Working Paper

Multi-Objective Optimization in Negotiation Support

Andrzej P. Wierzbicki, Marek Makowski

WP-92-007
February 1992

IIASA International Institute for Applied Systems Analysis □ A-2361 Laxenburg □ Austria
Telephone: +43 2236 715210 □ Telex: 079 137 iiasa a □ Telefax: +43 2236 71313

Multi-Objective Optimization in Negotiation Support

Andrzej P. Wierzbicki, Marek Makowski

WP-92-007
February 1992

IIASA International Institute for Applied Systems Analysis ▫ A-2361 Laxenburg ▫ Austria
Telephone: +43 2236 715210 ▫ Telex: 079 137 iiasa a ▫ Telefax: +43 2236 71313

Foreword

This paper gives the methodological complement to a contribution by the authors to a special issue of the *Theory and Decision* Journal titled *Systems Support for International Negotiation: Implications for Application.* The special issue will contain contributions to the Scoping Conference on Systems Analysis Techniques for International Negotiations held at IIASA in Laxenburg on October 9-10, 1991. The paper gives a good overview of the developments of multi-objective analysis with a strong emphasis on its practical applicability.

Abstract

The paper reviews the methodology of multi-objective modeling and optimization used in decision support based on computerized analytical models (as opposed to logical models used in expert systems) that represent expert knowledge in a given field. The essential aspects of this methodology relate to its flexibility: modeling and optimization methods are treated not as goals in themselves but as tools that help a sovereign user (an analyst or a decision maker) to interact with the model, to generate and analyze various decision options, to learn about possible outcomes of these decisions. Although the application of such methods in negotiation and mediation support is scarce yet, their flexibility increases essentially the chances of such applications. Various aspects of negotiation and mediation methods related to multi-objective optimization and game theory are also reviewed.

Contents

Multi-Objective Optimization in Negotiation Support

Andrzej P. Wierzbicki,* *Marek Makowski*

1 Introduction

We are living at the turn of an epoch of civilization variously characterized as the transition to the post-industrial, service or information society. One of the main features of this transition relates (see e.g. Toffler (1980) or Wierzbicki (1983), (1987)) to the change in the basic concepts of understanding time, space, cause and effect relationships, of understanding uncertainty and chaos, of computer processing of information and representations of knowledge, of the nature of human decisions and of the possibilities of supporting them by computers. While the change of these basic concepts originally resulted from developments in physics (the relativity of space and time), telecommunications and automatic control (the concept of feedback that made the mechanical understanding of cause and effect obsolete), further elements of this change are related to the challenges of the information age and have resulted in developments in systems and decision sciences. For example, we understand today that various models can be used to represent uncertainty and the probabilistic models used, e.g in physics, are by no means absolute, because deterministic nonlinear models of sufficient complexity can also be used for this purpose, and can result in chaotic behavior or even in order emerging out of chaos (as in fractal geometry - see e.g. Gleick (1987)). Thus, much of the philosophical discussions from the first half of our century about the indeterminism of the universe seem to be based on insufficient conceptual premises.

Together with this change, a significant development of the methodology of decision analysis and support can be observed in the past decades. Today we understand that a decision support system should never replace a human, sovereign decision maker; even in operational, repetitive decision situations, computerized decision support can at most relieve us from standard calculations and fill in the details of a suggested decision. In strategic decision situations, typical for negotiations, decision support systems can play a different role: they can represent the knowledge of specialized analysts about the substantive aspects of a decision situation and thus provide a laboratory ground, a proxy of the real world for studying various impacts of selected decisions as well as possible developments of the negotiation process. Although their informational or educational role can be considerable, their value depends crucially on the validity of knowledge representation contained in them. Knowledge is represented in decision support systems in the form of computerized models, either of the logical (in expert systems), or analytical (in analytical systems) or procedural type (expressing e.g. the knowledge about a rational organization of the decision or negotiation process). Today, we also know a lot about the art and science of mathematical modeling - see e.g. Wierzbicki (1984) - which combines the knowledge in modeling methodology (properties of various types of models, methods

*Institute of Automatic Control, Warsaw University of Technology Warsaw, Poland

of model validation and parameter estimation, sensitivity analysis etc) with expertise in a given substantive field that is actually crucial for the value of the model.

However, even a model that represents the best knowledge in a given field has two important limitations. One limitation is that the model must be always simplified to some extent, and never represents all of the pertinent aspects of a given decision situation. And we cannot make such models more adequate by increasing their complexity, because this might also increase their sensitivity, make the parameter estimation less reliable, or introduce unintended chaotic effects. We can usually select sufficiently simple models that represent the most salient aspects of operational, repetitive decision situations. But even in such cases we replace human judgment by automated decisions only in well-tested cases and with sufficient safeguards (e.g. in technical automatic control systems with negative feedback that is a strong safeguard principle against various inadequacies of models). In strategic decision situations, however, the intuition of a human decision maker remains irreplaceable and the knowledge representation by a computerized model can only help to enhance this intuition by allowing the decision maker to learn about possible consequences of various hypothetical actions. The fact that master experts in a given field (including especially international negotiations) make their decisions intuitively, by using the subconscious parts of their brains, is sufficiently well documented experimentally - see e.g. "Mind over Machine" by H. and S. Dreifus (1986), a book intended as a critique of the concepts of artificial intelligence, but actually opening a road to a rational understanding and even research of the intuitive, subconscious human decision making. This does not mean that decision support systems based on computerized models are useless, for they can be used, for example, to train master experts.

Another important limitation of computerized models is (paradoxically) that they are constructed by expert analysts in a given substantive field of knowledge. Such experts are usually sufficiently familiar with the art and science of model building (at least, for classes of models typically used in their field), but their expertise seldom extends to the relatively new methodology of using such models in decision support, which in turn significantly influences the way the models should be formulated, validated and used. Thus, the cooperation of a team of specialists (in a given substantive field, of model building, of decision support) is necessary in order to construct a good decision support system. Moreover, such cooperation is not sufficient yet: the value of a decision support system is in the eyes of its user, hence the system must be "user-oriented" - the ultimate user must also decisively cooperate in the development of the system.

With all of the above reservations, this paper concentrates on a part of the modern methodology of decision support based on computerized models, in particular, on models of analytical or possibly also the procedural type. When using models of the analytical type, we can often exploit the optimization techniques for tentatively selecting decision options. However, optimization in such cases should always be treated as a flexible tool of model analysis and decision support, never as a goal in itself, since attempts to model the preferences of individual decision makers by a single objective function (value or utility function) are never fully adequate and should be treated as rough approximations only: the reservations concerning substantive models that represent knowledge in a given field apply doubly to preferential models that represent human preferences. When treated as a tool, the flexibility of optimization techniques can be increased considerably through multi-objective formulation. Thus, the paper attempts to describe in simple (though hopefully not too simplistic) terms the basic concepts of multi-objective optimization, some selected results from the corresponding mathematical theory, their relations to representing knowledge by analytical modeling and to decision support, the related possibilities of analyzing multi-objective games, negotiation and mediation processes.

This paper was presented at the *Systems Analysis Techniques for International Negotiations*, Scoping Conference, held at IIASA on October 9-10, 1991. However, due to the space limitations for the proceedings of this Conference (to be published in a special issue of the *Theory and Decision* Journal in 1992), only part of the presented paper has been submitted for the proceedings. This Working Paper aims primarily at presenting a short overview of multicriteria optimization (this part has not been included for the proceedings). An updated and extended description of a prototype of a multi-objective mediation support system MCBARG, will be published as a separate Working Paper.

2 Multi-objective modeling and simulation.

While single-objective mathematical optimization models are in a sense closed and distinct from simulation models that are typically used for analytical representations of knowledge in a given substantive field, multi-objective optimization models can be formulated as a natural, open extension of simulation models. If we admit that a decision maker in the real world can have multiple objectives and then we simulate a part of this world by a model, we can simply treat various quantities represented by variables of the model as measures of possible objectives, while the final selection of the objectives will be made by the user - the analyst or the decision maker. The model might not be complete in the sense that it might not represent all concerns of the user, but then either it must be reformulated even for simulation purposes, or its incompleteness must be overtly admitted and accounted for in the analysis.

Thus, an analytical model of the substantive aspects of a decision situation typically contains:

- actions or decisions represented by decision variables;
- potential objectives represented by outcome variables;
- various intermediate variables (state variables, balance variables etc.) that are essential for a flexible model formulation;
- parametric variables or parameters that might remain constant during model simulations but are essential for model validation and alternative model variants;
- constraining relations (inequalities, equations etc.) that determine the set of admissible decisions and are usually divided into direct decision constraints that involve only decision variables and indirect constraints that involve also outcome and intermediate variables;
- outcome relations that determine how the outcome variables depend on the decision variables (often not directly, with the help of intermediate variables and equations such as state equations in dynamic models, often with the help of recursive or even implicit formulae);
- a representation of model uncertainty (in probabilistic, fuzzy set, set-valued etc. terms); if such a representation is not explicit, we often call such a model "deterministic" and assume that it represents average situations.

While the typical models for single-objective optimization specify only one optimized outcome ("the" objective function) and treat all constraints inflexibly, multi-objective modeling and optimization allows a flexible choice of objective variables between the outcome variables (if necessary, also between decision variables) and a much more elastic interpretation of constraints.

It is well known that (particularly indirect) constraints that are represented in single-objective optimization with a standard form, say, of an inequality, intend to model two quite different classes of phenomena of the real world. One of these classes contains balances that must be satisfied such as the balance of energy in a physical model, or domains of model validity such as the edges of a table for a model of motion of a ball; these are so-called hard constraints. The other class contains balances that we would like to satisfy, such as the balance in a budget sheet; these constraints can be violated (at an appropriate cost) and are called soft constraints. Soft constraints can be modeled even in single-objective optimization by appropriate penalty terms in the objective function; but then the question what are their permissible violations calls for additional judgment. In multi-objective modeling and optimization, soft constraints are most naturally interpreted as additional objectives and their evaluation is thus included in the overall evaluation of a multi-objective solution.

Having formulated a multi-objective model, one has to estimate its parameters and validate it - that is, check whether the model represents adequately not only the formal, but also the intuitive side of expert knowledge in a given substantive field. While there are many methods of parameter estimation and formal model validation, depending on particular model type and described in a broad literature, the intuitive model validation relies usually on repetitive simulation: the model must be run many times by experts in the field of knowledge under changing assumptions about decisions (or their scenarios in case of dynamic models) or even parameters, and the obtained outcomes (or their trajectories in the dynamic case) must be compared against the formal knowledge and the intuition of the experts. It has often been stressed that most valuable are models that can produce also counter-intuitive results; but the experts must be able to internalize such results, that is, explain to themselves why these results are obtained and check with their intuition (also by additional research and experiments) whether these results can also occur in the real world; otherwise, counter-intuitive results are useless in learning.

The way that various constraints are treated during the simulation of a model is also essential for its validation. Typical approaches to simulation and existing simulation languages usually allow only for direct decision constraints that can be represented by admissible ranges of decision variables; they do not allow to include indirect constraints nor to distinguish hard and soft constraints. Moreover, expert users of simulation models are often interested in inverse simulation, in which desirable trajectories of model outcomes are specified by the user and decision variables should be chosen during the simulation to result in model outcomes close to the specified trajectories. Inverse simulation is particularly useful in scenario generation. Moreover, good simulation techniques should make it possible to perform sensitivity analysis of simulated solutions along with simulation runs. All these issues of simulation under constraints, inverse simulation, scenario generation and sensitivity analysis can be included in sufficiently sophisticated methods of simulation that use optimization techniques and multi-objective approaches as tools of simulation support. IIASA has contributed considerably to the development of such methods, see e.g. Kallio et al. (1980), Grauer et al. (1982), Makowski and Sosnowski (1984), Kurzhanski (1986).

3 Basic concepts of multi-objective optimization.

After or during the simulation of a model, its multi-objective analysis and optimization can also be performed to help in understanding the model and in selecting decision options that are interesting for the user. The basic concepts of multi-objective optimization are well described in several monographs (see e.g. Sawaragi et al. (1985), Yu (1985), Steuer (1986), Seo and Sakawa (1988)). However, we review them here shortly.

These concepts start with the set of admissible decisions X_0 in the decision space X. The decisions $x \in X$ can be of various character: simple logical decisions *yes* or *no*, quantitative decisions represented by elements of R^n, decision scenarios represented by sequences of decisions of the previous types, decision strategies represented by probability distributions of decisions or even by their dependence on observed decision outcomes. The admissible decisions $x \in X_0$ are such elements of the decision space that satisfy the (direct and hard indirect) constraints incorporated in a given model.

The outcomes of decisions are represented by outcome variables $y \in Y$, where Y is the outcome space; in dynamic models, we often interpret the elements of Y as entire sequences or trajectories of outcomes. The outcomes are determined by outcome (and intermediate) relations of a given model, denoted shortly by $h : X_0 \longrightarrow Y$ thus $y = h(x)$; the set Y_0 of attainable outcomes contains such outcomes that can be results of admissible decisions, $Y_0 = h(X_0)$. One should not be misled by the deceptive simplicity of this notation: the function h represents here a model that can be quite complicated, thus the set Y_0 is usually not known explicitly, we can only obtain its elements y by running a simulation of the model for some admissible decision x; additionally, an assessment of admissibility of a decision may be also a complex task. However, it is convenient to explain the concepts of multi-objective optimization as if the set Y_0 were known; usually, we know only some of its general properties.

General properties of the set of attainable outcomes Y_0 are actually decisive for a basic classification of model types. If this set is a convex polyhedron (defined by a set of linear inequalities), then the model is linear – or piece-wise linear that might be reduced to linear; if not, then the model is nonlinear, convex or nonconvex depending on the convexity of Y_0. If the set Y_0 is discrete (consists of separate elements), then the model is discrete. Dynamic models – that include dynamic relations such as state equations – can be subdivided into two classes: essentially dynamic models in which the set Y_0 contains outcome trajectories, and dynamic models with static outcomes in which the elements of the set Y_0 correspond to outcomes in a chosen time-instant. If the set Y_0 consists of outcome trajectories that can occur with some probability, then the model is called stochastic, etc.

Between the outcome variables y of a model, the selection of objectives (objective outcomes, objective variables, criteria) z and thus the determination of the space of objectives Z should be left to the user. Therefore, Z is a subspace of Y, $Z \subseteq Y$, $z = f(x)$, where $f : X_0 \longrightarrow Z$ is the corresponding restriction of the function $h : X_0 \longrightarrow Y$, and the set of attainable objectives $Z_0 = f(X_0)$ is the corresponding projection of Y_0 on Z. This point is usually omitted in classical presentations of multi-objective theory, where no distinction between the space of outcomes and the space of objectives is made; but this distinction is essential for multi-objective simulation and decision support. It should be also the model user who determines what to do with the objectives: whether to maximize them, or minimize, or either stabilize or softly constrain at a user-prescribed level. This specifications define a so-called partial preordering of the objective space – a partial model of the preferences of the user, flexible enough to be further modified and/or specified.

Because the objective space Z can have a rather general nature (it can even be infinite-dimensional; precisely speaking, we only need to assume it to be a Banach space, while the set X_0 should be compact and the function f continuous to obtain a compact Z_0), it is useful to assume that the partial preordering specified by the user is represented by a positive cone $Q \subseteq Z$. The positive cone can be simply interpreted as the set of objectives improving when compared to the origin of the space Z; in case, say, of two scalar objectives each to be maximized, the cone $Q = R^2_+$ is the positive quarter of the plane $Z = R^2$.

The basic difference between single-objective and multi-objective optimization is that while we look for a uniquely determined "best" outcome and the corresponding decisions in the single-objective case, the multi-objective case eliminates only such decisions that result in outcomes which can be improved in the sense of the positive cone. Such decisions and outcomes that cannot be improved in a specified sense are called non-dominated (or also Pareto-optimal if all objectives are to be maximized or minimized, or generalized Pareto-optimal in other cases). However, there are usually many non-dominated outcomes and decisions for a given model, there is no single solution for the multi-objective formulation, and we reserve for the user (an analyst or the decision-maker) the right to select between them.

Actually, there are several variants of defining non-dominated or Pareto-optimal solutions; we explain only those of them that are important for applications. A Pareto-optimal decision for the case of maximizing two objectives is such that we cannot improve one objective without deteriorating the other one; equivalently (but more generally for other

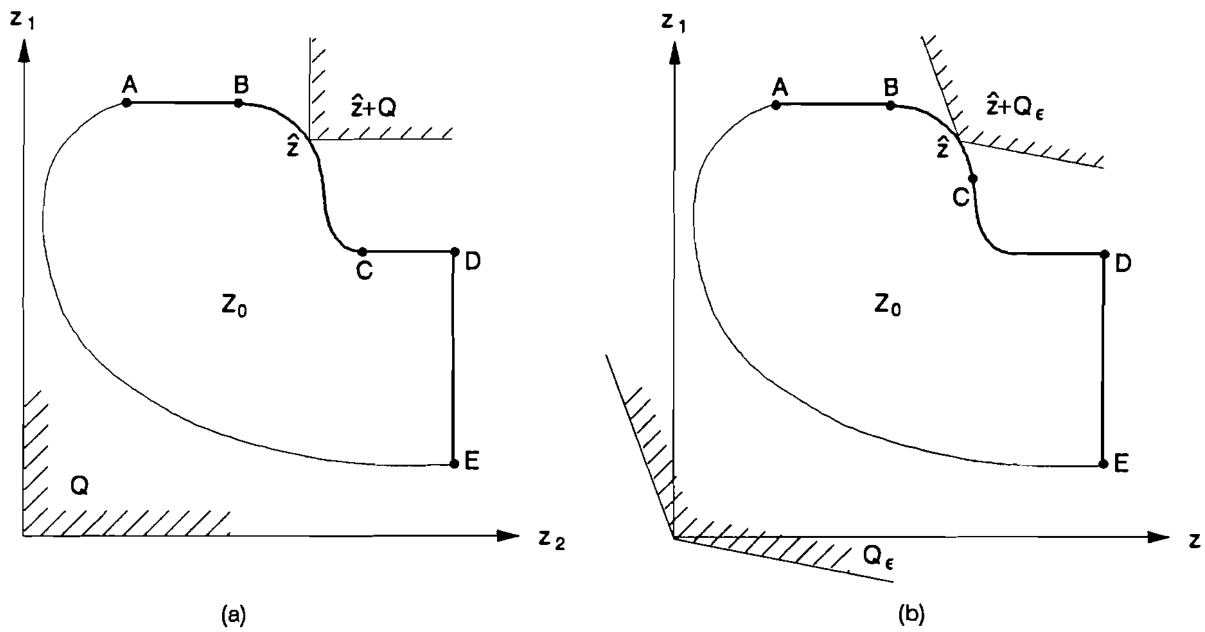

Figure 1: An example of the set $\hat{Z}_0$ of Pareto-optimal objective outcomes (1a) and the set $\hat{Z}_0^\epsilon$ of properly Pareto-optimal objective outcomes (1b).

cases) this means that the positive cone Q when shifted to a nondominated objective outcome $\hat{z}$ does not intersect with the set of attainable objectives Z_0 except at $\hat{z}$, see Fig. 1a. Note, that Pareto-optimal solutions are located on the curve ABCDE: the segment BC contains properly Pareto-optimal solutions, whereas segments AB, CD and DE contain non-properly Pareto-optimal solutions.

However, Pareto-optimal decisions and outcomes can also include such that have infinite (or very large) trade-off coefficients – that is, coefficients defining how much should we deteriorate one objective in order to improve another one by a unit. Thus, more important in applications are properly Pareto-optimal (more generally, properly non-dominated) decisions and outcomes, for which trade-off coefficients are bounded. There are several ways of defining properly non-dominated outcomes; a useful one – see Wierzbicki (1977), (1986) – is to enlarge slightly the positive cone Q to a cone Q_ϵ (with a small parameter ϵ that is related to the bound – approximately $1/\epsilon$ – on trade-off coefficients and defines how much the cone Q is enlarged), then to define the properly nondominated outcomes as before but with respect to the cone Q_ϵ (see Fig. 1b). Note, that the set of properly Pareto-optimal solutions represented in Fig. 1b by the segment DC is for the slightly enlarged cone Q_ϵ smaller than the corresponding set BC for the cone Q (see Fig. 1a).

The essential question in multi-objective optimization is how to compute properly nondominated outcomes and decisions for a given model - not randomly selected ones, but such that correspond to somehow specified preferences of the user. Historically, many ways of such computations have been proposed, differing in assumptions about the specification of user preferences and about basic properties of the model. The oldest and simplest assumptions for the case of Pareto optimization of n objectives in models with convex Z_0 were that the user should specify multipliers or weighting coefficients $\alpha_i > 0$ for all objectives to be maximized (or minimized, while changing only the signs of the multipliers). Then a corresponding weighted sum of the objectives:

$$s_1(z,\alpha) = \sum_{i=1}^{n} \alpha_i z_i = \sum_{i=1}^{n} \alpha_i f_i(x); \qquad \alpha = (\alpha_1, \ldots, \alpha_i, \ldots, \alpha_n) \tag{1}$$

should be single-objectively maximized with respect to $x \in X_0$ (under all constraints) in order to obtain a Pareto-optimal outcome and the corresponding decisions. This technique, presented in most textbooks as "the way" of dealing with multiple objectives, creates the unfortunate impression that there is nothing more to multi-objective optimization than specifying weighting coefficients; but this technique is possibly the worst one because the function $s_1(z,\alpha)$ – that constitutes the simplest example of the so-called scalarizing functions – has many drawbacks.

The most important drawback of this technique is that the user cannot effectively control the selection of Pareto-optimal outcomes when changing the weighting coefficients α which in the function $s_1(z,\alpha)$ should be treated as controlling parameters at the disposal of the user. If we consider the simplest case of a linear model with two objectives to be maximized – see Fig. 2a – the user can only select the vertices A and B by changing the weighting coefficient value from α' to α'' and cannot select any of the points on the Pareto-optimal edge joining these vertices. Moreover, this technique results in skipping many Pareto-optimal outcomes in most nonconvex cases (see Fig. 2b for an example of a discrete model) and becomes rather complicated in applications to dynamic models: the intuition of the user can deal effectively with trade-offs and weighting coefficients only for a small number of objectives.

Another technique of communicating with the user and computing nondominated outcomes consists in the identification of the value or utility function of the user: we maximize the following function :

$$s_2(f(x),\alpha) = u(f(x),\alpha) \tag{2}$$

with respect to $x \in X_0$, where $u(f(x),\alpha)$ is an assumed form of the multiattribute utility or value function, with parameters α that must be identified by questioning the user

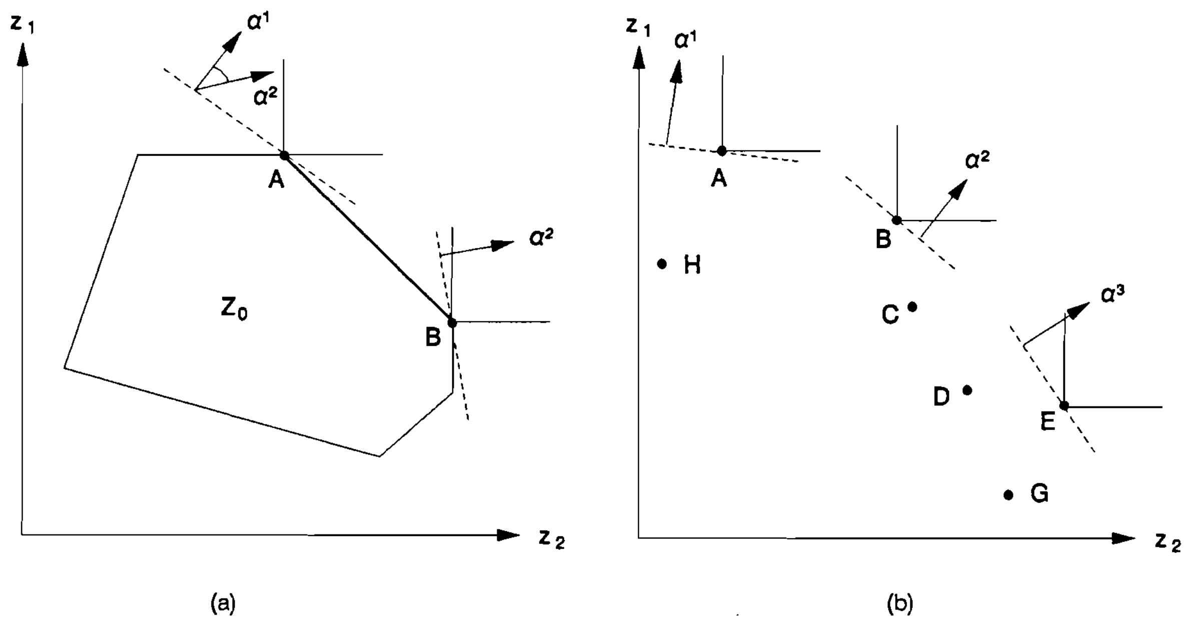

Figure 2: The impossibility of selecting all Pareto-optimal outcomes by maximizing a weighted sum of objectives: the cases of a linear model (2a) and of a discrete model (2b).

about his preferences. Although very well theoretically developed – see Keeney and Raiffa (1976) – this technique has turned out to be impractical in interactive sessions of users with analytical models. Expert users rely on their intuition and thus expect to learn during such interactive sessions; therefore, they are apt to change their preferences, hence the identification of their value functions u should be repeated many times which would simply be too time-consuming. Moreover, many experienced decision-makers – particularly practitioners of negotiations – dislike revealing too much about their preferences.

The preferences of a decision-maker could be much more simply indicated – while not fully revealed – by specifying some desirable levels of objective outcomes. There are several techniques of multi-objective optimization using this way of communication with the user, see Salukvadze (1971), (1974), Zeleny (1973), Haimes et al. (1974). Many practical applications are related to a group of techniques called goal programming (see Charnes and Cooper (1977), Ignizio (1978)), based on the following general idea: a goal vector w in the objective space Z is specified by the user and a distance function

$$s_3(f(x), w) = \| f(x) - w \| \tag{3}$$

is minimized with respect to $x \in X_0$, thus resulting in an objective outcome that is in some sense the closest one to the specified goals. This technique can be refined in various ways – by an appropriate selection of the norm $\| \cdot \|$ defining the distance, by using weighting coefficients as additional controlling parameters. However, this technique has one basic drawback: it can give misleading, dominated results when the set of attainable objective outcomes is nonconvex or even in convex, linear cases when the user specifies attainable objective goals (in which case $z = f(x) = w$ obviously minimizes the norm, but z can be dominated by other elements of Z_0).

To overcome this drawback, Salukvadze and Zeleny used goals that are sufficiently far away from the nondominated frontier of the set Z_0 (at least as far as the so-called ideal point obtained by maximizing all objectives separately); Haimes used other ways of dealing with this drawback while only using attainable goals. However, there is an effective way of overcoming this drawback while allowing for arbitrary (attainable or not) goals: we must abandon the use of a simple norm and use instead special scalarizing functions (often constructed with the help of a norm) that remain monotone in the sense of the positive cone Q even if goals are attainable.

This group of techniques has been developed by Wierzbicki (1977), (1982), Steuer and Choo (1983), Nakayama and Sawaragi (1984), Korhonen and Laakso (1986) and many others, in close cooperation with IIASA. Because attainable and dominated goals should not be actually treated as goals (it is better then to improve them), the desirable levels w_i of objective outcomes are called aspiration levels, while $w = (w_1, \ldots, w_i, \ldots, w_n)$ is called aspiration level point or reference point in these approaches. There are many forms of scalarizing functions that use reference points w as controlling parameters; to be useful in applications to various types of models with possibly a complicated structure of the space of objective outcomes Z and of the positive cone Q, they should have two general properties (see Wierzbicki (1986)):

a) the scalarizing functions $s(z, w)$ should have appropriate monotonicity properties with respect to z in the sense of the positive cone Q or the slightly enlarged cone Q_ϵ;

b) if the reference point w happens to coincide with a nondominated attainable outcome $\hat{z}$, then the scalarizing function $s(z, w)$ should (nonlinearly) separate – see e.g. Fig. 3a – the set Z_0 from the positive cone Q (or the slightly enlarged cone Q_ϵ) shifted to the point $w = \hat{z}$.

Scalarizing functions that posses these two general properties are called order-consistent achievement functions and can be applied to multi-criteria analysis and optimization of linear and nonlinear, even nonconvex, discrete or dynamic models, even with infinite-dimensional objective outcome spaces. The nondominated objective outcomes obtained by maximizing such achievement functions have the following properties:

(i) if the user has overestimated the outcomes of admissible decisions (as defined by the analytical model) and there are no admissible decisions with outcomes equal to or better than the reference point w, then the nondominated, attainable objective outcomes obtained by maximizing an achievement function are (uniformly in some sense) as close to the point w as possible;

(ii) if the user has underestimated the outcomes of admissible decisions and there are such decisions with outcomes better than the reference point w, then the nondominated outcomes obtained by maximizing an achievement function are uniformly better (as much as possible while remaining attainable) than the point w;

(iii) if the user – by a chance or as a result of learning – is just right and there is an admissible and nondominated decision with outcomes equal to the reference point, then the nondominated outcomes obtained by maximizing an achievement function just coincide with the point w (in this case, these techniques do not try to correct the user, but only tell him which detailed decisions should be used to obtain the desired outcome).

For the case of maximization of n objectives, the following basic form of an order-consistent scalarizing function has been widely used:

$$s_4(f(x), w) = \min_{1 \le i \le n}((f_i(x) - w_i)/(\tilde{z}_i - w_i) + \rho \sum_{i=1}^{n}(f_i(x) - w_i)/(\tilde{z}_i - w_i)) \qquad (4)$$

where $\tilde{z} = (\tilde{z}_1, ..\tilde{z}_i, ..\tilde{z}_n)$ is a point sufficiently far away from the nondominated frontier (an approximation of the so-called ideal or utopia point, whereas the reference points are restricted by $w \le \tilde{z}$), $\rho > 0$ is a small parameter related to ϵ (e.g. $\rho = \epsilon/n$). The func-

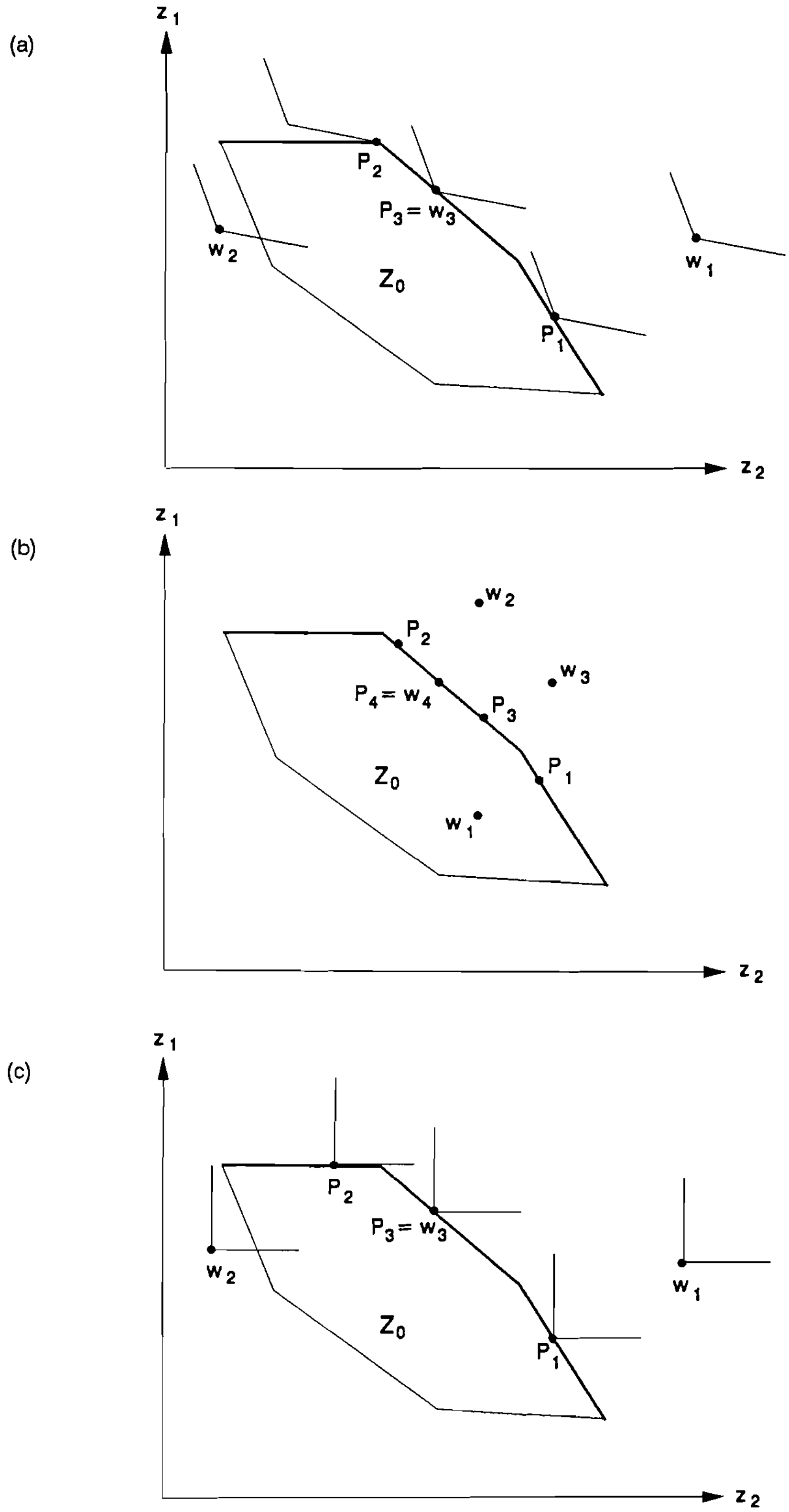

Figure 3: The results of maximization of function (4) for a linear model with two maximized objective outcomes: (3a) the separation property (b) and the properties (i), (ii), (iii); (3b) the continuous controllability of the selection of nondominated outcomes; (3c) the possibility of obtaining weakly nondominated outcomes if $\rho = 0$.

tion (4) is maximized with respect to $x \in X_0$ to obtain properly Pareto-optimal objective outcomes that have the above properties (i), (ii), (iii). This function is nondifferentiable at such x that $f(x) = w$; but it can be shown (see Wierzbicki (1986)) that this nondifferentiability is an essential property of this function, related to the general property b). Precisely because of this property the maxima of this function are parametrically controllable: the user can continuously influence his selection of nondominated outcomes by changing the reference point w, even if the desired outcomes are located on a linear edge of the attainable set, see Fig. 3b. This function also has been used often in its simpler form with $\rho = 0$, but such simplification is not advisable: the maxima of such a simplified function are only weakly Pareto-optimal, that is, it might happen that some of the objective outcomes can be improved without deteriorating other objectives, see Fig. 3c.

In the case of linear models, the maximization of a nondifferentiable function such as (4) can be equivalently restated as a linear programming problem with special structural properties that can be exploited when solving it numerically. Special forms of such a function have also been developed (Wierzbicki (1986), Ogryczak et al. (1989) and others) for the case when, instead of using only one reference point, a separate reservation point and an aspiration point are used as controlling parameters by the user.

For the case of nonlinear models, the use of a nondifferentiable function such as (4) might cause some difficulties; nondifferentiable optimization techniques (see e.g. Kiwiel and Stachurski (1989)) can be used in such a case. Another way out of this difficulty – used in the decision analysis and support system DIDAS-N for nonlinear models, see e.g. Kreglewski et al. (1989) – is to develop differentiable approximations of function (4). One of such approximations can be constructed with the help of the l_p norm concept[1] of the difference $f(x) - w$:

$$s_5(f(x), w) = 1 - \left(\frac{1}{n}\sum_{i=1}^{n}((\tilde{z}_i - f_i(x))/(\tilde{z}_i - w_i))^p\right)^{1/p} \tag{5}$$

Modifications of the function (4) are especially important for the case of linear dynamic models (see e.g. Kallio et al. (1980), Makowski and Sosnowski (1984), Lewandowski et al. (1989)). One of the main advantages of the reference point techniques is that reference trajectories can be used to indicate preferences of the user in the case of dynamic outcomes of a model. An experienced user is perfectly at ease when evaluating intuitively outcome trajectories and specifying reference trajectories for them, while he would be lost when evaluating trade-offs and specifying weighting coefficients for such trajectories. Therefore, reference trajectories have found applications from the very beginning of the development of reference point techniques (Kallio et al. (1980)), have been used to generate dynamic scenarios for expert evaluation (Messner and Strubegger, (1985)) and found implementation in various model-based decision support systems or multi-criteria model analysis and optimization systems (DIDAS, see e.g. Rogowski (1989), or HYBRID, see e.g. Makowski and Sosnowski (1989)). Moreover, reference trajectories turned out to be particularly useful when applied with dynamic stochastic models, in which case they result in an essential generalization of the classical formulations of stochastic optimization (see Ruszczynski (1991), Karbowski et al. (1991)).

As an example, we present here the main ideas of the multi-criteria modelling and optimization system HYBRID (cf Makowski and Sosnowski (1989)). HYBRID can be considered as a mathematical programming package which includes all the functions necessary for the solution of multicriteria LP problems and single-criteria linear-quadratic problems.

[1] But it is not, precisely speaking, the l_p norm.

HYBRID is oriented towards an interactive mode of operation in which a sequence of problems is to be solved under varying conditions (e.g., different objective functions, reference points, values of constraints or bounds). Criteria for multiobjective problems may be easily defined and updated with the help of the package. Besides that HYBRID offers many options useful for diagnosis of a problem being solved. HYBRID is developed in two versions: one for UNIX (implemented on Sun Sparc 1+ and on VAX 6210) and one for a PC compatible with IBM PC.

HYBRID, like any mathematical programming package, should not be used directly as a tool in a real decision making situation (cf e.g. a discussion of problems of design and implementation of decision support systems by Makowski (1991)). Specialized software should be developed for each real-life case. HYBRID can however be used as a basis for such an application. HYBRID-FMS (cf Makowski and Sosnowski (1991)) is an example of a specialized DSS – based on the HYBRID system – for the problem of designing flexible manufacturing systems. It is composed of a specialized editor that allows for the modification of data (extracted from a data base) that define a particular instance of the problem. The specialized problem generator is used for the creation of a corresponding multicriteria optimization problem in a form suitable for a solver. The optimization problem is nonlinear, therefore a specialized solver has been designed and implemented in order to allow for the efficient solution of sequences of optimization problems. All modules of software are controlled by a driver and a uniform user interface (which also includes a context sensitive help) is implemented. Thus the software is easy to be used also by a user who has little computer experience.

One of the implementations of HYBRID (cf Makowski and Sosnowski (1988)) is specially useful for dynamic problems; this covers a wide area of applications of operation research. Many optimization problems in economic planning over time, production scheduling, inventory, transportation, control of dynamic systems can be formulated as linear dynamic problems. Such problems are also called multistage or staircase linear programming problems. A dynamic problem can be formulated as an equivalent large static LP and any commercial LP code may be used for solving it, but application of a specialized package has two advantages: first, a specialized algorithm exploits the structure of a dynamic problem and therefore is much more efficient, second, the user has the advantage of handling a problem as a dynamic one which results in an easy way of formulation of criteria and of interpretation of results. Since the first argument is of more technical nature, let us give an illustration of the second one.

Consider a problem of controlling, in order to prevent a flood, a water system (Kreglewski et al. (1985)) which consists of three general purpose reservoirs supplying water to the main river reach. The model consists of water balance equations for selected points and for each time period. The capacities of reservoirs are also constrained. The goal of the system dispatcher is to operate the reservoirs in such a way that the flood peaks on the main river do not coincide. If we use a static model we would have to deal with different variables for every point and time period, i.e. x_{it} would be an i-th variable (such as flow or storage) for the t-th period of time. However, in many situations it is more practical for a user to consider a whole trajectory as one variable, i.e. $x_i = \{x_{it}\}$, $t = 1, \dots, T$, where T is the number of periods. Criteria z_k of the following types (defined in the i-th point) have been chosen for evaluation of different control strategies:

FOL which corresponds to following a given reference (desired) trajectory of water flow:

$$z_k = \max_{t=1,\dots,T}(abs(x_{it} - \bar{x}_{it})) \longrightarrow \min$$

where x_i is a selected state or control variable, $\bar{x}_i$ —its reference trajectory

SUP which corresponds to minimization of the maximal (over time) difference between the flow and a corresponding reference trajectory:

$$z_k = \max_{t=1,\dots,T} (x_{it} - \bar{x}_{it}) \longrightarrow \min$$

DER which corresponds to minimization of the water flow changes (in consecutive time periods)

$$z_k = \max_{t=1,\dots,T} (abs(x_{it} - x_{i,t-1})) \longrightarrow \min$$

Obviously, a user does not enter the above formulae. All what he has to do is to select a relevant criterion type for a given point and to specify a reference trajectory for a variable selected at this point. Criteria values give (for each point separately) the corresponding values of a maximum (over time) difference between the actual and reference trajectories. Such information, in this case, is sufficient to assess a strategy of controlling the system of reservoirs for a given reference point[2]. Such a strategy may be either an optimal solution computed by the package or any strategy given by a user (in the latter case also its feasibility is assessed by the package). A reference point in this case is interpreted as the maximum (over time) difference between actual and desired values of flows and should not be confused with a corresponding reference trajectory.

4 Multi-objective games and bargaining; simulating multi-objective negotiation processes.

It is well known (see e.g. Rapoport (1989)) that normative game theory, while very far developed theoretically both for noncooperative and cooperative solution concepts, does not sufficiently well represent the complexity of practically observed behavior of decision makers in gaming, bargaining or negotiation situations. This observation stimulated research on modifications and extensions of game theory to obtain more practical tools for studying decision situations in which multiple decision makers might have conflicting interests. Especially interesting results in this direction, useful for understanding negotiating behavior, are connected to the concept of evolution of cooperation (see Axelrod, (1984)).

However, the practical use of computerized models of substantive aspects of a decision situation with conflicting interests has until now been restricted almost entirely to simulated gaming. Users (students, analysts, decision makers) participating in such a gaming exercise can play the roles of decision makers by entering their decisions to the simulation model and then observe the simulated results of these decisions. While very valuable as learning exercise, such simulated gaming has one essential drawback: users must rely on their individual intuition only, are usually denied more advanced decision support - whereas in real life any important, strategic decision is based on decision support provided by analysts, by discussions at executive boards, etc. This creates the challenge of introducing decision support tools into simulated gaming - see e.g. Wierzbicki (1989). Similarly as more advanced model simulation should include today multi-objective optimization tools, simulated gaming should include game-theoretical tools. However, such tools must not

[2]But a user may request information about a variable value for any time instance.

be treated as normative prescriptions, they should help only in analyzing the simulated game; therefore, they should also take into account possible multi-objective formulations of the game. Unfortunately, while game theory from its very beginning admitted multiple objectives of the players, it almost always assumed the possibility of aggregating them by value or utility functions. The basic solution concepts in game theory - starting, say, with the Nash noncooperative equilibrium concept - were not sufficiently generalized to the multi-objective case; only recently (see e.g. Wierzbicki (1990)) such extensions are attempted.

Multi-objective equilibria in a game are necessarily not uniquely defined, there might be many of them. Unilateral selection of such equilibria by players can lead to conflict escalation. If one player selects an equilibrium that seems rational to him, but another player aims at a different equilibrium, and both pursue strategies that should lead to the outcomes desired by them, this usually results in non-equilibrium outcomes that are much worse - even disastrous - for both the players. This fact is well known even in single-objective game theory for cases with multiple equilibria, as for example the so-called game of chicken (see e.g. Axelrod (1984)). However, since conflict escalation processes occur also in real life, the multi-objective formulation of a game can help in understanding such processes. For example, during a gaming simulation, appropriate conflict coefficients can be computed to inform the participants how far they are from a noncooperative or even cooperative equilibrium.

Together with game theory, bargaining theory has been developed (see e.g. Roth (1979)) concentrating on certain cooperative solutions to bargaining games which could be useful in supporting negotiations, in particular mediation and arbitration. Multi-objective aspects of bargaining have been also studied, (see e.g. Krus et al. (1990), Krus and Bronisz (1991)) which has led to the development of a prototype multi-objective mediation support system MCBARG.

Although there exists already notable methodological reflection on the art and science of negotiations (see e.g. Raiffa (1982)), the development of the methodology of using analytical models of substantive aspects of a multi-actor decision situation for supporting negotiations, mediation and arbitration is still in the beginning stages; however, some applications are known - see e.g. the multi-objective analysis of multilateral gas trade by Messner (1985) - and some conclusions can be already drawn. Such models can be used in many modes, corresponding to various procedural assumptions about negotiation situations:

a) Analytical models, combined with role-playing and gaming simulation, preferably in multi-objective formulation and supplemented with various decision-support tools, can be used by one side team preparing for negotiations. While such preparation might be very valuable, extreme caution should be exercised when preparing the substantive analytical model and when playing the role of the opposite side. The experts and analysts of one side are apt to misperceive the concerns and objectives of the opposite side; thus, the substantive model prepared by them might not include objective outcomes of the opposite side and not be adequate for gaming simulation. When role playing, ideological indoctrination of one side can lead to serious misrepresentation of the behavior of the opposite side (this was observed by one of the authors during an international conference on gaming; it should be noted that ideological indoctrination is not restricted to totalitarian societies, even more dangerous is a subconscious ideological bias in representatives of democratic societies which are convinced that they are free of such biases).

b) Analytical models representing internationally accepted knowledge in a given field, validated by experts from various sides, can be much more useful in supporting both the preparation for and the actual conduct of negotiations. This, however, assumes quite a different procedural model of negotiations: the sides in the conflict must be prepared for "getting to yes" (see Fisher and Ury (1981)), accept a mediating (at least in terms of gathering relevant knowledge and information) role of an impartial institution, preferably international, send their experts to participate in validating the model, be prepared to take seriously the outcomes simulated by the model. If these prerequisites are met, then the resulting model can have profound impact on the course of negotiations, such as in the known case of negotiating the law of the sea agreements; even more profound effects in terms of influencing the attitudes of high-level decision-makers had the (actually not computerized, only generally formulated) model of nuclear winter agreed upon by American and Soviet experts.

c) Perhaps not the contemporary generation of negotiators, but future generations more exposed to concepts and techniques of the information society might accept further extensions of the procedural role of knowledge formalized in models and decision support systems in negotiations and mediation. To achieve this, the following conditions should be satisfied:
   (i) The principle of user sovereignty should be particularly stressed and strictly observed when constructing decision support for negotiations (diplomats quite rightly stress the role of their intuition and negotiating skills; thus, a decision support system must only augment and enhance these skills).
   (ii) Prototype negotiation and mediation support systems - including various procedural variants, such as support in unilateral decisions, support in evaluation of multilateral role-playing, support in mediation and arbitration according to various procedural schemes - should be introduced as a part of training young negotiators.
   (iii) The methodology of developing and using analytical models that represent substantive aspects of multi-actor decision situations with conflicting interests should be further advanced, along with appropriate developments of multi-objective, multi-actor decision support. IIASA can play a considerable role in this development, particularly when concentrating on models in such substantive fields in which the Institute has a tradition of excellence (e.g. international environmental studies) and combining this with further methodological developments.

5 Conclusions.

At the beginning of the information age, one of the main challenges is to combine human expertise and intuition with more formalized knowledge, while preserving the strengths of both parts. In our increasingly more complex and fastly changing world, there are also increasingly more problems whose analysis might profit from knowledge formalization in the form of computerized models; but we also need more flexible tools of eliciting pertinent aspects of such formalized knowledge if we wish to combine them with human intuition.

Can multi-objective modelling, optimization and decision support be really useful for negotiations and mediation ? The answer to this question is positive. For this purpose, the use of multi-objective model formulation, optimization and game theory treated not as goals or normative prescriptions, but as flexible tools, might be especially useful. Using this methodology one can design and implement rather flexible tools (further on referred to as DSS) that can be tailored to specific needs. However, to make a successful implementation of a DSS, many conditions should be met. Below we list only few critical conditions (for a more complete discussion see e.g. Makowski (1991)):

- A DSS should serve as a tool for making a fast, but reliable analysis of large amounts of data and logical relations in order to provide results that can support experts or decision makers. However, such an analysis is problem specific and covers only a part of the issues related to negotiations or mediations. Therefore, for each case a careful study should be made for identification of that part, for which it is possible and desired to design and implement a DSS.
- We must be humble before the power of human expertise and intuition. Therefore, a DSS should clearly be only a supportive tool, under the full control of a user. The user should be aware of its function (especially about its limitations) and of the underlying mathematical model. A user must be convinced that a DSS is not aimed at replacing his/her knowledge or experience, but be sure that it is just a tool for performing sophisticated or cumbersome calculations and analysis of a problem that is specified with a user. A user should be sure that a DSS provides him/her with a useful analysis thus allowing him/her to concentrate on that part of the negotiations or mediation that is not formalized and covered by a DSS.
- We must be modest in claims of usefulness of a DSS for any application. However, the usefulness of a DSS, particularly in the negotiation field, can be improved only through real-life applications.
- A team which develops a DSS should work in close cooperation with future user(s) of a specific DSS. As observed by many authors, this is a critical condition for any real-life application of any DSS. Case studies for applications of DSS have to be carefully chosen. They should contain a subproblem that is complicated enough to justify development of a DSS and that is simple enough to be covered by a DSS.

One can ask why such applications have not been already made. We think that the reason is twofold. First, the proposed methodology is fairly new. It is still not well represented in text-books. Many applications of multi-objective optimization use techniques that have many drawbacks (some of them are discussed in Section 3), thus spreading a misleading judgement that such techniques are not useful. Second, *"serious"* applications of computers used to be restricted to specialists well trained in operating systems. This second argument is now vanishing. Rapid development of computer hardware will result, in about five years, in a palm-top type personal computer that has a computational power of nowaday's workstation. Usage of networks and sharing of data, together with technology of cellular communication, will become universal, thus making collaborative computing and computing on the move also popular. Development of so called *user friendly* software (which includes multi-media presentation and communication) together with powerful but relatively cheap hardware will result in proliferation of computers to many new areas and applications thus making – in a perspective of just few years – the use of computers by almost anyone natural. However, it should be stressed that specification, estimation and verification of a mathematical model (which underlies any DSS) must be left to specialists, who should work together with future users.

To test usefulness of this perspective, interdisciplinary cooperation, research and real-life (or at least realistic) testing applications are needed. The authors of this paper are convinced that the PIN Project at IIASA can play a key role in organizing such activities. What can be prepared methodologically and experimentally today, might be applied tomorrow by new generations of people of the information age.

References.

Axelrod, R. (1984): The Evolution of Cooperation. Basic Books, New York.

Charnes, A. and W.W. Cooper (1977): Goal programming and multiple objective optimization. J. Oper. Res. Soc., Vol. 1 (1977) pp. 39-54.

Dreifus, H. and S. Dreifus (1986): Mind over Machine: The Role of Human Intuition in the Era of Computers.

Fisher, R. and W. Ury (1981): Getting to Yes. Negotiating Agreement Without Giving In. Houghton Mifflin, Boston.

Gleick, J. (1987): Chaos - Making a New Science. Viking, New York.

Grauer, M., A. Lewandowski and L. Schrattenholzer (1982): Use of the reference level approach for the generation of efficient energy supply strategies. WP-82-19, IIASA, Laxenburg.

Haimes, Y.Y. and W.A. Hall (1974): Multiobjectives in water resource systems analysis: the surrogate trade-off method. Water Resource Research, Vol. 10, pp. 615-624.

Ignizio, J.P. (1978) : Goal programming - a tool for multiobjective analysis. Journal for Operational Research, Vol. 29 (1978) pp. 1109-1119.

Kallio, M., A. Lewandowski and W. Orchard-Hays (1980): An implementation of the reference point approach for multi-objective optimization. WP-80-35, IIASA, Laxenburg.

Karbowski, A., T. Rogowski and J. Sobczyk (1991): DIDAS-L - A Dynamic Interactive Decision Support System for Linear Problems. IIASA Workshop on User-Oriented Methodology and Techniques of Decision Analysis and Support, Serock near Warsaw, September 1991 (to be published).

Keeney, R. and H. Raiffa (1976): Decisions with Multiple Objectives: Preferences and Value Trade-Offs. J. Wiley, New York.

Kiwiel, K.C., and A. Stachurski (1989): Issues of Effectiveness Arising in the Design of a System of Nondifferentiable Optimization Algorithms. In A. Lewandowski and A.P. Wierzbicki, eds.: Aspiration Based Decision Support Systems. Springer Verlag, Berlin - Heidelberg.

Korhonen, P. and J. Laakso (1986): Solving a generalized goal programming problem using visual interactive approaches. European Journal of Operational Research 26, pp. 355-363.

Kreglewski, T., Lewandowski, A. and T. Rogowski (1985). Dynamic Extension of the DIDAS system and its Application in Flood Control. In M. Grauer, M. Thompson, A.P. Wierzbicki, eds.: Plural Rationality and Interactive Decision Processes, Springer Verlag.

Kreglewski, T., J. Paczynski, J. Granat and A.P. Wierzbicki (1989): IAC-DIDAS-N: A Dynamic Interactive Decision Analysis and Support System for Multicriteria Analysis of Nonlinear Models. In A. Lewandowski and A.P. Wierzbicki, eds.: Aspiration Based Decision Support Systems. Springer Verlag, Berlin - Heidelberg.

Krus, L., P. Bronisz and B. Lopuch (1990): MCBARG - Enhanced, A System Supporting Multicriteria Bargaining, CP-90-06, IIASA, Laxenburg, Austria.

Krus, L. and P. Bronisz (1991): Multicriteria Mediation Support, paper prepared for *Systems Analysis Techniques for International Negotiations*, Scoping Conference, October 9-10, 1991, IIASA, Laxenburg, Austria.

Kurzhanski, A.B. (1986): Inverse problems in multiobjective stochastic optimization. In Y. Sawaragi, K. Inue and H. Nakayama, eds.: Towards Interactive and Intelligent Decision Support Systems, Springer Verlag, Berlin - Heidelberg.

Lewandowski, A., T. Kreglewski, T. Rogowski and A.P. Wierzbicki (1989): Decision Support Systems of DIDAS Family. In A. Lewandowski and A.P. Wierzbicki, eds.: Aspiration Based Decision Support Systems. Springer Verlag, Berlin - Heidelberg.

Makowski, M. and J.S. Sosnowski (1988): A Mathematical Programming Package for Multicriteria Dynamic Linear Problems HYBRID. Methodological and User Guide to Version 3.03, WP-88-02, IIASA, Laxenburg, Austria.

Makowski, M. and J.S. Sosnowski (1989) Mathematical Programming Package Hybrid. In: A. Lewandowski and A.P. Wierzbicki, eds.: Aspiration Based Decision Support Systems. Springer Verlag, Berlin - Heidelberg.

Makowski, M. and J.S. Sosnowski (1991): Hybrid-FMS: An element of DSS for designing Flexible Manufacturing Systems, in: Multiple Criteria Decision Support, P. Korhonen, A. Lewandowski, J. Wallenius (Eds.), Lecture Notes in Economics and Mathematical Systems, Vol. 356, Springer Verlag.

Makowski M. (1991): Selected Issues od design and implementation of Decision Support Systems, WP-91-16, IIASA, Laxenburg, Austria.

Messner, S. (1985): Natural gas trade in Europe and interactive decision analysis. In G. Fandel et al., eds.: Large Scale Modeling and Interactive Decision Analysis. Springer Verlag, Berlin - Heidelberg.

Nakayama, H. and Y. Sawaragi (1983): Satisficing trade-off method for multi-objective programming. In M. Grauer and A.P. Wierzbicki, eds.: Interactive Decision Analysis, Springer Verlag, Berlin - Heidelberg.

Ogryczak, W., K. Studzinski and K. Zorychta (1989): Solving Multiobjective Distribution-Location Problems with the DINAS System. In A. Lewandowski and A.P. Wierzbicki, eds.: Aspiration Based Decision Support Systems. Springer Verlag, Berlin - Heidelberg.

Rapoport, A. (1989): Decision Theory and Decision Behavior. Kluwer Academic Publishers, Dordrecht.

Rogowski, T. (1989): Dynamic Aspects of Multiobjective Optimization in Decision Support Systems. In A. Lewandowski and A.P. Wierzbicki, eds.: Aspiration Based Decision Support Systems. Springer Verlag, Berlin - Heidelberg.

Ruszczynski, A. (1991): Reference Trajectories in Multistage Stochastic Optimization Problems. IIASA Workshop on User-Oriented Methodology and Techniques of Decision Analysis and Support, Serock near Warsaw, September.

Salukvadze, M.E. (1971): On the optimization of vector functionals, 1. Programming optimal trajectories, 2. Analytic construction of optimal controllers. Automation and Remote Control, Vol. 31 (1971) No. 7, 8.

Salukvadze, M.E. (1974): On the existence of solutions in problems of optimization under vector-valued criteria. JOTA, Vol. 13, No. 2, pp. 203- 217.

Sawaragi, Y., H. Nakayama and T. Tanino (1985): Theory of Multiobjective Optimization. Academic Press, Orlando Fl.

Seo, F. and M. Sakawa (1988): Multiple Criteria Decision Analysis in Regional Planning: Concepts, Methods and Applications. D. Reidel Publishing Company, Dordrecht.

Steuer, R.E. and E.V. Choo (1983): An interactive weighted Chebyshev procedure for multiple objective programming. Mathematical Programming, Vol. 26, pp. 326-344.

Steuer, R.E. (1986): Multiple Criteria Optimization: Theory, Computation and Application. J. Wiley, New York.

Strubegger, M. (1985): An approach for integrated energy - economy decision analysis: the case of Austria. In G. Fandel et al., eds.: Large Scale Modeling and Interactive Decision Analysis. Springer Verlag, Berlin - Heidelberg.

Toffler, A. (1980) The Third Wave. W.Morrow, New York.

Wallenius, H. (1991): Implementing Interactive Multiple Criteria Decision Methods in Public Policy, Academic dissertation, Faculty of Social Sciences, University of Jyväskylä, Jyväskylä, Finland.

Wierzbicki, A.P. (1977): Basic properties of scalarizing functionals for multiobjective optimization. Mathematische Operationsforschung und Statistik, s. Optimization, Vol. 8, pp. 55-60.

Wierzbicki, A.P. (1982): A mathematical basis for satisficing decision making. Mathematical Modelling, Vol. 3, pp. 391-405.

Wierzbicki, A.P. (1983): Changing Cultural Paradigms, Options (IIASA), No 4, p. 10.

Wierzbicki, A.P. (1984): Models and Sensitivity of Control Systems. Elsevier, Amsterdam.

Wierzbicki, A.P. (1986): On the completeness and constructiveness of parametric characterizations to vector optimization problems. OR-Spektrum, Vol. 8, pp. 73-87.

Wierzbicki, A.P. (1987): Education for a new cultural era of informed reason. In J.G. Richardson, ed.: Windows on Creativity and Invention, Lomond Publications, Mt. Airy, Maryland.

Wierzbicki, A.P. (1989): Multiobjective decision support for simulated gaming. In A.G. Lockett and G. Islei, eds.: Improving Decision Making in Organisations, Proceedings, Manchester 1988. Springer Verlag, Berlin - Heidelberg.

Wierzbicki, A.P. (1990): Multiple criteria solutions in noncooperative game theory. Part III: Theoretical foundations. Kyoto Institute of Economic Research, Discussion Paper No. 288.

Zeleny, M. (1973): Compromise programming. In J.L. Cochrane and M. Zeleny, eds.: Multiple Criteria Decision Making. Univ. of South Carolina Press, Columbia S. Carolina.

Yu, P.L. (1985): Multiple-Criteria Decision Making - Concepts, Techniques and Extensions. Plenum Press, New York and London.