

# Working Paper

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WP-93-72

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# Incomplete Information and the Cost-Efficiency of Ambient Charges

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## **Abstract.**

The established opinion is that in the face of uncertain information on pollution control costs, environmental agencies cannot set levels of ambient charges enabling the reaching of desired concentration levels at receptor sites in a cost-effective way. Although a trial-and-error procedure could finally result in the attainment of concentration standards this is generally not cost-effective. This paper proves that environment agencies can develop adaptive procedures that enable the achievement of the standards at minimum costs. The proof is based on ideas of non-monotonic optimization. The adaptation mechanisms are applied in a case study of charges for acidification in the Netherlands. The results show that the iterative procedure approaches the cost- minimum fairly quickly but that over and undershooting may occur underway. The number of iterations and extent of overshooting can be reduced by using available knowledge on the violation of ambient concentrations at receptors and by a simulation of polluters responses to charges.

**Key-words:** cost-effective emission charges, pollution charges, uncertainty, non-monotonic optimization, adaptation, market mechanism.

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## **1. Introduction.**

Targets of environmental policy should be met and pollution control costs should not be higher than is strictly necessary. Baumol and Oates [2] argued that a uniform emission charge can be a cost effective instrument when the objective of the environmental authority is to reduce total emissions at sources to a specified target level when complete information on pollution control costs is given to every single firm and household. However, if the authority has full information on the costs of reducing emissions it will also be possible to design cost-effective emission standards for each individual source making it unclear why a charge should be preferred. The choice between the two instruments appears to be particularly important when the authority does not possess full information on abatement costs. Under conditions of cost uncertainty the dilemma seems to be that of a choice between a cost-effective but environmentally ineffective emission charge on the one hand and an environmentally effective emission standard that is cost ineffective on the other. In their article, Baumol and Oates tried to solve the dilemma by proposing a trial-and-error adjustment mechanism: if actual emissions are below, respectively above, the emission target at the initial emission charge the tax should be raised, respectively lowered. Baumol and Oates expect that emitters will adjust their emissions and that after a number of steps the target level for total emissions would be approached.

A uniform emission charge for all sources will generally not be cost effective (Tietenberg [9],[10]). Since the contribution of different sources to concentration usually differs, the emission charge has to be different for each source and should be tailored to its location. Bohm and Russell [3] add uncertainty on costs and suggest that if there is more than one receptor, the authority will not be able to set the appropriate level of ambient charge to meet environmental standards at each receptor at minimum cost. If there is only one receptor, trial-and-error can be used since in this case the ratios of the (source specific) emission charges are equal to the transfer coefficients of the sources to the one receptor. The only action needed is to set an initial level and change all charges with the same percentage until the ambient standard is met. In case of multiple receptors, the environmental agency would have to know

not only the transfer coefficients but also the costs of emission reduction in order to determine the shadow prices of each receptor. Bohm and Russel conclude that although trial-and-error can result in attainment of the ambient standards at each receptor there is no guaranty that these standard will be met at minimal cost, if the environmental authority is uncertain about pollution control costs.

The aim of this paper is to demonstrate that the vector of cost-effective emission charges can be identified. An adjustment mechanism is introduced analogous to the trial-and-error type of procedure suggested by Baumol and Oates. The difference is that in our paper the adjustment mechanism uses the difference (generally random) between actual and target concentrations of pollutants as a signal for the adjustment of ( emission ) charges instead of the gap between actual and target emissions. It will be demonstrated by way of non-monotonic optimization techniques that the search process converges and that the environmental authority is able to determine a cost effective set of emission taxes without knowledge of pollution control costs.

This paper is organized in 6 Sections. Section 2 formulates the problem of setting emission standards and charges under the assumption that the environmental agency has imperfect information on pollution control costs. Section 3 then proceeds to define an adjustment mechanism that enables the agency to discover the cost minimizing vector of emission charges without perfect knowledge of the costs. It is assumed that the sources have perfect information. In section 4 the cost effectiveness of emission standards and emission charges is discussed for the case where the environmental agency not only has imperfect information, but costs are inherently stochastic and vary over time. Section 5 extends this discussion to the case of uncertainties involved in the transfer coefficients. We show that under conditions of cost uncertainty emission charges are environmentally effective and more cost effective than emission standards. If in addition to that there is uncertainty about the transfer coefficients, a system of pollution charges may be preferable to the emission charges. Section 6 applies the theory in an empirical setting to illustrate how the adjustment mechanism functions. Conclusions are given in section 6.

## 2. Deterministic Pollution Control.

Let  $i = 1, \dots, n$  be sources of emission (firms and households, regions or countries) and  $j = 1, \dots, m$  receptors. Each polluter  $i$  emits a single pollutant at the rate  $x_i$ . The emission vector  $x = (x_1, \dots, x_n)$  is mapped into concentrations at receptors by a transport equation, which is described by a transfer matrix  $H = \{h_{ij}\}$ ,  $i = 1, \dots, n$ ,  $j = 1, \dots, m$ , where  $h_{ij}$  stands for the contribution made by one unit of emission of source  $i$  to the concentration of pollutants at point  $j$ . Ambient concentrations  $q_j$  at the receptor points  $Q = (q_1, \dots, q_m)$  impose constraints on emission rates

$$\sum_{i=1}^n x_i h_{ij} \leq q_j, j = 1, \dots, m, \quad (1)$$

$$x_i \geq 0, i = 1, \dots, n. \quad (2)$$

The environmental authority's problem is to achieve target concentrations  $Q$  at minimum total cost for polluters: that is to choose the vector  $x = (x_1, \dots, x_n)$  to minimize the total cost

$$\sum_{i=1}^n f_i(x_i) \quad (3)$$

subject to constraints (1)-(2).

The solution of this problem  $x^* = (x_1^*, \dots, x_n^*)$  defines the cost effective set of emission levels which can be imposed on sources as emission standards. The implementation of such a policy of direct regulation requires that the regulatory authority has full information on parameters  $h_{ij}$ ,  $q_j$  and cost functions  $f_i(x)$ .

By using classical duality results Tietenberg [10] has demonstrated that there exists a vector  $\lambda = (\lambda_1, \dots, \lambda_m)$  of shadow prices (taxes) at receptors which corresponds with the vector of cost - effective emission standards. The simple linear structure of the pollution transport equations allows the calculation of a vector of emission charges  $u_i$ . The emission charge can be written as a function of transfer coefficients and the shadow prices  $\lambda_j$ ,  $j = 1, \dots, m$  at the receptors:

$$u_i = h_{i1}\lambda_1 + h_{i2}\lambda_2 + \dots + h_{im}\lambda_m. \quad (4)$$

If optimal emission charges  $u_i = u_i^*$  associated with optimal  $\lambda_j = \lambda_j^*$  are imposed they will induce sources to adjust emissions in an optimal way:

$$f_i(x_i) + u_i x_i = f_i(x_i) + \left( \sum_{j=1}^m \lambda_j h_{ij} \right) x_i = \min, x_i \geq 0 \quad (5)$$

for  $i = 1, \dots, n$ , where  $\lambda_j = \lambda_j^*$ ,  $j = 1, \dots, m$ , or  $u_i = u_i^*$ . It results in the cost effective emission  $x_i^*$ ,  $i = 1, \dots, n$ .

The main question is how the environmental authority can determine the cost-minimizing charges without knowledge of the cost functions where there is more than one receptor (Bohm and Russel, [3]). In principle the authority could easily find a vector of high enough emission taxes to meet the target concentrations, but there is, however, no guaranty that the vector of emission charges is cost effective.

### 3. Adjustment mechanism.

This section describes an adjustment mechanism capable of implementing the cost-effective policy through the adaptation of the emission taxes in successive steps to equilibrium shadow prices. The proposed procedure decomposes the pollution control problem into two types of decision problems. The first choice problem is that of the environmental authority having to decide on the tax and adjust its level, given information on the discrepancy between actual and target concentrations. The procedure for adjusting taxes is based on non-monotonic, deterministic, and stochastic optimization techniques, which do not require information on the total cost of controlling emissions. The proof will be given that actually the environmental authority can concentrate only on monitoring target concentrations

$$\Gamma_j(x) = \sum_{i=1}^n x_i h_{ij} - q_j, j = 1, \dots, m$$

without bothering about costs. The second optimization problem is that of individual cost minimizing polluters which choose their emission level given the emission charge that is imposed by the environmental agency. The formal description of the adjustment mechanism is the following.

Suppose  $\lambda^0 = (\lambda_1^0, \dots, \lambda_m^0)$  is a vector of initial taxes on concentrations (or deposition) at receptor points and let  $\lambda^k = (\lambda_1^k, \dots, \lambda_m^k)$  be the vector of pollution taxes at step  $k$  of the adaptive process. Each polluter adjusts its emission level  $x_i^k, i = 1, \dots, n$  by minimizing its

individual cost function  $f_i(x) + u_i^k x$  at current values of pollution taxes  $\lambda_j^k, j = 1, \dots, m$ , which are translated into emission taxes  $u_i^k$  by the equation (4):  $u_i^k = \sum_{j=1}^m \lambda_j^k h_{ij}, i = 1, \dots, n$ . The agency observes the ambient concentrations generated by the vector  $x^k$ ; it calculates the difference  $\Gamma_j(x(\lambda)) = \sum_{i=1}^n x_i^k h_{ij} - q_j, j = 1, \dots, m$  between actual and target concentrations and adjusts the pollution taxes in the next step according to the formula

$$\lambda_j^{k+1} = \max\left\{0, \lambda_j^k + \rho_k \left(\sum_{i=1}^n x_i^k h_{ij} - q_j\right)\right\}, \quad (6)$$

where  $j = 1, \dots, m$  and  $k=0, 1, \dots$ . The step size multiplier  $\rho_k$  is chosen such as to ensure the convergence of the sequence  $\lambda^k, k = 0, 1, \dots$  to the optimal dual variables  $\lambda^* = (\lambda_1^*, \dots, \lambda_m^*)$ .

It is important to observe that the direction of the movement

$$\Gamma(x^k) = (\Gamma_1(x^k), \dots, \Gamma_m(x^k))$$

in (6) is a subgradient of the following, in general, nondifferentiable function (see appendix 2).

$$\gamma(\lambda) = \min_{x \geq 0} \left[ \sum_{i=1}^n f_i(x_i) + \sum_{j=1}^m \lambda_j \left( \sum_{i=1}^n x_i h_{ij} - q_j \right) \right] = L(x(\lambda), \lambda). \quad (7)$$

The minimization of  $\gamma(\lambda), \lambda \geq 0$  is the dual minimax problem to the original pollution control problem (1)-(3). Under usual assumptions on the convexity of the functions  $f_i(x)$  the minimax problem (6) is equivalent to the original problem (1)-(3).

The Lagrange function

$$L(x, \lambda) = \sum_{i=1}^n f_i(x_i) + \sum_{j=1}^m \lambda_j \left( \sum_{i=1}^n x_i h_{ij} - q_j \right)$$

of the problem (1)-(3) may be considered as an environmental indicator providing a link between emissions  $x$  and desired environmental quality. The shadow prices  $\lambda_j$  convert the indicators  $\Gamma_j(x)$  or constraints (1) to the same unit of measurement as the objective function (3). Indicators  $\Gamma_j$  and shadow prices (taxes)  $\lambda_j$  play an essential role in the design of the tax adjust mechanisms that will be discussed in this section .

The procedure (6) can be interpreted as a kind of "market system". According to this scheme cost minimizing polluters independently adjust emissions to the current taxes  $u_i^k$  minimizing the cost function (5). The agency learns what the current pollutions  $\Gamma(x^k)$  are and reacts as Walrasian auctioneer would. If he observes an "excess demand" for available environmental user space ( $\Gamma_j > 0$ ) he will raise the price  $\lambda_j$  for users. In the other case the



price will be lowered. Prices are a signal for users of environmental space to adjust their emissions accordingly.

The procedure (6) can be modified easily to include any additional a priori information on possible tax values. For example, if we know an upper bound  $\bar{\lambda}_j$ ,  $j = 1, \dots, m$  on optimal values  $\lambda_j^*$ , the process (6) is modified as the following:

$$\lambda_j^k = \min \left[ \max \left\{ 0, \lambda_j^k + \rho_k \left( \sum_{i=1}^n x_{i=1}^k h_{ij} - q_j \right) \right\}, \bar{\lambda}_j \right]. \quad (8)$$

If the vector  $\lambda^*$  has to belong to a convex set  $\Lambda$ , the vector of new taxes  $\lambda^{k+1}$  is calculated as

$$\lambda^{k+1} = \text{Pr } j_{\Lambda} \left[ \lambda^k + \rho_k \Gamma(\lambda^k) \right], \quad k = 0, 1, \dots \quad (9)$$

where  $\text{Pr } j$  is the symbol of the projection operation on the set  $\Lambda$ :

$$\text{Pr } j_{\Lambda} [y] = \arg \left\{ \min \|y - \lambda\|^2, \lambda \in \Lambda \right\}.$$

Examples of such operation on the set  $\lambda \geq 0$  and on hypercube  $0 \leq \lambda \leq \bar{\lambda}$  are given by eqs. (6),(8). The main problem concerning the adjustment mechanisms (6), (8) and (9) is that value  $\rho_k$ ,  $k = 0, 1, \dots$  can not be chosen in order to decrease the total cost  $\sum_{i=1}^n f_i(x_i)$  by passing from  $x^k$  to  $x^{k+1}$  since this function is unknown to the agency. Besides this, the implementation of a monotonic optimization procedure, which guarantees an improvement of the objective function at each adjustment step, runs into difficulties since, in general, the direction of the movement  $\Gamma(x^k)$  in (6) is a subgradient of the non-differentiable function  $\gamma(\lambda)$  at  $\lambda = \lambda^k$ . In appendix 1 we outline the proof of the theorem showing that procedures (6), (8) and (9) converge under rather general conditions for the values of the step size multipliers  $\rho_k$ . Indeed it may be any sequence  $\rho_0, \rho_1, \dots$  such that  $\rho_k \geq 0$ ,  $\sum_{k=0}^{\infty} \rho_k = \infty$ , for example  $\rho_k = c/k$ , where  $c$  is a positive constant. These conditions guarantee a convergence independently of the initial vector of taxes  $\lambda^0$ . The procedure does not lead to monotonic improvements of total costs since the choice of the step size multipliers  $\rho_0, \rho_1, \dots$  is based on excess concentrations ( positive or negative ) and not on the calculation of the total cost. Nevertheless the process enables to find optimal taxes (see appendix 1). We also show that the convergence takes place even for nonconvex functions  $f^i(x)$ ,  $i = 1, \dots, n$

#### 4. Imperfect information of individual sources.

The deterministic pollution control problem discussed in section 3 assumed that the individual sources have accurate information on costs. An implicit assumption in section 3 was that the conditions determining costs remain unchanged during the whole period of adjusting the environmental tax. Dynamic convergence to equilibrium (9) requires only the monitoring of environmental indicators  $\Gamma_j(x^k) = \sum_{i=1}^n x_i^k h_{ij} - q_j$  at emission levels  $x^k = (x_1^k, \dots, x_n^k)$ , with polluters minimizing costs  $f_i(x) + u_i^k x$  at current emission charges  $u_i^k = \sum_{j=1}^m \lambda_j^k h_{ij}$ .

In this section we drop the assumption of unchanged conditions. For each source the costs of pollution control depend on a large number of factors. Some of them are of a rather general kind, like weather conditions, prices of inputs; others are more specific; for example types of fuel and other raw material used, type and age of existing pollution control equipment, its state of maintenance, quality of operating personal, knowledge of available new abatement technology. Even in such a case sources can not predict exact values of pollution control costs. It will be assumed that the agency forms expectation about pollution control costs and sets out to minimize the expected value of aggregate pollution control costs.

Actually the "state of the world" that affects cost will change during the adjustment period. We assume that the polluter is informed about the current local conditions and minimizes his costs given his certain information about factors affecting the cost-function. We shall show that in the face of these assumptions the decentralized strategy of emission taxes results in lower expected total pollution control costs than the centralized policy of setting emission standards.

Suppose  $f_i(x_i, v_i)$  is the cost function of the source  $i$ , where  $x_i$  is an emission level and  $v = (v_1, \dots, v_n)$  is a random vector representing variables that affect the source costs. It is plausible to assume that under such circumstances the agency is able to compute only expected costs  $F_i(x_i) = E f_i(x_i, v_i)$ . We assume the existence of all necessary integrals. For example, we can keep in mind the case that  $v$  has a finite number of values.

Let us first look at a policy of setting emission standards. The minimum cost emission standard strategy for the environmental agency is found by solving the stochastic optimization problem: minimize

$$F(x) = \sum_{i=1}^n \text{E}f_i(x_i, v_i) \quad (10)$$

subject to

$$\sum_{i=1}^n x_i h_{ij} \leq q_j, \quad j = 1, \dots, m, \quad i = 1, \dots, n. \quad (11)$$

If expected costs  $F_i(x) = \text{E}f_i(x, v_i)$ ,  $i = 1, \dots, n$  are convex functions, we can derive, using duality theory, the existence of pollution charges  $\lambda^* = (\lambda_1^*, \dots, \lambda_m^*)$  and emission charges  $u_i^* = \sum_{j=1}^m \lambda_j^* h_{ij}$  such that the least cost optimal strategy  $x_i^*$  minimizes

$$\text{E}f_i(x, v_i) + u_i x, \quad x \geq 0 \quad (12)$$

for  $u_i = u_i^*$ ,  $i = 1, \dots, m$ . Equation (12) represents the agency's expectation of the emission reductions induced by charges  $u_i^*$ . However, the agency's expectation of emissions will usually not coincide with the actual reactions of polluters. Since each polluter  $i$  knows the situation  $v_i$  and costs at the source, he actually adjusts emission to the level  $x_i(u_i, v_i)$  minimizing

$$f_i(x, v_i) + u_i x, \quad x \geq 0. \quad (13)$$

Therefore the environmental indicators  $\Gamma_j(x^k)$  of the procedure (6) are calculated at random vector of emission levels  $x^k = (x_1^k, \dots, x_n^k)$  minimizing cost functions (13) at random situations  $v_i^k$ ,  $i = 1, \dots, n$ . In this case procedure (6) generates a sequence  $\lambda^0, \lambda^1, \dots$  of random pollution charges and corresponding emission charges  $u^0, u^1, \dots$ . It can be shown (appendix 2) that the direction of movement  $\Gamma(x^k) = (\Gamma_1(x^k), \dots, \Gamma_n(x^k))$  in (6) is a stochastic subgradient of the following function  $\gamma^1(\lambda)$ , which is similar to function  $\gamma(\lambda)$  defined by eq. (6):

$$\begin{aligned} \gamma^1(\lambda) &= \min_{x(v) \geq 0} \sum_{i=1}^n \text{E} \left[ f_i(x_i(v), v_i) + \left( \sum_{j=1}^m \lambda_j h_{ij} \right) x_i(v) \right] - \sum_{j=1}^m \lambda_j q_j = \\ &= \sum_{i=1}^n \text{E} \min_{x_i \geq 0} \left[ f_i(x_i, v_i) + \left( \sum_{j=1}^m \lambda_j h_{ij} \right) x_i \right] - \sum_{j=1}^m \lambda_j q_j \end{aligned} \quad (14)$$

In other words, the vector  $\Gamma(x^k)$  is obtained formally by taking the gradient (with respect to variables  $\lambda = (\lambda_1, \dots, \lambda_m)$ ) under the symbol of the mathematical expectation for observed  $v_1^k, \dots, v_n^k$  and calculated  $x_1^k, \dots, x_n^k$ . More precisely, it is possible to show (appendix 2) that the random vector  $\Gamma(x^k)$  is an unbiased estimate of a subgradient  $\gamma_\lambda^1$  of function  $\gamma^1(\lambda)$  at  $\lambda = \lambda^k$ :  $\text{E}[\Gamma(x^k) | x^k, \lambda^k] = \gamma_\lambda^1(\lambda^k)$ , where  $\text{E}[\cdot | \cdot]$  is the conditional expectation symbol.

From the convergence of the stochastic quasigradient (see, for example, Ermoliev and Wets (1987)) methods we can derive the convergence of the random sequence  $\lambda^0, \lambda^1, \dots$  to the minimal value of the function  $\gamma^1(\lambda)$ ,  $\lambda \geq 0$ . The proof of this fact is close to the one given in the appendix 1. Let us show that the expected cost of the emission charge strategy  $u_i^* = \lim_{k \rightarrow \infty} \sum_{j=1}^n \lambda_j^k h_{ij}$  is less than the expected costs of the emission standard strategy derived from the problem (10)-(11).

It is easy to see that the minimization of  $\gamma^1(\lambda)$  is a dual problem to the following: find emission vector  $x(v) = (x_1(v), \dots, x_n(v))$  that minimizes the expectation functional

$$F(x(\cdot)) = \sum_{i=1}^n \text{E}f_i(x_i(v), v_i) \quad (15)$$

subject to

$$\sum_{i=1}^n \text{E}x_i(v)h_{ij} \leq q_j, j = 1, \dots, m, x_i(v) \geq 0, i = 1, \dots, n. \quad (16)$$

Suppose that  $x^*(\cdot)$  is an optimal solution of this problem. Consider the Lagrange function

$$L(x(v), \lambda) = \sum_{i=1}^n \text{E}f_i(x_i(v), v_i) + \sum_{j=1}^m \lambda_j \left( \sum_{i=1}^n \text{E}x_i(v)h_{ij} - q_j \right).$$

The existence of a saddle point  $(x^*(v), \lambda^*)$  such that

$$L(x^*(v), \lambda) \leq L(x^*(v), \lambda^*) \leq L(x(v), \lambda^*) \quad (17)$$

for all  $x(v) \geq 0, \lambda \geq 0$  follows from the convexity of the function  $f_i(\cdot, v_i)$  for any fixed  $v$ .

Since decision  $x(\cdot)$  in eqs. (15)-(16) depends on concrete situations  $v$  at sources, the optimal value of the objective functions in eqs.(15)-(16) is smaller than the optimal value of the objective function in eqs.(10)-(11). Hence

$$F(x^*) \geq F(x^*(\cdot)) = L(x^*(v), \lambda^*) = \max \gamma^1(\lambda).$$

## 5. Uncertainty on transfer coefficients.

In the addition to the uncertainty involved in pollution control costs there exists uncertainty about transfer coefficients. Transmission of pollutants by air is affected by factors such as wind direction and speed and weather conditions. The concentration of effluents in water depends on the volume and velocity of the flow. It is plausible to assume that the source

has better information on the specific conditions that determine the actual value of the transfer coefficients than the agency has. Let us show that in such situations the policy of pollution charges may be better than emission charges.

Suppose that the agency is able to monitor values of random indicators

$$\Gamma_j(x) = \sum_{i=1}^n x_i \eta_{ij} - q_j, \quad j = 1, \dots, n.$$

On the basis of these values the agency pursues the pollution charges policy: it announces values of shadow prices  $\lambda_j$  instead of emission's taxes  $u_i$ .

Assume that each source  $i$  has better information on the transfer coefficients  $\eta_{ij}$  rather than averaged values  $h_{ij} = E\eta_{ij}$ , for example, some conditional expectations  $\zeta_{ij}$  of  $\eta_{ij}$  given a particular weather situation at the source. We assume that each source  $i$  knows random values  $\zeta_{ij}$ ,  $j = 1, \dots, m$ , and let  $w$  be the collection of all uncertainties  $w = \{v_i, \zeta_{ij}, i = 1, \dots, n, j = 1, \dots, m\}$ . Then the emission level is adjusted to the level minimizing

$$f_i(x, v_i) + \left( \sum_{j=1}^m \lambda_j \eta_{ij} \right) x, \quad x \geq 0. \quad (19)$$

The first term represents the direct cost of reducing emissions and the second term the expenditure of the polluter on pollution charges.

In contrast to the situation described by the random function (13), each source now chooses optimal levels  $x_i^k$  at an announced level of pollution charges  $\lambda_j^k$  on the basis of the information on events  $\zeta_{ij}$  and  $v_i^k$  affecting the cost function  $f_i$ . The resulting random vector  $\Gamma(x^k)$  of the procedure (6) in this case is a stochastic subgradient (see appendix 2) of similar to  $\gamma^1(\lambda)$  function

$$\begin{aligned} \gamma^2(\lambda) &= \min_{y(w) \geq 0} \sum_{i=1}^n E \left[ f_i(y_i(w), v_i) + \left( \sum_{j=1}^m \lambda_j \eta_{ij} \right) y_i(w) \right] - \sum_{j=1}^m \lambda_j q_j = \\ &= \min_{y(w) \geq 0} \sum_{i=1}^n E \left\{ E \left[ f_i(y_i(w), v_i) + \left( \sum_{j=1}^m \lambda_j \eta_{ij} \right) y_i(w) \mid w \right] \right\} = \\ &= \sum_{i=1}^n E \min_{y_i \geq 0} \left[ f_i(y_i, v_i) + \left( \sum_{j=1}^m \lambda_j \eta_{ij} \right) y_i \right] - \sum_{j=1}^m \lambda_j q_j. \end{aligned} \quad (20)$$

From general results on the convergence of stochastic quasigradient methods it is possible again to derive the convergence with probability 1 of random sequence  $\lambda^0, \lambda^1, \dots$  generated by eq.(6) to the vector  $\lambda^*$  minimizing  $\gamma^2(\lambda)$ ,  $\lambda \geq 0$ . It is easy to check that the

minimization of  $\gamma^2(\lambda)$ ,  $\lambda \geq 0$  is a dual to the following problem: find an emission vector  $y(w) = (y_1(w), \dots, y_n(w))$  minimizing the expectation functional

$$F(y(\cdot)) = \sum_{i=1}^n E f_i(y_i(w), v_i) \quad (21)$$

subject to

$$\sum_{i=1}^n E y_i(w) \eta_{ij} \leq q_j, \quad j = 1, \dots, m, \quad y_i(w) \geq 0, \quad i = 1, \dots, n. \quad (22)$$

Let  $(y^*(\omega), \lambda^*)$  be a saddle point of the corresponding Lagrange function  $L(y(\omega), \lambda)$ . The feasible solutions of the problem (16)-(17) are chosen independently of values  $\{\eta_{ij}\}$ . Therefore for the optimal values  $F(x^*(\cdot))$ ,  $F(y^*(\cdot))$  of problems (16)-(17), (21)-(22) the following inequality holds

$$F(x^*(\cdot)) \geq F(y^*(\cdot)) = L(y^*(\omega), \lambda^*) = \max \gamma^2(\lambda).$$

In other words, the pollution charge policy (19) leads up to more cost-effective average outcomes.

## 6. A numerical example

Acidification is one of the major problems in Europe and the Netherlands. Ammonia emissions are a major source of acid rain in the Netherlands are ammonia emissions. These emissions mainly result from livestock farming and fertilizer use and are generally transported over short distances (50% is deposited within 100 kilometers of the source). This implies that the major sources of ammonia deposition in the Netherlands are in the Netherlands, Belgium, France, Western Germany, Ireland, Luxembourg and the United Kingdom which contribute to four receptor areas (grid size 150 x 150 km). The Netherlands' policy is to reduce acid deposition to 2400 equivalents of acid/hectare in the year 2000 (VROM, [11]). After subtracting the expected contribution from sulfur and nitrogen oxides in the year 2000, targets for ammonia deposition can be formulated for each grid (Table 1).

For the adaptative charge mechanism data are needed on transfer coefficients and deposition levels. These levels can be simulated by solving subproblems (5) for each source and given current taxes  $u_i$  or  $\lambda_j$  calculated according to the eqs.(7). Transfer coefficients for ammonia are based on European Program for Monitoring and Evaluation. (Sandnes and Styve,

[8]). The transfer coefficients for the four Dutch receptors, based on the average meteorology for 1985, 1987, 1988, 1989 and 1990, are displayed in Table 2. The transfer coefficients clearly show the short travel distances of ammonia. For example, line 1 shows that a fraction 0.366 (36.6%) of Belgian emissions is deposited in the Dutch receptor no. 3 (location 20-15) and only 0.3% of France's  $\text{NH}_3$  comes down in the same receptor. The costs for controlling ammonia emissions are based on the RAINS model of IIASA (Alcamo et al., [1]). RAINS stands for Regional Acidification Information and Simulation. RAINS distinguishes the following options for controlling ammonia emissions: low ammonia manure application, ammonia poor stable systems, covering manure storage, cleaning stable air, low nitrogen fodder and industrial stripping. For each country the potential and costs of their techniques are calculated accounting for country- and technology-specific factors (Klaassen, [6],[7]). These options are then compiled in national cost functions which rank the options according to their marginal costs and volume of emissions removed.

To simulate the adaptative ambient charge mechanism, a computer program was written by V.Kirilyuk from V. Glushkov Institute of Cybernetics, Kiev, Ukraine. It is based on the package developed by A. Gaivoronski [5]. The program simulates the behavior of the environmental agency which maximizes, according to eq.(6), the Lagrange function  $\gamma(\lambda)$  on the basis of observed deposition levels. Differences between actual and target depositions (excess deposition) lead to changes in ambient (or deposition) charges. These changes are translated in emission charges using the transfer coefficient. The simulation assumes that the agency has imperfect knowledge of costs and perfect knowledge of transfer coefficients. The sources have perfect (deterministic) cost knowledge (as in section 3.).

Two simulations were carried out:

1. the agency starts with initial deposition charges of zero (scenario 1)
2. the agency starts with a deposition charge of 100,000 DM/kton  $\text{NH}_3$  deposited in grids 3 and 4 (scenario 2).

The reason for scenario 2 is that the agency knows immediately that without any control the deposition targets at receptors 1 and 2 are already met. Figures 1 and 2 show the

values of both the total (annual) pollution control costs and the Lagrange function as a function of the number of iterations. Figure 1 shows that after 12 iterations the total cost and Lagrange function converges to around 2.9 billion DM/year (optimal value). Figure 2 clearly shows the impact of starting from an initial deposition charge of 100,000 DM/kton NH<sub>3</sub> deposited at each receptor. In this case only seven iterations would be necessary to approach the cost-minimum solution. Such an initial charge could be set if the environmental agency would ask the individual source how they would respond to a certain emission charge before setting the initial charges. If sources would overestimate their emission reductions in order to reduce the charge level, the adaptation process would take longer and, as Figure 1 clearly indicates, costs would be higher than necessary during the period of adaptation.

Figures 3 and 4 show the evolution of the emission charges over time for both scenarios. Obviously, starting from a zero charge would initially lead (see step 2) very high charges. This is especially so for Belgium and the Netherlands which have large impacts on the deposition at the receptors located in the Netherlands. For Belgium the charges would reach 1596 x 1000 DM/ton NH<sub>3</sub> at step 2 and would then gradually decrease to their final level of 72 x 1000 DM/ton ammonia controlled. Such overshooting might cause problems. Firstly, during the adaptation period, charges and hence pollution control costs would be higher than necessary. Secondly, if the fixed costs element of pollution control costs is high, high charges may induce inflexibility since investments already taken are sunk costs. As Figure 5 shows, starting from a deposition charge level of 100,000 DM/kton NH<sub>3</sub> deposited at receptors 3 and 4 would reduce the number of interactions to only seven. Moreover, the emission charges would not fluctuate greatly over time. After some (small) overshooting at step 1 and underscoring at step 2, the emission charges in all countries would gradually increase to the level where the deposition constraints are met at minimum costs.

In conclusion, a system of adaptive deposition charges can be formulated that converges to the cost-minimum solution even if the environmental agency has imperfect knowledge on the costs. If some information on possible costs is available to the agency prior to setting the initial charge, the adaptation can proceed faster and lead to less significant



fluctuations in costs and emission charge levels than if the agency starts with initial deposition charges equal to zero.

## **7. Conclusions.**

This paper discussed the feasibility of an emission charge policy applied to non uniformly dispersed pollutants when the environmental authority's information on pollution control costs and transfer coefficients is uncertain. We have shown that, contrary to established opinion, the emission charge policy is both environmentally effective and cost effective. It has also been demonstrated that under cost uncertainty deposition targets can be met but pollution control cost will be lower if emission charges are imposed instead of emission standards. The pollution charges are more cost effective than emission charges when the uncertainty is involved in transfer coefficients.

It has been shown that an adjustment mechanism can be designed which resembles a Walrasian process of "tatonnement". In a stepwise way taxes are adjusted such that ambient concentration converges to the target levels and simultaneously total costs of pollution control are minimized without the environmental agency having full information on costs and/or transfer coefficients. In a mathematical sense the procedure deals with the solution of the deterministic and stochastic minimax type problems. In the deterministic case this problem is equivalent to the original pollution control problem assuming the cost functions are convex. In general cases with stochastic costs and transfer coefficients the solution of the minimax problems leads to more cost effective policies than the emission standard strategy. Since the pollution control costs are unknown to the environmental authority the proposed adjustment mechanisms are based on non-monotonic (deterministic and stochastic) techniques. By starting from an arbitrary chosen vector of shadow prices, which are translated into emission taxes, the authority observes the emission (or deposition) levels of polluters and assesses also the environmental effectiveness of the solution by calculating the gaps between actual depositions and deposition targets. When discrepancies are observed, shadow prices and corresponding

emission taxes are adjusted according to eq.(8) until all gaps are closed. The vector of emissions then is both cost effective and environmentally effective.

Let us notice that this procedure can also be viewed as a "dialog" between the environmental agency and polluters to investigate optimal taxes in order to avoid possible troubles with investment in the case of overshooting taxes. The agency announces current taxes and polluters report corresponding levels of emissions to the agency. The agency adjusts taxes according to the proposed adjustment rule and announces new taxes and so on. Such a preliminary, possibly highly computerized, dialog may provide a fairly accurate approximation of taxes to be implemented in practice.

Our conclusion from the result is that there is no real difference in the applicability of emission taxes, whether the environmental targets are formulated in terms of the emission goal or as a set of ambient concentration standards. Under both types of environmental policy emission charges can be applied as an instrument that is both cost and environmentally effective and that is superior to an emission standard policy that could be environmentally effective but less cost effective.

Table 1. Ammonia targets (kton NH<sub>3</sub>/grid/yr)

Receptor	1	2	3	4
Location (EMEP x-y)	19-15	19-16	20-15	20-16
Target	54	43	38	30 <sup>1)</sup>

1) Infeasible. For calculation increased to 45.

Table 2. Transfer coefficients (fraction of emissions deposited at receptor sites)

Receptor	1	2	3	4
EMEP x/y	(19-15)	(19-16)	(20-15)	(20-16)
Source				
BEL	0.116	0.142	0.366	0.269
FRA	0.003	0.001	0.003	0.005
FRG-W	0.001	0.002	0.016	0.048
IRE	0.002	0.002	0.001	0.002
LUX	0.001	0.001	0.005	0.004
UK	0.007	0.006	0.004	0.004
NET	0.074	0.115	0.190	0.183

Figure 1. Total costs and Lagrange function ( $10^6$  DM/yr) with zero initial charges (scenario 1)

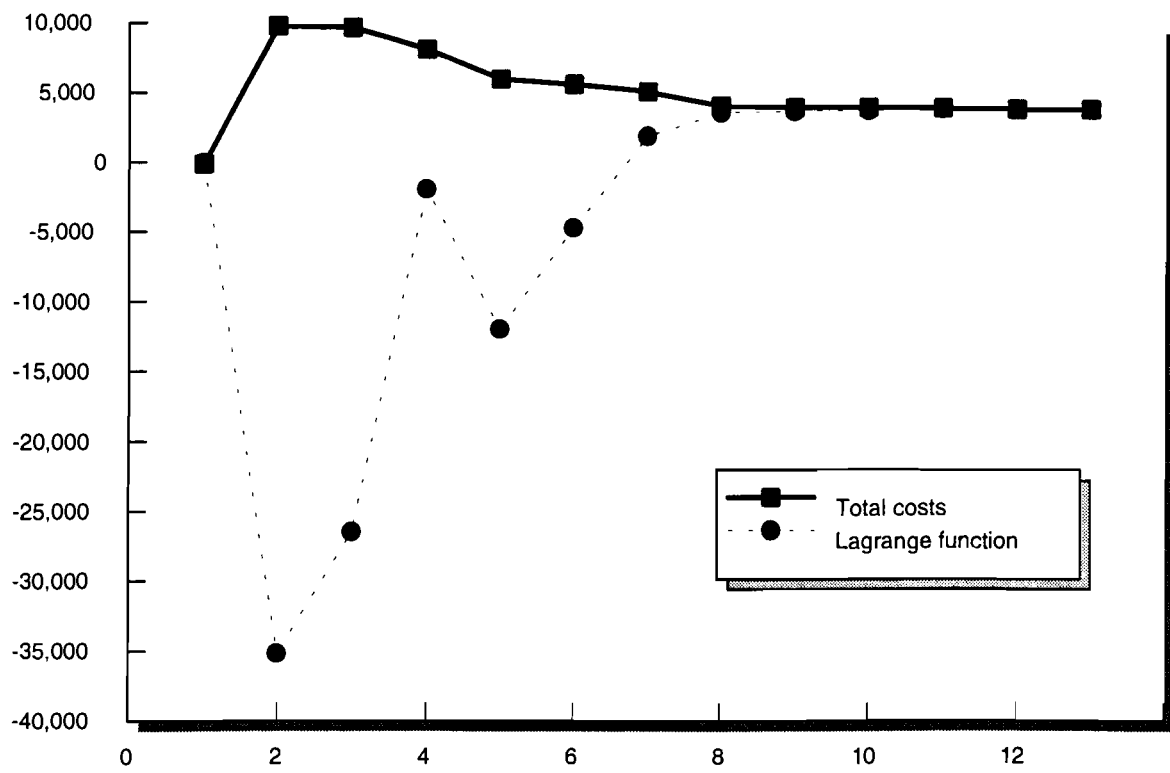


Figure 2. Total costs and Lagrange function ( $10^6$  DM/yr) with zero initial charges (scenario 2)

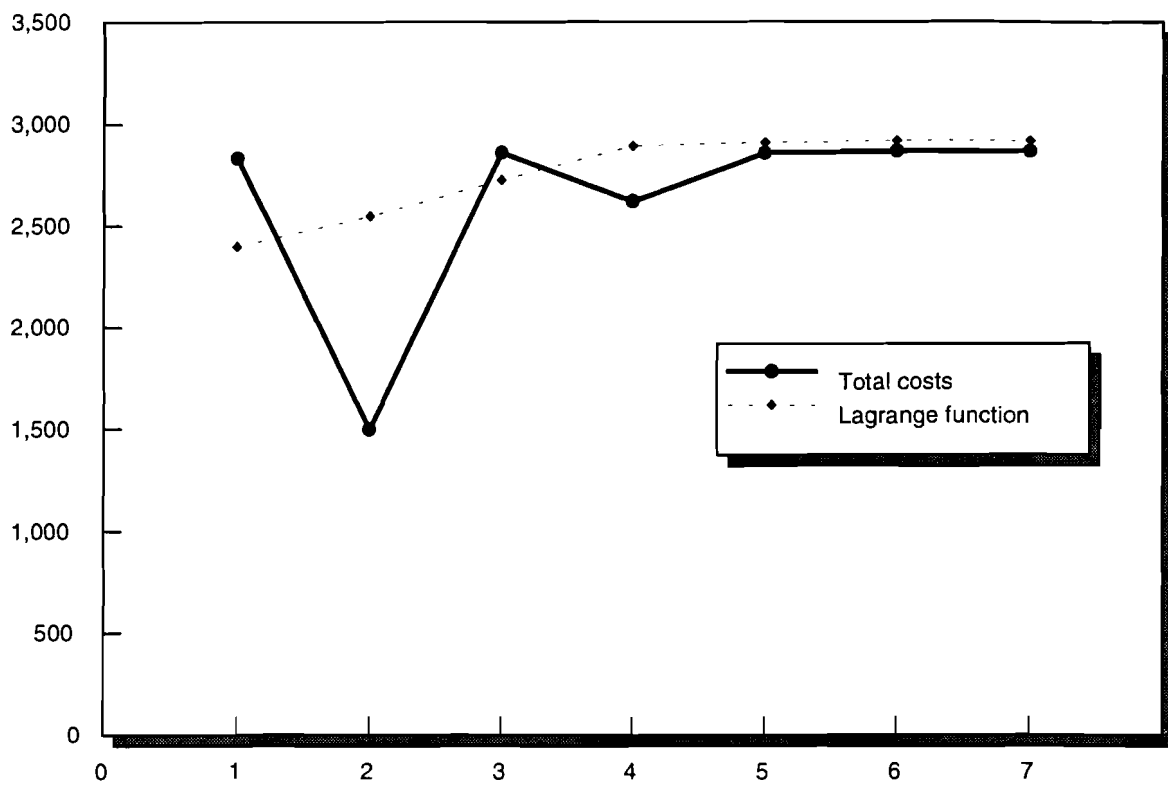


Figure 3a. Emission charges (1000 DM/ton) with scenario (1) (iteration 1 - 5)

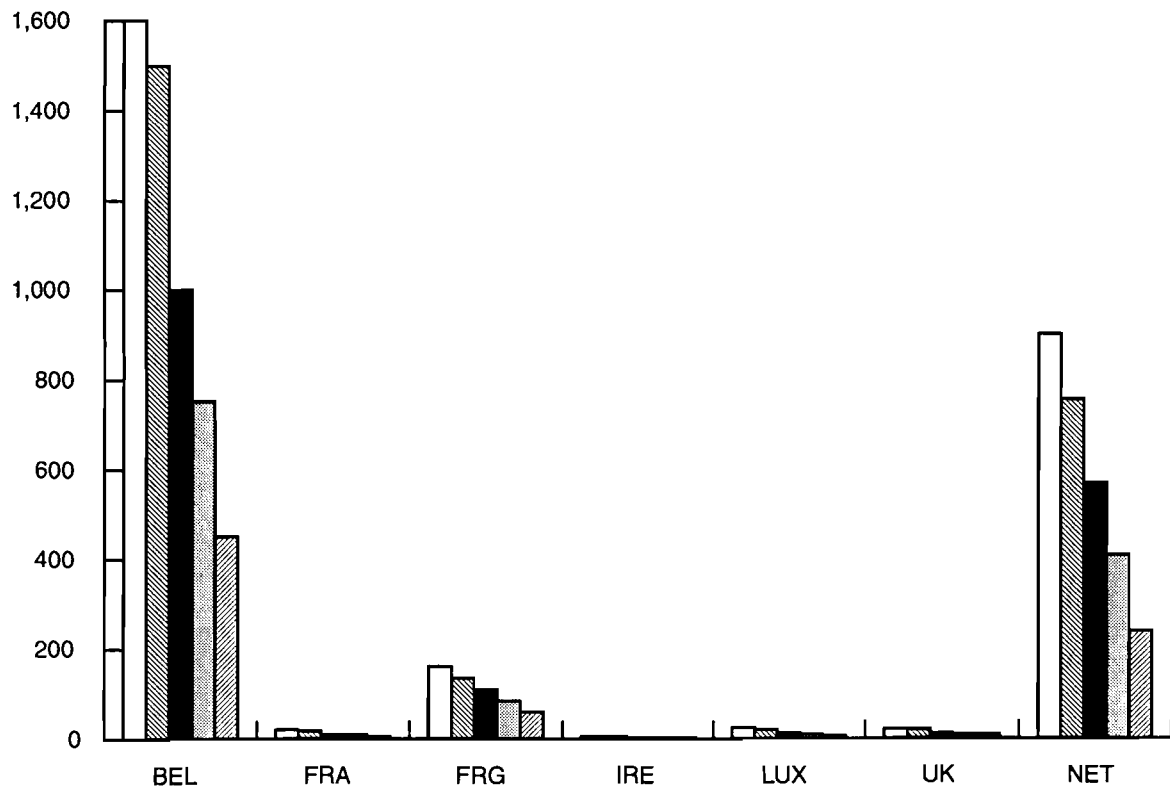


Figure 3b. Emission charges (1000 DM/ton) with scenario (1) (iteration 6 - 12)

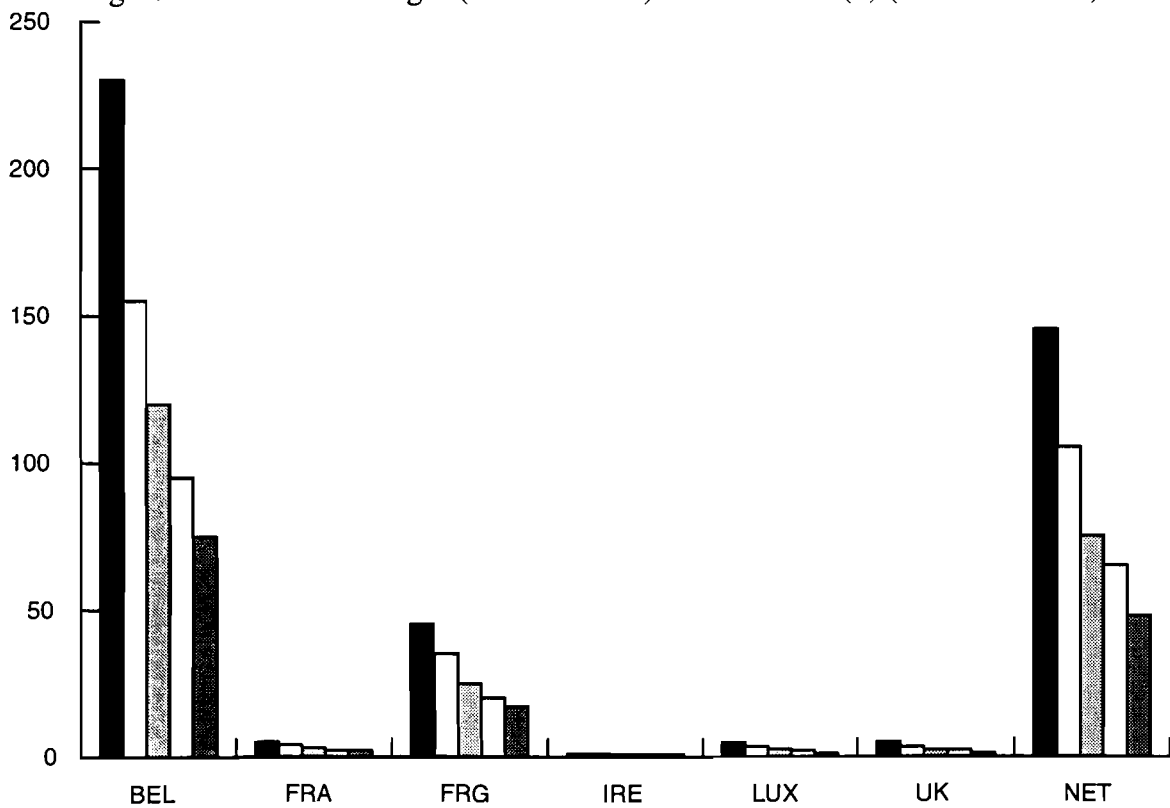
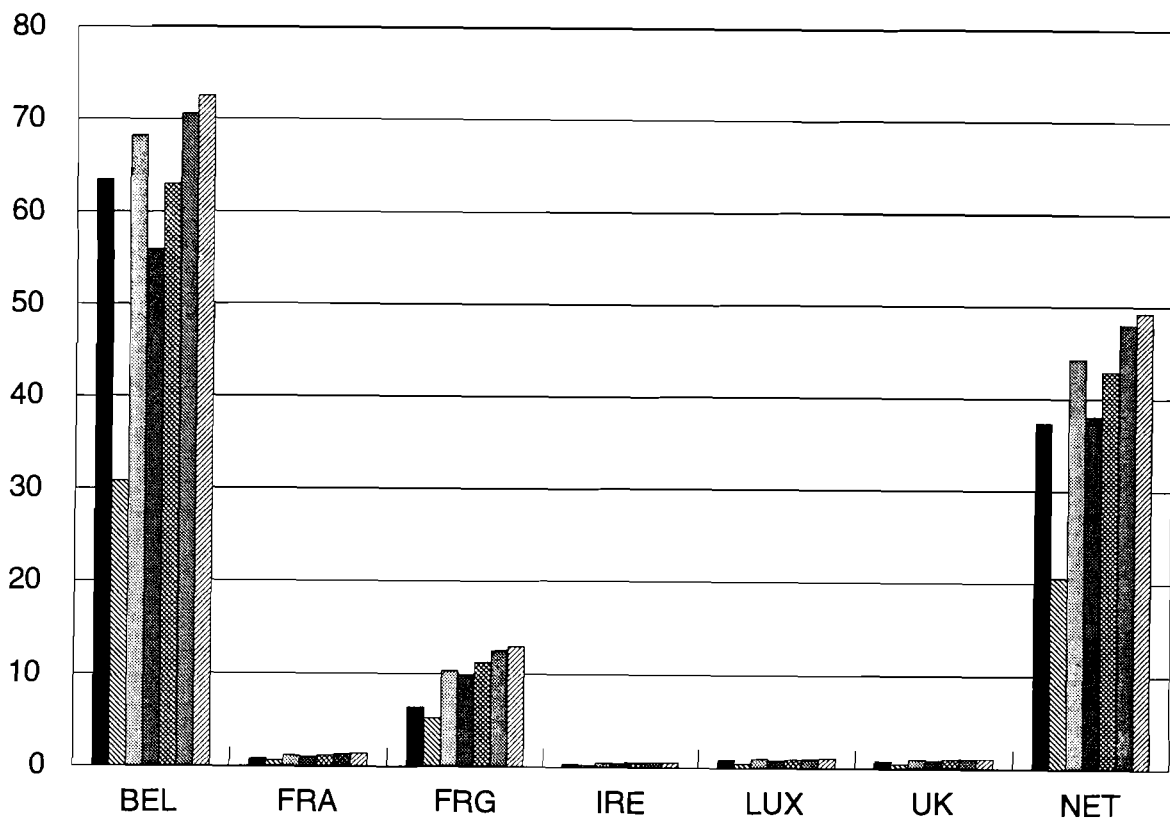


Figure 4. Emission charges (1000 DM/ton) with scenario 2 (iteration 1 - 7)



## Appendix 1.

### Asymptotic properties of the adjustment mechanisms.

Various requirements on the convergence of sequences  $\lambda^1, \lambda^2, \dots$  and  $x^0, x^1, \dots$  generated by rules (6),(8)-(9) can be derived from rather general results on the convergence of stochastic quasigradient methods (see, for example, Ermoliev and Wets [4]). Let us outline the proof only for the case where  $\Gamma(x^k)$  is the deterministic vector and the sequence  $\rho_0, \rho_1, \dots$  satisfies the condition

$$\rho_k \geq 0, \sum_{k=0}^{\infty} \rho_k = \infty, \sum_{k=0}^{\infty} \rho_k^2 < \infty \quad (23)$$

in particular,  $\rho_k = c/k$  with  $c$  constant.

Suppose that  $\rho_k, k = 0, 1, \dots$  satisfy conditions (23). We imply also the existence of optimal solutions  $x^0, x^1, \dots$  defined according to eq.(5). Then the sequence of  $\lambda^k$  defined by the rule (9), converges to the optimal dual variables (taxes)  $\lambda^* = (\lambda_1^*, \dots, \lambda_m^*)$ . If in addition  $x_i^k, k = 0, 1, \dots$  is the unique solution of the minimization problem (5) and  $f_i$  are convex functions then the sequence  $x_i^0, x_i^1, \dots$  converges to the cost effective emissions  $x_i^*, i = 1, \dots, n$ .

**Proof.** From the projection operation definition follows that

$$\|\lambda^* - \lambda^{k+1}\|^2 \leq \|\lambda^* - \lambda^k - \rho_k \Gamma(x^k)\|^2 \leq \|\lambda^* - \lambda^k\|^2 - 2\rho_k (\Gamma(x^k), \lambda^* - \lambda^k) + \rho_k^2 \|\Gamma(x^k)\|^2, \quad (24)$$

where  $\|\cdot\|$  denotes the Euclidean norm and  $(a, b)$  is the scalar product of vectors  $a$  and  $b$ . Since  $\Gamma(x^k)$  is a subgradient of the concave function  $\gamma(\lambda)$  at  $\lambda^k$  then for any vector  $\lambda$ :

$$\gamma(\lambda^*) - \gamma(\lambda^k) \geq (\Gamma(x^k), \lambda^* - \lambda^k) \quad (25)$$

and hence

$$(\Gamma(x^k), \lambda^* - \lambda^k) \geq 0,$$

where  $\gamma(\lambda^*) = \max_{\lambda \geq 0} \gamma(\lambda)$ . Therefore from the inequality (24) we have

$$\|\lambda^* - \lambda^{k+1}\|^2 \leq \|\lambda^* - \lambda^k\|^2 + \rho_k^2 \|\Gamma(x^k)\|^2.$$

From the problem formulation follows that  $x_i(\lambda) \leq x_i(0)$  for each  $i=1, \dots, n$  and  $\lambda \geq 0$ . Therefore, we can assume that  $\|\Gamma(x^k)\| < \text{const}$ . In other words for some constant  $C$

$$\|\lambda^* - \lambda^{k+1}\|^2 \leq \|\lambda^* - \lambda^k\|^2 + \rho_k^2 C.$$



Define  $\delta_k = \|\lambda^* - \lambda^k\|^2 + C \sum_{s=k}^{\infty} \rho_s^2$ . From the above inequality follows that  $\delta_{k+1} \leq \delta_k$  and hence the sequence  $\{\delta_k\}$  converges. Since  $\sum_{k=0}^{\infty} \rho_k^2 < \infty$  the sequence  $\{\|\lambda^* - \lambda^k\|^2\}$  is also convergent. From inequality (24) and taking into account that  $\|\Gamma(x^k)\| < C$  (for some constant  $C$ ) we have

$$0 \leq \|\lambda^* - \lambda^0\|^2 - 2 \sum_{k=0}^{\infty} \rho_k (\Gamma(x^k), \lambda^* - \lambda^k) + C \sum_{k=0}^{\infty} \rho_k^2.$$

Hence  $\sum_{k=0}^{\infty} \rho_k (\Gamma(x^k), \lambda^* - \lambda^k) < \infty$ . Since  $\sum_{k=0}^{\infty} \rho_k = \infty$ , there exists a sequence  $\{\lambda^{k_l}\}$  such that  $(\Gamma(x^{k_l}), \lambda^* - \lambda^{k_l}) \rightarrow 0, l \rightarrow \infty$ . Therefore  $\gamma(\lambda^{k_l}) \rightarrow \gamma(\lambda^*), l \rightarrow \infty$ .

From this fact and the convergence of the sequence  $\{\|\lambda^* - \lambda^k\|^2\}$  follows the desired result:  $\lambda^k \rightarrow \lambda^*, k \rightarrow \infty$ .

Suppose  $f_i$  are convex functions. Consider now the sequence of emissions:  $x^k = \underset{x \geq 0}{\operatorname{argmin}} L(x, \lambda^k)$ , where  $L(x, \lambda)$  is the Lagrange function of the pollution control problem. In other words, each component  $x_i^k$  of the vector  $x^k = (x_1^k, \dots, x_n^k)$  is the solution of the subproblem (5) for  $\lambda_j = \lambda_j^k, j = 1, \dots, m$ . Let us show that  $x^k \rightarrow x^*, k \rightarrow \infty$

From the definition of  $\gamma(\lambda)$  we have  $L(x^k, \lambda^k) = \gamma(\lambda^k)$ . From the convergence  $\lambda^k \rightarrow \lambda^*, k \rightarrow \infty$  and the inequality (25) follows that  $\gamma(\lambda^k) \rightarrow \gamma(\lambda^*)$ . Therefore from the duality theory follows that

$$L(x^k, \lambda^k) = \gamma(\lambda^k) \rightarrow \gamma(\lambda^*) = L(x^*, \lambda^*).$$

Suppose that  $x^k$  does not converge to  $x^*$ , that is to say there exists a sequence  $x^{k_l} \rightarrow \bar{x} \neq x^*$ . Then

$$L(x^{k_l}, \lambda^{k_l}) = \underset{x \geq 0}{\operatorname{argmin}} L(x, \lambda^{k_l}) \rightarrow L(\bar{x}, \lambda^*)$$

Since  $x^* = \underset{x \geq 0}{\operatorname{argmin}} L(x, \lambda^*)$  is uniquely defined,  $\bar{x} = x^*$ . The contradiction completes the proof.

## Appendix 2. On the calculation of stochastic subgradients.

Functions  $\gamma(\lambda), \gamma^1(\lambda), \gamma^2(\lambda)$  belong to the following family of functions. Suppose  $\varphi(x, \lambda, \theta)$  is a function defined on  $x \in R^n, \lambda \in R^n$  and  $\theta \in \Theta$ , where  $(\Theta, \mathfrak{S}, P)$  is a probability

space. Assume that  $\varphi(x, \cdot, \theta)$  is a concave continuously differentiable function,  $\varphi_\lambda$  is its gradient and there exist all necessary integrals which are considered further.

Consider maximization of the function

$$g(\lambda) = E \min_x E[\varphi(x, \lambda, \theta) | A] = E\varphi(x(\lambda, w), \lambda, \theta)$$

where  $E[\varphi | A]$  is the conditional expectation with respect to a given family of events  $\omega \in A \subset \mathfrak{S}$ . Let us show that the vector

$$\xi = \varphi_\lambda(x, \lambda, \theta) \Big|_{x=x(\lambda, w)}$$

is an unbiased estimate of a subgradient  $g_\lambda$  (stochastic subgradient of  $g_\lambda$ ):

$$g(\mu) - g(\lambda) \leq (E[\xi | \lambda], \mu - \lambda), \quad \forall \mu.$$

Indeed from the concavity of  $\varphi(x, \cdot, \theta)$  follows

$$\begin{aligned} E[\varphi(x(\mu, w), \mu, \theta) | A] - E[\varphi(x(\lambda, w), \lambda, \theta) | A] &\leq \\ E[\varphi(x(\lambda, w), \mu, \theta) | A] - E[\varphi(x(\lambda, w), \lambda, \theta) | A] &\leq (E[\varphi_\lambda(x(\lambda, w), \lambda, \theta) | A], \mu - \lambda). \end{aligned}$$

The desired result follows now by taking the conditional expectation (for fixed  $\mu, \lambda$ ) in both sides of this inequality:  $g(\mu) - g(\lambda) \leq (E\varphi_\lambda(x(\lambda, w), \lambda, \theta), \mu - \lambda)$ .

## References

1. **J. Alcamo, R. Shaw and L. Hordijk** (eds.) 1990 *The RAINS model of acidification, science and strategies in Europe*. Kluwer Academic Publishers, Dordrecht/Boston/London .
2. **W. Baumol and W.Oates** 1971 *The use of standards and prices for protection of the environment*. *The Swedish Journal of Economics*, 73 (March 1971), pp.42-54.
3. **P. Bohm and C.S.Russel** 1985 *Comparative analysis of alternative policy instruments*. In: *A.V. Kneese and J.L.Sweeney (Eds) Handbook of natural resource and energy economics. Vol. 1*, pp. 395-460. Amsterdam, New York, Oxford: North-Holland.
4. **Y. Ermoliev and R. Wets** (Eds.) 1988 *Numerical techniques for Stochastic Optimization*. Springer - Verlag, Computational Mathematics.
5. **A. Gaivoronski** 1988 *Implementation of Stochastic Quasigradient Methods*, In: *Y.Ermoliev and R. Wets (Eds.) Numerical techniques for Stochastic Optimization*. Springer - Verlag, Computational Mathematics.

6. **G. Klaassen** 1991 *Costs of controlling ammonia emissions in Europe. Status Report SR-91-03. IIASA, Laxenburg, Austria.*
7. **G. Klaassen** 1994 (forthcoming) *Options and costs of controlling ammonia emissions in Europe. European Review of Agricultural Economics.*
8. **H. Sandnes and H. Styve** 1992 *Calculated budgets for airborne acidifying components in Europe, 1985, 1987, 1988, 1989, 1990 and 1991. EMEP/SC-W report 1/92. Meteorological Synthesizing Center-West, the Norwegian Meteorological Institute, Oslo, Norway.*
9. **T.H. Tietenberg** 1974 *Derived decision rules for pollution control in a general equilibrium space economy, J. Environ: Econ. Manag. 1, 1-16.*
10. **T.H. Tietenberg** 1978 *Spatially differentiated air pollutant emission charges: an economic and legal analysis. The American Economic Review Land Economics V54, pp.265-277*
11. **VROM** 1989 *National Environmental Policy Plan Plus. Ministry of Housing, Physical Planning and Environment, The Hague, The Netherlands.*