COMMENTS ON THE BUDWORM FOR FOREST ECOLOGY MODEL

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Comments on the Budworm

For Forest Ecology Model

The simulation model described by STANDER is being studied to see of it can be restated as a linear programming model.

As a first step I investigated the relationships embodied in Figures 4 and 5, which give the probabilities and the number of eggs surviving to third instars as a function of their density per 10 square feet of foliage.

If it cannot be formulated as a linear program, can a variant of such a model be developed, which will make it possible to practically compute an optimal policy?

For extremely low worm density of eggs, foliage is not limiting and the survival rate depends on other factors. The input-out model is quite simple:

eggs	1
foliage	α_o
instars	$-p_o$

which may be interpreted if we input one egg and α_o square feet (or more) of foliage per egg, then p_o is the proportion that survive to become third instar larvae.

These proportions hold for x_o eggs as long as

$$\alpha_o \cdot x_o \leq 10 \text{ (sq. ft. of foliage)}$$

Once, however, there is a competition for foliage among the worms the input of foliage per worm drops as well as the probability of survival. At density $\theta \geq \frac{10}{\alpha_o}$ the input-output coefficients are

eggs	1
foliage	α_θ
instars	$-p_\theta$

According to Fig. 5, the empirical data assumes

$$\text{For large } \theta \begin{cases} \theta . a_\theta = 10 & \text{or} \quad a_\theta = 10/\theta \\ \theta . p_\theta = \text{Constant} & \text{or} \quad p_\theta = \text{constant} / \theta \end{cases}$$

The general law appears to be that as the food supply per worm decreases to very low levels, the probability of its survival is proportional to its food supply ($10/\theta$).

For intermediate levels of density θ, Figure 4 states $P_\theta = a - b\theta$, a straight line and the number of eggs surviving is given by $\theta . p_\theta = (a-b\theta)\theta$, (parabolic in form) as in Figure 5,

$$\text{For intermediate } \theta: \begin{cases} \theta . a_\theta = 10 \\ \theta . p_\theta = \theta(a-b\theta) \end{cases}$$

The linear program takes the discrete form for θ_o , $\theta_1, \ldots$ $\ldots, \theta_n$ where θ_n is upper bound on range of θ.

Problem: Find $x_i \geq 0$, Max z: $\sum_{i=1}^{n} x_i = \theta$

$$\sum_{i=1}^{n} \frac{10}{\theta_i} x_i \leq 10$$

$$\sum_{i=1}^{n} p_{\theta_i} x_i = z \quad \text{(Max)}$$

It is important that θ_n be set beyond the range that could ever be exceeded in an application.

If one observes the shape of the curve giving the number of eggs surviving to become third stage instars as a function of eggs density, one observes that it is not convex. This means that it can not be represented by a simple linear program and that a special variant of the simplex method would have to be used that involves the concept of specially "ordered sets" (in order to get around the non-convexity). In what follows, we assume the latter approach is tractable and would be applied. (See Figure 5 from the report,copy of which is attached).

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Figure 5. Numbers of eggs surviving to third instar stage.