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Foreword

The simulation of production costs for power systems is a key factor in the capacity expansion as well as some other questions raised in connection with planning for power systems. This paper develops a probabilitic model for production cost simulation that is able to handle uncertainty of both generating units and peak load forecast.

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PRODUCTION COSTING SIMULATION IN THERMAL POWER SYSTEMS USING THE MIXTURE OF CONDITIONAL LOAD DISTRIBUTION FUNCTIONS

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Abstract: In the 1980's the cumulant method became popular in reliability type algorithms for production cost evaluation, particularly in the evaluation of loss-of-load probability (LOLP), energy not served (ENS) and expected energy generation (EEG) of a set of generating units belonging to an electric power system. We developed a probabilistic model which is able to handle the uncertainty of both generating units and peak load forecast. In order to model the load including peak load forecast uncertainty we use conditional probability distributions. We show that the cumulant method is still applicable, as we can compute all the moments of the load duration curve (load distribution) without discretizing the density function of peak load forecast.

Keywords: electric power system, production cost simulation,

I. INTRODUCTION

For a thermal power system the prediction of the expected energy generation of the units, the loss-of-load probability and the expected value of unserved energy are important aspects both in system operation and planning.

For a more realistic and accurate simulation of system operation, multiblock representation of units' forced outage was introduced (see [1], [3], [5], [6]). However in the literature the maximum load level is usually fixed at a given value by the load duration curve. It means that the probability with which load exceeds the given value is equal to zero. With this model (representation) of the load duration curve the peak load forecast uncertainty and extreme load values cannot be taken into account.

We point out here that in the paper we write about load duration curve which is often called in the literature as inverted, normalised load duration curve. In the figure, system load (the argument of the function) is shown on the horizontal axis and probability with which load exceeds the corresponding load value (dependent variable) is shown on the vertical axis.

It is very important that we have an accurate approximation for the distribution of peak load values (the tail of the load duration curve), as it can have influence on the maintenance scheduling plan. It is obvious that fewer changes of the load duration curve, around the minimum load value, doesn't modify the number of units to be loaded, only the expected energy generation of some units changes. On the contrary, if we perturbe the load duration curve around the peak load value, keeping its original shape, the number of units to be loaded changes in order to meet the prescribed LOLP limit. This fact can significantly influence the maintenance scheduling plan.

In the model of the paper we suppose that the tail of the load duration curve (i.e. the maximum value of load) can change. The load duration curve, for a given peak load value, is a piecewise linear function, joining three parts. The first two parts are the same for all values of peak load. The peak load value can change according to a well-known distribution (e.g. uniform or exponential). Above the first interval the load duration curve is equal to 1. The second interval represents the expected base load domain of the power system, the third one simulates peak load forecast uncertainty (see Figure 1.).

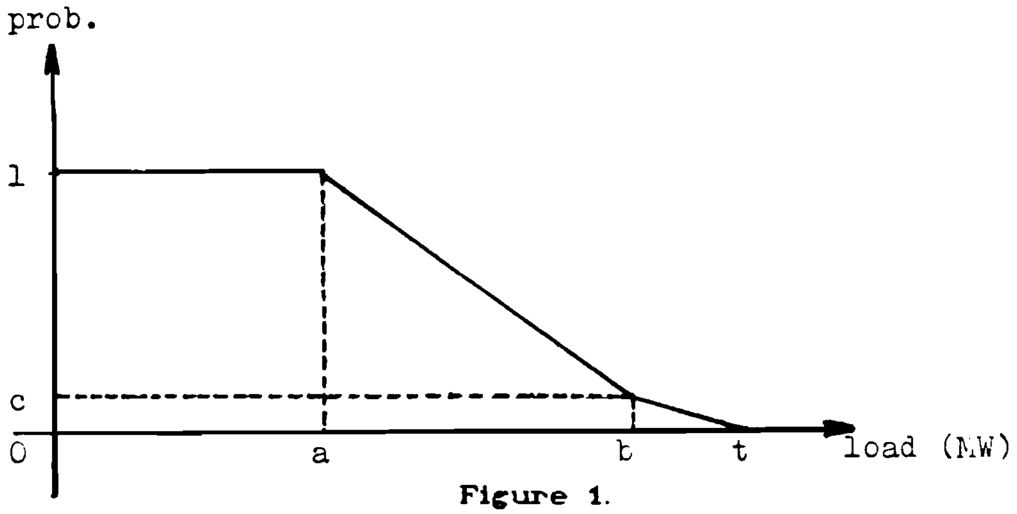

Figure 1.

The parameters of the load duration curve (minimum load value, expected maximum value, the probability of the event that the load exceeds the previously mentioned value, parameter(s) of the distribution of peak load forecast) can be defined by the energy planner.

Using this representation the exact shape of the load duration curve becomes unknown, but the cumulant method is still applicable, as we can compute all the moments of the resulting load distribution.

At the Hungarian Electricity Board a program package has been implemented on an IBM PC XT or AT to check the system reliability level. The user of the package is enabled to simulate several availability-situations of the power system, by setting units available or unavailable, changing the maintenance scheduling plan, the parameters of the load duration curve.

The realistic and easy-to-handle choice for the distribution of peak load forecast is as follows:

- uniform distribution (with given maximum load or with given mean of the maximum load),
- exponential distribution (with given mean of the maximum load or with given right endpoint of the interval the probability of which is greater than 0.999.

II. MOMENTS OF THE RESULTING LOAD DURATION CURVE

Let X be the random variable representing the load, T be the random variable of peak load forecast, $p(t)$ be the probability density function of T with existing moments $(j=1,\dots,J)$:

$$p(t) \geq 0,$$

$$\int_{-\infty}^{\infty} p(t)\, dt = 1,$$

$$\int_{-\infty}^{\infty} t^j p(t)\, dt < +\infty, \qquad j=1,\dots,J.$$

Let $LDC(x)$, $F(x)$, and $f(x)$ denote the load duration, the load distribution and the load density function, respectively:

$$F(x) = 1 - LDC(x),$$

$$f(x) = F'(x).$$

Let $LDC(x|t)$ be the conditional load duration function (supposing $T = t$) and denote by X_t the corresponding random variable. Then the load duration function $LDC(x)$ is the integral of load duration functions $LDC(x|t)$ depending on parameter t. Using the following notations:

$$F(x|t) = 1 - LDC(x|t),$$

$$f(x|t) = \frac{dF(x|t)}{dt},$$

we have:

$$LDC(x) = \int_{-\infty}^{\infty} LDC(x|t)\, p(t)\, dt, \tag{1}$$

$$F(x) = \int_{-\infty}^{\infty} F(x|t)\ p(t)\ dt,$$

$$f(x) = \int_{-\infty}^{\infty} f(x|t)\ p(t)\ dt.$$

It is obvious that the explicit form of the load duration curve depends on the probability density function $p(t)$, and we are seldom able to transform it to a quite simple formula.

In order to use the cumulant method we need only the cumulants of the random variable representing the load. Cumulants are polynomials of the central moments and thus polynomials of the moments, as well (see Kendall and Stuart [2]). Therefore we need only the moments of the random variable of the load:

$$M(X^k) = \int_{-\infty}^{\infty} x^k \left[\int_{-\infty}^{\infty} f(x|t)\ p(t)\ dt \right] dx. \qquad (2)$$

By virtue of the Fubini theorem (see Rudin [4]) the order of integrals can be changed in (2), if the k-th moment ($k=1,\dots,K$) of the variable X_t exists and it is finite for all possible t value; i.e. if the reversed integral is finite. This last assumption holds in the cases we examine later. By changing the order of the integral we have the following formula:

$$M(X^k) = \int_{-\infty}^{\infty} p(t) \left[\int_{-\infty}^{\infty} x^k f(x|t)\ dx \right] dt.$$

As the inner integral is equal to $M(X_t^k)$:

$$M(X^k) = \int_{-\infty}^{\infty} p(t)\ M(X_t^k)\ dt. \qquad (3)$$

There are several cases when it is quite easy to compute $M(X^k)$. One of them is as follows: $M(X_t^k)$ is a polynomial of the variable t and we know all the needed moments of the random variable T (see Example 1. and Example 2. below). If

$$M(X_t^k) = \sum_{j=1}^{n_k} c_{kj} t^j$$

and

$$max \{n_k : k=1,\dots,K\} \le J$$

hold, then

$$M(X^k) = \int_{-\infty}^{\infty} p(t) \sum_{j=1}^{n_k} c_{kj} t^j \, dt,$$

$$M(X^k) = \int_{\infty}^{\infty} \sum_{j=1}^{n_k} c_{kj} \, t^j \, p(t) \, dt,$$

$$M(X^k) = \sum_{j=1}^{n_k} c_{kj} \int_{-\infty}^{\infty} t^j \, p(t) \, dt,$$

$$M(X^k) = \sum_{j=1}^{n_k} c_{kj} \, M(T^j) \, dt.$$

As illustration, consider the following examples:

Example 1.: X_t is of normal distribution, t can be either the mean or the standard deviation of X_t. Denote the k-th moment by m_k and the variance of the distribution by s^2. Then

$$m_2 = s^2 + m_1^2 ,$$

and

$$m_k = m_1 m_{k-1} + s^2 (k-1) m_{k-2}$$

are valid, and by using the above recursion-formula, it is easy to see that $M(X_t^k)$ is the polynomial of either the mean or the standard deviation.

Example 2.: Let X_t be of uniform distribution, and t one of the two endpoints of the interval of possible values. If the interval in question is $[r,s]$, then

$$M(X_t^k) = \frac{(s^{k+1}-r^{k+1})}{(s-r)(k+1)}$$

$$M(X_t^k) = \frac{s^k+s^{k-1}r+...+sr^{k-1}+r^k}{k+1},$$

which is a polynomial of either r or s.

Example 3.: Let X_t be of exponential distribution the parameter of which is t, T be of uniform distribution on the interval $[r,s]$ $(r,s$ are fixed). Then

$$M(X_t^k) = \frac{k!}{t^k},$$

$$M(X^k) = \int_r^s \frac{k!}{t^k(s-r)} dt,$$

$$M(X^k) = \frac{k!}{s-r} \int_r^s t^{-k} dt.$$

$$M(X^k) = \begin{cases} \dfrac{\ln s - \ln r}{s - r}, & \text{for } k = 1, \\[2ex] \dfrac{-k!}{(k-1)(s-r)} \left[\dfrac{1}{s^{k-1}} - \dfrac{1}{r^{k-1}}\right], & \text{for } k > 1. \end{cases}$$

III. SPECIFICATION OF $LDC(x|t)$ AND T USED IN THE PROGRAM PACKAGE

In our model construction: let a and b be the endpoints of the interval where the load is simply uniformly distributed. Let c $(0<c<1)$ be the probability of the event of the load being greater than b or equal to it. In order to follow the nature of the practical problem, suppose that

$$p(t) = 0, \quad \text{for} \quad t < b.$$

We can define the load duration function $LDC(x|t)$ as follows (see Figure 1.):

$$LDC(x|t) = \begin{cases} 1, & \text{for} \quad x < a, \\ 1 + (x-a)(c-1)/(b-a), & \text{for} \quad a \le x < b, \\ c - c(x-b)/(t-b), & \text{for} \quad b \le x < t, \\ 0, & \text{for} \quad x \ge t. \end{cases}$$

Then applying (1) we have

$$LDC(x) = \begin{cases} 1, & \text{for} \quad x < a, \\ 1 + (x-a)(c-1)/(b-a), & \text{for} \quad a \le x < b, \\ \int_{-\infty}^{\infty} [c - c(x-b)/(t-b)]\, p(t)\, dt, & \text{for} \quad x \ge b. \end{cases}$$

For a fixed t, $f(x|t)$ is as follows:

$$f(x|t) = \begin{cases} (1-c)/(b-a), & \text{for} \quad a \le x < b, \\ c/(t-b), & \text{for} \quad b \le x < t, \\ 0 & \text{otherwise}. \end{cases}$$

Moments of X_t can be expressed as follows:

$$M(X_t^k) = \int_{-\infty}^{\infty} x^k f(x|t)\, dx,$$

$$M(X_t^k) = \int_a^b x^k \frac{1-c}{b-a} dx + \int_b^t x^k \frac{c}{t-b} dx =$$

$$= \frac{1-c}{b-a} \int_a^b x^k dx + \frac{c}{t-b} \int_b^t x^k dx =$$

$$= \frac{(1-c)(b^{k+1}-a^{k+1})}{(k+1)(b-a)} + \frac{c(t^{k+1}-b^{k+1})}{(k+1)(t-b)} =$$

$$= \frac{(1-c)(b^{k+1}-a^{k+1})}{(k+1)(b-a)} + \frac{c(t^k+t^{k-1}b+...+tb^{k-1}+b^k)}{k+1} .$$

Consequently the moments of X can be expressed by means of the moments of T. Substituing in (3) we obtain:

$$M(X^k) = \frac{(1-c)(b^{k+1}-a^{k+1})}{(k+1)(b-a)} +$$

$$+ \frac{c}{k+1} \left[M(T^k)+M(T^{k-1})b+...+M(T)b^{k-1}+b^k\right].$$

In the program package T is supposed to be of exponential distribution, the possible values of which are greater than b or equal to it. Let d $(d>0)$ be the parameter of T. In this case the density function is

$$p(t) = \begin{cases} 0, & \text{for } t < b, \\ d\, exp(-d(t-b)), & \text{for } t \geq b. \end{cases}$$

We think that the assumption of exponentiality is close to the nature of the peak load distribution. We suppose that T could be modellised with uniform distribution as well, but we have no numerical experience for this case yet.

The expected value of T is

$$M(T) = b + \frac{1}{d},$$

and the reader can easily verify (using the method of partial integration) the following recursion formula

$$M(T^j) = b^j + \frac{j}{d} M(T^{j-1}), \qquad \text{for} \quad j \geq 2.$$

This completes the details of computation of $M(X^k)$.

In order to speed up the calculations we used a recursion formula for the quantities

$$v_k = \frac{b^k - a^k}{b - a}$$

and

$$w_k = M(T^{k-1}) + M(T^{k-2})b + \ldots + b^{k-1}$$

as well. It is obvious that

$$v_{k+1} = v_k a + b^k,$$

$$w_{k+1} = M(T^k) + w_k b.$$

Since

$$M(X^k) = \frac{(1-c)(b^{k+1} - a^{k+1})}{(k+1)(b-a)} +$$

$$+ \frac{c}{k+1} \left[M(T^k) + M(T^{k-1})b + \ldots + M(T)b^{k-1} + b^k \right],$$

we obtain

$$M(X^k) = \frac{(1-c)v_{k+1} + cw_{k+1}}{k+1}.$$

$M(X^k)$ can be expressed by means of v_k and w_k as well:

$$M(X^k) = \frac{(1-c)(v_k a + b^k) + c[M(T^k) + w_k b]}{k+1}.$$

IV. CONCLUSIONS

Following the method described in the paper we need not discretize the density function $p(t)$, and in this way there is no need for evaluating loss-of-load probability, energy not served and expected energy generation of units repeatedly for all the impulse values of peak load forecast.

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