GLOBAL MODELS AND GLOBAL MECHANISMS IV: TOWARDS AN INTERDISCIPLINARITY IN INVESTIGATIONS ON LONG-TERM SOCIETAL DEVELOPMENT

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GLOBAL MODELS AND GLOBAL MECHANISMS IV:
Towards an Interdisciplinarity in Investigations on Long-Term Societal Development

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Part I-III of this working paper described strategies for an improvement of the knowledge of global mechanisms which can be used as behavioral equations. Results were described which form a consistent framework of relationships, and describe processes of long-term societal development. These processes are located in different fields, and are covered by different disciplines. The following paper shows ways to improve and extend these first results by an interdisciplinary research.

I. The "Formula Bank".

In February, 1975, nine IIASA scientists agreed to propose an interdisciplinary activity, "Methodological and Empirical Components for Comprehension of Global Processes", and wrote the following memo:

"The worldwide scientific discussion about the future is growing in significance and IIASA's part in this discussion will increase. In this discussion, global models are becoming of increasing importance. IIASA can therefore not avoid the confrontation and must for this and other reasons deal with global processes.

The confrontation with global models should take place not in competition with them but at the level of methodological and empirical foundations.

These foundations include substantive and formal aspects: empirically observed characteristics of the processes of the real world including methodological questions of empirical investigations, purpose of modelling and modellers bias and mathematical methods that can be used for modelling of global processes and for constructing global models.

In the first stage IIASA should deal with the substantive aspects of empirical investigations of global processes and the related formal aspects.

Interdisciplinary activity "Methodological and empirical components for comprehension of global processes"

1. Objective:

   1.1 Provide a network of empirically observed relations in the field of long-term societal operation and development and in that of the boundary conditions, that can be used at first for explanation of the long-term development in certain fields in the past and later may serve as basis for forecasting. These results can then be made available in a form appropriate for model builders and users. The results could be used as a bank of tested components for modelling of global processes.

   1.2 Relate the proposed activity to the IIASA project teams. The results may provide a framework of the most important linkages between the subsystems investigated by the different project teams.

2. We propose to achieve these objectives in the following way:

   2.1 Continue informal discussions among those IIASA scientists interested in the project.

   2.2 Search literature and find names of scientists and organizations involved in pertinent studies throughout the world and contact them.

   2.3 Prepare workshop on "Global Processes"; the subjects could be:

   -- Methodological questions
   -- Working examples of empirically observed processes
   -- Applications of these examples for concrete questions of IIASA-project-teams or model builders
   -- Organization for continuing contacts.

Proposed date: A small IIASA workshop with a few scientists from neighbouring countries: May 1975.

Workshop, "Global Processes": October 1975.

3. Dissemination of results: We will investigate the best way and implement:

   3.1 To keep IIASA project people informed about our discussions.

   3.2 To publish in a way appropriate for IIASA.

   3.3 To establish a formula bank:
   The formula bank should be a continuously up-dated catalogue of formulas, functions, and data about global processes with associated documentation. It should provide a framework of the most important linkages between the subsystems investigated by the different project teams and could be used as a bank of tested components for modelling of global processes.

4. Responsibility:

   Dr. Millendorfer will collect information for the formula bank, organize the permanent work in this field and stimulate the informal scientific discussions and workshops."

Since this memo, considerations are more and more concentrated on the formula bank:

1. The formula bank is a collection of all such relations regarding long-term societal development, that:

   -- can be used as behavioral equations of models,

   -- are based on empirical observations which are arrived at by appropriate

   -- statistical methods, i.e. multiregressional analysis, etc.,

   -- have resulted in theoretically meaningful relationships.

2. The formula bank is started on the base of form-sheets where examples of global processes are described. The further development of the formula bank should be done in three steps:

(a) as a "clearing house" for scientists engaged in global modeling. In this concept model builders have interest as the response--i.e. the Bariloche group--to our letters show;

(b) as a more consistent network of empirically observed processes for long-term societal development;

(c) as consistent sets of explanatory hypotheses leading to an "objective"--i.e. by all model builders accepted--base for models and theories of long-term societal development.

3. The formula bank should serve as a nucleus for the iterative process of interdisciplinary teamwork described in Part I of this working paper. The examples for empirically observed relationships collected in a formula bank in the appendix are only an illustration of the basic idea, and of course they are not yet a useful instrument for the model builder because they are only a few incomplete examples (i.e. the point "integration hypothesis" is described in only a few examples) and not structured for the proposed task. These examples were discussed with participants of the last IIASA conference on global models in order to find the best criteria for selecting the formula and for structuring the collected material. The result is:

- The general criteria for selecting empirically observed relationships in the literature is the possibility to use them as behavioral equations in models in any field of long-term societal development.
- In the first step this criteria should be reduced to the possible application of the formulas to certain fields, treated by IIASA project teams, i.e. Food and Agriculture, Energy, and Health.
- Another question is the underlying theory for selecting variables. Here there seems to be a pluralism of theories or even a useful pragmatic approach. The theoretical work of IIASA should be basically only to prove the applied statistical methodology, and to show connections of the observed relationships to other relationships and to various existing theories.
- The structuring of the formulas should follow pragmatic documentation criteria. (How to find a wanted formula in the shortest time.)

The examples show the difference of the proposed documentation to usual documentations, i.e. the book of Meadows is split into a lot of single observations with different keywords. Under the same keyword, single observations of different authors are to be found. If we would have had the formula bank in the last global modelling conference, some of the

open questions could have been better understood, i.e. the question of diminishing results of fertilizers, observed by different authors.

The idea of the formula bank was well accepted by the model builders, especially Mallmann and Roberts, and should be realized in a way useful for model builders in and outside of IIASA. For this purpose, the computerization should be considered.

II. Interdisciplinary Cooperation with STUDIA.

A. Basic Considerations

1. STUDIA's research

STUDIA's work is based on an iterative empirical and system theoretical learning process concerning the questions of long-term societal development. The result of this work is a framework of equations and relationships centered on the general production function. The latter can be understood as a linkage to and a modification of the neoclassical economic theory explaining the residuals of the Cobb-Douglas function due to non-economic factors. The framework there serves as a nucleus for the development of a consistent set of hypotheses as an approach to theory of societal development. In this context, more detailed studies on different subsystems of society were made. For example, sector analysis in a world-wide, cross comparison, studies of the technological and medical progress, psycho-sociological studies using various indicators, etc. The results of these basic studies were

applied to practical questions like prognosis for Austria, family studies, perspective studies for the Austrian iron and metal industry, etc.

2. STUDIA's role in an interdisciplinary cooperation

The self-understanding of STUDIA is that of a generalist working on a very large basis of empirical data, reducing the redundancy of the information of this data base and concentrating this information on a framework of consistent relationships in close cooperation with experts in various fields. This framework can be understood as a starting point for a larger, interdisciplinary cooperation and as the first contribution of the generalist to an iterative process between generalists and specialists in different fields, describing the interfaces between the different disciplines. The specialists study, on the one side, their particular interests using this framework as a link to other disciplines and by their results contribute to the improvement of this framework.

On the basis of the improved framework as improved interdisciplinary cooperation leads to the next step of the iterative process, etc. STUDIA would like to extend its collaboration with experts in the different fields and can offer general results in the question of societal development and experience as a generalist in an interdisciplinary cooperation.

3. Basic concept for a possible cooperation

IIASA's work is concerned with questions of long-term development of different subsystems of society using formal mathematical tools and system theoretical concepts. In the various project teams, detailed problems are treated by different specialists in an interdisciplinary project. A closer cooperation between the various project teams is planned. In this context, STUDIA could serve as a generalist linking the fields of different disciplines by the framework mentioned in point 1, in the following three ways:

3.1 Introduction of STUDIA's detailed results into the work of the teams, for example, the sectoral production function into the work of the industrial systems group or the main factors of health development into the planned health model.

3.2 Introduction of STUDIA's general results as linkages between the subsystems of society investigated by the various project teams. For example, the planned formula bank as an extension of the framework mentioned in point 1, will contain a lot of relationships between different project teams and could therefore serve as a better understanding of interdisciplinary interfaces.

3.3 Further development of the framework mentioned in point 1 and point 2 in the direction of an empirical system--theoretical basis for global modelling.

4. Cooperation with IIASA project teams

4.1 Biomedical Group--STUDIA is working on a study for the main factors of health development sponsored by the Austrian National Bank. Where data is available, age corrected mortality rates of all countries are applied in various methods of multivariate analysis. This reduces redundancy and finds the basic patterns of health development. These results would serve as an input to the planned dynamic health model of IIASA. (See attached paper).

4.2 Industrial Systems--STUDIA has completed the first part of a study on various industrial sectors and has developed in this context a secotral production function (article in a UNIDO publication). The results of this study and the sectoral production function could serve as a contribution to cope with problems of industrial development.

4.3 Energy--A first attempt to understanding energy consumption in a larger, societal framework, the "new societal equations" could be improved and expanded. The cooperation with Dr. Marchetti should be continued.

5. Cooperative research

Independent of these existing projects, new projects could be created. In particular, the field of human behavior

which has, up to now, been neglected in all world models and where a seriously based discussion could be developed. A promising approach seems to be in statistical indicators for social-psychologically determined behavior patterns. STUDIA began the study of these problems within the health project, a project of Austrian families and prognosis for Austria.

Another example for a possible new project is the question of the optimum relationship between educational and capital investment or between the different categories of education in development planning or in world models.

Conclusion

A relatively small number of social scientists in the world still use the described iterative method for investigations of long-term societal development. A collection of their results could be a first step to extending the knowledge of global mechanisms. This increased knowledge would provide a substitution of non-verified theoretical assumptions by "observed facts" for behavioral equations, not only in world models but also in models in the different special fields like energy, food and agriculture, etc. Further research in a cooperation between generalists and specialists as outlined should lead to a consistent network of quantitatively described processes, a framework for a better understanding long-term societal development, linking together the different fields where IIASA teams are involved.

APPENDIX

Examples of Relationships To Collect in the Formula Bank

Agriculture : 7, 8, 9, 10, 11, 12
- and land development : 9
- fertiliser : 12
- yield : 7
- tractors : 10
- climate : 8
- fertiliser consumption : 11

Ammonia production : 6

Cities (distribution of population)
- in the world : 24
- Canada : 25
- GB : 26
- Germany : 27
- India : 28
- UK : 29
- US : 30, 31, 32
- urban for rural living : 23

Climate (and plant growth) : 8

Consumption : 1, 2, 3, 4
- of copper (US) : 2
- electric energy : 3
- energy : 4
- fertiliser : 11
- steel : 1
- steel (US) : 2

Copper (US) : 2

Death rates : 14

Efficiency : 6, 5
- of the ammonia production : 6
- prime movers : 5

Economic growth : 17, 18, 19
- birth rates : 18
- psychosomatic : 17
- size of families : 19

Energy : 3, 4, 5, 6
consumption of el. energy : 3
_____, _____ energy (total): 4
efficiency of prime movers : 5
in technology : 6

Expectation of life : 13, 14, 15
age specific : 14
in growth zones : 15
and nutrition : 13

Family planning : 19

Fertiliser : 6, 7, 11, 12
ammonia production : 6
fertiliser production: 12
yield function : 7
consumption : 17

Land (potentially arable) : 9

Nutrition (and life expectancy) : 13

Plant production : 8

Population : 13, 18, 19
birth rate : 18
family size : 19
economic growth: 18
nutrition : 13

Psychosomatic (ulcer) : 14

Social progress (wages) : 16

Society (urban for rural living) : 23

Steel : 1, 2, 21, 22
consumption (world) : 1
US : 2
substitution of processes : 21, 22

Substitution : 20, 21, 22, 23
steel : 21, 22
societal : 23

Technological forecasting : 20

Wages : 16

Yield (function) : 7

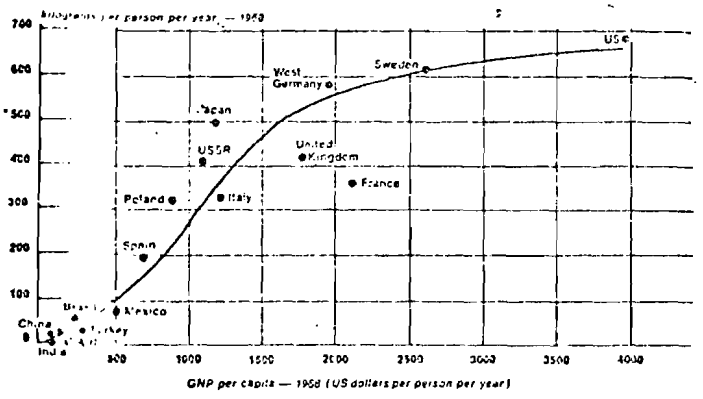
IIASA FORMULA BANK	CONSUMPTION of STEEL	FORM Nº 1

(1) Keyword: steel consumption, GNP per capita, consumption.

(2) Source: D. Meadows, "Limits to Growth", Universe Books, New York, 1972.

(3) Functional Relationship (formula, diagram).

WORLD STEEL CONSUMPTION AND GNP PER CAPITA



1968 steel consumption per person in various nations of the world

Abscissa: GNP per capita (US$). (Linear)

Ordinata: Kilogram per person, per year. (Linear)

(4) Type of analysis, source of basic statistics.

(a) Cross section comparison of GNP per capita vs. steel consumption from different nations.

(b) Data: Steel consumption from UN Department of Economics and Social Affairs, Statistical Yearbook, 1969 (New York: UN, 1970).

GNP per capita from World Bank Atlas (Washington, DC: International Bank for Reconstruction and Development, 1970).

IIASA

FORMULA BANK

CONSUMPTION of STEEL & COPPER

FORM

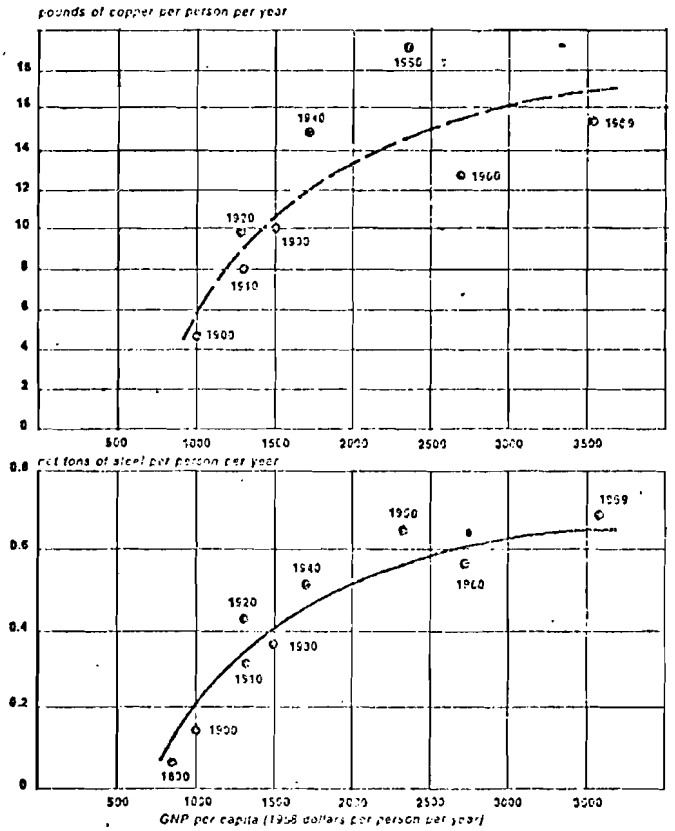
№ 2

(1) Keyword: consumption, increase of consumption, copper consumption (US), steel consumption (US).

(2) Source: D. Meadows, "Limits to Growth", Universe Books, New York, 1972.

(3) Functional Relationships (formula, diagram).

Figure 30 US COPPER AND STEEL CONSUMPTION AND GNP PER CAPITA



Abscissa: GNP per capita (US$). (Linear).

Ordinata: Consumption of copper (steel) per person per year in pounds (copper), net tons (steel). (Linear).

(4) Type of analysis, source of basic statistics.

(a) Comparison of time series with elimination of time.

(b) Data: copper and steel consumption from metal statistics (Somerset, N.J.: American Metal Market Company, 1970). Historical population and GNP from US Department of Commerce, US Economic Growth (Washington, D.C.: Government Printing Office, 1969).

IIASA FORMULA BANK | CONSUMPTION of ENERGY (ELECTRIC) | FORM No 3

(1) Keyword: Energy, electricity, per capita income, world

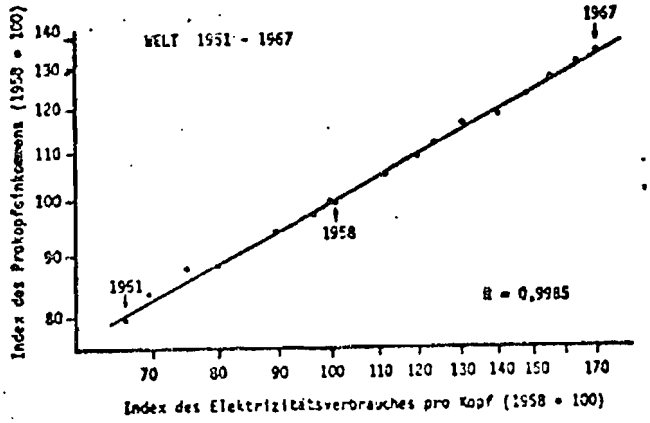
(2) Source: H. Millendorfer, Ch. Gaspari Immaterielle und materielle Faktoren der Entwicklung, Ansätze zu einer allgemeinen Produktionsfunktion, Zeitschrift für Nationalökonomie 31 Bd. Wien 1971

(3) Functional relationship: (description of variables; formula, diagramms; significance)

$y = c.m^{0,518}$ $R = 0,9985$

$y = c.m^{0,5}$ straight line in the diagram

y ... per capita income of the world 1951 - 1967 (index)
m ... per capita electricity consumption of the world 1951-67 (index)



Abscissa: Index of electricity consumption (1958=100). (Logarithmic)

Ordinata: Index of per capita income (1958=100). (Logarithmic)

(4) Type of analysis: contextual mapping, comparison of time series with elimination of time

data:

United Nations World Energy Supplies 1929 - 1966

" " Statistical Yearbook 1968

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FORMULA BANK

CONSUMPTION of ENERGY

FORM

No 4

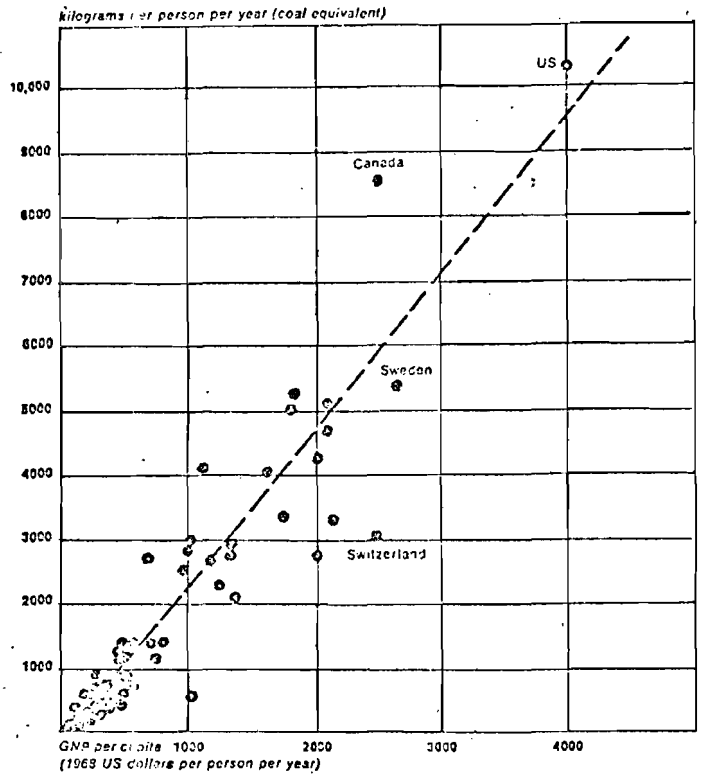
(1) Keyword: energy consumption, per capita income.

(2) Source: D. Meadows, "Limits to Growth", Universe Book, New York, 1972.

(3) Functional Relationship (formula, diagrams ...).

Formula for per capita income of the world and per capita electricity consumption of the world (time series). See Form No.:

ENERGY CONSUMPTION AND GNP PER CAPITA



Although the nations of the world consume greatly varying amounts of energy per capita, energy consumption correlates fairly well with total output per capita (GNP per capita). The relationship is generally linear, with the scattering of points due to differences in climate, local fuel prices, and emphasis on heavy industry.

Abscissa: GNP (US$ per person per year). (Linear)

Ordinata: Energy consumption (Kilogramms per person per year--coal equivalent). (Linear)

(4) Type of Analysis; source of basic statistics.

(a) Cross section comparison of energy consumption per capita vs. GNP per capita from different countries.

(b) Data: Energy consumption from UN Department of Economic and Social Affairs, Statistical Yearbook 1969 (New York: UN, 1970).

GNP per capita from World Bank Atlas (Washington, DC: International Bank for Reconstruction and Development, 1970).

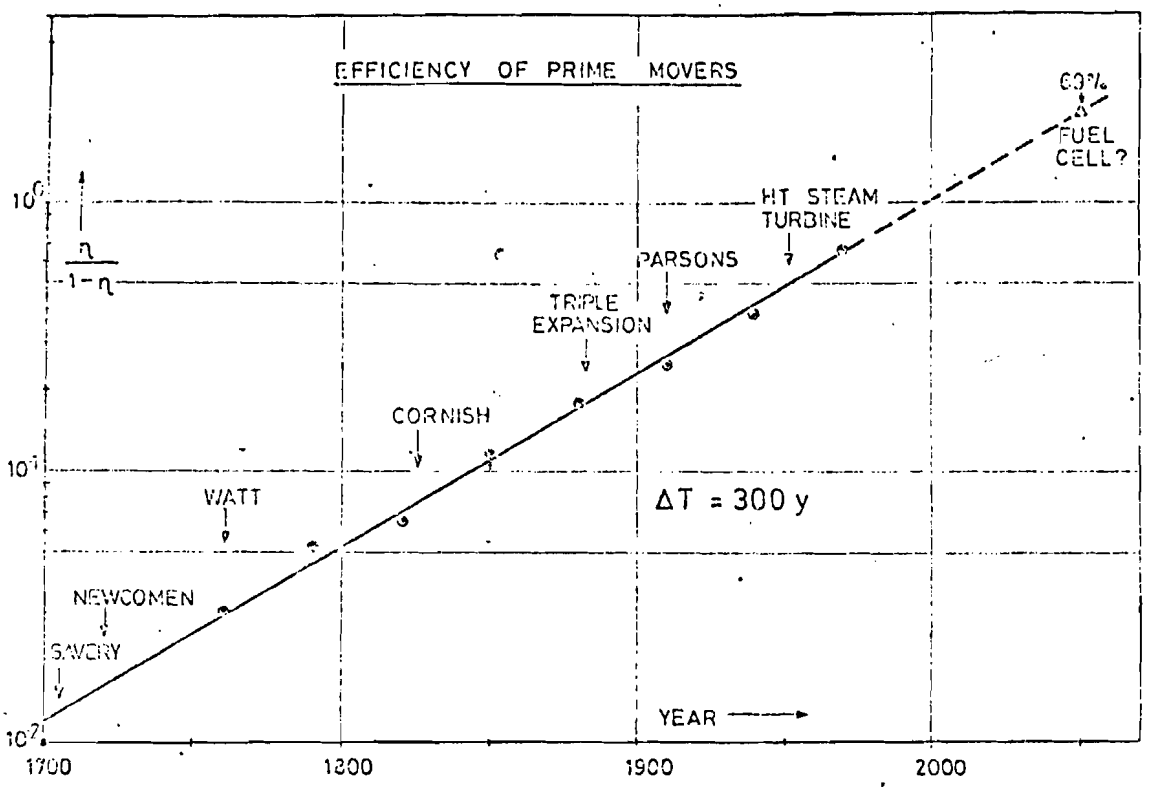
IIASA FORMULA BANK	ENERGY EFFICIENCY	FORM № 5

(1) Keyword: energy, efficiency, prime movers

(2) Source:

(3) Functional Relationship (formula, diagram)

$$\ln(\eta/(1-\eta)) = \alpha t + c$$



abscissa: year (linear)

ordinata: $\frac{\eta}{1-\eta}$ efficiency (logarithmic)

(4) Type of analysis, significance and source of basic statistics

(a) Type of analysis: Time series of prime movers efficiency.

(b) Source:

IIASA FORMULA BANK	TECHNOLOGICAL EFFICIENCY FERTILIZER PRODUCTION	FORM No 6

(1) Keyword: efficiency, fertilizer, ammonia product

(2) Source:

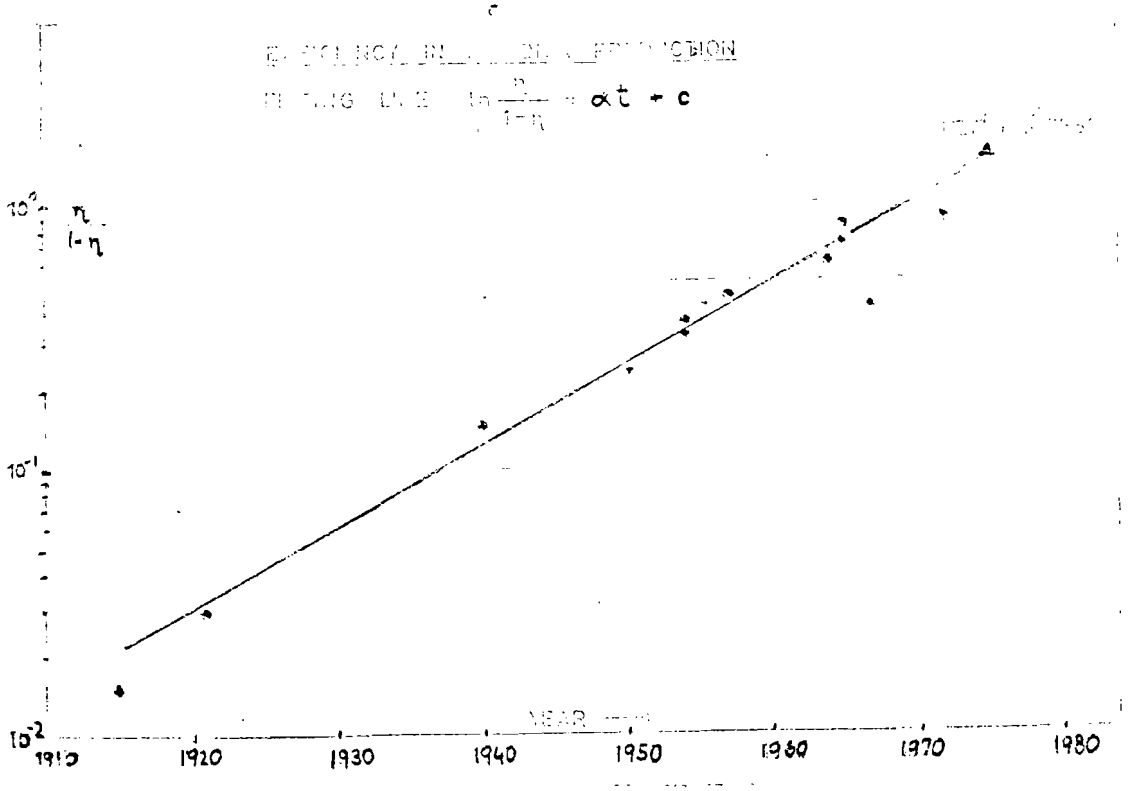
(3) Functional Relationship (formula, diagram)

$$\ln(\eta/1-\eta) = \alpha t + c$$

η --- efficiency of the ammonia production process (theoretical minimum 4180 kcal/kgNH_3)

α&c --- constants

t --- time



abscissa: year (linear)

ordinata: $\frac{\eta}{1-\eta}$ efficiency (logarithmic)

(4) Type of analysis, source of basic statistics

(a) Type of analysis

Time series of different ammonia production processes vs. efficiency of these processes (date of invention?)

(b) Source

Data from M. Slesser

(1) Keyword: yield function, agricultural yield

(2) Source: Latin American World Model, Food Sector
Fundacion Bariloche

(3) Functional Relationships:

$$RENDK_t = 16.5737 - 13.4721 \cdot e^{-3.8374\ FHA_t} \leq REMAX$$

RENDK . . . aggregated agricultural yield (10^6kcal/ha).

FHA. . . . fertilizers consumed per ha of arable land (tons/ha).

16,5757 ? ?

REMAX . . . $10 \cdot 10^6$kcal/ha = maximum yield.

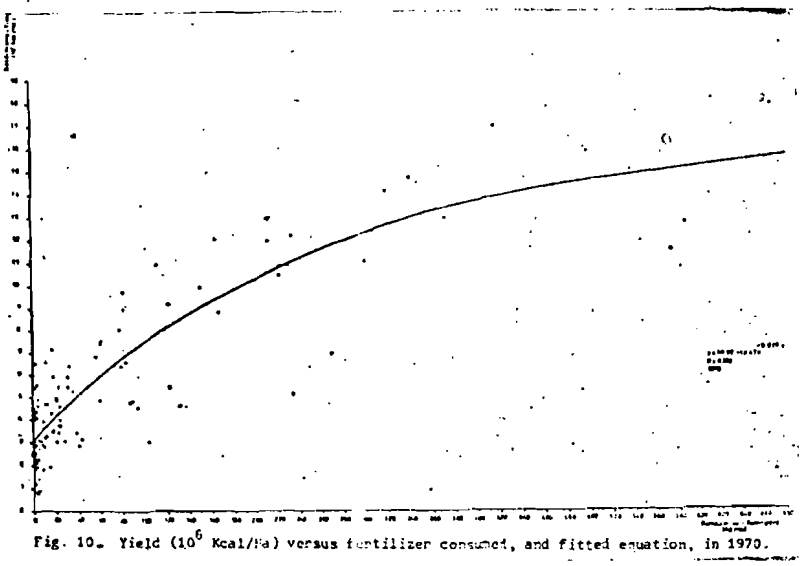


Fig. 10. Yield (10^6 Kcal/Ha) versus fertilizer consumed, and fitted equation, in 1970.

TABLE 9 - Values of KPROT per block

BLOCK	KPROT
Developed countries	0.0331
Latin America	0.0348
Asia	0.0336
Africa	0,0338

$$RENDP_t = RENDK_t \cdot KPROT$$

RENDP . . . aggregated agricultural yields (tons protein/ha)

RENDK . . . aggregated agricultural yields (10^6kcal/ha)

KPROT . . . . conversion factor for each block (tons protein/10^6kcal)

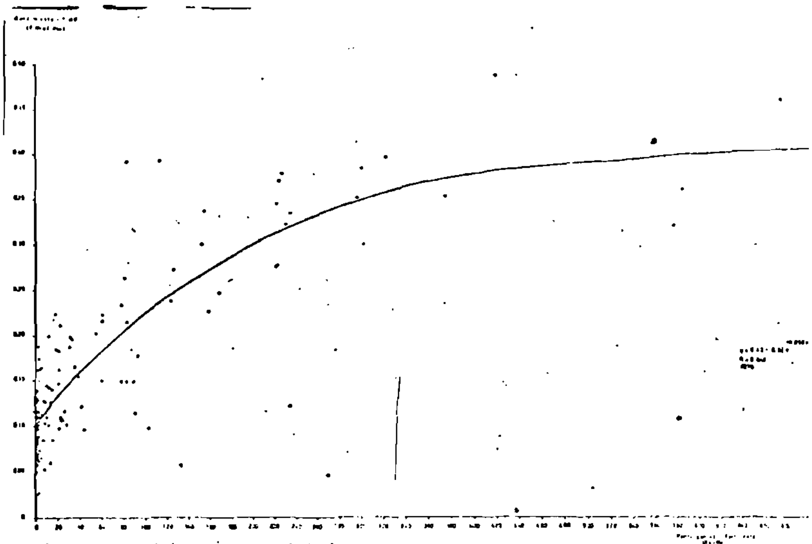


Fig. 11. Yield (Tons of protein/Ha) versus fertilizer consumed, and fitted equation, [illegible]

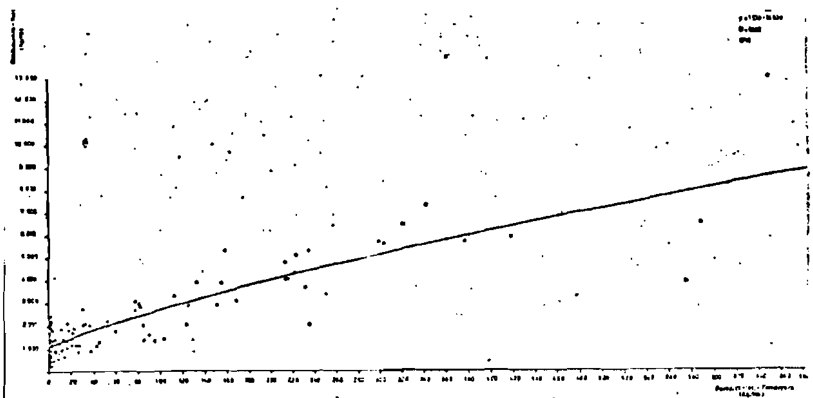


Fig. 9. Yield (Tons of crop/Ha) versus fertilizer consumed, and fitted equation, in [illegible]

9

(4) Type of analysis, significance and basic statistics

(a) Type of analysis

Exponential function fitted to a cross-section of 107 countries

Region	Year	Fertilizers (Kg/Ha)	Yield (T/Ha)	Yield (10^6 [illegible]/Ha)	Yield (TProt./Ha)
Developed countries	[illegible]	22.89	1.84	4.70	0.14
	1965	54.23	[illegible]	[illegible]	[illegible]
	1970	78.96	[illegible]	7 1	0.15
Latin America	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	15.14	1.27	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	0.17
Asia	[illegible]	1.17	[illegible]	[illegible]	[illegible]
	1965	4.46	[illegible]	[illegible]	0.14
	1970	19.59	1.33	[illegible]	0.15
[illegible]	[illegible]	2.75	0.89	3.46	0.05
	[illegible]	[illegible]	[illegible]		
	[illegible]	[illegible]			
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	1970	45.07	1.54	[illegible]	0.19

IIASA	AGRICULTURE	FORM
FORMULA BANK		№ 8

1) Keyword: Agriculture, climate, plant-production, Earth

2) Source: N.I. Bazilevich, L.Y. Rodin, N.N. Rozov
Geographical Aspects of Biological Productivity
Soviet Geography, Review & Translation, May 1971

3) Functional relationship: (description of variables; formula; diagramms; significance)

$$Kp = W \cdot T_v / 36\ R$$

Kp ... hydrothermal potential of productivity (production of vegetation i.e. primary production of green plants)
W .... mean annual productive moisture supply of an area in millimeters i.e. precipitation minus runoff
T_v ... length of the growing season in 10 day periods
R .... mean of annual radiation balance in kcal/cm^2

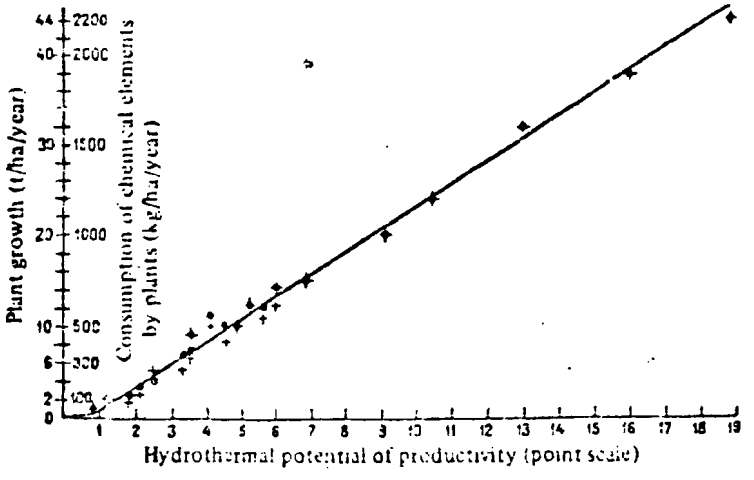


Fig. 5. Correlation between the hydrothermal potential of plant productivity and the actual plant growth (dots) and the consumption of chemical elements by plants (crosses) (in most cases dots and crosses coincide). After A. M. Ryabchikov, 1968.

4) Type of analysis, significance and sources of basic statistics

a) cross section comparison of the different bioclimatic regions of the world.

b) data on plant productivity obtained during the International Biological Program (Various Authors)

data on bioclimatic regions: Physical-Geographic Atlas of the World, 1964

Various Authors i.e. "Recent estimates of some of the Earth Sources" 1964, Great Soviet Encyclopedia Annual Moscow.

(1) Keyword: Cost of land development, potentially arable land.

(2) Source: Latin American World Model, Food Sector
Fundación Bariloche.

(3) Functional Relationships:

$$COSTC_t = 1200 + 8800 \cdot e^{-16,87\ TIRE_t}$$

$COSTC_t$ . . . unit cost of land development ($/ha)

$TIRE_t$ . . . fraction of potentially arable land not yet utilized.

$$TIRE_t = \frac{TCNU_t}{TPCU_t}$$

TCNU . . .potentially arable land, not yet utilized (ha).

TPCU . . .potentially arable land, total (ha).

Abscissa: fraction of potentially arable land remaining. (lin.)

Ordinata: unit cost of land development (10^3 $/ha). (lin.)

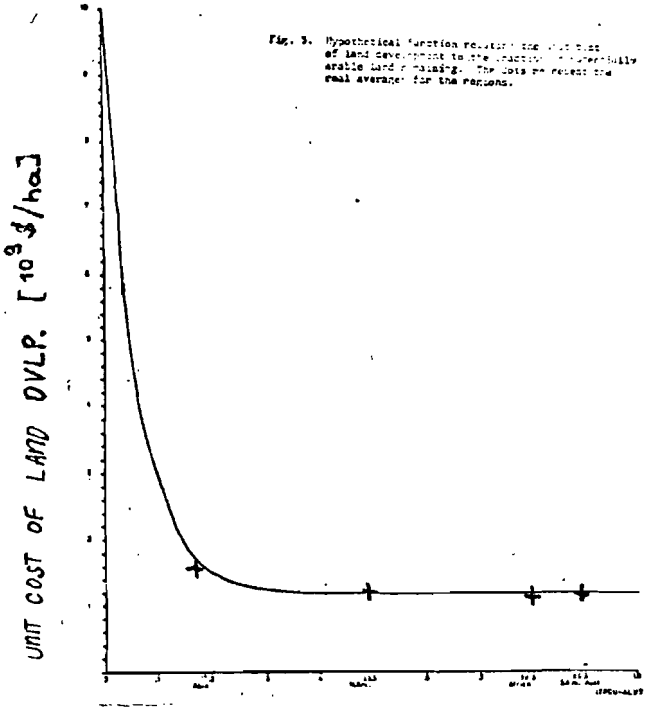


Fig. 9. Hypothetical function relating the unit cost of land development to the fraction of potentially arable land remaining. The dots represent the real average for the regions.

(4) Type of analysis, significance and basic statistics.

(a) Type of analysis: Cross section of different regions compared with hypothetical functions.

(b) Reference: U.N. 1970, Statistical Yearbook, New York

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FORMULA BANK

AGRICULTURE

FORM

No 10

(1) Keyword: agriculture, tractor, national income

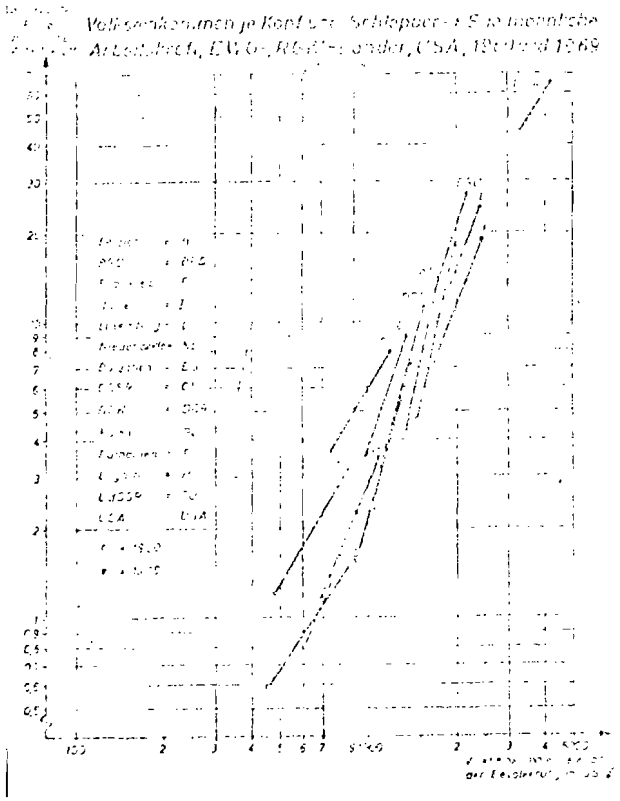
(2) Source: A. Weber, Arbeits- und bodensparende Technologien, Agrarwirtschaft, Feb. 1973.

(3) Functional relationship: (description of variables; formula, diagram)

$$\ln Q = -6{,}138 + 2{,}227 \log Y \qquad R^2 = 0{,}933$$

Q ... horsepower of tractors per agricultural worker(male)

Y ... national income



(4) Type of analysis:

Time series and cross section analysis of EWG RGW countries and USA.

Data: National income: International Bank for Reconstruction and Development, horsepower of tractors, workers: Statistisches Jahrbuch über Ernährung, Landwirtschaft und Forsten der Bundesrepublik Deutschland.

IIASA FORMULA BANK

AGRICULTURE FERTILIZER CONSUMPTION

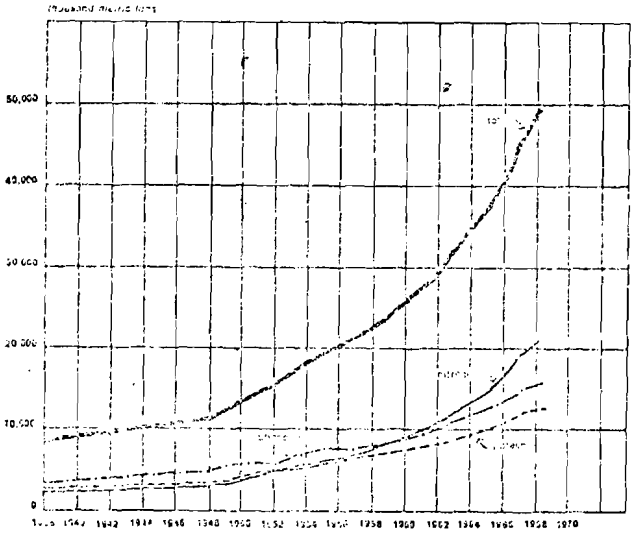
FORM № 11

(1) Keyword: fertilizer consumption, agriculture consumption

(2) Source: D. Meadows, "Limits to Growth", Universe Books, New York, 1972.

(3) Functional relationships (formula, diagram)

Figure 2 WORLD FERTILIZER CONSUMPTION



Abscissa: years. (Linear)

Ordinata: thousand metric tons. (Linear)

(4) Type of analysis, source of basic statistics:

(a) Time series analysis of world fertilizer consumption.

(b) U.N. Department of Economic and Social Affairs, Statistical Yearbook, 1955, 1960 and 1970 (New York: UN -956, 1961 and 1971).

(1) Keyword: fertilizer production, agricultural inputs, erosion control, soil fertility control

(2) Source: Latin American World Model, Food Sector
Fundacion Bariloche

(3) Functional Relationships:

$$CAF_t = CAIA_t \cdot (1-KFER-KERO-KINS)$$

CAF. . . available capital for fertilizer production increase ($).

CAIA. . .available capital for agricultural inputs increase ($).

KFER. . .fraction of available capital for agricultural inputs, allocated to soil fertility maintenance (0.10).

KERO. . .fraction of available capital for agricultural inputs, allocated to soil erosion control (0.05).

KINS. . .fraction of available capital for agricultural inputs, allocated to inputs other than fertilizers (0.20).

TABLE 10 - Estimated capital investment costs for improved seeds, fertilizers, and pesticides to increase agricultural production in developing countries - Rearranged from (1) -

% increase in agricultural production	Capital investments (10^6 $)			Total capital (10^9 $)	Capital for fertilizers as % of total capital
	Seeds	Fertilizers	Pesticides		
0					
10	30	2500	100	2.63	95.1
20	60	5000	240	5.30	94.3
30	90	7500	380	7.97	94.1
40	120	10000	540	10.66	93.8
50	150	13000	710	13.86	93.8
60	180	16000	910	17.09	93.6
70	210	20000	1150	21.36	93.6
80	240	23500	1410	25.15	93.4
90	270	27000	1670	28.94	93.
100	300	30500	1910	32.71	93.2

(4) Type of analysis, significance, basic statistics.

Reference: The President's Science Advisory Committee. Panel on the World Food Supply, 1967, "The World Food Problem" (3 vols), Washington, D.C.

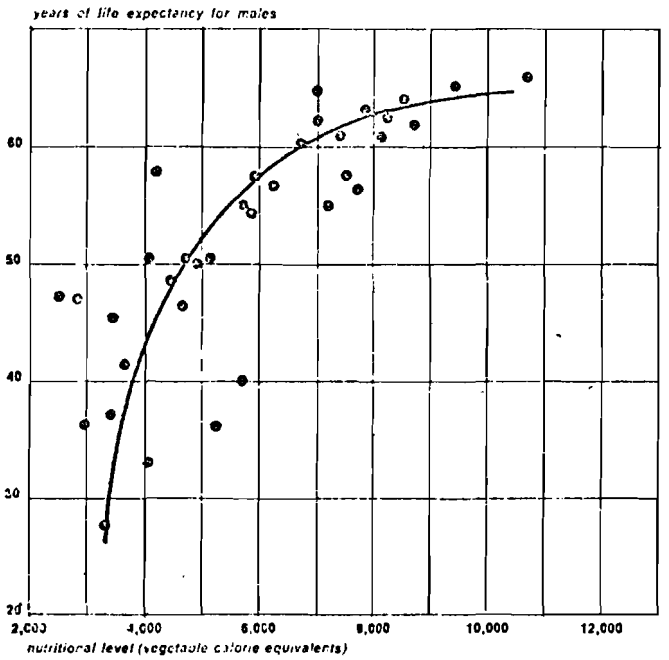
IIASA FORMULA BANK	EXPECTATION OF LIFE & NUTRITION	FORM No 13

(1) Keyword: life expectancy, nutrition, food, population

(2) Source: D. Meadows: "Limits to Growth", Universe Books, New York, 1972.

(3) Functional relationships (formulas, diagram ...)

NUTRITION AND LIFE EXPECTANCY



Abscissa: nutritional level (vegetable calorie equivalents). (Linear)

Ordinata: years of life expectancy for males. (Linear).

(4) Type of analysis, source of basic statistics.

(a) Cross-section analysis (different nations) from average life expectancy and nutritional level in 1953.

(b) Data: M. Cépède, F. Houbart, and L. Grond, "Population and Food," New York: Sheed and Wand, 1964.

EXPECTATION OF LIFE (AGE SPECIFIC)

(1) Keywords: aging, death rates, age specific death rates.

(2) Source: Brookhaven Lecture Series, "The Biology of Aging", by Howard J. Curtis. No. 34, March 18, 1964.

(3) Functional relationships (diagram).

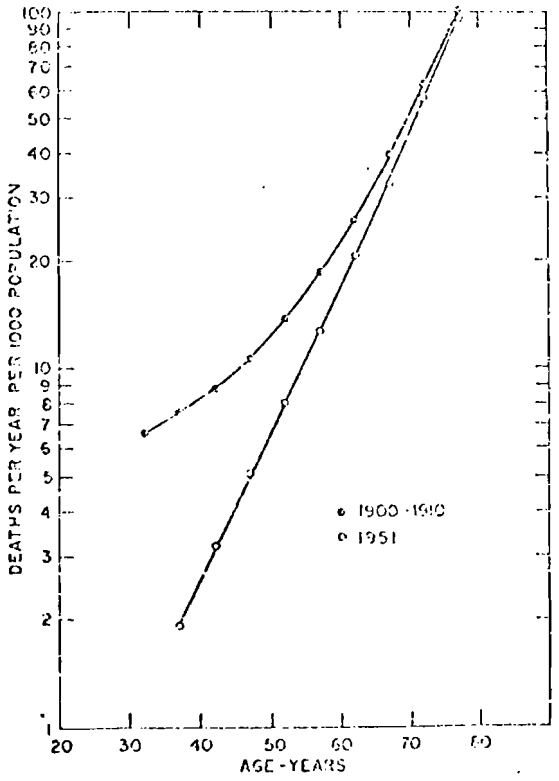


Figure 5. Log age-specific death rate vs. age for males in Sweden for 1900-1910 and for 1951 (data from Jones).

(4) Type of analysis, significance and sources of basic statistics.

(a) Cross section comparison of age-specific death rate curves at two different times. (ø1900-1910 and 1951).

(b) H. B. Jones, "A Special Consideration of the Aging Process, Disease, ..."
in: Advances in Biological and Medical Physics, Vol. 4, pp 281-336, Academic Press, N.Y. 1956.

(1) Keywords: expectation of life, social progress, trends of growth.

(2) Source: Hart-Hornell and Hilda Hertz, "Expectation of Life as an Index of Social Progress", in American Sociological Review, 9 (1944) pp.609-621.

(3) Functional Relationships:

$$Yc = k_1 + k_2/[1+10^{g(d_i-d_a)}]$$

Yc ... average expectation of life (value to be concluded on the curve);

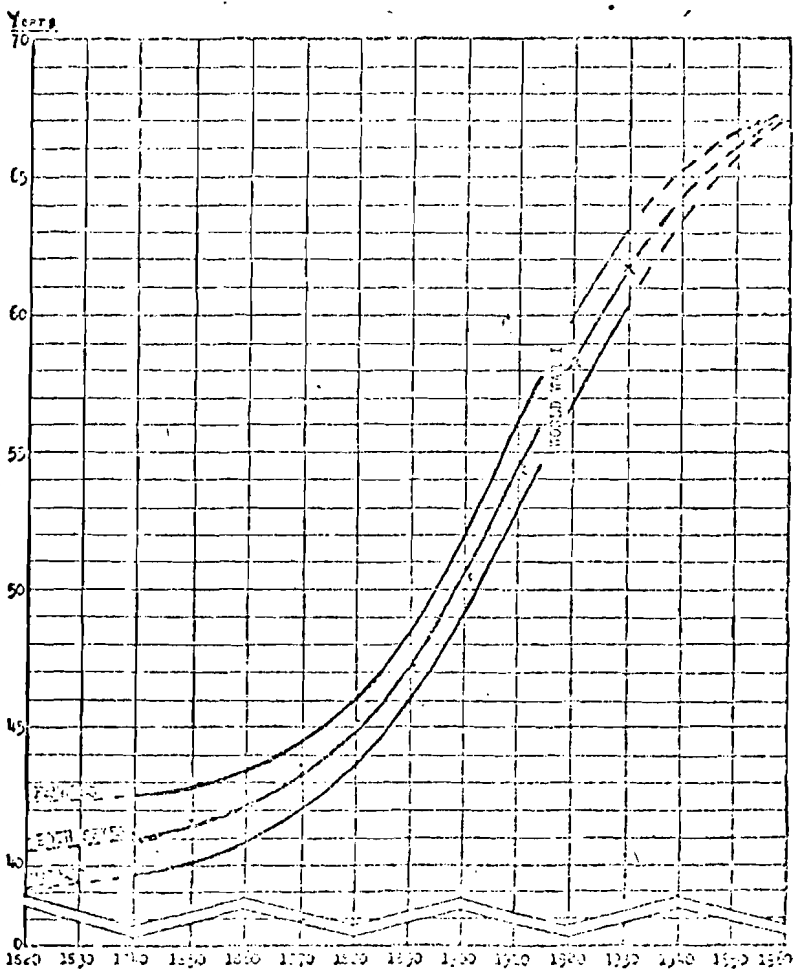
k_1 ... lower limit of growth zone;

k_2 ... width of the growth zone;

g ... growth logarithm;

d_a ... any assigned data during the period for which the formula is valid;

d_i ... the date of inflection.



Average Expectations of Life at Birth in Seven Euro-American Countries, [illegible]

CONSTANTS OF LOGISTIC CURVES FITTED TO EXPECTATION OF LIFE AT BIRTH

Variable	Areas and Sexes	Dates of Data		Constants of Logistic Curve				
				Inflection Date	Growth Zone			Growth Increment (y)
		First	Last		Lower Limit (c_1)	Width (k_2)	Upper Limit (k_1+k_2)	
Expectation of life	7 countries, both	1840	1930	1915	40.38	31.68	72.06	.0215
Expectation of life	7 countries, both	1840	1930	1911	40.45	28.92	69.37	.0235
Expectation of life	7 countries, males	1840	1930	1915	38.85	31.82	70.67	.0210
Expectation of life	7 countries, females	1840	1930	1908	42.27	26.02	68.29	.0274
Expectation of life	7 countries, females	1840	1930	1915	41.97	31.79	73.01	.0205
Expectation of life	13 countries, both	1880	1930	1915	36.99	35.29	72.28	.0183
Expectation of life	6 countries, both	1880	1935	1911	38.98	30.16	69.14	.0226

(4) Type of analysis significance and sources of basic statistics.

(a) Time series analysis of life expectation data.

(b) i. Pitirim Sorankin: "Social and Cultural Dynamcs", Vol. II, pp. 134, 164, 169.

ii. Statistical Abstract for the U.K. 1924-38, p. 307.

iii. Statistical Abstract of the U.S., p. 952.

TRENDS of GROWTH

(1) Keywords: wages, real wages, trends of growth, social progress, progress.

(2) Source: Hart Bornell and Hilda Hertz, "Expectation of Life as an Index of Social Progress," in American Sociological Review, 9 (1944) pp. 609-621.

(3) Functional Relationships:

$$W_{GB\ US} = 39.0 + \frac{81.5}{1+10^{.0231\ (1883-da)}}$$

$W_{GB\ US}$ ... index of real wages in GB and US (1914 = 100).

da ..... any assigned data during the period for which the formula is valid.

For interpretation of empirical data used in the formula see form No.

Abscissa: years (linear)

Ordinata: indices (linear)

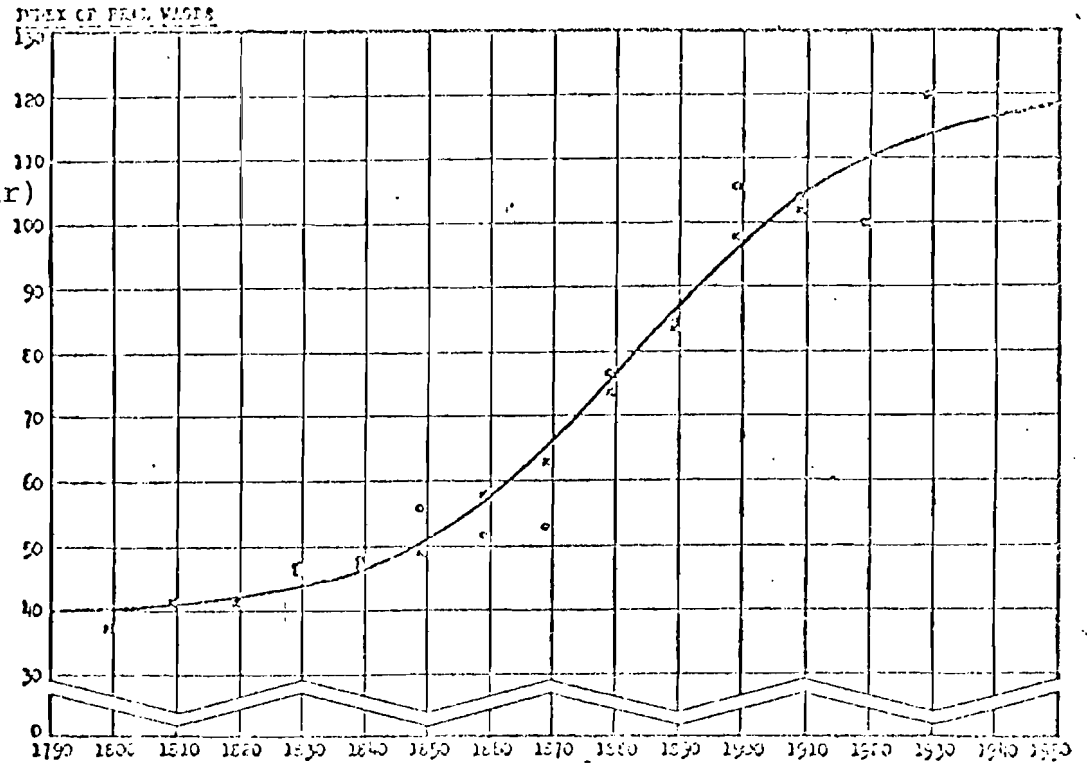


FIGURE 2. Index Numbers of Real Wages of Factory Employees (1914 = 100) in Great Britain and the United States, 1799-1929, with Logistic Curve Fitted, 1790-1950.

(4) Type of analysis, significance and source of basic statistics:

(a) type of analysis of real wage indices;

(b) source: Hart Hornell, "Techniques of Social Progress," 1929, p. 139.

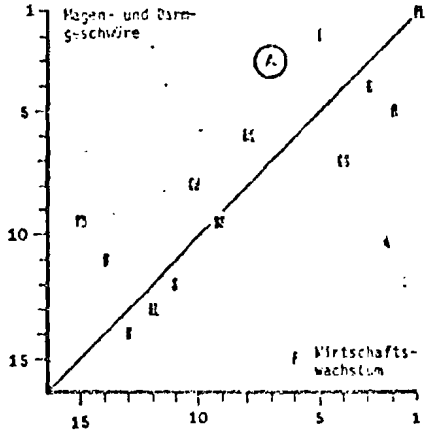
(1) Keyword:

psychosomatic, social psychology, economic growth.

(2) Source:

C. Gaspari, H. Millendorfer: Prognosen für Österreich
Verlag für Geschichte und Politik, Wien 1973.

(3) Functional relationship: (description of variables; formula diagrams; significance).



Ordinata: rank order of economic growth rates 1955-64.

Abscissa: rank order of age corrected mortality rates of ulcer 1955-64.

Letters: European countries marked by car-sticker.

(4) Type of analysis, signficance and basic statistics.

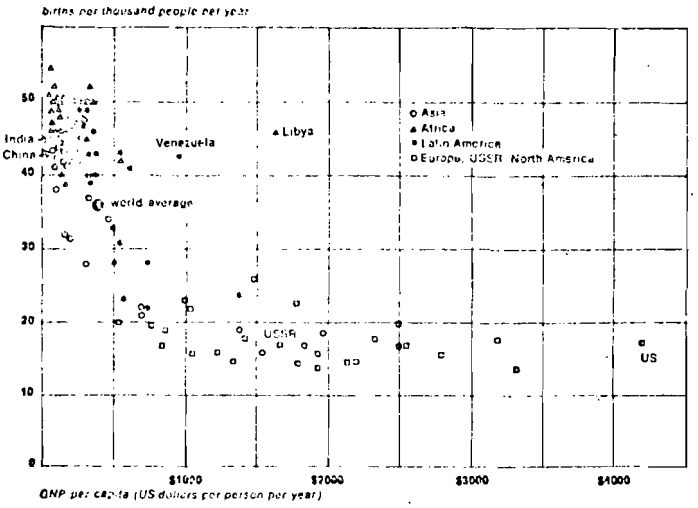
Cross-section comparison of rank order of European countries 1965 data: economic growth rates: Yearbook of National Accounts Statistics 1967, NY 1968.

(1) Keyword: birth rate, per capita income.

(2) Source: D. Meadows, "Limits to Growth", Universe Books, New York, 1972.

(3) Functional relationships (formula diagrams ...)

BIRTH RATES AND GNP PER CAPITA



Abscissa: per capita income (linear).

Ordinata: birth rate (linear).

(4) Type of analysis; source of basic statistics and significance.

(a) Cross section comparision of birth rates vs. per capita income from all countries of the world.

(b) US Agency for International Development, Population Program Assistance, Washington, D.C. 1970

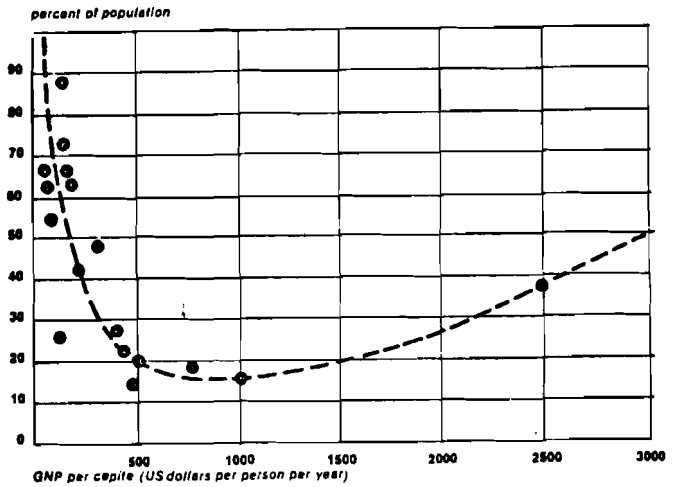
IIASA FORMULA BANK	ECONOMIC GROWTH AND POPULATION (FAMILIES)	FORM № 19

(1) Keyword: family, family planning, GNP per person.

(2) Source: D. Meadows, "Limits to Growth", Universe Books, New York, 1972.

(3) Functional Relationship (formula, diagram).

FAMILIES WANTING FOUR OR MORE CHILDREN AND GNP PER CAPITA



Respondents to family planning surveys in seventeen different countries indicated how many children they would like to have. The percentage of respondents desiring large families (four or more children) shows a relationship to average GNP per capita.

Abscissa: GNP (US$ per person per year). (Linear)

Ordinata: Percentage of respondents desiring large families (four or more children). (Linear)

rp

(4) Type of analysis, source of basic statistics.

(a) Cross section comparison of 17 different countries (percent of population wanting four or more children and GNP per capita).

(b) Bernard Berelson, et al., Family Planning and Population Programs (Chicago: University of Chicago & Press, 1965).

(1) Keyword: technological forecasting, technological change, substitution, steel

(2) Source: P.H. PRY
"A Simple Substitution Model of Technological Change"
General Electric Technical Information Services
No. 70-C-215/1970

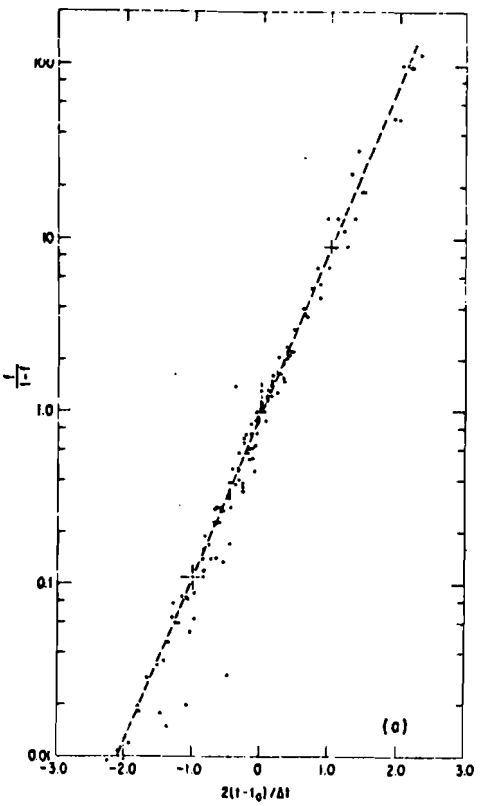
(3) Functional relationships:

$$f/(1-f) = \exp((2\ln 9/\Delta T)(T-T0))$$

f ... fraction of new product or service that was substituted at a time T;

To ... is the time <u>when</u> the substitution is half complete;

ΔT ... "take over time" is the time required from the substitute fraction f to go from 0.1 to 0.9



Figs. 9 (a, b) Fit of substitution model function to substitution data for all 17 cases vs normalized units of time.

Ordinata: $f/(1-f)$ (logarithmic)
Abscissa: $2(t-t_o)/\Delta t$ (linear)
is a dimensionless parameter which normalizes all of the data of 17 substitutions to a single mathematical form.

(4) Type of analysis, significance and sources of basic statistics.

(a) Type analysis: a test of the technological forecast model (comparison of 17 time series data from different substitution processes).

(b) All the data are from 17 cases of substitutions taking place over the last, almost 100 years.

References:

1. Statistical Abstracts of the United States (1969). U.S. Department of Commerce.

2. IIT Research Institute, Ceramics Bull. No. 24 (Nov.-Dec. 1969).

3. Chemical Economics Handbook, Stanford Research Institute.

5. Markets for Plastics, Rosato, Falton, and Rosata, Van Nostrand-Reinhold, 1969.

6. Chemical Week, September 26, 1969 p.69.

TAKEOVER TIMES (τ) AND SUBSTITUTION MIDPOINTS, t_0, FOR A NUMBER OF SUBSTITUTION CASES

Substitution	Units	ΔT-Year	t_0-Year	Reference Source
Synthetic/Natural-Rubber	Pounds	58	1955	1
Synthetic/Natural-Fibers	Pounds	58	1969	1
Plastic/Natural-Leather	Equiv. Hides	57	1957	1
Margarine/Natural-Butter	Pounds	56	1957	1
Electric Arc/Open Hearth - Specialty Steels	Tons*	47	1947	2
Water Based/Oil Based-House Paint	Gallons	43	1967	3
Open Hearth/Bessemer - Steel	Tons	42	1907	2
Sulfate/Tree-tapped-Turpentine	Pounds	42	1959	3
TiO_2/PbO-ZnO - Paint Pigments	Pounds	26	1949	3
Plastic/Hardwood - Residence Floors	Square Feet	25	1966	1
Plastic/Other-Pleasure Boat Hulls	Hulls	20	1966	4
Organic/Inorganic - Insecticides	Pounds	19	1946	3
Synthetic/Natural-Tire Fibers	Pounds	17.5	1948	1
Plastics/Metal - Cars	Pounds*	16	1981	4
BOF/Open Hearth - Steel	Tons	10.5	1968	2
Detergent/Natural - Soap (U.S.)	Pounds*	8.75	1951	5
Detergent/Natural - Soap (Japan)	Pounds*	8.25	1962	5

IIASA FORMULA BANK	TECHNOLOGICAL SUBSTITUTION	FORM № 21

(1) Keyword: technological forecasting, technological change, substitution, steel

(2) Source: P.H. PRY
"A Simple Substitution Model of Technological Change"
General Electric Technical Information Services
No. 70-C-215/1970

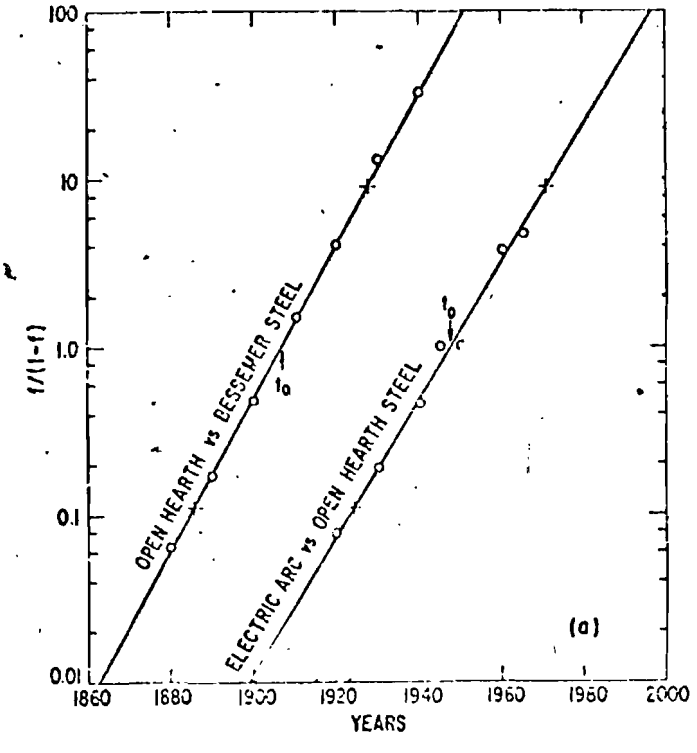
(3) Functional relationships:

$$f/(1-f) = \exp((2\ln 9/\Delta T)(T-To))$$

f ... fraction of new product or service that was substituted at a time T;

To ... is the time when the substitution is half complete;

ΔT ... "take over time" is the time required from the substitute fraction f to go from 0.1 to 0.9.



Abscissa: years (linear)
Ordinata: 1/(1-f) (logarithmic)

(4) Type of analysis, significance and sources of basic statistics.

(a) time series of steel substitution processes.

(b) Source: IIT Research Institute Ceramics Bull.
No. 24/1969.

(1) Keyword: technological forecasting, technological change, substitution

(2) Source: R.H. PRY
"Forecasting the Diffusion of Technology", General Electric Technical Information Series No. 73CRD220, July 1973.

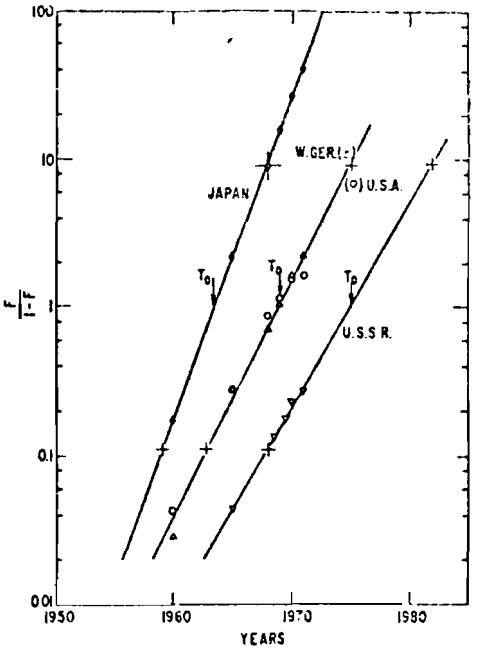
(3) Functional relationships:

$$f/(1-f) = \exp((2\ln 9/\Delta T)(T-To))$$

f ... fraction of new product or service that was substituted at a time T;

To ... is the time when the substitution is half complete;

ΔT ... "take over time" is the time required from the substitute fraction f to go from 0.1 to 0.9.



U.S.A.: ΔT = 10,5
T_o = 1968

Abscissa: years (linear)
Ordinata: f/(1-f) (logarithmic)

Fig. 3 Substitution plot of BOF for open hearth and Bessemer steel production in Japan, U.S.S.R., West Germany, and U.S.A. since 1960.

(4) Type of analysis, significance and source of basic statistical data:

(a) time series of statistical data.

(b) Source: Manual International Steel-Statistics 1971. Country Books, British Steel Corp., Statistical Services, 12 Addiscombe Road, Droyden, Great Britain.

(1) Keyword:

substitution, urban living, rural living, society.

(2) Source: R.H. PRY

"Forecasting the Diffusion of Technology", General Electric Technical Information Series No. 73CRD220, July 1973.

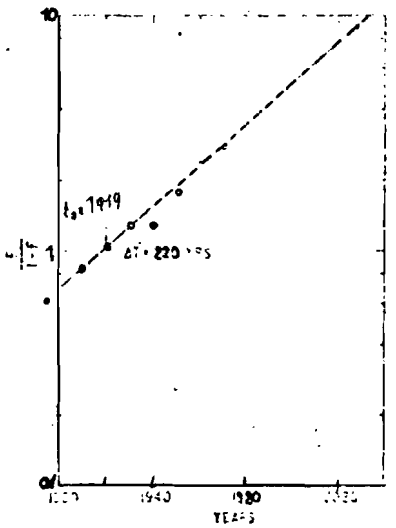
(3) Functional relationships:

$$f/(1-f) = \exp((2\ln 9/\Delta T)(T-To))$$

f ... fraction of new product or service that was substituted at a time T;

To ... is the time when the substitution is half complete;

ΔT ... "take over time" is the time required from the substitute fraction f to go from 0.1 to 0.9.



Substitution plot of urban for rural living in U.S.

abscissa: years (linear).
ordinata: f/(1-f) (logarithmetic).

(4) Type of analysis, significance and sources of basic statistics

(a) time series of basic societal substitution

(b) statistical abstracts of the U.S., U.A. Department of Commerce (1972).

IIASA FORMULA BANK	CITIES DISTRIBUTION OF POPULATION	FORM № 24

(1) Keyword: cities, communities, distribution of population, least effort, Force of Unification, Force of Diversification.

(2) Source: Zipf, G.K., "Human Behavior and the Principle of Least Effort," 1949, N.Y.

(3) Functional Relationships:

$$r.P^q = K$$

r ... rank number of the community when ranked in the order of decreasing population size;

q ... the ratio of the magnitude of the theoretical Force of Diversification divided by the magnitude of the theoretical Force of Unification. (EMPIRICAL CONSTANT)

P.... Population size of a community with a discrete rank number.

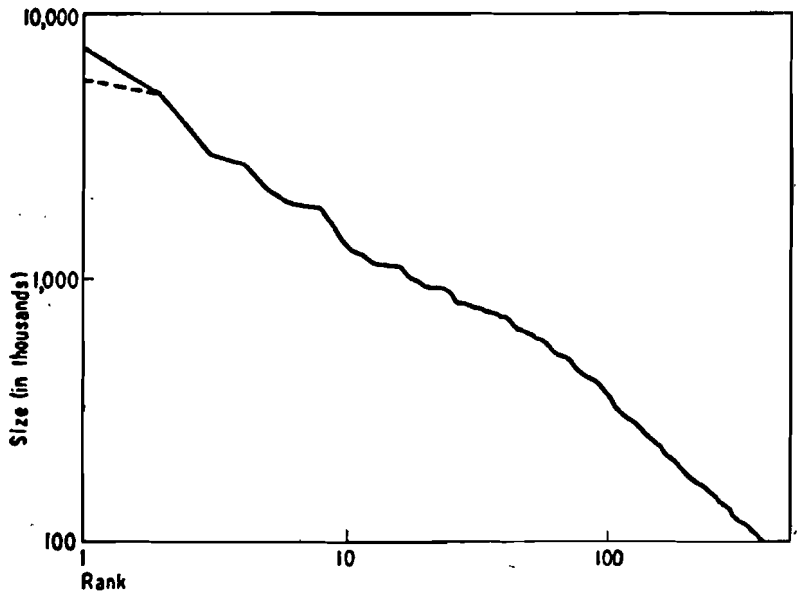


Fig. 10-8. Cities of the world (about 1920) with at least 100,000 inhabitants, ranked in the decreasing order of population size.

(4) Type of analysis, signficance and basic statistics.

(a) Type of analysis:

Cross section comparison of communities

(b) Source:

G. K. Zipf, "National Unit and Disunity," Bloomington Ind.: Principion Press, 1941, pp. 135f., 140f.

(5) Interpreting Hypothesis:

The hypothesis says, that the actual location of the population will depend upon the extent to which persons are moved to materials (Force of Diversification) and materials are moved to persons (Force of Unification).

The Force of Diversification makes for a larger n number of communities of smaller P population size, and the Force of Unification makes for a smaller n number of communities of larger P population size.

The n number of different communities, when ranked, r, in the order of their decreasing P size, will follow the equation:

$$(1) \quad r.P^q = K$$

IIASA FORMULA BANK	CITIES DISTRIBUTION OF POPULATION (CDN)	FORM № 25

(1) Keyword: cities, communities, distribution of population, least effort, Force of Unification, Force of Diversification.

(2) Source: Zipf, G.K., "Human Behavior and the Principle of Least Effort," 1949, N.Y.

(3) Functional Relationships:

$$r.p^q = K$$

r ... rank number of the community when ranked in the order of decreasing population size;

q ... the ratio of the magnitude of the theoretical Force of Diversification divided by the magnitude of the theoretical Force of Unification. (EMPIRICAL CONSTANT)

P.... Population size of a community with a discrete rank number.

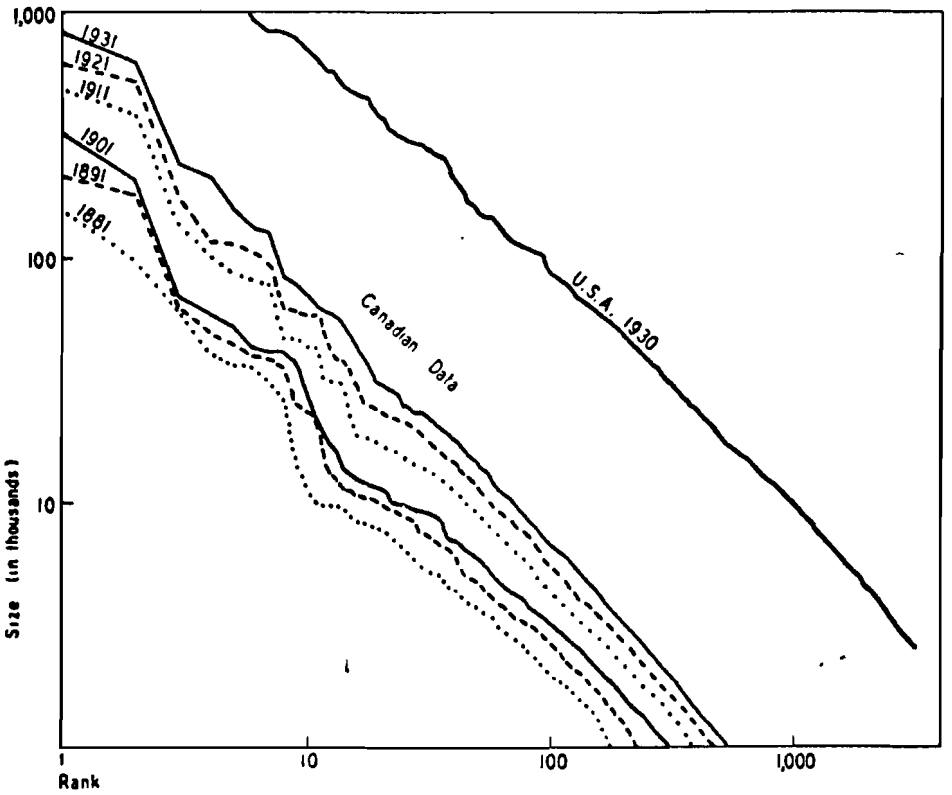


Fig. 10-1. Canada 1881-1931. Communities of 1,000 or more persons, ranked in the decreasing order of population size.

(4) Type of analysis, signficance and basic statistics.

(a) Type of analysis:

A time series comparison of communities cross-sections.

(b) Source:

G. K. Zipf, "National Unit and Disunity," Bloomington Ind.: Principion Press, 1941, pp. 135f., 140f.

(5) Interpreting Hypothesis:

The hypothesis says, that the actual location of the population will depend upon the extent to which persons are moved to materials (Force of Diversification) and materials are moved to persons (Force of Unification).

The Force of Diversification makes for a larger n number of communities of smaller P population size, and the Force of Unification makes for a smaller n number of communities of larger P population size.

The n number of different communities, when ranked, r, in the order of their decreasing P size, will follow the equation:

$$(1) \quad r.P^q = K$$

IIASA FORMULA BANK	CITIES DISTRIBUTION OF POPULATION (GB)	FORM No 26

(1) Keyword: cities, communities, distribution of population, least effort, Force of Unification, Force of Diversification.

(2) Source: Zipf, G.K., "Human Behavior and the Principle of Least Effort," 1949, N.Y.

(3) Functional Relationships:

$$r.P^q = K$$

r ... rank number of the community when ranked in the order of decreasing population size;

q ... the ratio of the magnitude of the theoretical Force of Diversification divided by the magnitude of the theoretical Force of Unification. (EMPIRICAL CONSTANT)

P.... Population size of a community with a discrete rank number.

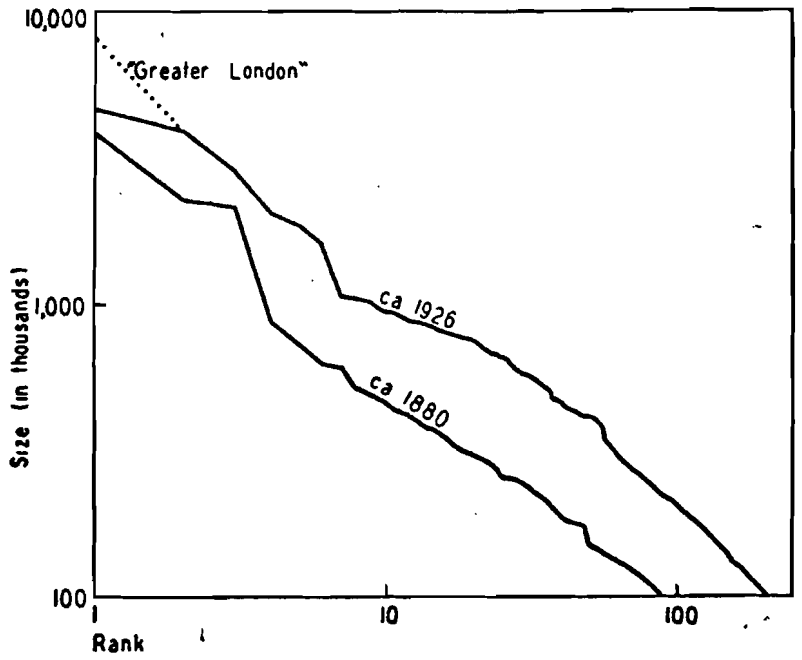


Fig. 10–11. Cities in Europe of not less than 100,000 inhabitants in 1880 and 1920 (approximately).

(4) Type of analysis, signficance and basic statistics.

(a) Type of analysis:

A time series comparison of communities cross-sections.

(b) Source:

G. K. Zipf, "National Unit and Disunity," Bloomington Ind.: Principion Press, 1941, pp. 135f., 140f.

(5) Interpreting Hypothesis:

The hypothesis says, that the actual location of the population will depend upon the extent to which persons are moved to materials (Force of Diversification) and materials are moved to persons (Force of Unification).

The Force of Diversification makes for a larger n number of communities of smaller P population size, and the Force of Unification makes for a smaller n number of communities of larger P population size.

The n number of different communities, when ranked, r, in the order of their decreasing P size, will follow the equation:

$$(1) \quad r.P^q = K$$

IIASA FORMULA BANK	CITIES DISTRIBUTION OF POPULATION (D)	FORM Nº 27

(1) Keyword: cities, communities, distribution of population, least effort, Force of Unification, Force of Diversification.

(2) Source: Zipf, G.K., "Human Behavior and the Principle of Least Effort," 1949, N.Y.

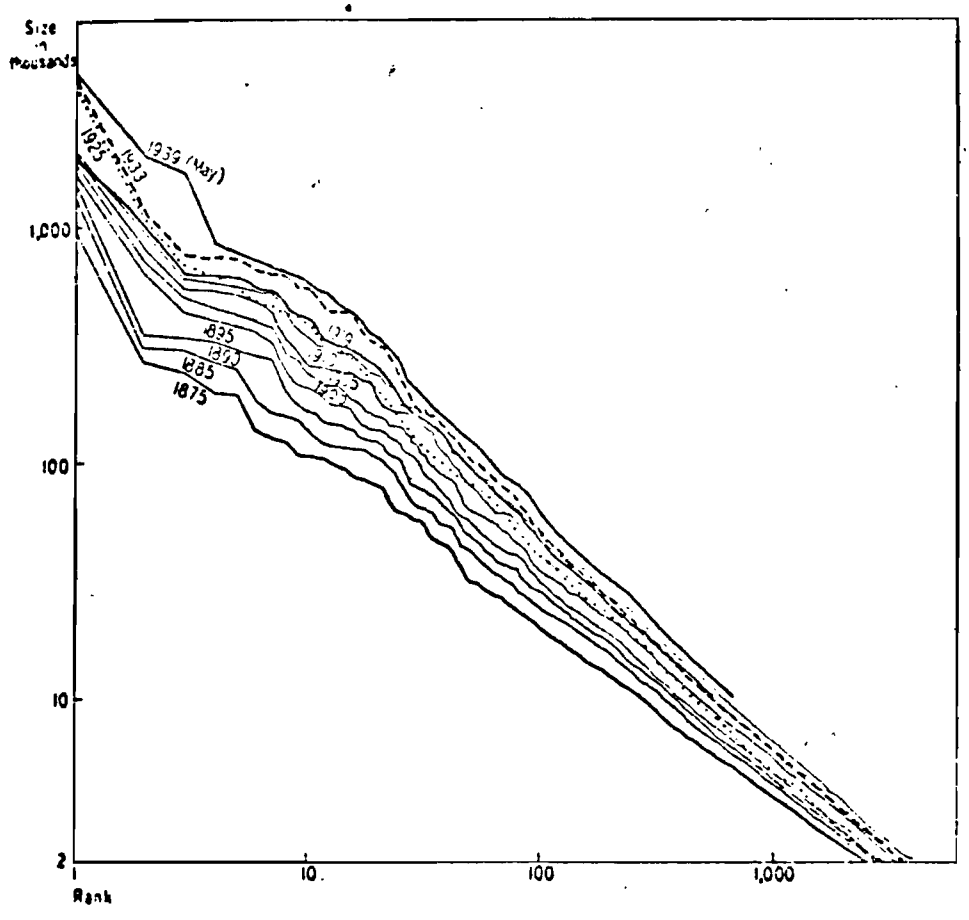
(3) Functional Relationships:

$$r.P^q = K$$

r ... rank number of the community when ranked in the order of decreasing population size;

q ... the ratio of the magnitude of the theoretical Force of Diversification divided by the magnitude of the theoretical Force of Unification. (EMPIRICAL CONSTANT)

P.... Population size of a community with a discrete rank number.



Germany, 1875–1939 (May). Communities ranged in the decreasing order of population size.

(4) Type of analysis, signficance and basic statistics.

(a) Type of analysis:

A time series comparison of communities cross-sections.

(b) Source:

G. K. Zipf, "National Unit and Disunity," Bloomington Ind.: Principion Press, 1941, pp. 135f., 140f.

(5) Interpreting Hypothesis:

The hypothesis says, that the actual location of the population will depend upon the extent to which persons are moved to materials (Force of Diversification) and materials are moved to persons (Force of Unification).

The Force of Diversification makes for a larger n number of communities of smaller P population size, and the Force of Unification makes for a smaller n number of communities of larger P population size.

The n number of different communities, when ranked, r, in the order of their decreasing P size, will follow the equation:

$$(1) \quad r.P^q = K$$

IIASA FORMULA BANK	CITIES DISTRIBUTION OF POPULATION (INDIA)	FORM Nº 28

(1) Keyword: cities, communities, distribution of population, least effort, Force of Unification, Force of Diversification.

(2) Source: Zipf, G.K., "Human Behavior and the Principle of Least Effort," 1949, N.Y.

(3) Functional Relationships:

$$r.P^q = K$$

r ... rank number of the community when ranked in the order of decreasing population size;

q ... the ratio of the magnitude of the theoretical Force of Diversification divided by the magnitude of the theoretical Force of Unification. (EMPIRICAL CONSTANT)

P.... Population size of a community with a discrete rank number.

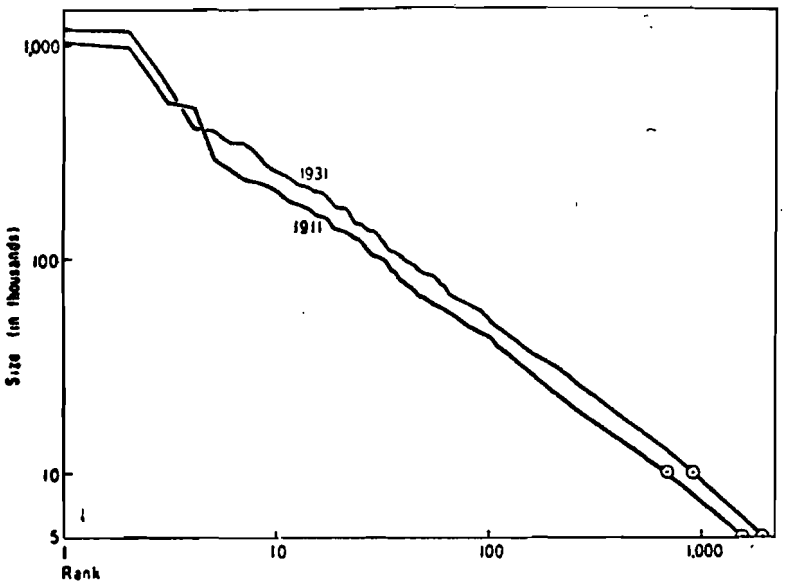


Fig. 10-7. India in 1911 and 1931. Communities ranked in the decreasing order of population size.

(4) Type of analysis, signficance and basic statistics.

(a) Type of analysis:

A time series comparison of communities cross-sections.

(b) Source:

G. K. Zipf, "National Unit and Disunity," Bloomington Ind.: Principion Press, 1941, pp. 135f., 140f.

(5) Interpreting Hypothesis:

The hypothesis says, that the actual location of the population will depend upon the extent to which persons are moved to materials (Force of Diversification) and materials are moved to persons (Force of Unification).

The Force of Diversification makes for a larger n number of communities of smaller P population size, and the Force of Unification makes for a smaller n number of communities of larger P population size.

The n number of different communities, when ranked, r, in the order of their decreasing P size, will follow the equation:

$$(1) \quad r.P^q = K$$

IIASA FORMULA BANK	CITIES DISTRIBUTION OF POPULATION (UK)	FORM No 29

(1) Keyword: cities, communities, distribution of population, least effort, Force of Unification, Force of Diversification.

(2) Source: Zipf, G.K., "Human Behavior and the Principle of Least Effort," 1949, N.Y.

(3) Functional Relationships:

$$r.P^q = K$$

r ... rank number of the community when ranked in the order of decreasing population size;

q ... the ratio of the magnitude of the theoretical Force of Diversification divided by the magnitude of the theoretical Force of Unification. (EMPIRICAL CONSTANT)

P.... Population size of a community with a discrete rank number.

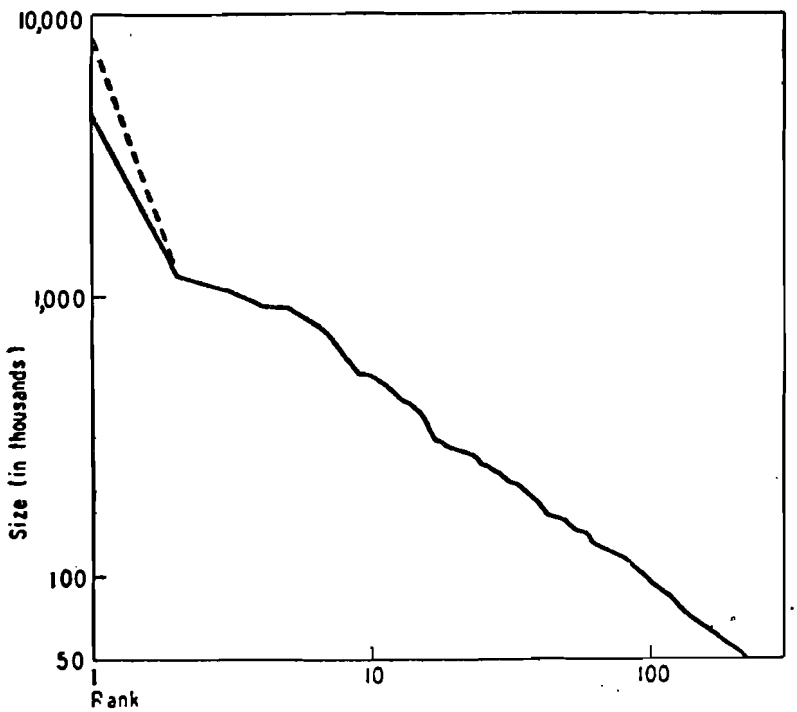


Fig. 10-9. British empire (about 1921). Communities with at least 50,000 persons, ranked in the decreasing order of population size.

(4) Type of analysis, signficance and basic statistics.

(a) Type of analysis:

Cross section comparison of communities

(b) Source:

G. K. Zipf, "National Unit and Disunity," Bloomington Ind.: Principion Press, 1941, pp. 135f., 140f.

(5) Interpreting Hypothesis:

The hypothesis says, that the actual location of the population will depend upon the extent to which persons are moved to materials (Force of Diversification) and materials are moved to persons (Force of Unification).

The Force of Diversification makes for a larger n number of communities of smaller P population size, and the Force of Unification makes for a smaller n number of communities of larger P population size.

The n number of different communities, when ranked, r, in the order of their decreasing P size, will follow the equation:

$$(1) \quad r.P^q = K$$

IIASA FORMULA BANK	CITIES DISTRIBUTION OF POPULATION (U S)	FORM No 30

(1) Keyword: cities, communities, distribution of population, least effort, Force of Unification, Force of Diversification.

(2) Source: Zipf, G.K., "Human Behavior and the Principle of Least Effort," 1949, N.Y.

(3) Functional Relationships:

$$r.P^q = K$$

r ... rank number of the community when ranked in the order of decreasing population size;

q ... the ratio of the magnitude of the theoretical Force of Diversification divided by the magnitude of the theoretical Force of Unification. (EMPIRICAL CONSTANT)

P.... Population size of a community with a discrete rank number.

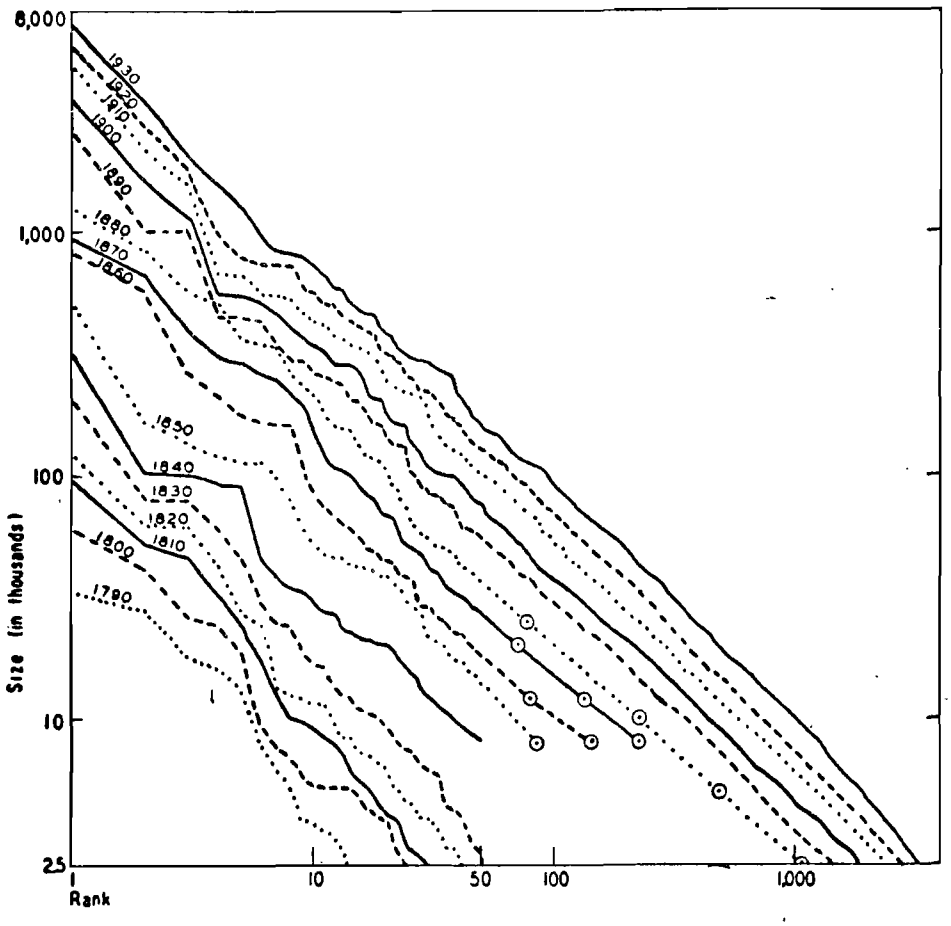


Fig. 10-2. U. S. A. 1790-1930. Communities of 2500 or more inhabitants ranked in the decreasing order of population size.

(4) Type of analysis, signficance and basic statistics.

(a) Type of analysis:

A time series comparison of communities cross-sections.

(b) Source:

G. K. Zipf, "National Unit and Disunity," Bloomington Ind.: Principion Press, 1941, pp. 135f., 140f.

(5) Interpreting Hypothesis:

The hypothesis says, that the actual location of the population will depend upon the extent to which persons are moved to materials (Force of Diversification) and materials are moved to persons (Force of Unification).

The Force of Diversification makes for a larger n number of communities of smaller P population size, and the Force of Unification makes for a smaller n number of communities of larger P population size.

The n number of different communities, when ranked, r, in the order of their decreasing P size, will follow the equation:

$$(1) \quad r.P^q = K$$

IIASA FORMULA BANK	CITIES DISTRIBUTION OF POPULATION (US)	FORM Nº 31

(1) Keyword: cities, communities, distribution of population, least effort, Force of Unification, Force of Diversification.

(2) Source: Zipf, G.K., "Human Behavior and the Principle of Least Effort," 1949, N.Y.

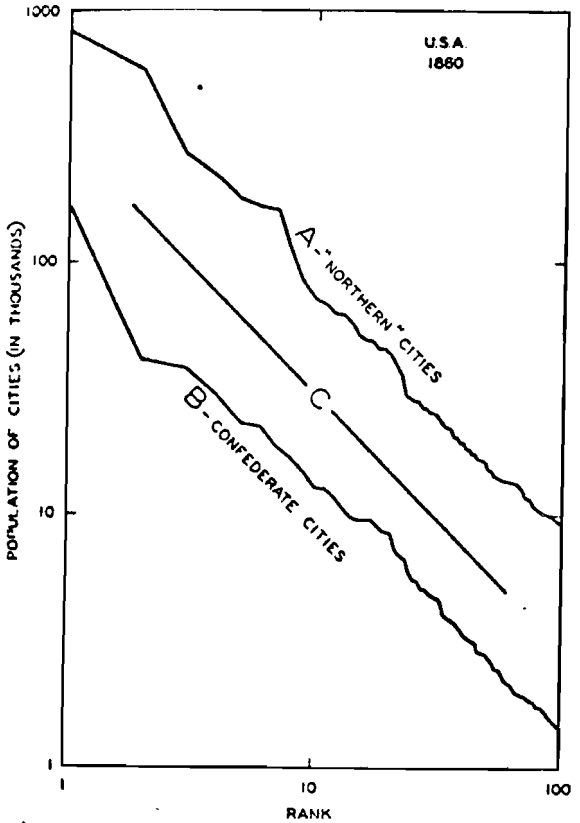
(3) Functional Relationships:

$$r.P^q = K$$

r ... rank number of the community when ranked in the order of decreasing population size;

q ... the ratio of the magnitude of the theoretical Force of Diversification divided by the magnitude of the theoretical Force of Unification. (EMPIRICAL CONSTANT)

P.... Population size of a community with a discrete rank number.



U. S. A. 1860. (A) "Northern" cities, and (B) CONFEDERATE cities, ranked in the decreasing order of population size.

(4) Type of analysis, signficance and basic statistics.

(a) Type of analysis:

Cross section comparison of communities

(b) Source:

G. K. Zipf, "National Unit and Disunity," Bloomington Ind.: Principion Press, 1941, pp. 135f., 140f.

(5) Interpreting Hypothesis:

The hypothesis says, that the actual location of the population will depend upon the extent to which persons are moved to materials (Force of Diversification) and materials are moved to persons (Force of Unification).

The Force of Diversification makes for a larger n number of communities of smaller P population size, and the Force of Unification makes for a smaller n number of communities of larger P population size.

The n number of different communities, when ranked, r, in the order of their decreasing P size, will follow the equation:

$$(1) \quad r.P^q = K$$

IIASA FORMULA BANK	CITIES DISTRIBUTION OF POPULATION (U.S.)	FORM Nº 32

(1) Keyword: cities, communities, distribution of population, least effort, Force of Unification, Force of Diversification.

(2) Source: Zipf, G.K., "Human Behavior and the Principle of Least Effort," 1949, N.Y.

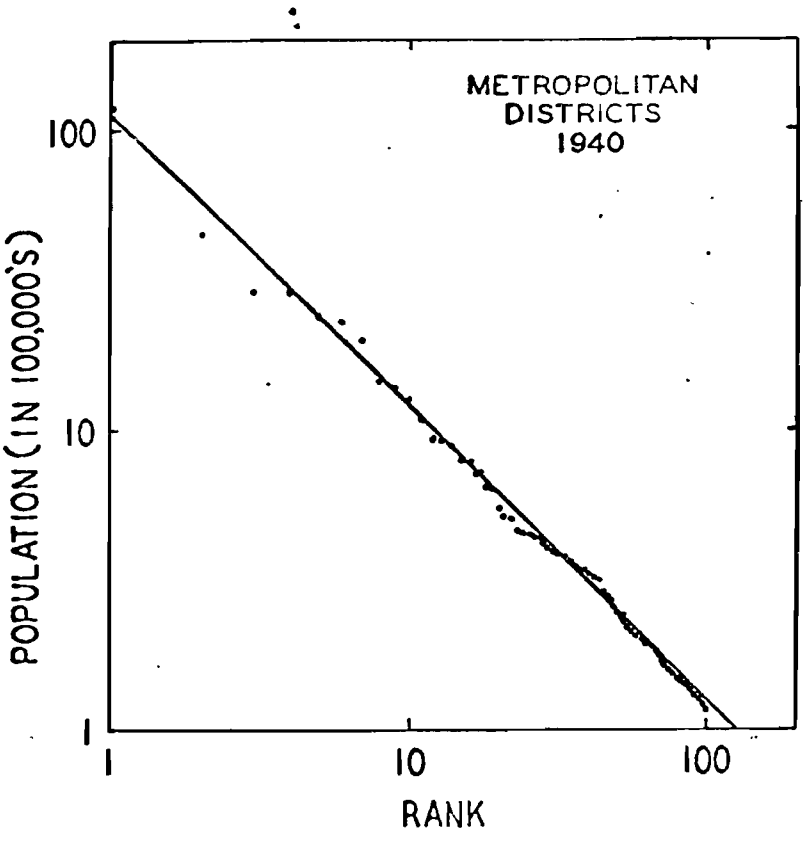
(3) Functional Relationships:

$$r.p^q = K$$

r ... rank number of the community when ranked in the order of decreasing population size;

q ... the ratio of the magnitude of the theoretical Force of Diversification divided by the magnitude of the theoretical Force of Unification. (EMPIRICAL CONSTANT)

P.... Population size of a community with a discrete rank number.



Metropolitan districts. One hundred largest in the U. S. A. in 1940, ranked in the order of decreasing population size.

(4) Type of analysis, signficance and basic statistics.

(a) Type of analysis:

Cross section comparison of communities

(b) Source:

G. K. Zipf, "National Unit and Disunity," Bloomington Ind.: Principion Press, 1941, pp. 135f., 140f.

(5) Interpreting Hypothesis:

The hypothesis says, that the actual location of the population will depend upon the extent to which persons are moved to materials (Force of Diversification) and materials are moved to persons (Force of Unification).

The Force of Diversification makes for a larger n number of communities of smaller P population size, and the Force of Unification makes for a smaller n number of communities of larger P population size.

The n number of different communities, when ranked, r, in the order of their decreasing P size, will follow the equation:

$$(1) \quad r.P^q = K$$