Working Paper

USING PROBABILISTIC SAFETY ANALYSIS (PSA) TO OPTIMIZE OPERATION SCHEDULES

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WP-90-30
July 1990

IIASA

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Foreword

This is the first report of a work on time dependent probabilities initiated as a cooperation between the International Atomic Energy Agency (IAEA) and IIASA in 1990. The treatment of the underlying mathematical model is rather theoretical, but the intent has been to cover a broad range of applications. Originally the formulation was initiated by the problem of optimization of test intervals at nuclear power plants. There have however been also other applications proposed to be treated within the proposed modelling framework. One specific problem is the selection of the most suitable time instant for a major repair or retrofitting at a plant. The time horizon of the model can be selected either short stretching only over a few weeks or very long to encompass the complete life time of a depository of spent nuclear fuel. The advantage with the problem formulation is that it enables the inclusion also of monetary considerations connected to risks and the actions for decreasing them. The intent in formulating the model is that it will be used for a computerized optimization of selected decision variables.

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S. Uryas'ev and H. Valerga

1 Introduction

Probabilistic safety analysis (PSA) is generally used to determine weak points of a plant and make recommendations for possible design changes. The same technique can also be used to assess technical specifications and to develop guidance for operator training and accident management. The calculation of optimal test intervals for stand-by systems is one of the problems that can be solved with this approach. These intervals influence the global safety of the plant. The optimization of operation schedules is quite a complex problem due to the sophisticated dynamical interrelations of the variables and the large dimension.

Some analytical formulae exist to calculate the optimal test intervals for one isolated component (see [2, 7, 8, 15, 18, 19] and others). More advanced analytical expressions also exist for the case when the problem can be decomposed and test intervals for different groups of components can be treated independently. For example, this is the case when a group of components can be treated as one "super" component [2, 18].

There are several tools (computer codes) to assess the global unavailability of a plant, taking into account dependencies between test intervals of the components (see, for example, FRANTIC [17, 5], and SOCRATES [20] and MARELA[10]). These codes can be used to compare different variants, but not to optimize with respect to model parameters.

The International Atomic Energy Agency (IAEA) is developing and coordinating the computer code PSAPACK [11] for fault/event tree analysis. It is planned to upgrade PSAPACK to an operational safety tool [1]. In this framework, a module to help the user choose the correct values for the test intervals would be quite desirable. This module would optimize operational schedules, taking into account relationships between different groups of components. This paper briefly discusses a model and optimization technique for this module.

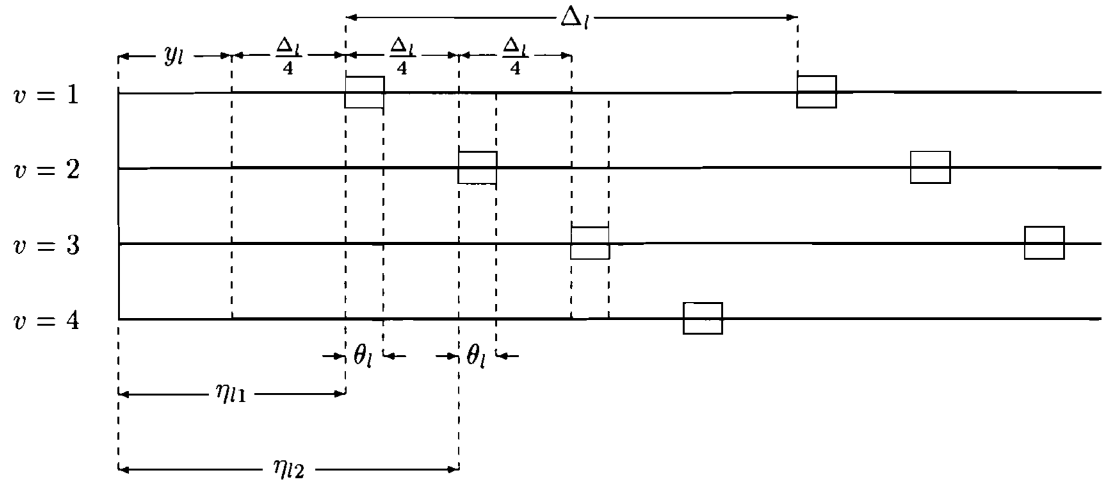

Figure 1: Time schedule for the set l.

2 General Description of the Model

We consider a global system failure in the time interval $0 \leq t \leq T$. The system consists of some components, the set of which we designate by A. To guarantee the low unavailability of the system, some groups of components should be tested periodically. The set A can be divided into $L+1$ subsets U_l $(0 \leq l \leq L)$. Here U_l $(1 \leq l \leq L)$ is a subset of components with the same interval between tests, and U_0 is a subset of components that should not be tested during the time interval $[0, T]$. The subset U_l could be divided into redundant groups of components. We suppose in this paper that during testing, a component is not available (at least not immediately). Later we shall consider components with unavailability less than one during the testing. These redundant groups should not be tested simultaneously as this could lead to high unavailability of the system. Let us designate by V_l the amount of redundant groups of components in U_l, and by v a group in U_l $(1 \leq v \leq V_l)$. We also define M_{lv} as the number of components in the group v from the set l. Let m be the component number in the group. We designate by $a_{lvm} \in A$ the component m in group v from the set l.

As an example, let us assume that for the subset U_l the amount V_l is equal to 4 (see Figure 1). The time schedule for groups of components is staggered. The time interval between the start of successive periodic tests of the same group of components is equal to Δ_l. The shortest interval between the beginning of tests of different groups is equal to Δ_l/V_l. We designate a shift in the schedule by y_l and η_{lv} is the first periodic inspection interval of group v from set l. Test intervals in set l are staggered and η_{lv} satisfies the equation (see Figure 1)

$$\eta_{lv} = y_l + \frac{\Delta_l}{V_l} v, \quad l = 1, \ldots, L, \quad v = 1, \ldots V_l, \; . \tag{1}$$

We consider that a fault tree was constructed for the plant in the standard way (see, for example, [12]). The fault tree can be represented in terms of Boolean equations; these equations can then be used to determine "minimal cut sets".

Any fault tree will consist of a finite number of minimal cut sets that are unique for the top event. The minimal cut set expression for the top event can be written in the general form

$$M = M_1 \vee M_2 \vee \cdots \vee M_W \; , \tag{2}$$

where M is the top event, and M_w ($w = 1, \ldots, W$) are the minimal cut sets. Here and below we assume that all random variables are specified on the probability space $(P, \Im, \Omega)$. Each minimal cut set consists of a combination of specific component failures, and hence the general minimal cut set can be expressed as

$$M_w = \bigwedge_{(lvm) \in C_w} \alpha_{lvm} \; ,$$

where α_{lvm} is an event of failure of the component a_{lvm}, and C_w is a set of components in the cut set w $(1 \le w \le W)$. Thus equation (2) can be represented as

$$M = \bigvee_{w=1}^{W} \bigwedge_{(lvm) \in C_w} \alpha_{lvm} \; . \tag{3}$$

Let us designate by $p_{lvm}(t)$ the unavailability of the component a_{lvm} at time t . We assume that all events α_{lvm} are mutually stochastically independent and the probability of each cut set M_w is considerably less than 1 , i.e.

$$P(M_w) \ll 1, \quad w = 1, \ldots, W \; .$$

If we take into account only linear terms, then

$$P(M) = P\left(\sum_{w=1}^{W} \prod_{(lvm) \in C_w} \alpha_{lvm} \right) \approx \sum_{w=1}^{W} \prod_{(lvm) \in C_w} P(\alpha_{lvm}) = \sum_{w=1}^{W} \prod_{(lvm) \in C_w} p_{lvm}(t) \; . \tag{4}$$

Let us define

$$p(t) = \sum_{w=1}^{W} \prod_{(lvm) \in C_w} p_{lvm}(t) \; . \tag{5}$$

Later we will use the value $p(t)$ to calculate approximately the failure probability of the whole system.

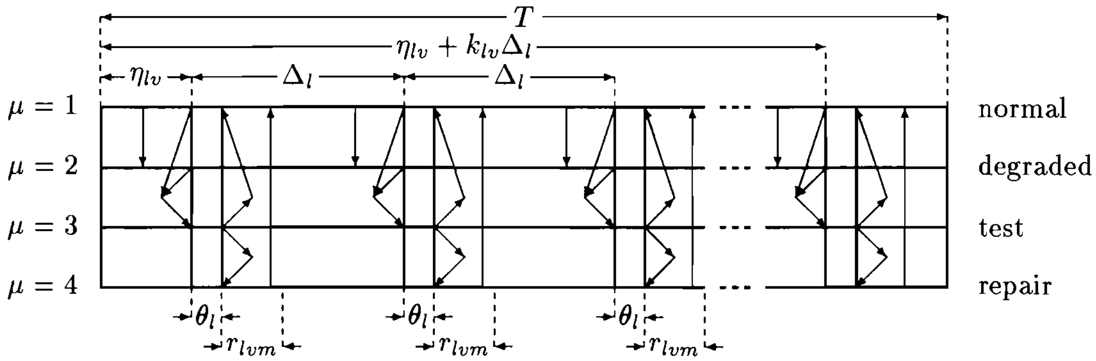

Figure 2: Time schedule for the set l.

3 Behavior of the Instantaneous Unavailability Curve for One Component

In this section we describe the instantaneous unavailability of the tested components. We consider that each component a_{lvm}, $l = 1, \ldots, L$ at time t can be in one of the following four states (see Figure 2):

- $\mu = 1$ – *normal state* (stand-by state, component is available);
- $\mu = 2$ – *degraded state* (stand-by state, with latent failure);
- $\mu = 3$ – *test* (or maintenance) *state* ;
- $\mu = 4$ – *repair state.*

A component is available in a normal state and unavailable in others. After a failure, the component enters the degraded state. To identify the failure, the component is tested periodically (test state). If it appears that the component has failed, then it is repaired (repair state).

It is assumed that in the test state ($\mu = 3$), the component is unavailable. In many practical cases the component is available (with some probability) at this state. The test by-pass may fail and the component may be unavailable (with some probability) after the test. These possibilities are not yet included in the model, but will be incorporated later.

During their lifetime, the test components periodically pass three different phases. Each phase defines an interval; for this reason we divide the time interval $0 \leq t \leq T$ into three types of intervals:

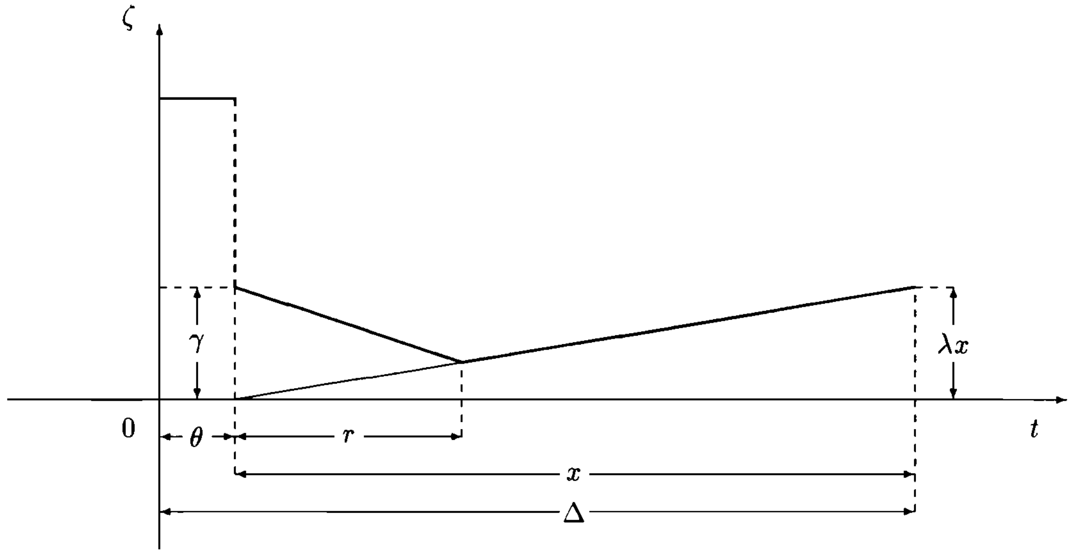

Figure 3: The function ζ.

- *Test* (or maintenance) duration *intervals* are intervals in which the component is tested (or serviced) and is in the test state ($\mu = 3$). These intervals are designated by θ_l. Here, θ_l are deterministic variables chosen by the plant operator or the designer.

- *Repair intervals*, following the test intervals, are for potential repairs. During these intervals the component could be in the normal state ($\mu = 1$), degraded state ($\mu = 2$), or repair state ($\mu = 4$). The component changes from the repair state to the normal state during repair intervals of length r_{lvm}. The component can also go from a normal state to a degraded state during these intervals.

- *Normal intervals* are intervals in which no components are being tested or repaired. During these intervals a component can be in a normal or degraded state. After a failure, it goes from a normal to a degraded state.

The random changes of states can be described by a Markov chain with continuous time and discrete states. During the normal and repair intervals, we consider that the probability of changing from state μ to state ν in the time interval δt is equal to

$$\lambda_{\nu\mu}(t)\delta t + o(\delta t) ,$$

where $o(\delta t)/\delta t \to 0$ for $\delta t \to 0$, i.e. this is a Markov nonstationary chain with continuous time, and coefficients $\lambda_{\mu\nu}(t)$. We designate by $p^{\mu}_{lvm}(t)$ the probability that the component a_{lvm} is in the state μ at time t.

On each interval $[\eta_{lv} + k\Delta_l + \theta_l, \eta_{lv} + (k+1)\Delta_l]$, probabilities satisfy the system of differential equations (see, for example, Freedman [4], Seneta [13])

$$\dot{p}^{\mu}_{lvm}(t) = \sum_{\nu=1}^{4} \lambda_{\mu\nu}(t) p^{\nu}_{lvm}(t) - p^{\mu}_{lvm}(t) \sum_{\nu=1}^{4} \lambda_{\nu\mu}(t), \quad \mu = 1, \ldots, 4 . \tag{6}$$

We define by k_{lv} the maximal number of scheduled tests for group v from set l during the time interval T

$$k_{lv} = \max\{k \colon \eta_{lv} + k\Delta_l \leq T - \Delta_l / V_l - \theta_l\}. \tag{7}$$

The initial conditions for this system of equations are changed at the points

$$\eta_{lv} + k\Delta_l + \theta_l, \quad k = 0, \ldots, k_{lv}$$

according to the time schedule. The initial conditions reflect the actions of the operational schedule. Since some coefficients in equation (6) are equal to zero, we have the system:

$$\dot{p}^{1}_{lvm}(t) = \lambda_{14}(t) p^{4}_{lvm}(t) - \lambda_{21}(t) p^{1}_{lvm}(t) , \tag{8}$$

$$\dot{p}^{2}_{lvm}(t) = \lambda_{21}(t) p^{1}_{lvm}(t) , \tag{9}$$

$$\dot{p}^{4}_{lvm}(t) = -\lambda_{14}(t) p^{4}_{lvm}(t) . \tag{10}$$

Since $p^{1}_{lvm}(t) \approx 1$, equation (9) can be approximated as

$$\dot{p}^{2}_{lvm}(t) = \lambda_{21}(t) . \tag{11}$$

For the component a_{lvm} we assume that $\lambda_{21}(t)$ is constant on the interval $[0, T]$ and is designated by λ_{lvm} . Equation (11) implies

$$p^{2}_{lvm}(t) = \lambda_{lvm}[t - (\eta_{lv} + k\Delta_l + \theta_l)] \tag{12}$$

on the interval $[\eta_{lv} + k\Delta_l + \theta_l \ , \ \eta_{lv} + (k+1)\Delta_l]$.

Since the repair of component a_{lvm} is carried out during the time r_{lvm}, the coefficient λ_{14} cannot be a constant. We assume that

$$\lambda_{14}(t) = (-t + \eta_{lv} + k\Delta_l + \theta_l + r_{lvm})^{-1}$$

on the interval $[\eta_{lv} + k\Delta_l + \theta_l \ , \ \eta_{lv} + k\Delta_l + \theta_l + r_{lvm}]$ and

$$\lambda_{14}(t) = 0$$

on the interval $[\eta_{lv} + k\Delta_l + \theta_l + r_{lvm} \ , \ \eta_{lv} + (k+1)\Delta_l]$ for $k = 0, \ldots, k_{lv}$. With this coefficient, equation (10) implies

$$p^{4}_{lvm}(t) = \left(\frac{\eta_{lv} + k\Delta_l + \theta_l - t}{r_{lvm}} + 1 \right) p^{4}_{lvm}(\eta_{lv} + k\Delta_l + \theta_l) \tag{13}$$

on the interval $[\eta_{lv} + k\Delta_l + \theta_l \ , \ \eta_{lv} + k\Delta_l + \theta_l + r_{lvm}]$ and

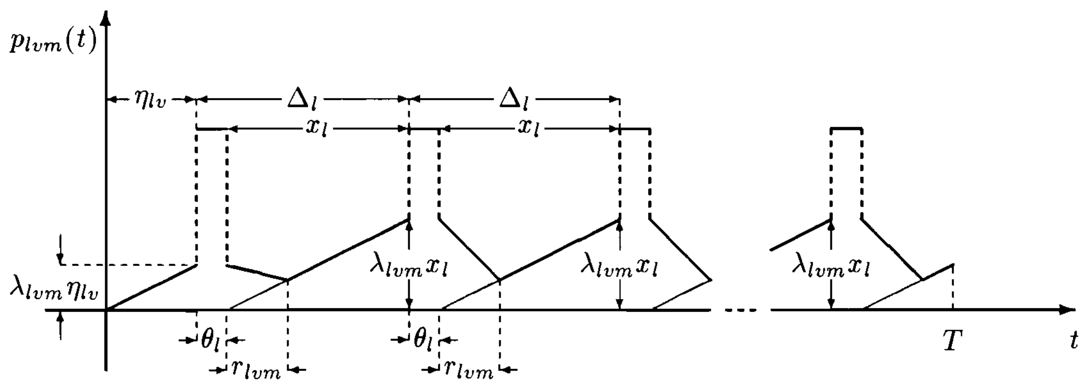

Figure 4: Time schedule for the set l.

$$p^4_{lvm}(t) = 0 \tag{14}$$

on the interval $[\eta_{lv} + k\Delta_l + \theta_l + r_{lvm}\ ,\ \eta_{lv} + (k+1)\Delta_l]$ for $k = 0, \ldots, k_{lv}$. Equations (12)–(14) define the unavailability of the component a_{lvm}. To describe one period of the component unavailability $p_{lvm}(t)$, let us introduce a function $\zeta(\theta, \gamma, r, \lambda, x, t)$ on the interval $[0, \Delta]$ (see Figure 3)

$$\zeta(\theta,\gamma,r,\lambda,x,t) = \begin{cases} 1, & \text{if } 0 \le t < \theta, \\ (t-\theta)(\lambda - \gamma/r) + \gamma, & \text{if } \theta \le t < \theta + r, \\ (t-\theta)\lambda, & \text{if } \theta + r \le t \le \theta + x. \end{cases} \tag{15}$$

With the function ζ, the unavailability $p_{lvm}(t)$ of the component a_{lvm} is given by the formula (see Figure 4)

$$p_{lvm}(t) = \begin{cases} \lambda t, & \text{if } 0 \le t < \eta_{lv}; \\ \zeta(\theta_l, \lambda_{lvm}\eta_{lv}, r_{lvm}, \lambda_{lvm}, x_l, t - \eta_{lv}), & \text{if } \eta_{lv} \le t < \eta_{lv} + \Delta_l; \\ \zeta(\theta_l, \lambda_{lvm}x_l, r_{lvm}, \lambda_{lvm}, x_l, t - (k-1)\Delta_l - \eta_{lv}), & \text{if } \eta_{lv} + (k-1)\Delta_l \le t, \\ & \quad t < \eta_{lv} + k\Delta_l, \\ & \quad k = 2, \ldots, k_{lv}; \\ \zeta(\theta_l, \lambda_{lvm}x_l, r_{lvm}, \lambda_{lvm}, & \\ T - \eta_{lv} - k_{lv}\Delta_l - \theta_l, t - \eta_{lv} - k_{lv}\Delta_l), & \text{if } \eta_{lv} + k_{lv}\Delta_l \le t, \\ & \quad t \le T\,. \end{cases} \tag{16}$$

Formulae (16) and (5) are used to calculate the pointwise unavailability of the whole system.

4 Calculation of Average Unavailability

We integrate the function $p(t)$ with respect to t to calculate the average unavailability p_u of the whole system

$$p_u = \frac{1}{T}\int_0^T p(t) = \frac{1}{T}\int_0^T \sum_{w=1}^{W} \prod_{(lvm)\in C_w} p_{lvm}(t)\ . \tag{17}$$

Here we consider that T is very large in comparison with $\Delta_l, l = 1, \ldots, L$. For this case we can omit the beginning and end of the operation interval and consider only the periodical part of the schedule for each component. The average unavailability p_u can be approximated as

$$p_u \approx \lim_{T\to\infty} \frac{1}{T}\int_0^T \sum_{w=1}^{W} \prod_{(lvm)\in C_w} p_{lvm}(t) \stackrel{\text{def}}{=} \tilde{p}_u\ . \tag{18}$$

Further we consider three cases where the approximate unavailability of the function $\tilde{p}_u$ can be expressed in terms of the limit of average unavailability $\lim_{T\to\infty}\frac{1}{T}\int_0^T p_{lvm}(t)$ of the components a_{lvm}.

Since one period of the function $p_{lvm}(t)$ for $l > 0$ is specified by the function

$$\zeta(\theta_l, \lambda_{lvm}x_l, r_{lvm}, \lambda_{lvm}, x_l, t - (k-1)\Delta_l - \eta_{lv})\ ,$$

consequently (see Figure 4)

$$\frac{1}{T}\int_0^T p_{lvm}(t) \approx \lim_{T\to\infty}\frac{1}{T}\int_0^T p_{lvm}(t) = \frac{1}{\Delta_l}\int_0^{\Delta_l} \zeta(\theta_l, \lambda_{lvm}x_l, r_{lvm}, \lambda_{lvm}, x_l, t)\ .$$

It is not difficult to calculate the integral of the function ζ ; to designate this integral we introduce a new function ψ

$$\psi(\theta, r, \lambda, x) \stackrel{\text{def}}{=} \frac{1}{\theta + x}\int_0^{\theta+x} \zeta(\theta, \lambda x, r, \lambda, x, t) = \frac{\theta + 0.5\lambda x(r + x)}{\theta + x}\ . \tag{19}$$

With the previous designation, the average unavailability of the component a_{lvm} is approximated by the following function ψ_{lvm}

$$\psi_{lvm} = \begin{cases} p_{lvm}\ , & \text{if } l = 0\ , \\ \psi(\theta_l, r_{lvm}, \lambda_{lvm}, x_l)\ , & \text{otherwise}\ . \end{cases} \tag{20}$$

System with Independent Components. Let us consider that in each subset of components U_l, $l = 1, \ldots, L$ there is only one tested component a_{l11}. We say that tested components are *independent* if in each cut set there is at most one tested component. For this case the unavailability estimate $\tilde{p}_u$ in formula (18) is equal to

$$\tilde{p}_u = \lim_{T\to\infty}\frac{1}{T}\int_0^T \sum_{l=1}^{L} K_l\, p_{l11}(t) + \text{constant} =$$

$$\sum_{l=1}^{L} K_l \lim_{T\to\infty} \frac{1}{T} \int_0^T p_{l11}(t) + \text{constant} = \sum_{l=1}^{L} K_l \psi_{l11} + \text{constant}\ , \tag{21}$$

here K_l, $l = 1, \ldots, L$ are some constants.

System with Independent Series Groups. Let us consider that in each subset U_l, $l = 1, \ldots, L$ there is only one group of components. We say that all tested groups in the system are *independent series groups* if the unavailability estimate $\tilde{p}_u$ for this system can be represented as

$$\tilde{p}_u = \lim_{T\to\infty} \frac{1}{T} \int_0^T \sum_{l=1}^{L} K_l \sum_{m=1}^{M_{l1}} p_{l1m}(t) + \text{constant} =$$

$$\sum_{l=1}^{L} K_l \sum_{m=1}^{M_{l1}} \lim_{T\to\infty} \frac{1}{T} \int_0^T p_{l1m}(t) + \text{constant} =$$

$$\sum_{l=1}^{L} K_l \sum_{m=1}^{M_{l1}} \psi_{l1m} + \text{constant}\ .$$

This means that all components from the same group belong to the same train and the availability of this group at any time is the product of the availabilities of all components in the group. The unavailability $\tilde{p}_u^l$ of the group from U_l can be represented approximately as

$$\tilde{p}_u^l = \sum_{m=1}^{M_{l1}} \psi(\theta_l, r_{l1m}, \lambda_{l1m}, x_l) =$$

$$(\theta_l + x_l)^{-1} \sum_{m=1}^{M_{l1}} [\theta_l + 0.5\lambda_{l1m} x_l (r_{l1m} + x_l)] =$$

$$M_{l1}(\theta_l + x_l)^{-1} \left[\theta_l + 0.5(M_{l1}^{-1} \sum_{m=1}^{M_{l1}} \lambda_{l1m})\, x_l \left(\frac{\sum_{m=1}^{M_{l1}} \lambda_{l1m} r_{l1m}}{\sum_{m=1}^{M_{l1}} \lambda_{l1m}} + x_l\right)\right] =$$

$$M_{l1}\psi\left(\theta_l, \frac{\sum_{m=1}^{M_{l1}} \lambda_{l1m} r_{l1m}}{\sum_{m=1}^{M_{l1}} \lambda_{l1m}}, M_{l1}^{-1} \sum_{m=1}^{M_{l1}} \lambda_{l1m}, x_l\right)\ . \tag{22}$$

The last formula shows that a system with independent series groups can be reduced to a system with independent "super" components. Each "super" component corresponds to a series group and has a repair time of $\left(\sum_{m=1}^{M_{l1}} \lambda_{l1m} r_{l1m}\right) \left(\sum_{m=1}^{M_{l1}} \lambda_{l1m}\right)^{-1}$, a test duration interval of θ_l and a failure rate of $M_{l1}^{-1} \sum_{m=1}^{M_{l1}} \lambda_{l1m}$. Analogous results for the series system were achieved in the work of Vaurio [18].

System with Random Starts of Tests. Now we consider the case when each subset U_l, $l = 1, \ldots, L$ has only one component (subset U_0 can have several components). For each $l = 1, \ldots, L$ the shift of schedule y_l is a random variable uniformly distributed on the interval $[0, \Delta_l]$. All shifts are independent. With this condition, the expected unavailability $E\tilde{p}_u$ can be calculated as

$$E\tilde{p}_u = \lim_{T\to\infty} \frac{1}{T} \int_0^T \sum_{w=1}^{W} \prod_{(l1m)\in C_w} p_{l1m}(t) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \sum_{w=1}^{W} \prod_{(l1m)\in C_w} Ep_{l1m}(t) \ .$$

If $l > 0$ and $\Delta_l/4 + 2\Delta_l \le t \le \Delta_l/4 + k_{lv}\Delta_l$, then the mathematical expectation $Ep_{l1m}(t)$ is equal to the integral of the function $p_{l1m}(t)$ over one time period, i.e.

$$p_{l1m}(t) = \psi_{l1m}.$$

Consequently,

$$E\tilde{p}_u = \lim_{T\to\infty} \frac{1}{T} \int_0^T \sum_{w=1}^{W} \prod_{(l1m)\in C_w} \psi_{l1m} = \sum_{w=1}^{W} \prod_{(l1m)\in C_w} \psi_{l1m} \ . \tag{23}$$

The last formula also can be considered as a good approximation of the unavailability (17) for the general case.

5 Optimization of Test Intervals: Special Cases

Here we consider optimization problems for the unavailability estimate $\tilde{p}_u$ with respect to the test intervals $x_1, \dots, x_L$

$$\tilde{p}_u(x) \to \min_{x\in X} \ , \tag{24}$$

subject to the constraints

$$f_i(x) \le 0 \ , \quad i = 1, \dots, I \ , \tag{25}$$

where $x = (x_1, \dots, x_L) \in R^L$ and $X = \{x \in R^L\colon \underline{x}_l \le x_l \le \overline{x}_l \ , \text{ for } l = 1, \dots, L \ \}$; for $l = 1, \dots, L$, the constants $\underline{x}_l$ and $\overline{x}_l$ are the lower and upper bounds, respectively, of the test interval x_l. Constraints (25) (possibly nonlinear) can take into account relations between test intervals of different groups of components. For example, it could be the following constraints

$$x_i = x_j = \dots = x_k \ ,$$

where $i, j, \dots, k$ are some group of numbers from the set $l = 1, \dots, L$. In this case we can introduce a new variable $\hat{x}$

$$\hat{x} = x_i = x_j = \dots = x_k$$

and reduce the dimension of the problem.

System with Independent Components. First let us consider the optimization problem for the function (21) with the simplest constraints $x \in X$

$$\tilde{p}_u = \sum_{l=1}^{L} K_l \ \psi_{l11} + \text{constant} \to \min_{x\in X\subset R_L} \ . \tag{26}$$

Since we take into account only simplest constraints $x \in X$, problem (26) can be reduced to the *independent* problem

$$\psi_{l11} \rightarrow \min_{\underline{x}_l \leq x_l \leq \overline{x}_l} , \quad l = 1, \ldots, L . \tag{27}$$

With standard algebraic formulae we can calculate

$$\nabla_{x_l} \psi_{lvm} = \begin{cases} 0 , & \text{if } l = 0 , \\ (\theta_l + x_l)^{-2}[\theta_l(0.5\lambda_{lvm} r_{lvm} - 1) + \lambda_{lvm} x_l(\theta_l + 0.5x_l)] , & \text{otherwise} . \end{cases} \tag{28}$$

For one tested component the average unavailability function $\psi(\theta_l, r_{lvm}, \lambda_{lvm}, x_l)$ can be minimized analytically. To find an optimum point without constraints we use necessary conditions of extremum

$$\nabla_{x_l} \psi(\theta_l, r_{lvm}, \lambda_{lvm}, x_l) = 0 . \tag{29}$$

Formulae (28) and (29) imply

$$\theta_l(0.5\lambda_{lvm} r_{lvm} - 1) + \lambda_{lvm} x_l(\theta_l + 0.5x_l) = 0 , \tag{30}$$

and consequently

$$x_l^2 + 2\theta_l x_l + \theta_l(-2/\lambda_{lvm} + r_{lvm}) = 0 . \tag{31}$$

Equation (31) has only one positive root

$$x_l^{\text{root}} = -\theta_l + \sqrt{\theta_l^2 + 2\theta_l/\lambda_{lvm} - r_{lvm}\theta_l} . \tag{32}$$

If $\lambda_{lvm} r_{lvm} \ll 1$ and $\theta_l r_{lvm} \ll 1$ then we get from (32) an approximation

$$x_l^{\text{root}} \approx \sqrt{\frac{2\theta_l}{\lambda_{lvm}}} . \tag{33}$$

The last value is well-known in the literature (see [2, 7, 18]). After the calculation of x_l^{root} we can consider the constraints $\underline{x}_l \leq x_l \leq \overline{x}_l$, and the optimal value for problem (27) is given by the formula

$$x_l^* = \begin{cases} \overline{x}_l , & \text{if } \; x_l^{\text{root}} \geq \overline{x}_l , \\ \underline{x}_l , & \text{if } \; x_l^{\text{root}} \leq \underline{x}_l , \\ x_l^{\text{root}} , & \text{otherwise} \quad . \end{cases} \tag{34}$$

Let us now consider the optimization problem (26) with nonlinear constraints (25). Generally speaking this problem cannot be solved analytically; numerical techniques are needed. Gradient nonlinear programming methods can be used for this purpose (see, for example, [9, 16]). The gradient of the function $\tilde{p}_u(x_1, \ldots, x_L)$ can be calculated with the formula

$$\nabla_x \tilde{p}_u(x) = \sum_{l=1}^{L} K_l \nabla_{x_l} \psi_{l11} , \tag{35}$$

and formula (28).

System with Independent Series Groups. It was shown in the previous section that a system with *independent series groups* can be reduced to a system with independent "super" components. Therefore for the optimization of unavailability of this system we can use the same technique as for a system with independent components.

System with Random Starts of Tests. Let us consider the following optimization problem for a system with *random starts of tests*

$$E\tilde{p}_u = \sum_{w=1}^{W} \prod_{(l1m)\in C_w} \psi_{l1m} \rightarrow \min_{x\in X\subset R_L} , \tag{36}$$

$$X = \{x \in R^L \colon \underline{x}_l \leq x_l \leq \overline{x}_l \text{ , for } l = 1,\dots,L \} ,$$

subject to

$$f_i(x) \leq 0 , \quad i = 1,\dots,I . \tag{37}$$

This problem cannot be solved analytically in the general case; we use gradient methods to solve it. The gradient of the function $E\tilde{p}_u$ is calculated as follows:

$$\nabla_x E\tilde{p}_u(x) = \sum_{w=1}^{W} \sum_{(l1m)\in C_w} \psi_{l1m}^{-1} \nabla_{x_l} \psi_{l1m} \prod_{(l1m)\in C_w} \psi_{l1m} . \tag{38}$$

The gradient $\nabla_{x_l}\psi_{l1m}$ in this equation is given in the expression (28). The objective function $E\tilde{p}_u(x)$ and the gradient $\nabla_x E\tilde{p}_u(x)$ can be calculated during one run through the cut sets.

6 Calculation of Average Unavailability: General Case

Let us describe formulae to calculate the unavailability (17) in the general case

$$p_u = \frac{1}{T}\int_0^T \sum_{w=1}^{W} \prod_{(lvm)\in C_w} p_{lvm}(t) = \frac{1}{T}\sum_{w=1}^{W} \int_0^T \prod_{(lvm)\in C_w} p_{lvm}(t) . \tag{39}$$

Unavailability p_u can be calculated through the functions p^w

$$p^w \stackrel{\text{def}}{=} \int_0^T \prod_{(lvm)\in C_w} p_{lvm}(t) . \tag{40}$$

Since $p_{lvm}(t)$ is a partially linear function, then $\prod_{(lvm)\in C_w} p_{lvm}(t)$ is a partially polynomial function. We consider that

$$[\, 0 = t_0 < t_1 < \cdots < t_J < t_{J+1} = T \,] \tag{41}$$

and on each interval $[t_j, t_{j+1})$ the function $\prod_{(lvm)\in C_w} p_{lvm}(t)$ is polynomial. By $[t_j, t_{j+1})$ we designate the interval that includes the point t_j and excludes the point t_{j+1}. The partition (41) depends upon the cut set w. The function p^w can be represented as

$$p^w = \sum_{j=0}^{J_w} \int_{t_j}^{t_{j+1}} \prod_{(lvm)\in C_w} p_{lvm}(t) = \sum_{j=0}^{J_w} p^{wj} , \tag{42}$$

$$p^{wj} \stackrel{\text{def}}{=} \int_{t_j}^{t_{j+1}} \prod_{(lvm)\in C_w} p_{lvm}(t) .$$

On the interval $[t_j, t_{j+1})$ the polynomial function

$$\prod_{(lvm)\in C_w} p_{lvm}(t) = b_0 + b_1 t + \cdots + b_\Phi t^\Phi = \sum_{\phi=0}^{\Phi} b_\phi t^\phi \tag{43}$$

can be integrated analytically and

$$p^{wj} = \int_{t_j}^{t_{j+1}} \sum_{\phi=0}^{\Phi} b_\phi t^\phi = \sum_{\phi=0}^{\Phi} b_\phi (t_{j+1}^{\phi+1} - t_j^{\phi+1}) . \tag{44}$$

The coefficients b_ϕ and the number ϕ in these formulae depend upon j, w. Combining (39), (40), (42), and (44) we can calculate

$$p_u = \frac{1}{T} \int_0^T \sum_{w=1}^{W} p^w = \frac{1}{T} \sum_{w=1}^{W} \sum_{j=0}^{J_w} p^{wj} .$$

This last formula can be used for fast numerical calculation of the function p_u.

7 Optimization of Operational Schedules

Here we consider the optimization problem for the unavailability function (39) in the general case

$$p_u(y, x) = \frac{1}{T} \sum_{w=1}^{W} p^w(y, x) \quad \rightarrow \quad \min_{\substack{y\in Y \\ x\in X}} , \tag{45}$$

subject to

$$f_i(y, x) \le 0 , \quad i = 1, \ldots, I , \tag{46}$$

where

$$Y = \{y \in R^L\colon \underline{y}_l \le y_l \le \overline{y}_l \text{ , for } l = 1, \ldots, L \} ,$$

$$X = \{x \in R^L\colon \underline{x}_l \le x_l \le \overline{x}_l \text{ , for } l = 1, \ldots, L \} ,$$

and the function $p^w(y, x)$ is given by equation (42). In comparison with the optimization problem (27) and (36) here we also take into account the first inspection intervals $y_1, \ldots, y_L$. This problem is very complex from the numerical point of view. To calculate the objective function $p_u(y, x)$ we must calculate the functions $p^{wj}(y, x)$ for each $w = 1, \ldots, W$; $j = 1, \ldots, J$. Further, the functions $p_{lvm}(t)$ are discontinuous; consequently the unavailability $p_u(y, x)$ is a nonsmooth function with respect to the variables y, x. The function $p_{lvm}(t)$ is multiextremal. To find a local extremum of the problem (45), (46) some nonsmooth or stochastic quasi-gradient methods could be used (see, for example, [3, 6, 14, 16]). Heuristic procedures can be developed to move from one local extremum to another.

References

[1] IAEA (1988): *Advisory Group Meeting Report on the "Development of Computer Software for Using PSA in NPP Operational Safety Management"*, IAEA Internal Report, Vienna, Sept., 1988.

[2] Apostolakis, G. and T. Chu (1980): The Unavailability of Systems under Periodic Tests and Maintenance, *Nuclear Technology*, **50**, Mid-Aug.

[3] Ermoliev, Yu. (1983): Stochastic Quasi-Gradient Methods and Their Applications to System Optimization. *Stochastic*, **4**.

[4] Freedman, D. (1971): *Markov Chains*, Holden Day, San Francisco.

[5] Ginsburg, T. and J.T. Powers (1986): *FRANTIC-III - a Computer Code for the Time-Dependent Reliability Analysis*, Brookhaven National Laboratory (BNL), Technical Report 3230, 8-20-86.

[6] Lemarechal, C., J.J. Strodiat and A. Bihain (1981): On a Bundle Algorithm for Nonsmooth Optimization, in O.L. Mangasarian, R.R. Meyer, and S.M. Robinson, eds., *Nonlinear Programming 4*, Academic Press, New York.

[7] Lofgren, E., F. Varcolik and W.E. Vesely (1981): *Optimum Test Intervals for Online Testing.* NUREG/CR - 2158 SAI 528951-S.

[8] Mankamo, T. and U. Pulkkinen (1988): *Test Interval Optimization of Stand-by Equipment.* Research Notes 892. Technical Research Centre of Finland.

[9] Murtagh, B.A. and M.A. Saunders (1982): A Projected Lagrangian Algorithm and its Implementation for Sparse Nonlinear Constraints, *Mathematical Programming Study*, **16**, pp. 84–117.

[10] Papazoglou, I.A.(1988): *A Code for Marcovian Reliability Analysis of Systems. User's Guide.* Demokritos. The Greek Atomic Energy Commission.

[11] Bojadiev, A., L. Lederman and H. Valerga (1989): *PSAPACK: An Event/Fault Tree Package for PSA Using PC*, PSA "89 ANS ENS Topical Meeting", Pittsburgh, USA, April 1989.

[12] Robert, N.H., W.E. Vesely, D.F. Haasl and F.F. Goldberg (1981): *Fault Tree Handbook*, System and Reliability Research Office of Nuclear Regulatory Research, U.S. Nuclear Regulatory Commission, NUREG-0492, January 1981.

[13] Seneta, E. (1981): *Non-Negative Matrices and Markov-Chains*, Springer Verlag, New York.

[14] Shor, N.Z. (1985): *Minimization Methods for Non-Differentiable Functions*, Springer-Verlag.

[15] Sim, S.H. and J. Endrenyi (1988): Optimal Preventive Maintenance with Repair. *IEEE Trans. on Reliability*, **37**, April, pp. 92-96.

[16] Uryas'ev, S. (1990): Adaptive Variable Metric Algorithms for Nondifferentiable Optimization Problems, in A. Bensoussan and J.L. Lions, eds., *Analysis and Optimization of Systems*, Proc. of 9th International Conference, Antibies, June, 1990, Springer-Verlag, pp. 432–441.

[17] Vesely, W.E. and F.F. Goldberg (1977): *FRANTIC - A Computer Code for Time-Dependent Unavailability Analysis*, NUREG-0193.

[18] Vaurio, J.K (1979): Unavailability of Components with Inspection and Repair, *Nucl. Eng. Des.*, **54**, 309.

[19] Vaurio, J.K (1982): *Practical Availability Analysis of Stanby Systems*, Proc. of Annual Reliability and Maintainability Symposium, pp. 125–131.

[20] Wagner, D.P., L.A. Minton, S.E. Rose and D.J. Hesse (1987): *PC-SOCRATES Version 1.02 User's Guide*, Report, Electric Power Research Institute, California, USA, September.