of no particular help to introduce repairers for the repairers and so forth, ending up in a useless infinite regress. The way out of this loop is to make the repair components self-replicating. In this way, new copies of the repair mechanism are continually being produced, and it is unnecessary to assume the repair functions are immortal or to fall into an infinite regress of repairers to insure survivability of the firm. Let us see how to introduce the idea of replication into the foregoing framework.

Since the replication operation involves reproducing the genetic component Φ_f from the metabolic activity of the firm, it follows that the replication map, call it β_γ, must be such that

$$\beta_\gamma : H\ (\Omega, \Gamma) \rightarrow H(\Gamma, H(\Omega, \Gamma)),$$

if it exists at all. The question is: how can such a map β_γ be constructed from the basic metabolic components Ω, Γ, and $H(\Omega, \Gamma)$ of the firm? To see how this is done, it is easiest to consider a somewhat more general situation.

Let X and Y be arbitrary sets. Then for each $x \varepsilon X$, we can define a map

$$\hat{x}: H(X, Y) \rightarrow Y$$

by the rule

$$\hat{x}(f) = f(x),$$

for all $f \varepsilon H(X, Y)$. Thus, we have an *embedding* of X into the set $H(H(X, Y), Y)$. Now, *assume* that the map $\hat{x}$ has a left inverse $\hat{x}^{-1}$, so that

$$\hat{x}^{-1}: Y \longrightarrow H(X, Y).$$

Then, we clearly have

$$\hat{x}^{-1}\hat{x}\ (f) = f$$

for all $f \varepsilon H(X, Y)$.

Returning now to our replication situation, we set

$$X = \Gamma, \quad Y = H(\Omega, \Gamma),$$

and apply the foregoing general argument to obtain for each $\gamma \varepsilon \Gamma$, a map $\beta_\gamma = \hat{\gamma}^{-1}$ with the property that

$$\beta_\gamma : H(\Omega, \Gamma) \longrightarrow H(\Gamma, H(\Omega, \Gamma))$$

for all $\hat{\gamma}$ possessing a left inverse. In short, the metabolic activity of the firm can be used to reproduce its repair component if the technical condition on the invertibility of the map $\hat{\gamma}$ is satisfied. The economic interpretation of this condition is that $\hat{\gamma}$ is invertible if different innovations and R&D activities (i.e. different genetics mechanisms $\Phi_{f_1} \neq \Phi_{f_2}$) give rise to different production and marketing functions (i.e. different metabolic processes $f_1 \neq f_2$). In the industrial context we are examining, this seems to be a reasonably defensible assumption that will be accepted for the remainder of the paper.

Before entering into a more thorough discussion of the implications of the (M,R)-sysem as a paradigm for industrial structuring and operation, it is of interest to consider the actual meaning of the replication process described by the map β_γ. We have seen that the repair mechanism Φ_f basically provides the prescription by which revenues are used to support and renew the production-marketing process f. By the same token, the replication process β_γ gives the instructions by which the genetic process Φ_f is duplicated. Thus, since Φ_f corresponds to innovation/R&D, we can only conclude that β_γ corresponds to the diffusion of innovation/R&D. In short, β_γ is a prescription for growth of the firm by development of new divisions. Alternately, it could represent the start-up procedure of new firms that spin-off from the parent corporation. In either case, the innovation and "know-how" of the parent firm is transferred to a new organization and is then used as part of the metabolic operation f to produce revenue from environmental inputs in the usual way.

4. Consequences of the (M,R)-Framework for a Single Firm

The minimal structure introduced thus far to define an (M,R)-system is already sufficient to shed light on a variety of interesting questions surrounding the way a firm can respond to changes in its operating environment, the possibility for innovation to occur through environmental effect, the circumstances under which environmental changes can be reversed, feedback, and so on. In this section we sketch the way in which these issues appear within the (M,R)-framework, and consider the conclusions that can be drawn about firm behavior from this structure.

A. Stable Metabolic Operations in Changing Environments - imagine the situation in which the firm's "usual" input ω of raw materiais, labor, etc. is disturbed to a new input $\bar{\omega}$. The condition for stable operation of the firm is for the environment ω to be such that

$$\Phi_f \; (f(\omega)) = f \;, \qquad (*)$$

i.e. the metabolic structure f is stable in the environment ω in the sense that the repair mechanism Φ_f always regenerates f when the environmental input is ω. We would say that all $\omega \in \Omega$ satisfying (*) form a stable environment for the firm.

Now suppose that the new environment $\bar{\omega} \neq \omega$. Then (*) will hold only if either

$$f(\omega) = f(\bar{\omega}) \qquad or \qquad \Phi_f(f(\bar{\omega})) = f.$$

The first case is trivial in the sense that the observed revenues of the firm are invariant to the change of environmental inputs. If $f(\omega) \neq f(\bar{\omega})$

then the firm's revenues are not stable with respect to the change of environment and we must consider the repair mechanism to see whether or not the environmental alterations can be compensated for in the sense that

$$\Phi_f \; (f(\bar{\omega})) = \bar{f} \neq f ,$$

with $\bar{f}(\bar{\omega}) = f(\omega)$, i.e. will the genetic mechanism produce a new metabolism $\bar{f}$ which will duplicate the revenues of f, but with the input $\bar{\omega}$ rather than ω. In this case, the entire metabolic activity of the firm would be permanently altered if we had

$$\Phi_f(\bar{f}(\bar{\omega})) = \bar{f}.$$

On the other hand, if we had $\bar{f}(\bar{\omega}) = f(\omega)$ or, more generally,

$$\Phi_f(\bar{f}(\bar{\omega})) = f ,$$

then the firm's metabolism would only undergo periodic changes in time.

Finally, we could have the situation in which

$$\Phi_f(\bar{f}(\bar{\omega})) = \hat{f} \neq f, \bar{f}$$

and, iterating this process, we see that an environmental change may cause the firm to wander about in the set $H(\Omega, \Gamma)$, changing its production-marketing procedures through a sequence of metabolic processes $f^{(1)}, f^{(2)}, f^{(3)}, \cdots$. This "hunting process will terminate if either

(i) there exists an N such that

$$\Phi_f(f^{(N)}(\bar{\omega})) = f^{(N)}$$

or

(ii) there exists an N such that

$$\Phi_f \ (f^{(N)}(\bar{\omega})) = f^{(N-k)} \ , \quad k = 1,2,\ldots,N-1 \ .$$

In case (i) the firm becomes stable in the new environment $\bar{\omega}$, while in case (ii) the firm undergoes periodic changes in its metabolic structure. If no such N exists, the firm is unstable and aperiodic. (Note: This last possibility can occur only if the set $H(\Omega\, ,\Gamma)$ of possible production-marketing procedures is infinite).

B. "Lamarckian" Changes in the Repair Process - the above discussion of metabolic changes was undertaken subject to the tacit assumption that the repair map Φ_f remains unchanged. It is of interest to inquire as to whether or not an environmental change $\omega \longrightarrow \bar{\omega}$ can generate a "Lamarckian" type of genetic change in Φ_f through the replication process described in the last section. If such a change were indeed possible, then it would imply that the actual innovation/R&D process, which regenerates the metabolic activity f, could be affected by environmental changes alone.

To examine this question, suppose we have the environmental change $\omega \to \bar{\omega}$. Then the replication map β_γ associated with the input ω and the output $\gamma = f(\omega)$ is changed to $\beta_{\bar{\gamma}}$, where $\bar{\gamma} = f(\bar{\omega})$. Recalling that

$$\beta_\gamma = \hat{\gamma}^{-1} \ , \ \beta_{\bar{\gamma}} = \hat{\bar{\gamma}}^{-1} \ ,$$

and

$$\hat{\gamma}(\Phi_f) = \Phi_f(f(\omega)) \ , \ \hat{\bar{\gamma}}(\Phi_f) = \Phi_f(f(\bar{\omega})) \ ,$$

after applying β_γ, $\beta_{\bar{\gamma}}$, respectively to the last two relations, we find that

$$\beta_{\gamma}(\Phi_f(f(\omega))) = \beta_{\bar{\gamma}}\ (\Phi_f(f(\bar{\omega}))) = \Phi_f\ ,$$

showing that the new replication map $\beta_{\bar{\gamma}}$ replicates the existing repair component Φ_f exactly. Thus, an environmental change alone can have no effect upon the repair map Φ_f.

Now we ask whether it is possible for a change in the metabolic production-marketing procedures to result in a change of the firm's "genotype." Suppose we replace the metabolic activity f by some other production-marketing process $b\ \varepsilon\ H(\Omega\ ,\Gamma)$. By definition,

$$\hat{b}(\omega)\ (\Phi_f) = \Phi_b(h(\omega))\ .$$

Assuming $\hat{b}(\omega)$ is invertible, we apply $\hat{b}^{-1}(\omega)$ to both sides of the above relation to obtain

$$\beta_{b(\omega)}\ (\Phi_f(b(\omega))) = \Phi_f\ .$$

Thus, the induced replication map reproduces the original repair component of the firm under all conditions. In short, no Lamarckian changes in the metabolic component, either in the environment Ω or in the metabolic set $H(\Omega\ ,\Gamma)$, can result in changes in the firm's repair mechanism. Such "genetic" changes can only come about through a direct intervention in the genetic code itself (mutation) and not via indirect metabolic alterations.

C. Feedback as an Environmental Regulator - the environmental changes discussed thus far have been assumed to be generated by actions external to the firm; however, it may often be the case that the firm's output of revenue is employed as one of the environmental inputs, i.e. ω is a function of γ, $\omega = \omega(\gamma)$. In this event, the firm actually creates part of its own environment and, as a consequence, can partially regulate its own structural alterations. An important aspect of this general process is to understand as to what degree adverse environmental disturbances can be "neutralized" by suitably chosen feedback policies. This question is a special case of the more general problem of "reachability", in which we ask about the possibility of attaining any pre-defined metabolic structure by means of a sequence of environmental changes. This is a topic we shall return to later in connection with discussing the dynamical aspects of the firm's morphology.

5. Global Industry as a Network of (M,R)-Systems

So far we have only considered a single firm F as an (M,R)-system. If we connect several firms together, with the outputs of some firms serving as inputs to others and the repair mechanisms of each firm requiring the output from at least one firm, then we have the structural basis for characterization of an entire industry. Such a network of interdependent firms gives rise to a number of significant questions involving the birth, growth and death of an industry and of the individual firms comprising the industry. In this section, we consider how these issues arise naturally within the context of an (M,R)-network and the way in which our earlier (M,R)-formalism for a single firm can be extended to form a basis for modeling an entire industry.

In order to fix ideas, consider the specific (M,R)-network depicted in Figure 1. The square blocks labeled $F_1, F_2, \cdots, F_6$ represent the metabolic processes of the individual firms, while the ovals, denoted $R_1, R_2, \cdots, R_6$ represent the respective firms' repair mechanisms. The requirements that we impose for any such network are modest:

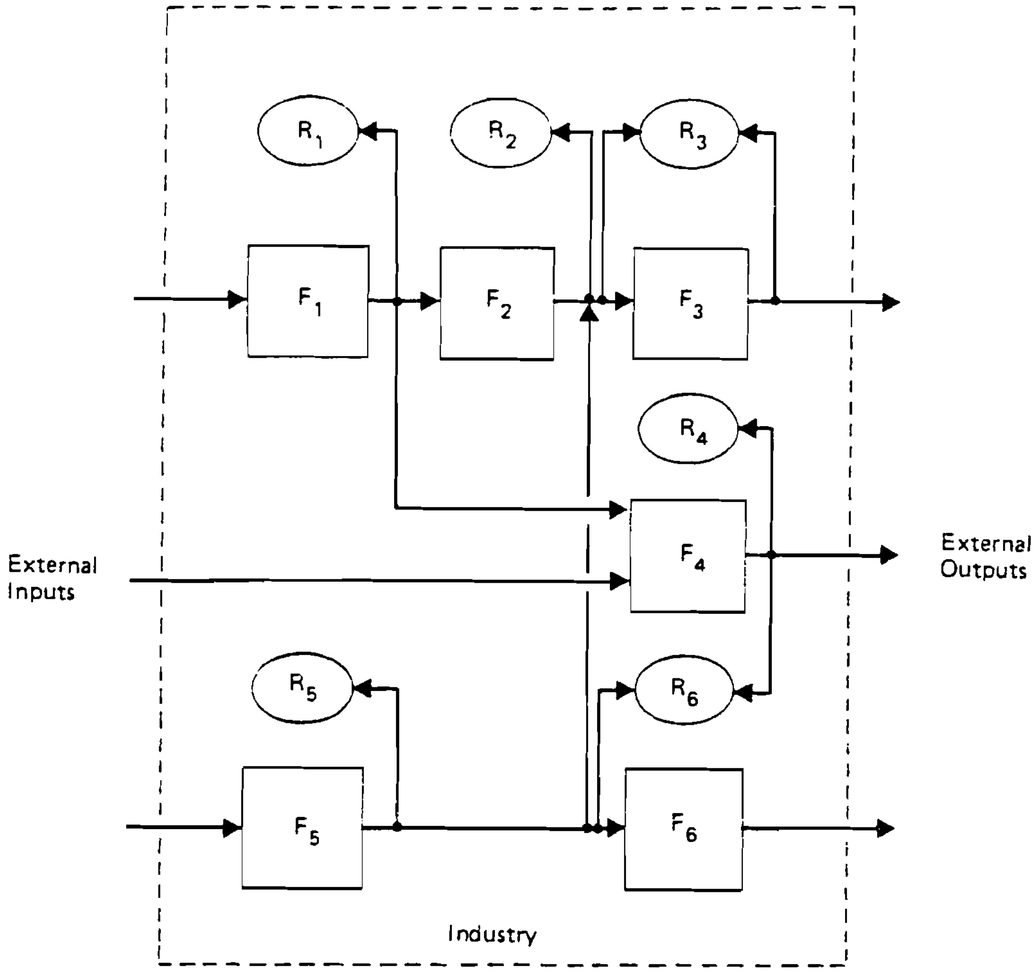


FIGURE 1. A Typical (M,R)-Network

i) each firm must receive at least one input, either from the external world or from the output of another firm;

ii) each firm produces at least one output;

iii) each repair mechanism receives the output of at least one firm in the network.

We have stated the requirements for an (M,R)-network in quite general terms. In a typical industry, such as the automotive industry, it is likely that each firm will receive its input from the outside world and discharge at least part of its output of revenues back to the external world in the form of payment for goods and services and returns to shareholders. Also, the repair mechanisms will, for the most part, receive only the output from their corresponding firm since the case of one firm devoting its resources toward supporting another (as with the firm F_4 of Figure 1) seems rather improbable under most circumstances, although certainly not impossible.

The first general issue to consider for an (M,R)-network is the dependency structure. We are concerned with the question of how the removal of a given firm from the network affects the existence and operation of other firms in the industry. For instance, referring to Figure 1 we see that the failure of F_5 results in the failure of firm F_6, as well, since F_6 receives its only input from F_5. Furthermore, the failure of F_5 *may* influence the operation of F_3, as F_3 receives part of its input from F_5. Thus, in this case we would consider firms F_3 and F_6 to comprise the ***dependency set*** of firm F_5. Any firm whose failure affects either the existence or operation of *all* firms in the industry will be called a *central* firm, i.e. the dependency set of a central firm is the entire industry.

Since we know that in the absence of the repair mechanism any firm will go out of existence after some finite lifetime, it is clear that other firms in the dependency set of a given firm will also go out of existence when that firm does. However, with the repair mechanism in operation, it is quite possible that a given firm could "come back to life" even after its initial demise. For example, firm F_6 in Figure 1 may cease metabolic operation and be removed from the network; however, the repair mechanism R_6 receives its necessary inputs from firms F_4 and F_5 indicating that whatever "shock" caused the extinction of F_6, the firm will be re-inserted into the network after some characteristic delay time depending upon the repair mechanism R_6. In other words, copies of F_6 will continue to be manufactured even after the removal of F_6 from the network. Firms like F_6 will be called ***re-establishable***, while all other firms are termed ***non-reestablishable*** (e.g. $F_1, F_5, \cdots$). There is an important relationship between the notion of re-establishability and the concept of a central component expressed by the following result.

Theorem 1. Every (M,R)-network must contain at least one non-reestablishable firm.

Corollary. If an (M,R)-network contains only one non-reestablishable firm, then that firm is a central component. The proofs of these results can be found in the papers cited in the references.

The significance of this result is twofold:

i) every industry must contain at least one firm whose metabolic failure cannot be repaired. This conclusion follows only from the connective structure of the (M,R)-network and is completely independent of the specific industry, the procedures of the firms, their products or marketing strategies. It is solely a consequence of the meaning of the metabolism-repair functions and the replication process.

ii) in order to be "resilient" to unforeseen disturbances, one would desire an industry to consist of a large number of re-establishable firms. On the other hand, the above results show that if only a small number of firms are non-reestablishable, then there is a high likelihood that one of them will be a central component whose failure will destroy the entire industry. Thus, an industry with a large number of re-establishable firms will be able to survive many types of shocks and surprises, but there will be certain types of disturbances that will effectively cripple the whole industry. Consequently, it may be better to have an industry with a relatively large number of non-reestablishable firms if it is desirable to protect the industry from complete breakdown.

6. Time-Lags and Dynamics

Up to this point, it has been assumed that the metabolic and repair functions of the firm take place instantaneously, i.e. inputs are transformed into revenues immediately and there is no delay in either repairing the metabolic process itself or in the replication of the repair mechanism. Needless to say, these assumptions are pure fiction; production of revenue and repair/replication takes time and the delays involved often spell the difference between success or failure for a firm.

While there is no space here to enter into a detailed discussion of the matter, let us simplify the situation by assuming only two types of delays. The first we term the ***production delay***, corresponding to the time required to transform a given input of materials, manpower and knowledge into an observable amount of revenue or the time required for a repair function to restore a metabolic operation. The second type of delay we shall call the *transport delay*. It corresponds to the time needed to transport the output of a firm to where it can be utilized as the input to either another firm or a repair mechanism (or to the external world).

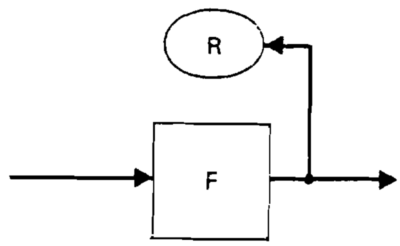


Figure 2. A Single Firm

As an illustration of how time-lags can influence the behavior of an (M,R)-model of a firm, consider the case of the single firm F depicted in Figure 2. The firm is clearly non-reestablishable in the sense discussed earlier. If the combined delay time of the production delay of R and the transport delay from F to R is T units, and if at time $t = 0$ F produces an output and is then removed from the network, T units later R will produce a copy of F and F will be built back into the network even though F is graph-theoretically non-reestablishable. However, if F is not just removed from the industrial network, but is suppressed for an amount of time $t \geq T$, then irreversible damage will have occurred and F will be removed from the network forever. The interplay between the various time-lags involved when several firms are coupled together into an industry is a delicate matter and will be taken up in a later paper.

Closely related to the time-lag problem is the matter of system dynamics. There are several issues surrounding this topic, not all of them mutually consistent. For simplicity, let us consider here only the case of a single firm F modeled as an (M,R)-system. Abstractly, the diagram for F is

$$\begin{array}{ccccccc} \Omega & \xrightarrow{f} & \Gamma & \xrightarrow{\Phi_f} & H(\Omega\,,\Gamma) & \xrightarrow{\beta_\gamma} & H(\Gamma\,,H(\Omega\,,\Gamma)) \\ & {\scriptstyle g}\searrow & \nearrow{\scriptstyle h} & & & & \\ & & X & & & & \end{array}$$

and ask in what manner F can be regarded as a dynamical system. If it were not for the repair and replication maps Φ_f and β_γ, this would be a straightforward question addressable via normal system-theoretic realization theory procedures, i.e. we would have the problem of constructing a canonical internal model of the firm

$$\begin{aligned} \dot{x} &= p(x\,,u) \\ y &= h(x), \end{aligned}$$

whose input/output behavior duplicates that of the given metabolic map f . Techniques for handling this question are readily available in the mathematical system theory literature [3-5].

Let us ignore for the moment the factorization of the firm's metabolism f through the production-marketing maps g and h, and consider the (M,R)-system

$$\Omega \xrightarrow{f} \Gamma \xrightarrow{\Phi_f} H(\Omega\,,\Gamma)$$

where $f \in H(\Omega\,,\Gamma)$, $\Phi_f \in H(\Gamma\,,H(\Omega\,,\Gamma))$. We wish to show how this abstract model of a firm can be considered as a sequential machine, i.e. as a discrete-time dynamical input/output system.

Let us recall that a sequential machine M is a composite $M = (A, B, S, \delta, \lambda)$, where A, B, and S are sets (possibly infinite), while $\delta : A \times S \longrightarrow S$, $\lambda : S \times A \longrightarrow B$ are maps. We interpret A as the input alphabet of M, B as the output set, S as the set of states, with δ and λ the state- transition and output maps of the machine, respectively. At each discrete instant of time $t = 0, 1, 2, \cdots$, M receives an input symbol from A, emits an output in B and the state is changed according to the rule δ, and the process continues from the time $t + 1$. Further details on the properties of sequential machines can be found, for example, in [4, 6].

In order to characterize the firm F as a sequential machine, we make the identifications

$$A = \Omega, \quad B = \Gamma, \quad S = H(\Omega, \Gamma),$$

$$\delta(\omega, f) = \Phi_f(f(\omega)), \quad \lambda(f, \omega) = f(\omega).$$

Thus, in general any firm can be regarded as a sequential machine in which the set of "states" of the machine correspond to the set of possible "phenotypes" of the firm, while the input and output sets of the machine are the inputs and revenues of the firm, respectively.

Putting the above ideas together, we arrive at the following scheme for characterizing the dynamics of the firm:

i) regarding the firm as a sequential machine formed from the elements $F = (\Omega, \Gamma, H(\Omega, \Gamma), \Phi_f, f)$, we compute the metabolic process f

at the time $t = 0$;

ii) using f , Ω , Γ, we employ realization theory to form a canonical model for the state space X and the production/marketing maps g and h;

iii) Let $t \rightarrow t + 1$ and use the sequential machine to calculate the new metabolism f. If it is the same as at the previous time-step, then continue to use the earlier X, g and h; if f changes, calculate a new canonical production/marketing model and continue the process with the new model until the next time period.

It is of interest to note that in the above scheme, a change of metabolism implies that the production process, the marketing procedure and/or the actual product has been changed. This can come about only if the repair map Φ_f fails to reproduce f. We have already seen that this may come about only by means of environmental changes, in general, unless the replication map fails to exist. But this last situation depends entirely upon the size of the set $H(\Omega, H(\Omega, \Gamma))$, the space of all possible repair maps. If it is either too large or too small, then no replication is possible. It would take us too far afield to enter into the details of this argument here, but the implications are that it is only in a highly restricted class of categories that replicating (M,R)-systems exist, and it is within this class that we must search for viable models of industrial growth and decline.

7. Attainable Production-Marketing Processes

The arguments given earlier show that the metabolic component of a firm can be altered by changes in its environmental inputs, while such changes leave the firm's repair mechanism unaffected. By turning these arguments around, we can investigate the degree to which environmental changes can be used in order to bring the firm's production and marketing processes to some *pre-assigned* state. An important special case of this "reachability" question is to ask if a metabolic structure reached by some sequence of environmental changes can be *reversed* by another appropriate sequence of changes in the environment. Questions of the above type strike to the heart of many important industrial issues having to do with the way in which changes in materials, men, and machines can be employed to affect the overall productive capabilities of the firm.

In terms of machines, the reachability problem can be stated as: given a machine in a specified initial state, does there exist a sequence of inputs that will bring the machine to some preassigned state (perhaps also at a preassigned time)? In general, the answer to this question is no. Machines having this "complete reachability" property are called ***strongly connected***, and we can ask whether or not machines that correspond to (M,R)-systems are strongly connected.

Generally speaking, machines corresponding to (M,R)-systems may fail to be strongly connected; hence, there may exist abstract "firms" that may be unreachable from any initial configuration by any sequence of environmental alterations. In the usual theory of sequential machines, this difficulty can be formally by-passed by enlarging the set of inputs and by appropriately extending the maps δ and λ. For (M,R)-systems this is a much more subtle business for the following reasons:

(a) the output set Ω and the state set $S = H(\Omega, \Gamma)$ are related in the (M,R)-systems and we cannot enlarge Ω without also enlarging S;

(b) by extending the maps Φ_f and f in the (M,R)-systems, we move the mappings from the sets $H(\Omega, \Gamma)$ and $H(\Gamma, H(\Omega, \Gamma))$ to new sets $H(\Omega', \Gamma)$, $H(\Gamma, H(\Omega', \Gamma))$, respectively. But this last set must possess certain properties in order for replication to be possible and this property is by no means implied by the replicability of the original system.

8. Prospects and Conclusions

The development of (M,R)-systems as a theoretical framework for the study of industrial growth and re-structuring has only been tentatively sketched in the preceding pages to the degree necessary to demonstrate feasibility of the idea. To transform the basic idea into a working tool to study, for instance, the evolution and development of the world automotive industry, requires a substantial research effort on both the theoretical, as well as applied fronts. It will be necessary to give concrete meaning and structure to the various abstract components composing the (M,R)-network (the elements Ω, Γ, $H(\Omega, \Gamma)$, Φ_f, etc.), as well as work out the various connectivity structures that link the individual firms comprising an industry. Such activities form the basis of the applied component of any implementation of the (M,R)-framework for a specific industry. Some complementary evolutionary ideas are given for the auto industry by Businaro in [9] and their connection with (M,R)-systems merit much further study. See also the general evolutionary ideas in [10, 11].

But there are also a number of purely theoretical aspects of the (M,R)-formalism that need further study if the overall structure is to bear the weight of providing the foundation for such an investigation of individual dynamics. We have already touched upon some of these issues in passing, but it is worthwhile to re-examine them again as the basis for a future research agenda.

i) Lamarckian changes - we have seen that changes in the firm's repair mechanism Φ_f cannot come about by environmental alterations alone, as long as certain invertibility assumptions on the replication procedure hold. This assumption, and its resultant conclusion, are quite acceptable in the biological context but rest upon much shakier ground in our industrial setting. It certainly seems plausible that at least certain types of environmental changes could give rise to a change of the firm's "genotype." At this stage it is unclear exactly how to modify the mathematical setting given above to accommodate such "Lamarckian" changes.

ii) Networks and Time-Lags - the manner in which time-lags, in both firm operations and in transport from one firm to another, affect the overall behavior of an industry is critical for determination of the long-term growth or decline of given firms within the industry. We have already seen simple examples in which time-lags can result in either the permanent extinction of a firm or, conversely, in its "resurrection" after being theoretically "dead." The interdependencies of lags of different types and lengths is a topic that cannot be ignored if the (M,R)-framework is to be used to gain insight into the behavior of real industries.

Dynamics - the procedure outlined in the text for regarding an (M,R)-system as a sequential machine is *one way* to introduce dynamical considerations into the overall formalism. There may be many other non-equivalent approaches, each leading to a different view of the dynamical behavior of a firm. Even accepting the approach given here for a single firm, there still arises the question of what will be the dynamical behavior of a *collection* of such firms, i.e. an industry. Obviously, the answer to such a question depends upon the connective and dependency structure of the network, which in turn takes us back to some of the time-lag considerations discussed earlier.

iv) adaptation and selection - if an (M,R)-network is to provide a mathematical metaphor for the *evolution* of an industry, then it must possess some means to accommodate the concepts of genetic variability and adaptive selection. We have already spoken of the need to be able to incorporate genetic changes in the repair map Φ_f into the mathematical machinery of (M,R)-systems. A natural candidate for the selection mechanism is to impose some sort of optimality criterion upon the possible abstract firms that may result from genetic "mutations." Production efficiency, profitability, survivability are logical possibilities, but so also are less economic-oriented criteria like degree of re-establishability and level of centrality, criteria suggested more by the functional role of a firm in a network than by its economic performance as an isolated unit.

v) categories and the comparison of industrial structures - a basic question in the study of industrial evolution and change is to ask if the processes at work modifying one industry can be used in any way to infer information about the forces influencing another; if we understand the dynamics that shape, say, the chemical industry, can that knowledge be used to understand, for instance, the evolution of the automotive industry? In order to answer such a question, we must have a systematic procedure for *comparing* the industries and a means for deciding whether they are abstractly equivalent. The (M,R)-system framework provides a means for making such comparisons through the mathematical apparatus termed "category theory" [7, 8]. Briefly, any collection of sets $A, B, C, \cdots$, such that to each ordered pair (A, B) we have another set $H(A, B)$, the mappings from A to B, is called a *category* provided certain primitive assumptions are satisfied for the set of mappings $H(A, B)$. We will defer any technical discussion of these matters to another paper, but it is important here to observe that every (M,R)-system is a category, in which the objects Ω, Γ are the sets and the metabolic maps $H(\Omega, \Gamma)$ are the mappings of the category. Thus, every firm can be regarded as a category, and by extending the sets and the mappings, so can every industry. If we change the sets Ω, Γ and/or the mappings $H(\Omega, \Gamma)$ obtaining a different firm, then we have a new category, and the machinery of category theory allows us to compare the structures of the two categories by means of mappings called *functors*. Roughly speaking, a functor is a sort of dictionary allowing us to translate the structure of one category into that of another, and conversely. This is exactly the type of tool that is needed to compare one firm or one industry with another. The systematic exploitation of this idea in the context of industrial structural change within the above (M,R)-framework offers the promise of unlocking many key features responsible for the dynamics underlying the evolution and development of modern global industries.

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