WORKING PAPER

BIVOPROB:
A Computer Program for Maximum-Likelihood Estimation of Bivariate Ordered-Probit Models for Censored Data

Charles A. Calhoun

June 1989
WP-89-38

IIASA
International Institute
for Applied Systems Analysis

BIVOPROB:
A Computer Program for Maximum-Likelihood Estimation of Bivariate Ordered-Probit Models for Censored Data

Charles A. Calhoun

June 1989
WP-89-38

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS
A-2361 Laxenburg, Austria

Foreword

Despite the large number of models devoted to the statistical analysis of censored data, relatively little attention has been given to the case of censored discrete outcomes. In this paper, Charles Calhoun presents a technical description and user's guide to a computer program for estimating bivariate ordered-probit models for censored and uncensored data. The model and program are currently being applied in an analysis of World Fertility Survey data for Europe and the United States, and the results of this work will be described in a forthcoming IIASA working paper.

Nathan Keyfitz
Leader
Population Program

Acknowledgements

Initial development of the computer program BIVOPROB was undertaken while the author was Research Associate in the Program in Demographic Studies of The Urban Institute. The program has since been revised and extended in several ways, thus necessitating a new version of the user's guide. This working paper documents the latest version of the program, with the intention of making it accessible to researchers in the IIASA National Member Countries and the scientific community in general. Support for this work was provided by the International Institute for Applied Systems Analysis. The views expressed are entirely those of the author.

Contents

BIVOPROB:
A Computer Program for Maximum-Likelihood Estimation of Bivariate Ordered-Probit Models for Censored Data

Charles A. Calhoun

1. Introduction

BIVOPROB is a Fortran program for maximum-likelihood estimation of bivariate ordered-probit models. The model generalizes the univariate ordered-probit framework (McKelvey and Zavoina, 1975) to the case of two ordered-discrete or ordered-categorical dependent variables. The sample likelihood function is based on a linear simultaneous-equations model for two latent normal random variables, and two sets of threshold parameters that relate the continuous latent variables to observed discrete outcomes. The use of a simultaneous-equations model for latent dependent variables makes it possible to estimate and test causal relationships that determine the discrete random variables. The model can be viewed as a special case of the linear structural-relations probit model for binary or ordered-categorical data (Muthen, 1979, 1983).

BIVOPROB can be used to estimate five alternative models depending on the censoring status of the discrete-dependent variables: (1) an uncensored bivariate ordered-probit model in which the discrete-dependent variables are assumed to be observed without error; (2) a model of lower-limit endogenous censoring in which one discrete-dependent variable is a lower bound on the observed value of the other; (3) a model of upper-limit endogenous censoring in which one discrete-dependent variable is an upper bound on the observed value of the other; (4) a discrete endogenous-switching model in which only the minimum of the two discrete-dependent variables is observed; and (5) a two-limit bivariate ordered-probit model in which the values of one or both of the discrete-dependent variables are known only to lie between minima and maxima that are exogenous to the model. The program includes options for reparameterizing the thresholds as linear functions of observed covariates, and for using sampling weights in estimation.

Section 2 summarizes the basic model. Section 3 presents structural and reduced-form versions of the model and issues of parameter identification. Maximum-likelihood estimation and a joint likelihood-ratio test are discussed in sections 4 and 5. The modifications to the likelihood function that are required for estimating the different censoring models are given in section 6. Section 7 provides instructions on installing and running the computer program BIVOPROB.

2. The Bivariate Ordered-Probit Model

It is assumed that ordered-discrete or ordered-categorical random variables Y_1^* and Y_2^* are determined by the following system of simultaneous latent random variables and threshold equations:

$$Z_1^* = \gamma_1 Z_2^* + X_1\beta_1 + u_1 \tag{1}$$

$$Z_2^* = \gamma_2 Z_1^* + X_2\beta_2 + u_2 \tag{2}$$

$$Y_1^* = \begin{cases} 0 & \text{if } Z_1^* \le \mu_0 \\ 1 & \text{if } \mu_0 < Z_1^* \le \mu_1 \\ 2 & \text{if } \mu_1 < Z_1^* \le \mu_2 \\ \vdots & \\ C & \text{if } \mu_{C-1} < Z_1^* \end{cases} \tag{3}$$

$$Y_2^* = \begin{cases} 0 & \text{if } Z_2^* \le \delta_0 \\ 1 & \text{if } \delta_0 < Z_2^* \le \delta_1 \\ 2 & \text{if } \delta_1 < Z_2^* \le \delta_2 \\ \vdots & \\ D & \text{if } \delta_{D-1} < Z_2^* \ . \end{cases} \tag{4}$$

Latent variables Z_1^* and Z_2^* are unobserved continuous outcomes for which only the discrete indicators Y_1^* and Y_2^* are potentially observable. Z_1^*, Z_2^*, and the random disturbances u_1 and u_2 are assumed to always be unobservable. In this section it is assumed that the discrete outcomes for Y_1^* and Y_2^* are uncensored. The case where censoring of one of the four types described in the introduction prevents us from observing the actual outcomes for the discrete variables is considered in section 6.

X_1 and X_2 are row vectors of observed explanatory variables that are assumed to be distributed independently of the unobserved random disturbances u_1 and u_2. β_1 and β_2 are column vectors of unknown regression coefficients whose elements correspond to the variables in X_1 and X_2. γ_1 and γ_2 are unknown scalar parameters that account for the direct effects of Z_1^* and Z_2^* outcomes on each other. Differences in the latent variables result in different discrete values of Y_1^* and Y_2^* depending on the location of Z_1^* and Z_2^* *vis-a-vis* the unknown threshold parameters μ_0, μ_1, μ_2, ..., μ_{C-1} and δ_0, δ_1, δ_2, ..., δ_{D-1}. The threshold parameters can also be expressed as linear functions of observed covariates, in which case the scalar parameters μ_i and δ_j are replaced by linear forms $X_3\mu_i$ and $X_4\delta_j$, where X_3 and X_4 are row vectors of observed covariates satisfying the same assumptions as X_1 and X_2 with regard to independence from u_1 and u_2. The model is completed by assuming that random disturbances u_1 and u_2 are distributed bivariate normal with zero means and covariance matrix Σ.

3. Structural and Reduced-Form Models and Identification

Identification of simultaneous-equations probit models is discussed in Muthen (1979) and Maddala (1983). The main points as they apply to the bivariate ordered-probit model are summarized in this section. The assumption that the explanatory variables X_1 and X_2 are measured without error and are independent of u_1 and u_2 implies that the parameters β_1, β_2, γ_1, and γ_2 are identified, up to a constant of proportionality, under the same rank and order conditions that hold for ordinary linear simultaneous equations. Simultaneous equations (1) and (2) may be written in matrix notation as follows:

$$Z^*\Gamma = X\beta + u \tag{5}$$

where

$$Z^* = \begin{bmatrix} Z_1^* & Z_2^* \end{bmatrix} \tag{6a}$$

$$X = \begin{bmatrix} X_1 & X_2 \end{bmatrix} \tag{6b}$$

$$\beta = \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix} \tag{6c}$$

$$\Gamma = \begin{bmatrix} 1 & -\gamma_2 \\ -\gamma_1 & 1 \end{bmatrix} \tag{6d}$$

$$u = \begin{bmatrix} u_1 & u_2 \end{bmatrix}. \tag{6e}$$

The reduced-form of the simultaneous (structural) equations in (5) is given by

$$Z^* = X\Pi + v \tag{7}$$

where

$$\Pi = \beta\Gamma^{-1} \tag{8a}$$

and

$$v = u\Gamma^{-1} . \tag{8b}$$

When u is bivariate normal with zero mean vector and covariance matrix Σ, then v is bivariate normal with zero mean vector and covariance matrix

$$\Omega = (\Gamma^{-1})'\Sigma(\Gamma^{-1}) . \tag{9}$$

As with any probit model, the variances of the latent variables Z_1^* and Z_2^* cannot be estimated with data on ordered outcomes for Y_1^* and Y_2^*. This implies that the parameters of an estimable reduced-form model are identified only up to a pair of unknown constants of proportionality. If we define the diagonal matrix Δ with elements $(\sqrt{\omega_{11}})^{-1}$ and $(\sqrt{\omega_{22}})^{-1}$, where ω_{11} and ω_{22} are the main-diagonal elements of Ω, then $\Pi\Delta$ and $\Delta\Omega\Delta$ are the reduced-form parameters that can be estimated. Under the usual rank and order conditions for identification of linear simultaneous equations, the structural parameters that can be estimated are $\Delta\Gamma\Delta^{-1}$, $\beta\Delta$, and $\Delta\Sigma\Delta$. These restrictions are imposed by post-multiplying equations (5) and (7) by Δ. This is equivalent to assuming that the reduced-form disturbances have a bivariate standard-normal distribution.

Normalization of the reduced-form variances to one implies additional restrictions on structural variances σ_{11} and σ_{22}. Writing the reduced-form variances in terms of γ_1, γ_2, and the elements of Σ, and setting these expressions equal to one, produces two equations in σ_{11} and σ_{22} which can be solved in terms of σ_{12}, γ_1, and γ_2:

$$\sigma_{11} = \frac{(1-\gamma_1\gamma_2)^2(1-\gamma_1^2) + (2\gamma_2\gamma_1^2-2\gamma_1)\sigma_{12}}{1 - \gamma_1^2\gamma_2^2} \tag{10a}$$

$$\sigma_{22} = \frac{(1-\gamma_1\gamma_2)^2(1-\gamma_2^2) + (2\gamma_1\gamma_2^2-2\gamma_2)\sigma_{12}}{1 - \gamma_1^2\gamma_2^2} . \tag{10b}$$

Equations (10a) and (10b) can be used to eliminate σ_{11} and σ_{22} from the expression for the reduced-form covariance given by:

$$\omega_{12} = \frac{(1+\gamma_1\gamma_2)\sigma_{12}+\gamma_1\sigma_{22}+\gamma_2\sigma_{11}}{(1-\gamma_1\gamma_2)^2} . \tag{11}$$

Restrictions on the unknown threshold parameters consist of setting μ_0 and δ_0 equal to zero, in which case a constant term can be included in the explanatory variable vectors X_1 and X_2. Identification of the other threshold parameters requires that there be observed outcomes in the categories above and below each threshold. Otherwise, empty cells must be collapsed with adjacent ones until all unidentified threshold parameters are eliminated. The threshold covariate vectors X_3 and X_4 should each include at least a constant term to insure that there is a difference between the thresholds for the zero categories and higher-order outcomes.

4. Maximum-Likelihood Estimation

Full-information maximum-likelihood estimates of the structural parameters of the simultaneous latent-variables equations are found by deriving the likelihood function for the corresponding reduced-form model, expressing each reduced-form parameter as a function of structural parameters, and then maximizing the resulting sample likelihood function over the structural and threshold parameters. As discussed in the previous section on identification, the variances of the unobserved latent variables Z_1^* and Z_2^* cannot be estimated from data on ordered outcomes for Y_1^* and Y_2^*, so it is assumed that the reduced-form model is based on a bivariate standard-normal distribution. The contribution to the sample likelihood of an observation with discrete outcomes $Y_1^*=i$ and $Y_2^*=j$ is given by

$$l = \int_{A(i-1)}^{A(i)} \int_{B(j-1)}^{B(j)} \varphi(a,b;\omega_{12})\, db\, da \tag{12}$$

where $\varphi(a,b;\omega_{12})$ is the density function for the bivariate standard-normal distribution with correlation ω_{12},

$$A(i) = X_3\mu_i - \frac{X_1\beta_1 + \gamma_1 X_2\beta_2}{1-\gamma_1\gamma_2} \tag{13}$$

$$B(j) = X_4\delta_j - \frac{X_2\beta_2 + \gamma_2 X_1\beta_1}{1-\gamma_1\gamma_2} \tag{14}$$

and $\mu_{-1}=\delta_{-1}=-\infty$, $\mu_0=\delta_0=0$, and $\mu_C=\delta_D=\infty$. The likelihood function for a sample of independent observations is found by taking the product of the individual likelihood contributions defined by (12).

Maximization of the likelihood function with respect to the structural and threshold parameters requires the use of numerical methods for iterative optimization (Dennis and Schnabel, 1983; Gill, Murray, and Wright, 1981). BIVOPROB uses subroutines for steepest-descent and Davidon-Fletcher-Powell (*DFP*) iterations programmed by Gruvaeus and Joreskog (1970). The *DFP* method uses the likelihood function and gradient vector to compute the hessian matrix of second derivatives at each iteration. This gives an approximation to the information matrix that can be used to compute statistical tests based on the asymptotic normality of maximum-likelihood estimators. Bivariate cumulative-normal probabilities are computed using the method of Owen (1956).

5. A Joint Likelihood-Ratio Test

This section describes a joint likelihood-ratio test of a model with explanatory variables (in addition to constant terms) and non-zero bivariate normal correlation parameter ω_{12}, against the model with no explanatory variables and $\omega_{12}=0$. The test-statistic is given by

$$LR = 2ln(\frac{L(\hat{\theta})}{L(\hat{\theta}_0)}) \tag{15}$$

where $L(\hat{\theta})$ is the likelihood value of the unrestricted model, $L(\hat{\theta}_0)$ is the likelihood value of the restricted model, and $\hat{\theta}$ and $\hat{\theta}_0$ are the vectors of maximum-likelihood parameter estimates for each model. The test statistic *LR* is distributed (asymptotically) χ^2 with r degrees of freedom, where r is the difference in the number of parameters in $\hat{\theta}$ and $\hat{\theta}_0$.

It is possible to compute maximum-likelihood estimates for the restricted model directly from the observed sample proportions, because there are exactly C-1 parameters and independent sample proportions for the first equation, and D-1 parameters and independent sample proportions for the second equation. The maximum-likelihood estimates of $\hat{\theta}_0$ are given by

$$\hat{\beta}_1 = -\Phi^{-1}(P_1(0)) \tag{16a}$$

$$\hat{\mu}_j = \Phi^{-1}(\sum_{i=0}^{j} P_1(i)) + \hat{\beta}_1 \qquad \text{for } j=1,2,...,C-1 \tag{16b}$$

and

$$\hat{\beta}_2 = -\Phi^{-1}(P_2(0)) \tag{17a}$$

$$\hat{\delta}_j = \Phi^{-1}(\sum_{i=0}^{j} P_2(i)) + \hat{\beta}_2 \qquad \text{for } j=1,2,\ldots,D-1 \tag{17b}$$

where Φ^{-1} is the inverse univariate standard-normal cumulative distribution function, and $P_i(j)$ is the observed sample proportion with $Y_i^*=j$. The contribution to the sample likelihood function of the restricted model of an observation with $Y_1^*=i$ and $Y_2^*=j$ is given by

$$l = \int_{\hat{\mu}_{i-1}-\hat{\beta}_1}^{\hat{\mu}_i-\hat{\beta}_1} \varphi(a)\,da \int_{\hat{\delta}_{j-1}-\hat{\beta}_2}^{\hat{\delta}_j-\hat{\beta}_2} \varphi(b)\,db \tag{18}$$

where $\varphi(z)$ is the univariate standard-normal density function evaluated at z, and $\hat{\mu}_{-1}=\hat{\delta}_{-1}=-\infty$, $\hat{\mu}_0=\hat{\delta}_0=0$, and $\hat{\mu}_C=\hat{\delta}_D=\infty$. The test statistic LR is computed automatically by BIVOPROB whenever the uncensored bivariate ordered-probit model is estimated. The values of $\Phi^{-1}(p)$ are computed using a rational-approximation formula from Abramowitz and Stegun (1976). For tests against less restrictive models with explanatory variables or non-zero bivariate-normal correlation, or in models with censoring, the likelihood-ratio test requires that both the restricted and unrestricted versions of the model be estimated using the iterative optimization procedure.

6. Censoring Models

The current version of BIVOPROB can be used to estimate the uncensored bivariate ordered-probit model and four types of censoring models. Applications of bivariate ordered-probit censoring models to substantive problems in demography are discussed in Calhoun (1989a, 1989b, 1989c). Censoring occurs when the actual discrete outcomes for Y_1^* or Y_2^* cannot be observed. It is assumed that partial information about Y_1^* and Y_2^* is available, and that this information can be expressed in terms of the observed values of discrete random variables Y_1 and Y_2. In a full-information approach based on latent variables, censoring can be incorporated with relatively simple modifications to the sample likelihood function. Table 1 summarizes the observed data and contributions to the likelihood function for the uncensored bivariate ordered-probit model and the four models of censoring. The models can be summarized as follows:

Model 1: Uncensored Bivariate Ordered-Probit

This model was discussed in the previous sections.

Model 2: Lower-Limit Endogenous Censoring

The second model in Table 1 shows the contribution to the sample likelihood function of outcomes for which the observed value Y_2 is limited to a value that is greater than or equal to the observed (uncensored) value of Y_1^*. The only difference between the likelihood function for model 1 and that for model 2 is that integration over values of Z_2^* is not limited below when $Y_1^* \leq Y_2^*$.

Model 3: Upper-Limit Endogenous Censoring

The third model in Table 1 shows the analogous situation where Y_2 is limited to values that are less than or equal to the observed (uncensored) values of Y_1^*. Models 2 and 3 can be seen as discrete cases of plane truncation analogous to those for continuously distributed normal random variables considered by Tallis (1965). Here the truncation relationship is defined by the restriction $Y_1 \leq Y_2$ or $Y_1 \geq Y_2$.

Model 4: Discrete Endogenous-Switching

The fourth model in Table 1 is the discrete counterpart to the endogenous switching-regression model for continuous data (Fair and Jaffee, 1972; Goldfeld and Quandt, 1973; Maddala and Nelson, 1975). It is assumed that $Y_1 = Y_1^*$ is observed when $Y_1^* = min(Y_1^*, Y_2^*)$ and that $Y_2 = Y_2^*$ is observed when $Y_2^* = min(Y_1^*, Y_2^*)$. It is also assumed that independent information is available to indicate which of the two ordered-probit equations generated the observation.

Model 5: Two-Limit Bivariate Ordered-Probit Model

The fifth model in Table 1 gives the contribution to the likelihood function when the actual discrete outcomes are known only to lie between observed minimum and maximum values given by $Y_1^{\min}$ and $Y_1^{\max}$ for Y_1^*, and $Y_2^{\min}$ and $Y_2^{\max}$ for Y_2^*. The limit values can vary from observation to observation, but are assumed to be exogenous to the model. This model extends the univariate two-limit probit model (Rosett and Nelson, 1975) to one with two-equations having more than three outcomes and limits that can vary by observation.

All of the censoring models are assumed to have the same underlying structure given by equations (1) to (4). The distributions of Z_1^*, Z_2^*, u_1, and u_2 are defined over the population. Y_1^* and Y_2^* are interpreted as the potential values of the discrete outcomes that would be observed in the absence of censoring, while the observed discrete outcomes given by Y_1 and Y_2 may be defined only for certain values of Y_1^* and Y_2^*, or only for selective subsamples of the population, depending on the type of censoring involved.

Computational formulas for calculating the integrals in Table 1 are given in Tables 2.1 to 2.5. These are used in subroutine FCTGR for computing the likelihood function values for each of the models.

7. Running BIVOPROB

This section provides instructions on installing and running BIVOPROB. The program has been written in Fortran, and is currently being used on VAX 11 and 6200 series mainframe computers, and IBM-compatible personal computers. There are approximately 2000 lines of code and comments in the program. There is a subroutine (USER) that can be modified for user-defined recodes and variable transformations. A user-supplied missing data code (XMISS) is used to control the selection of data for analysis. Instructions on how to modify subroutine USER are given in the program. USER loads the data into the vector XDATA, which is located in common storage region /DAT/. The size of XDATA can be increased or decreased depending on the size of the data set and hardware capacity. The initial size of XDATA has been set at 500,000 cells (cases x variables). Included on the diskette containing the source code is an executable version of the program called BIVOPROB.EXE for use on an IBM-compatible personal computer equipped with a math coprocessor. The size of XDATA has been set to 50000 in BIVOPROB.EXE.

Files for Input-Output

The user must create three (3) files that will contain the control cards and model options (SETUP.FLE), the input data (DATAIN.FLE), and the starting values for the maximum-likelihood procedure (START.FLE). The estimation results are directed to OUTPUT.FLE, and the estimated parameters and information matrix from the last run are output to VALUES.FLE. OUTPUT.FLE and VALUES.FLE do not have to exist prior to running the program. If they do exist, they will be overwritten.

The program includes an option for using the values returned to VALUES.FLE to restart the program if additional iterations are desired. The filenames used for these I/O devices can be changed by editing the OPEN statements in the driver program BIVOPROB.

Starting Values for Maximum-Likelihood Estimation

The user provides starting values only for the coefficient vectors β_1 and β_2. These should be given in the file START.FLE in free format (i.e., separated by spaces or commas). The values for β_1 and β_2 should be given in two sets that each begin on a separate line. Each set can be continued on additional lines if necessary.

The starting values of the other parameters are computed by the program. Integers are used as starting values for the threshold parameters. When X_3 and X_4 include covariates in addition to the constant terms, integers are used as the starting values of the coefficients of the constants, and the other coefficients are set to zero. The starting values for γ_1, γ_2, and ω_{12} or σ_{12} are always set to zero.

Good starting values for β_1 and β_2 may help in avoiding a local maximum. On the other hand, accurate starting values have the disadvantage that convergence could be achieved before an accurate estimate of the information matrix is obtained. In this case the parameter estimates and likelihood function values will be correct, but the reported variances, standard errors, and t-statistics should be ignored. Zeros have been found to work well in most cases. If zeros do not work, as indicated by a failure to converge in the steepest-decent iterations, it may suffice to use the means of the dependent variables as starting values for the coefficients of the constant terms, with all other elements of β_1 and β_2 set to zero.

Convergence Limits

The iterative procedure that is used by BIVOPROB to obtain maximum-likelihood estimates depends on several criteria for convergence. The user specifies the maximum number of iterations as an input to the program. The values of other convergence criteria are assigned in subroutine USER, and can be altered to increase or decrease the accuracy of the results, and, conversely, increase or decrease the time required by the program. In most cases, only one of these parameters, *EPS*, will be changed by the user. *EPS* determines the relative magnitude of the maximum gradient element at convergence. A description of the other parameters and their function can be found in Gruvaeus and Joreskog (1970).

Regardless of whether or not the convergence criterion implied by *EPS* is satisfied, upon termination the program prints the current values of all parameters and statistics, including the gradient. One can examine the gradient elements to see if they are acceptable, and to determine which variables might be deleted from the model in order to improve the overall fit.

Control-Card Sequence

The following list gives the sequence of values that must be supplied in the file SETUP.FLE in order to run the program:

$NVARIN$ $NVAR$ $MODEL$
$IREAD$ $INAME$ $XMISS$
$ISTART$ $IDFP$ $MXITER$
$DATFMT$ (include only when $IREAD = 1$ *or* 2)
$NAME(I)$, I=1,...,$NVAR$ (include only when $INAME = 1$)
$IG1$ $IG2$ (if $MODEL$ = 1, 2, 3, *or* 4)
$IG1L$ $IG1U$ $IG2L$ $IG2U$ (if $MODEL = 5$)
$IMAX1$ $IMAX2$
$NVAR1$ $NVAR12$ $NVAR2$
$NUM(I)$, I=1,...,$NVAR1$
$NUM(I)$, $I=NVAR1+1,...,NVAR1+NVAR12$
$NUM(I)$, $I=NVAR1+NVAR12+1,...,NVAR1+NVAR12+NVAR2$
$NVMU1$ $NVMU2$
$NUM(I)$, $I=NVAR1+NVAR12+NVAR2+1,...,$
$NVAR1+NVAR12+NVAR2+NVMU1$
$NUM(I)$, $I=NVAR1+NVAR12+NVAR2+NVMU1+1,...,$
$NVAR1+NVAR12+NVAR2+NVMU1+NVMU2$
$IGAM1$ $IGAM2$
$IRHO$
$IWGHT$

Each row of items listed above must start on a new line. Except for the data format statement ($DATFMT$) and variable names ($NAME(I)$,I=1,$NVAR$) all items are given in free format (i.e., separated by commas or spaces), and may be continued on as many lines as desired. The values listed above are defined as follows:

NVARIN Number of variables in the data set to be read from DATAIN.FLE.

NVAR Number of variables returned by subroutine USER for analysis. See the instructions provided in subroutine USER for adding and recoding variables.

MODEL Model version number.

1 = Uncensored bivariate ordered-probit model.

2 = Lower-limit endogenous censoring model.

3 = Upper-limit endogenous censoring model.

4 = Discrete endogenous-switching model.

5 = Two-limit bivariate ordered-probit model.

IREAD Format of the data.

1 = Formatted, integer data. Must provide *DATFMT*.

2 = Formatted, real data. Must provide *DATFMT*.

3 = Free format, integer data.

4 = Free format, real data.

INAME Equals 1 if variable names are to be specified by the user, equals 0 otherwise. If *INAME*=0 then *VAR NO* is the prefix assigned as the variable name for all variables, which are then identified only by number.

XMISS The missing data code required by subroutine USER for selecting the cases for analysis. Only cases with missing data for the variables that are actually used in estimating the model will be rejected.

ISTART Equals 1 if the start values for the maximum-likelihood iterations are read from VALUES.FLE, equals 0 if start values are read from START.FLE. This must be set to 0 for the first attempt to estimate a given model specification. At the end of each run the current parameter values and estimated information matrix are output to VALUES.FLE.

IDFP Equals 1 if technical output from *DFP* is to be written to OUTPUT.FLE, 0 otherwise. Technical output includes a summary of steepest-descent and Davidon-Fletcher-Powell iterations.

MXITER Maximum number of iterations. If convergence is not achieved in *MXITER* iterations, the results at that point are printed and the current values of the parameters and hessian matrix are output to VALUES.FLE. The program can be restarted by setting *ISTART*=1 if additional iterations are desired.

DATFMT Fortran format in 80 characters or less. Include only if *IREAD*=1 *or* 2. Must be enclosed in parentheses. For example, (2*X*,20*I*6,4*X*,3*I*4), if *IREAD*=1. Data must be all integers or all real numbers.

$NAME(I)$	Variable names in $(10A8)$ format for I=1,...,$NVAR$. A constant term should be included in the data set or created in subroutine $USER$.
$IG1$	Variable number of first dependent variable when $MODEL$=1, 2, *or* 3. When $MODEL$=4 then $IG1$ is the variable number of the observed values of $\min(Y_1^*, Y_2^*)$.
$IG2$	Variable number of second dependent variable when $MODEL$=1, 2, *or* 3. When $MODEL$=4 then $IG2$ is the variable number of the indicator of which equation generated the observed values of $\min(Y_1^*, Y_2^*)$. This variable should take the value 1 or 2 depending on whether the observation is from equation 1 or equation 2.
$IG1L$	Variable number of lower-limit $Y_1^{\min}$ for Y_1^* when $MODEL$=5.
$IG1U$	Variable number of upper-limit $Y_1^{\max}$ for Y_1^* when $MODEL$=5.
$IG2L$	Variable number of lower-limit $Y_2^{\min}$ for Y_2^* when $MODEL$=5.
$IG2U$	Variable number of upper-limit $Y_2^{\max}$ for Y_2^* when $MODEL$=5.
$IMAX1$	Maximum value of the first dependent variable.
$IMAX2$	Maximum value of the second dependent variable.
$NVAR1$	Number of explanatory variables that appear ONLY in the first structural equation.
$NVAR12$	Number of explanatory variables (including constants) that appear in BOTH structural equations. Do not double count -- if the constant term is the only variable common to both equations, then $NVAR12$=1.
$NVAR2$	Number of explanatory variables that appear ONLY in the second structural equation.
$NUM(I)$	Explanatory variable numbers for X_1 and X_2 given in three sets. The first set are the numbers of the $NVAR1$ variables appearing only in structural equation 1. The second set are the numbers of the $NVAR12$ variables appearing in both structural equations. The third set are the numbers of the $NVAR2$ variables appearing only in structural equation 2. The list for each set must start on a new line. If $NVAR1$, $NVAR12$, or $NVAR2$ are zero, no numbers are provided for the corresponding line. BLANK LINES SHOULD NOT BE INCLUDED.
$NVMU1$	Number of explanatory variables for the first-equation threshold values. If $IMAX1$=1 then the program automatically sets $NVMU1$=0. When $IMAX1$>1 then the list of explanatory variables for the first-equation thresholds should always include at least a constant term.

NVMU2	Number of explanatory variables for the second-equation threshold values. If *IMAX2*=1 then the program automatically sets *NVMU2*=0. When *IMAX2*>1 then the list of explanatory variables for the second-equation thresholds should always include at least a constant term.
NUM(I)	Explanatory variable numbers for X_3 and X_4 given in two sets. The first set are the numbers of the *NVMU1* explanatory variables for the first-equation threshold values. The second set are the numbers of the *NVMU2* explanatory variables for the second-equation threshold values. If *IMAX1*=1 or *IMAX2*=1 then no variable numbers are given for the corresponding line. BLANK LINES SHOULD NOT BE INCLUDED.
IGAM1	Indicator for whether Z_2^* appears in structural equation 1. If *IGAM1*=1, then γ_1 is estimated. If *IGAM1*=0, then γ_1 is constrained to zero.
IGAM2	Indicator for whether Z_1^* appears in structural equation 2. If *IGAM2*=1, then γ_2 is estimated. If *IGAM2*=0, then γ_2 is constrained to zero.
IRHO	If *IRHO*=1 then σ_{12} or ω_{12} is estimated. If *IRHO*=0, then σ_{12} or ω_{12} is constrained to be zero.
IWGHT	Number of the variable used to weight the likelihood values in estimation. If *IWGHT*=0 then estimation is unweighted. The weights are automatically scaled to sum to one.

References

Abramowitz, M. and Stegun, I.A. (1976) *Handbook of Mathematical Functions*, New York: Dover.

Calhoun, C.A. (1989a) "Estimating the Distribution of Desired Family Size and Excess Fertility," *Journal of Human Resources*, (forthcoming, Fall 1989).

Calhoun, C.A. (1989b) "Bivariate Ordered-Probit Models for Censored Data with Applications in Demography," presented at the Third Annual Conference of the European Society for Population Economics, L'Université de l'UAP, Domaine de Frémigny, Bouray sur Juine, France, June 8-10, 1989.

Calhoun, C.A. (1989c) "Desired and Excess Fertility in Europe and the United States: Indirect Estimates from World Fertility Survey Data," IIASA WP-89-xx, Laxenburg, Austria: International Institute for Applied Systems Analysis (forthcoming).

Dennis, J.E. and Schnabel, R.B. (1983) *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Englewood Cliffs: Prentice Hall.

Fair, R.C. and Jaffee, D.M. (1972) "Methods of Estimation for Markets in Disequilibrium," *Econometrica*, 42(1):177-190.

Gill, P.E.; Murray, W.; and Wright, M.H. (1981) *Practical Optimization*, New York: Academic Press.

Goldfeld, S.M. and Quandt, R.E. (1973) "The Estimation of Structural Shifts by Switching Regressions," *Annals of Economic and Social Measurement*, 2:475-485.

Gruvaeus, G.T. and Joreskog, K.G. (1970) "A Computer Program for Minimizing a Function of Several Variables," unpublished manuscript, Princeton: Educational Testing Service.

McKelvey, R.D. and Zavoina, W. (1975) "A Statistical Model for the Analysis of Ordinal Level Dependent Variables," *Journal of Mathematical Sociology*, 4:103-120.

Maddala, G.S. (1983) *Limited-Dependent and Qualitative Variables in Econometrics*, New York: Cambridge University Press.

Maddala, G.S. and Nelson, F. (1975) "Switching Regression Models with Exogenous and Endogenous Switching," *Proceedings of the American Statistical Association* (Business and Economics Section), pp. 423-426.

Muthen, B. (1979) "A Structural Probit Model with Latent Variables," *Journal of the American Statistical Association*, 74:807-11.

Muthen, B. (1983) "Latent Variable Structural Equation Modeling with Categorical Data," *Journal of Econometrics*, 22:43-65.

Owen, D. (1956) "Tables for Computing Bivariate Normal Probabilities," *Annals of Mathematical Statistics*, 27:1075-1090.

Rosett, R.N. and Nelson, F.D. (1975) "Estimation of the Two-Limit Probit Regression Model," *Econometrica*, 43(1):141-146.

Tallis, G.M. (1965) "Plane Truncation in Normal Populations," *Journal of the Royal Statistical Society, Series B*, 27(2):301-307.

Table 1
Bivariate Ordered-Probit Models
Observed Data and Likelihood Functions

Observed Data	Contribution to Sample Likelihood*
1. Uncensored Bivariate Ordered-Probit Model	
$Y_1 = Y_1^*$, $Y_2 = Y_2^*$	$\int_{A(Y_1-1)}^{A(Y_1)} \int_{B(Y_2-1)}^{B(Y_2)} \varphi(a,b;\, \omega_{12})\,db\,da$
2. Lower-Limit Endogenous Censoring Model	
$Y_1 = Y_1^*$, $Y_2 = Y_2^*$ if $Y_2^* > Y_1^*$	$\int_{A(Y_1-1)}^{A(Y_1)} \int_{B(Y_2-1)}^{B(Y_2)} \varphi(a,b;\, \omega_{12})\,db\,da$
$Y_1 = Y_1^*$, $Y_2 = Y_1^*$ if $Y_2^* \leq Y_1^*$	$\int_{A(Y_1-1)}^{A(Y_1)} \int_{-\infty}^{B(Y_2)} \varphi(a,b;\, \omega_{12})\,db\,da$
3. Upper-Limit Endogenous Censoring Model	
$Y_1 = Y_1^*$, $Y_2 = Y_2^*$ if $Y_2^* < Y_1^*$	$\int_{A(Y_1-1)}^{A(Y_1)} \int_{B(Y_2-1)}^{B(Y_2)} \varphi(a,b;\, \omega_{12})\,db\,da$
$Y_1 = Y_1^*$, $Y_2 = Y_1^*$ if $Y_2^* \geq Y_1^*$	$\int_{A(Y_1-1)}^{A(Y_1)} \int_{B(Y_2-1)}^{\infty} \varphi(a,b;\, \omega_{12})\,db\,da$
4. Discrete Endogenous-Switching Model	
$Y_1 = min(Y_1^*, Y_2^*)$ if $Y_1^* \leq Y_2^*$	$\int_{A(Y_1-1)}^{A(Y_1)} \int_{B(Y_1-1)}^{\infty} \varphi(a,b;\, \omega_{12})\,db\,da$
$Y_2 = min(Y_1^*, Y_2^*)$ if $Y_1^* > Y_2^*$	$\int_{A(Y_2)}^{\infty} \int_{B(Y_2-1)}^{B(Y_2)} \varphi(a,b;\, \omega_{12})\,db\,da$
5. Two-Limit Bivariate Ordered-Probit Model	
$Y_1^{min} \leq Y_1^* \leq Y_1^{max}$, $Y_2^{min} \leq Y_2^* \leq Y_2^{max}$	$\int_{A(Y_1^{min}-1)}^{A(Y_1^{max})} \int_{B(Y_2^{min}-1)}^{B(Y_2^{max})} \varphi(a,b;\, \omega_{12})\,db\,da$

* See Section 4 of the main text for the definitions of A(i) and B(j).

Table 2.1
Computational Formulas for Likelihood Values

Model 1: Uncensored Bivariate Ordered-Probit Model

Observed Data		**Likelihood Value***
$Y_1 = 0$	$Y_2 = 0$	$F(0,0)$
$Y_1 = 0$	$0 < Y_2 < D$	$F(0,Y_2)-F(0,Y_2-1)$
$Y_1 = 0$	$D \le Y_2$	$G(0)-F(0,D-1)$
$0 < Y_1 < C$	$Y_2 = 0$	$F(Y_1,0)-F(Y_1-1,0)$
$0 < Y_1 < C$	$0 < Y_2 < D$	$F(Y_1,Y_2)-F(Y_1,Y_2-1)-F(Y_1-1,Y_2)+F(Y_1-1,Y_2-1)$
$0 < Y_1 < C$	$D \le Y_2$	$G(Y_1)-G(Y_1-1)-F(Y_1,D-1)+F(Y_1-1,D-1)$
$C \le Y_1$	$Y_2 = 0$	$H(C-1)-F(C-1,0)$
$C \le Y_1$	$0 < Y_2 < D$	$H(Y_2)-H(Y_2-1)-F(C-1,Y_2)+F(C-1,Y_2-1)$
$C \le Y_1$	$D \le Y_2$	$1-G(C-1)-H(D-1)+F(C-1,D-1)$

* The functions F(i,j), G(i), and H(j) are the cumulative normal probabilities given by

$$F(i,j) = \Phi(A(i),B(j);\omega_{12})$$
$$G(i) = \Phi(A(i))$$
$$H(j) = \Phi(B(j))$$

where $\Phi(a,b;\omega_{12})$ is the bivariate standard-normal *cdf* with correlation parameter ω_{12}, $\Phi(a)$ is the univariate standard-normal *cdf*,

$$A(i) = X_3\mu_i - \frac{X_1\beta_1 + \gamma_1 X_2\beta_2}{1-\gamma_1\gamma_2}$$

$$B(j) = X_4\delta_j - \frac{X_2\beta_2 + \gamma_2 X_1\beta_1}{1-\gamma_1\gamma_2}$$

$$\omega_{12} = \frac{(1+\gamma_1\gamma_2)\sigma_{12}+\gamma_1\sigma_{22}+\gamma_2\sigma_{11}}{(1-\gamma_1\gamma_2)^2}$$

and $\mu_{-1}=\delta_{-1}=-\infty$, $\mu_0=\delta_0=0$, and $\mu_C=\delta_D=\infty$. The structural-variance parameters σ_{11} and σ_{22} satisfy the following restrictions:

$$\sigma_{11} = \frac{(1-\gamma_1\gamma_2)^2(1-\gamma_1^2) + (2\gamma_2\gamma_1^2-2\gamma_1)\sigma_{12}}{1-\gamma_1^2\gamma_2^2}$$

$$\sigma_{22} = \frac{(1-\gamma_1\gamma_2)^2(1-\gamma_2^2) + (2\gamma_1\gamma_2^2-2\gamma_2)\sigma_{12}}{1-\gamma_1^2\gamma_2^2}.$$

Table 2.2
Computational Formulas for Likelihood Values

Model 2: Lower-Limit Endogenous Censoring Model

Observed Data		Likelihood Value*
$Y_1 = 0$	$Y_2 = 0$	$F(0,0)$
$Y_1 = 0$	$0 < Y_2 < D$	$F(0, Y_2) - F(0, Y_2 - 1)$
$Y_1 = 0$	$D \leq Y_2$	$G(0) - F(0, D-1)$
$0 < Y_1 < C$	$0 < Y_2 < D$ $Y_2 > Y_1$	$F(Y_1, Y_2) - F(Y_1, Y_2 - 1) - F(Y_1 - 1, Y_2) + F(Y_1 - 1, Y_2 - 1)$
$0 < Y_1 < C$	$0 < Y_2 < D$ $Y_2 \leq Y_1$	$F(Y_1, Y_1) - F(Y_1 - 1, Y_1)$
$0 < Y_1 < C$	$D \leq Y_2$ $Y_2 > Y_1$	$G(Y_1) - G(Y_1 - 1) - F(Y_1, D-1) + F(Y_1 - 1, D-1)$
$0 < Y_1 < C$	$D \leq Y_2$ $Y_2 \leq Y_1$	$G(Y_1) - G(Y_1 - 1)$
$C \leq Y_1$	$Y_2 < D$ $Y_2 \leq Y_1$	$H(C-1) - F(C-1, 0)$
$C \leq Y_1$	$0 < Y_2 < D$ $Y_2 > Y_1$	$H(Y_2) - H(Y_2 - 1) - F(C-1, Y_2) + F(C-1, Y_2 - 1)$
$C \leq Y_1$	$D \leq Y_2$ $Y_2 > Y_1$	$1 - G(C-1) - H(D-1) + F(C-1, D-1)$
$C \leq Y_1$	$D \leq Y_2$ $Y_2 \leq Y_1$	$1 - G(Y_1 - 1)$

* See Table 2.1 for definitions of F(i,j), G(i), and H(j).

Table 2.3
Computational Formulas for Likelihood Values

Model 3: Upper-Limit Endogenous Censoring Model

Observed Data		**Likelihood Value***
$Y_1 = 0$	$0 \leq Y_2$	$G(0)$
$0 < Y_1 < C$	$Y_2 = 0$	$F(Y_1,0)-F(Y_1-1,0)$
$0 < Y_1 < C$	$0 < Y_2 < D$ $Y_2 < Y_1$	$F(Y_1,Y_2)-F(Y_1,Y_2-1)-F(Y_1-1,Y_2)+F(Y_1-1,Y_2-1)$
$0 < Y_1 < C$	$0 < Y_2 < D$ $Y_2 \geq Y_1$	$G(Y_1)-G(Y_1-1)-F(Y_1,Y_1-1)+F(Y_1-1,Y_1-1)$
$0 < Y_1 < C$	$D \leq Y_2$	$G(Y_1)-G(Y_1-1)-F(Y_1,D-1)+F(Y_1-1,D-1)$
$C \leq Y_1$	$Y_2 \leq 0$	$H(C-1)-F(C-1,0)$
$C \leq Y_1$	$0 < Y_2 < D$ $Y_2 < Y_1$	$H(Y_2)-H(Y_2-1)-F(C-1,Y_2)+F(C-1,Y_2-1)$
$C \leq Y_1$	$0 < Y_2 < D$ $Y_2 \geq Y_1$	$1-G(C-1)-H(C-1)+F(C-1,C-1)$
$C \leq Y_1$	$D \leq Y_2$ $Y_2 \geq Y_1$	$1-G(C-1)-H(D-1)+F(C-1,D-1)$

* See Table 2.1 for definitions of F(i,j), G(i), and H(j).

Table 2.4
Computational Formulas for Likelihood Values

Model 4: Discrete Endogenous-Switching Model

Observed Data*		**Likelihood Value****
$Y_1 = min(Y_1^*, Y_2^*)$	$Y_1 = 0$	$G(0)$
$Y_1 = min(Y_1^*, Y_2^*)$	$0 < Y_1 < C$ $Y_1 < D$	$G(Y_1)-G(Y_1-1)-F(Y_1,Y_1-1)+F(Y_1-1,Y_1-1)$
$Y_1 = min(Y_1^*, Y_2^*)$	$0 < Y_1 < C$ $Y_1 \geq D$	$G(Y_1)-G(Y_1-1)-F(Y_1,D-1)+F(Y_1-1,D-1)$
$Y_1 = min(Y_1^*, Y_2^*)$	$C \leq Y_1$ $C < D$	$1-G(C-1)-H(C-1)+F(C-1,C-1)$
$Y_1 = min(Y_1^*, Y_2^*)$	$C \leq Y_1$ $C \geq D$	$1-G(C-1)-H(D-1)+F(C-1,D-1)$
$Y_2 = min(Y_1^*, Y_2^*)$	$Y_2 = 0$	$H(0)$
$Y_2 = min(Y_1^*, Y_2^*)$	$0 < Y_2 < D$ $Y_2 < C$	$H(Y_2)-H(Y_2-1)-F(Y_2,Y_2)+F(Y_2,Y_2-1)$
$Y_2 = min(Y_1^*, Y_2^*)$	$0 < Y_2 < D$ $Y_2 \geq C$	$H(Y_2)-H(Y_2-1)-F(C-1,Y_2)+F(C-1,Y_2-1)$
$Y_2 = min(Y_1^*, Y_2^*)$	$D \leq Y_2$ $D < C$	$1-G(D)-H(D-1)+F(D,D-1)$
$Y_2 = min(Y_1^*, Y_2^*)$	$D \leq Y_2$ $D \geq C$	$1-G(C-1)-H(D-1)+F(C-1,D-1)$

* $Y_1=min(Y_1^*,Y_2^*)$ when $Y_1^* \leq Y_2^*$ and $Y_2=min(Y_1^*,Y_2^*)$ when $Y_1^* > Y_2^*$.

** See Table 2.1 for definitions of F(i,j), G(i), and H(j).

Table 2.5
Computational Formulas for Likelihood Values

Model 5: Two-Limit Bivariate Orderd-Probit Model

Observed Data		Likelihood Value*
$Y_1^{\min} = 0$ $Y_1^{\max} < C$	$Y_2^{\min} = 0$ $Y_2^{\max} < D$	$F(Y_1^{\max}, Y_2^{\max})$
$Y_1^{\min} = 0$ $Y_1^{\max} < C$	$0 < Y_2^{\min} < D$ $0 < Y_2^{\max} < D$	$F(Y_1^{\max}, Y_2^{\max}) - F(Y_1^{\max}, Y_2^{\min} - 1)$
$Y_1^{\min} = 0$ $0 < Y_1^{\max} < C$	$0 < Y_2^{\min}$ $D \leq Y_2^{\max}$	$G(Y_1^{\max}) - F(Y_1^{\max}, Y_2^{\min} - 1)$
$0 < Y_1^{\min} < C$ $0 < Y_1^{\max} < C$	$Y_2^{\min} = 0$ $Y_2^{\max} < D$	$F(Y_1^{\max}, Y_2^{\max}) - F(Y_1^{\min} - 1, Y_2^{\max})$
$0 < Y_1^{\min} < C$ $0 < Y_1^{\max} < C$	$0 < Y_2^{\min} < D$ $0 < Y_2^{\max} < D$	$F(Y_1^{\max}, Y_2^{\max}) - F(Y_1^{\max}, Y_2^{\min} - 1) - F(Y_1^{\min} - 1, Y_2^{\max})$ $+ F(Y_1^{\min} - 1, Y_2^{\min} - 1)$
$0 < Y_1^{\min}$ $0 < Y_1^{\max} < C$	$0 < Y_2^{\min}$ $D \leq Y_2^{\max}$	$G(Y_1^{\max}) - G(Y_1^{\min} - 1) - F(Y_1^{\max}, Y_2^{\min} - 1)$ $+ F(Y_1^{\min} - 1, Y_2^{\min} - 1)$
$0 < Y_1^{\min}$ $C \leq Y_1^{\max}$	$Y_2^{\min} = 0$ $Y_2^{\max} < D$	$H(Y_1^{\min} - 1) - F(Y_1^{\min} - 1, Y_2^{\max})$
$0 < Y_1^{\min}$ $C \leq Y_1^{\max}$	$0 < Y_2^{\min} < D$ $0 < Y_2^{\max} < D$	$H(Y_2^{\max}) - H(Y_2^{\min} - 1) - F(Y_1^{\min} - 1, Y_2^{\max})$ $+ F(Y_1^{\min} - 1, Y_2^{\min} - 1)$
$0 < Y_1^{\min}$ $C \leq Y_1^{\max}$	$0 < Y_2^{\min}$ $D \leq Y_2^{\max}$	$1 - G(Y_1^{\min} - 1) - H(Y_2^{\min} - 1) + F(Y_1^{\min} - 1, Y_2^{\min} - 1)$
$0 < Y_1^{\min}$ $C \leq Y_1^{\max}$	$Y_2^{\min} = 0$ $D \leq Y_2^{\max}$	$1 - G(Y_1^{\min} - 1)$
$0 < Y_1^{\min} < C$ $0 < Y_1^{\max} < C$	$Y_2^{\min} = 0$ $D \leq Y_2^{\max}$	$G(Y_1^{\max}) - G(Y_1^{\min} - 1)$
$Y_1^{\min} = 0$ $Y_1^{\max} < C$	$Y_2^{\min} = 0$ $D \leq Y_2^{\max}$	$G(Y_1^{\max})$
$0 < Y_1^{\min}$ $C \leq Y_1^{\max}$	$0 < Y_2^{\min}$ $D \leq Y_2^{\max}$	$1 - H(Y_2^{\min} - 1)$
$Y_1^{main} = 0$ $C \leq Y_1^{\min}$	$0 < Y_2^{\min} < C$ $0 < Y_2^{\max} < D$	$H(Y_2^{\max}) - H(Y_2^{\min} - 1)$
$Y_1^{\min} = 0$ $C \leq Y_1^{\max}$	$Y_2^{\min} = 0$ $Y_2^{\max} < D$	$H(Y_2^{\max})$
$Y_1^{\min} = 0$ $C \leq Y_1^{\max}$	$Y_2^{\min} = 0$ $D \leq Y_2^{\max}$	1

* See Table 2.1 for definitions of F(i,j), G(i), and H(j).

Recent Working Papers Produced in IIASA's Population Program

Copies may be obtained at a cost of US $ 5.00 each from IIASA's Publications Department.

WP-88-10, *On the Concentration of Childbearing in China, 1955–1981* by W. Lutz. February 1988.

WP-88-13, *Beyond "The Average American Family": U.S. Cohort Parity Distributions and Fertility Concentration* by M. King and W. Lutz. March 1988.

WP-88-23, *Understanding Medical and Demographic Trends with MEDDAS* by M. Rusnak and S. Scherbov. April 1988.

WP-88-32, *Kinship Patterns and Household Composition of the Elderly: Hungarian Women, 1984* by D. Wolf. April 1988.

WP-88-36, *"DIAL" – A System for Modeling Multidimensional Demographic Processes* by S. Scherbov and V. Grechucha. May 1988.

WP-88-44, *Kin Availability and the Living Arrangements of Older Unmarried Women: Canada, 1985* by D. Wolf, T. Burch, and B. Matthews. June 1988.

WP-88-46, *Population Futures for Europe: An Analysis of Alternative Scenarios*, by D. Wolf, B. Wils, W. Lutz, and S. Scherbov. June 1988.

WP-88-90, *Comparative analysis of Completed Parity Distributions: A Global WFS-Perspective*, by W. Lutz. October 1988.

WP-88-104, *Future Regional Population Patterns in the Soviet Union: Scenarios to the Year 2050*, by S. Scherbov and W. Lutz. November 1988.

WP-88-120, *AIDS and HIV Surveillance in Europe*, by M. Artzrouni and G. Heilig. December 1988.

WP-88-124, *DDMSLT: A Computer Program for Estimating the Duration-Dependent Multistate Life Table Model*, by C. Calhoun. December 1988.

WP-89-05, *Multi-State Analysis of Family Dynamics in Austria: Scenarios to the Year 2030*, by W. Lutz and S. Scherbov. January 1989.

WP-89-06, *The Demographic Dimensions of Divorce: The Case of Finland*, by W. Lutz, B. Wils, and M. Nieminen. January 1989.

WP-89-18, *Markets as Queues, with an Application to Education*, by R. Boylan. February 1989.

WP-89-19, *Living Arrangements and Family Networks of Older Women in Italy*, by D. Wolf and A. Pinnelli. February 1989.

WP-89-27, *Reconciling Economic and Ecological Theory on Population*, by Nathan Keyfitz. March 1989.

WP-89-28, *Multistate Life Table with Duration-Dependence: An Application to Hungarian Female Marital History*, by Alain Belanger. April 1989.

WP-89-34, *Recent Trends in Living Arrangements in Fourteen Industrialized Countries*, by J.-P. Gonnot and G. Vukovich. May 1989.

WP-89-35, *Averaging Life Expectancy*, by E. Andreev, W. Lutz, and S. Scherbov. June 1989.

WP-89-37, *Measuring Fertility Responses to Policy Measures in the German Democratic Republic*, by T. Büttner and W. Lutz. June 1989.