

# ***WORKING PAPER***

## **A NONLINEAR MULTICRITERIA MODEL FOR STRATEGIC FMS SELECTION DECISIONS**

*Markku Kuula  
Antonie Stam*

August 1989  
WP-89-62

**A NONLINEAR MULTICRITERIA MODEL FOR  
STRATEGIC FMS SELECTION DECISIONS**

*Markku Kuula* \*  
*Antonie Stam* \*\*

August 1989  
WP-89-62

\* Helsinki School of Economics  
Runeberginkatu 14-16  
SF-00100 Helsinki, Finland

\*\* Department of Management Sciences  
and Information Technology  
College of Business Administration  
University of Georgia  
Athens, Georgia, 30602 U.S.A.

*Working Papers* are interim reports on work of the International Institute for Applied Systems Analysis and have received only limited review. Views or opinions expressed herein do not necessarily represent those of the Institute or of its National Member Organizations.

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS  
A-2361 Laxenburg, Austria

## **FOREWORD**

In recent years, the area of flexible manufacturing has generated considerable interest among practitioners and modelers. The current paper proposes a multicriteria decision support framework to address the important and complicated problem of selecting an appropriate flexible manufacturing system. The paper extends previous research conducted at IIASA which utilized linear simplifications of the model formulation, by considering a more general class of nonlinear models. The DIDAS-N package, which was recently developed in part at IIASA was used to illustrate the framework.

Alexander B. Kurzhanski  
Chairman  
System and Decision Science Program

## ABSTRACT

The strategic decision of selecting an optimal flexible manufacturing system (FMS) configuration is a complicated question which involves evaluating tradeoffs between a number of different, potentially conflicting criteria such as annual production volume, flexibility, production and investment costs, and average throughput of the system. Recently, several structured approaches have been proposed to aid management in the FMS selection process. While acknowledging the nonlinear nature of a number of the relationships in the model, notably between batch size and the number of batches produced of each part, these studies used linear simplifications to illustrate the decision dynamics of the problem. These linear models were shown to offer useful analytical tools in the FMS pre-design process. Due to the nonlinearities of the true relationships, however, the tradeoffs between the criteria could not fully be explored within the linear framework.

This paper builds on the two-phase decision support framework proposed by Stam and Kuula (1989), and uses a modified nonlinear multicriteria formulation to solve the problem. The software used in the illustration can easily be implemented, is user-interactive and menu-driven. The methodology is applied to real data from a Finnish metal product company, and the results are compared with those obtained in previous studies.

## INTRODUCTION

Over the past ten or fifteen years, the concept of building flexible manufacturing systems (FMS) has received increasing interest in the industrial and academic community (Buzacott and Yao 1986; Jaikumar 1986; Ranta, Koskinen and Ollus 1988). The main motivation for switching from a traditional system to an FMS is to introduce a considerable amount of flexibility into the manufacturing and production process, in order to enable the company to more effectively and efficiently compete in the ever more competitive market place.

Manufacturing flexibility can be defined in a number of different ways (Ranta 1989), and at various levels of the organization. At the lowest level, operational flexibility refers to the ability to produce parts in different batch sizes and quantities, while maintaining a flexible schedule which allows for changing routing procedures in the plant. This type of flexibility depends on the characteristics of the specific production system and machinery, and can be realized by acquiring the appropriate machines and production organization. Operational flexibility is a necessary condition for guaranteed short delivery times and customized production. It is also a must for higher levels of flexibility. At the middle level, product and production flexibility allows for rapid introduction of new products and timely modifications of existing products without a need for major changes in the production system (Ranta 1989). At the highest level, flexibility is related to the company's capability to adapt to long-term changes in its industrial environment, necessitating that the total structure of the company is flexible and that long term considerations such as economic risk and the need for adaptation are taken into account when making the investment decisions.

Thus, the problem of selecting the appropriate FMS design is very complex and poses a strategic question which is typically addressed at the highest managerial level of the organization. In this paper, only a part of this comprehensive decision problem is addressed. We assume that management has already made the decision in principle to switch to an FMS, and has gathered general information about the various different available FMS designs. Stam and Kuula (1989) propose a methodology where the decision process is divided into two distinct phases. In the pre-screening phase, a preliminary analysis is performed to narrow the list of candidate configurations to a manageable number of perhaps three or four. At this stage, only rough estimates of costs and benefits are required, and qualitative as well as quantitative evaluation criteria can be used. This is important, because many of the considerations in the pre-screening phase are related to higher level types of flexibility such as long term planning goals of the firm, and these factors are often either of a qualitative nature or difficult to quantify. While this phase is

important, we will not focus on pre-screening the alternatives in this paper. For a detailed description and illustration, the interested reader is referred to Stam and Kuula (1989).

As mentioned above, we assume that the pre-screening phase has resulted in the selection of a few "most attractive" alternative FMS designs. In the second phase proposed by Stam and Kuula (1989), each of the remaining candidate configurations is analyzed in detail, using quantitative criteria, many of which are predominantly related to the lower levels of flexibility of operations and production. The analysis in this paper is concerned with this second phase, therefore does not comprehensively cover all aspects of the FMS selection problem, and should be complemented by other types of analysis, for instance related to financial and organizational feasibility studies and investment risk analyses.

For each of the FMS designs under consideration, a separate multiobjective programming model is formulated with performance and cost characteristics which are specific to the particular configuration. Various scenarios involving different combinations of batch size and number of batches for each part are explored, evaluating their effects on such relevant criteria as total annual production volume, system utilization rate, annual production and investment costs, and several measures of flexibility. The model structure is similar to that suggested by Ranta and Alabian (1988), Ranta (1989) and Stam and Kuula (1989). These studies used a case study based on real data from a Finnish metal product company, and the data of this case will be used in our illustration as well. All of the model formulations proposed in the above studies were nonlinear in nature, in particular the relationship between batch size and the number of batches. The illustrations in all of these studies, however, were simplified to the linear case by either fixing the batch size or the number of batches in the model analysis. In our illustration below, on the other hand, the analysis will be truly nonlinear (in fact bilinear), using the recently developed powerful nonlinear multicriteria mathematical programming package IAC-DIDAS-N (Kreglewski, Paczynski, Granat and Wierzbicki 1988). Thus, in contrast with the previously mentioned studies, in our analysis the tradeoffs of the criteria can fully be explored and explicitly evaluated within the model framework for an infinite number of combinations of batch size and number of batches produced. Our methodology further differs from Ranta and Alabian (1988) in that their study was not based on multicriteria optimization techniques, but rather on a random hitting algorithm which attempts to match specified regions in the decision space and outcome space.

The remainder of the paper is organized as follows. First, the case background is briefly discussed, followed by the mathematical programming formulation of our illustration example. Next, the illustration itself is presented using the configuration data for one particular candidate FMS system, and the results are compared with those obtained in previous studies which have used the same case. The last section of the paper consists of

previous studies which have used the same case. The last section of the paper consists of concluding remarks.

### CASE BACKGROUND AND MODEL FORMULATION

The case study of a Finnish metal producing company which we used has previously been described by Ranta and Alabian (1988) and Ranta (1989). The name of the company is not revealed for reasons of confidentiality. Rather than the complete system, a section of the factory in which 80 different parts are produced is analyzed. The data used in our analysis are the same as those used in the above studies and in Stam and Kuula (1989). The particular FMS design to which the data apply consists of one turning machine, two machine centers and one grinding machine, as well as automatic transportation and warehouses for system integration. Following the forementioned three studies, a subset of 13 representative parts was selected from the family of 80 parts.

The formulation of the model as a multicriteria mathematical programming problem closely follows that by Stam and Kuula (1989) and Ranta (1989). Rather than introducing a general formulation first, as was done in these studies, we immediately introduce the specific model tailored to the data available for our particular application. As mentioned above, the particular FMS configuration we will analyze consists of  $m=4$  machines which are used to produce  $n=13$  different parts.

The decision variables in our problem are the batch size and the number of batches produced annually of part  $i$ , denoted by  $b_i$  and  $v_i$ , respectively, yielding a total annual production volume  $V_i = b_i * v_i$  for part  $i$ , and a total annual production volume of  $V = \sum_i V_i$ . The two major kinds of constraints relate to the scarce resources, time and costs. We discuss these resources next.

#### Time

Let the actual tooling time of part  $i$  on machine  $j$  be  $T_{ij}$  minutes, and the unit overhead time including changing, checking, repairing and waiting  $t_{ij}$  minutes. Thus the total time (in minutes) machine  $j$  is used annually,  $T_j$ , is given by (1):

$$T_j = \sum_i (T_{ij} + t_{ij}) * b_i * v_i, \quad (j = 1, \dots, m) \quad (1)$$

Several recent case studies (Kuivanen, Lepisto and Tinsanen 1988, Lakso 1988, Norros, Toikka and Hyotylainen 1988) have found the machine disturbance time or technical

nonavailability time  $T_{dj}$  to depend on part complexity, the number of batches of each part, the size and complexity of the software needed and a personnel training factor. As indicated by Ranta (1989), the part complexity coefficients ( $d_{ij}^g$ ) and batch number coefficients ( $d_{ij}^b$ ) of the time components of the disturbance time in general will be machine-dependent. In our application, however, this was not the case, and the coefficients were equal across parts, so that we use  $d_g = d_{ij}^g$  and  $d_b = d_{ij}^b$ . Similarly, the software complexity scaling coefficients  $d_s^s$  and training factor  $d_{pl}^{pl}$  are equal across machines, so that  $d_s = d_j^s$  and  $d_{pl} = d_j^{pl}$  for all  $j$ . Moreover, all four machines have approximately the same software complexity  $S$ .  $T_{dj}$  can then be expressed as follows:

$$T_{dj} = d_g * \sum_i g_i + d_b * \sum_i v_i + d_s * S - d_{pl} * PL \quad (j = 1, \dots, m) \quad (2)$$

Denoting the minimum required and maximum possible number of minutes of operation of machine  $j$  by  $T_{jMIN}$  and  $T_{jMAX}$ , respectively, then using (1) and (2) the following holds:

$$T_{jMIN} \leq T_j + T_{dj} \leq T_{jMAX} \quad (j = 1, \dots, m) \quad (3)$$

Equation (3) can be viewed as a measure of utilization of machine  $j$ . For the system as a whole, the utilization constraint (4) includes a batch change time of  $r_i$  minutes for part  $i$ , so that the total batch change time equals  $T_b = \sum_i r_i * v_i$ , so that

$$T_{MIN} \leq T + T_d + T_b \leq T_{MAX} \quad (4)$$

where  $T = \sum_j T_j$ ,  $T_d = \sum_j T_{dj}$ , and  $T_{MIN}$  and  $T_{MAX}$  are the lower and upper bounds on the utilization of the system. In our application, no lower bounds for the system and machine utilization were used.

### Costs

All cost figures are in U.S. dollars. The total annual costs of the FMS,  $C$ , divide into machine costs ( $C_M$ ), tool costs ( $C_L$ ), parts pallet costs ( $C_P$ ), software costs ( $C_S$ ), transportation costs ( $C_T$ ) and other costs ( $C_O$ ). Thus,  $C$  can be written as (5),

$$C = C_M + C_L + C_P + C_S + C_T + C_O \quad (5)$$

Assuming only the direct investment costs are included in the machine costs, and adjusting these costs by discounting and pro-rating them over the planned lifetime of the machine,  $C_M$  can be expressed as (6),

$$C_M = \sum_j e_j * M_j, \quad (6)$$

where  $M_j$  represents the adjusted direct investment costs of machine  $j$  per unit produced, and  $e_j$  is the relative efficiency of machine  $j$ .

The tool costs depend on the complexity of the parts and the number of tools needed, so that (7) follows,

$$C_L = q_g * \sum_i g_i + q_l * \sum_i L_i, \quad (7)$$

where  $g_i$  is the complexity of part  $i$ , as measured by the form of the part, precision and other characteristics,  $L_i$  is the number of parts needed to produce part  $i$ , while  $q_g$  and  $q_l$  are scaling coefficients.

The parts pallet costs depend on part complexity, batch size and the number of batches produced annually of each part:

$$C_P = p_g * \sum_i g_i + p_b * \sum_i b_i + p_v * \sum_i v_i, \quad (8)$$

where  $p_g$ ,  $p_b$  and  $p_v$  are scalar values.

Software costs have been shown to depend on numerical control (NC)-programs, scheduling and communication algorithms, and on the amount of interfaces needed (Ranta 1989). Thus,  $C_S$  can be written as in (9),

$$C_S = s_g * \sum_i g_i + (s + s_v) * \sum_i v_i + s_l * \sum_i L_i + s_e * \sum_j e_j, \quad (9)$$

where  $s_g$ ,  $s$ ,  $s_v$ ,  $s_l$  and  $s_e$  are appropriate constant coefficients. In their order of appearance in (9), the terms refer to software complexity, the annual number of batches produced, tool management and machine efficiency.

Data on internal transportation costs were not available for our case study, and are not included in our analysis. Finally, the remaining costs  $C_0$  consist of personnel training costs  $C_{TR}$ , which depend on the number of employees to be trained and on residual costs  $C_{RES}$ , and can be represented as (10),

$$C_D = C_{TR} + C_{RES} = c_{PL} * PL + C_{RES} \quad (10)$$

where  $c_{PL}$  is the average annual training cost per employee.

#### Other Constraints

Due to economic considerations of demand and supply, minimum ( $V_{iMIN}$ ) and maximum ( $V_{iMAX}$ ) levels were established for the annual quantity ( $V_i$ ) produced of part  $i$ , as formulated in (11),

$$V_{iMIN} \leq V_i \leq V_{iMAX} \quad (11)$$

#### Objectives

A number of relevant criteria can be used to evaluate the costs and benefits of a proposed FMS configuration (see e.g., Ranta and Alabian 1988, Ranta 1989, and Stam and Kuula 1989). One important performance measure is the total annual production volume of the system. Ranta (1989) suggests weighting the contribution to the company of producing one unit of part  $i$  by a relative importance coefficient  $w_i$ , so that the criterion of maximizing weighted annual production is given by (12):

$$\text{maximize WEIGHTED\_PRODUCTION} = \sum_i w_i * b_i * v_i \quad (12)$$

If all weights  $w_i$  equal unity, then (12) reduces to maximizing the total physical production volume, counting a unit of each part equally. Stam and Kuula (1989) note that the linear combination of production quantities in expression (12) may not facilitate a meaningful interpretation. Rather, if the part family can be partitioned into  $k$  different groups  $G_1, \dots, G_k$  which internally have reasonably homogeneous characteristics, in particular in terms of batch size and part complexity, then it may be more relevant to consider tradeoffs between total production quantities of these  $k$  groups. Such a set of objectives is represented by (13):

$$\text{maximize PRODUCTION\_GROUP\_h} = \sum_{i \in G_h} b_i * v_i \quad (h = 1, \dots, k) \quad (13)$$

For instance, as we will see below, in the second part of our illustration the 13 parts under consideration can be aggregated into three different groups. A second criterion is to minimize the total costs  $C$ :

$$\text{minimize } C \quad (14)$$

where  $C$  is defined in (5) above. Another performance criterion of interest is the system utilization rate, expressed as the ratio of the time during which the machines are actively producing ( $T$ ) to the physical maximum annual production time ( $T_{MAX}$ ), multiplied by 100,

$$\text{maximize UTILIZATION\_RATE} = 100 * T / T_{MAX} \quad (15)$$

Of course utilization rates can be formulated for each machine separately as well, but in our illustration this was not done. An alternative measure of system utilization including disturbance time ( $T_d$ ) and batch change time ( $T_b$ ) could have been used instead of (15), but was not.

System flexibility can be represented in a number of different ways, for instance by part complexity  $g_i$  as measured by the number of facets of the part, the precision needed in machining the part and other factors, by the number of tools needed to produce a part, and the average batch size. These measures of flexibility are given in criteria (16), (17) and (18) below:

$$\text{maximize FLEXIBILITY\_1} = f_g * \sum_i g_i * b_i * v_i \quad (16)$$

$$\text{maximize FLEXIBILITY\_2} = \sum_i L_i * b_i * v_i \quad (17)$$

$$\text{maximize FLEXIBILITY\_3} = -\sum_i b_i / n \quad (18)$$

The coefficient  $f_g$  in (16) is scalar-valued. The negative sign on the right-hand side of (18) is due to the fact that smaller batch sizes imply a higher flexibility.

## ILLUSTRATION

The illustration consists of two separate model formulations, both of which were analyzed using the nonlinear multicriteria software package IAC-DIDAS-N 3.2 (Kreglewski, Paczynski, Granat and Wierzbicki 1988), also known as DIDAS-N. This package will run on IBM/PC/XT/AT and compatible microcomputers, uses a convenient spreadsheet format, and facilitates an interactive solution process based on the reference point method (Wierzbicki 1982, Lewandowski and Wierzbicki 1988). The interactive methodology underlying DIDAS-N uses the concepts of satisficing solutions and bounded rationality (March and Simon 1958), and has been shown to be consistent with the process of human decision making. For a detailed discussion of various aspects of the reference point method the interested reader is referred to Lewandowski and Wierzbicki (1988).

In the interactive solution process, the decision maker specifies aspiration and reservation levels for each of the criteria. The aspiration level of a criterion represents the level which the decision maker would like to achieve, if possible, while the reservation level is the worst level which would be acceptable to the decision maker. DIDAS-N uses the specified aspiration and reservation levels as the basis for solving a multicriteria optimization problem to find a Pareto optimal or nondominated solution which reaches the aspiration levels as closely as possible (using the Tchebycheff norm), while satisfying the reservation levels for the criteria. A solution is said to be Pareto optimal if none of the criteria can be improved without sacrificing at least one of the remaining criteria.

The solution which results is presented to the decision maker, who can subsequently modify the aspiration and reservation levels according to his preferences and the information contained in the solution. For instance, if the aspiration levels are uniformly exceeded in the solution presented by DIDAS-N, the decision maker can obtain solutions which are better than he had anticipated, and may want to raise his expectations by selecting higher aspiration levels. On the other hand, if the reservation levels of some criteria are too tight and unattainable, the decision maker may choose to relax at least some of these levels. DIDAS-N will then propose a revised solution based on the modified aspiration and reservation levels. In this way the decision maker is able to interactively explore various types of tradeoffs between the criteria. At any point of the analysis he can inspect and evaluate the relevant decision variables and constraints on the screen. It is also possible to graphically display the tradeoffs between the criteria in the form of bargraphs.

The first model formulation is Problem 1, and uses the same three criteria as Stam and Kuula (1989): maximize the total production volume in equation (12) with equal weights ( $w_i = 1$ ), minimize the total costs (14), and maximize flexibility as measured by (16). Problem 1 was analyzed because it provides a direct comparison with previous results obtained for the simplified linear formulation of Stam and Kuula (1989) in which the batch sizes were fixed to 5 for all parts. The formulation in Problem 1 is not realistic, and as will be discussed below, a more detailed set of criteria was used in Problem 2. For practical reasons, in both problems an upper bound of 20 was imposed on the batch size of each part.

Tables 1 and 2 contain the data for our illustration. The first column of Table 1 is the index for the parts, followed by the previously defined parameters related to the minimum and maximum production volumes for each part, the complexity coefficients, machining and overhead times, batch change times and the number of tools needed for the production of each part. A concise definition of all parameters is given in Appendix B.

**Table 1.** Part family, maximum and minimum production boundaries, part complexity, tooling an overhead times, batch change times and numbers of tools needed in production

$i$	$V_{iMIN}$	$V_{iMAX}$	$g_i$	$T_{i1}$	$t_{i1}$	$T_{i2}$	$t_{i2}$	$T_{i3}$	$t_{i3}$	$T_{i4}$	$t_{i4}$	$r_i$	$L_i$
1	500	700	4	20	2.0	20	2.0	20	2.0	8	4.0	4.0	50
2	2000	2500	2	12	1.6	6	1.2	6	1.2	4	2.0	2.0	50
3	1500	2000	3	20	2.0	14	2.0	14	2.0	8	4.0	4.0	50
4	1500	2000	4	20	2.0	20	2.0	20	2.0	8	4.0	4.0	50
5	1000	1200	4	40	1.2	10	1.2	10	1.2	8	4.0	4.0	50
6	100	300	6	20	1.6	20	2.0	40	2.0	20	4.8	4.8	50
7	200	300	8	40	2.0	40	2.4	60	2.4	40	6.0	6.0	50
8	3000	3500	2	12	1.6	6	1.2	6	1.2	4	2.0	2.0	50
9	3000	3500	2	12	1.6	6	1.2	6	1.2	4	2.0	2.0	50
10	1500	2000	3	12	0.8	8	0.8	8	0.8	8	2.0	2.0	50
11	200	300	9	48	4.0	60	4.0	60	4.0	80	6.0	6.0	100
12	150	250	10	60	5.0	45	5.0	45	5.0	80	6.0	6.0	100
13	100	200	10	0	0.0	40	5.0	60	5.5	50	8.0	8.0	100

Table 2 provides the remaining coefficients related to disturbance time, time constraints, cost, flexibility and efficiency parameters.

**Table 2.** Disturbance coefficients and time constraints, cost and flexibility coefficients and efficiency coefficients

$d^b$	$d^g$	$d^s$	$d^{PL}$	$s$	$PL$	$T_{jMAX}$	$s_g$	$s$	$s_v$	$s_e$	$h$	$p_g$	$p_b$	$p_v$	$M_j$	$q_g$	$q$	$c_{PL}$	$f_g$	$e_1$	$e_2$	$e_3$	$e_4$
3	40	0.05	3	1	100	316800	0.5	10	20	2	3	10	3	200	100	5	10	100	10	3	3	3	6

### Problem 1

First we discuss Problem 1. Initially the utopia and nadir values for each criterion were calculated. The utopia value or selfish solution of a criterion is the best possible value for this criterion if all other criteria are ignored. The nadir value of a criterion is its worst possible value over the set of efficient solutions. As in general the nadir values are very difficult to compute, DIDAS-N approximates them by the worst values found among all solutions calculated. Since the different criteria are conflicting, the utopia values for all criteria combined can usually not be attained. The utopia and nadir values are important because these provide the decision maker with valuable information about the relevant ranges of the objective functions. Next, DIDAS-N determines a "neutral" solution, representing an initial suggested solution which is used to start the interactive decision process. The utopia and nadir values as well as the initial solution are given in Table 3.

**Table 3: Utopia, Nadir Values and Initial Solution for the Three Criteria Problem**

Criterion	Production (Max)	Cost (Min)	Flexibility (Max)
Utopia Value	17,672	189,709	535,479
Nadir Value	13,325	1,497,770	392,250
Initial Solution	17,257	497,722	506,520

DIDAS-N also suggests modified aspiration and reservation levels based on the initial "neutral" solution. If the decision maker wishes to explore the dynamics of the tradeoffs between the criteria, he can adjust the suggested aspiration and reservation levels, after which DIDAS-N re-solves the problem and presents a new solution. The information is presented in the format of Table 4, providing the utopia and nadir values, the current solution, and the associated suggested aspiration and reservation levels. Of course, the current solution in Table 4 is also the initial neutral solution in our case, because no other solutions have been calculated as of yet.

**Table 4: Utopia, Nadir Values and Aspiration and Reservation Values for the Current (Initial) Solution, just prior to Calculating Solution 2, for the Three Criteria Problem (Problem 1)**

Criterion	Production (Max)	Cost (Min)	Flexibility (Max)
Utopia Value	17,672	189,709	535,479
Aspiration Level	17,370	395,032	513,352
Current Solution	17,257	497,722	506,520
Reservation Level	17,143	600,412	499,687
Nadir Value	13,325	1,497,770	392,250

Suppose the decision maker judges the cost level of \$497,722 in the initial solution of Table 4 to be too high, and wishes to emphasize the cost minimization criterion by tightening the aspiration and reservation levels to \$200,000 and 250,000, respectively. Note that the modified aspiration level is lower than the suggested level of \$395,032 in Table 4, because the cost criterion is a minimization criterion. The decision maker is willing to lower the

reservation levels for production and flexibility from 17,143 units and 499,687 to 17,000 and 450,000, respectively. The resulting nondominated Solution 2 in Table 5 has a much lower cost (\$216,904) than the initial solution, but the production volume and flexibility are lower as well.

**Table 5: Selected Results of the Interactive Decision Process for the Three Criteria Problem (Problem 1)**

Criterion	Solution			
	Initial	2	3	4
Production	17,488	17,051	14,750	16,363
Cost	497,722	216,904	189,709	258,741
Flexibility	506,520	465,322	435,000	511,267
Machine Times				
T <sub>1</sub>	313,043	316,792	269,110	301,974
T <sub>2</sub>	214,987	231,482	185,480	223,898
T <sub>3</sub>	224,410	247,120	193,530	238,483
T <sub>4</sub>	188,775	212,712	164,580	204,584
Production Volumes				
v <sub>1</sub>	634.7	684.7	500.0	631.8
v <sub>2</sub>	2,445.3	2,211.0	2,000.0	2,194.9
v <sub>3</sub>	1,803.9	1,737.5	1,500.0	1,725.1
v <sub>4</sub>	1,793.7	1,763.9	1,500.0	1,900.0
v <sub>5</sub>	1,189.3	1,169.1	1,000.0	1,034.9
v <sub>6</sub>	125.6	285.6	100.0	238.5
v <sub>7</sub>	231.9	292.7	200.0	286.3
v <sub>8</sub>	3,500.0	3,211.0	3,000.0	3,021.0
v <sub>9</sub>	3,500.0	3,211.0	3,000.0	3,021.0
v <sub>10</sub>	1,787.8	1,737.5	1,500.0	1,567.3
v <sub>11</sub>	214.8	299.4	200.0	300.0
v <sub>12</sub>	150.2	249.5	150.0	243.0
v <sub>13</sub>	110.9	198.7	100.0	199.5
Batch Information				
(b <sub>1</sub> , v <sub>1</sub> )	(11.7, 54.5)	(19.6, 34.9)	(20.0, 25.0)	(12.1, 52.1)
(b <sub>2</sub> , v <sub>2</sub> )	(13.8, 176.7)	(20.0, 110.5)	(20.0, 100.0)	(17.0, 129.2)
(b <sub>3</sub> , v <sub>3</sub> )	(12.5, 130.4)	(20.0, 86.9)	(20.0, 75.0)	(15.9, 101.6)
(b <sub>4</sub> , v <sub>4</sub> )	(15.4, 116.4)	(20.0, 88.2)	(20.0, 75.0)	(18.4, 103.3)
(b <sub>5</sub> , v <sub>5</sub> )	(12.9, 91.9)	(19.5, 59.9)	(20.0, 50.0)	(14.1, 73.6)
(b <sub>6</sub> , v <sub>6</sub> )	(11.3, 11.1)	(19.6, 14.5)	(20.0, 50.0)	(9.8, 24.4)
(b <sub>7</sub> , v <sub>7</sub> )	(12.8, 18.1)	(18.9, 15.5)	(20.0, 10.0)	(10.4, 27.6)
(b <sub>8</sub> , v <sub>8</sub> )	(5.4, 649.3)	(20.0, 160.5)	(20.0, 150.0)	(17.1, 176.4)
(b <sub>9</sub> , v <sub>9</sub> )	(5.3, 649.2)	(20.0, 160.5)	(20.0, 150.0)	(17.1, 176.4)
(b <sub>10</sub> , v <sub>10</sub> )	(12.5, 142.9)	(20.0, 86.9)	(20.0, 75.0)	(14.6, 107.4)
(b <sub>11</sub> , v <sub>11</sub> )	(12.9, 16.6)	(20.0, 15.0)	(20.0, 10.0)	(12.1, 24.7)
(b <sub>12</sub> , v <sub>12</sub> )	(13.6, 11.1)	(20.0, 12.5)	(20.0, 7.5)	(10.9, 22.2)
(b <sub>13</sub> , v <sub>13</sub> )	(11.1, 10.0)	(20.0, 10.0)	(20.0, 5.0)	(10.1, 19.8)

Further emphasis on the cost criterion at the expense of production and flexibility leads to Solution 3, where cost is at its utopia value and the other criteria are at a rather low level. Note that in Solution 3 the batch sizes are at their highest level, and that the production volumes are at their lower bounds. This was to be expected because it is less expensive to produce in large batches, and to produce as few units total as possible. Even though the decision maker will like the low cost associated with Solution 3, he may want to achieve a better production volume and flexibility. Increasing the aspiration and reservation levels for these criteria while relaxing these levels for cost leads to Solution 4. In this solution, flexibility is improved from 435,000 to 511,267, and production from 14,750 to 16,363 units, in exchange for a cost increase of \$ 41,737 from \$ 216,904 to \$ 258,741. The tradeoffs between criteria and the differences between the various solutions in terms of their criteria levels can also be depicted graphically as in Figure 1.

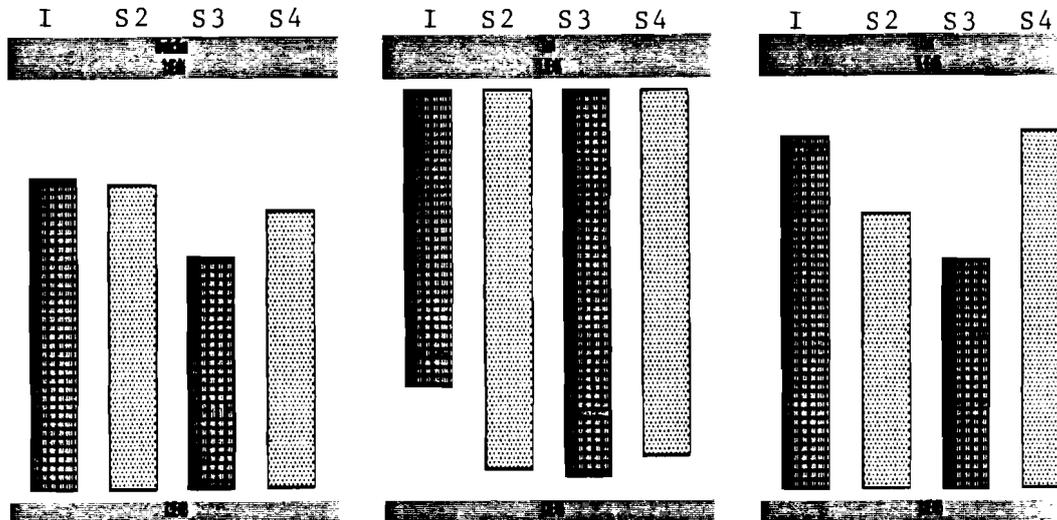


Figure 1: Graphical Display of the Tradeoffs for Selected Results, for the Three Criteria Problem (Problem 1)

In Figure 1, the initial "neutral" solution is indicated by I, and Solution 2, 3 and 4 by S<sub>2</sub>, S<sub>3</sub> and S<sub>4</sub>, respectively. The first and third criteria, production and flexibility, are to be maximized, and the height of the vertical bars indicates their levels. Cost is a minimization criterion, and its value is given by the distance between the vertical bar and the horizontal line at the bottom of the picture. For instance, Solution 3 (S<sub>3</sub>) has the lowest cost level, but also the lowest production volume and flexibility.

Summarizing the illustration of Problem 1, we see that the decision maker can evaluate a variety of Pareto optimal tradeoffs between the criteria by varying their aspiration and reservation levels, enabling him to better understand the dynamics of the multicriteria problem. In the solutions presented in Table 5, most of the batch sizes are relatively large. This is reasonable given the objectives used in the formulation of Problem 1, because on the one hand larger batch sizes imply lower production costs, and on the other hand the fact that small batch sizes reflect a higher flexibility is not explicitly included in the flexibility criterion. In the analysis of Problem 2 which follows below, batch size is explicitly included in (18) as one of the measures of flexibility, and as will be seen, solutions with lower batch sizes will result.

The second illustration is called Problem 2, and represents a more realistic extension of Problem 1. The 13 parts can be partitioned into three groups with relatively homogeneous characteristics. For group 1, consisting of parts 2, 8, 9 and 10, the most likely batch size in practice is between 15 and 20. Group 2 includes parts 1, 3, 4 and 5. These parts are typically produced in batches of 10 to 15 units. Group 3 contains the remaining parts 6, 7, 11, 12 and 13, for which the typical batch size is less than 10. Since each group represents parts of a different nature, it is meaningful to maximize the production volume of each group separately. As the composite measure of flexibility used in Problem 1 does not capture some important aspects of flexibility, the three different measures given in (16) through (18) are used in Problem 2, reflecting the potential of part complexity, the number of tools needed to produce a part, and the average batch size, respectively. Maximizing the utilization rate (15) and minimizing the cost (14) complete the set of criteria used in Problem 2.

### Problem 2

The utopia and nadir values, as well as the initial "neutral" solution and suggested reservation and aspiration levels, are given in Table 6. From Table 6 we see for instance that the worst Pareto optimal value obtained for the system utilization rate is 66.80 percent, and the maximum utilization rate possible, calculated by ignoring all other objectives, is 81.19 percent. The initial (current) solution presented to the manager is repeated in the sixth column from the right of Table 7. Table 7 also provides detailed information on the relevant decision variables (batch size and number of batches, for each part), the production volume for each part and the machine times. The costs of \$ 454,804 are moderately high in the initial solution, but suppose the manager is willing to accept higher costs level to improve the flexibility measures, so that he relaxes the aspiration level for costs suggested by DIDAS-N from \$ 366,439 to \$ 475,000 and increases the reservation levels of the flexibility measures to 515,000 for flexibility 1 (part complexity), to 17,350 for flexibility 2 (tool utilization), and to -12.0 for flexibility 3 (average batch size). The resulting revised solution is given by Solution 2 in Table 7. As expected, the cost level has increased somewhat, but

the flexibility measures have improved at the same time. The utilization rate has improved as well, from 78.18 percent to 79.50 percent. Further inspection of Solution 2 shows that the increase in flexibility is primarily due to a shift in production from group 1 to groups 2 and 3. Note that the total production increased by 107 units from 16,702 to 16,809 units.

**Table 6: Utopia, Nadir Values and Aspiration and Reservation Values for the Current (Initial) Solution, just prior to Calculating Solution 2, for the Eight Criteria Problem (Problem 2)**

Criterion	Prod1 (Max)	Prod2 (Max)	Prod3 (Max)	Util (Max)	Cost (Min)	Flex1 (Max)	Flex2 (Max)	Flex3 (Max)
Utopia Value	11,500	5,900	1,350	81.19	189,709	523,720	17,996	-1.83
Aspiration Value	10,851	5,393	1,141	79.18	366,439	511,810	17,551	-10.96
Current Solution	10,527	5,139	1,056	78.18	454,804	505,860	17,328	-15.52
Reservation Level	10,203	4,885	932	77.18	543,170	499,900	17,106	-20.00
Nadir Value	9,500	4,500	750	66.80	1,500,000	435,000	15,200	-20.00

Suppose the manager next wishes to produce considerably more units of group 1. By increasing the aspiration level of this criterion, Solution 3 is obtained. The costs in Solution 3 are slightly lower than in Solution 2 (\$ 477,095 versus \$ 477,404), and the production in group 1 has increased considerably, by 881 units, while the production of groups 2 and 3 is at a lower level. The total production in Solution 3 increased by a net volume of 434 units over Solution 2.

By appropriate adjustment to the aspiration and reservation levels of the cost criterion, the effects of decreasing the total costs can be analysed. Solution 4 has much lower costs of \$ 310,923, while the production levels of groups 1 and 2 have decreased in comparison with Solution 3. In addition to the lower production levels, the decrease in costs is due to the fact that parts were produced in much larger batch sizes, as seen from the average batch size criterion, which decreased to -18.29 from -11.08 and from the batch information for the individual parts. Solution 5 represents a middle-of-the-road solution with total costs of \$ 382,827 and a total production of 16,847 units for all three groups combined.

As a final analysis, a separate model was formulated where the batch sizes for each group were restricted according to their typical or most likely values. The ranges of these values

**Table 7: Selected Results of the Interactive Decision Process for the Eight Criteria Problem (Problem 2)**

Criterion	Solution					
	Initial	2	3	4	5	6*
Production1	10,527	10,080	10,961	10,500	10,942	10,793
Production2	5,139	5,588	5,236	5,000	4,879	5,246
Production3	1,056	1,141	1,046	1,048	1,026	1,038
Utilization	78.18	79.50	78.62	78.25	78.11	79.03
Costs	454,804	477,404	477,095	310,923	382,827	283,354
Flexibility1	505,860	515,390	517,280	503,740	508,136	515,340
Flexibility2	17,328	17,359	17,793	17,294	17,490	17,694
Flexipility3	-15.52	-11.29	-11.08	-18.29	-15.68	-12.58
Machine Times						
T <sub>1</sub>	316,800	316,800	316,800	316,122	316,800	316,800
T <sub>2</sub>	217,523	223,511	219,921	217,676	216,737	221,822
T <sub>3</sub>	229,359	239,434	233,945	227,783	227,915	234,335
T <sub>4</sub>	196,354	196,462	194,838	199,345	197,786	197,480
Production Volumes						
V <sub>1</sub>	615.0	587.9	560.2	670.6	573.0	692.4
V <sub>2</sub>	2,219.0	2,000.0	2,018.1	2,000.0	2,106.1	2,003.0
V <sub>3</sub>	1,719.8	2,000.0	1,812.8	1,500.7	1,569.3	1,756.8
V <sub>4</sub>	1,684.3	2,000.0	1,862.6	1,668.8	1,623.1	1,797.2
V <sub>5</sub>	1,119.6	1,000.0	1,000.0	1,159.5	1,113.7	1,000.0
V <sub>6</sub>	141.6	300.0	296.2	100.3	115.9	215.7
V <sub>7</sub>	272.3	291.1	200.0	200.1	266.7	204.9
V <sub>8</sub>	3,320.0	3,167.5	3,495.8	3,500.0	3,492.7	3,447.3
V <sub>9</sub>	3,315.7	3,314.0	3,447.0	3,500.0	3,353.9	3,460.3
V <sub>10</sub>	1,672.2	1,598.6	1,999.9	1,500.0	1,989.3	1,882.5
V <sub>11</sub>	248.4	200.0	200.0	297.4	266.1	266.9
V <sub>12</sub>	209.5	150.0	150.0	249.8	204.9	150.0
V <sub>13</sub>	181.5	200.0	200.0	200.0	172.1	200.0
Batch Information						
(b <sub>1</sub> , v <sub>1</sub> )	(17.7, 34.8)	(13.2, 44.6)	(12.6, 44.4)	(20.0, 33.5)	(18.4, 31.1)	(14.2, 48.9)
(b <sub>2</sub> , v <sub>2</sub> )	(14.4, 154.3)	(12.8, 156.0)	(12.9, 156.0)	(20.0, 100.0)	(17.6, 120.0)	(16.3, 123.1)
(b <sub>3</sub> , v <sub>3</sub> )	(15.9, 108.4)	(17.5, 114.1)	(15.9, 113.9)	(20.0, 75.0)	(20.0, 78.5)	(15.0, 117.2)
(b <sub>4</sub> , v <sub>4</sub> )	(17.8, 94.5)	(19.5, 102.6)	(18.2, 102.4)	(20.0, 83.4)	(20.0, 81.2)	(15.0, 119.8)
(b <sub>5</sub> , v <sub>5</sub> )	(16.0, 69.9)	(13.4, 74.5)	(13.4, 74.5)	(20.0, 58.0)	(20.0, 55.7)	(12.6, 79.7)
(b <sub>6</sub> , v <sub>6</sub> )	(18.7, 7.4)	(10.9, 27.6)	(10.7, 27.6)	(20.0, 5.0)	(12.9, 9.0)	(10.0, 21.6)
(b <sub>7</sub> , v <sub>7</sub> )	(19.4, 14.0)	(10.7, 27.3)	(7.6, 26.2)	(20.0, 10.0)	(15.6, 17.1)	(6.2, 32.8)
(b <sub>8</sub> , v <sub>8</sub> )	(5.3, 627.0)	(5.0, 627.5)	(5.6, 627.5)	(8.9, 393.8)	(7.0, 500.0)	(16.3, 211.6)
(b <sub>9</sub> , v <sub>9</sub> )	(5.3, 626.9)	(5.3, 627.4)	(5.5, 627.4)	(8.9, 393.8)	(6.7, 500.0)	(16.4, 211.6)
(b <sub>10</sub> , v <sub>10</sub> )	(13.8, 120.8)	(12.9, 123.7)	(16.1, 124.1)	(20.0, 75.0)	(13.3, 149.3)	(20.0, 94.1)
(b <sub>11</sub> , v <sub>11</sub> )	(19.1, 12.8)	(8.7, 22.9)	(8.7, 22.9)	(20.0, 14.9)	(17.5, 15.2)	(7.7, 34.5)
(b <sub>12</sub> , v <sub>12</sub> )	(18.6, 11.1)	(7.9, 19.1)	(7.9, 19.1)	(20.0, 12.5)	(16.7, 12.2)	(5.4, 27.7)
(b <sub>13</sub> , v <sub>13</sub> )	(20.0, 8.9)	(8.9, 22.6)	(8.9, 22.6)	(20.0, 10.0)	(18.2, 9.5)	(8.5, 23.4)

\*: Solution 6 was calculated using different bounds on the batch sizes, representing the most likely batch sizes for each group. Therefore, Solution 6 cannot directly be compared with the other solutions in this table.

were introduced above. One representative solution for this modified formulation is given by Solution 6. All criteria in this solution are at an attractive level, and the solution almost dominates the initial solution of the original model. Note, however, that the outcomes cannot directly be compared, because the formulations are not identical, and therefore the efficient set (i.e., the set of nondominated solutions) may not be the same.

A final remark about the solutions presented in Tables 3 through 7 is in order. Due to the nonlinear nature of DIDAS-N all solutions are approximate. The inaccuracy was generally found to be reasonably small. For instance, replicating the linear problem formulation solved by Stam and Kuula (1989) with batch sizes fixed to 5 using DIDAS-N yielded a solution with virtually identical criterion values, except for flexibility criterion which was about 3 percent from the exact solution. One disadvantage of DIDAS-N is that it employs only one solver based on the projected conjugate gradient method and penalty shift functions. While this approach is effective and efficient for many types of problems, it may not be the best approach for bilinear problems such as our FMS formulation. Perhaps due to accuracy problems or local optimal solutions, several times during the interactive decision process strictly dominated solutions were obtained. Additionally, some of the calculations took more computer time (up to half an hour) than is reasonable from the user's point of view in an interactive session. Thus, the computational performance in terms of speed and accuracy, and the flexibility as far as the types of nonlinear functions which can be used, of other nonlinear multicriteria procedures should be investigated. The availability of nonlinear multicriteria software, however, is very limited.

Recently, a specialized version of the HYBRID (Makowski and Sosnowski 1988) package, HYBRID-FMS (Makowski and Sosnowski 1989) has been developed to model the three criteria bilinear FMS problem of Ranta (1989). HYBRID, a member of the DIDAS family of multicriteria procedures (see Lewandowski 1988), uses the reference point method and was originally designed to solve linear, dynamic and quadratic multicriteria problems. The original HYBRID package is available for IBM/PC/XT/AT and compatible microcomputers with a math coprocessor, and is also available in a mainframe version. HYBRID-FMS, however, is only available on the microcomputer. In its current state, HYBRID-FMS is inflexible and can only be applied to Ranta's bilinear formulation with three criteria, i.e., our Problem 1, and as a result cannot be used for other model formulation such as the more realistic Problem 2. Preliminary analyses indicate that HYBRID-FMS solves Problem 1 faster (between 30 and 150 seconds) and gives more accurate solutions than DIDAS-N, so that a future extension allowing for more general problem formulations appears very promising. The authors of the package are currently developing such an extension of HYBRID-FMS.

DIDAS-N and HYBRID-FMS cannot handle integer variables, so that for instance the batch sizes in each solution are real-valued. One possible way to interpret the solutions is

to round the values to the nearest integer. In this case, the property of optimality may be lost, but other researchers have found that for most problems the resulting solutions will still be close to optimal (see e.g., Lodish 1976). Alternatively, the solution can be interpreted in general terms as long run averages or target values upon which to base production planning. In this situation fractional values for batch size and number of batches produced are reasonable.

## CONCLUSIONS

The issue of selecting the most appropriate FMS configuration poses management with a complex strategic decision problem in which many quantitative and qualitative criteria are to be considered. In this paper we propose using a powerful nonlinear multicriteria optimization model to aid management in the qualitative aspects of the decision process. As shown in our illustration, the model can be used interactively to explore the various tradeoffs between the relevant criteria. The nonlinear nature of the model formulation is more realistic than previously proposed linear simplifications of the problem. Our approach was illustrated using data from a specific real decision situation, but with minor modifications the methodology is applicable to a general class of problems. Therefore the methodology should provide a valuable contribution to the balancing of various quantitative aspects which affect the overall decision of acquiring a flexible manufacturing system.

Future research should further examine the viability of the proposed modeling framework using other real data sets and applications. Multicriteria optimization packages other than DIDAS-N should be explored as well to address issues such as accuracy and speed of computation. One promising alternative appears the more general extension of HYBRID-FMS which is currently under development.

## REFERENCES

- J.A. Buzacott and D.D. Yao, "Flexible Manufacturing Systems: A Review of Analytical Models," Management Science, 32 (1986) 890-905.
- R. Jaikumar, "Postindustrial Manufacturing," Harvard Business Review, 6 (1986) 67-79.
- T. Kreglewski, J. Paczynski, J. Granat and A.P. Wierzbicki, "IAC-DIDAS-N: A Dynamic Interactive Decision Analysis and Support System for Multicriteria Analysis of Nonlinear Models with Nonlinear Model Generator Supporting Model Analysis," WP-88-112, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1988.
- R. Kuivanen, J. Lepisto and R. Tinsanen, "Disturbances in Flexible Manufacturing," 3rd IFAC/IFIP/IEA/IFORS Conference on Man-Machine Systems, June 14-16, 1988, Oulu, Finland.
- T. Lakso, "The Influence of FMS-Technology on the Efficiency of NC-Controlled Machine Tools," Publication #50, Tampere University of Technology, Tampere, Finland, 1988.
- A. Lewandowski, "Short Software Descriptions," WP-88-109, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1988.
- A. Lewandowski and A.P. Wierzbicki, "Theory, Software and Testing Examples in Decision Support Systems," WP-88-071, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1988.
- L.M. Lodish, "Assigning Salesmen to Accounts to Maximize Profits," Journal of Marketing Research, 13 (1976) 440-444.
- M. Makowski and J. Sosnowski, "User Guide to a Mathematical Programming Package for Multicriteria Dynamic Linear Problems: HYBRID Version 3.1," WP-88-111, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1988.
- M. Makowski and J. Sosnowski, "HYBRID-FMS: An Element of DSS for Designing Flexible Manufacturing Systems," presented at the International Workshop on Multiple Criteria Decision Support, Helsinki, August 1989.
- J. March and H. Simon, "Organizations," New York, John Wiley, 1958.
- L. Norros, K. Toikka and R. Hyotylainen, "FMS Design from the Point of View of Implementation Results of a Case Study," 3rd IFAC/IFIP/IEA/IFORS Conference on Man-Machine Systems, June 14-16, 1988, Oulu, Finland.
- J. Ranta, "FMS Investment as a Multiobjective Decision-Making Problem," Unpublished Manuscript, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1989.
- J. Ranta and A. Alabian, "Interactive Analysis of FMS Productivity and Flexibility," WP-88-098, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1988.

J. Ranta, K. Koskinen and M. Ollus, "Flexible Production Automation and Computer Integrated Manufacturing: Recent Trends in Finland," Computers and Industry, 4 (1988) 1, 55-76.

A. Stam and M. Kuula, "Selecting a Flexible Manufacturing System Using Multiple Criteria Analysis," WP-89-048, International Institute for Applied Systems Analysis, Laxenburg, Austria, 1989.

A.P. Wierzbicki, "A Mathematical Basis for Satisficing Decision Making," Mathematical Modelling, 3 (1982) 391-405.

## APPENDIX A: List of Equations Used in the Paper

$$T_j = \sum_i (T_{ij} + t_{ij}) * b_i * v_i, \quad (j = 1, \dots, m) \quad (1)$$

$$T_{dj} = d_g * \sum_i g_i + d_b * \sum_i v_i + d_s * S - d_{pl} * PL \quad (j = 1, \dots, m) \quad (2)$$

$$T_{jMIN} \leq T_j + T_{dj} \leq T_{jMAX} \quad (j = 1, \dots, m) \quad (3)$$

$$T_{MIN} \leq T + T_d + T_b \leq T_{MAX} \quad (4)$$

$$C = C_M + C_L + C_P + C_S + C_T + C_O \quad (5)$$

$$C_M = \sum_j e_j * M_j, \quad (6)$$

$$C_L = q_g * \sum_i g_i + q_l * \sum_i L_i, \quad (7)$$

$$C_P = p_g * \sum_i g_i + p_b * \sum_i b_i + p_v * \sum_i v_i, \quad (8)$$

$$C_S = s_g * \sum_i g_i + (s + s_v) * \sum_i v_i + s_l * \sum_i L_i + s_e * \sum_j e_j, \quad (9)$$

$$C_O = C_{TR} + C_{RES} = c_{pl} * PL + C_{RES} \quad (10)$$

$$V_{iMIN} \leq V_i \leq V_{iMAX} \quad (11)$$

$$\text{maximize WEIGHTED\_PRODUCTION} = \sum_i w_i * b_i * v_i \quad (12)$$

$$\text{maximize PRODUCTION\_GROUP\_h} = \sum_{i \in G_h} b_i * v_i \quad (h = 1, \dots, k) \quad (13)$$

$$\text{minimize } C \quad (14)$$

$$\text{maximize UTILIZATION\_RATE} = 100 * T / T_{MAX} \quad (15)$$

$$\text{maximize FLEXIBILITY\_1} = f_g * \sum_i g_i * b_i * v_i \quad (16)$$

$$\text{maximize FLEXIBILITY\_2} = \sum_i L_i * b_i * v_i \quad (17)$$

$$\text{maximize FLEXIBILITY\_3} = -\sum_i b_i / n \quad (18)$$

**APPENDIX B: List of Criteria, Decision Variables, Model Parameters and Coefficients**

<b>Criteria of Problem 1:</b>	<b>Description:</b>
Maximize WEIGHTED_PRODUCTION	total weighted production volume of the system per period
Minimize COST	total direct investment cost of the system per period
Maximize FLEXIBILITY	total flexibility of the system

<b>Criteria of Problem 2:</b>	<b>Description:</b>
Maximize PRODUCTION_GROUP_h	total production volume for group h of the system per period (h=1,2,3)
Minimize COST	total direct investment cost of the system per period
Maximize UTILIZATION_RATE	percentage of total available time during which machines are producing
Maximize FLEXIBILITY_1	flexibility measured by potential of part complexity
Maximize FLEXIBILITY_2	flexibility measured by potential of number of tools needed
Maximize FLEXIBILITY_3	flexibility measured by average batch size

<b>Decision Variables:</b>	<b>Description:</b>
$b_i$	batch size, part i
$v_i$	number of batches produced per period, part i

<b>Indices:</b>	<b>Description:</b>
$i \in \{1, \dots, n\}$	the set of parts
$j \in \{1, \dots, m\}$	the set of machines

<b>Cost Components:</b>	<b>Description:</b>
$C_M$	machine costs per period
$C_L$	tool costs per period
$C_P$	parts pallet costs per period
$C_S$	software costs per period
$C_T$	transportation costs per period
$C_O$	other costs per period

## Model Parameters and Coefficients

### Parameter/Coefficient: Description:

$M_j$	direct annual investment costs, machine $j$
$g_i$	measure of complexity of part $i$
$L_i$	number of tools needed to produce part $i$
$T_{ij}$	unit tooling time of part $i$ on machine $j$
$t_{ij}$	unit overhead time of part $i$ on machine $j$
$T_j$	total time machine $j$ is in operation annually
$S_j$	complexity of the software needed
$T_{dj}$	total annual nonavailable (disturbance) time of machine $j$
$PL$	number of employees to be trained annually
$T_{jMAX}$	maximum minutes machine $j$ can operate annually
$T_{jMIN}$	required minimum minutes machine $j$ should operate annually
$r_i$	unit batch change time for part $i$
$T_{MAX}$	maximum minutes all machines combined can operate annually
$T_{MIN}$	required minimum minutes all machines combined should operate annually
$T$	total time all machines combined are in operation annually
$T_d$	total annual nonavailable (disturbance) time of all machines combined
$T_b$	total annual batch change time
$e_j$	efficiency of machine $j$
$w_i$	relative importance weight of producing part $i$

## Scaling Coefficients

### Model Parameters: Contribution to:

$q_g$	tool cost of part complexity $g_i$
$q_l$	tool cost of number of tools needed $L_i$
$p_g$	parts pallet cost of part complexity $g_i$
$p_b$	parts pallet cost of batch size $b_i$
$p_v$	parts pallet cost of number of batches produced $v_i$
$s_g$	software costs of part complexity $g_i$
$s$	software costs of total number of batches produced
$s_v$	software costs of number of batches produced $v_i$
$s_l$	software costs of number of tools needed $L_i$
$s_e$	software costs of machine efficiency $e_j$
$c_{PL}$	training costs per employee
$d_g$	nonavailability of part complexity
$d_b$	nonavailability of batch size
$d_s$	nonavailability of software size and complexity
$d_{PL}$	nonavailability of personal training
$f_g$	total flexibility, Problem 1