ON AN APPROACH TO THE PROBLEM OF HYDROTECHNICAL DEVELOPMENT IN RIVER BASINS

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Abstract

On the basis of network modelling of the flow in a river basin, we consider the problem of hydrotechnical development of a basin and formulate it like a simulation one for the case of a limited investment for development.

Suppose we have a network diagram for a river basin, which consists of nodes i ($i \varepsilon R_1$, where R_1 is the set of all nodes) and arcs F_{ij} (arc F_{ij} connects nodes i and j and directs from i to j). An example of a diagram is shown in Figure 1.

Each node corresponds to the point of

- runoff inflow into the river system,
- tributary inflow,
- water transfer from another river basin,
- location of storage reservoir, pumping station, or some other hydrotechnical constructions,
- location of the users of different kinds,
- water take off for some other river basin and so on.

For each arc F_{ij} we prescribe the value of the water which flows from node i to node j. They are restricted by natural physical limits

$$\underline{F_{ij}} \leq F_{ij}(t) \leq \overline{F_{ij}} \quad , \qquad \forall_i , i \; \varepsilon \; R_1 \tag{1}$$

where $\underline{F_{ij}}$ and $\overline{F_{ij}}$ are minimum and maximum values of the flow, correspondingly.

Some arcs could have one of the indexes equal to zero. If the first index is zero, it means that this arc gives the inflow into the river system. If the second one is zero, it means that this arc gives the outflow from the system. The nodes with only ingoing or outgoing arcs we will call fictitious ones. For set R_2 of all other nodes ($R_2 = R_1 - R_3$, where R_3 is a set of all fictitious nodes) we will have the dynamic equations

$$W_i(t + 1) = W_i(t) + \sum_j F_{ji}(t) - \sum_K F_{iK}(t) \quad , \qquad \forall_i \in R_2 \quad , \tag{2}$$

which describe the volume of the water stored in node i at the time interval t. Values $W_i(t)$ are also subjected to natural physical constraints

$$\underline{W_i} \leq W_i(t) \leq \overline{W_i} \qquad \forall_i \in R_2 \tag{3}$$

where $\underline{W_i}$ and $\overline{W_i}$ are minimum and maximum volumes of water which could be stored. For points i, where there are no storage reservoirs, or where their construction is not planned, we have

$$\underline{W_i} = \overline{W_i} = 0 \quad . \tag{4}$$

The problem of hydrotechnical development in the river basin could be formulated in a few different ways. Let us consider the case when the total capital investment is given and equal S* and we would like to use them for building hydrotechnical constructions of such types, size and location to minimize losses of the users from the shortage of the water and exploitation expenses of all hydrotechnical facilities in the river basin system during the planned period of exploitation.

If we plan to build storage reservoir in point i, then its capacity $\tilde{W}_i$ should be found as a solution of our problem. $\bar{W}_i$ corresponds to reasonable geographical limitation for storage reservoir capacity, and of course

$$\tilde{W}_i \leq \bar{W}_i \tag{5}$$

For the point i where the pumping station or channel should be built the situation will be analogous. Maximum value of the flow through the pumping station or channel $\tilde{F}_{ij}$ will depend on their size and should be found from the solution of our problem. We also have

$$\tilde{F}_{ij} \leq \bar{F}_{ij} \tag{6}$$

where $\bar{F}_{ij}$ is the characteristic of physical possibilities.

Let us suppose for simplicty that capital investment S_i for building hydrotechnical constructions in point i ($i \in \Gamma_1$, where Γ_1 is the set of all points, where hydrotechnical constructions will be built) is some function of maximum volume of the water to be stored, $\tilde{W}_i$, or maximum flow, $\tilde{F}_{ij}$, we would like to pump, that is

$$S_i = S_i(\tilde{F}_{ij}, \tilde{W}_i) \quad , \qquad \forall i \in \Gamma_1 \quad . \tag{7}$$

The values S_i should be subjected to the constraint

$$\sum_{i \in \Gamma_1} S_i(\tilde{F}_{ij}, \tilde{W}_i) \leq S^* \tag{8}$$

and the objective function will have the form

$$\min I = \sum_{t=T_1}^{T_2} \Big[\sum_{i \in \Gamma_2} \alpha_i^t P_i^t (d_i(t) - \sum_j F_{ji}(t)) + \\ + \sum_{i \in \Gamma_3} \beta_i^t E_i^t(\tilde{W}_i, \tilde{F}_{ij}, W_i(t), F_{ij}(t)) \Big] \tag{9}$$

where

P_i^t = function of the water users losses from the water shortage,

E_i^t = exploitation expenses,

Γ_2 = set of all water users

Γ_3 = set of all hydrotechnical construction under exploitation,

T_1 = beginning of exploitation

T_2 = end of exploitation

α_i^t and β_i^t = discount coefficients

$d_i(t)$ = demand of the water users located in node i

Demand $d_i(t)$ could change with the time. We assume that

$$d_i(t) \geq \sum_i F_{ji}(t) \qquad (10)$$

i.e.total amount of the water supplied to the user from different point j never exceeds the user demand. We will accept that

$$\begin{array}{ll} P_i^t = 0 & \text{when} \quad \omega_i(t) = 0 \\ P_i^t \quad 0 & \text{when} \quad \omega_i(t) > 0 \end{array} \qquad (11)$$

where $\omega_i(t) = d_i(t) - \sum_j F_{ji}(t)$ is a shortage of the water.

We will not discuss here the form and structure of the functions P_i^t and E_i^t but they seem to depend very much on the users nature and type of hydrotechnical construction. Coefficients α_i^t and β_i^t decrease with the time and it shows that losses and exploitation expenses now are much more important than in future.

Block-scheme of solution of this problem is shown on Figure 2. This block-scheme just describes the procedure of simulation of development in the basin. It gives the possibility to compare different alternatives of development.

The simulation procedure is the following. We fix the points for building a hydrotechnical construction and also values of investment in each point.

Then after checking the constraint (8), we could find values $\tilde{F}_{ij}$ and $\tilde{W}_i$ by using relations (7). Of course, these values should satisfy the physical constraints (5) and (6)

Now we are ready for solution of optimization part of the problem. We should find such $F_{ij}(t)$ and $W_i(t)$ to minimize the objective function (9). Values $F_{ij}(t)$ and $W_i(t)$ should be restricted by constraints

$$O \leq F_{ij} \leq \tilde{F}_{ij} \tag{12}$$

$$O \leq W_i \leq \tilde{W}_i \tag{13}$$

Ater solution of this sybproblem we find maximal flows and volumes of water in the points of hydrotechnical constructions

$$\tilde{F}_{ij} = \max_t F_{ij}(t) \quad , \qquad \forall i \in \Gamma_1 \tag{14}$$

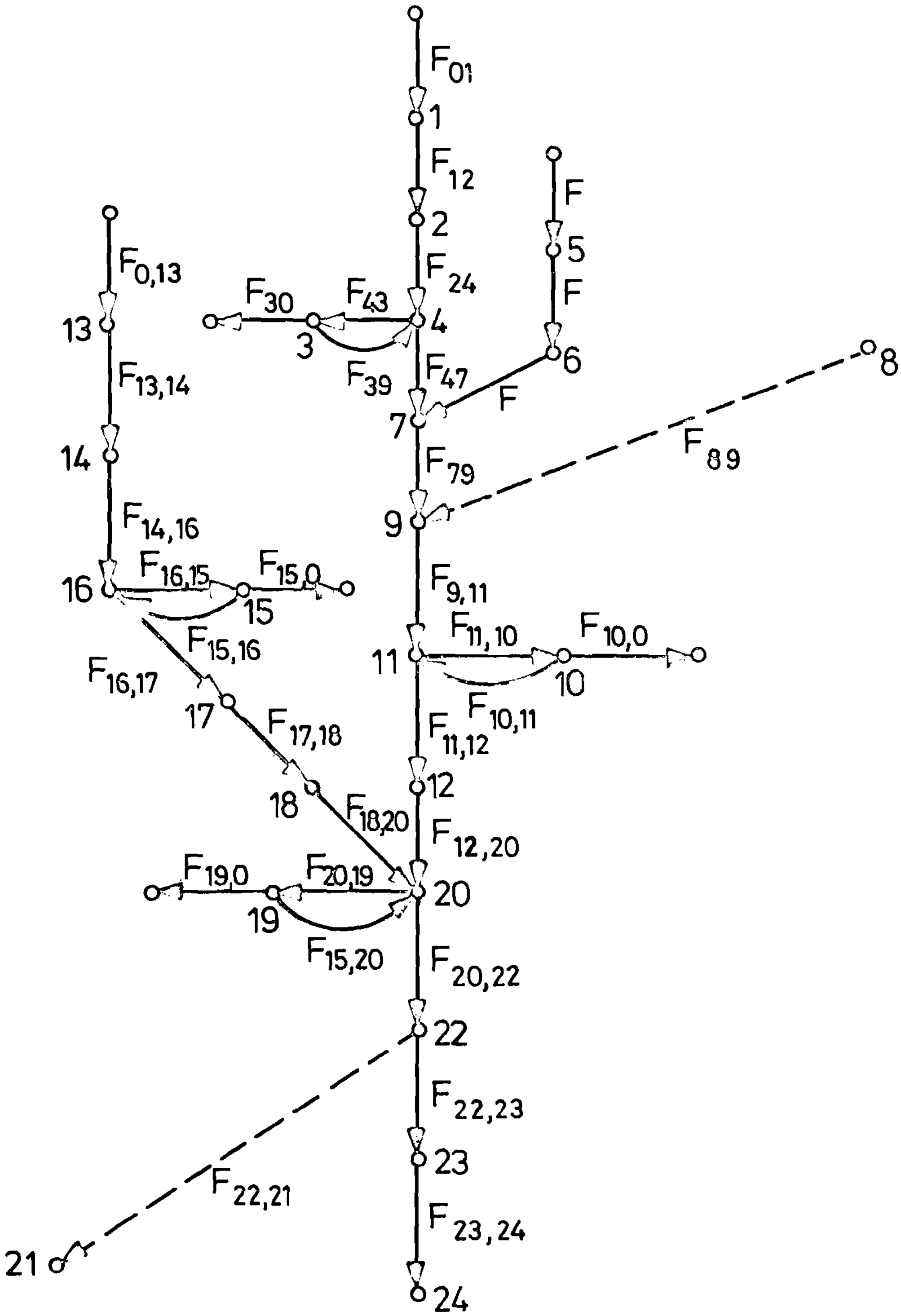
$$\tilde{W}_i = \max_t W_i(t) \qquad \forall i \in \Gamma_i \tag{15}$$

and calculate the differences

$$\delta\tilde{F}_{ij} = \tilde{F}_{ij} - \tilde{F}'_{ij} \cdot \tag{16}$$

$$\delta\tilde{W}_i = \tilde{W}_i - \tilde{W}'_i \tag{17}$$

These differences together with the value of the objective function should be used for estimation of different alternatives of hydrotechnical development in the river basin.



F01
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F15,0
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F23,24
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FIGURE 1.

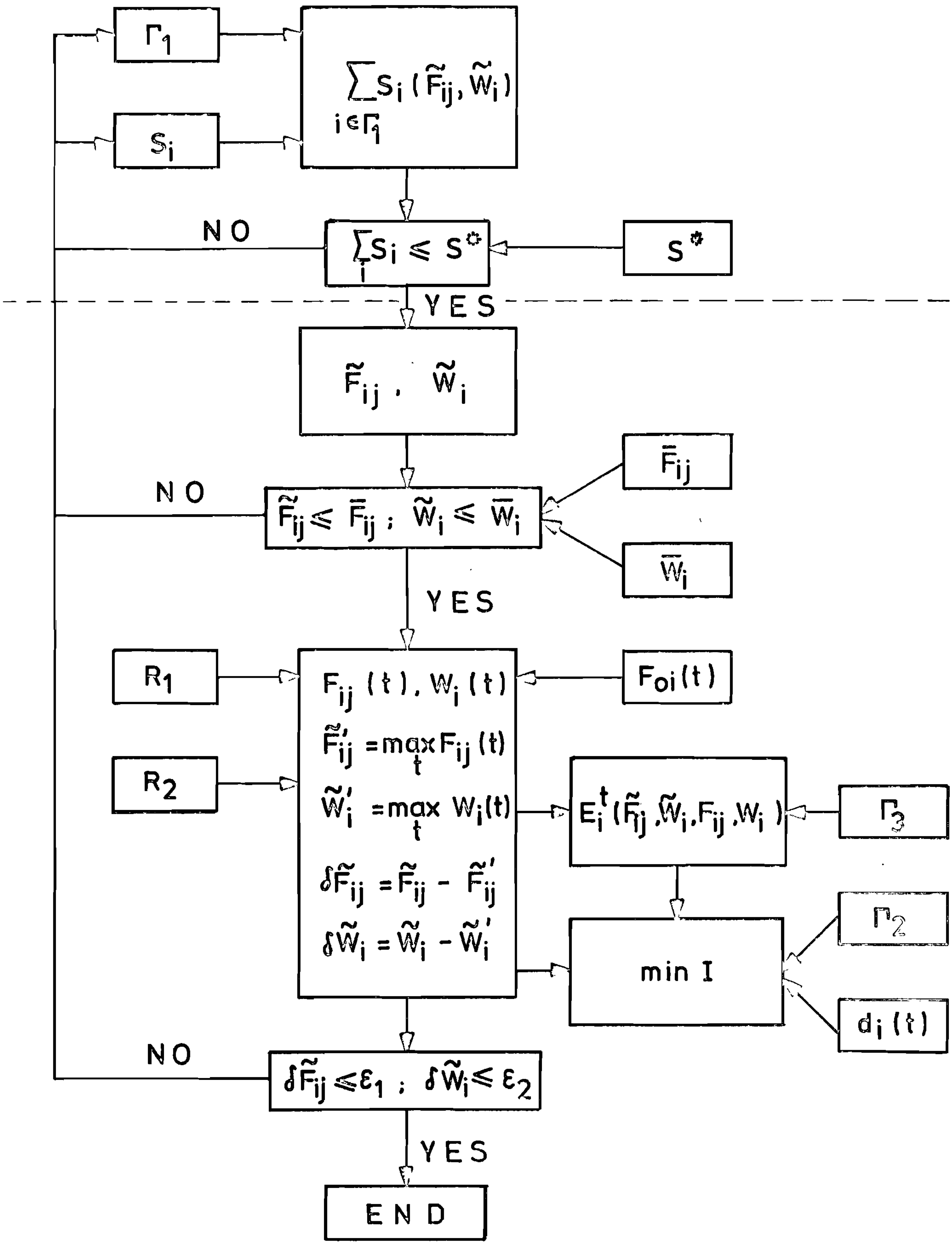
Γ_1
$\sum_{i \in \Gamma_1} s_i(\tilde{F}_{ij}, \tilde{W}_i)$
s_i
NO
$\sum_i s_i \leqslant S^*$
S^*
YES
$\tilde{F}_{ij}\ ,\ \tilde{W}_i$
NO
$\tilde{F}'_{ij} \leqslant \bar{F}_{ij}\ ;\ \tilde{W}_i \leqslant \bar{W}_i$
$\bar{F}_{ij}$
$\bar{W}_i$
YES
R_1
$F_{ij}(t),\ W_i(t)$
$\tilde{F}'_{ij} = \max_t F_{ij}(t)$
$\tilde{W}'_i = \max_t W_i(t)$
$\delta\tilde{F}_{ij} = \tilde{F}_{ij} - \tilde{F}'_{ij}$
$\delta\tilde{W}_i = \tilde{W}_i - \tilde{W}'_i$
$F_{oi}(t)$
R_2
$E_i^t(\tilde{F}_{ij}, \tilde{W}_i, F_{ij}, W_i)$
Γ_3
Γ_2
min I
$d_i(t)$
NO
$\delta\tilde{F}_{ij} \leqslant \varepsilon_1\ ;\ \delta\tilde{W}_i \leqslant \varepsilon_2$
YES
END

FIGURE 2