WORKING PAPER

COMPUTATION OF ELECTRIC ENERGY EXCHANGE BETWEEN TWO POWER SYSTEMS

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IIASA
International Institute
for Applied Systems Analysis

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FOREWORD

The expected energy exchange between cooperating systems is an important information supporting capacity expansion planning for electric power systems. A model based on engineering considerations and a program system on IBM/PC-XT or AT compatibles has been developed for the stochastic analysis of the electric energy exchange between two interconnected power systems. Data required for the analysis are expected generation and load in the individual systems, and interties data. This work has been carried out in the frame of the IIASA Contracted Study "Modeling of interconnected power systems".

Alexander B. Kurzhanski
Chairman
System and Decision Sciences Program

CONTENTS

1. INTRODUCTION

The utilization of power systems has to be adopted to management objectives. The main thrust of electric power system operation are the security and cost of operation and quality of supply. One of the ways to fulfill the management objectives is the cooperation among electric power systems.

The purpose of this paper is to present methods for the computation of electric energy exchange between cooperating systems. The stochastic nature of the availability of generating units and transmission lines and the randomness in consumer demand are taken into account.

The approach employed here is based on a paper by T. Terstyánszky [1] and on the papers [2, 3, 4].

The main features discussed are listed below:

- statistical analysis of the random effects influencing electric power exchange,
- stochastic modeling of the interconnected systems consisting of submodels for the individual components.
- computational procedures for the solution of the model.
- possibilities for further development.

The capability of calculating the expected electric energy exchange between cooperating systems can be utilized both for short-term production planning and long-term capacity expansion planning. For a short-term horizon better utilization of the existing equipment can be achived. In the case of long-term planning, a large amount of alternatives for the development of the energy system are analyzed, compared and an appropriate variant selected. In this case the energy exchange is calculated for the individual alternatives .

2. MAIN CHARACTERISTICS OF ELECTRIC ENERGY EXCHANGE

In most countries the electric power industry has realized the economic and technical advantages of cooperation. The exchange of electric power became part of the international trade, too. But in contrast with other commodities electricity has the very specific character as it can't be stored in large quantities. This implies that in an emergency situation (caused e.g. by forced outages) the risk of breaking the continuity of supply increases considerably. In this case the supply continuity for customers can be ensured by power transfer from other power systems. The quantity of electric power exchanged among the cooperating countries or utilities is the result of a compromise between a concern for electric power independence, the economic advantages of cooperation, and the technical problems arising from different strategies of frequency control.

The overall amount of electic energy exchange for the year 1985 can be seen on Figures 1 and 2 for the Western and Eastern European countries, respectively (see also [5,6]). The relative importance of electric power exchange for selected European countries is shown in Fig. 3 [7].

Two diagrams are presented to show the stochastic character of the power exchange:

Fig 4. shows the aggregate sum of simultaneous loads of electric power transfer connections across state borders in the West European UCPTE system, over a 6 years period for a specified time of a month.

The power exchange can be seen in details on Fig. 5, for the case of the Hungarian inter-ties. This figure shows the load pattern of one-hour average scheduled export-import on long-term agreement, in comparison with the actual load which contains also the short-term random power exchange performed on the basis of mutual assistance between the Hungarian Power System and the Interconnected Power System of CMEA.

Considering the unmistakably undeterministic nature of electric power exchange a stochastic modeling approach is an apropppriate way of dealing with the problem. The model and computational methods presented in this paper are developed for the estimation of power exchange based both on long term and short term agreements.

3. STATISTICAL ANALYSIS OF ELECTRIC ENERGY EXCHANGE.

The base load of tie-lines between cooperating power systems is fixed by long term agreements. The actual load of lines however, differs from this assigned transfer level as a result of changes in operating conditions such as forced outage of units.

As the power exchange is of a stochastic nature, its amount can be considered as a random variable. To provide a basis for model-development a statistical analysis of transferred power between a large system having installed capacity of approximately 70000 MW and a small system with 4000 MW was carried out.

To ensure sufficiently large sample sizes actual power data of transfers for daily high load periods in one year have been taken into account. (Fig. 6.). The analysis was repeated for 5 consecutive years.

The hypothesis to be tested was the following: The power exchange is a normally distributed random variable. The assumpion underlying our investigation was that the distribution function belongs to the Pearson family of probability distributions [8].

Results of the analysis confirmed in all cases considered that the short-term electric power exchange is normally distributed, since the corresponding constants in the differential equation identifying Pearson' family were found to be approximately zero.

For the sake of a more accurate evaluation, it is appropriate to represent data in relative units. The mean value representing the systems' cooperation fluctuates in principle around zero, therefore it is reasonable to examine the short term power exchange in per unit representation with respect to the standard deviation (see Fig. 7).

In order to verify the hypothesis concerning the distribution of the power exchange a chi-square-test was carried out, which confirmed the hypothesis of normality.

4. PROBLEM FORMULATION AND SOLUTION METHODOLOGY

4.1 A MATHEMATICAL MODEL OF COOPERATING POWER SYSTEMS

A power system is designed in such a way, that mostly it can satisfy the consumer demand, moreover it has a reserve for the case of failures, so usually it is possible to give assistance for the cooperating partner if it is in trouble, and vice versa.

In the model presented both energy systems are reduced to a node; each with its own generation and load and with no internal transmission limitations. Their energy consumptions, the available power plant capacities and the transmission capacity of the tie-line system are considered to be random variables [2].

For the evaluation of the exchange of energy between the cooperating systems the following variables will be introduced:

- γ_1 and γ_2 denote the random variables of the aggregated available capacity of generating units in the first and second system, respectively.
  $g_1(z)$ and $g_2(z)$ denote their density functions, $G_1(z)$ and $G_2(z)$ the distribution functions.

- δ_1 and δ_2 denote the random variables of the consumer demand in the first and second system, respectively.

- Let s be the deterministic variable of the scheduled exchange of energy predefined according to long term agreement. In the model this type of energy exchange is taken into account as a demand. Let μ_1 and μ_2 denote the energy demand modified by s

$$\mu_1 = \delta_1 + s$$
$$\mu_2 = \delta_2 - s \qquad (1)$$

Let $h_1(x)$ and $h_2(x)$ be their density functions, $H_1(x)$ and $H_2(x)$ the distribution functions.

- ε_1 and ε_2 are random variables of deficiency/excess of energy for both systems

$$\varepsilon_1 = \gamma_1 - \mu_1$$
$$\varepsilon_2 = \gamma_2 - \mu_2 \qquad (2)$$

If the $\varepsilon_i<0$ $(i=1,2)$ then there is deficiency of energy in the ith system, if $\varepsilon_i=0$ then the demand and the generated energy are balanced, otherwise there is energy excess in the system.

Let $f_1(x)$ and $f_2(x)$ be their density functions, $F_1(z)$ and $F_2(z)$ the distribution functions.

ε_1 and ε_2 can be assumed to be independent random variables because they mainly depend on internal characteristics of the individual systems.

- Let ξ denote the random variable of energy exchange flowing through the transmission lines connecting the systems. Let $F(z)$ denote the distributon function of ξ. The value of ξ can vary between the maximal demand of the first system multiplied by -1, and the maximal demand of the second system. A positive ξ means energy flowing from the first system to the second one.

In our model we assume the following assistance policy: Within a system no consumer limitations are imposed due to deficiency in the other system. Assistance is only given when energy reserve is available.

There are other possibilities for an assistance policy, e.g. the systems could share the costs of damage and limitation of consumers in a predefined ratio. However we think that this policy is not a feasible one in the normal economical, accounting relations between different countries.

According to the assistance policy assumed above the exchange of energy is determined by the following expression:

$$\xi = \begin{cases} \min(\varepsilon_1, -\varepsilon_2) & \text{if } \varepsilon_1>0 \text{ and } \varepsilon_2<0 \\ \max(\varepsilon_1, -\varepsilon_2) & \text{if } \varepsilon_1<0 \text{ and } \varepsilon_2>0 \\ 0 & \text{otherwise} \end{cases} \qquad (3)$$

The main problem addressed in this paper is the computation of the distribution of variable ξ.

The distribution function of ξ is the following function.

$$F(z) = P(\xi<x) = \begin{cases} P(\min(\varepsilon_1,-\varepsilon_2)<z,\ \varepsilon_1>0,\ \varepsilon_2<0)+ \\ +P(\max(\varepsilon_1,-\varepsilon_2)<z,\ \varepsilon_1<0,\ \varepsilon_2>0)+ \\ +P(\varepsilon_1\varepsilon_2>0) & \text{if } z>0 \\ \\ P(\max(\varepsilon_1,-\varepsilon_2)<z,\ \varepsilon_1<0,\ \varepsilon_2>0) & \text{if } z\le 0 \end{cases} \qquad (4)$$

It is easy to see that this function can be expressed in the simple form given below:

$$F(z) = \begin{cases} 1 - P(\varepsilon_1>z,\ \varepsilon_2<-z) & \text{if } z>0 \\ \\ P(\varepsilon_1<z,\ \varepsilon_2>-z) & \text{if } z\le 0 \end{cases} \qquad (5)$$

Using the distribution functions of ε_1 and ε_2 the function can be rewritten as follows

$$F(z) = \begin{cases} 1 - (1-F_1(z))\ F_2(z) & \text{if } z>0 \\ \\ F_1(z)\ (1 - F_2(z)) & \text{if } z\le 0 \end{cases} \qquad (6)$$

Function F(z) has a nonzero jump at z=0 of the following magnitude

$$F_0 = 1 + 2F_1(0)F_2(0) - F_1(0) - F_2(0) \qquad (7)$$

expressing the probability of the event that no energy exchange occurs. This is the case when either both systems can produce the sufficient amount of energy to satisfy their consumers or both systems have energy deficiency.

As the distribution function has a jump there does not exist a corresponding density function. To characterise probabilities of the different amounts of energy exchange the following function will be introduced and calculated.

- For $z<0$ it expresses, in the same way as usual histograms, the probabilities of the amount of energy transmitted from system 2 to system 1.
- For $z=0$ its value is the probability of no power exchange.
- For $z>0$ it expresses the probabilities of the amount of energy transmitted from system 1 to system 2.

This function will be called exchange probability function and denoted by f(z). As it plays same role as histograms we will also refer to it as exchange histogram. Using the formula given below for F(z) an explicit form can be obtained by differentiation:

$$f(z) = \begin{cases} f_1(-z)\ [1-F_2(z)] + f_2(z)\ F_1(-z) & \text{if } z>0 \\ f_2(-z)\ [1-F_1(z)] + f_1(z)\ F_2(-z) & \text{if } z<0 \\ 1 + 2F_1(0)F_2(0) - F_1(0) - F_2(0) & \text{if } z=0 \end{cases} \qquad (8)$$

For computation of the exchange histogram f(z) it is necessary to determine the density functions $f_1(z)$ and $f_2(z)$. Since ε_1 and ε_2 are the sum of random variables as given above, they can be computed by convolution integrals. The corresponding distribution functions can be obtained afterwards by integration. In the computations histograms are used as empirical density functions.

The main steps of the calculation are the following:

- Computation of the availability histogram of generating units for both energy systems, which means calculation of the histograms corresponding to $g_1(z)$ and $g_2(z)$.

- Computation of the deficiency-excess histograms $f_1(z)$ and $f_2(z)$ on the basis of the availability histograms $g_1(z)$ and $g_2(z)$. of generating units, and the daily variation of consumer demand. The consumer demand includes scheduled export-import.

- Knowing the deficiency-excess histogram for both systems, the program calculates the probabilities of different amounts of energy exchange between the two systems. In this way the histogram of the energy exchange is calculated. In this step the cooperating line system is considered to be an ideal connection without capacity limitations and without any failure or outage.

- In the last step transmission line system limitations and possibilities of failure are taken into account.

The computation of the power exchange presupposes the modeling of the generating units, the consumer-demand, the scheduled export-import, as well as the model of the tie-line system. These components are described as follows.

4.2 ELEMENTS OF THE POWER SYSTEM

4.2.1 GENERATORS

The required data for the capacity-availability characteristics of the generating blocks are summarized as follows.

- maximum capacity of the individual blocks (P_i in [MW]),

- self-consumption of the blocks (S_i in [%]),
- failure probability of the blocks (p_i in [1]).

Denote by N_i the capacity of the i-th block decreased by its selfconsumption, where i denotes the index of the block.

4.2.2 CONSUMER-DEMAND

The consumer demand is described by the random variables δ_1 and δ_2 which are assumed to be normally distributed. The time horizon is subdivided into periods. The expected values and standard deviations are considered to be constant along each time-period, so the expected daily load-curve is a stepwise constant function of time. Therefore the consumer demands are modeled by

- their stepwise constant function expected values (D_k in [MW]),
- their standard deviations (σ_k in[1]).

where k denotes the time-interval index.

The deviation σ_{max} is given with respect to the peak D_{max}, in percentage of the peak value. The deviations for the individual periods are calculated by the following formula:

$$\sigma_k = \sigma_{max} \sqrt{\frac{D_k}{D_{max}}} \tag{9}$$

This equation expresses the tendency, that a lower load is associated with a lower deviation.

The determination of the consumer-demand is based on the forecasting of daily load curves. The calculation of the energy exchange is usually carried out for a longer time-interval, e.g. for one year. Our program allows for selecting six seasons with optional lengths, and it is possible to select different daily-load curves to all of the seasons. Fig 9. shows an example of a typical daily load curve.

4.2.3 SCHEDULED EXPORT-IMPORT BASED ON LONG TERM AGREEMENTS

The scheduled export-import energy exchange, denoted above by s, is modeled as a consumer-demand. It is assumed to be deterministic, and its variation within the time horizon is considered as a stepwise constant function. Therefore the scheduled export-import is modeled by a stepwise constant function with values E_k in [MW], where k denotes the time-interval index.

As for the consumer-demand model it is here also possible to select different curves, according to the seasons.

In both energy systems the generating units must serve the algebraic sum of the consumer-demand and the scheduled export-import.

4.2.4 TIE-LINES

The data required for the description of the cooperating lines are the following.

- maximum transfer capacities (L_i in[MW]),
- failure probabilities (l_i in[1]).

where i denotes the index of cooperating line.

4.3 THE COMPUTATIONAL PROCEDURE

4.3.1 COMPUTATION OF THE AGGREGATED CAPACITY-AVAILABILITY HISTOGRAM OF GENERATING UNITS

The probability distribution of the availability of generators is a discrete distribution, so its density is calculated in the form of a histogram. Let we consider one of the power systems individually.

Let ϑ_i, $i=1,\ldots,n$ be the following random variables describing unit availability for the ith unit

$$\vartheta_i = \begin{cases} 0 & \text{with probability } p_i \\ N_i & \text{with probability } 1-p_i \end{cases} \tag{10}$$

where n is the number of generating units in the system.

Denote by $\hat{\vartheta}_i$ the partial sums of ϑ_i's

$$\hat{\vartheta}_i = \sum_{i=1}^{j} \vartheta_i \qquad j=1,\dots,n. \tag{11}$$

The calculation of the aggregated capacity availability histogram will be carried out in the following way:

For $\hat{\vartheta}_1 = \vartheta_1$ the histogram is known.

Knowing the histogram for $\hat{\vartheta}_i$ ($1 \le i \le n-1$) the histogram for $\hat{\vartheta}_{i+1}$ can be computed by convolution since $\hat{\vartheta}_{i+1} = \hat{\vartheta}_i + \vartheta_{i+1}$. Using this procedure the histogram for $\hat{\vartheta}_n$ is the aggregated capacity availability histogram g(x).

Denote d the length of the subintervals in the histogram, and

$$k = \mathrm{entier}\left(\sum_{i=1}^{n} N_i / d\right) + 1$$

is the number of the subintervals in the histogram.

The first step:

$$g_{1,1}(x) = \begin{cases} p_1 & \text{if } x < d \\ 1-p_1 & \text{if } a_0 < x < b_0 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

where $a_0 = d\ \mathrm{entier}(N_1/d)$ and
$b_0 = d\ (\mathrm{entier}(N_1/d)+1)$

general step: Let already be (i) machines in the system. Then the next machine (number i+1) is added to the system by the following cycle, with $m=k-1,\ k-2,\dots,1$.

$$g_{i+1,m}(x) = \begin{cases} g_{i,m+1}(x) + g_{i,m+1}(x-N_{i+1})\ (1-p_{i+1}) & \text{if } a_{i,m} < x < b_{i,m} \\ g_{i,m+1}(x)\ p_{i+1} & \text{if } (m-1)\ d < x < m\ d \\ g_{i,m+1}(x) & \text{otherwise} \end{cases} \tag{13}$$

where $a_{i,m} = d\,(m-1) + d\,\text{entier}(N_{i+1}/d)$

$a_{i,m} = d\,m + d\,\text{entier}(N_{i+1}/d)$,

and $g_{i+2,k}(x) = g_{i+1,1}(x)$

(N_i is the capacity, decreased by the selfconsumption; and p_i the failure probability).

Performing the general step for every machine in any order, the function g(x), i.e.the capacity-availability histogram of the generators is obtained.

The computation of the exact distribution of random variables γ_1 and γ_2 (aggregated capacity availability) is computationally infeasible for systems of realistic size because of the combinatorial exploison. It is clear that the computational effort needed to get the results heavily depends on the length of subintervals in the histogram.

Fig. 11. shows a capacity-availability histogram, calculated by the program, while Fig. 12. shows the distibution function.

4.3.2 COMPUTATION OF DEFICIENCY-EXCESS HISTOGRAM

For both systems, the energy produced by the machines must serve the algebraic sum of the energy demanded by the consumers and the scheduled exchange of energy between the two systems. Since the comsumption is supposed to be normally distributed, therefore the sum of consumption and scheduled export/import, denoted above by μ_1 and μ_2 are normally distributed, as well. The expected value is $M_k = D_k + E_k$ and the standard deviation σ_k.

Denoting by $h_k(x)$ the density function of the random variable, having the parameters defined above, the following convolution integral gives the deficiency-excess density function:

$$f_k(x) = \int_0^{hmax} h_k(y-x)\, dG_k(y) \qquad (14)$$

$k=1,2$, where $G_k(y)$ denotes the distribution function of the capacity-availability of generators.

This calculation can be performed for both power systems, so we have the functions which describe the probability of energy deficiency or excess for both systems.

4.3.3 COMPUTATION OF EXCHANGE POWER

The exchange histogram is calculated according to the expression(8). This expression refers to a given time interval which is a subinterval in the daily load curve. The exchange histogram for a given period of time, e.g. for a day or for a year, can be determined by summation of the histograms for these subintervals.

4.3.4 COMPUTATION OF EXCHANGE POWER FOR REAL CONNECTION

Let ω_i $i=1,\dots,k$ denote the following random variables, describing transmission capacity availability for the ith tie-line.

$$\omega_i = \begin{cases} 0 & \text{with probability } l_i \\ L_i & \text{with probability } 1-l_i \end{cases} \qquad (15)$$

where k is the number of transmission lines in the tie-line system.

The method for the determination of aggregated transmission capacity availabilty histogram is very similiar to the procedure applied for the aggregated capacity availability histogram for the generating units. Denote $p(x)$ the histogram of the aggregated capacity availability, $P(x)$ its distribution function, and $Q(x)=1-P(x)$.

Fig 10. shows an example of the distribution of the available transfer-capacity of the cooperating-line system.

Taken into account the probability distribution of the capacity of the cooperating line system, i.e. the possibility of unexpected break-down or outages, the histogram of the exchange power which can be transmitted on an ideal line system (8) is modified according to the following relations:

$$f'(z) = f(z) * Q(|z|) \quad \text{if } |z| \neq M \qquad (16)$$

$$f'(M) = p(M_i) * \int_{M_i}^{f_{2max}} f(x)\, dx \qquad (17)$$

$$f'(-M_i) = p(M_i) * \int_{-M_i}^{-f_{1max}} f(x)\, dx \qquad (18)$$

Fig. 13. shows an example on the histograms of exchange power. Fig. 13/a shows the histogram for cooperation lines with unlimited capacity. Fig. 13/b shows the case of real connections, while Fig. 13/c shows the not realised exchange power resulting from limitations or failure of the transmission system.

5. REALIZATION ON IBM/PC

5.1 OVERVIEW OF THE PROGRAM SYSTEM

The algorithm outlined above has been implemented on an IBM/PC. To provide a convenient user interface with mult-window facility and graphical representation of results graphics toolkits are utilized.

Typical applications of the system require a series of runs with input data which are variants of a base case data set. Data structures are designed to support this kind of usage. Input data for the different parts of the model are stored in different data-files, and the system is endowed with convenient data retrieval, modification and storage facilities. The result of the computations appears in a graphic form on the screen, which can be copied to a dot matrix printer.

The program-system consists of three programs. The actual computations are performed by the program COOPER, while results are displayed by the programs GENER and EXCHANGE.

Running time of the program largely depends on the size of the problem. For systems with 10000 MW generation capacity and assuming a 24-periods subdivision for the daily load curves, the running time is around 10 minutes on an IBM/PC AT with arithmetic coprocessor and turbo cards. Collecting data is a crucial point in the real-life applications of the system. Usually users have data for their own country or utility, but obtaining correct data from the cooperating partner is not always an easy task.

5.2 RUNNING THE PROGRAM

The first action to be performed when using the system is the loading of the graphics drivers. This can be performed simply by executing the files WM and GRAPHICS. The first of these supports window-manipulations, the second provides a print-screen copy utility.

The general pattern of data input or modification is the following: Data are to be typed into specified fields on the screen, data input into a specific field is closed by pressing <RETURN>, closing input for a particular screen can be performed by pressing the <F10> function-key.

The main computational processes are performed by the program COOPER, which displays during run time the required user interactions and information on the current stage of computations.

The actions to be performed in a typical user session are listed below.

1/ Execute the files WM and GRAPHICS, and afterwards COOPER, by simply typing their names on subsequent system prompts.

2/ Press <F10>, a menu appearsfor modificatons of general parameters of the interconnected systems.

3/ Perform modifications,if needed.

4/ Press F10 to close input.

5/ A prompt appears asking whether the new data set is to be saved. Save data set if necessary.

6/ Press <F10>, a menu appears for modifications in generator data (capacity, self-consumption and failure probability)of the first power system.

7/ Perform modifications,if needed.

8/ Press F10 to close input.

9/ A prompt appears asking whether the new data set is to be saved. Save data set if necessary.

11/ Carry out steps 6 - 9 for the second system.

12/ Press <F10>, a menu appears on modifications in the scheduling of consumer demand and export-import, as well as in their maxima.

13/ Perform modifications,if needed.

14/ Press F10 to close input.

15/ A prompt appears asking whether the new data set is to be saved. Save data set if necessary.

16/ Press <F10> for possible modifications in tie-line capacity-distribution. Transmittable power should be specified in decsreasing order of probability.

17/ Perform modifications,if needed.

18/ Press F10 to close input.

19/ A prompt appears asking whether the new data set is to be saved. Save data set if necessary.

20/ Press <F10>, the computation are started.

21/ After running the program COOPER, execute the file GENER by typing its name. This program displays the availability histograms and distributions for both systems.

22/ To display the histograms and distributions of the exchange power for transmission lines of unlimited capacity, with real connections, as well as the not realised exchange power, execute the file EXCHANGE by typing its name.

6. POSSIBILITIES OF FURTHER DEVELOPMENT

In the year 1987 the program system for calculation of exchange power between two cooperating systems has been developed. We intend to make further improvements on this model including the following items:

- To get a more accurate load-model, the daily load curves should be incorporated into the model by taking into account their dependency on the type of the day (first workday after a holiday, general workday, saturday, holiday, etc.).

- The present program works with the probability distribution function of transmission capability between the two systems. It would be desirable to develop a method for the computation of this distribution from the transmission capacities and breakdown probabilities of the individual tie-lines.

- The energy produced by the generating units is decreasing according to the scheduled maintenance. Usually the stochastic analysis of exchange power is performed for long term predictions (5 - 20 years), so the maintenance schedule is not known when the analysis is carried out. Since the capacity-availability of the generating units depends on the selection of generating units having different failure probabilities, it is worthwhile to incorporate a simulation of maintenance scheduling.

The possibilities of development outlined above refer to further developments of the two-point model. We intend to perform further research to develop a stochastic model for the analysis of interconnected systems consisting of more than two systems.

Prekopa [9] developed a general mathematical programming model for capacity expansion planning of transportation type networks. Prekopa and Boros [10] developed an algorithm for the computation of LOLP for a discrete moment in cooperating systems. We will investigate the possibilities of integrating this technique into our model consisting of more than two energy systems.

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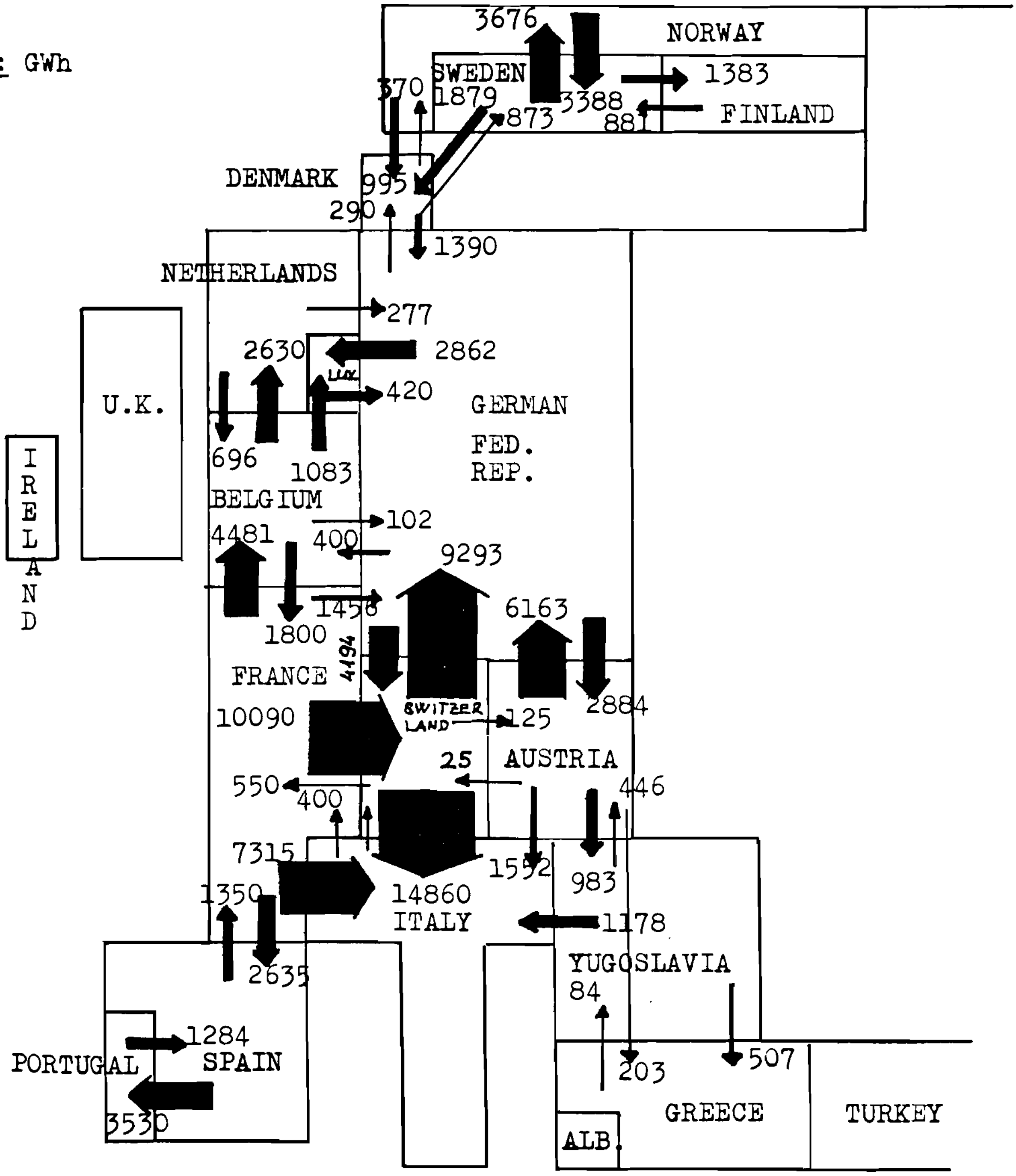

Fig.1. Electric energy exchange among the West-European countries in 1985.

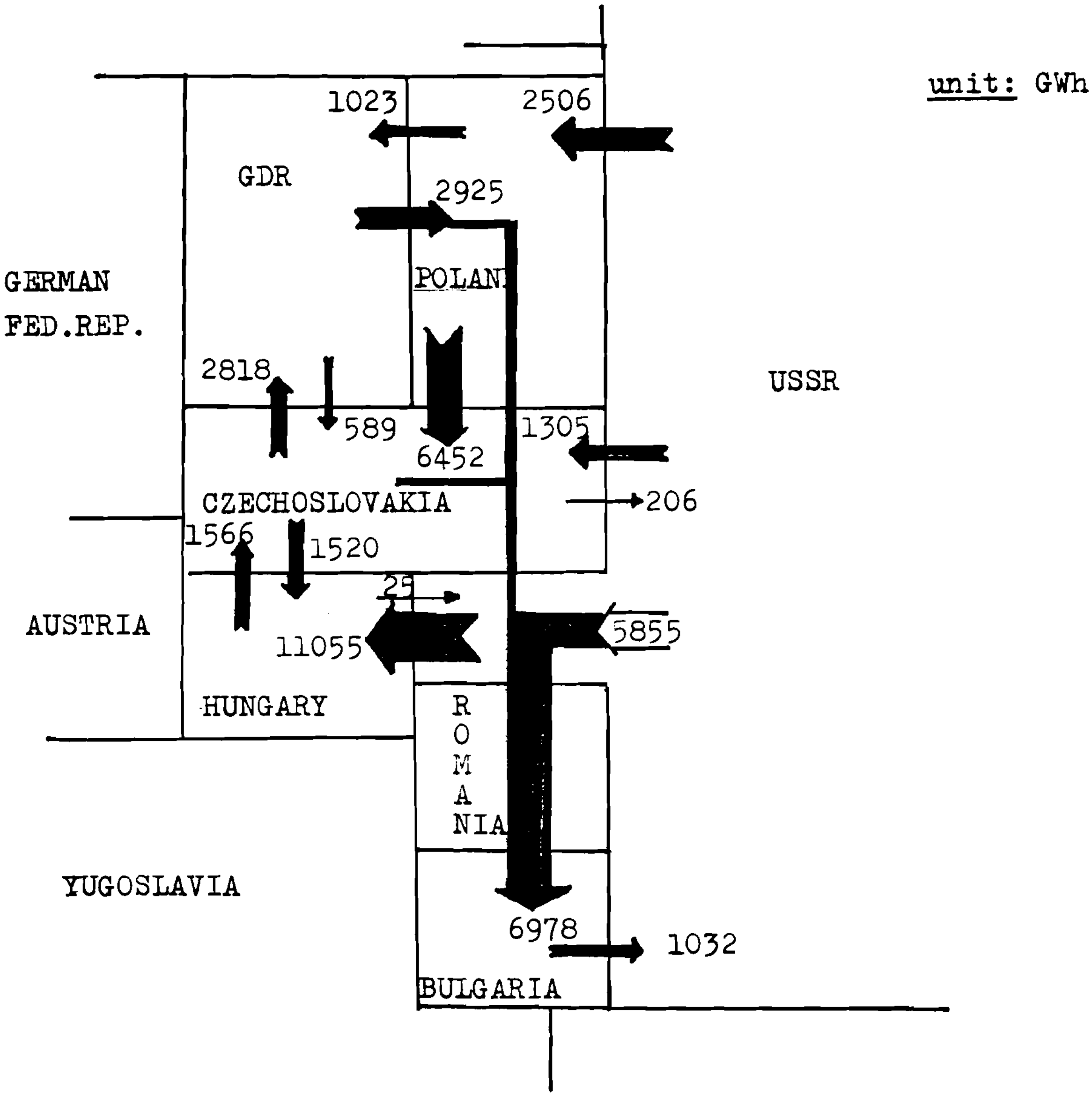

Fig.2. Electric energy exchange among the East-European countries in 1985.

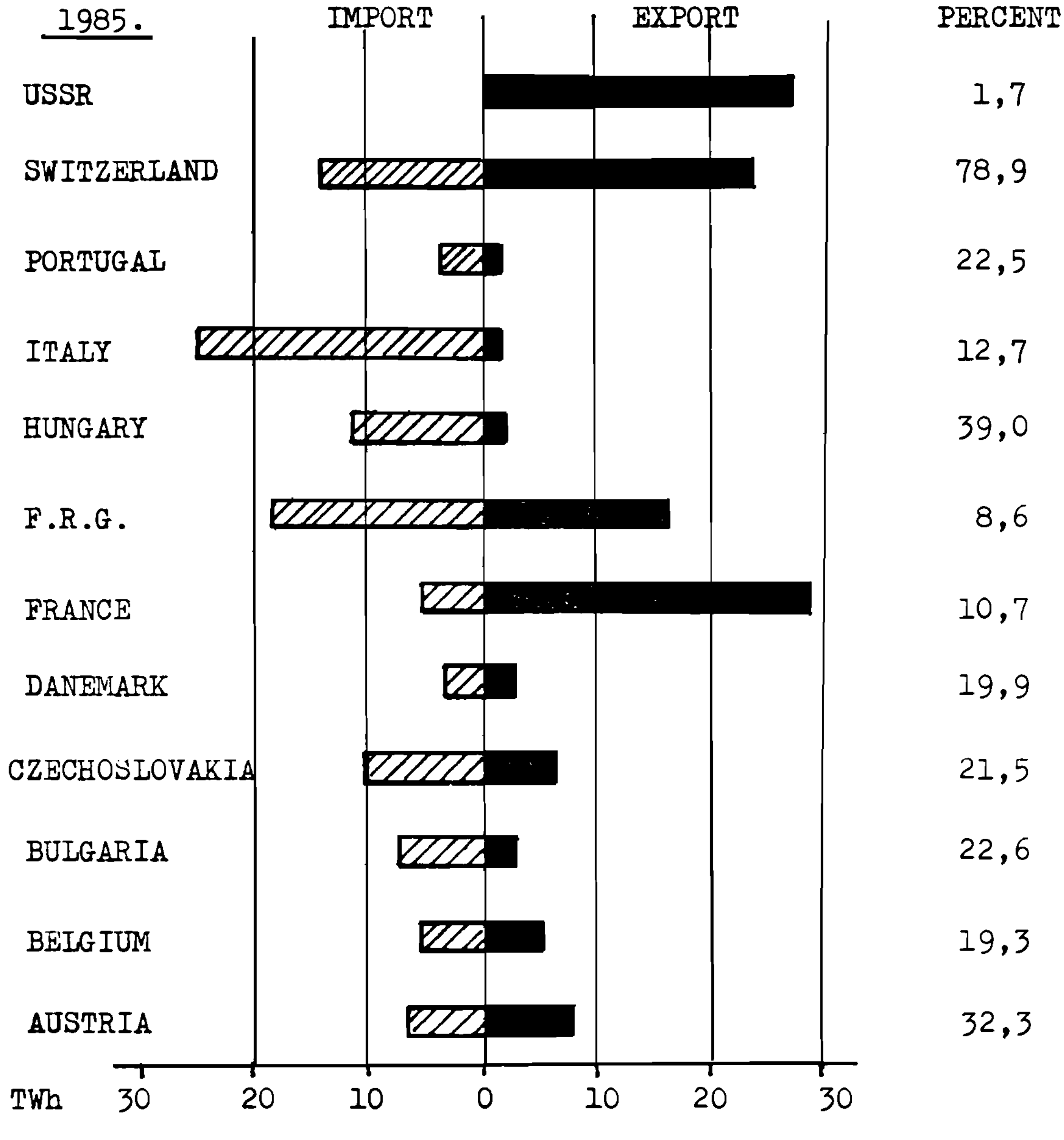

Fig.3. Electric energy exchange in several European countries and its proportion related to gross electric energy consumption.

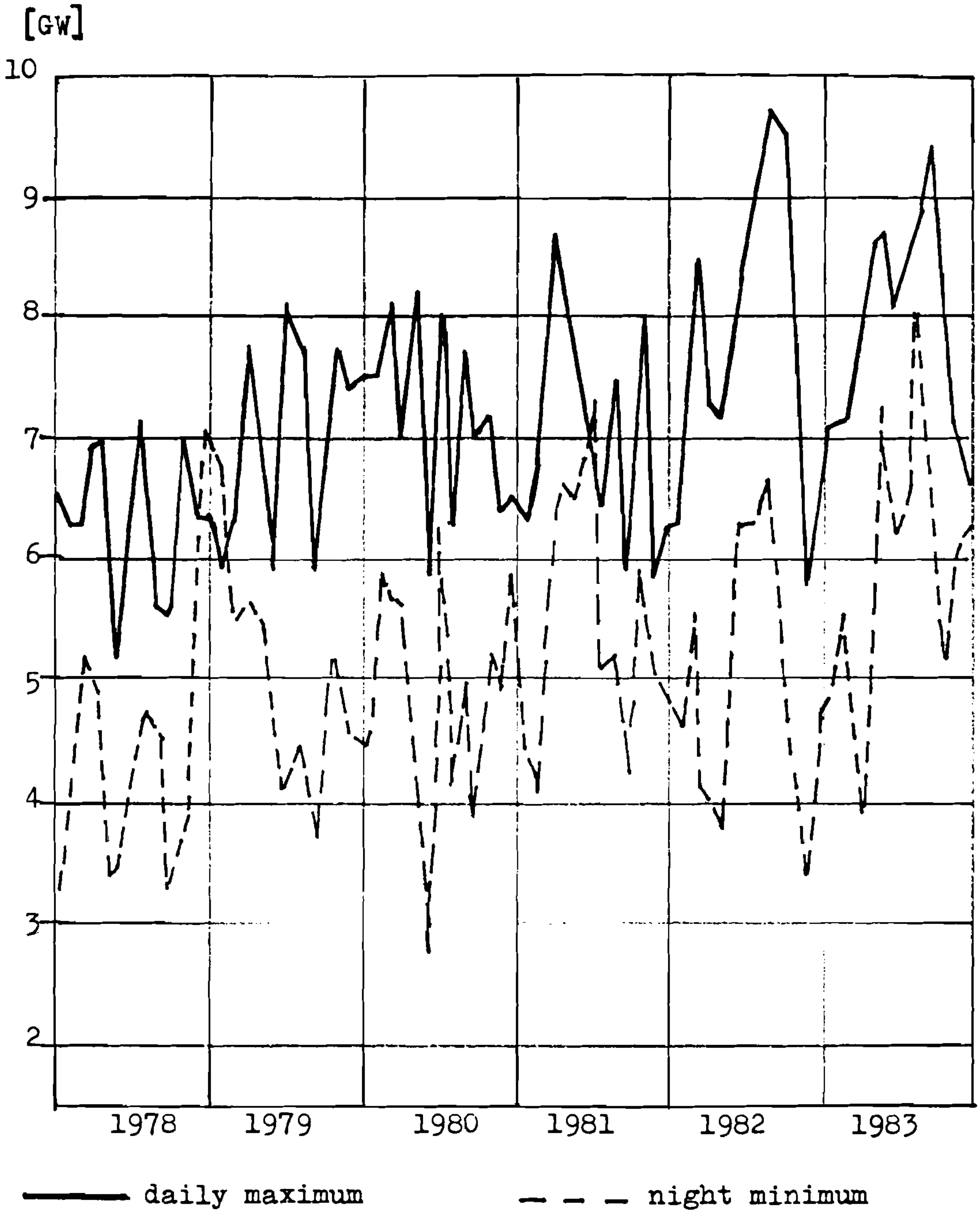

Fig.4. Aggregate sum of simultaneous load of electric power flow across state borders in UCPTE system.

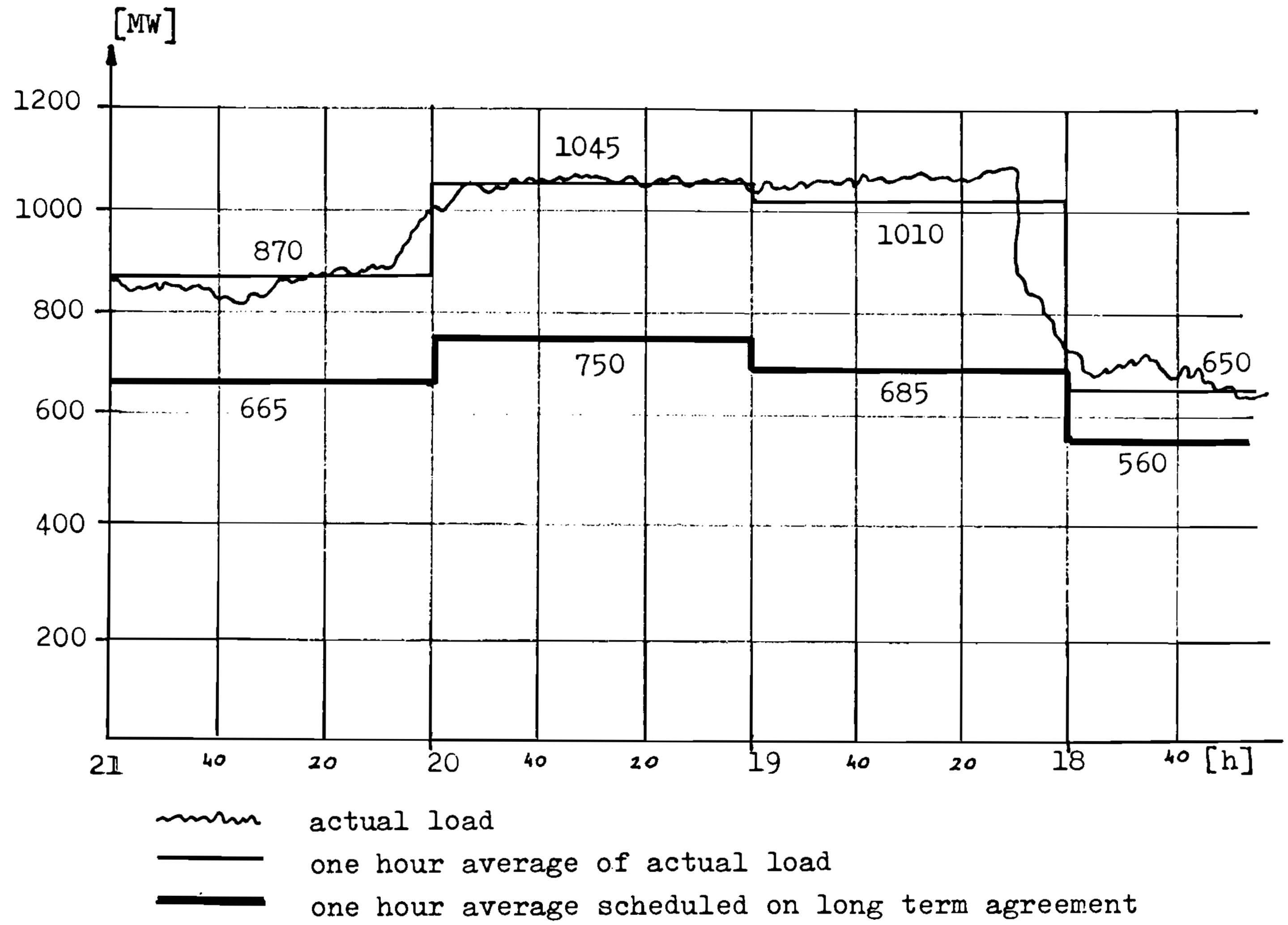

Fig.5. Load on the Hungarian interties
/example selected at random/

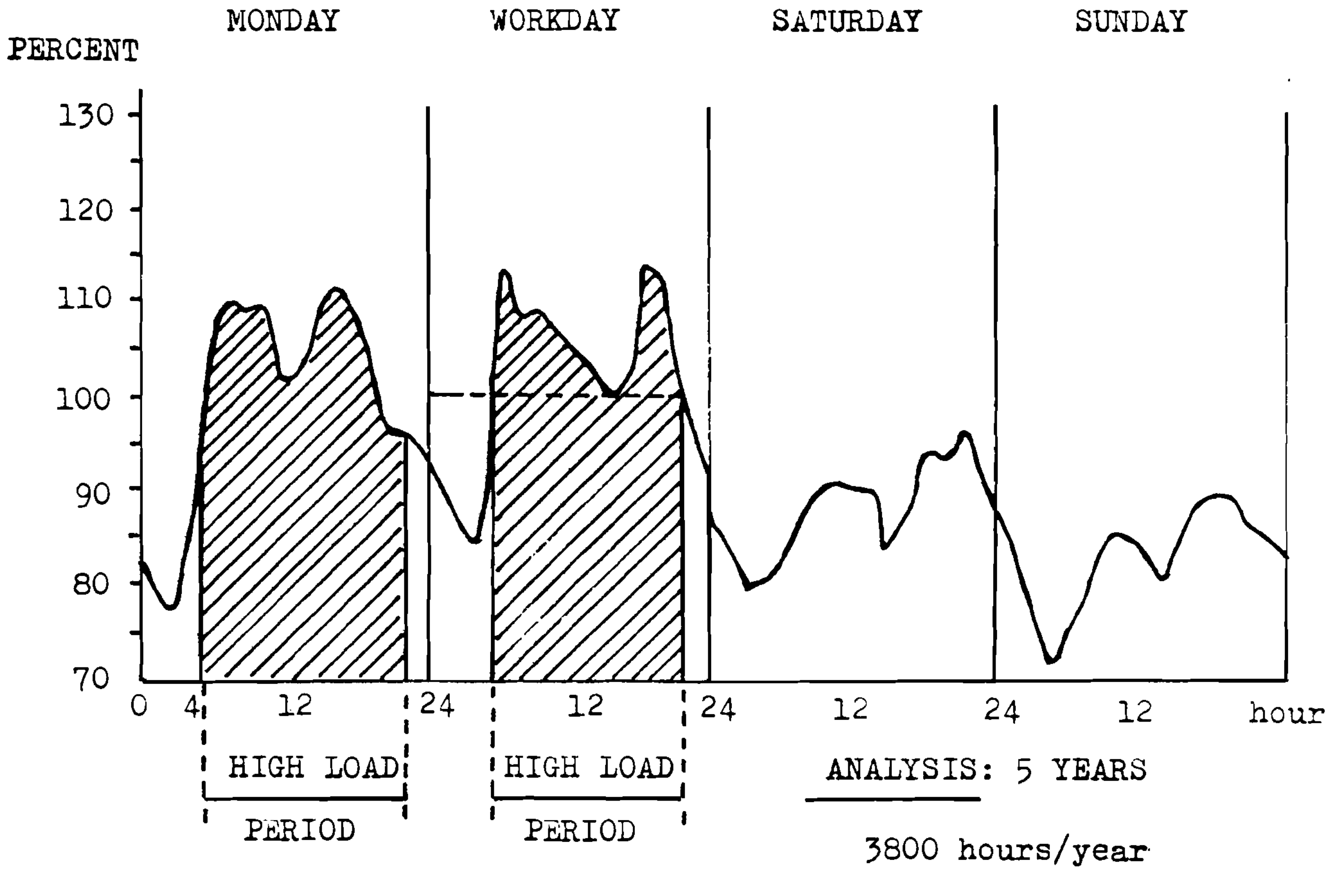

Fig. 6. Load curve in Hungary
december 1986.

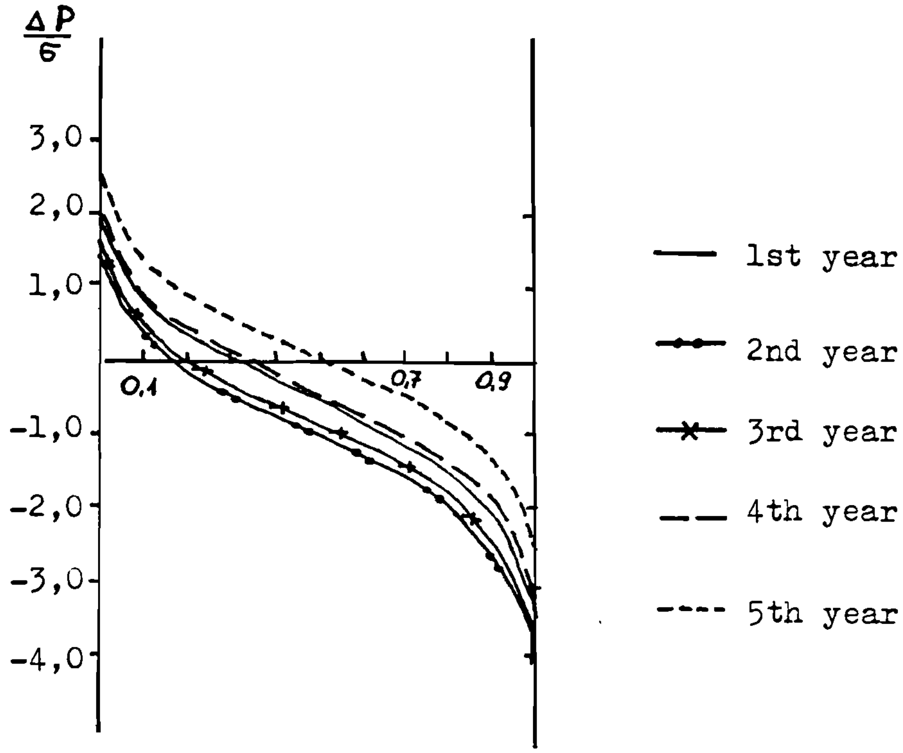

Fig.7. Distribution functions of short-term power exchange.

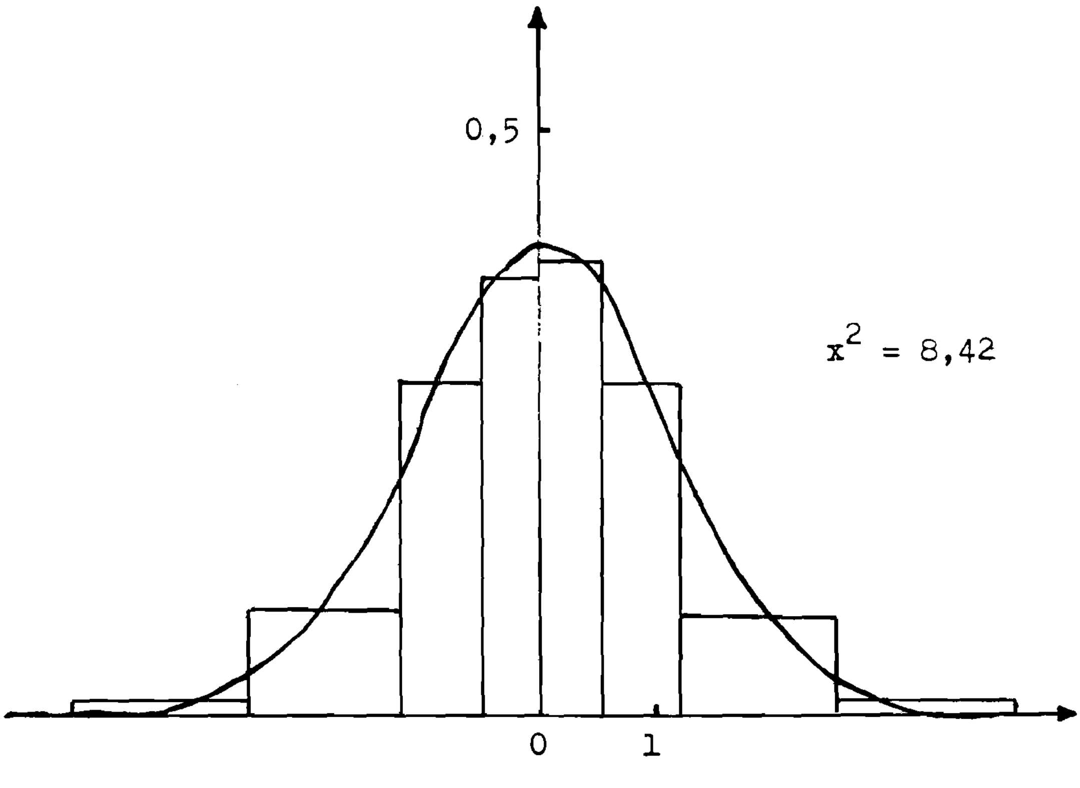

Fig.8. Checking hypothesis for distribution.
Theoretical and empirical density function.

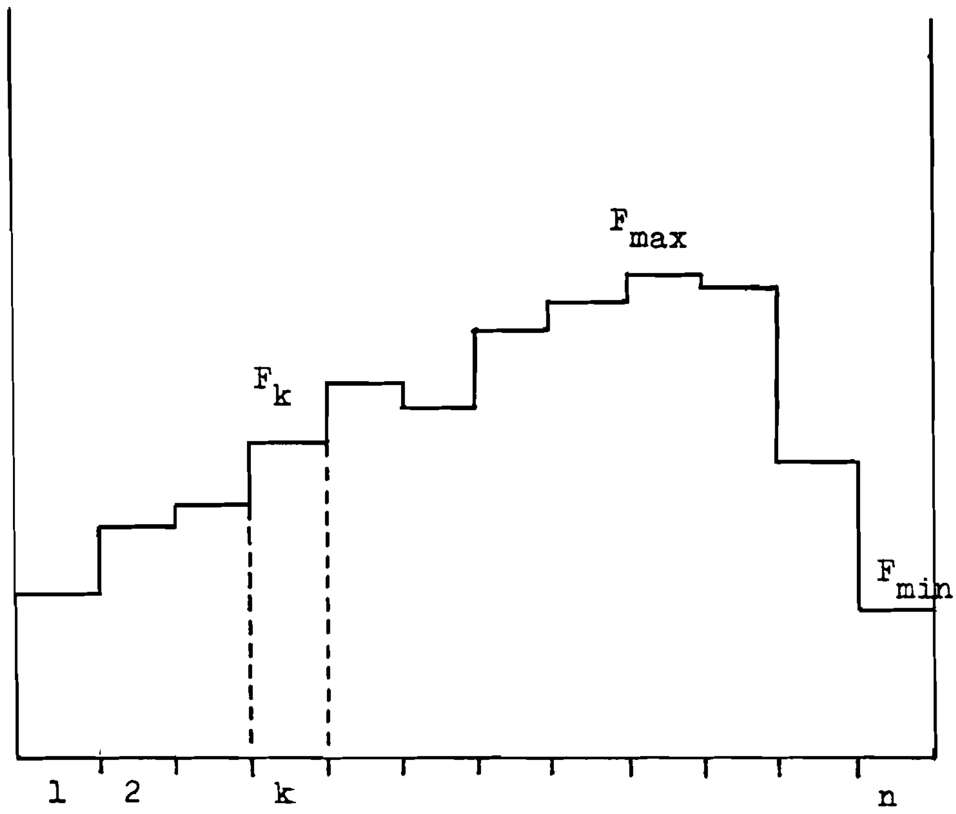

Fig.9. Daily load curve

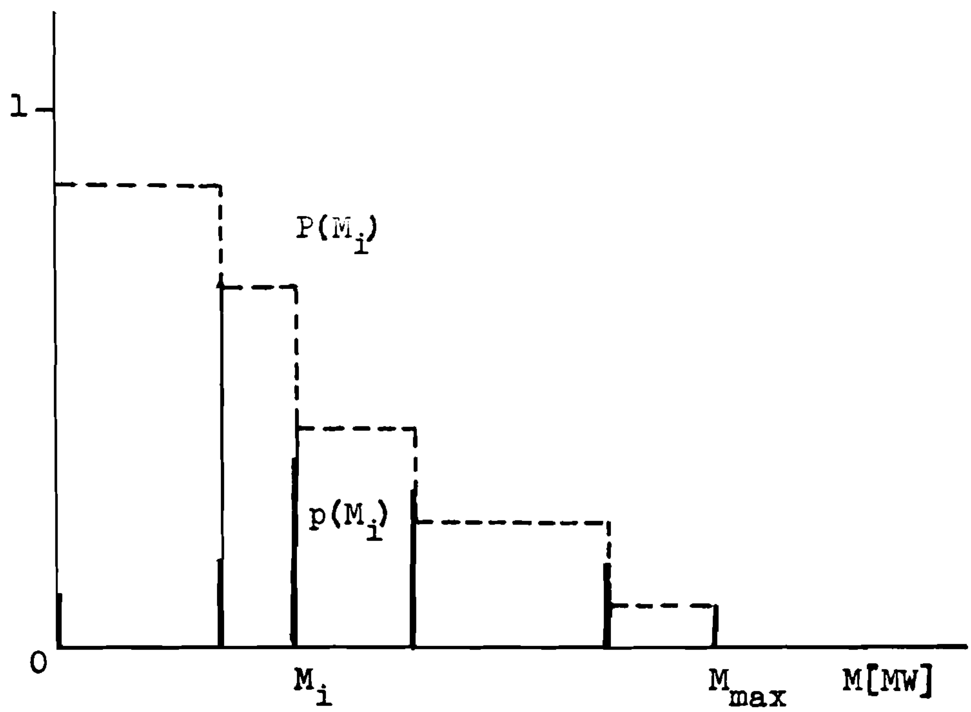

Fig.10. Capacity of a transmission line system

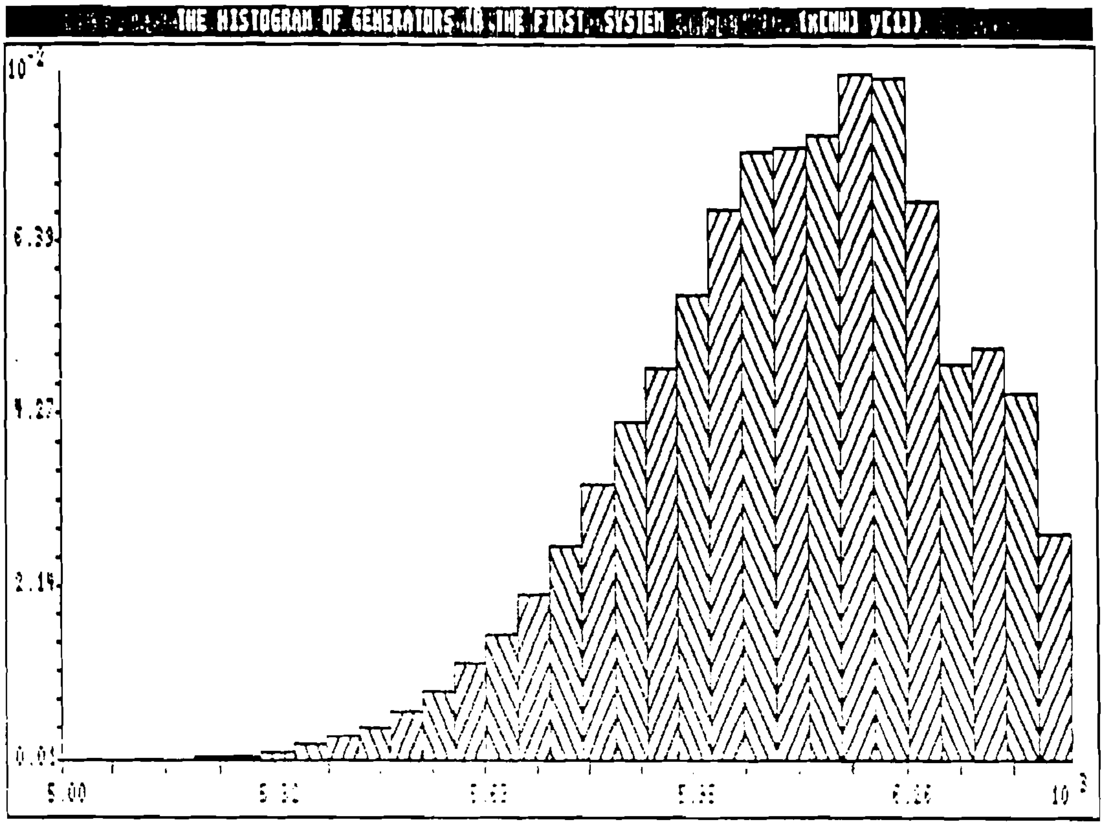

Fig.11. Capacity-availability histogram of generating units.

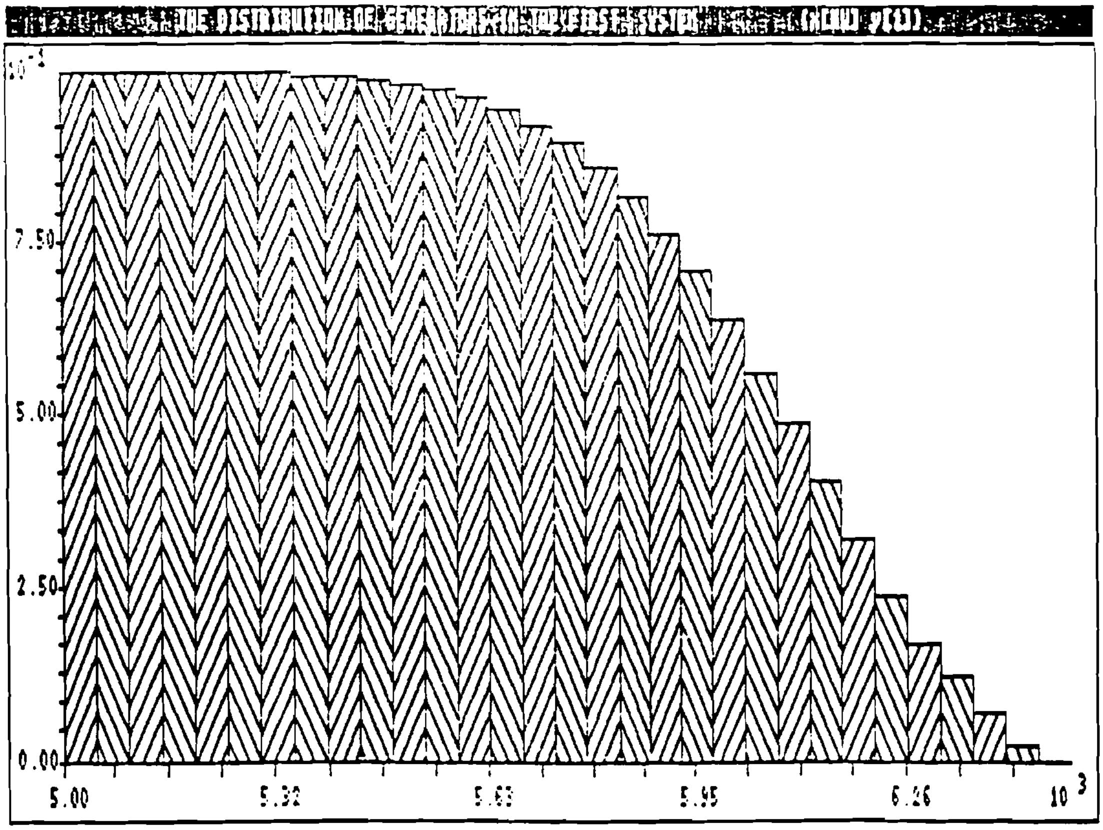

Fig.12. Capacity-availability distribution of generating units.

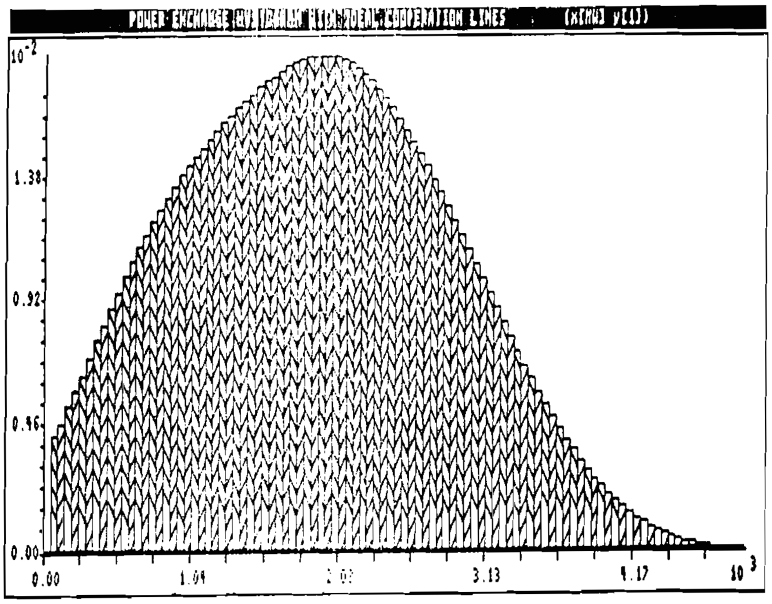

Fig.13/a Demanded exchage of energy

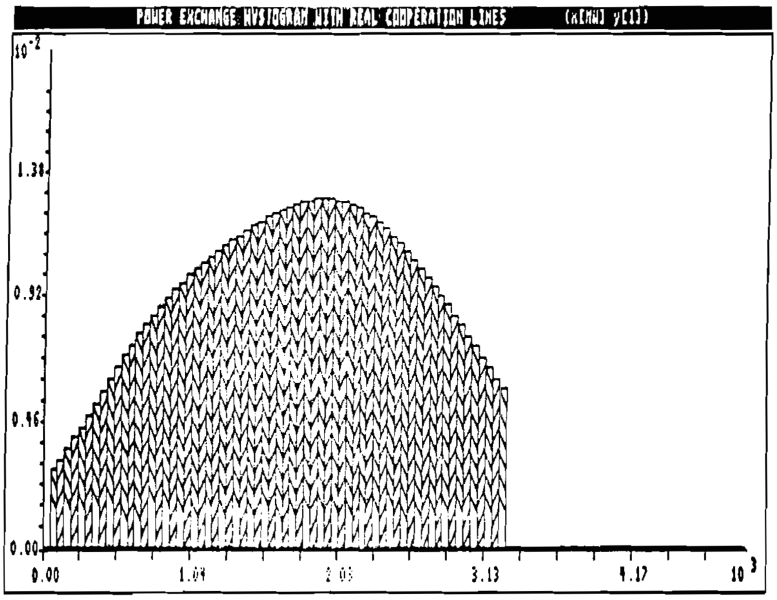

Fig.13/b. Realized exchange of energy

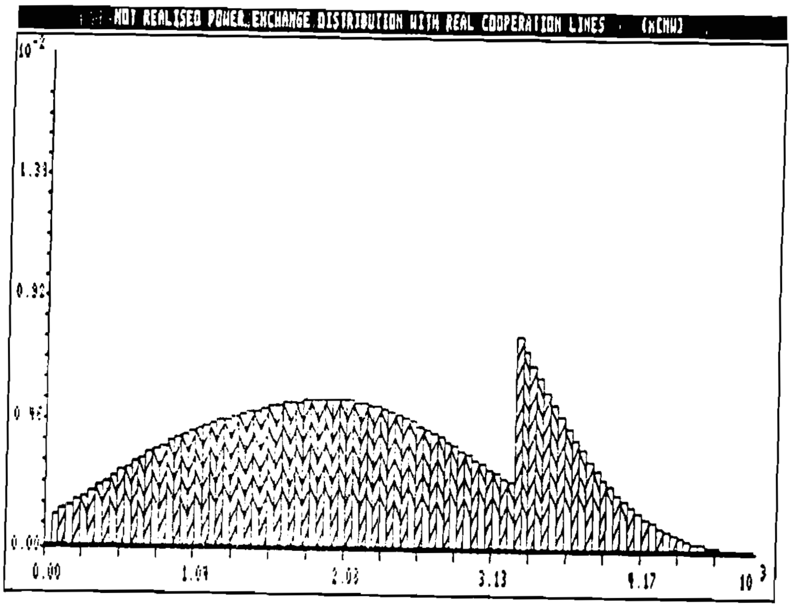

Fig.13/c. Not realized exchange of energy

Fig.13. Energy exchange histograms